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Integrability of D1-brane on group manifold

Josef Klusoň

Department of Theoretical Physics and Astrophysics, Faculty of Science, Masaryk University, Kotlářská 2, 611 37, Brno, Czech Republic

E-mail: klu@physics.muni.cz

Abstract: This paper is devoted to the analysis of the integrability of D1-brane on group manifold. We consider D1-brane as principal chiral model, determine corresponding equations of motions and find Lax connection. Then we calculate the Poisson brackets of Lax connection and we find that it has similar structure as in case of principal chiral model. As the second example we consider more general background with non-zero NS-NS two form. We again show that D1-brane theory is integrable on this background and determine Poisson brackets of Lax connection.

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Contents

1 Introduction and summary

One of the greatest achievements in string theory is the discovery of the integrability of the classical string sigma model on $AdS_5 \times S^5$ background [1].[1] The integrability of the string is based on the existence of the Lax connection that leads to the existence of the infinite tower of conserved charges. However as was stressed in [6] the integrability means that we have infinite tower of conserved charges that are also in involution which means that they Poisson commute with each other. On the other hand even if the theory possesses Lax formulation, there is a well known problem in determining the Poisson brackets of the conserved charges due to the existence of non-ultra local terms in these Poisson brackets. One possibility how to resolve this problem is in the prescription of regularizing of these problematic brackets that was known from the work by Maillet [7, 8], for alternative possibility, see recent work [9, 10].

As is well known string theories contain another extended objects, as for example Dp-branes, NS5-branes, etc. It is also well known that Type IIB superstring theory is invariant under S-duality that maps theory at week coupling to the theory at strong coupling and where for example fundamental string is mapped to D1-brane.Then we can ask the question whether integrability is also preserved under S-duality. Of course, this is very difficult problem in the full generality due to the fact that it is not clear whether classical description can be applied when the coupling constant is strong. On the other hand we would like to see whether D1-brane, that propagates on some group manifold, possesses integrable structure as fundamental string does. The goal of this paper is to answer this question. We study D1-brane on the background where string propagating on given background is defined by principal chiral model that is well known to be integrable. On the other hand the dynamics of D1-brane is governed by Dirac-Born-Infeld action with presence of the gauge field so that the integrability of given theory has to be checked. We explicitly construct corresponding Lax pair in case of D1-brane on the group manifold and show that obeys the flatness conditions on condition when all fields obey the equations of motion. We further

[1] For recent review of integrability in the context of string theory, see [2–5].

JHEP09(2014)159

proceed to the Hamiltonian formalism and calculate the Poisson bracket between spatial components of Lax connection and we find that it has the same form as in case of principal chiral model with exceptions that coefficients depend on the momentum conjugate to the spatial component of the gauge field and we also find that the resulting Poisson bracket contains the constraint that corresponds to the gauge invariance of given theory.

As the next step we proceed to the analysis of the integrability of D1-brane on the manifold that could be described as Wess-Zummino-Witten model which means that there is non-trivial b_{NS} field. We determine corresponding Hamiltonian for D1-brane and determine its constraint structure. Then we construct Lax connection that obeys the flatness condition when all fields are on shell. Note that the parameters in the Lax connection, that were determined from the requirement of its flatness, depends on the constant (on-shell) value of the momenta conjugate to the spatial component of the gauge field. When we proceed to the Hamiltonian formalism we perform the of-shell extension of this expression. Then we calculate the Poisson bracket between spatial components of the Lax connections and we find that it takes the same form as in case of the WZW model with the exception that now the parameters depend on the momenta conjugate to the spatial component of the gauge field and also it is proportional to the secondary constraint that forces this momentum to be spatial independent.

In summary, we find that D1-brane possesses the same integrability as the fundamental string on given background. The fact that D1-brane is integrable could be considered as the first step in the analysis of the more general configurations of D1-branes on group manifold. More explicitly, it would be very interesting to discuss non-abelian DBI action for collection of N D1-branes on given background [11]. We can expect that when we have collection of N D1-branes that are far away that the resulting theory should be integrable since it effectively reduces to the collections of N abelian D1-brane actions. However it would be nice to see what happens when we move these branes closer. In this case we can expect that the integrability is lost. We hope to return to this problem in future.

The organization of this paper is as follows. In the next section 2 we consider D1-brane as principal chiral model and determine corresponding Lax connection. In section 2.1 we calculate the Poisson brackets between spatial components of Lax connection. In section 3 we consider D1-brane as WZW model. Finally in section 4 we determine Hamiltonian formalism for given theory and calculate the Poisson bracket of spatial components of Lax connection.

