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Kahler independence of the G_2-MSSM

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Abstract: The G_2-MSSM is a model of particle physics coupled to moduli fields with interesting phenomenology both for colliders and astrophysical experiments. In this paper we consider a more general model — whose moduli Kahler potential is a *completely arbitrary* G_2-holonomy Kahler potential and whose matter Kahler potential is also more general. We prove that the vacuum structure and spectrum of BSM particles is largely unchanged in this much more general class of theories. In particular, gaugino masses are still suppressed relative to the gravitino mass and moduli masses. We also consider the effects of higher order corrections to the matter Kahler potential and find a connection between the nature of the LSP and flavor effects.

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1 Introduction

From several theoretical points of view, the existence of moduli fields seems inevitable. For instance, supersymmetry may be the mechanism responsible for stabilizing the scale of the Standard Model. Supersymmetry requires supergravity, whose only (known) reasonable UV completion seems to be String theory; and along with string theory come extra dimensions and their moduli. In fact, since string/M theory contains no dimensionless parameters, moduli appear necessary to explain the observed values of various couplings in nature. From the bottom up, moduli appear in various theories with "dynamical couplings" as well as in Inflation — the inflaton field is usually a neutral scalar field aka a modulus. For all of these reasons and more, moduli physics and phenomena must be considered seriously.

In a series of papers, [1–4], a very detailed model of moduli physics coupled to matter has been described. The G_2-MSSM model, largely inspired by M theory compactifications on manifolds of G_2-holonomy, is a model in which strong gauge dynamics in the hidden sector generates a potential which both stabilizes all the moduli fields and simultaneously generates a hierarchically small scale — thus solving (most of) the hierarchy problem. The model has an interesting spectrum: moduli have masses in the 50-100 TeV region, scalar superpartners and higgsinos have masses in the 10's of TeV region, whilst gauginos, which are the lightest BSM particles have masses of order 100's of GeV. Direct production of gluinos and electroweak gauginos are the dominant new physics channels at the LHC. The nature of the LSP is also very interesting as it is a neutral Wino. Moreover, its production in the early universe is dominated by the decays of the moduli fields (ie non-thermal production) and can naturally account for the observed fraction of dark matter today. The moduli and gravitino problems are avoided due to the gravitino mass scale being one to two orders of magnitude larger than the TeV scale. One drawback of the model is the fine tuning between the 10's of TeV scale and M_Z and is the reason the model solves most of the hierarchy problem and not all of it.

However, the G_2-MSSM model, as defined in [3], is based on some specific assumptions about the moduli and matter Kahler potentials, albeit with the claim that these are general enough to incorporate all of the essential ingredients of more general Kahler potentials (and hence G_2-manifolds). Thus far, there has been no serious study of these assumptions and it is the main aim of this paper to undertake this. The main result that we prove here is that the mass spectrum of the theory depends very weakly on the specific form of the *moduli* Kahler potential; in fact the spectrum depends on the Kahler potential for moduli *only* through the fact that it is the Log of a homogeneous function (the volume of the extra dimensions); the precise nature of this homogeneous function is fairly irrelevant as we will see.

We also discuss the Kahler potential for *charged matter fields*. We give three consistent arguments for calculating the moduli dependence of the matter kinetic terms in 4d Einstein frame. Whilst non-trivial, these modifications do not change the results of [1–3] much. More importantly, we also consider higher order terms in the matter Kahler potential, in particular the terms which are usually considered troublesome for flavor physics in theories of gravity mediated susy breaking. Whilst we expect that such operators will be suppressed, if they enter with large coefficients they can affect the mass spectrum: 1) they can directly alter the scalar and higgsino masses, which are typically large; 2) they can indirectly (via threshold effects from the higgsinos) alter the nature of the LSP. In particular, we find that the LSP can also be a Bino in some cases. This provides a connection between flavor physics and the nature of the LSP in models of this sort.

The paper is organized as follows. The next section describes some simple properties of the moduli space metric for general G_2-manifolds which will be important for our later considerations. Section 3 is devoted to the Kahler potential for charged matter fields. Following this, we re-do the analysis of Moduli stabilization from [1, 2] in this much more

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general context. In section 5 we compute the mass spectrum and susy breaking couplings in the minimum of the potential and demonstrate that it is almost identical to that of the original G_2-MSSM. In section 6 we present a further generalization of the construction. In section 7 we renormalize the Lagrangian down to the Electroweak scale and give the spectrum there.

2 General properties of moduli space metrics on G_2 holonomy manifolds

In this section we describe some very general and simple properties of the moduli space metric of G_2-manifolds. It is these simple properties, which will allow us to draw very general conclusions.

The metric $g(X)$ on a G_2 holonomy manifold X can be expressed in terms of the associative three-form Φ as

$$g_{ij} = (\det s)^{-\frac{1}{9}}\, s_{ij} \tag{2.1}$$

with

$$s_{ij} = \frac{1}{144}\Phi_{ikl}\Phi_{jnm}\Phi_{rst}\hat{\epsilon}^{klnmrst}\,, \quad \hat{\epsilon}^{12\ldots7} = +1. \tag{2.2}$$

Expanding Φ in terms of basis harmonic three-forms $\phi_i \in H^3(X, \mathbf{Z})$ (modulo torsion) we obtain

$$\Phi = \sum_{i=1}^{N} s_i \phi_i\,, \quad N = b^3(X) = dim\left(\mathcal{M}(X)\right)\,, \tag{2.3}$$

where s_i are geometric moduli corresponding to the perturbations of the internal metric. The complexified moduli space $\mathcal{M}(X)$ of a G_2 holonomy compactification manifold X has holomorphic coordinates z_i given by

$$z_i = t_i + i s_i\,, \tag{2.4}$$

where t_i are the axions parameterizing the zero modes of the 11-dimensional supergravity three-form C_3. The classical moduli space metric (not including possible quantum corrections) can be derived from the following Kahler potential [5, 6]

$$\hat{K} = -3\ln 4\pi^{1/3} V_X\,, \tag{2.5}$$

where the dimensionless volume $V_X \equiv Vol(X)/l_M^7$ is a homogeneous function of s_i of degree $7/3$ and l_M is the 11d Planck length. The homogeneity of V_X is the key property that we will utilize in what follows. In terms of the associative three-form Φ, the volume is given by [5, 6]

$$V_X = \frac{1}{7}\int_X \Phi \wedge *\Phi\,. \tag{2.6}$$

Define the following derivatives with respect to the moduli

$$\hat{K}_i \equiv \frac{\partial \hat{K}}{\partial s_i} \quad \text{and} \quad \hat{K}_{ij} \equiv \frac{\partial^2 \hat{K}}{\partial s_i \partial s_j}\,. \tag{2.7}$$

The matrix $\hat{K}_{ij}$, the Hessian of $\hat{K}$ is related to the actual Kahler metric $\hat{G}_{i\bar{j}}$ which controls the kinetic terms as $4\hat{G}_{i\bar{j}} = \hat{K}_{i\bar{j}}$, where in the Hessian we simply replace index j with $\bar{j}$. Since V_X is a homogeneous function of degree 7/3, the first derivative of $\hat{K}$ defined above has the following property

$$\sum_{i=1}^{N} s_i \hat{K}_i = -7\,. \tag{2.8}$$

Differentiating (2.8) with respect to s_j we obtain an important property of the metric $\hat{K}_{ij}$

$$\sum_{i=1}^{N} s_i \hat{K}_{ij} = -\hat{K}_j\,, \text{ and since } \hat{K}_{ij} \text{ is symmetric } \sum_{j=1}^{N} s_j \hat{K}_{ij} = -\hat{K}_i\,. \tag{2.9}$$

Let us now introduce a set of dual coordinates $\{\tau_i\}$ defined by

$$\tau_i \equiv \frac{\partial V_X}{\partial s_i}\,. \tag{2.10}$$

Note that the variables $\{\tau_i\}$ are homogeneous functions of $\{s_i\}$ of degree 4/3 . Using the homogeneity of the volume together with the definition (2.10) we can express the volume V_X as

$$\frac{7}{3}V_X = \sum_{i=1}^{N} s_i \frac{\partial V_X}{\partial s_i} = \sum_{i=1}^{N} s_i \tau_i\,, \;\Rightarrow\; V_X = \frac{3}{7}\sum_{i=1}^{N} s_i \tau_i\,, \tag{2.11}$$

and

$$\hat{K}_i = \frac{\partial \hat{K}}{\partial s_i} = -\frac{3}{V_X}\frac{\partial V_X}{\partial s_i} = -\frac{3\tau_i}{V_X}\,. \tag{2.12}$$

Combining (2.3), (2.6) and (2.11) we can reexpress the dual variables as

$$\tau_i = \frac{1}{3}\int_X \phi_i \wedge *\Phi = \frac{1}{3}\int_{[\tau_i]} *\Phi\,, \tag{2.13}$$

where for each harmonic basis three-form $\phi_i \in H^3(X, \mathbf{Z})$ we introduced a Poincare dual four-cycle $[\tau_i] \in H_4(X)$. We now use the above duality to make a particularly convenient choice of the basis harmonic three-forms. In particular, we choose a basis $\{\phi_i\} \in H^3(X)$ such that the periods of the fundamental co-associative four-form $*\Phi$ over the Poincare dual four-cycles are positive definite:

$$\int_{[\tau_i]} *\Phi > 0\,. \tag{2.14}$$

This choice of a basis becomes obvious when we recall that for a generic basis four-cycle $[\tau_i] \in H_4(X)$

$$Vol([\tau_i]) \geq \frac{1}{3}\int_{[\tau_i]} *\Phi = \tau_i\,, \tag{2.15}$$

where the above relation becomes an equality if and only if the corresponding four-cycle is co-associative.

All geometric moduli describing the fluctuations of the internal metric must be massive in order to satisfy constraints from fifth force experiments and cosmology. At the same time the vacuum expectation values of the moduli (coordinates s_i on the moduli space) must be fixed in the region of the moduli space where the geometric description makes sense. In section 4 we describe a way to stabilize the moduli, which ensures that we find isolated minima that satisfy these conditions automatically.

It turns out that for our purposes it is convenient to introduce a set of "angular" variables a_i defined by

$$a_i \equiv -\frac{1}{3} s_i \hat{K}_i = \frac{s_i \tau_i}{V_X}\,, \text{ no sum over i}\,. \tag{2.16}$$

We see that a_i are scale-independent and satisfy

$$\sum_{i=1}^{N} a_i = \frac{7}{3}\,. \tag{2.17}$$

Thus, we can also parameterize the moduli space $\mathcal{M}(X)$ by a subset of $N-1$ variables a_i plus one volume, e.g. the volume of the manifold V_X. Differentiating the a_i allows us to introduce the matrix

$$P_{ij} \equiv -s_j \frac{\partial a_i}{\partial s_j}\,, \text{ no sum over j}\,. \tag{2.18}$$

which has components

$$P_{ij} = \frac{1}{3}\delta_{ij} s_j \hat{K}_i + s_i s_j \frac{1}{3}\hat{K}_{ij}\,, \text{ no sum over i, j}\,. \tag{2.19}$$

P_{ij} has the following contraction properties, which follow from (2.17) and the fact that a_i are homogeneous of degree zero

$$\sum_{i=1}^{N} P_{ij} = 0\,, \text{ and } \sum_{j=1}^{N} P_{ij} = 0\,. \tag{2.20}$$

We can then write

$$\hat{K}_{ij} = \frac{3a_j}{s_i s_j}\Delta_{ij}\,, \tag{2.21}$$

where the matrix Δ_{ij} is defined as

$$\Delta_{ij} \equiv \delta_{ij} + \frac{P_{ij}}{a_j}\,, \tag{2.22}$$

and satisfies the following contraction properties

$$\sum_{i=1}^{N} \Delta_{ij} = 1\,, \text{ and } \sum_{j=1}^{N} \Delta_{ij} a_j = a_i\,, \tag{2.23}$$

where we used (2.20) to derive (2.23). Note that parameters a_i defined in (2.16) are the components of an eigenvector a of the non-Hermitian matrix Δ with unit eigenvalue.

