Non-local formulation of ghost-free bigravity theory

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ABSTRACT: We study the ghost-free bimetric theory of Hassan and Rosen, with parameters β_i such that a flat Minkowski solution exists for both metrics. We show that, expanding around this solution and eliminating one of the two metrics with its own equation of motion, the remaining metric is governed by the Einstein-Hilbert action plus a non-local term proportional to $W_{\mu\nu\rho\sigma}(\Box - m^2)^{-1}W^{\mu\nu\rho\sigma}$, where $W_{\mu\nu\rho\sigma}$ is the Weyl tensor. The result is valid to quadratic order in the metric perturbation and to all orders in the derivative expansion. This example shows, in a simple setting, how such non-local extensions of GR can emerge from an underlying consistent theory, at the purely classical level.

Contents

1 Introduction

The study of infrared modifications of General Relativity (GR) is motivated both by its intrinsic conceptual interest and by the aim of explaining the observed accelerated expansion of the Universe. A natural way of modifying the theory in the infrared is to add a mass term. In this direction, significant progresses have been made in recent years with the construction of a ghost-free theory of massive gravity, the dRGT theory [1, 2] (see also [3–15], and [16, 17] for reviews). Such a theory involves, beside the dynamical metric $g_{\mu\nu}$, a non-dynamical reference metric $f_{\mu\nu}$ which is needed to construct a mass term. A natural subsequent step is to promote $f_{\mu\nu}$ to a dynamical field. This leads to bimetric theories. Ghost-free massive gravity has been generalized to a ghost-free bimetric theory by Hassan and Rosen [18]. Conceptual aspects of bigravity have been investigated in [19–21], and its cosmological consequences have been studied e.g. in [22–30]. The Hassan-Rosen bimetric theory is defined by the action

$$S = \int d^4x \, M_g^2 \sqrt{-g} R(g) + \int d^4x \, M_f^2 \sqrt{-f} R(f) - 2 \, M_f^2 m^2 \int d^4x \, \sqrt{-g} \sum_{n=0}^{4} \beta_n e_n(\mathbb{X}) \quad (1.1)$$

where β_i are general real coefficients, m is a parameter with the dimension of mass, $\mathbb{X}^\nu_\mu = (\sqrt{\mathrm{g^{-1}f}})^\nu_\mu$ and the $e_i(\mathbb{X})$ polynomials are given by

$$e_0 = I \quad e_1 = [\mathbb{X}] \quad e_2 = \frac{1}{2}([\mathbb{X}]^2 - [\mathbb{X}^2]) \quad e_3 = \frac{1}{6}([\mathbb{X}]^3 - 3[\mathbb{X}][\mathbb{X}^2] + 2[\mathbb{X}^3])$$

$$e_4 = \frac{1}{24}([\mathbb{X}]^4 - 6[\mathbb{X}]^2[\mathbb{X}^2] + 8[\mathbb{X}][\mathbb{X}^3] + 3[\mathbb{X}^2]^2 - 6[\mathbb{X}^4]), \quad (1.2)$$

where the bracket denotes the trace of the matrix and, for simplicity, we have restricted ourselves to $D = 4$ space-time dimensions.

The purpose of this paper is to show how this theory can be recast into a non-local form involving only one metric. Working up to terms quadratic in the curvature and choosing the parameters β_i such that the theory admits a background solution $\bar{g}_{\mu\nu} = \bar{f}_{\mu\nu} = \eta_{\mu\nu}$, we will find that the action (1.1) is classically equivalent to the action

$$ S' = M_{\text{pl}}^2 \int d^4x \sqrt{-g}\, R(g) \, - \, \frac{M_f^2}{2} \int d^4x\, \sqrt{-g}\, W_{\mu\nu\rho\sigma} \frac{1}{\Box - m^2} W^{\mu\nu\rho\sigma} + \mathcal{O}(R_{\mu\nu\rho\sigma}^3)\,, \quad (1.3) $$

where $W^{\mu\nu\rho\sigma}$ is the Weyl tensor constructed with the metric $g_{\mu\nu}$, and $M_{\text{pl}}^2 = M_g^2 + M_f^2$. In order to get this result, we will integrate out $f_{\mu\nu}$ by using its own equations of motion, linearized over Minkowski, and we will then covariantize the result.[1] Our analysis will complement the study performed by Hassan, Schmidt-May and von Strauss [21], where $f_{\mu\nu}$ is rather eliminated using the equation of motion of $g_{\mu\nu}$; we will comment below on the relation between the two approaches.

This result reveals an interesting relation between bigravity and Stelle's higher derivative gravity. The term $W_{\mu\nu\rho\sigma}(\Box - m^2)^{-1}W^{\mu\nu\rho\sigma}$ can be seen as a UV completion of a term $-(1/m^2)W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}$. In the infrared limit $(\Box - m^2)^{-1} \simeq -1/m^2$ and, neglecting also cubic and higher order terms, eq. (1.3) reduces to

$$ S_{\text{Stelle}} = M_{\text{pl}}^2 \int d^4x \sqrt{-g}\, R(g) \, + \, c_W \int d^4x \sqrt{-g}\, W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}\,, \quad\quad (1.4) $$

(where $c_W = M_f^2/(2m^2)$), which is the action of Stelle's theory [31, 32]. Stelle's theory has 7 propagating degrees of freedom, organized into a massless spin-2 graviton and a massive ghost-like spin-2 state. The original bigravity theory also has a massless and a massive graviton, but is ghost-free. Therefore, this construction provides an explicit example of how to embed Stelle's higher-derivative gravity into a ghost-free theory (as already discussed from a different point of view in [21]). The non-local expression (1.3) is also useful to investigate the relation, and the differences, between this non-local formulation of bigravity, and non-local modifications of General Relativity such as those that have been discussed in [33–41].

The paper is organized as follows. In section 2 we express the fluctuations of the metric $f_{\mu\nu}$ in terms of that of $g_{\mu\nu}$. The non-local action is computed in section 3. We conclude with a discussion of our results in section 4. In appendix A we compare our results with that of ref. [21] and in appendix B we extend the computation to the interaction with matter. We use the signature $(-, +, +, +)$ and units $\hbar = c = 1$.