2 D1-brane on group manifold

Our goal is to show that D1-brane on the group manifold defines an integrable theory. Let us consider following D1-brane action

$$S = -T_{D1} \int d\sigma d\tau \sqrt{-\det \mathbf{A}} \,, \qquad (2.1)$$

where

$$\mathbf{A}_{\alpha\beta} = g_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta}\,,$$
$$g_{\alpha\beta} = g_{MN}\partial_\alpha x^M \partial_\beta x^N\,, \quad F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha\,, \quad x^\alpha \equiv (\tau,\sigma)\,, \tag{2.2}$$

and where D1-brane tension is equal to

$$T_{D1} = \frac{1}{g_s(2\pi\alpha')}\,, \tag{2.3}$$

where g_s is constant string coupling constant. When we consider group manifold G we presume that g_{MN} has the form

$$g_{MN} = E_M^{\ A} E_N^{\ B} K_{AB}\,, \tag{2.4}$$

where for the group element $g \in G$ we have

$$g^{-1}dg = E_M^{\ A} T_A dx^M\,, \tag{2.5}$$

where T_A is basis of Lie Algebra $\mathcal{G}$ of the group G. Finally x^M are coordinates on the group manifold G. Now from this definition it is clear that we can write $g_{\alpha\beta}$ as

$$g_{\alpha\beta} = J_\alpha^A J_\beta^B K_{AB}\,, \tag{2.6}$$

where

$$J_\alpha = g^{-1}\partial_\alpha g = J_\alpha^A T_A\,, J_\alpha^A = E_M^{\ A}\partial_\alpha x^M\,. \tag{2.7}$$

It is useful to rewrite the D1-brane action into the form

$$S = -T_{D1}\int d^2\sigma\sqrt{-\det g - (2\pi\alpha')^2(F_{\tau\sigma})^2}\,, \tag{2.8}$$

where $\det g = g_{\tau\tau}g_{\sigma\sigma} - g_{\tau\sigma}g_{\tau\sigma}$. Now the equation of motion for A_τ, A_σ that follow from (2.8) have the form

$$\partial_\tau\left[\frac{2\pi\alpha' F_{\tau\sigma}}{\sqrt{-\det g - (2\pi\alpha')^2(F_{\tau\sigma})^2}}\right] = 0\,,$$
$$\partial_\sigma\left[\frac{2\pi\alpha' F_{\tau\sigma}}{\sqrt{-\det g - (2\pi\alpha')^2(F_{\tau\sigma})^2}}\right] = 0 \tag{2.9}$$

and consequently we obtain

$$\frac{2\pi\alpha' F_{\tau\sigma}}{\sqrt{-\det g - (2\pi\alpha')^2(F_{\tau\sigma})^2}} = \Pi\,, \tag{2.10}$$

where Π is a constant. In order to derive the equation of motion for J_α^A let us consider the variation of g as $\delta g = g\delta X$, $\delta X = \delta X^A T_A$. Then we obtain

$$\delta J_\alpha^A = J_\alpha^B f_{BC}{}^A \delta X^C + \partial_\alpha \delta X^A\,, \tag{2.11}$$

where ${f_{BC}}^A$ are structure coefficients of Lie algebra $\mathcal{G}$ defined by $[T_A, T_B] = {f_{AB}}^C T_C$. Then we find that the equations of motion for the current J^A take the form

$$\partial_\alpha \left[J^A_\beta g^{\beta\alpha} \frac{\det g}{\sqrt{-\det g - (2\pi\alpha')^2 F^2_{\tau\sigma}}} \right] = 0 \,. \tag{2.12}$$

Deriving both equations of motion for J^A_α and A_α we can now proceed to the construction of the flat current for D1-brane on the group manifold. Before we do it note that when we consider equations of motion we demand that all fields are on-shell. This fact however implies that we should replace $F_{\tau\sigma}$ in (2.12) with the value that follows from (2.10) so that (2.12) simplifies considerably

$$\partial_\beta [J^A_\alpha g^{\alpha\beta} \sqrt{-\det g}] = 0 \,. \tag{2.13}$$

This result suggests that it is natural to consider the flat current in the same form as in case of the fundamental string moving on the same group manifold. Explicitly, we consider flat current in the form

$$L_\alpha = \frac{1}{1-\Lambda^2} [J^A_\alpha - \Lambda \epsilon_{\alpha\beta} g^{\beta\omega} J^A_\omega \sqrt{-\det g}] \tag{2.14}$$

or in components (using $\epsilon_{\alpha\beta} = \epsilon_{\tau\sigma} = -\epsilon_{\sigma\tau} = 1$)

$$\begin{aligned} L_\sigma &= \frac{1}{1-\Lambda^2} [J^A_\sigma + \Lambda g^{\tau\alpha} J^A_\alpha \sqrt{-\det g}] \,, \\ L_\tau &= \frac{1}{1-\Lambda^2} [J^A_\tau - \Lambda g^{\sigma\alpha} J^A_\alpha \sqrt{-\det g}] \,, \end{aligned} \tag{2.15}$$

where Λ is spectral parameter. Then we calculate

$$\partial_\tau L^A_\sigma - \partial_\sigma L^A_\tau = -\frac{1}{1-\Lambda^2} J^B_\tau J^C_\sigma {f_{BC}}^A \tag{2.16}$$

using equations of motion for J^A_α and A_α. On the other hand we have

$$L^B_\tau L^C_\sigma {f_{BC}}^A = \frac{1}{1-\Lambda^2} J^B_\tau J^C_\sigma {f_{BC}}^A \,. \tag{2.17}$$