We can compute the formal inverse of the Hessian metric, $\hat{K}^{ij}$. By definition of the inverse it must satisfy

$$\sum_{j=1}^{N} \hat{K}^{ij}\hat{K}_{jk} = \delta^i_k\,, \tag{2.24}$$

and using (2.21) it can be expressed as

$$\hat{K}^{ij} = \frac{s_i s_j}{3a_i}(\Delta^{-1})^{ij}\,, \tag{2.25}$$

where the inverse matrix $(\Delta^{-1})^{ij}$ satisfies

$$\sum_{j=1}^{N}(\Delta^{-1})^{ij}\Delta_{jk} = \delta^i_k\,. \tag{2.26}$$

Symbolically we can express Δ^{-1} as

$$\Delta^{-1} = \frac{1}{1+\frac{P}{a}}\,, \tag{2.27}$$

which in terms of components translates into

$$(\Delta^{-1})^{ij} = \delta^{ij} - P_{ij}\frac{1}{a_j} + P_{il}\frac{1}{a_l}P_{lj}\frac{1}{a_j} - P_{il}\frac{1}{a_l}P_{lm}\frac{1}{a_m}P_{mj}\frac{1}{a_j} + \ldots\,. \tag{2.28}$$

Using (2.20) and (2.28) we derive the following properties of the inverse matrix Δ^{-1}

$$\sum_{i=1}^{N}(\Delta^{-1})^{ij} = 1\,,\ \text{and}\ \sum_{j=1}^{N}(\Delta^{-1})^{ij}a_j = a_i\,, \tag{2.29}$$

which could have also been obtained directly from (2.23). Note that although we do not have a closed form expression for the components $(\Delta^{-1})^{ij}$, the contraction properties in (2.29) are what will ultimately allow us to derive explicit expressions for the terms in the soft breaking lagrangian — since such couplings depend only on the contractions and not the precise details of the functional form of V_X. Before going on to the details of these calculations, we first must consider the Kahler potential for matter fields in M theory.

3 Kahler potential for charged chiral matter

In this section we re-visit the Kahler potential for charged matter fields in M theory. In practice, the absence of a useful microscopic formulation makes it difficult to compute the moduli dependence of the Kahler potential for these fields in general. Below we outline three arguments for the structure of the Kahler potential - first from dimensional reduction, second based on the scaling properties of physical Yukawa couplings and the third based on the form of the threshold corrections to the physical gauge coupling. Happily, all three methods agree.

3.1 Kahler potential from dimensional reduction

In M theory, charged chiral matter is localized near conical singularities [7–10]. These are literally points in the seven extra dimensions. Because of this, we expect that the kinetic terms for the chiral matter fields should be "largely independent of bulk moduli fields" that the G_2 manifold X has. The precise meaning of this statement will be clarified below in terms of the scaling property of the kinetic term. They could, of course depend on local moduli inherent to the conical singularity, but, since, in a supersymmetric theory, a single chiral multiplet in a complex representation of the gauge group usually has no D or F-flat directions [11], there are typically no such local moduli.

There is a subtlety in the above general arguments. Since, in four dimensions, a scalar field kinetic term is not invariant under Weyl rescalings of the metric, one has to pick a Weyl gauge. We will argue that the correct Weyl gauge for the statement above is NOT the 4d Einstein frame. Therefore, the kinetic term for chiral matter will be non-trivial in the 4d Einstein frame, which is the standard one in which to define the Kahler potential.

Since the physics of a conical singularity in M theory does not introduce any new scale, asides from the 11d Planck scale, the only reasonable Weyl frame is the 11d Einstein frame. Therefore the lagrangian density in the 11d frame is

$$L \sim M_{11}^9 \sqrt{g_{11}} R + \delta_7 \wedge \partial_M \phi \partial_N \phi^\dagger g^{MN} \kappa(s_i) \sqrt{g_{11}} + \ \ldots , \tag{3.1}$$

where δ_7 is a delta function peaked at the position of the matter multiplet containing the scalar field ϕ and has mass dimension seven. $\kappa(s_i)$ is a homogeneous function of the moduli of degree zero which will generally be of order one and vary adiabatically,[1] i.e.

$$\sum_{k=1}^{N} s_k \frac{\partial \kappa(s_i)}{\partial s_k} = 0 \,. \tag{3.2}$$

The above property implies that $\kappa(s_i)$ remains invariant when the moduli are rescaled as $s_i \to \lambda s_i$, thus explicitly implementing the idea that in the 11d frame the kinetic term of a matter field localized at a point $p \in X$ is "largely independent of bulk moduli". A particularly simple example satisfying (3.2) is when $\kappa(s_i) = const$. Integrating this over X leads to a 4d density

$$L_4 \sim V_X M_{11}^2 \sqrt{g_4} R_4 + \kappa(s_i) g_4^{\mu\nu} \partial_\mu \phi \partial_\nu \phi^\dagger \sqrt{g_4} \,, \tag{3.3}$$

where V_X is the volume of the extra dimensions in 11d units. This is the Lagrangian in 11d Einstein frame. If we now Weyl rescale into the 4d Einstein frame we find

$$L_4 \sim \frac{1}{2\kappa_4^2} \sqrt{g_E} R_E + \frac{\kappa(s_i)}{V_X} g_E^{\mu\nu} \partial_\mu \phi \partial_\nu \phi^\dagger \sqrt{g_E} \,, \tag{3.4}$$

where the subscript E indicates that we are using the 4d Einstein frame metric.

We have only considered the Einstein-Hilbert and kinetic terms of the matter fields. Including all the other terms would give the 4d supergravity Lagrangian in Einstein frame.

[1] In the toroidal Type IIA compactifications with intersectiong D6-branes, the analog of κ is a scale invariant function that depends on the relative intersection angles θ_i^α [12].

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In particular, from this we would read off that the Kahler metric for the multiplet containing ϕ is

$$\tilde{K}_{\phi\bar{\phi}} = \frac{\kappa(s_i)}{V_X}\,. \tag{3.5}$$

If we introduce dimensionless fields $\hat{\phi}$ as in $\phi = m_{pl}\hat{\phi}$, the Kahler potential is

$$\tilde{K} = \kappa(s_i)\frac{\phi\phi^\dagger}{V_X} = \kappa(s_i)\frac{\hat{\phi}\hat{\phi}^\dagger}{V_X}m_{pl}^2\,. \tag{3.6}$$

As we will see, this is consistent with the arguments given in the next subsection.

3.2 Kahler metric from the properties of the physical Yukawa couplings

Here we will describe an alternative way of deducing the volume dependence of the Kahler metric for charged chiral matter. This method is due to Conlon, Cremades and Quevedo [13] and utilizes the relation between the physical (normalized) Yukawa couplings $Y_{\alpha\beta\gamma}$ and the unnormalized Yukawa couplings $Y'_{\alpha\beta\gamma}$ that appear in the supergravity superpotential. Recall that in G_2 compactifications of M theory, a superpotential Yukawa coupling $Y'_{\alpha\beta\gamma}$ between the multiplets α, β, γ that are localized at three co-dimension seven singularities is induced by an M2-brane instanton wrapping a supersymmetric three-cycle connecting the three singular points. The absolute value of the Yukawa coupling is given by

$$|Y'_{\alpha\beta\gamma}| \sim e^{-2\pi V_{\alpha\beta\gamma}}\,, \tag{3.7}$$

where

$$V_{\alpha\beta\gamma} = \sum_i m_i^{\alpha\beta\gamma} s_i\,, \tag{3.8}$$

is the volume of the supersymmeric three-cycle. After we diagonalize the Kahler metric for the matter fields and go to the canonical basis, the relation between the absolute values of the physical and unnormalized Yukawa couplings is simply a rescaling by $e^{K/2}\left(\tilde{K}_\alpha\tilde{K}_\beta\tilde{K}_\gamma\right)^{-1/2}$

$$|Y_{\alpha\beta\gamma}| = e^{K/2}\,|Y'_{\alpha\beta\gamma}|\left(\tilde{K}_\alpha\tilde{K}_\beta\tilde{K}_\gamma\right)^{-1/2} \sim |Y'_{\alpha\beta\gamma}|\left(V_X^3\tilde{K}_\alpha\tilde{K}_\beta\tilde{K}_\gamma\right)^{-1/2}\,. \tag{3.9}$$

On the other hand, one can construct perfectly well-defined seven-dimensional local models where the G_2 manifold is non-compact, e.g. an ALE-fibration over a three-sphere or a quotient thereof, in which case $V_X \to \infty \Rightarrow m_{pl}/M_{11} \to \infty$ and gravity is effectively decoupled. Such models can also contain charged chiral matter fields and since their interactions are determined locally, the corresponding physical Yukawa couplings should not vanish when gravity is decoupled. Local models of this type can be obtained via lifting effective theories on intersecting D6-branes in Type IIA to M theory.

Therefore, locality implies that the physical Yukawa couplings should be independent of the overall volume V_X in the limit $V_X \to \infty$. For that to happen, the Kahler metrics $\tilde{K}_\alpha$, $\tilde{K}_\beta$, $\tilde{K}_\gamma$ in (3.9) must scale with the volume V_X as

$$\tilde{K}_\alpha \sim \tilde{K}_\beta \sim \tilde{K}_\gamma \sim \frac{1}{V_X}\,, \tag{3.10}$$

which is in perfect agreement with the form of the Kahler metric derived in the previous subsection.

3.3 Consistency check for the Kahler metric

In this section we confirm the form of the Kahler metric for charged chiral matter by comparing the threshold corrections to the physical gauge couplings in G_2 compactifications of M theory with the general results in $\mathcal{N}=1$ $D=4$ supergravity.

Let us first consider a hidden sector containing a pure glue SU(N) supersymmetric Yang-Mills theory. Using the notation in [14, 15] we have the following relation for the gauge coupling at one loop

$$\frac{16\pi^2}{g^2(\mu)} = \frac{16\pi^2}{g_M^2} - 3N\ln\left(\frac{\Lambda^2}{\mu^2}\right) + \mathcal{S}\,, \tag{3.11}$$

where $\mathcal{S}$ are the one-loop threshold corrections, $g(\mu)$ is the physical gauge coupling and g_M is the tree-level Wilsonian gauge coupling. In our convention g_M is related to the gauge kinetic function f as

$$\frac{4\pi}{g_M^2} = \mathrm{Im} f\,. \tag{3.12}$$

Recall that the Wilsonian gauge coupling gets renormalized at one loop only. On the other hand the physical coupling $g(\mu)$ is renormalized to all orders. The threshold corrections come from massive states and are independent of the scale μ. Based on the topological arguments [14, 15], the threshold corrections due to Kaluza Klein modes in G_2 compactifications of M theory are rather simple and can be calculated even without knowing the G_2 metric! Friedmann and Witten [14, 15] explicitly computed one-loop threshold corrections due to the heavy Kaluza-Klein modes living on a supersymmetric cycle $\mathcal{Q}$ with $b_1(\mathcal{Q}) = b_2(\mathcal{Q}) = 0$ and a non-trivial fundamental group. Such corrections come in a form of linear combinations of Ray-Singer analytic torsions [16–18], which are topological invariants of $\mathcal{Q}$. For the case at hand, the threshold corrections are given by

$$\mathcal{S} = 2N\ln V_{\mathcal{Q}}\Lambda^3 + 2\sum_i \mathcal{T}_i \mathrm{Tr}_{\mathcal{R}_i} Q^2\,, \tag{3.13}$$

where $V_{\mathcal{Q}}$ is the volume of the supersymetric cycle $\mathcal{Q}$, $\mathcal{T}_i$ are the Ray-Singer torsions corresponding to different irreducible representations of the fundamental group and Q are the generators of SU(N). Here, the cutoff dependence appears as a correction due to the zero mode contributions transforming in the trivial representation of $\pi_1(\mathcal{Q})$. Once the threshold corrections are included explicitly, a somewhat unexpected cancellation of the Λ-dependence occurs [14, 15] and the one-loop relation can be written as

$$\frac{16\pi^2}{g^2(\mu)} = \frac{16\pi^2}{g_M^2} - 3N\ln\left(\frac{1}{V_{\mathcal{Q}}^{2/3}\mu^2}\right) + \mathcal{S}' \tag{3.14}$$

where

$$\mathcal{S}' = 2\sum_i \mathcal{T}_i \mathrm{Tr}_{\mathcal{R}_i} Q^2\,. \tag{3.15}$$

In fact, the cancellation of the Λ-dependence occurs for any supersymmetric cycle $\mathcal{Q}$ with $b_1(\mathcal{Q}) = b_2(\mathcal{Q}) = 0$ [14, 15].

Now we would like to consider a more general case when the gauge theory is a supersymmetric QCD with N_f flavors of chiral matter fields Q_α transforming in N of SU(N) plus N_f flavors of $\tilde{Q}_\alpha$ transforming in $\overline{N}$. Each chiral matter field transforming in a complex representation arises from a separate co-dimension seven conical singularity on X with each singular point $P_i \in \mathcal{Q}$. It was argued in [14, 15] that the singularities producing charged chiral matter fields have no effect on the KK harmonics of the seven-dimensional vector multiplet. Moreover, since the conical singularities introduce no new scale below the eleven-dimensional Planck scale M_{11}, the effective cutoff scale for these multiplets is naturally M_{11}. Including such multiplets into the running is straightforward and results in

$$\frac{16\pi^2}{g^2(\mu)} = \frac{16\pi^2}{g_M^2} - 3N \ln\left(\frac{1}{V_{\mathcal{Q}}^{2/3}\mu^2}\right) + N_f \ln\left(\frac{M_{11}^2}{\mu^2}\right) + \mathcal{S}' \,. \tag{3.16}$$

In addition to the KK thresholds, there may be some unknown corrections due to possible charged massive matter fields with masses of order M_{11}. At this point we cannot say with certainty whether such massive charged M theory modes are present in the spectrum but we cannot exclude this possibility either. Just like the KK thresholds, these corrections cannot be holomorphic functions of the chiral multiplets z_i describing the moduli of X since the axion partners of the geometric moduli decouple from the computations of the threshold corrections. However, there may be some non-holomorphic as well as constant contributions from such massive charged states. For now we will simply assume that they are constant and result in a slight shift of the tree-level gauge coupling. On the other hand, moduli dependent contributions may arise from non-perturbative corrections due to membrane instantons but they will be exponentially suppressed and can be safely neglected.

