

Inflationary baryogenesis in a model with gauged baryon number

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ABSTRACT: We argue that inflationary dynamics may support a scenario where significant matter-antimatter asymmetry is generated from initially small-scale quantum fluctuations that are subsequently stretched out over large scales. This scenario can be realised in extensions of the Standard Model with an extra gauge symmetry having mixed anomalies with the electroweak gauge symmetry. Inflationary baryogenesis in a model with gauged baryon number is considered in detail.

KEYWORDS: Cosmology of Theories beyond the SM, Gauge Symmetry, Anomalies in Field and String Theories

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1 Introduction

It is a standard lore that dynamical generation of matter-antimatter asymmetry must happen after an inflationary epoch, since any asymmetry generated before that is diluted away due to the rapid spacetime expansion. In order to produce a significant asymmetry, during inflation, the production rate of baryonic charge must exceed its dilution rate. Actually, inflationary dynamics may support such a scenario: if a large baryonic charge density is created due to small-scale quantum fluctuations, it will typically be stretched out over large scales due to inflation. This basic observation has been realised in a model of inflationary leptogenesis [1], where a lepton asymmetry is produced during inflation due to the gravitational birefringence through a gravitational lepton number anomaly coupled to an extra pseudoscalar field.

In this paper we argue that the inflationary baryogenesis scenario can be realised in extensions of the Standard Model with an anomalous gauge symmetry which has mixed anomalies with the electroweak gauge symmetry.¹ This anomalous theory can also be viewed as an effective low-energy theory, which admits a fundamental completion free of gauge anomalies. The obvious candidates for such an anomalous gauge theory are gauged baryon (B) and lepton (L) numbers, or any linear combination thereof except for $(B - L)$. In the present paper we consider a model with gauged B -number in detail.

The basic three Sakharov's conditions for dynamical baryogenesis [5] are satisfied in our model as follows. As in the Standard Model the baryon number is not conserved because

¹In the early universe, when the expansion rate is faster than processes with fermion chirality flip, the gauged anomaly may effectively appear within the Standard Model [2]. Indeed, it has been argued in [3] that anomalous production of the right-handed electron number is possible through the hypercharge anomaly. An inflationary version of the above scenario is discussed in [4].

of the mixed electroweak - B anomaly. On top of this, $U(1)_B$ gauge invariance requires a pseudoscalar field, that describes the longitudinal polarization of the baryonic photon, to couple to the anomaly. In the cosmological setting these interactions spontaneously violate \mathcal{CP} invariance and lead to the \mathcal{CP} -asymmetric out-of-equilibrium production of electroweak gauge bosons with different polarizations. In particular, during an inflationary epoch the produced particles form a Bose-Einstein condensate with a large correlation length which supports the generation of a non-zero baryon number through the anomaly.

The rest of the paper is organized as follows. In the next section we describe a model with gauged B -number. In section 3, we present quantization of the weak gauge bosons in an inflationary spacetime. In section 4, we compute the generated baryon asymmetry. Section 5 is reserved for conclusions. Finally, some technical details of our calculations and useful formulas are delegated to appendices A and B.

2 A model with gauged B -number

Let us consider an extension of the Standard Model with gauged symmetry $SU(3) \times SU(2) \times U(1)_Y \times U(1)_B$. We assume that no extra fermions and scalars are introduced beyond those in the Standard Model, except for a field (or fields) which drives inflation in the early universe. The detailed dynamics of the inflaton field(s) is not essential in our analysis. Since the gauged baryon number $U(1)_B$ is anomalous, the associated gauge boson carries three degrees of freedom,² that is, it is necessarily massive. A scalar field $\theta(x)$ that describes the longitudinal degree of freedom of a massive baryonic photon X_μ can be used to cancel out anomalies without introducing new matter fermions [6–8].³ The addition to the Lagrangian density, describing the Standard Model, then reads:

$$\begin{aligned} \frac{1}{\sqrt{-g}}\mathcal{L}_B = & -\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}X_{\mu\nu}X_{\alpha\beta} + \frac{1}{2}f_B^2g^{\mu\nu}(g_BX_\mu - \partial_\mu\theta)(g_BX_\nu - \partial_\nu\theta) \\ & + \frac{3\theta(x)}{32\pi^2} \left[g_1^2B_{\mu\nu}\tilde{B}^{\mu\nu} - g_2^2W_{\mu\nu}^a\tilde{W}^{a\mu\nu} \right] - \end{aligned} \tag{2.1}$$

where $X_{\mu\nu}$, $B_{\mu\nu}$ and $W_{\mu\nu}^a$ ($a = 1, 2, 3$; summation under the repeated weak isospin indices is assumed throughout the paper) denote field strengths for $U(1)_B$, $U(1)_Y$ and $SU(2)$ gauge bosons with corresponding coupling constants g_B , g_1 , and g_2 , respectively; f_B is a parameter that defines the mass of the baryonic photon, $m_B = g_B f_B$; $\tilde{B}^{\mu\nu}(\tilde{W}^{a\mu\nu}) = \frac{1}{2\sqrt{-g}}\epsilon^{\mu\nu\rho\sigma}B_{\rho\sigma}(W_{\rho\sigma}^a)$ is the dual field strength, and $\epsilon^{\mu\nu\rho\sigma}$ is the Levi-Civita tensor. In eq. (2.1) and in what follows, we omitted interactions of X_μ with the baryonic current J_B^μ , since these are not relevant to our discussion. Note that the second term on the r.h.s. of eq. (2.1) is the familiar term of the Stueckelberg formalism [10] for a massive gauge boson.

Note, that the terms in the second line of eq. (2.1) are introduced to maintain gauge invariance of the full quantum theory under $U(1)_B$ transformations. Indeed, while they are not invariant under $U(1)_B$ gauge transformations, $X_\mu \rightarrow X_\mu + (1/g_B)\partial_\mu\alpha$ and $\theta(x) \rightarrow$

²See [6] for a review of anomalous gauge theories.

³This mechanism of anomaly cancellation has been originally suggested in 10d anomalous gauge theories in [9].

$\theta(x) + \alpha(x)$, their variance cancels out against the gauge variation of the functional measure of quark fields within the path integral quantization framework. It is clear that the above model can also be viewed as an effective low-energy approximation of an anomaly-free theory [11–15], where additional fermionic fields, which cancel the $[SU(2)]^2 - U(1)_B$ and $[U(1)_Y]^2 - U(1)_B$ mixed anomalies, are integrated out. Then, according to t’Hooft’s anomaly matching condition [16], the terms restoring gauge invariance necessarily appear in the low-energy theory.

A remark related to the above $U(1)_B$ gauge invariance is in order. In principle one may locally fix the gauge such that $\theta(x) = 0$,⁴ so that the theory with θ field is equivalent (within the perturbation theory) to a theory with purely massive X_μ coupled to quarks without the θ field (‘unitary gauge’). Nevertheless, we find it to be more convenient if θ is manifestly present as in eq. (2.1), since the longitudinal physical degree of freedom of the massive baryonic photon, which plays a crucial role in our analysis, is easily identifiable in this case.

The metric tensor in eq. (2.1) describes a homogeneous and spatially flat cosmological spacetime, and hence, in conformal coordinates can be written as: $g_{\mu\nu} = a^2(\tau)\eta_{\mu\nu}$ and $g \equiv \det(g_{\mu\nu})$. The scale factor $a(\tau)$ during inflation reads:

$$a(\tau) = -1/H_{\text{inf}}\tau, \tag{2.2}$$

where H_{inf} is an expansion rate ($H_{\text{inf}} \cong \text{const.}$) and $\tau \in [-\infty, 0]$ is the conformal time.