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[1]In the following, in fully covariant expressions it is understood that $\Box$ is the d'Alembertian computed with respect to the full metric $g_{\mu\nu}$, while in linearized expression it is understood that $\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$ is the flat-space d'Alembertian, and similarly for its inverse $\Box^{-1}$.

2 Elimination of the second metric

2.1 Computation of $l_{\mu\nu}$

The equations of motion derived from eq. (1.1) are

$$\frac{M_g^2}{M_f^2}G_{\mu\nu}(g) + m^2\sum_{n=0}^{3}(-1)^n\beta_n g_{\mu\lambda}\mathbb{Y}_{(n)\nu}^{\lambda}(\mathbb{X}) = 0\,, \tag{2.1}$$

$$G_{\mu\nu}(f) + m^2\sum_{n=0}^{3}(-1)^n\beta_{4-n}f_{\mu\lambda}\mathbb{Y}_{(n)\nu}^{\lambda}(\mathbb{X}^{-1}) = 0\,, \tag{2.2}$$

where $\mathbb{Y}_{(n)}(\mathbb{X}) = \sum_{r=0}^{n}(-1)^r\mathbb{X}^{n-r}e_r(\mathbb{X})$, and we neglect for the moment matter sources (the extension to matter sources is performed in appendix B). In order to obtain an effective action involving only the metric $g_{\mu\nu}$, we eliminate $f_{\mu\nu}$ by using its own equation of motion. This involves the inversion of a differential operator, which in practice can only be done by expanding around a simple background, such as Minkowski. The result can then be covariantized and, as long as one truncates the theory to quadratic order, the covariantization can be done uniquely. Thus, in order to simplify the problem, we choose the coefficients β_n in eq. (1.1) in such a way that there exists a solution of the equations of motion with $\bar{g}_{\mu\nu} = \bar{f}_{\mu\nu} = \eta_{\mu\nu}$. This can be obtained for instance setting [20]

$$\beta_0 = \beta_4 + 2\beta_3 - 2\beta_1; \quad \beta_2 = -\frac{\beta_1}{3} - \frac{\beta_4}{3} - \beta_3; \qquad (\beta_1,\beta_3,\beta_4)\in\mathbb{R} \tag{2.3}$$

Imposing that only one of the three remaining free parameters i.e. β_1 is different from zero, eq. (2.3) implies $\beta_0 = -2\beta_1$ and $\beta_2 = -\beta_1/3$. In the following we adopt for definiteness this choice and, for later convenience, we set $\beta_1 = 3$ (in any case, different choices of the β_i, satisfying eq. (2.3), can be reabsorbed in the definition of m^2). Then the potential term in eq. (1.1) becomes

$$\sum_{n=0}^{4}\beta_n e_n(\mathbb{X}) = -6e_0(\mathbb{X}) + 3e_1(\mathbb{X}) - e_2(\mathbb{X})\,, \tag{2.4}$$

and the equations for $f_{\mu\nu}$ and $g_{\mu\nu}$ become

$$\frac{M_g^2}{M_f^2}G_{\mu\nu}(g) - m^2\left[6g_{\mu\lambda}\mathbb{Y}_{0\nu}^{\lambda}(\mathbb{X}) + 3g_{\mu\lambda}\mathbb{Y}_{1\nu}^{\lambda}(\mathbb{X}) + g_{\mu\lambda}\mathbb{Y}_{2\nu}^{\lambda}(\mathbb{X})\right] = 0\,, \tag{2.5}$$

$$G_{\mu\nu}(f) - m^2\left[3f_{\mu\lambda}\mathbb{Y}_{3\nu}^{\lambda}(\mathbb{X}^{-1}) + f_{\mu\lambda}\mathbb{Y}_{2\nu}^{\lambda}(\mathbb{X}^{-1})\right] = 0\,. \tag{2.6}$$

We now expand $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $f_{\mu\nu} = \eta_{\mu\nu} + l_{\mu\nu}$. Then eqs. (2.5) and (2.6) give

$$\frac{M_g^2}{M_f^2}\mathcal{E}_{\mu\nu\rho\sigma}h^{\rho\sigma} + m^2(h_{\mu\nu} - \eta_{\mu\nu}h) = m^2(l_{\mu\nu} - \eta_{\mu\nu}l)\,, \tag{2.7}$$

$$\mathcal{E}_{\mu\nu\rho\sigma}l^{\rho\sigma} + m^2(l_{\mu\nu} - \eta_{\mu\nu}l) = m^2(h_{\mu\nu} - \eta_{\mu\nu}h)\,, \tag{2.8}$$

where for the Lichnerowicz operator we use the convention

$$\mathcal{E}_{\mu\nu\rho\sigma}h^{\rho\sigma} = -\Box h_{\mu\nu} + \eta_{\mu\nu}\Box h - \partial_\mu\partial_\nu h - \eta_{\mu\nu}\partial_\rho\partial_\sigma h^{\rho\sigma} + \partial^\rho\partial_\nu h_{\rho\mu} + \partial^\rho\partial_\mu h_{\rho\nu}\,. \tag{2.9}$$

It is also convenient to define the tensor

$$S_{\mu\nu\rho\sigma} = \frac{1}{2} \left(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} \right) - \eta_{\mu\nu}\eta_{\rho\sigma} \,. \tag{2.10}$$

Equation (2.8) can then be rewritten as

$$(\mathcal{E}_{\mu\nu\rho\sigma} + m^2 S_{\mu\nu\rho\sigma})l^{\rho\sigma} = m^2 S_{\mu\nu\rho\sigma}h^{\rho\sigma} \,. \tag{2.11}$$