Our next task is to independently verify that the Kahler metric for the charged chiral matter fields matches the previously obtained result (3.5). Here we will use a strategy similar to the one in [12, 19] and compare (3.16) with the corresponding one-loop expression in $\mathcal{N}=1$ $D=4$ supergravity given by [20, 21]

$$\frac{16\pi^2}{g^2(\mu)} = \frac{16\pi^2}{g_M^2} - (3N - N_f)\ln\left(\frac{m_{pl}^2}{\mu^2}\right) - (N - N_f)\,\hat{K} + 2N \ln\left(\frac{1}{g_M^2}\right) - 2N_f \ln\left(\tilde{K}_{\alpha\bar{\alpha}}\right) . \tag{3.17}$$

In the above expression, $\hat{K} = -3\ln 4\pi^{1/3}V_X$ is the Kahler potential for the moduli and $\tilde{K}_{\alpha\bar{\alpha}}$ is the Kahler metric for the charged chiral matter fields. We can use the definition of the four-dimensional Newton's constant $\kappa_4 = \sqrt{8\pi G_N} = 1/m_{pl}$ in terms of the eleven-dimensional gravitational coupling κ_{11}

$$\kappa_4^2 \equiv \frac{\kappa_{11}^2}{V_X l_M^7} \,, \tag{3.18}$$

in combination with the common convention $2\kappa_{11}^2 = (2\pi)^8 M_{11}^{-9}$ and $M_{11} = 2\pi/l_M$ to obtain

$$M_{11}^2 = \frac{\pi m_{pl}^2}{V_X} \,. \tag{3.19}$$

Using the above relations together with $4\pi/g_M^2 = V_Q/l_M^3$ we have from (3.17)

$$
\begin{aligned}
\frac{16\pi^2}{g^2(\mu)} &= \frac{16\pi^2}{g_M^2} - (3N-N_f)\ln\left(\frac{V_X M_{11}^2}{\pi\mu^2}\right) + 3\,(N-N_f)\ln 4\pi^{1/3}V_X + 2N\ln\left(\frac{V_Q}{4\pi l_M^3}\right) - 2N_f\ln\left(\tilde{K}_{\alpha\bar{\alpha}}\right) \\
&= \frac{16\pi^2}{g_M^2} - (3N-N_f)\ln\left(\frac{M_{11}^2}{4\pi^{4/3}\mu^2}\right) - 2N_f\ln 4\pi^{1/3}V_X + 2N\ln\left(\frac{V_Q}{4\pi l_M^3}\right) - 2N_f\ln\left(\tilde{K}_{\alpha\bar{\alpha}}\right) \\
&= \frac{16\pi^2}{g_M^2} - 3N\ln\left(\frac{1}{V_Q^{2/3}\mu^2}\right) + N_f\ln\left(\frac{M_{11}^2}{\mu^2}\right) - 2N_f\ln\left(V_X\tilde{K}_{\alpha\bar{\alpha}}\right) - \ln\left((2\pi)^{4N}(8\pi)^{2N_f}\right) .
\end{aligned}
\tag{3.20}
$$

The appearance of the last term is most likely due to the convention used to define M_{11} in terms of κ_{11} as well as the ambiguity in defining the relation between l_M and M_{11}. Thus, we shall regard this term as an artifact and ignore it in further discussion. Comparing (3.20) with the expression on the right hand side in (3.16) we conclude that up to a constant multiplicative factor, Kahler metric for the charged chiral matter fields Q_α is

$$
\tilde{K}_{\alpha\bar{\alpha}} \sim \frac{1}{V_X}\,, \tag{3.21}
$$

which precisely matches the result obtained in the previous subsections. On the other hand, the constant term $\mathcal{S}'$ in (3.16) has no corresponding analog in (3.20) and represents a genuine threshold correction to the Wilsonian gauge coupling g_M.

In the framework of $\mathcal{N}=1$ $D=4$ supergravity, the RG-invariant scale where super QCD with $N > N_f$ becomes strongly coupled is

$$
\Lambda^{3N-N_f} = m_{pl}^{3N-N_f} e^{-\frac{8\pi^2}{g_M^2}-\frac{S'}{2}} e^{-(N-N_f)\frac{\hat{K}}{2}}\,, \tag{3.22}
$$

where the second exponential factor is due to the local SUSY. The Affleck-Dine-Seiberg effective superpotential [22, 23] W should be identified with

$$
e^{\hat{K}/2}W = \frac{(N-N_f)\tilde{\Lambda}^{\frac{3N-N_f}{N-N_f}}}{\det(Q\tilde{Q})^{\frac{1}{N-N_f}}}\,, \tag{3.23}
$$

where the gauge coupling inside $\tilde{\Lambda}$ is complexified. Using (3.22), up to an overall numerical constant we obtain

$$
W \sim (N-N_f)m_{pl}^{\frac{3N-N_f}{N-N_f}}\det(Q\tilde{Q})^{-\frac{1}{N-N_f}} e^{i\frac{2\pi}{N-N_f}f} e^{-\frac{S_a'}{2(N-N_f)}}\,. \tag{3.24}
$$

In (3.24), the dimensionful chiral matter fields[2] Q can be expressed in terms of dimensionless fields $\hat{Q}$ as

$$
Q = m_{pl}\hat{Q}\,, \quad \text{and} \quad \tilde{Q} = m_{pl}\hat{\tilde{Q}}\,. \tag{3.25}
$$

Then, the superpotential becomes

$$
W = \tilde{C}(N-N_f)\,m_{pl}^3\det(\hat{Q}\hat{\tilde{Q}})^{-\frac{1}{N-N_f}} e^{i\frac{2\pi}{N-N_f}f} e^{-\frac{S_a'}{2(N-N_f)}}\,, \tag{3.26}
$$

[2]Here we suppressed the flavor index.

where $\tilde{C}$ is an overall numerical constant. In our further notation, we also define the following constants

$$C \equiv \tilde{C}\, e^{-\frac{S'_a}{2(N-N_f)}}\,, \quad \text{and} \quad A \equiv (N - N_f)\, C\,. \tag{3.27}$$

Let us now consider the case of $N_f = 1$ flavors. Introducing an effective meson degree of freedom

$$\phi \equiv \sqrt{2\hat{Q}\hat{\tilde{Q}}}\,, \tag{3.28}$$

we can rewrite the superpotential in terms of ϕ as

$$W = A\, m_{pl}^3 \phi^{-\frac{2}{N-1}} e^{i\frac{2\pi}{N-1}f}\,, \tag{3.29}$$

where we have absorbed the factor of $2^{1/(N-1)}$ into the normalization constant C. Along the D-flat direction we have $\hat{Q} = \hat{\tilde{Q}}$ and the Kahler potential for the matter fields can be rewritten in terms of the effective meson fields ϕ as

$$\tilde{K} = \kappa(s_i)\frac{\hat{Q}^\dagger\hat{Q}}{V_X}m_{pl}^2 + \kappa(s_i)\frac{\hat{\tilde{Q}}^\dagger\hat{\tilde{Q}}}{V_X}m_{pl}^2 = \kappa(s_i)\frac{\bar{\phi}\phi}{V_X}m_{pl}^2\,. \tag{3.30}$$

3.4 Higher order terms

Based on three independent arguments we have been able to deduce the volume dependence of the Kahler metric for charged chiral matter fields localized at co-dimension seven singularities. Denoting the visible sector charged chiral matter fields by Q^α their Kahler potential is then given by

$$\tilde{K} = \frac{\kappa_{\alpha\bar{\beta}}(s_i)Q^\alpha Q^{\dagger\bar{\beta}}}{V_X}\,. \tag{3.31}$$

In the regime where the size of the supersymmetric cycle supporting the visible sector is large (this assumption is justified in the context of the MSSM where the corresponding volume is $\alpha_{GUT}^{-1} \approx 25$) we can perform a systematic expansion of $\kappa_{\alpha\bar{\beta}}(s_i)$ in the inverse volume of the cycle (weak coupling) so that in the leading order $\kappa_{\alpha\bar{\beta}}(s_i)$ is a homogeneous function of s_i of degree λ, satisfying

$$\sum_{i=1}^{N} s_i \frac{\partial\kappa_{\alpha\bar{\beta}}(s_i)}{\partial s_i} = \lambda\kappa_{\alpha\bar{\beta}}(s_i)\,. \tag{3.32}$$

Based on the property that a given charged chiral matter multiplet is localized at a point $p \in X$ we expect that $\lambda = 0$, i.e. $\kappa_{\alpha\bar{\beta}}(s_i)$ is scale invariant in the leading order. Therefore, when the moduli are simultaneously scaled up by an overall positive constant, the ratio $\kappa_{\alpha\bar{\beta}}(s_i)/V_X \ll 1$. Nevertheless, for the sake of generality we will keep λ as a free parameter for the time being. In our derivation of the Kahler potential we so far neglected possible higher order contributions to the visible sector matter Kahler potential of the form[3]

$$\delta\tilde{K} = c_{\alpha\bar{\beta}}(s_i)Q_c^\alpha Q_c^{\dagger\bar{\beta}}\phi_c\bar{\phi}_c + \ldots = c_{\alpha\bar{\beta}}(s_i)\frac{Q^\alpha Q^{\dagger\bar{\beta}}}{V_X}\frac{\phi\bar{\phi}}{V_X} + \ldots, \tag{3.33}$$

[3] Here we set $m_{pl} = 1$ and treat all matter fields as dimensionless in units of m_{pl}.

which gravitationally couple the hidden sector meson to the visible sector fields Q_α. In the above expression, the subscript c denotes canonically normalized matter fields in the 4-dimensional Einstein frame. Such couplings can create problems if the meson F-term is quite large (which is true in the G_2-MSSM) because they can induce flavor changing neutral currents. This is the flavor problem of gravity mediated susy breaking models.[4] These terms were neglected in our previous work [3].

Technically, computing the unknown coefficients $c_{\alpha\bar{\beta}}(s_i)$ from the underlying theory is difficult, goes well beyond the scope of this work and our aim here is *not* to explain the flavor structure of the supersymmetry breaking Lagrangian. Rather, we would like to understand the effect that the presence of such terms might have on other sectors of the theory, e.g. their effect on superpartner masses and couplings. For these purposes it is sufficient to assume that the flavor structure of the Kahler metric is completely determined by the matrix $\kappa_{\alpha\bar{\beta}}(s_i)$, so that

$$c_{\alpha\bar{\beta}}(s_i) = \kappa_{\alpha\bar{\beta}}(s_i)\,\frac{c(s_i)}{3}\,, \tag{3.34}$$

where we introduced the factor of 1/3 for future convenience. As we will see in the later sections, whilst this does not introduce any flavor violation, the point will be that the effect of such terms on the mass spectrum will be similar even if we introduced flavor violating terms, as should become clear eventually.[5]

Actually, such a form might arise from an expansion of the Kahler potential if the visible and hidden sectors were completely sequestered. Though we do not expect M theory to be sequestered, it can be useful to think of the sequestering as an extreme limit in a more general model.

A sequestered Kahler potential has the form

$$K^{seq} = -3\ln\left(4\pi^{1/3}V_X - \frac{1}{3}\phi\bar{\phi} - \frac{1}{3}\kappa_{\alpha\bar{\beta}}Q^\alpha Q^{\dagger\bar{\beta}}\right)\,, \tag{3.35}$$

and the Kahler metric for the visible sector is given by

$$K^{seq}_{\alpha\bar{\beta}} = \frac{\kappa_{\alpha\bar{\beta}}}{4\pi^{1/3}V_X - \frac{1}{3}\phi\bar{\phi}}\,. \tag{3.36}$$

Absorbing the factor of $4\pi^{1/3}$ into the definition of the fields and expanding the above expression in powers of ϕ^2/V_X we obtain

$$K^{seq}_{\alpha\bar{\beta}} = \frac{\kappa_{\alpha\bar{\beta}}}{V_X}\left(1 + \frac{\phi\bar{\phi}}{3V_X}\right) + \ldots\,. \tag{3.37}$$

[4] Note that the Kahler metric derived from (3.31) introduces no flavor problems since, as we will see from explicit computations, the mass matrix for the scalars will be proportional to $\kappa_{\alpha\bar{\beta}}(s_i)$ and therefore diagonalization and canonical normalization of the kinetic terms automatically results in universal scalar masses.