To proceed further we make the following simplifying assumptions. We assume that $g_B \ll 1$, and thus $\theta(x)$ and X_μ fields essentially decouple from each other. The smallness of the $U(1)_B$ coupling constant implies the baryonic photon is relatively light, $m_B/f_B \ll 1$, and hence we will not be interested in its dynamics during inflation. We also ignore the dynamics of the hypercharge gauge field B_μ as it is less relevant compared to the dynamics of weak isospin fields W_μ^a , due to the fact that $g_2 > g_1$. Furthermore, as we are interested in small quantum fluctuations of $SU(2)$ gauge bosons around a trivial (vacuum) configuration, we ignore self-interactions of W_μ^a restricting to the linearized approximation. For the $\theta(x)$ field we only consider a classical homogeneous background configuration, $\theta(\tau, \vec{x}) = \theta(\tau)$, and ignore quantum fluctuations over it. With these assumptions the Lagrangian terms being considered significantly simplify to:

$$\mathcal{L} = -\frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma}W_{\mu\nu}^aW_{\rho\sigma}^a + \frac{a^2(\tau)}{2}(\phi'(\tau))^2 - \frac{3g_2^2}{64\pi^2 f_B}\phi(\tau)\epsilon^{\mu\nu\rho\sigma}W_{\mu\nu}^aW_{\rho\sigma}^a, \tag{2.3}$$

where $\phi(\tau) \equiv f_B\theta(\tau)$ and $\phi' \equiv d\phi/d\tau$.

The equation of motion for $\phi(\tau)$ that follows from the above Lagrangian reads:

$$(a^2\phi')' = 0, \tag{2.4}$$

where we have ignored terms quadratic in W_μ^a . From eq. (2.4) we obtain:

$$\phi'(\tau) = \frac{\phi'_0}{a^2(\tau)}, \tag{2.5}$$

⁴There may exist a topological obstruction to imposing this gauge condition globally in spacetime because of the presence of vortex excitations around which $\theta(x)$ has a non-trivial winding number. However, within the perturbative framework this complication is irrelevant; hence we ignore this non-perturbative effect here.

where ϕ'_0 is an integration constant associated with the ‘field velocity’ at the start of inflation $\tau = \tau_0$, $a(\tau_0) = 1$. Plugging eq. (2.5) into the linearized equation of motion for the W_μ^a gauge fields we obtain:

$$\left(\partial_\tau^2 - \vec{\nabla}^2\right) W^{ai} + \kappa\tau^2 \epsilon^{ijk} \partial_j W_k^a = 0, \tag{2.6}$$

where

$$\kappa = \frac{3g_2^2 \phi'_0 H_{\text{inf}}^2}{8\pi^2 f_B}, \tag{2.7}$$

and we have adopted the gauge where $W_0^a = \partial_i W_i^a = 0$. Note that the first and the last terms in eq. (2.6) have opposite \mathcal{P} and, hence, \mathcal{CP} parities. This is the source of \mathcal{CP} violation in our model which is one of the necessary Sakharov’s conditions [5].

The simplifying assumptions made in this section allow us to undertake an analytical treatment of the problem in expense of the accuracy of the calculations. Our final results must be understood as an order of magnitude estimation.

3 Quantum fluctuations of the weak gauge bosons during inflation

To quantize the model described in the previous section we promote the weak gauge boson fields to operators:

$$W_i^a = \int \frac{d^3\vec{k}}{(2\pi)^{3/2}} \sum_\alpha \left[F_\alpha(\tau, k) \epsilon_{i\alpha} \hat{a}_\alpha^a e^{i\vec{k}\cdot\vec{x}} + F_\alpha^*(\tau, k) \epsilon_{i\alpha}^* \hat{a}_\alpha^{a\dagger} e^{-i\vec{k}\cdot\vec{x}} \right], \tag{3.1}$$

where creation, $\hat{a}_\alpha^{a\dagger}(\vec{k})$, and annihilation, $\hat{a}_\alpha^a(\vec{k})$, operators satisfy canonical commutation relations:

$$\left[\hat{a}_\alpha^a(\vec{k}), \hat{a}_\beta^{b\dagger}(\vec{k}') \right] = \delta_{\alpha\beta} \delta^{ab} \delta^3(\vec{k} - \vec{k}'), \tag{3.2}$$

and

$$\hat{a}_\alpha^a(\vec{k})|0\rangle_\tau = 0, \tag{3.3}$$

where $|0\rangle_\tau$ is an instantaneous vacuum state at time τ .