Collecting there results together we find

$$\partial_\tau L^A_\sigma - \partial_\sigma L^A_\tau + L^C_\tau L^D_\sigma {f_{CD}}^A = 0 \,. \tag{2.18}$$

In other words we have shown that the Lax connection is flat which is the necessary condition for the theory to be integrable. As the next step in the proof of the integrability we determine the Poisson brackets between spatial components of Lax connection and show that it has the right form for the existence of infinite number of conserved charges that are in involution.

2.1 Hamiltonian analysis and Poisson brackets of Lax connection

In this section we develop the Hamiltonian formalism for the action (2.8) and calculate the Poisson bracket between spatial components of Lax connection. First of all we determine momenta conjugate to x^M, A_α from (2.8)

$$
\begin{aligned}
p_M &= T_{D1}\frac{g_{MN}\partial_\alpha x^N g^{\alpha\tau}\det g}{\sqrt{-\det g-(2\pi\alpha')^2(F_{\tau\sigma})^2}}\,,\\
\pi^\sigma &= -T_{D1}\frac{(2\pi\alpha')^2 F_{\tau\sigma}}{\sqrt{-\det g-(2\pi\alpha')^2(F_{\tau\sigma})^2}}\,,\qquad \pi^\tau\approx 0\,.
\end{aligned}\tag{2.19}
$$

Using these results we find that the theory possesses two primary constraints

$$
\begin{aligned}
\mathcal{H}_\tau &= p_M g^{MN}p_N+\frac{1}{(2\pi\alpha')^2}\pi^\sigma g_{\sigma\sigma}\pi^\sigma+T_{D1}^2\partial_\sigma x^M g_{MN}\partial_\sigma x^N\approx 0\,,\\
\mathcal{H}_\sigma &= p_M\partial_\sigma x^M\approx 0\,.
\end{aligned}\tag{2.20}
$$

On the other hand the bare Hamiltonian has the form

$$
H_E=\int d\sigma(p_M\partial_\tau x^M+\pi^\sigma\partial_\tau A_\sigma-L_{D1})=-\int d\sigma\pi^\sigma\partial_\sigma A_\tau \tag{2.21}
$$

and hence the Hamiltonian including all constraints has the form

$$
H=\int d\sigma(\lambda_\tau\mathcal{H}_\tau+\lambda_\sigma\mathcal{H}_\sigma+A_\tau\partial_\sigma\pi^\sigma+v_\tau\pi^\tau)\,,\tag{2.22}
$$

where of course the requirement of the preservation of the constraint $\pi^\tau\approx 0$ gives the secondary constraint

$$
\mathcal{G}=\partial_\sigma\pi^\sigma\approx 0\,.\tag{2.23}
$$

We are not going to determine the algebra of constraints which will be performed in the section devoted to the analysis of D1-brane on the group manifold with non-zero NS-NS two form. We rather express the spatial component of the flat current L_σ as function of the canonical variables

$$
L_\sigma^A(\Lambda)=\frac{1}{1-\Lambda^2}\left[E_M^A\partial_\sigma x^M-\Lambda\frac{K^{AB}E_B^M p_M}{\sqrt{T_{D1}^2+\left(\frac{\pi^\sigma}{2\pi\alpha'}\right)^2}}\right]\,.\tag{2.24}
$$

Now using the canonical Poisson brackets

$$
\left\{x^M(\sigma),p_N(\sigma')\right\}=\delta_N^M\delta(\sigma-\sigma')\,,\quad \left\{A_\sigma(\sigma),\pi^\sigma(\sigma')\right\}=\delta(\sigma-\sigma')\tag{2.25}
$$

we obtain

$$
\begin{aligned}
\left\{L_\sigma^A(\Lambda),L_\sigma^B(\Gamma)\right\} &= \frac{\Lambda+\Gamma}{(1-\Lambda^2)(1-\Gamma^2)}K^{AB}\frac{1}{\sqrt{T_{D1}^2+\left(\frac{\pi^\sigma}{2\pi\alpha'}\right)^2}}\partial_\sigma\delta(\sigma-\sigma')-\\
&\quad-\frac{\Gamma}{(1-\Lambda^2)(1-\Gamma^2)}\frac{K^{AB}}{(T_{D1}^2+\left(\frac{\pi^\sigma}{2\pi\alpha'}\right)^2)^{3/2}}\frac{\pi^\sigma}{(2\pi\alpha')^2}\mathcal{G}\delta(\sigma-\sigma')+
\end{aligned}
$$