[5] Generically, the absence of flavor changing neutral currents implies that the off-diagonal entries in the mass matrix for the canonically normalized squarks and sleptons are suppressed, though in particular models even stronger constraints are necessary, e.g. the requirement that the diagonal entries are nearly degenerate, depending on the spectrum and the A-terms [24].

Comparing the above expression with (3.33) we can read off the coefficients

$$c^{seq}_{\alpha\bar{\beta}} = \frac{1}{3}\kappa_{\alpha\bar{\beta}}\,, \tag{3.38}$$

which corresponds to (3.34) when $c(s_i) = 1$. Hence, function $c(s_i)$ in (3.34) is the measure of deviation of the matter Kahler potential from the exactly sequestered form. As was pointed out in [25, 26], sequestering is not at all generic in string/M theory and presumably G_2 compactifications of M theory are no exception. We thus will regard the value of $c(s_i)$ in a given vacuum as a parameter and consider the theory for various values of $c(s_i)$.

Combining all of the previous considerations, the visible sector matter Kahler metric and its inverse take the form

$$\begin{aligned} \tilde{K}_{\alpha\bar{\beta}} &= \frac{\kappa_{\alpha\bar{\beta}}(s_i)}{V_X}\left(1 + c(s_i)\frac{\phi\bar{\phi}}{3V_X}\right)\,, \\ \tilde{K}^{\alpha\bar{\beta}} &= \kappa^{\alpha\bar{\beta}}(s_i)\frac{V_X}{\left(1 + c(s_i)\frac{\phi\bar{\phi}}{3V_X}\right)} \approx \kappa^{\alpha\bar{\beta}}(s_i)V_X\left(1 - c(s_i)\frac{\phi\bar{\phi}}{3V_X}\right)\,, \end{aligned} \tag{3.39}$$

where $\kappa^{\alpha\bar{\beta}}(s_i)$ satisfies

$$\kappa^{\alpha\bar{\beta}}(s_i)\kappa_{\bar{\beta}\gamma}(s_i) = \delta^{\alpha}_{\gamma}\,. \tag{3.40}$$

Combining (3.32) with the above we conclude that $\kappa^{\alpha\bar{\beta}}(s_i)$ is a homogeneous function of the moduli of degree $-\lambda$. Function $c(s_i)$ will be typically assumed to take values in the range

$$0 \leq c(s_i) \leq 1\,. \tag{3.41}$$

However, as long as the Kahler metric is positive-definite, one may also consider the regime when $c(s_i) < 0$. Diagonalizing the Kahler metric of the visible sector we obtain

$$\tilde{K}_{\alpha}\delta_{\alpha\bar{\beta}} = \mathcal{U}^{\dagger}_{\alpha\gamma}\tilde{K}_{\gamma\bar{\rho}}\mathcal{U}_{\bar{\rho}\bar{\beta}}\,, \tag{3.42}$$

where the eigenvalues $\tilde{K}_{\alpha}$ are given by

$$\tilde{K}_{\alpha} = \frac{\kappa_{\alpha}(s_i)}{V_X}\left(1 + c(s_i)\frac{\phi\bar{\phi}}{3V_X}\right)\,, \tag{3.43}$$

and $\kappa_{\alpha}(s_i)$ are homogeneous functions of degree λ that satisfy (3.32). In computing the anomaly mediated contribution to the gaugino masses, it will be necessary to compute various derivatives of $\ln\tilde{K}_{\alpha}$. For this purpose, it turns out that it is very convenient to express $\ln\tilde{K}_{\alpha}$ as

$$\ln\tilde{K}_{\alpha} = \ln\kappa_{\alpha}(s_i) - \ln V_X + \ln\left(1 + c(s_i)\frac{\phi\bar{\phi}}{3V_X}\right) \approx \frac{1}{3}K + \ln\kappa_{\alpha}(s_i) + (c(s_i) - 1)\,\frac{\phi\bar{\phi}}{3V_X} + \text{const}\,, \tag{3.44}$$

where K is the Kahler potential in (4.5).

4 Moduli stabilization

In this section we reconsider the problem of moduli stabilization with the much more general moduli and matter Kahler potentials introduced in the previous section. We will be working in the framework of $\mathcal{N} = 1$ $D = 4$ effective supergravity and will demonstrate that all the moduli can be stabilized self-consistently in the regime where the supergravity approximation is valid. Recall that in the compactifications we study here, non-Abelian gauge fields arise from co-dimension four singularities [7, 27–30]. In other words, there exist three-dimensional submanifolds $\mathcal{Q}$ inside the G_2-manifold X, along which there is an orbifold singularity of A-D-E type.

The basic idea is that strong dynamics in the hidden sector breaks supersymmetry, stabilizes the moduli and generates a small scale. In this context we would like to highlight some important properties that distinguish G_2 compactifications from other known corners of the string landscape. First, unlike four-dimensional Calabi-Yau compactifications, where one typically has to deal with several different types of moduli, e.g. complex structure, Kahler moduli, the dilaton, vector bundle moduli, etc., which are typically stabilized via different mechanisms, in G_2 compactifications of M theory all deformations of the internal metric of the manifold X are completely captured by the periods s_i of the associative three-form Φ. Since all s_i are on an equal footing the task of moduli stabilization is dramatically simplified as one can use a *single mechanism* to stabilize *all* geometric moduli.[6] Second, all the complexified moduli $z_i = t_i + is_i$ enjoy a Peccei-Quinn-type shift symmetry, which is inherited from the gauge symmetry associated with the three-form C_3 of the eleven-dimensional supergravity. In the absence of tree-level flux contributions this symmetry is exact at the perturbative level but it can be broken by non-perturbative effects. Therefore, in the fluxless sector of the theory, the entire superpotential is purely non-perturbative and depends upon all the moduli s_i. Therefore, one naturally expects exponential hierarchies to be generated, once the moduli are stabilized.[7]

The simplest possibility consistent with the supergravity approximation is a hidden sector with two gauge groups $\mathrm{SU}(P + N_f)$ and $\mathrm{SU}(Q)$ where the first is super QCD with $N_f = 1$ flavor of quarks Q and $\tilde{Q}$ transforming in a complex (conjugate) representation of $\mathrm{SU}(P + 1)$ (the corresponding associative cycle $\mathcal{Q}$ contains two isolated singularities of co-dimension seven) and the second hidden sector with the gauge group $\mathrm{SU}(Q)$ is a "pure glue" super Yang-Mills theory. One can easily consider more general gauge groups without much qualitative difference. One can also consider a setup with charged matter in both hidden sectors. However, as was demonstrated in [1, 2], in such cases, one of the two F-terms coming from the matter fields in the hidden sectors is always suppressed relative

[6]For those G_2 compactifications of M theory that are dual to the four-dimensional vacua of the Heterotic string, the dilaton and the vector bundle moduli on the Heterotic side are mapped to some of the geometric moduli s_i on the M theory side.

[7]To contrast this, recall that in Type IIB orientifold compactifications, because the complex structure moduli *do not* possess a shift symmetry, the superpotential generically receives unsuppressed perturbative contributions. Furthermore, with the exception of some toroidal examples, the precise dependence of the non-perturbative contributions in Type IIB orientifolds on the complex structure moduli is currently unknown.

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to the other and thus does not contribute to the quantities relevant for phenomenology. A single hidden sector gauge theory is also enough to stabilise the moduli, though the vacuum is not in a place where supergravity is trustable!

Therefore, the non-perturbative effective superpotential generated by the strong gauge dynamics in the hidden sectors is given by

$$W = A_1 \phi^a e^{ib_1 f} + A_2 e^{ib_2 f} . \tag{4.1}$$

The matter field ϕ represents an effective meson degree of freedom defined in (3.28) in terms of the chiral matter fields $\hat{Q}$ and $\hat{\tilde{Q}}$. The coefficients b_1, b_2 and a are

$$b_1 = \frac{2\pi}{P}, \quad b_1 = \frac{2\pi}{Q}, \quad a = -\frac{2}{P} . \tag{4.2}$$

In [1, 2] it was explained that if one uses a superpotential of the form (4.1), de Sitter vacua arise only when $Q > P$ (if we include matter in both hidden sectors dS vacua exist without such condition). Hence, we will keep this in mind from now on.

In (4.1) we explicitly assumed that the associative cycles supporting both hidden sectors are in proportional homology classes which results in the gauge kinetic function being given by essentially the same integer combination of the moduli z_i for both hidden sectors

$$f = \sum_{i=1}^{N} N_i z_i , \tag{4.3}$$

were

$$\mathrm{Im}(f) = V_{\mathcal{Q}} \equiv \int_{\mathcal{Q}} \Phi = \sum_{i=1}^{N} N_i s_i \tag{4.4}$$

is the volume of the corresponding associative cycle with the integers N_i specifying the homology class. This possibility may naturally arise when the three-cycle $\mathcal{Q}$ has a non-trivial fundamental group, e.g. $\mathcal{Q} = S^3/\mathbf{Z_q}$, so it can support discrete Wilson lines. Then, just like the visible sector GUT is broken to the Standard Model, the unified hidden sector gauge group can be broken to a product subgroup $\mathrm{SU}(N+M) \to \mathrm{SU}(N) \times \mathrm{SU}(M) \times \mathrm{U}(1)$ while $N+M$ and $\overline{N+M}$ matter multiplets, localized at two distinct co-dimension seven singularities, give rise to $(N,1)+(1,M)$ plus the conjugate.[8] Unless the singularities are extremely close, the supersymmetric mass terms of the vector-like pairs are exponentially suppressed by the corresponding membrane instanton. Thus, one obtains two hidden supersymmetric QCD gauge theories with light vector-like matter supported along the same three-cycle $\mathcal{Q}$. As mentioned above, since one of the two matter F-terms is always suppressed relative to the other [1, 2], one obtains virtually the same results in the simplified scenario where one of the hidden sectors is a "pure glue" supersymmetic Yang-Mills gauge theory.

While one can certainly consider possibilities where the gauge kinetic functions f_1 and f_2 are not proportional, the results in [1, 2] taught us that unless $f_1 \propto f_2$ it is more difficult to stabilize all the moduli in the regime where the supergravity approximation is

[8] Alternatively, one may also consider a hidden $\mathrm{SO}(2(N+M)) \to \mathrm{SU}(N) \times \mathrm{SU}(M) \times \mathrm{U}(1) \times \mathrm{U}(1)$ with charged chiral matter in $2(N+M)$ giving rise to $(N,1)+(\bar{N},1)+(1,M)+(1,\bar{M})$.

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valid. Thus, obtaining solutions which we can trust is the main reason for choosing to consider the case where $f_1 = f_2 = f$. Obviously, progress in the more general cases would be welcome.

Typical examples for three-cycles supporting non-Abelian gauge fields in G_2-manifolds are spheres and their quotients such as Lens spaces $S^3/\mathbf{Z_q}$ considered in [14, 15]. The expression in (4.1) can in principle contain many additional non-perturbative contributions if X contains other rigid associative cycles. In that respect, the two terms included in (4.1) should be regarded as the leading order exponentials. As long as Q and P are large enough compared to the Casimirs from the other gauge groups, the remaining terms will be exponentially suppressed in general. This is particularly true for the membrane instanton corrections to (4.1) which come with exponentials containing $b_i = 2\pi$. On the other hand, some such instantons induce Yukawa interactions among the visible sector matter fields and are therefore implicitly assumed to be part of the full superpotential.

The total Kahler potential - moduli plus hidden sector matter, is given by

$$K = -3\ln 4\pi^{1/3} V_X + \kappa(s_i)\frac{\bar{\phi}\phi}{V_X}\,. \tag{4.5}$$

In what follows we first consider a simplified case where the function $\kappa(s_i)$ is a pure constant, i.e.

$$\kappa(s_i) = 1\,. \tag{4.6}$$

However, in section 6 we will generalize our results to the case where $\kappa(s_i)$ is a homogeneous function satisfying (3.2). The important point is that even then the functional form of the soft breaking terms remains virtually unchanged compared to the simplified case, thus validating our approach. In general, (4.5) must include the contributions to the Kahler potential from all matter sectors including the visible sector as described in the previous section. However, since the visible sector fields will obtain zero vacuum expectation values (vevs), they can be dropped for the purposes of stabilizing moduli.