The operator acting on l is precisely the Fierz-Pauli operator. We know that for $m \neq 0$ it is invertible and the inverse is

$$Q_{\mu\nu\rho\sigma} = -\frac{1}{\Box - m^2} \left[\frac{1}{2} \left(\Pi_{\mu\rho}\Pi_{\nu\sigma} + \Pi_{\mu\sigma}\Pi_{\nu\rho} \right) - \frac{1}{3}\Pi_{\mu\nu}\Pi_{\rho\sigma} \right] \,, \tag{2.12}$$

where $\Pi_{\mu\nu} = \eta_{\mu\nu} - m^{-2}\partial_\mu\partial_\nu$. Then

$$\begin{aligned}
l^{\mu\nu} &= m^2 Q^{\mu\nu\alpha\beta} S_{\alpha\beta\rho\sigma}h^{\rho\sigma} \\
&= \frac{1}{\Box - m^2} \left(\partial^\mu\partial_\alpha h^{\nu\alpha} + \partial^\nu\partial_\alpha h^{\mu\alpha} - \partial^\mu\partial^\nu h \right) - \frac{m^2}{\Box - m^2}h^{\mu\nu} \\
&\quad + \frac{1}{3}\frac{1}{\Box - m^2} \left(\eta^{\mu\nu} + \frac{2\partial^\mu\partial^\nu}{m^2} \right) (\Box h - \partial_\alpha\partial_\beta h^{\alpha\beta}) \,.
\end{aligned} \tag{2.13}$$

This expression can be rewritten in terms of the linearized Ricci tensor $\mathcal{R}_{\mu\nu}$ and of the linearized Ricci scalar $\mathcal{R}$ (we use calligraphic letters to denote quantities linearized over Minkowski), which are given by

$$\mathcal{R}_{\mu\nu} = \frac{1}{2} \left(\partial^\alpha\partial_\mu h_{\nu\alpha} + \partial^\alpha\partial_\nu h_{\mu\alpha} - \Box h_{\mu\nu} - \partial_\mu\partial_\nu h \right) \,, \tag{2.14}$$

$$\mathcal{R} = \partial^\mu\partial^\nu h_{\mu\nu} - \Box h \,. \tag{2.15}$$

Then eq. (2.13) becomes

$$l_{\mu\nu} = h_{\mu\nu} + \frac{1}{\Box - m^2} \left[2\mathcal{R}_{\mu\nu} - \frac{1}{3} \left(\eta_{\mu\nu} + 2\frac{\partial_\mu\partial_\nu}{m^2} \right) \mathcal{R} \right] \,. \tag{2.16}$$

Taking the trace we get

$$l = h - \frac{2\mathcal{R}}{3m^2} \,, \tag{2.17}$$

so the trace l is a local function of $h_{\mu\nu}$. Plugging these two expressions in eq. (2.7) we get a non local equation for $h_{\mu\nu}$,

$$\frac{M_g^2}{M_f^2}\mathcal{E}_{\mu\nu}{}^{\rho\sigma}h_{\rho\sigma} - \frac{2}{3}\eta_{\mu\nu}\mathcal{R} - \frac{m^2}{\Box - m^2} \left[2\mathcal{R}_{\mu\nu} - \frac{1}{3} \left(\eta_{\mu\nu} + 2\frac{\partial_\mu\partial_\nu}{m^2} \right) \mathcal{R} \right] = 0 \,. \tag{2.18}$$

It is straightforward to check that the divergence of the left-hand side vanishes identically, as it should. Therefore, when $h_{\mu\nu}$ is coupled to the matter energy-momentum tensor $T_{\mu\nu}$, energy-momentum conservation, $\partial^\mu T_{\mu\nu} = 0$, is automatically assured.

2.2 Helicity decomposition of the metric perturbations

It is instructive to repeat the above computation by first decomposing the metric perturbations $h_{\mu\nu}$ and $l_{\mu\nu}$ into their scalar, vector and tensor components,

$$h_{\mu\nu} = h_{\mu\nu}^{TT} + \frac{1}{2}(\partial_\mu \epsilon_\nu^T + \partial_\nu \epsilon_\mu^T) + \partial_\mu \partial_\nu \alpha + \frac{1}{3}\eta_{\mu\nu}s\,, \tag{2.19}$$

$$l_{\mu\nu} = l_{\mu\nu}^{TT} + \frac{1}{2}(\partial_\mu l_\nu^T + \partial_\nu l_\mu^T) + \partial_\mu \partial_\nu \beta + \frac{1}{3}\eta_{\mu\nu}u\,, \tag{2.20}$$

where $h_{\mu\nu}^{TT}$ is the transverse-traceless part, $\partial_\mu h_{\mu\nu}^{TT} = 0$, $\eta^{\mu\nu}h_{\mu\nu}^{TT} = 0$, ϵ_μ^T is a transverse vector, $\partial^\mu \epsilon_\mu^T = 0$, and α and s are scalar under rotation (and similarly for the decomposition of $l_{\mu\nu}$). We also define $v_{\mu\nu} = h_{\mu\nu} - l_{\mu\nu}$ and we decompose it as

$$v_{\mu\nu} = v_{\mu\nu}^{TT} + \frac{1}{2}(\partial_\mu v_\nu^T + \partial_\nu v_\mu^T) + \partial_\mu \partial_\nu \gamma + \frac{1}{3}\eta_{\mu\nu}c \tag{2.21}$$

so, of course, $v_{\mu\nu}^{TT} = h_{\mu\nu}^{TT} - l_{\mu\nu}^{TT}$, $v_\mu^T = \epsilon_\mu^T - l_\mu^T$, $\gamma = \alpha - \beta$ and $c = s - u$. In term of these variables the quadratic Einstein-Hilbert actions take the form (see e.g. [8] or appendix B of [33])

$$S_{EH_1}^{(2)} + S_{EH_2}^{(2)} = \frac{1}{4}\int d^4x\ M_g^2\left(h_{\mu\nu}^{TT}\Box h_{\mu\nu}^{TT} - \frac{2}{3}s\Box s\right) + M_f^2\left(l_{\mu\nu}^{TT}\Box l_{\mu\nu}^{TT} - \frac{2}{3}u\Box u\right)\,, \tag{2.22}$$

while, after some integrations by part, the interaction term coming from the dRGT potential takes the form

$$S_{\text{int}} = \frac{-M_f^2 m^2}{4}\int d^4x\left(v_{\mu\nu}^{TT}v^{TT\mu\nu} - \frac{1}{2}v_\mu^T\Box v^{T\mu} - 2c\Box\gamma - \frac{4}{3}c^2\right)\,. \tag{2.23}$$