JHEP09(2014)159

$$+\frac{1}{(1-\Lambda^2)(1-\Gamma^2)}\frac{(\Lambda+\Gamma)}{\sqrt{T_{D1}^2+\left(\frac{\pi^\sigma}{2\pi\alpha'}\right)^2}}E_M^D f_{CD}{}^A K^{BC}\partial_\sigma x^N\delta(\sigma-\sigma')+$$
$$-\frac{\Lambda\Gamma}{(1-\Lambda^2)(1-\Gamma^2)}\frac{1}{T_{D1}^2+\left(\frac{\pi^\sigma}{2\pi\alpha'}\right)^2}K^{BD}f_{DC}{}^A K^{CF}E_F^P p_P\delta(\sigma-\sigma') \quad (2.26)$$

using

$$\partial_N E_M^A-\partial_M E_N^A+E_N^B E_M^C f_{BC}{}^A=0\,,$$
$$f(\sigma')\partial_\sigma\delta(\sigma-\sigma')=f(\sigma)\partial_\sigma\delta(\sigma-\sigma')+\partial_\sigma f(\sigma)\delta(\sigma-\sigma')\,. \quad (2.27)$$

Now we demand that the expression proportional to the delta function is equal to

$$-\left(AL_\sigma^C(\Lambda)-BL_\sigma^C(\Gamma)\right)f_{CD}{}^A K^{DB}\delta(\sigma-\sigma')\,. \quad (2.28)$$

Comparing (2.28) with (2.26) we determine A and B as

$$A=\frac{\Gamma^2}{(1-\Gamma^2)(\Gamma-\Lambda)\sqrt{T_{D1}^2+\left(\frac{\pi^\sigma}{2\pi\alpha'}\right)^2}}\,,$$
$$B=\frac{\Lambda^2}{(1-\Lambda^2)(\Gamma-\Lambda)\sqrt{T_{D1}^2+\left(\frac{\pi^\sigma}{2\pi\alpha'}\right)^2}}\,. \quad (2.29)$$

In other words we find the final result

$$\left\{L_\sigma^A(\Lambda),L_\sigma^B(\Gamma)\right\}=\frac{\Lambda+\Gamma}{(1-\Lambda^2)(1-\Gamma^2)}K^{AB}\frac{1}{\sqrt{T_{D1}^2+\left(\frac{\pi^\sigma}{2\pi\alpha'}\right)^2}}\partial_\sigma\delta(\sigma-\sigma')-$$
$$-\frac{\Gamma^2}{(1-\Gamma^2)(\Gamma-\Lambda)\sqrt{T_{D1}^2+\left(\frac{\pi^\sigma}{2\pi\alpha'}\right)^2}}L_\sigma^C(\Lambda)f_{CD}{}^A K^{DB}\delta(\sigma-\sigma')-$$
$$-\frac{\Lambda^2}{(1-\Lambda^2)(\Gamma-\Lambda)\sqrt{T_{D1}^2+\left(\frac{\pi^\sigma}{2\pi\alpha'}\right)^2}}L_\sigma^C(\Gamma)f_{CD}{}^A K^{DB}\delta(\sigma-\sigma')-$$
$$-\frac{\Gamma}{(1-\Lambda^2)(1-\Gamma^2)}\frac{K^{AB}}{(T_{D1}^2+\left(\frac{\pi^\sigma}{2\pi\alpha'}\right)^2)^{3/2}}\frac{\pi^\sigma}{(2\pi\alpha')^2}\mathcal{G}\delta(\sigma-\sigma')\,. \quad (2.30)$$

We see that this Poisson bracket has similar form as in case of the principal chiral model even if the coefficients in front of L_σ on the right side of the Poisson bracket depend on the canonical variable π^σ and the right side of the Poisson bracket contains the secondary constraint $\mathcal{G}\approx 0$. On the other hand the presence of this constraint implies that π^σ does not depend on σ on the constraint surface $\mathcal{G}\approx 0$. Further, the equation of motion for π^σ implies that it does not depend on τ as well. In other words π^σ is constant on shell that physically counts the number of fundamental strings.

3 D1-brane on the background with non-trivial NS-NS field

In this section we consider more general possibility when D1-brane is embedded on the group manifold with non-trivial b_{NS} field. In other words we consider the background that