The standard $\mathcal{N} = 1$ $D = 4$ supergravity scalar potential is given by

$$V = e^K \left(K^{n\bar{m}} F_n \bar{F}_{\bar{m}} - 3|W|^2\right)\,, \tag{4.7}$$

where the F-terms are

$$\begin{aligned} F_i = \partial_i W + W\partial_i K &= iN_i e^{ib_2\vec{N}\cdot\vec{t}}\left(-b_1 A_1 \phi_0^a e^{-b_1\vec{N}\cdot\vec{s}} + b_2 A_2 e^{-b_2\vec{N}\cdot\vec{s}}\right) \qquad (4.8)\\ &\quad + i\frac{3a_i}{2s_i}\left(1+\frac{\phi^2}{3V_X}\right) e^{ib_2\vec{N}\cdot\vec{t}}\left(-A_1\phi^a e^{-b_1\vec{N}\cdot\vec{s}} + A_2 e^{-b_2\vec{N}\cdot\vec{s}}\right)\\ F_\phi = \partial_\phi W + W\partial_\phi K &= -\,e^{ib_2\vec{N}\cdot\vec{t}-i\theta} a A_1 \phi_0^{a-1} e^{-b_1\vec{N}\cdot\vec{s}} \qquad (4.9)\\ &\quad + \frac{\phi_0}{V_X} e^{ib_2\vec{N}\cdot\vec{t}-i\theta}\left(-A_1\phi^a e^{-b_1\vec{N}\cdot\vec{s}} + A_2 e^{-b_2\vec{N}\cdot\vec{s}}\right)\,. \end{aligned}$$

In the above we used

$$\frac{\partial K}{\partial z_i} = \frac{1}{2i}\frac{\partial K}{\partial s_i}\,, \tag{4.10}$$

together with the definition of a_i in (2.16) in combination with

$$\frac{\partial}{\partial s_i}\frac{1}{V_X} = \frac{\hat{K}_i}{3V_X}\,. \tag{4.11}$$

We also parameterized the meson field ϕ as

$$\phi = \phi_0 e^{i\theta}\,, \tag{4.12}$$

and fixed one combination of the axions and the meson phase θ

$$\cos((b_1 - b_2)\vec{N}\cdot\vec{t} + a\theta) = -1\,. \tag{4.13}$$

Before we proceed to constructing de Sitter vacua it is instructive to take a step back and consider a simpler case where the first non-perturbative term in the superpotential is also a pure gaugino condensate arising from a "pure glue" supersymmetric Yang-Mills theory. In this case one possible solution corresponds to a supersymmetric AdS extremum described by the following set of equations

$$F_i = 0\,, \quad \Rightarrow \quad N_i\left(-b_1A_1e^{-b_1\vec{N}\cdot\vec{s}} + b_2A_2e^{-b_2\vec{N}\cdot\vec{s}}\right) + \frac{3a_i}{2s_i}\left(-A_1e^{-b_1\vec{N}\cdot\vec{s}} + A_2e^{-b_2\vec{N}\cdot\vec{s}}\right) = 0\,, \tag{4.14}$$

which is equivalent to

$$s_i = -\frac{3a_i}{2N_i}\frac{\left(-A_1e^{-b_1\vec{N}\cdot\vec{s}} + A_2e^{-b_2\vec{N}\cdot\vec{s}}\right)}{\left(-b_1A_1e^{-b_1\vec{N}\cdot\vec{s}} + b_2A_2e^{-b_2\vec{N}\cdot\vec{s}}\right)}\,. \tag{4.15}$$

Using the contraction property (2.17) we can find from (4.15) that the volume $\vec{N}\cdot\vec{s}$ of the hidden sector three-cycle can be determined by solving the following transcendental equation

$$\vec{N}\cdot\vec{s} = -\frac{7}{2}\frac{\left(-A_1e^{-b_1\vec{N}\cdot\vec{s}} + A_2e^{-b_2\vec{N}\cdot\vec{s}}\right)}{\left(-b_1A_1e^{-b_1\vec{N}\cdot\vec{s}} + b_2A_2e^{-b_2\vec{N}\cdot\vec{s}}\right)}\,. \tag{4.16}$$

In the limit when $\vec{N}\cdot\vec{s} \gg 1$, the approximate solution is given by

$$\begin{gathered} V_Q = \vec{N}\cdot\vec{s} \approx \frac{1}{b_1 - b_2}\ln\left(\frac{A_1b_1}{A_2b_2}\right) > 0\,, \\ \text{when } \; b_1 > b_2\,\&\, A_1b_1 > A_2b_2\,, \;\text{ or }\; b_1 < b_2\,\&\, A_1b_1 < A_2b_2\,. \end{gathered} \tag{4.17}$$

The moduli vevs can then be found from

$$s_i = \frac{3a_i}{7N_i}V_Q\,, \quad \Rightarrow \quad \tau_i = N_i\frac{7V_X}{3V_Q}\,, \tag{4.18}$$

where the seven-dimensional volume is stabilized at

$$V_X = V_Q^{7/3}\left(\frac{3}{7}\right)^{7/3} V_X(s_k)\Big|_{s_k = \frac{a_k}{N_k}}\,. \tag{4.19}$$

Note from (4.18) that at the extremum, the periods of the co-associative four-form $\tau_i \sim N_i$ up to a positive constant. Recall that here, N_i are the integers representing the homology class of $\mathcal{Q}$:

$$N_i = \int_{[\mathcal{Q}]} \phi_i \,, \quad \phi_i \in H^3(X,\, Z)\,, \tag{4.20}$$

where the harmonic three-form ϕ_i is Poincare dual to a four-cycle $[\tau_i]$. Therefore, from (4.18) we see that extremization of the supergravity scalar potential dynamically stabilizes the co-associative four-form $*\Phi$ to be proportional to the integral homology class of the associative three-cycle $\mathcal{Q}$:

$$F_i = 0 \quad \Rightarrow \quad *\Phi = \alpha \cdot PD_X(\mathcal{Q})\,, \;\; 0 < \alpha \in R\,. \tag{4.21}$$

Therefore, in the basis specified by (2.14) the integers N_i must be positive definite

$$N_i > 0\,, \;\; \forall i = 1, \,\dots\; N\,. \tag{4.22}$$

In order to determine the values of a_i at the minimum we substitute our expressions for s_i (4.18) into the definition of a_i in (2.16) to get a system of N transcendental equations, which then completely determine a_i in principle

$$\hat{K}_i\Big|_{s_i = \frac{a_i}{N_i}} + 3N_i \; = 0\,. \tag{4.23}$$

Note that the dependence on $V_{\mathcal{Q}}$ in (4.23) is gone due to the scaling property of the volume V_X. Hence, we have recast the problem of determining the moduli vevs at the minimum into a problem of determining the values of a_i. Obviously, obtaining general analytic solutions for a_i from (4.23) is impossible in practice, since V_X has not been specified. However, precisely because the moduli vevs at the minimum are given by (4.18), it turns out that in order to compute the quantities relevant for particle physics, one does not need to know the values of a_i explicitly. All one actually needs to know are the contraction properties (2.17) and (2.29).

Therefore, the results we derive will be valid for any singular manifold of G_2 holonomy containing an associative three-cycle $\mathcal{Q}$ that contributes to the non-perturbative superpotential in a form of at least two gaugino condensates, whose integral homology class in the basis (2.14) is specified by positive integers. By explicitly checking in explicit toy examples, both numerically and analytically it seems that, for a given form of V_X, an isolated solution indeed exists.

In principle, there exists an alternative way of determinimg a_i more directly, although in the long run it may be more practical to solve the system (4.23). Namely, suppose one can reexpress the volume V_X in terms of the dual variables τ_i defined in (2.10).[9] With respect to τ_i the volume V_X is a homogeneous function of degree $7/4$. Then, we find

$$\frac{\partial V_X}{\partial \tau_i} = \sum_{j=1}^{N} \frac{\partial V_X}{\partial s_j}\frac{\partial s_j}{\partial \tau_i} = \sum_{j=1}^{N} \frac{\partial V_X}{\partial s_j}\frac{\partial s_i}{\partial \tau_j} = \sum_{j=1}^{N} \tau_j \frac{\partial s_i}{\partial \tau_j} = \frac{3}{4} s_i\,, \tag{4.24}$$

[9] One needs to ensure that in the new basis the signature of the Hessian $\frac{\partial^2 V_X}{\partial \tau_i \partial \tau_j}$ remains Lorentzian.

where we used the property that s_i are homogeneous functions of τ_i of degree $3/4$ and the symmetry of the Jacobi matrix

$$\frac{\partial \tau_i}{\partial s_j} = \frac{\partial^2 V_X}{\partial s_j \partial s_i} = \frac{\partial \tau_j}{\partial s_i}. \tag{4.25}$$

Then, using (2.16), (4.24) and (4.18) we obtain

$$a_i = \frac{s_i \tau_i}{V_X} = \frac{4\tau_i}{3V_X}\frac{\partial V_X}{\partial \tau_i} = \frac{4}{3}N_i \frac{\partial \ln V_X}{\partial \tau_i}\Big|_{\tau_i = N_i}, \tag{4.26}$$

where in the final step we used the property that the "angular" variables a_i do not scale. Then we can re-express the moduli vevs (4.18) as

$$s_i = \frac{4}{7}V_{\mathcal{Q}}\frac{\partial \ln V_X}{\partial \tau_i}\Big|_{\tau_i = N_i}. \tag{4.27}$$

Recall that all the integers N_i that describe the homology of the hidden sector associative cycle $\mathcal{Q}$ are fixed for a given manifold X. Therefore, according to (4.27) once we specify the microscopic details such as V_X and N_i, the vevs of *all* the moduli s_i are automatically determined in terms of a *single parameter* - the volume $V_{\mathcal{Q}}$ of the three-cycle $\mathcal{Q}$. Therefore, all masses and couplings, being functions of s_i, are also fixed in terms of $V_{\mathcal{Q}}$, including $\alpha_{GUT}^{-1} = \sum N_i^{vis} s_i = const \times V_{\mathcal{Q}}$.[10]

In an explicit realistic compactification, one could automatically determine the proportionality constant between α_{GUT}^{-1} and $V_{\mathcal{Q}}$ from the integers N_i^{vis} specifying the homology of the visible sector GUT three-cycle. Then, using the bottom-up MSSM value $\alpha_{GUT}^{-1} \approx 25$ one would be able to fix the volume $V_{\mathcal{Q}} \approx 25/const$ as well as *all* remaining couplings, including the visible sector Yukawa couplings and masses! Thus, given a realistic G_2 compactification one could *in principle* make genuine predictions and quickly rule out models that do not satisfy experimental constraints.[11] This extreme rigidity of fluxless G_2 vacua is quite remarkable and runs in stark contrast to the flexibility found for flux compactifications, where for a given manifold one can perform a very large scan over the integer fluxes and generate distributions of masses and couplings [31–33].

In order to illustrate how the system (4.23) is realized in practice we give a couple of explicit examples, though we stress that we have checked many more general examples than just those given here. Let us first consider a particularly simple N-parameter family of Kahler potentials consistent with G_2 holonomy where the volume V_X is given by

$$V_X = \prod_{i=1}^{N} s_i^{n_i}, \text{ where } \sum_{i=1}^{N} n_i = \frac{7}{3}. \tag{4.28}$$

[10] In our discussion we are neglecting all the subleading effects, e.g. the threshold corrections to α_{GUT} due to the Kaluza-Klein modes [14, 15] as well as possible Coleman-Weinberg-type loop corrections to (4.27).

[11] The current lack of explicit realistic G_2 examples presents a great challenge in implementing these ideas. However, we shall demonstrate here that one can nevertheless make significant progress in computing several quantities relevant for particle physics, e.g. soft terms in the supersymmetry breaking lagrangian, without relying on either a specific functional form of V_X or the microscopic details of the MSSM embedding.

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In this case the solutions to (4.23) are simply constants given by

$$a_i = n_i \,. \tag{4.29}$$

In fact, this example represents the class of Kahler potentials considered in the previous work [1, 2] and the solutions are discussed in detail there.