The corresponding equations of motion are

$$\Box v^{T\mu} = 0\,, \tag{2.24}$$

$$\Box c = 0\,, \tag{2.25}$$

$$(\Box - m^2)l_{\mu\nu}^{TT} = -m^2 h_{\mu\nu}^{TT}\,, \tag{2.26}$$

$$\frac{2}{3}\Box u + m^2\left(\Box\gamma + \frac{4}{3}c\right) = 0\,. \tag{2.27}$$

Equation (2.24) implies $\Box l_\mu^T = \Box \epsilon_\mu^T$. We solve it with the boundary condition that, when $\epsilon_\mu^T = 0$, we must have $l_\mu^T = 0$. Then, $\Box v^{T\mu} = 0$ implies $v^{T\mu} = 0$. Similarly, $\Box c = 0$ implies $c = 0$. Therefore we get

$$l_\mu^T = \epsilon_\mu^T\,, \qquad u = s\,. \tag{2.28}$$

The other two equations give

$$l_{\mu\nu}^{TT} = -\frac{m^2}{\Box - m^2}h_{\mu\nu}^{TT} \tag{2.29}$$

and

$$\beta = \alpha + \frac{2s}{3m^2}\,. \tag{2.30}$$

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This decomposition allows us to appreciate that the non-locality in the relation between $h_{\mu\nu}$ and $l_{\mu\nu}$ only appears in the tensor sector. The equivalence with the result found in eq. (2.16) is easily proved inverting the decomposition (2.19), which gives (in $D = 4$ space-time dimensions) [33]

$$\alpha = -\frac{1}{3}\frac{1}{\Box}\left(\eta^{\mu\nu} - \frac{4}{\Box}\partial^\mu\partial^\nu\right)h_{\mu\nu}\,, \tag{2.31}$$

$$s = \left(\eta^{\mu\nu} - \frac{1}{\Box}\partial^\mu\partial^\nu\right)h_{\mu\nu}\,, \tag{2.32}$$

$$\epsilon_\mu^T = \frac{2}{\Box}\left(\delta_\mu^\rho - \frac{\partial_\mu\partial^\rho}{\Box}\right)\partial^\sigma h_{\rho\sigma}\,, \tag{2.33}$$

$$h_{\mu\nu}^{\mathrm{TT}} = h_{\mu\nu} - \frac{1}{3}\left(\eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box}\right)h - \frac{1}{\Box}(\partial_\mu\partial^\rho h_{\nu\rho} + \partial_\nu\partial^\rho h_{\mu\rho})$$
$$+\frac{1}{3}\,\eta_{\mu\nu}\frac{1}{\Box}\partial^\rho\partial^\sigma h_{\rho\sigma} + \frac{2}{3}\frac{1}{\Box^2}\partial_\mu\partial_\nu\partial^\rho\partial^\sigma h_{\rho\sigma}\,. \tag{2.34}$$

Under linearized diffeomorphisms $h_{\mu\nu} \to h_{\mu\nu} - (\partial_\mu\xi_\nu + \partial_\nu\xi_\mu)$, decomposing $\xi_\mu = \xi_\mu^T + \partial_\mu\xi$, we have $\epsilon_\mu^{\mathrm{T}} \to \epsilon_\mu^{\mathrm{T}} - 2\xi_\mu^T$ and $\alpha \to \alpha - 2\xi$, while $h_{\mu\nu}^{\mathrm{TT}}$ and s are invariant. Thus we can choose the gauge so that $\epsilon_\mu^T = \alpha = 0$, and this leaves no residual gauge symmetry. Since $h_{\mu\nu}^{\mathrm{TT}}$ and s are invariant, it is possible to express them in terms of the linearized Ricci scalar and Ricci tensor (recall that, in linearized theory, the Riemann tensor is gauge-invariant rather than covariant). Indeed, eqs. (2.32) and (2.34) can be rewritten as

$$s = -\frac{1}{\Box}\mathcal{R}\,, \tag{2.35}$$

$$h_{\mu\nu}^{TT} = \frac{2}{3}\frac{\partial_\mu\partial_\nu}{\Box^2}\mathcal{R} + \frac{1}{3}\frac{\eta_{\mu\nu}}{\Box}\mathcal{R} - \frac{2}{\Box}\mathcal{R}_{\mu\nu}\,. \tag{2.36}$$

Substituting eqs. (2.28)–(2.30) into eq. (2.20), and expressing α, s, ϵ_μ^T and $h_{\mu\nu}^{\mathrm{TT}}$ in terms of $h_{\mu\nu}$ using eqs. (2.31), (2.33), (2.35) and (2.36) it is straightforward to show that one recovers eq. (2.16).

3   Non-local action

We can now describe the dynamics entirely in term of $h_{\mu\nu}$. Note that, since $l_{\mu\nu}$ has been expressed in terms of $h_{\mu\nu}$ by using its own equation of motion, it is legitimate to substitute eq. (2.16) (or, equivalently, eqs. (2.28)–(2.30)) directly into the action. We find convenient to work with the variables that appear in the helicity decomposition. The quadratic Einstein-Hilbert term of the second metric becomes

$$S_{EH_2} = \frac{M_f^2}{4}\int d^4x\,\left[m^4 h_{\mu\nu}^{TT}\frac{1}{(\Box - m^2)}h^{TT} + m^6 h_{\mu\nu}^{TT}\frac{1}{(\Box - m^2)^2}h^{TT} - \frac{2}{3}s\Box s\right]\,. \tag{3.1}$$

The term S_{int}, given in eq. (2.23), greatly simplifies thanks to eq. (2.28), and becomes

$$S_{\mathrm{int}} = -\frac{M_f^2 m^2}{4}\int d^4x\,\left[h_{\mu\nu}^{TT}h^{TT\mu\nu} + m^4 h_{\mu\nu}^{TT}\frac{1}{(\Box - m^2)^2}h^{TT\mu\nu} + 2m^2 h_{\mu\nu}^{TT}\frac{1}{\Box - m^2}h^{TT\mu\nu}\right]\,. \tag{3.2}$$