JHEP09(2014)159

corresponding to the fundamental string as WZW model which is integrable. The presence of this field modifies the action in the following way

$$S = -T_{D1}\int d^2\sigma\sqrt{-\det g - ((2\pi\alpha')F_{\tau\sigma}+b_{\tau\sigma})^2}\,, \tag{3.1}$$

where

$$b_{\alpha\beta} \equiv b_{MN}\partial_\alpha x^M\partial_\beta x^N = -b_{\beta\alpha} \tag{3.2}$$

since $b_{MN} = -b_{NM}$. Now the equation of motion for x^M has the form

$$\begin{aligned}
&-\partial_\alpha\left[\frac{g_{MN}\partial_\beta x^N g^{\beta\alpha}\det g}{\sqrt{-\det g-((2\pi\alpha')F_{\tau\sigma}+b_{\tau\sigma})^2}}\right]+\frac{\partial_M g_{KL}\partial_\alpha x^K\partial_\beta x^L g^{\beta\alpha}\det g}{2\sqrt{-\det g-((2\pi\alpha')F_{\tau\sigma}+b_{\tau\sigma})^2}}+\\
&+\frac{((2\pi\alpha')F_{\tau\sigma}+b_{\tau\sigma})}{\sqrt{-\det g-((2\pi\alpha')F_{\tau\sigma}+b_{\tau\sigma})^2}}\partial_M b_{KL}\partial_\tau x^K\partial_\sigma x^L-\\
&-\partial_\tau\left[\frac{b_{MN}\partial_\sigma x^N((2\pi\alpha')F_{\tau\sigma}+b_{\tau\sigma})}{\sqrt{-\det g-((2\pi\alpha')F_{\tau\sigma}+b_{\tau\sigma})^2}}\right]+\partial_\sigma\left[\frac{b_{MN}\partial_\tau x^N((2\pi\alpha')F_{\tau\sigma}+b_{\tau\sigma})}{\sqrt{-\det g-((2\pi\alpha')F_{\tau\sigma}+b_{\tau\sigma})^2}}\right]=0
\end{aligned} \tag{3.3}$$

while the equations of motion for A_τ, A_σ take the form

$$\begin{aligned}
&\partial_\tau\left[\frac{(2\pi\alpha')F_{\tau\sigma}+b_{\tau\sigma}}{\sqrt{-\det g-((2\pi\alpha')F_{\tau\sigma}+b_{\tau\sigma})^2}}\right]=0\,,\\
&\partial_\sigma\left[\frac{(2\pi\alpha')F_{\tau\sigma}+b_{\tau\sigma}}{\sqrt{-\det g-((2\pi\alpha')F_{\tau\sigma}+b_{\tau\sigma})^2}}\right]=0\,.
\end{aligned} \tag{3.4}$$

These equations imply

$$\frac{(2\pi\alpha')F_{\tau\sigma}+b_{\tau\sigma}}{\sqrt{-\det g-((2\pi\alpha')F_{\tau\sigma}+b_{\tau\sigma})^2}}=\Pi\,,\quad \Pi=\mathrm{const}\,. \tag{3.5}$$

Now with the help of the solution of the equation of motion for A_α given in (3.5) we obtain that the equation of motion for x^M takes the form

$$\begin{aligned}
\sqrt{1-\Pi^2}\partial_\alpha[g_{MN}\partial_\beta x^N g^{\alpha\beta}\sqrt{-\det g}]-\frac{1}{2}\sqrt{1-\Pi^2}\partial_M g_{KL}\partial_\alpha x^K\partial_\beta x^L g^{\beta\alpha}\sqrt{-\det g}+\\
+\Pi H_{MKL}\partial_\tau x^K\partial_\sigma x^L=0\,,
\end{aligned} \tag{3.6}$$

where

$$H_{MNK}=\partial_M b_{NK}+\partial_N b_{KM}+\partial_K b_{MN}\,. \tag{3.7}$$

To proceed further note that in case of the WZW model H_{MNK} obeys following relation

$$H_{MNK}E^M_A E^N_B E^K_C=\kappa f_{ABC}\,, \tag{3.8}$$

where κ is a constant. Then we can write

$$H_{MKL}\partial_\tau x^K\partial_\sigma x^L=\kappa E_M^A f_{ABC}J^B_\tau J^C_\sigma\,. \tag{3.9}$$

JHEP09(2014)159

Using this result we finally find that the equation of motion for x^M has the form

$$\sqrt{1+\Pi^2}K_{AB}\partial_\alpha[J^B_\beta g^{\beta\alpha}\sqrt{-\det g}]+\Pi\kappa f_{ABC}J^B_\tau J^C_\sigma=0\,. \tag{3.10}$$

Let us consider Lax connection in the form

$$\begin{aligned} L^A_\tau &= AJ^A_\tau+B\sqrt{-\det g}g^{\sigma\alpha}J^A_\alpha\,, \\ L^A_\sigma &= AJ^A_\sigma-B\sqrt{-\det g}g^{\tau\alpha}J^A_\alpha\,, \end{aligned} \tag{3.11}$$

where parameters A,B will be determined by the requirement that the Lax connection should be flat. Explicitly, we have

$$\partial_\tau L^A_\sigma-\partial_\sigma L^A_\tau=-AJ^B_\tau J^C_\sigma f_{BC}{}^A+B\frac{\Pi}{\sqrt{1+\Pi^2}}\kappa f_{BC}{}^A J^B_\tau J^C_\sigma \tag{3.12}$$

using the equation of motion for x^M and also $\partial_\tau J^A_\sigma-\partial_\sigma J^A_\tau+J^C_\tau J^D_\sigma f_{CD}{}^A=0$. At the same time we have

$$f_{BC}{}^A L^B_\tau L^C_\sigma=(A^2-B^2)f_{BC}{}^A J^B_\tau J^C_\sigma\,. \tag{3.13}$$