In eq. (3.1), two vectors $\vec{\epsilon}_\alpha$ ($\alpha = +, -$) describe two helicity states (we treat the weak bosons as massless particles, since $m_W \ll H_{\text{inf}}$) and they are in fact complex conjugates of each other, i.e. $\vec{\epsilon}_+^* = \vec{\epsilon}_-$. The equations for the mode functions, $F_\pm(\tau, k)$ [$k \equiv |\vec{k}|$], straightforwardly follow from eq. (2.6):

$$F_\pm'' + (k^2 \mp \kappa\tau^2 k) F_\pm = 0. \tag{3.4}$$

According to this equation, towards the end of inflation ($\tau_{\text{end}} \simeq 0$) all the modes with $k \gg \mu = |\kappa|\tau_{\text{end}}^2$ approach \mathcal{CP} -symmetric flat spacetime plane waves:

$$F_\pm(\tau, k) \xrightarrow{\tau \rightarrow 0} \frac{1}{\sqrt{2k}}. \tag{3.5}$$

These also include large wavelength superhorizon modes $k|\tau_{\text{end}}| \ll 1$, which are of our prime interest. The field operator eq. (3.1) for $\tau \rightarrow 0$ becomes:

$$W_i^a = \int \frac{d^3\vec{k}}{(2\pi)^{3/2} \sqrt{2k}} \sum_\alpha \left[\epsilon_{i\alpha} \hat{b}_\alpha^a e^{-ik|\tau| + i\vec{k}\cdot\vec{x}} + \epsilon_{i\alpha}^* \hat{b}_\alpha^{a\dagger} e^{ik|\tau| - i\vec{k}\cdot\vec{x}} \right]. \tag{3.6}$$

The nonzero term $\propto \kappa\tau^2 k$ in eq. (3.4) is responsible for \mathcal{CP} -asymmetric ($F_+ \neq F_-$) solutions:

$$F_+(\tau, k) = C_1 D_{-\frac{1}{2}(1-\Omega_k)} \left(\frac{\sqrt{2}k\tau}{\sqrt{\Omega_k}} \right) + C_2 D_{-\frac{1}{2}(1+\Omega_k)} \left(\frac{i\sqrt{2}k\tau}{\sqrt{\Omega_k}} \right), \quad (3.7)$$

and

$$F_-(\tau, k) = C_3 D_{-\frac{1}{2}(1+i\Omega_k)} \left(\frac{\sqrt{2}ik\tau}{\sqrt{\Omega_k}} \right) + C_4 D_{-\frac{1}{2}(1-i\Omega_k)} \left(\frac{i\sqrt{2}ik\tau}{\sqrt{\Omega_k}} \right), \quad (3.8)$$

where $D_\nu(z)$ is the parabolic cylinder function and $\Omega_k = \left(\frac{k^3}{\kappa}\right)^{1/2}$. The integration constants $C_{1,2,3,4}$ are defined through the Wronskian normalization condition and by matching eqs. (3.7), (3.8) with plane wave modes according to eq. (3.5). For superhorizon modes ($k|\tau| \rightarrow 0$), which are of our prime interest, they are given in appendix A, eqs. (A.1)–(A.4).