Since we have solved the equation for $l_{\mu\nu}$ without the need of fixing the gauge, the resulting non-local action for $h_{\mu\nu}$ is invariant under linearized diffeomorphisms, and in fact it depends only on the invariant quantities $h_{\mu\nu}^{TT}$ and s. We can now use eqs. (2.35) and (2.36) and, upon use of the linearized Bianchi identity $\partial^\mu \mathcal{R}_{\mu\nu} = (1/2)\partial_\nu \mathcal{R}$, we get

$$S_{EH_2}^{(2)} + S_{\text{int}} = -M_f^2 \int d^4x \left[\frac{1}{6}\mathcal{R}\frac{1}{\Box}\mathcal{R} + \mathcal{R}_{\mu\nu}\frac{m^2}{\Box(\Box - m^2)}\mathcal{R}^{\mu\nu} - \frac{1}{3}\mathcal{R}\frac{m^2}{\Box(\Box - m^2)}\mathcal{R} \right] . \quad (3.3)$$

Observe that, since the term in square bracket is already $\mathcal{O}(h^2)$, at the quadratic order at which we are working we could simply replace d^4x by $d^4x\sqrt{-g}$. Using $\frac{m^2}{\Box(\Box - m^2)} = -\frac{1}{\Box} + \frac{1}{\Box - m^2}$ we can rewrite $S_{EH_2}^{(2)} + S_{\text{int}} = S_B + S_W$ where:

$$S_B = M_f^2 \int d^4x \left[\mathcal{R}_{\mu\nu}\frac{1}{\Box}\mathcal{R}^{\mu\nu} - \frac{1}{2}\mathcal{R}\frac{1}{\Box}\mathcal{R} \right] , \quad (3.4)$$

$$S_W = -M_f^2 \int d^4x \left[\mathcal{R}_{\mu\nu}\frac{1}{\Box - m^2}\mathcal{R}^{\mu\nu} - \frac{1}{3}\mathcal{R}\frac{1}{\Box - m^2}\mathcal{R} \right] . \quad (3.5)$$

The first term can also be rewritten as

$$S_B = M_f^2 \int d^4x \left(\mathcal{R}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\mathcal{R} \right) \frac{1}{\Box}\mathcal{R}^{\mu\nu}$$
$$= M_f^2 \int d^4x\, \mathcal{G}^{\mu\nu}\frac{1}{\Box}\mathcal{R}^{\mu\nu} , \quad (3.6)$$

where $\mathcal{G}^{\mu\nu}$ is the linearized Einstein tensor. As first observed in [42], despite its non-local appearance, S_B is local with respect to $h_{\mu\nu}$, and is just a way of rewriting the quadratic part of the Einstein-Hilbert action. Indeed, using eqs. (2.14) and (2.15) and performing some integration by parts,

$$\int d^4x\, \mathcal{G}^{\mu\nu}\frac{1}{\Box}\mathcal{R}^{\mu\nu} = \frac{1}{4}\int d^4x\, h_{\mu\nu}\mathcal{E}^{\mu\nu\rho\sigma}h_{\rho\sigma} . \quad (3.7)$$

Thus in the end, putting together $S_B + S_W$ with the quadratic Einstein-Hilbert term of the first metric S_{EH_1}, we get

$$S_2 \equiv S_{EH_1}^{(2)} + S_{EH_2}^{(2)} + S_{\text{int}} \quad (3.8)$$
$$= \frac{M_g^2 + M_f^2}{4}\int d^4x\, h_{\mu\nu}\mathcal{E}^{\mu\nu\rho\sigma}h_{\rho\sigma} - M_f^2 \int d^4x \left[\mathcal{R}_{\mu\nu}\frac{1}{\Box - m^2}\mathcal{R}^{\mu\nu} - \frac{1}{3}\mathcal{R}\frac{1}{\Box - m^2}\mathcal{R} \right] .$$

The non-local term can be rewritten in terms of the linearized Weyl tensor $\mathcal{W}^{\mu\nu\rho\sigma}$ observing that

$$2\left(\mathcal{R}_{\mu\nu}\frac{1}{\Box - m^2}\mathcal{R}^{\mu\nu} - \frac{1}{3}\mathcal{R}\frac{1}{\Box - m^2}\mathcal{R} \right) = \mathcal{W}_{\mu\nu\rho\sigma}\frac{1}{\Box - m^2}\mathcal{W}^{\mu\nu\rho\sigma}$$
$$- \left(\mathcal{R}_{\mu\nu\rho\sigma}\frac{1}{\Box - m^2}\mathcal{R}^{\mu\nu\rho\sigma} - 4\mathcal{R}_{\mu\nu}\frac{1}{\Box - m^2}\mathcal{R}^{\mu\nu} + \mathcal{R}\frac{1}{\Box - m^2}\mathcal{R} \right) , \quad (3.9)$$

Consider now the quantity

$$\widetilde{\chi}_E \equiv \int d^4x\sqrt{-g} \left(R_{\mu\nu\rho\sigma}\frac{1}{\Box - m^2}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}\frac{1}{\Box - m^2}R^{\mu\nu} + R\frac{1}{\Box - m^2}R \right) . \quad (3.10)$$

If the factor $(\Box - m^2)^{-1}$ were not present this would be just the Gauss-Bonnet term, which is a topological invariant and does not contribute to the variation of the action. Because of the $(\Box - m^2)^{-1}$ factors this is no longer true. However, expanding over Minkowski space we find that

$$\int d^4x \sqrt{-g} \left[R_{\mu\nu\rho\sigma} \frac{1}{\Box - m^2} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} \frac{1}{\Box - m^2} R^{\mu\nu} + R \frac{1}{\Box - m^2} R \right] = \mathcal{O}(h^3). \quad (3.11)$$

Therefore in the end, to the order at which we are working, this term can indeed be neglected, and we end up with

$$S_2 = \frac{M_g^2 + M_f^2}{4} \int d^4x \, h_{\mu\nu} \mathcal{E}^{\mu\nu\rho\sigma} h_{\rho\sigma} - \frac{M_f^2}{2} \int d^4x \, \mathcal{W}_{\mu\nu\rho\sigma} \frac{1}{\Box - m^2} \mathcal{W}^{\mu\nu\rho\sigma}. \quad (3.12)$$

To the quadratic order at which we are working, this action has the obvious covariantization

$$S_2 = M_{\rm pl}^2 \int d^4x \sqrt{-g} \, R - \frac{M_f^2}{2} \int d^4x \sqrt{-g} \, W_{\mu\nu\rho\sigma} \frac{1}{\Box - m^2} W^{\mu\nu\rho\sigma}, \quad (3.13)$$

where the linearized Weyl tensor $\mathcal{W}^{\mu\nu\rho\sigma}$ has been promoted to the full Weyl tensor $W^{\mu\nu\rho\sigma}$, and $M_{\rm pl}^2 = M_g^2 + M_f^2$.