Then the requirement that the Lax current should be flat leads to the equation

$$A^2-B^2-A+\frac{\kappa\Pi B}{\sqrt{1+\Pi^2}}=0\,. \tag{3.14}$$

If we presume the solution in the form $B=-\Lambda A$ we find

$$A=\frac{1}{1-\Lambda^2}\left(1+\Lambda\kappa\frac{\Pi}{\sqrt{1+\Pi^2}}\right)\,,\quad B=-\frac{\Lambda}{1-\Lambda^2}\left(1+\Lambda\kappa\frac{\Pi}{\sqrt{1+\Pi^2}}\right)\,. \tag{3.15}$$

It is important to stress that the values of A and B were determined on condition when all fields obey the equations of motion which implies that

$$\frac{\Pi}{\sqrt{1+\Pi^2}}=\frac{(2\pi\alpha')F_{\tau\sigma}+b_{\tau\sigma}}{\sqrt{-\det g}}\,. \tag{3.16}$$

Then it is natural to propose off-shell form of the flat current when we use (3.16) in (3.11) so that

$$\begin{aligned} L^A_\tau &= \frac{1}{1-\Lambda^2}\left(1+\Lambda\kappa\frac{(2\pi\alpha')F_{\tau\sigma}+b_{\tau\sigma}}{\sqrt{-\det g}}\right)(J^A_\tau-\Lambda\sqrt{-\det g}g^{\sigma\alpha}J^A_\alpha)\,, \\ L^A_\sigma &= \frac{1}{1-\Lambda^2}\left(1+\Lambda\kappa\frac{(2\pi\alpha')F_{\tau\sigma}+b_{\tau\sigma}}{\sqrt{-\det g}}\right)(J^A_\tau+\Lambda\sqrt{-\det g}g^{\tau\alpha}J^A_\alpha)\,. \end{aligned} \tag{3.17}$$

Clearly this current is flat which is necessary condition for the integrability of D1-brane theory on the group manifold with non-zero b_{NS} field. However we have to also show that the Poisson bracket between spatial components of the Lax connection (3.17) has the right form in order to ensure an infinite number of conserved charges in involution.

4 Hamiltonian formalism

In this section we calculate the algebra of Poisson brackets of Lax connection for the model defined in the previous section. To do this we have to develop corresponding Hamiltonian formalism. From the action (3.1) we find

$$
\begin{aligned}
p_M &= T_{D1}\frac{1}{\sqrt{-\det g-((2\pi\alpha')F_{\tau\sigma}+b_{\tau\sigma})^2}}(g_{MN}\partial_\alpha x^N g^{\alpha\tau}\det g+((2\pi\alpha')F_{\tau\sigma}+b_{\tau\sigma})b_{MN}\partial_\sigma x^N)\,,\\
\pi^\sigma &= \frac{T_{D1}(2\pi\alpha')((2\pi\alpha')F_{\tau\sigma}+b_{\tau\sigma})}{\sqrt{-\det g-((2\pi\alpha')F_{\tau\sigma}+b_{\tau\sigma})^2}}\,, \quad \pi^\tau\approx 0\,.
\end{aligned} \tag{4.1}
$$

Using these definitions we find that the bare Hamiltonian is equal to

$$
H_E=\int d\sigma(p_M\partial_\tau x^M+\pi^\sigma\partial_\tau A_\sigma-\mathcal{L})=-\int d^2\sigma\pi^\sigma\partial_\sigma A_\tau \tag{4.2}
$$

while we have two primary constraints

$$
\begin{aligned}
\mathcal{H}_\tau &\equiv \left(p_M-\frac{\pi^\sigma}{(2\pi\alpha')}b_{MK}\partial_\sigma x^K\right)g^{MN}\left(p_K-\frac{\pi^\sigma}{(2\pi\alpha')}b_{NL}\partial_\sigma x^L\right)+\\
&\quad +T_{D1}^2 g_{MN}\partial_\sigma x^M\partial_\sigma x^N+\frac{(\pi^\sigma)^2}{(2\pi\alpha')^2}g_{MN}\partial_\sigma x^M\partial_\sigma x^N\approx 0\,,\\
\mathcal{H}_\sigma &\equiv p_M\partial_\sigma x^M\approx 0\,.
\end{aligned} \tag{4.3}
$$