Two sets of creation and annihilation operators, $\{\hat{a}_\alpha^a, \hat{a}_\alpha^{a\dagger}\}$ and $\{\hat{b}_\alpha^a, \hat{b}_\alpha^{a\dagger}\}$, in eqs. (3.1) and (3.6), are related through the Bogoliubov transformations:

$$\hat{b}_\alpha^a(\vec{k}) = \alpha_\alpha a_\alpha^{a\dagger}(\vec{k}) + \beta_\alpha^* \hat{a}_\alpha^a(\vec{k}) \quad (3.9)$$

$$\hat{b}_\alpha^{a\dagger}(\vec{k}) = \alpha_\alpha^* a_\alpha^a(\vec{k}) + \beta_\alpha \hat{a}_\alpha^{a\dagger}(\vec{k}) \quad (3.10)$$

The Bogoliubov coefficients for the superhorizon modes ($k|\tau_{\text{end}}| \approx 0$) of interest can be computed explicitly:

$$\alpha_\alpha = \frac{1}{2} + i\sqrt{\frac{1}{2k}} R_\alpha^* \quad \text{and} \quad \beta_\alpha = \frac{1}{2} - i\sqrt{\frac{1}{2k}} R_\alpha^*. \quad (3.11)$$

where $R_\alpha^* := F_\alpha^{*'} \Big|_{\frac{\kappa\tau_{\text{end}}^2}{k}, k|\tau_{\text{end}}| \rightarrow 0}$.

4 Computing the baryon asymmetry

We are now ready to compute the generated baryon number density. The anomalous non-conservation of baryonic current reads:

$$\partial_\mu (\sqrt{-g} j_B^\mu) = \frac{3g_2^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} W_{\mu\nu}^a W_{\rho\sigma}^a \equiv \frac{3g_2^2}{16\pi^2} \partial_\mu (\sqrt{-g} K^\mu), \quad (4.1)$$

where $K^\mu = \frac{1}{2\sqrt{-g}} \epsilon^{\mu\nu\rho\sigma} W_{\nu\rho}^a W_\sigma^a$ is a topological current. This equation implies that the net baryon number density $n_B = n_b - n_{\bar{b}} \equiv a^{-1}(\tau)_{\tau_0} \langle 0 | j_B^0 | 0 \rangle_{\tau_0}$ is related to the weak gauge boson Chern-Simons number density, $n_{CS} = \tau_0 \langle 0 | K^0(\tau) | 0 \rangle_{\tau_0}$, at the end of inflation, $\tau = \tau_{\text{end}}$:

$$n_B = \frac{3g_2^2}{16\pi^2} a(\tau_{\text{end}}) n_{CS}. \quad (4.2)$$

Here, $n_B(\tau_0) = n_{CS}(\tau_0) = 0$, at the start of inflation. Furthermore, we are interested in n_{CS} for large scale superhorizon modes ($k|\tau| \approx 0$), hence, we have:

$$n_{CS} = \frac{1}{a^4(\tau_{\text{end}})} \epsilon^{ijk} \lim_{k|\tau| \rightarrow 0} \langle 0 | W_i \partial_j W_k | 0 \rangle = \frac{3}{8\pi^2 a^4(\tau_{\text{end}})} \int_\mu^\Lambda k dk \left[|R_+|^2 - |R_-|^2 \right], \quad (4.3)$$

where

$$|R_+|^2 = \frac{\pi}{2} \sqrt{\frac{\kappa k}{2}} \left| C_1 \frac{2^{\frac{\Omega_k}{4}} (1 - \Omega_k)}{\Gamma\left(\frac{5 - \Omega_k}{4}\right)} + i C_2 \frac{(1 + \Omega_k)}{2^{\frac{\Omega_k}{4}} \Gamma\left(\frac{5 + \Omega_k}{4}\right)} \right|^2, \quad (4.4)$$

$$|R_-|^2 = \frac{\pi}{2} \sqrt{\frac{\kappa k}{2}} \left| C_3 \frac{(1 + i\Omega_k)}{2^{\frac{i\Omega_k}{4}} \Gamma\left(\frac{5 + i\Omega_k}{4}\right)} + i C_4 \frac{2^{\frac{i\Omega_k}{4}} (1 - i\Omega_k)}{\Gamma\left(\frac{5 - i\Omega_k}{4}\right)} \right|^2, \quad (4.5)$$

and Λ is an ultraviolet cut-off, and μ is an IR cut-off. We have found that the integral in eq. (4.3) is dominated by the dependence on μ given below, and is independent of Λ . This result can be understood as follows. Physically, the modes with large k are essentially \mathcal{CP} -invariant plane waves, thus the integrand in eq. (4.3) for those modes nullifies. Thus, the integral is effectively zero for large k modes. The IR cut-off is naturally given by $\mu = \kappa \tau_{\text{end}}^2$ which corresponds to the modes that were initially matched to the Minkowski planewave solutions, in eq. (3.5).