4 Discussion

We conclude with a few comments on our main result, eq. (3.13). First of all, we observe that, in the limit $m \to 0$, the result does not reduce to GR. This is a reflection of the vDVZ discontinuity of the original bigravity theory. In fact the original bigravity theory, when linearized over Minkowski, described a massless graviton, plus a massive graviton with a Fierz-Pauli mass term. The bigravity action goes smoothly into the action of GR in the limit $m \to 0$, but the discontinuity manifests itself when one computes the propagator. It is quite interesting to observe that, in our non-local formulation, after having eliminated the second metric with its own equations of motion, the discontinuity manifests itself directly at the level of the action, as we see from eq. (3.12). We can check that this discontinuity is just the vDVZ discontinuity by computing the propagator associated to the quadratic action (3.12). Using the explicit expression of the linearized Weyl tensor, eq. (3.12) reads

$$S_2 = \frac{M_{\rm pl}^2}{4} \int d^4x \, h_{\mu\nu} \left[\mathcal{E}^{\mu\nu\rho\sigma} - 2\tilde{\alpha}^2 \mathcal{F}^{\mu\nu\rho\sigma} \frac{1}{\Box - m^2} \right] h_{\rho\sigma}, \quad (4.1)$$

where $\tilde{\alpha} = M_f/M_{\rm pl}$ and

$$\mathcal{F}^{\mu\nu\rho\sigma} = \frac{1}{3} \partial^\mu \partial^\nu \partial^\rho \partial^\sigma - \frac{1}{4} \Box (\partial^\mu \partial^\rho \eta^{\nu\sigma} + \partial^\mu \partial^\sigma \eta^{\nu\rho} + \partial^\nu \partial^\rho \eta^{\mu\sigma} + \partial^\nu \partial^\sigma \eta^{\mu\rho})$$
$$+ \frac{1}{6} \Box (\partial^\mu \partial^\nu \eta^{\rho\sigma} + \partial^\rho \partial^\sigma \eta^{\mu\nu}) + \frac{1}{4} \Box^2 (\eta^{\mu\rho} \eta^{\sigma\nu} + \eta^{\mu\sigma} \eta^{\rho\nu}) - \frac{1}{6} \Box^2 \eta^{\mu\nu} \eta^{\rho\sigma}. \quad (4.2)$$

Since the above action is invariant under linearized diffeomorphisms, to invert the quadratic form in eq. (4.1) we must add a gauge fixing. Using the usual gauge-fixing term of linearized massless gravity, $\mathcal{L}_{\rm gf} \propto -(\partial^\nu \bar{h}_{\mu\nu})(\partial_\rho \bar{h}^{\rho\mu})$, where $\bar{h}_{\mu\nu} = h_{\mu\nu} - (1/2)h\eta_{\mu\nu}$, we find,

as expected, that the propagator is just the sum of the usual massless propagator of GR plus the propagator of a massive graviton with a Fierz-Pauli mass term. As in the usual Vainshtein mechanism, the vDVZ discontinuity will then be cured by the non-linearities due to the higher-order terms in the curvature.

Finally, it is interesting to compare eq. (3.13) with the non-local modification of gravity proposed in [35], which is based on the action

$$S_{\rm NL} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[R - \frac{1}{6} m^2 R \frac{1}{\Box^2} R \right] . \tag{4.3}$$

As discussed in [35, 40], this model has quite interesting cosmological properties. Non-local models of this type must be understood as derived from some fundamental non-local theory [34, 36],[2] and it is therefore natural to ask whether they could emerge from bigravity upon elimination of one of the two metrics. We see that the answer is negative. First of all, bigravity produces a different tensor structure, given by the Weyl squared term. Second, as we have seen the non-local term generated from bigravity does not vanish as $m \to 0$, contrary to the non-local term in eq. (4.3). In retrospect, the fact that the non-local term in eq. (4.3) could not have been generated by bigravity is a general consequence of the fact that the theory (4.3) has no vDVZ discontinuity [35, 38], while the non-local theory derived from bigravity inherits its vDVZ discontinuity.

Acknowledgments

A   Relation to the approach of Hassan, Schmidt-May and von Strauss

In this appendix we discuss the relation of our result to that obtained in [21]. In general, when we solve the equations of motion, we can eliminate $f_{\mu\nu}$ using its own equation of motion, $(\delta S/\delta f_{\mu\nu})_g = 0$, and then plugging the resulting expression into $(\delta S/\delta g_{\mu\nu})_f = 0$ or, alternatively, we can first obtain $f_{\mu\nu}$ by solving $(\delta S/\delta g_{\mu\nu})_f = 0$, and then plug it into $(\delta S/\delta f_{\mu\nu})_g = 0$. Obviously, these are equivalent and legitimate ways of solving the equations of motion. The issue is more subtle if we want to derive an equivalent effective action involving only $g_{\mu\nu}$. This point has been explained clearly in [21]: let $S' = S[g, f(g)]$ be the action obtained substituting $f_{\mu\nu}$ with its expression as a function of $g_{\mu\nu}$, obtained either from $(\delta S/\delta g_{\mu\nu})_f = 0$ or from $(\delta S/\delta f_{\mu\nu})_g = 0$. In both cases the variation of S' with respect to $g_{\mu\nu}$ is given by

$$\frac{\delta S'}{\delta g_{\mu\nu}(x)} = \left(\frac{\delta S}{\delta g_{\mu\nu}(x)} \right)_f + \int d^4y \frac{\delta f_{\rho\sigma}(y)}{\delta g_{\mu\nu}(x)} \left(\frac{\delta S}{\delta f_{\rho\sigma}(y)} \right)_g = 0 . \tag{A.1}$$

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[2]The same holds for the non-local model proposed in [43, 44], see [45] for a recent review. This model is however different, since it is rather constructed with a term $Rf(\Box^{-1}R)$ in the action, and it does not feature a mass scale m. Non-local actions have also been studied with motivation mostly coming from the UV, see e.g. [46–50].