One may consider more complicated examples such as

$$V_X = \sum_k V_k \,, \text{ where } V_k \equiv c_k \prod_{i=1}^{N} s_i^{n_i^k} \,, \text{ such that} \,\forall k \;\; \sum_{i=1}^{N} n_i^k = \frac{7}{3} \,. \tag{4.30}$$

In this case system (4.23) translates into

$$\sum_k \left(n_i^k - a_i\right) c_k \prod_{j=1}^{N} \left(\frac{a_j}{N_j}\right)^{n_j^k} = 0 \,. \tag{4.31}$$

In these cases one can check numerically that, for very generic sets of parameters $\{n_i^k,\, c_k,\, N_i\}$, the system of equations (4.31) yields positive solutions for a_i, where the Hessian matrix $H(V_X)_{ij}$ has Lorentzian signature.[12] For example, choosing $N_1 = 1$, $N_2 = 1$, $N_3 = 1$, $N_4 = 1$ we numerically compute a_i for the following toy examples with four moduli

$$\begin{aligned}
&V_X = s_1^{\frac{7}{9}} s_2^{\frac{7}{9}} s_3^{\frac{7}{18}} s_4^{\frac{7}{18}} - \frac{1}{3} s_1^1 s_2^{\frac{2}{3}} s_3^{\frac{1}{3}} s_4^{\frac{1}{3}} - \frac{1}{2} s_1^{\frac{1}{3}} s_2^1 s_3^{\frac{1}{2}} s_4^{\frac{1}{2}} \;\Rightarrow\; a_1 \approx 1.038 \,,\, a_2 \approx 0.648 \,,\, a_3 \approx 0.324 \,,\, a_4 \approx 0.324 \,,\\
&V_X = s_2^{\frac{14}{9}} s_3^{\frac{7}{18}} s_4^{\frac{7}{18}} + \frac{1}{3} s_1^1 s_2^{\frac{2}{3}} s_4^{\frac{2}{3}} + \frac{1}{2} s_1^{\frac{1}{3}} s_2^1 s_3^1 \;\Rightarrow\; a_1 \approx 0.051 \,,\, a_2 \approx 1.478 \,,\, a_3 \approx 0.459 \,,\, a_3 \approx 0.344 \,,
\end{aligned} \tag{4.32}$$

where for both examples

$$\operatorname{sign}\left(\frac{\partial^2 V_X}{\partial s_i \partial s_j}\right)\Big|_{s_i = \frac{a_i}{N_i}} = (+,\, -,\, -,\, -) \,, \tag{4.33}$$

which explicitly demonstrates that having positive solutions for a_i is fairly generic and more importantly is guaranteed when V_X is not just a randomly picked homogeneous function of degree $7/3$ but represents an actual volume of a G_2 manifold X.

We now go on to consider de Sitter vacua by including the charged chiral matter fields Q and $\tilde{Q}$ into the hidden sector. The superpotential and the Kahler potential are given by (4.1) and (4.5). In order to compute the scalar potential we need to compute the inverse Kahler metric. Using the Kahler potential (4.5) together with (2.16), (2.21), (4.10) and (4.11) we first obtain the following components for the Kahler metric

$$\begin{aligned}
K_{i\bar{j}} &= \frac{3a_{\bar{j}}}{4s_i s_{\bar{j}}}\left(1 + \frac{\phi_0^2}{3V_X}\right)\Delta_{i\bar{j}} + \frac{a_i a_{\bar{j}}}{4s_i s_{\bar{j}}}\frac{\phi_0^2}{V_X} \,, \\
K_{i\bar{\phi}} &= i\frac{a_i}{2s_i}\frac{\phi}{V_X} \,, \quad K_{\phi\bar{j}} = -i\frac{a_{\bar{j}}}{2s_{\bar{j}}}\frac{\bar{\phi}}{V_X} \,, \quad K_{\phi\bar{\phi}} = \frac{1}{V_X} \,.
\end{aligned} \tag{4.34}$$

[12] This is necessary if the homogeneous function V_X is the volume of a genuine G_2-manifold. This also guarantees positive kinetic terms for the moduli fields.

Note that on the right hand side of the above expressions $a_{\bar{j}}$ and $\Delta_{i\bar{j}}$ are the same real quantities defined previously with index j replaced by $\bar{j}$.

The inverse Kahler metric must satisfy the following set of equations

$$\begin{aligned} K^{i\bar{j}}\, K_{\bar{j}\,k} + K^{i\bar{\phi}} K_{\bar{\phi}k} &= \delta^i_k\,, \\ K^{i\bar{j}}\, K_{\bar{j}\,\phi} + K^{i\bar{\phi}} K_{\bar{\phi}\phi} &= 0\,, \\ K^{\phi\bar{j}}\, K_{\bar{j}\,\phi} + K^{\phi\bar{\phi}} K_{\bar{\phi}\phi} &= 1\,. \end{aligned} \tag{4.35}$$

After a little bit of work we obtain the following components for the inverse Kahler metric

$$\begin{aligned} K^{i\bar{j}} &= \frac{4 s_i s_{\bar{j}}}{3 a_i}\frac{(\Delta^{-1})^{i\bar{j}}}{1+\frac{\phi_0^2}{3V_X}}\,, \quad K^{i\bar{\phi}} = i\frac{2}{3}\frac{s_i\bar{\phi}}{1+\frac{\phi_0^2}{3V_X}}\,, \quad K^{\phi\bar{j}} = -i\frac{2}{3}\frac{s_{\bar{j}}\phi}{1+\frac{\phi_0^2}{3V_X}}\,, \\ K^{\phi\bar{\phi}} &= V_X\left(1+\frac{7}{3}\frac{1}{1+\frac{\phi_0^2}{3V_X}}\frac{\phi_0^2}{3V_X}\right). \end{aligned} \tag{4.36}$$

Note that despite the fact that the matter part of the Kahler potential in (4.5) is only given up to the quadratic order in $\frac{\phi_0^2}{V_X}$, we decided to keep all the higher order terms inside the inverse Kahler metric. This is self-consistent as long as the combination $\frac{\phi_0^2}{3V_X}$ appearing in the inverse Kahler metric is stabilized at a value sufficiently smaller than one such that the quartic and higher order terms are suppressed.

Now, putting all the pieces together we obtain the scalar potential

$$\begin{aligned} V = \frac{e^{\phi_0^2/V_X}}{64\pi V_X^3}\Bigg[&\frac{4}{3}\sum_{i=1}^{N}\sum_{\bar{j}=1}^{N}\frac{s_i s_{\bar{j}}\,N_i N_{\bar{j}}}{a_i}\frac{(\Delta^{-1})^{i\bar{j}}}{1+\frac{\phi_0^2}{3V_X}}\left(b_1 A_1\phi_0^a e^{-b_1\vec{N}\cdot\vec{s}} - b_2 A_2 e^{-b_2\vec{N}\cdot\vec{s}}\right)^2 \\ &+ 4\vec{N}\cdot\vec{s}\left(b_1 A_1\phi_0^a e^{-b_1\vec{N}\cdot\vec{s}} - b_2 A_2 e^{-b_2\vec{N}\cdot\vec{s}}\right)\left(A_1\phi_0^a e^{-b_1\vec{N}\cdot\vec{s}} - A_2 e^{-b_2\vec{N}\cdot\vec{s}}\right) \\ &+ 7\left(A_1\phi_0^a e^{-b_1\vec{N}\cdot\vec{s}} - A_2 e^{-b_2\vec{N}\cdot\vec{s}}\right)^2\left(1+\frac{\phi_0^2}{3V_X}\right) \\ &- \frac{4}{3}\left(\frac{b_1 A_1\phi_0^a e^{-b_1\vec{N}\cdot\vec{s}} - b_2 A_2 e^{-b_2\vec{N}\cdot\vec{s}}}{1+\frac{\phi_0^2}{3V_X}}\vec{N}\cdot\vec{s} + \frac{7}{2}\left(A_1\phi_0^a e^{-b_1\vec{N}\cdot\vec{s}} - A_2 e^{-b_2\vec{N}\cdot\vec{s}}\right)\right) \\ &\times\left(aA_1\phi_0^a e^{-b_1\vec{N}\cdot\vec{s}} + \frac{\phi_0^2}{V_X}\left(A_1\phi_0^a e^{-b_1\vec{N}\cdot\vec{s}} - A_2 e^{-b_2\vec{N}\cdot\vec{s}}\right)\right) + \frac{V_X}{\phi_0^2}\left(1+\frac{7}{3}\frac{1}{1+\frac{\phi_0^2}{3V_X}}\frac{\phi_0^2}{3V_X}\right) \\ &\times\left(aA_1\phi_0^a e^{-b_1\vec{N}\cdot\vec{s}} + \frac{\phi_0^2}{V_X}\left(A_1\phi_0^a e^{-b_1\vec{N}\cdot\vec{s}} - A_2 e^{-b_2\vec{N}\cdot\vec{s}}\right)\right)^2 \\ &-3\left(A_1\phi_0^a e^{-b_1\vec{N}\cdot\vec{s}} - A_2 e^{-b_2\vec{N}\cdot\vec{s}}\right)^2\Bigg]. \end{aligned} \tag{4.37}$$

To understand the minima of the potential we will use the techniques developed earlier in [1, 2]. Namely, we will work in the regime when the volume of the hidden sector associative cycle $V_Q = \vec{N}\cdot\vec{s}$ is large and expand our solutions in the inverse powers of this volume. This is equivalent to an expansion in the UV weak hidden sector gauge coupling. In this long a tedious procedure we utilize the methods developed in [1, 2], yet with some important modifications.

Since we are considering the simplified case by setting $\kappa(s_i) = 1$ in the Kahler potential for the effective meson field, the supersymmetry breaking F-term contributions are functions of V_X and $\vec{N} \cdot \vec{s}$ only and therefore the scale invariant "angular" coordinates a_i will remain the same as in the supersymmetric case. On the other hand, the "radial" coordinate parameterized by V_Q (or V_X) will be shifted. Reintroducing the notation of [1, 2]

$$\alpha \equiv \frac{A_1 \phi_0^a}{A_2} e^{-(b1-b2)\vec{N}\cdot\vec{s}}, \quad x \equiv \alpha - 1\,, \quad y \equiv b_1\alpha - b_2\,, \quad z \equiv b_1^2\alpha - b_2^2\,, \tag{4.38}$$

we therefore make the following ansatz for the moduli vevs at the minimum

$$s_i = \frac{a_i}{N_i}\frac{x}{y}L\,. \tag{4.39}$$

In this notation, the volume of the associative cycles supporting the hidden sector gauge groups is given by

$$V_Q = \vec{N} \cdot \vec{s} = \frac{x}{y}L\sum_{i=1}^{N} a_i = \frac{7}{3}\frac{x}{y}L\,, \tag{4.40}$$

in which case the moduli ansatz (4.39) can be rewritten as

$$s_i = \frac{a_i}{N_i}\frac{3}{7}V_Q\,. \tag{4.41}$$

Let us first assume that L is non-zero and finite when $y \to 0$. This assumption will be verified in this section by determining L explicitly. Then, we get from (4.40) and the definitions above

$$\begin{aligned} V_Q \to \infty \quad \Rightarrow \quad y \to 0 \quad \Rightarrow \quad \alpha = \frac{b_1}{b_2} + \mathcal{O}\left(\frac{1}{V_Q}\right) \\ \Rightarrow V_Q = \vec{N}\cdot\vec{s} = \frac{1}{b_1 - b_2}\ln\left(\frac{b_1 A_1 \phi_0^a}{b_2 A_2}\right) = \frac{1}{2\pi}\frac{PQ}{Q-P}\ln\left(\frac{Q A_1 \phi_0^a}{P A_2}\right). \end{aligned} \tag{4.42}$$

This fixes the value of the volume V_Q of the hidden sector three-cycle.