Then the extended Hamiltonian has the form

$$
H=\int d\sigma(\lambda_\tau\mathcal{H}_T+\lambda_\sigma\mathcal{H}_\sigma+A_\tau\partial_\sigma\pi^\sigma+v_\tau\pi^\tau)\,, \tag{4.4}
$$

where $\lambda_\tau,\lambda_\sigma,v_\tau$ are Lagrange multipliers corresponding to the primary constraints. Further, the requirement of the preservation of the primary constraint $\pi^\tau\approx 0$ implies the secondary constraint

$$
\mathcal{G}=\partial_\sigma\pi^\sigma\approx 0\,. \tag{4.5}
$$

Now we proceed to the analysis of the stability of the primary constraints $\mathcal{H}_\tau,\mathcal{H}_\sigma$. In fact, after some algebra we find

$$
\begin{aligned}
\{\mathcal{H}_\tau(\sigma),\mathcal{H}_\tau(\sigma')\} &= 8\left(T_{D1}^2+\left(\frac{\pi^\sigma}{2\pi\alpha'}\right)^2\right)\mathcal{H}_\sigma\partial_\sigma\delta(\sigma-\sigma')+\\
&\quad +4\left(T_{D1}^2+\left(\frac{\pi^\sigma}{2\pi\alpha'}\right)^2\right)\partial_\sigma\mathcal{H}_\sigma\delta(\sigma-\sigma')+\frac{8}{(2\pi\alpha')^2}\pi^\sigma\mathcal{G}\mathcal{H}_\sigma\delta(\sigma-\sigma')\,.
\end{aligned} \tag{4.6}
$$

We see that this Poisson brackets vanishes on the constraint surface $\mathcal{H}_\sigma\approx 0\,,\mathcal{G}\approx 0$. In the same way we find

$$
\{\mathcal{H}_\sigma(\sigma),\mathcal{H}_\tau(\sigma')\}=\mathcal{H}_\tau\partial_\sigma\delta(\sigma-\sigma')+\partial_\sigma\mathcal{H}_\tau\delta(\sigma-\sigma')+\frac{1}{(\pi\alpha')^2}\pi^\sigma\partial_\sigma x^M g_{MN}\partial_\sigma x^N\mathcal{G}\delta(\sigma-\sigma')\,, \tag{4.7}
$$

JHEP09(2014)159

where again right side of this Poisson bracket vanishes on the constraint surface. Finally we obtain

$$\left\{\mathcal{H}_\sigma(\sigma), \mathcal{H}_\sigma(\sigma')\right\} = 2\mathcal{H}_\sigma(\sigma)\partial_\sigma\delta(\sigma-\sigma') + \partial_\sigma\mathcal{H}_\sigma(\sigma)\delta(\sigma-\sigma') \tag{4.8}$$

and we again see that this Poisson bracket vanishes on the constraint surface. Collecting all these results we find that the constraints $\mathcal{H}_\tau \approx 0\,, \mathcal{H}_\sigma \approx 0$ are preserved during the time evolution of the system and no further constraints are generated. Now we are ready to proceed to the calculation of the Poisson bracket between spatial component of the Lax connection that expressed using canonical variables has the form

$$\begin{aligned} L^A_\sigma &= \frac{1}{1-\Lambda^2}\left(1+\Lambda\kappa\frac{\frac{\pi^\sigma}{2\pi\alpha'}}{\sqrt{T^2_{D1}+\left(\frac{\pi^\sigma}{2\pi\alpha'}\right)^2}}\right)\Bigg(J^A_\sigma - \\ &\quad -\Lambda K^{AB}E^M_B\frac{1}{\sqrt{T^2_{D1}+\left(\frac{\pi^\sigma}{2\pi\alpha'}\right)^2}}\left(p_M-\frac{\pi^\sigma}{2\pi\alpha'}b_{MN}\partial_\sigma x^N\right)\Bigg)\,. \end{aligned} \tag{4.9}$$

Then after some calculations we obtain

$$\begin{aligned} \left\{L^A_\sigma(\Lambda,\sigma), L^B_\sigma(\Gamma,\sigma')\right\} &= f(\Lambda)f(\Gamma)f_{FE}{}^AK^{EB}K^{FC}E^M_C\left(p_M-\frac{\pi^\sigma}{2\pi\alpha'}b_{MN}\partial_\sigma x^N\right)\delta(\sigma-\sigma') + \\ &\quad +f(\Lambda)f(\Gamma)\frac{\pi^\sigma}{2\pi\alpha'}\kappa f_{EF}{}^AK^{FB}J^E_\sigma\delta(\sigma-\sigma') - \\ &\quad -(h(\Lambda)f(\Gamma)+f(\Lambda)h(\Gamma))f_{EC}{}^AK^{CB}J^E_\sigma\delta(\sigma-\sigma') - \\ &\quad -(h(\Lambda)f(\Gamma)+f(\Lambda)h(\Gamma))K^{AB}\partial_\sigma\delta(\sigma-\sigma') - \\ &\quad -\left[h(\Lambda)\frac{d}{d\pi^\sigma}f(\Gamma)+f(\Lambda)\frac{d}{d\pi^\sigma}h(\Gamma)\right]K^{AB}\mathcal{G}\delta(\sigma-\sigma')\,, \end{aligned} \tag{4.10}$$