Finally, assuming that there was no significant entropy production after the reheating phase, we estimate the entropy density as: $s \simeq \frac{2\pi^2}{45} g^* T_{\text{rh}}^3$, where $g^*(T_{\text{rh}}) \sim 100$ and T_{rh} is the reheating temperature. We obtain the following simple expression for the baryon asymmetry parameter:

$$\eta_B = \frac{n_B}{s} \approx \frac{5g_2^2}{g^* \sqrt{2}\pi^7} \frac{\Gamma\left(\frac{3}{4}\right)^4}{\Gamma\left(\frac{5}{4}\right)^2} e^{-3N_e} \left(\frac{\kappa}{\mu T_{\text{rh}}^2} \right)^{\frac{3}{2}} \approx 4.1 \cdot 10^{-3} \frac{H_{\text{inf}} T_{\text{rh}}}{M_p^2}, \quad (4.6)$$

where $\tau_{\text{end}} = -\frac{1}{a(\tau_{\text{end}})H} = -\frac{e^{-N_{\text{inf}}}}{H_{\text{inf}}}$ and $g_2^2 \approx 4\pi/29$. The total number of e-folds N_e , that defines the dilution factor, includes the minimal number of e-folds required during inflation $N_{\text{inf}} \simeq 34 + \ln\left(\frac{T_{\text{rh}}}{100 \text{ GeV}}\right)$ and the number of e-folds during reheating $N_{\text{rh}} \simeq \frac{1}{3} \ln\left(\frac{45 H_{\text{inf}}^2 M_p^2}{4\pi^3 g^* T_{\text{rh}}^4}\right)$:

$$N_e = N_{\text{inf}} + N_{\text{rh}} \simeq 32 + \ln\left(\frac{T_{\text{rh}}}{100 \text{ GeV}}\right) + \frac{2}{3} \ln\left(\frac{H_{\text{inf}} M_p}{T_{\text{rh}}^2}\right) \quad (4.7)$$

Eq. (4.6) was obtained using a first order Taylor expansion around $\Omega_k = 0$. Interestingly, for the chosen IR cut-off $\mu = |\kappa| \tau_{\text{end}}^2$, the asymmetry parameter is not manifestly dependent on κ , due to the approximation adopted in our calculations. Indeed, in the opposite limit of vanishing $\kappa \rightarrow 0$ and $\Omega_k \rightarrow \infty$ leads to the F_{\pm} solutions to approach the flat spacetime limit, where the resulting asymmetry is 0. From eq. (4.6), the following requirement is obtained:

$$H_{\text{inf}} T_{\text{rh}} \approx 3 \times 10^{30} \text{ GeV}^2. \quad (4.8)$$

Hence, the desired value of $\eta_B \approx 8.5 \cdot 10^{-11}$ can be obtained as long as the Hubble rate and reheating temperature are suitable large as to satisfy eq. (4.8) (i.e. $H \sim 10^{14} \text{ GeV}$ and $T_{\text{rh}} \sim 10^{16} \text{ GeV}$). Although eq. (4.8) is an order of magnitude estimation, we expect the above mechanism to be phenomenologically viable if the ratio (r) of the inflationary tensor and scalar perturbation amplitudes satisfies $r \gtrsim 10^{-2}$.