We now go on to demonstrate that the ansatz for the moduli vevs (4.39) indeed represents the correct solution at the minimum of the scalar potential. In particular, we must verify our assumption that L is non-zero and finite in the limit $y \to 0$ by determining L self-consistently in this limit. Hence, we will now derive the equation for L and demonstrate explicitly that one of the possible solutions is indeed non-zero and finite in this limit. After minimizing the potential with respect to the moduli s_i and using the definitions (4.38) we obtain the following system of equations

$$\begin{aligned}\frac{\partial V}{\partial s_k} =& -\frac{3a_k}{s_k}\left(1+\frac{\phi^2}{3V_X}\right)V+\frac{e^{\phi^2/V_X}}{64\pi V_X^3}\left[\frac{4}{3}\frac{\partial}{\partial s_k}\left(\sum_{ij}\frac{s_is_jN_iN_j(\Delta^{-1})^{ij}}{a_i}\right)\frac{y^2}{1+\frac{\phi^2}{3V_X}}\right. \\ &-\frac{8}{3}\sum_{ij}\frac{s_is_jN_iN_j(\Delta^{-1})^{ij}}{a_i}\frac{N_kzy}{1+\frac{\phi^2}{3V_X}}+\frac{4}{3}\sum_{ij}\frac{s_is_jN_iN_j(\Delta^{-1})^{ij}}{a_i}\frac{y^2}{\left(1+\frac{\phi^2}{3V_X}\right)^2}\frac{\phi^2}{3V_X}\frac{a_k}{s_k} \\ &-4N_kxy-4N_k(\vec{N}\cdot\vec{s})y^2-4N_k(\vec{N}\cdot\vec{s})xz-7x^2\frac{\phi^2}{3V_X}\frac{a_k}{s_k} \\ &-2N_k\left(\frac{2}{3}\frac{y}{1+\frac{\phi^2}{3V_X}}-\frac{2}{3}(\vec{N}\cdot\vec{s})\frac{z}{1+\frac{\phi^2}{3V_X}}+\frac{2}{3}(\vec{N}\cdot\vec{s})\frac{y}{\left(1+\frac{\phi^2}{3V_X}\right)^2}\frac{\phi^2}{3V_X}\frac{a_k}{s_kN_k}-\frac{7}{3}y\right)\left(a\alpha+\frac{\phi^2}{V_X}x\right) \\ &+2N_k\left(\frac{2}{3}(\vec{N}\cdot\vec{s})\frac{y}{1+\frac{\phi^2}{3V_X}}+\frac{7}{3}x\right)\left(b_1a\alpha+\frac{\phi^2}{V_X}y\right) \\ &+\frac{a_k}{s_k}\frac{V_X}{\phi^2}\left(1+\frac{7}{3}\frac{1}{1+\frac{\phi^2}{3V_X}}\frac{\phi^2}{3V_X}\right)\left(a\alpha+\frac{\phi^2}{V_X}x\right)^2-\frac{7}{9}\frac{a_k}{s_k}\frac{1}{\left(1+\frac{\phi^2}{3V_X}\right)^2}\left(a\alpha+\frac{\phi^2}{V_X}x\right)^2 \\ &\left.-2N_k\frac{V_X}{\phi^2}\left(1+\frac{7}{3}\frac{1}{1+\frac{\phi^2}{3V_X}}\frac{\phi^2}{3V_X}\right)\left(a\alpha+\frac{\phi^2}{V_X}x\right)\left(ab_1\alpha+\frac{\phi^2}{V_X}\frac{a_k}{s_kN_k}x+\frac{\phi^2}{V_X}y\right)\right]A_2e^{-b_2\vec{N}\cdot s}=0\,, \end{aligned} \tag{4.43}$$

where in one of the intermediate steps we simplified

$$\frac{V_X}{\phi^2}\left(\frac{7}{3}\frac{1}{\left(1+\frac{\phi^2}{3V_X}\right)^2}\left(\frac{\phi^2}{3V_X}\right)^2\frac{a_k}{s_k}-\frac{7}{3}\frac{1}{1+\frac{\phi^2}{3V_X}}\frac{\phi^2}{3V_X}\frac{a_k}{s_k}\right)=-\frac{7}{9}\frac{1}{\left(1+\frac{\phi^2}{3V_X}\right)^2}\frac{a_k}{s_k}. \tag{4.44}$$

Multiplying (4.43) by $\frac{s_k}{a_kx^2}$ and using the explicit expression for the potential (4.37) in terms of the quantities (4.38) we obtain

$$\begin{aligned} &-3\left(1+\frac{\phi^2}{3V_X}\right)\left[\frac{4}{3}\sum_{ij}\frac{s_is_jN_iN_j(\Delta^{-1})^{ij}}{a_i}\frac{y^2}{x^2}\frac{1}{1+\frac{\phi^2}{3V_X}}+4(\vec{N}\cdot\vec{s})\frac{y}{x}+7\left(1+\frac{\phi^2}{3V_X}\right)\right. \\ &\left.-2\left(\frac{2}{3}(\vec{N}\cdot\vec{s})\frac{y/x}{1+\frac{\phi^2}{3V_X}}+\frac{7}{3}\right)\left(\frac{a\alpha}{x}+\frac{\phi^2}{V_X}\right)+\frac{V_X}{\phi^2}\left(1+\frac{7}{3}\frac{1}{1+\frac{\phi^2}{3V_X}}\frac{\phi^2}{3V_X}\right)\left(\frac{a\alpha}{x}+\frac{\phi^2}{V_X}\right)^2-3\right] \\ &+\frac{4}{3}\frac{s_k}{a_k}\frac{\partial}{\partial s_k}\left(\sum_{ij}\frac{s_is_jN_iN_j(\Delta^{-1})^{ij}}{a_i}\right)\frac{1}{1+\frac{\phi^2}{3V_X}}\frac{y^2}{x^2} \\ &-\frac{8}{3}\sum_{ij}\frac{s_is_jN_iN_j(\Delta^{-1})^{ij}}{a_i}\frac{zy}{x^2}\frac{1}{1+\frac{\phi^2}{3V_X}}\frac{N_ks_k}{a_k}+\frac{4}{3}\sum_{ij}\frac{s_is_jN_iN_j(\Delta^{-1})^{ij}}{a_i}\frac{y^2}{x^2}\frac{1}{\left(1+\frac{\phi^2}{3V_X}\right)^2}\frac{\phi^2}{3V_X} \\ &-4\frac{N_ks_k}{a_k}\frac{y}{x}-4\frac{N_ks_k}{a_k}(\vec{N}\cdot\vec{s})\frac{y^2}{x^2}-4\frac{N_ks_k}{a_k}(\vec{N}\cdot\vec{s})\frac{z}{x}-7\frac{\phi^2}{3V_X} \\ &-2\frac{N_ks_k}{a_k}\left(\frac{2}{3}\frac{y/x}{1+\frac{\phi^2}{3V_X}}-\frac{2}{3}(\vec{N}\cdot\vec{s})\frac{z/x}{1+\frac{\phi^2}{3V_X}}+\frac{2}{3}(\vec{N}\cdot\vec{s})\frac{y/x}{\left(1+\frac{\phi^2}{3V_X}\right)^2}\frac{\phi^2}{3V_X}\frac{a_k}{s_kN_k}-\frac{7}{3}\frac{y}{x}\right)\left(\frac{a\alpha}{x}+\frac{\phi^2}{V_X}\right) \end{aligned} \tag{4.45}$$

$$
\begin{aligned}
&+2\frac{N_k s_k}{a_k}\left(\frac{2}{3}(\vec{N}\cdot\vec{s})\frac{y/x}{1+\frac{\phi^2}{3V_X}}+\frac{7}{3}\right)\left(\frac{b_1 a\alpha}{x}+\frac{\phi^2}{V_X}\frac{y}{x}\right)\\
&+\frac{V_X}{\phi^2}\left(1+\frac{7}{3}\frac{1}{1+\frac{\phi^2}{3V_X}}\frac{\phi^2}{3V_X}\right)\left(\frac{a\alpha}{x}+\frac{\phi^2}{V_X}\right)^2-\frac{7}{9}\frac{1}{\left(1+\frac{\phi^2}{3V_X}\right)^2}\left(\frac{a\alpha}{x}+\frac{\phi^2}{V_X}\right)^2\\
&-2\frac{N_k s_k}{a_k}\frac{V_X}{\phi^2}\left(1+\frac{7}{3}\frac{1}{1+\frac{\phi^2}{3V_X}}\frac{\phi^2}{3V_X}\right)\left(\frac{a\alpha}{x}+\frac{\phi^2}{V_X}\right)\left(\frac{ab_1\alpha}{x}+\frac{\phi^2}{V_X}\frac{a_k}{s_k N_k}+\frac{\phi^2}{V_X}\frac{y}{x}\right)=0\,.
\end{aligned}
$$

At first sight it appears that finding an analytic expression for L from (4.45) is hopeless since a closed form for $(\Delta^{-1})^{i\bar{j}}$ is unknown and a_i have not been determined explicitly. However, upon further examination we notice that in order to find L from (4.45) we only need to know the contraction rules (2.17), (2.20) and (2.29). Indeed, using the the ansatz (4.39) together with the definition (2.18) and applying (2.17), (2.20) and (2.29) we first evaluate the terms

$$
\begin{aligned}
\frac{s_k}{a_k}\frac{\partial}{\partial s_k}\left(\sum_{ij}\frac{s_i s_j N_i N_j(\Delta^{-1})^{ij}}{a_i}\right) &= \frac{x^2}{y^2}L^2\frac{1}{a_k}\sum_j(\Delta^{-1})^{kj}a_j+\frac{x^2}{y^2}L^2\sum_i(\Delta^{-1})^{ik} \qquad (4.46)\\
&+\frac{x^2}{y^2}L^2\frac{1}{a_k}\sum_i\left(\frac{P_{ik}}{a_i}\sum_j(\Delta^{-1})^{ij}a_j\right)+\frac{x^2}{y^2}L^2\sum_j\left(a_j\sum_i\frac{\partial(\Delta^{-1})^{ij}}{\partial s_k}\right)\\
&=2\frac{x^2}{y^2}L^2+\frac{x^2}{y^2}L^2\frac{1}{a_k}\sum_i P_{ik}+\frac{x^2}{y^2}L^2\sum_j\left(a_j\frac{\partial}{\partial s_k}\sum_i(\Delta^{-1})^{ij}\right)=2\frac{x^2}{y^2}L^2\,,
\end{aligned}
$$

$$
\sum_{ij}\frac{s_i s_j N_i N_j(\Delta^{-1})^{ij}}{a_i}=\frac{x^2}{y^2}L^2\sum_i\sum_j(\Delta^{-1})^{ij}a_j=\frac{x^2}{y^2}L^2\sum_i a_i=\frac{7}{3}\frac{x^2}{y^2}L^2\,, \qquad (4.47)
$$

and then use the same ansatz (4.39) and contraction identities for the rest of the terms in (4.45) to obtain the following equation for L

$$
\begin{aligned}
&-3\left(1+\frac{\phi^2}{3V_X}\right)\Bigg[\frac{28}{9}L^2\frac{1}{1+\frac{\phi^2}{3V_X}}+\frac{28}{3}L+7\left(1+\frac{\phi^2}{3V_X}\right) \qquad (4.48)\\
&-2\left(\frac{14}{9}\frac{L}{1+\frac{\phi^2}{3V_X}}+\frac{7}{3}\right)\left(\frac{a\alpha}{x}+\frac{\phi^2}{V_X}\right)+\frac{V_X}{\phi^2}\left(1+\frac{7}{3}\frac{1}{1+\frac{\phi^2}{3V_X}}\frac{\phi^2}{3V_X}\right)\left(\frac{a\alpha}{x}+\frac{\phi^2}{V_X}\right)^2-3\Bigg]\\
&+\frac{8}{3}\frac{L^2}{1+\frac{\phi^2}{3V_X}}-\frac{56}{9}\frac{zx}{y^2}\frac{L^3}{1+\frac{\phi^2}{3V_X}}+\frac{28}{9}\frac{L^2}{\left(1+\frac{\phi^2}{3V_X}\right)^2}\frac{\phi^2}{3V_X}\\
&-4L-\frac{28}{3}L^2-\frac{28}{3}\frac{zx}{y^2}L^2-7\frac{\phi^2}{3V_X}\\
&-2L\left(\frac{2}{3}\frac{1}{1+\frac{\phi^2}{3V_X}}-\frac{14}{9}\frac{L}{1+\frac{\phi^2}{3V_X}}\frac{zx}{y^2}+\frac{14}{9}\frac{L}{\left(1+\frac{\phi^2}{3V_X}\right)^2}\frac{\phi^2}{3V_X}-\frac{7}{3}\right)\left(\frac{a\alpha}{x}+\frac{\phi^2}{V_X}\right)\\
&+2L\left(\frac{14}{9}\frac{L}{1+\frac{\phi^2}{3V_X}}+\frac{7}{3}\right)\left(\frac{b_1 a\alpha}{y}+\frac{\phi^2}{V_X}\right)
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{V_X}{\phi^2}\left(1+\frac{7}{3}\frac{1}{1+\frac{\phi^2}{3V_X}}\frac{\phi^2}{3V_X}\right)\left(\frac{a\alpha}{x}+\frac{\phi^2}{V_X}\right)^2-\frac{7}{9}\frac{1}{\left(1+\frac{\phi^2}{3V_X}\right)^2}\left(\frac{a\alpha}{x}+\frac{\phi^2}{V_X}\right)^2\\
&-2L\frac{V_X}{\phi^2}\left(1+\frac{7}{3}\frac{1}{1+\frac{\phi^2}{3V_X}}\frac{\phi^2}{3V_X}\right)\left(\frac{a\alpha}{x}+\frac{\phi^2}{V_X}\right)\left(\frac{ab_1\alpha}{y}+\frac{\phi^2}{V_X}\right)\\
&-2\left(1+\frac{7}{3}\frac{1}{1+\frac{\phi^2}{3V_X}}\frac{\phi^2}{3V_X}\right)\left(\frac{a\alpha}{x}+\frac{\phi^2}{V_X}\right)=0\,,
\end{aligned}
$$

Multiplying the above equation by $\frac{3}{28}(1+\frac{\phi^2}{3V_X})\frac{y^2}{zx}$ and taking the limit $y\to 0$ we obtain

$$
\frac{2}{3}L^3+L^2\left(1-\frac{a\alpha}{3x}\right)-L^2\frac{b_1a\alpha y}{3xz}-L\frac{b_1a\alpha y}{2xz}\left(1+\frac{\phi_0^2}{3V_X}\right)+L\frac{3b_1a\alpha y}{14xz}\left(1+\frac{10}{9}\frac{\phi_0^2}{V_X}\right)\left(\frac{a\alpha V_X}{\phi_0^2 x}+1\right)=0\,,
$$

where we dropped terms of $\mathcal{O}(y^2)$ and higher. A non-trivial solution can be obtained by solving the corresponding quadratic equation

$$
\frac{2}{3}L^2+L\left(1-\frac{a\alpha}{3x}\right)-L\frac{b_1a\alpha y}{3xz}-\frac{b_1a\alpha y}{2xz}\left(1+\frac{\phi_0^2}{3V_X}\right)+\frac{3b_1a\alpha y}{14xz}\left(1+\frac{10}{9}\frac{\phi_0^2}{V_X}\right)\left(\frac{a\alpha V_X}{\phi_0^2 x}+1\right)=0 \tag{4.49}
$$

which is analogous to the equation in the second line in (126) of [1, 2].