where

$$\begin{aligned} f(\Lambda) &= \frac{1}{1-\Lambda^2}\left(1+\Lambda\kappa\frac{\frac{\pi^\sigma}{2\pi\alpha'}}{\sqrt{T^2_{D1}+\left(\frac{\pi^\sigma}{2\pi\alpha'}\right)^2}}\right)\frac{\Lambda}{\sqrt{T^2_{D1}+\left(\frac{\pi^\sigma}{2\pi\alpha'}\right)^2}}\,, \\ h(\Gamma) &= \frac{1}{1-\Lambda^2}\left(1+\Lambda\kappa\frac{\frac{\pi^\sigma}{2\pi\alpha'}}{\sqrt{T^2_{D1}+\left(\frac{\pi^\sigma}{2\pi\alpha'}\right)^2}}\right)\,, \end{aligned} \tag{4.11}$$

and where we used the fact that

$$\partial_\sigma f \equiv \frac{df}{d\pi^\sigma}\mathcal{G}\,. \tag{4.12}$$

We again demand that the expression proportional to the delta function on the right side is equal to

$$-\left(AL^C_\sigma(\Lambda)-BL^C_\sigma(\Gamma)\right)f_{CD}{}^AK^{DB} \tag{4.13}$$

so that we find

$$\begin{aligned} A &= \frac{h(\Gamma)}{\sqrt{T^2_{D1}+(\frac{\pi^\sigma}{2\pi\alpha'})^2}}\frac{\Gamma^2}{\Gamma-\Lambda}\left[1-\kappa\Lambda\frac{\pi^\sigma}{2\pi\alpha'\sqrt{T^2_{D1}+(\frac{\pi^\sigma}{2\pi\alpha'})^2}}\right]\,, \\ B &= \frac{h(\Lambda)}{\sqrt{T^2_{D1}+(\frac{\pi^\sigma}{2\pi\alpha'})^2}}\frac{\Lambda^2}{\Gamma-\Lambda}\left[1-\kappa\Gamma\frac{\pi^\sigma}{2\pi\alpha'\sqrt{T^2_{D1}+(\frac{\pi^\sigma}{2\pi\alpha'})^2}}\right]\,. \end{aligned} \tag{4.14}$$

JHEP09(2014)159

In summary we obtain following form of the Poisson bracket between spatial component of the Lax connection

$$
\begin{aligned}
&\left\{L^A_\sigma(\Lambda,\sigma), L^B_\sigma(\Gamma,\sigma')\right\} = \\
&= -\frac{h(\Gamma)}{\sqrt{T^2_{D1}+(\frac{\pi^\sigma}{2\pi\alpha'})^2}}\frac{\Gamma^2}{\Gamma-\Lambda}\left[1-\kappa\Lambda\frac{\pi^\sigma}{2\pi\alpha'\sqrt{T^2_{D1}+(\frac{\pi^\sigma}{2\pi\alpha'})^2}}\right]L^C_\sigma(\Lambda)f_{CD}{}^A K^{DB}\delta(\sigma-\sigma')+ \\
&\quad +\frac{h(\Lambda)}{\sqrt{T^2_{D1}+(\frac{\pi^\sigma}{2\pi\alpha'})^2}}\frac{\Lambda^2}{\Gamma-\Lambda}\left[1-\kappa\Gamma\frac{\pi^\sigma}{2\pi\alpha'\sqrt{T^2_{D1}+(\frac{\pi^\sigma}{2\pi\alpha'})^2}}\right]L^C_\sigma(\Gamma)f_{CD}{}^A K^{DB}\delta(\sigma-\sigma')- \\
&\quad -\left(h(\Lambda)f(\Gamma)+f(\Lambda)h(\Gamma)\right)K^{AB}\partial_\sigma\delta(\sigma-\sigma')- \\
&\quad -\left(h(\Lambda)\frac{d}{d\pi^\sigma}f(\Gamma)+f(\Lambda)\frac{d}{d\pi^\sigma}h(\Gamma)\right)K^{AB}\mathcal{G}\delta(\sigma-\sigma')\,.
\end{aligned}
\tag{4.15}
$$

We see that the right side of this Poisson bracket has similar form (up to terms proportional to the primary constraints $\mathcal{G}$) as in case of WZW model. More explicitly, we find that it depends on the expression $\frac{\pi}{2\pi\alpha'}$ which is constant on-shell and determines the contribution of the fundamental strings to the resulting tension of the bound state of D1-brane and fundamental string and also to the coupling with b_{NS} two form. However the fact that the Poisson bracket between spatial components of the Lax connection takes the standard form implies that D1-brane on the group manifold with non-trivial NS-NS two form is integrable and possesses infinite number of conserved charges in involution, after appropriate regularization of the Poisson bracket (4.15).

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JHEP09(2014)159