The net baryon number density n_B eq. (4.2) generated during inflation evolves in the subsequent epochs. Besides the trivial dilution due to the expansion, which is cancelled

out in the asymmetry parameter eq. (4.6), there may be other processes that influence n_B . For example, non-perturbative $(B + L)$ -violating processes, which are thermally activated if $T_{\text{rh}} \gtrsim 100 \text{ GeV}$ [17], wash out any existing $(B + L)$ number, while preserving $(B - L)$ in thermal equilibrium. This means that part of the initial baryon number will be reprocessed into a lepton number, but n_B will remain of the same order of magnitude.

5 Conclusion

In this paper we have argued that a successful baryogenesis scenario can be realised during the inflationary epoch within a class of anomalous gauge theories. A model with gauged baryon number has been considered in detail. The large wavelength modes of electroweak gauge bosons, produced during inflation, form a Bose-Einstein condensate that supports non-zero net baryon number density n_B . We have found that the baryon number asymmetry parameter η_B has a simple dependence eq. (4.6) on the cosmological parameters H_{inf} and T_{rh} eq. (4.8), for which the experimental values can be accommodated. To obtain the desired asymmetry large scale inflation $H \sim 10^{14} \text{ GeV}$ and high reheating temperature $T_{\text{rh}} \sim 10^{16} \text{ GeV}$ are required. This is in accord with indications on the inflationary scale from the BICEP2 measurements of B-modes [18].

Several different versions of the model presented here are also possible. In fact, any model with an additional gauge symmetry having mixed anomalies with the electroweak symmetry can potentially provide a successful framework for inflationary baryogenesis. An interesting aspect of these classes of models is that hypothetical new physics behind the baryogenesis scenario may well be accessible at the LHC. It will be interesting to study collider phenomenology of these models as well.

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A Further details on coefficient derivations

F_+ coefficients, eq. (3.7). Matching superhorizon modes with the plane waves we obtain the following relation:

$$C_1 = \frac{\Gamma(\frac{3-\Omega_k}{4})}{2^{\frac{-1}{4}(1-\Omega_k)}\sqrt{\pi}} \left(\frac{1}{\sqrt{2k}} - C_2 \frac{2^{\frac{-1}{4}(1+\Omega_k)}\sqrt{\pi}}{\Gamma(\frac{3+\Omega_k}{4})} \right)$$

The Wronskian normalisation implies:

$$\sqrt{\frac{2}{\Omega_k}} C_1 C_2 \sin\left(\frac{\pi}{4}(1 + \Omega_k)\right) + C_2^2 \sqrt{\frac{\pi}{\Omega_k}} \frac{1}{\Gamma(\frac{1+\Omega_k}{2})} = \frac{1}{2k}$$

Solving the above conditions we find that the coefficients for F_+ modes are:

$$C_1 = \frac{2^{-\frac{1}{4}(1+\Omega_k)}\Gamma(\frac{3-\Omega_k}{4})}{\sqrt{\pi k}} - \frac{2^{-\frac{1}{2}(\Omega_k+3)}\Gamma(\frac{1+\Omega_k}{4})\Gamma(\frac{3-\Omega_k}{4})}{\Gamma(\frac{3+\Omega_k}{4})} \sqrt{\frac{\Omega_k}{\pi k}} \tag{A.1}$$

and

$$C_2 = \frac{\Gamma\left(\frac{1+\Omega_k}{4}\right)}{2\sqrt{2\pi}} \sqrt{\frac{\Omega_k}{k}} = \frac{\Gamma\left(\frac{1+\Omega_k}{4}\right)}{2\sqrt{2\pi}} \left(\frac{k}{\kappa}\right)^{\frac{1}{4}} \quad (\text{A.2})$$

F_- coefficients, eq. (3.8). Similarly as above we obtain the following relations from the matching,

$$C_4 = \frac{\Gamma\left(\frac{3-i\Omega_k}{4}\right)}{2^{\frac{-1}{4}(1-i\Omega_k)}\sqrt{\pi}} \left(\frac{1}{\sqrt{2k}} - C_3 \frac{2^{\frac{-1}{4}(1+i\Omega_k)}\sqrt{\pi}}{\Gamma\left(\frac{3+i\Omega_k}{4}\right)} \right),$$

and the Wronskian normalisation:

$$C_3^2 + |C_4|^2 + 2C_3 e^{\frac{-\pi\Omega_k}{4}} \sqrt{2\pi} \text{Im} \left(\frac{\sqrt{i}C_4^*}{\Gamma\left(\frac{1+i\Omega_k}{2}\right)} \right) = \frac{e^{\frac{-\pi\Omega_k}{4}}}{k} \sqrt{\frac{\Omega_k}{2}}$$