If $f_{\mu\nu}$ is a solution of $(\delta S/\delta f)_g = 0$, then the equations $\delta S'/\delta g = 0$ and $(\delta S/\delta g_{\mu\nu})_f = 0$ are equivalent. Thus S' is classically equivalent to S, as long as we are interested in the dynamics of $g_{\mu\nu}$ only. In contrast, if $f_{\mu\nu}$ is a solution of $(\delta S/\delta g)_f = 0$, the two actions are not equivalent. Solutions of $(\delta S/\delta f)_g = 0$ do satisfy $\delta S'/\delta g = 0$, but the converse is not necessarily true. The action S' also has spurious solutions characterized by $(\delta S/\delta f_{\mu\nu})_g = \chi_{\mu\nu}(x)$, with $\chi(x)$ a function such that

$$\int d^4y\, \frac{\delta f_{\rho\sigma}(y)}{\delta g_{\mu\nu}(x)} \chi_{\rho\sigma}(y) = 0\,. \tag{A.2}$$

Therefore, in this case extra conditions must be imposed to eliminate the spurious solutions, and the relation between the action S' and the original action is less direct. On the other hand, the equation $(\delta S/\delta g_{\mu\nu})_f = 0$ is algebraic in $f_{\mu\nu}$, and can always be solved. In contrast, solving with respect to $f_{\mu\nu}$ the equation $(\delta S/\delta f_{\mu\nu})_g = 0$ involves the inversion of a differential operator, which in practice can only be done by expanding around a simple background. The approach taken by Hassan, Schmidt-May and von Strauss [21] has been to eliminate $f_{\mu\nu}$ using the equation of motion of $g_{\mu\nu}$. Plugging the resulting expression into the action S it was found in ref. [21] that the resulting theory is given by the higher-derivative action

$$S^{\mathrm{HD}}_{(2)} = M_g^2 \int d^4x \sqrt{-g} \left[\Lambda + c_R R(g) - \frac{c_{RR}}{m^2} \left(R^{\mu\nu} R_{\mu\nu} - \frac{1}{3} R^2 \right) \right] + \mathcal{O}(m^{-4})\,, \tag{A.3}$$

where Λ, c_R and c_{RR} are some coefficients. By subtracting a Gauss-Bonnet term, similarly to what we have done in section 3, this action can be rewritten in terms of the Weyl tensor as [21]

$$S^{\mathrm{HD}}_{(2)} = M_g^2 \int d^4x \sqrt{-g} \left[\Lambda + c_R R(g) - \frac{c_{RR}}{2m^2} W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma} \right] + \mathcal{O}(m^{-4})\,. \tag{A.4}$$

The values of Λ, c_R and c_{RR} are given in [21] as functions of the β_i. With our choice $\beta_0 = -6$, $\beta_1 = 3$, $\beta_2 = -1$, $\beta_3 = \beta_4 = 0$ we have[3]

$$\Lambda = 0\,, \qquad c_R = \frac{M_g^2 + M_f^2}{M_g^2}\,, \qquad c_{RR} = \frac{M_g^2 + 2\, M_f^2}{M_f^2}\,. \tag{A.5}$$

Comparison with eq. (3.13) shows that the cosmological constant vanishes for both actions, and the coefficient of the Einstein-Hilbert term is the same. However, in the limit $\Box \ll m^2$, the coefficient of the $W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma}$ term in eq. (3.13) is $+M_f^2/(2m^2)$ while, in eq. (A.4), it is $-1/2\, M_g^2(M_g^2 + 2M_f^2)/(m^2\, M_f^2)$, so the two disagree, even in the sign. Actually, this is simply due to the fact that the action (A.4), by itself, is not equivalent to the original

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[3]We also take into account a difference in the definition of m^2. Comparing the actions we see that our m^2 is related to the parameter denoted m^2 in [21] by $m^2_{\mathrm{our}} M_f^2 = m^2_{\mathrm{their}} M_g^2$. Observe also that, to determine Λ, c_R and c_{RR}, we need to compute the parameter denoted by a in [21], which is determined by their eq. (2.19). For our values of β_i we get $a^2 - 3a + 2 = 0$, which has the solutions $a = 1$ and $a = 2$. We only retain $a = 1$, since only in this case we get $\Lambda = 0$. With $a = 2$ not only the coefficient of the Weyl term would differ, but also the cosmological term and the coefficient of the Einstein-Hilbert term.

bigravity action, since it has been obtained eliminating $f_{\mu\nu}$ with the equation of motion of $g_{\mu\nu}$, rather than with its own. As discussed in [21], and has we have recalled above, with this procedure the correspondence between the two theories is more indirect, and is only at the level of the equation of motion, once spurious solutions are eliminated. In contrast, the action (3.13) is indeed equivalent to the original bigravity action, up to quadratic orders in the curvature. This point can also be illustrated using a nice example given in appendix A1 of [21]. Consider in fact the theory with two scalar fields ϕ and ψ, given by

$$S[\phi, \psi] = \int d^4x \left[-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu\psi\partial^\mu\psi - \frac{\mu^2}{2}(\phi + \psi)^2 \right] . \tag{A.6}$$

(We do not need source terms for our purpose). Of course, one could diagonalize the action introducing $\Phi_0 = \phi - \psi$ and $\Phi_m = \phi + \psi$. However, it is instructive to rather integrate out ψ using either its equation of motion, or the one with respect to ϕ. The equation of motions are

$$\left(\frac{\delta S}{\delta \phi}\right)_\psi : \quad (\Box - \mu^2)\phi = \mu^2\psi , \tag{A.7}$$

$$\left(\frac{\delta S}{\delta \phi}\right)_\phi : \quad (\Box - \mu^2)\psi = \mu^2\phi . \tag{A.8}$$