These two equation determine the coefficients for F_- modes:

$$C_3 = \frac{1}{2\sqrt{2k}P(k)} \left(\sqrt{\Omega_k} e^{-\frac{\pi\Omega_k}{4}} - \frac{1}{\pi} \left| \Gamma\left(\frac{3-i\Omega_k}{4}\right) \right|^2 \right) \quad (\text{A.3})$$

$$C_4 = \frac{\Gamma\left(\frac{3-i\Omega_k}{4}\right)}{2^{\frac{-1}{4}(1-i\Omega_k)}\sqrt{2\pi k}} \left(1 - \frac{\sqrt{\pi}}{2^{\frac{1}{4}(5+i\Omega_k)}P(k)\Gamma\left(\frac{3+i\Omega_k}{4}\right)} \left(\sqrt{\Omega_k} e^{-\frac{\pi\Omega_k}{4}} - \frac{1}{\pi} \left| \Gamma\left(\frac{3-i\Omega_k}{4}\right) \right|^2 \right) \right), \quad (\text{A.4})$$

where

$$P(k) = \frac{2^{3/4}}{\sqrt{\pi}} \left(2\pi e^{-\frac{\pi\Omega_k}{4}} \text{Im} \left[\frac{\sqrt{i}}{2^{\frac{i\Omega_k}{4}}\Gamma\left(\frac{1+i\Omega_k}{4}\right)} \right] - \text{Re} \left[\frac{\Gamma\left(\frac{3-i\Omega_k}{4}\right)}{2^{\frac{i\Omega_k}{4}}} \right] \right)$$

B Useful properties of parabolic cylinder functions

Here we collect useful formulas and properties of special functions [19] used in the main text. The parabolic cylinder function is denoted $D_\nu(z)$. It is related to the confluent hypergeometric cylinder U and Whittaker W functions by the following,

$$\begin{aligned} D_\nu(z) &= 2^{\nu/2+1/4} z^{-1/2} W_{\nu/2+1/4, -1/4} \left(\frac{1}{2} z^2 \right) \\ &= \frac{2^{\nu/2} (-iz)^{1/4} (iz)^{1/4}}{\sqrt{z}} U \left(-\frac{1}{2}\nu, \frac{1}{2}, \frac{1}{2} z^2 \right) \end{aligned}$$

The following relation has been utilised: $D_\nu(z) = U\left(-\frac{1}{2} - \nu, z\right)$

The Wronskian identities for the parabolic cylinder function used are:

$$\begin{aligned} \mathcal{W}[U(a, z), U(a, -z)] &= \frac{\sqrt{2\pi}}{\Gamma\left(\frac{1}{2} + a\right)} \\ \mathcal{W}[U(a, z), U(-a, \pm iz)] &= \mp i e^{\pm i\pi\left(\frac{a}{2} + \frac{1}{4}\right)} \end{aligned}$$

The derivative of the parabolic cylinder function, in the $U(a, z)$ formalism, with respect to a variable τ is:

$$\frac{dU(a, z(\tau))}{d\tau} = -\frac{dz}{d\tau} \left[\left(a + \frac{1}{2} \right) U(a+1, z) + \frac{z}{2} U(a, z) \right]$$

When the argument z is set to zero, the above equation reads:

$$\frac{dU(a, 0)}{d\tau} = \frac{dz}{d\tau} \frac{\sqrt{\pi}}{2^{\frac{1}{2}(a-\frac{1}{2})}\Gamma(\frac{1}{2}(\frac{1}{2} + a))}$$

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