If we use $(\delta S/\delta\phi)_\psi$, ψ can be eliminated algebraically. Inserting it back in the action one finds the higher-derivative action [21]

$$S_{\mathrm{HD}} = \frac{1}{2\mu^4} \int d^4x \, \phi\Box(\Box - \mu^2)(\Box - 2\mu^2)\phi , \tag{A.9}$$

which to first non-trivial order in $\Box/\mu^2$ becomes

$$S_{\mathrm{HD}} \simeq \int d^4x \, \phi \Box \left(1 - \frac{3}{2}\frac{\Box}{\mu^2}\right)\phi . \tag{A.10}$$

If instead we eliminate ψ using its own equation of motion, we have the non-local expression $\psi = \mu^2(\Box - \mu^2)^{-1}\phi$. Inserting it in the action we get

$$S_{\mathrm{non-loc}} = \frac{1}{2} \int d^4x \, \phi\Box \left(1 + \frac{\mu^2}{\mu^2 - \Box}\right)\phi , \tag{A.11}$$

which, expanding to first non-trivial order in $\Box/\mu^2$, becomes

$$S_{\mathrm{non-loc}} \simeq \int d^4x \, \phi\Box \left(1 + \frac{\Box}{2\mu^2}\right)\phi . \tag{A.12}$$

We see that indeed the first correction is different (even in the sign) from that in eq. (A.10).

B Coupling with matter

In this appendix we extend the computation of the non-local action to the case of coupling with matter. In this case eqs. (2.7) and (2.8) become

$$\frac{M_g^2}{M_f^2}\mathcal{E}_{\mu\nu\rho\sigma}h^{\rho\sigma} + m^2(h_{\mu\nu} - \eta_{\mu\nu}h) = m^2(l_{\mu\nu} - \eta_{\mu\nu}l) - \frac{\kappa_1}{2}T_{\mu\nu} , \tag{B.1}$$

$$\mathcal{E}_{\mu\nu\rho\sigma}l^{\rho\sigma} + m^2(l_{\mu\nu} - \eta_{\mu\nu}l) = m^2(h_{\mu\nu} - \eta_{\mu\nu}h) - \frac{\kappa_2}{2}T_{\mu\nu} , \tag{B.2}$$

where we have introduced two generic couplings $\kappa_{1,2} \equiv k_{1,2}/M_f^2$. Then eq. (2.16) becomes

$$l_{\mu\nu} = h_{\mu\nu} + \frac{1}{\Box - m^2}\left[2\mathcal{R}_{\mu\nu} - \frac{1}{3}\left(\eta_{\mu\nu} + 2\frac{\partial_\mu\partial_\nu}{m^2}\right)\mathcal{R}\right] - \frac{\kappa_2}{2}\,Q_{\mu\nu\alpha\beta}T^{\alpha\beta}\,, \qquad \text{(B.3)}$$

where $Q_{\mu\nu\alpha\beta}$ is defined in eq. (2.12) and

$$Q_{\mu\nu\alpha\beta}T^{\alpha\beta} = -\frac{1}{\Box - m^2}\left(T_{\mu\nu} - \frac{1}{3}\eta^{\mu\nu}T\right)\,. \qquad \text{(B.4)}$$

Taking the trace of eq. (B.3) we get

$$l = h - \frac{2\mathcal{R}}{3m^2} - \frac{\kappa_2}{6}\frac{1}{\Box - m^2}T\,. \qquad \text{(B.5)}$$

We can now substitute back in the action eqs. (B.3) and (B.5). The result has the form $S_{\text{tot}} = S_2 + S_{\text{int}}$ where S_2 is given by eq. (3.13) and $S_{\text{int}} = \int d^4x\,\sqrt{-g}\,\mathcal{L}_{\text{int}}$ with

$$\begin{aligned}
\mathcal{L}_{\text{int}} = {}&\left(\frac{k_2}{32M_f}\right)^2\left[T_{\mu\nu}\frac{1}{\Box - m^2}T^{\mu\nu} - \frac{1}{3}T\frac{1}{\Box - m^2}T\right] \\
&- \left(\frac{k_2\,m}{8M_f}\right)^2\left[T_{\mu\nu}\frac{1}{(\Box - m^2)^2}T^{\mu\nu} - \frac{1}{3}T\frac{1}{(\Box - m^2)^2}T\right] \\
&- k_2\,m^2\left[T_{\mu\nu}\frac{1}{(\Box - m^2)^2}R^{\mu\nu} - \frac{1}{6}T\frac{1}{(\Box - m^2)^2}R\right] \\
&+ \frac{k_2}{2}\left[T_{\mu\nu}\frac{1}{\Box - m^2}R^{\mu\nu} - \frac{1}{6}T\frac{1}{\Box - m^2}R\right] - \frac{1}{2}(k_1 + k_2)\,T^{\mu\nu}\frac{1}{\Box}R_{\mu\nu}\,. \qquad \text{(B.6)}
\end{aligned}$$

The last term can be transformed observing that, at the linearized level,

$$\int d^4x\,T^{\mu\nu}\frac{1}{\Box}R_{\mu\nu} = \int d^4x\,T^{\mu\nu}\frac{1}{\Box}\left(\partial^\rho\partial_\mu h_{\nu\rho} - \Box h_{\mu\nu}\right)\,. \qquad \text{(B.7)}$$

The term $T^{\mu\nu}\Box^{-1}\partial^\rho\partial_\mu h_{\nu\rho}$ vanishes integrating by parts and using $\partial_\mu T^{\mu\nu} = 0$, and therefore in eq. (B.6) we can replace $-(1/2)(k_1 + k_2)\,T^{\mu\nu}\Box^{-1}\,R_{\mu\nu}$ by $+(1/2)(k_1 + k_2)\,T^{\mu\nu}h_{\mu\nu}$, which is the standard local coupling, with an effective Newton constant determined by $k_1 + k_2$. The remaining terms in eq. (B.6) provide genuinely non-local couplings.

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