Solving (4.49) to the first subleading order in y results in

$$
L=-\frac{3}{2}\left(1-\frac{a\alpha}{3x}\right)+y\frac{3b_1a\alpha}{14xz}\frac{1+\frac{a\alpha V_X}{\phi_0^2 x}}{1-\frac{a\alpha}{3x}}\left(1+\frac{\phi_0^2}{3V_X}\right). \tag{4.50}
$$

Hence, we see that this solution is non-zero and finite when $y\to 0$ and therefore is self-consistent. This is the solution describing the minimum of the potential. We must note that there is another possible solution of (4.49) for which $L\sim y\to 0$. In fact this other solution corresponds to the extremum at the top of the potential barrier and we will not discuss it further. Using (4.50) we can now compute the first subleading order correction to α to obtain

$$
\begin{aligned}
\alpha &= \frac{P}{Q}+\frac{7P\left(3(Q-P)-2\right)}{12\pi Q}\frac{1}{V_Q} \\
&= \frac{P}{Q}+\frac{7(Q-P)^2}{2Q^2}\left(1-\frac{2}{3(Q-P)}\right)\frac{P}{P_{\rm eff}}\,,
\end{aligned} \tag{4.51}
$$

where we have introduced

$$
P_{\rm eff}\equiv P\ln\left(\frac{QA_1\phi_0^a}{PA_2}\right). \tag{4.52}
$$

Using (4.51) we can express the solution for L from (4.50) as

$$
\begin{aligned}
L &= -\frac{3}{2}\left(1-\frac{2}{3(Q-P)}\right)+\frac{7}{2P_{\rm eff}}\left(1-\frac{2}{3(Q-P)}\right) \\
&\quad+\frac{3}{2P_{\rm eff}}\left(1+\frac{2V_X}{(Q-P)\phi_0^2}\right)\left(1+\frac{\phi_0^2}{3V_X}\right).
\end{aligned} \tag{4.53}
$$

In the leading order, the moduli vevs are given by

$$s_i = \frac{a_i}{N_i}\frac{3QP_{\rm eff}}{14\pi(Q-P)}\,. \tag{4.54}$$

We note that since a_i, N_i are positive, we need $P_{\rm eff} > 0$ if $Q > P$, so that there exists a local minimum with $s_i > 0$.

The next step is to determine the vev of the effective meson field by minimizing the potential with respect to ϕ_0. Let us first compute the potential at the minimum as a function of the meson. The result is given in equation (4.67) and the reader not interested in its derivation may proceed directly there. It turns out that since the moduli vevs at the minimum are proportional to a_i/N_i as in (4.41), explicit computation of the F-terms at the minimum and various contractions thereof while using the rules (2.17) and (2.29) becomes possible. Let us demonstrate some of these computations in detail. First we need to identify the gravitino mass in terms of our notation in (4.38). Using the usual definition in combination with (4.38) we have

$$m_{3/2} = e^{K/2}|W| = e^{K/2}|x|A_2 e^{-b_2\vec{N}\cdot\vec{s}}\,. \tag{4.55}$$

Because the existence of de Sitter vacua requires $Q-P > 0$ (see [1, 2] for details) we obtain using (4.51) that

$$x \approx \frac{P}{Q} - 1 < 0\,. \tag{4.56}$$

On the other hand, since $m_{3/2} > 0$ we can express the following combination in terms of the gravitino mass

$$e^{K/2}xA_2e^{-b_2\vec{N}\cdot\vec{s}} = -m_{3/2}\,. \tag{4.57}$$

We now multiply F_i in (4.8) by $e^{K/2}$ and using (4.38) and (4.57) express

$$\begin{aligned} e^{K/2}F_i &= iN_ie^{i\gamma_W}\left(-y - x\frac{3a_i}{2s_iN_i}\left(1+\frac{\phi_0^2}{3V_X}\right)\right)e^{K/2}A_2e^{-b_2\vec{N}\cdot\vec{s}} \\ &= iN_ie^{i\gamma_W}\left(\frac{y}{x} + \frac{3a_i}{2s_iN_i}\left(1+\frac{\phi_0^2}{3V_X}\right)\right)m_{3/2}\,, \end{aligned} \tag{4.58}$$

where γ_W denotes the overall phase of the superpotential. Using the ansatz (4.39) for s_i we obtain from (4.58)

$$\begin{aligned} e^{K/2}F_i &= iN_ie^{i\gamma_W}\left(\frac{y}{x} + \frac{3y}{2xL}\left(1+\frac{\phi_0^2}{3V_X}\right)\right)m_{3/2} \\ &= iN_ie^{i\gamma_W}\frac{7}{3V_Q}\left(L+\frac{3}{2}\left(1+\frac{\phi_0^2}{3V_X}\right)\right)m_{3/2}\,, \end{aligned} \tag{4.59}$$

where in the second line we used

$$\frac{x}{y}L = \frac{3}{7}V_Q\,, \tag{4.60}$$

obtained from (4.40). Similarly, we find from (4.8) using (4.38) together with (4.57)

$$e^{K/2}F_\phi = e^{i(\gamma_W-\theta)}\left(\frac{a\alpha}{\phi_0 x} + \frac{\phi_0}{V_X}\right)m_{3/2}\,. \tag{4.61}$$

Before computing $e^{K/2}F^i$ we would like to express the $K^{i\bar{j}}$ components of the inverse Kahler metric at the minimum using the ansatz (4.39) for $s_{\bar{j}}$ as follows

$$K^{i\bar{j}} = \frac{4s_i s_{\bar{j}}}{3a_i}\frac{(\Delta^{-1})^{i\bar{j}}}{1+\frac{\phi_0^2}{3V_X}} = \frac{xL}{y}\frac{4s_i a_{\bar{j}}}{3a_i N_{\bar{j}}}\frac{(\Delta^{-1})^{i\bar{j}}}{1+\frac{\phi_0^2}{3V_X}} = V_{\mathcal{Q}}\frac{4s_i a_{\bar{j}}}{7a_i N_{\bar{j}}}\frac{(\Delta^{-1})^{i\bar{j}}}{1+\frac{\phi_0^2}{3V_X}}\,. \tag{4.62}$$

Contracting (4.59) and (4.61) with the inverse Kahler metric and using the solution for L from (4.50) we then obtain

$$\begin{aligned} e^{K/2}F^i &= e^{K/2}K^{i\bar{j}}\,\bar{F}_{\bar{j}} + e^{K/2}K^{i\bar{\phi}}\bar{F}_{\bar{\phi}} \\ &= -ie^{-i\gamma_W}\frac{4s_i}{3a_i}\sum_{\bar{j}=1}^{N} a_{\bar{j}}(\Delta^{-1})^{i\bar{j}}\left(\frac{L}{1+\frac{\phi_0^2}{3V_X}}+\frac{3}{2}\right)m_{3/2} + ie^{-i\gamma_W}\frac{2}{3}\frac{s_i}{1+\frac{\phi_0^2}{3V_X}}\left(\frac{a\alpha}{x}+\frac{\phi_0^2}{V_X}\right)m_{3/2} \\ &= -is_i e^{-i\gamma_W}\frac{2yb_1 a\alpha}{7xz}\frac{1+\frac{a\alpha V_X}{\phi_0^2 x}}{1-\frac{a\alpha}{3x}}m_{3/2} \approx -ie^{-i\gamma_W}\frac{2s_i}{P_{\rm eff}}\left(1+\frac{a\alpha V_X}{\phi_0^2 x}\right)m_{3/2}\,, \end{aligned} \tag{4.63}$$

where in the last line we used (4.51) to plug into x, y, and z defined by (4.38) except for the combination $\left(1+\frac{a\alpha V_X}{\phi_0^2 x}\right)$ and kept the leading term in $1/P_{\rm eff}$. Note that in order to get from the second to third line in (5.18) we used the second contraction property in (2.29). Similarly, contracting (4.59) and (4.61) with the corresponding components of the inverse Kahler metric (4.36) we obtain

$$e^{K/2}F^{\phi} = e^{-i\gamma_W}\phi\left(1-\frac{7}{3P_{\rm eff}}\right)\left(1+\frac{a\alpha V_X}{\phi_0^2 x}\right)m_{3/2}\,. \tag{4.64}$$

Using the results (4.59), (4.61), (4.63) and (4.64) together with (4.50) and (4.51) we can compute the following contributions

$$\begin{aligned} e^K F^i F_i &= \frac{7}{P_{\rm eff}}\left(\frac{a\alpha}{x}+\frac{\phi_0^2}{V_X}\right)^2\left(\frac{V_X}{\phi_0^2}\right)^2\left[\frac{\phi_0^2}{3V_X}+\frac{1}{P_{\rm eff}}\left(1+\frac{\phi_0^2}{3V_X}\right)\right]m_{3/2}^2 \\ e^K F^{\phi}F_{\phi} &= \left(\frac{a\alpha}{x}+\frac{\phi_0^2}{V_X}\right)^2\frac{V_X}{\phi_0^2}\left(1-\frac{7}{3P_{\rm eff}}\right)m_{3/2}^2\,, \end{aligned} \tag{4.65}$$

where we also used $\vec{N}\cdot\vec{s} = V_{\mathcal{Q}}$ while performing the computations in the first line of (4.65).

Then, the potential at the minimum is given by

$$\begin{aligned} V_0 &= e^K(F^iF_i + F^{\phi}F_{\phi} - 3|W|^2) \\ &= \left(\frac{a\alpha}{x}+\frac{\phi_0^2}{V_X}\right)^2\frac{V_X}{\phi_0^2}m_{3/2}^2 + \frac{7}{P_{\rm eff}^2}\left(\frac{a\alpha}{x}+\frac{\phi_0^2}{V_X}\right)^2\left(1+\frac{\phi_0^2}{3V_X}\right)\left(\frac{V_X}{\phi_0^2}\right)^2 m_{3/2}^2 - 3m_{3/2}^2\,. \end{aligned} \tag{4.66}$$

Using (4.51) and dropping the terms of order $\mathcal{O}(1/P_{\rm eff}^2)$ we obtain the following expression for the leading contribution to the vacuum energy as a function of the meson field

$$V_0 = \left[\left(\frac{2}{Q-P}+\frac{\phi_0^2}{V_X}\right)^2 + \frac{14}{P_{\rm eff}}\left(1-\frac{2}{3(Q-P)}\right)\left(\frac{2}{Q-P}+\frac{\phi_0^2}{V_X}\right) - 3\frac{\phi_0^2}{V_X}\right]\frac{V_X}{\phi_0^2}m_{3/2}^2\,. \tag{4.67}$$

The polynomial in the square brackets in (4.67) is quadratic with respect to the canonically normalized meson vev squared $\phi_c^2 \equiv \phi_0^2/V_X$ with the coefficient of the $(\phi_0^2/V_X)^2$ monomial being positive $(+1)$ and therefore, the minimum V_0 is positive when the corresponding discriminant is negative. Tuning the cosmological constant to zero is then equivalent to setting the discriminant of the above polynomial to zero, which boils down to a simple condition

$$P_{\rm eff} = \frac{14(3(Q-P)-2)}{3(3(Q-P)-2\sqrt{6(Q-P)})}\,. \tag{4.68}$$

Note that $P_{\rm eff}$ defined in (4.52) is actually dependent on ϕ but because of the smallness of a and the Log dependence, it was safe to use the approximation $P_{\rm eff} \approx const$. This approximation turned out to be self-consistent since $P_{\rm eff}$ is fairly large. From (4.68) we see immediately that

$$P_{\rm eff} > 0 \;\Rightarrow\; Q-P \geq 3\,. \tag{4.69}$$

Minimizing (4.67) with respect to ϕ_c^2 and imposing the condition that the expression in the square brackets in (4.67) is tuned to zero, we obtain the meson vev at the minimum in the leading order

$$\phi_c^2 = \frac{\phi_0^2}{V_X} \approx \frac{2}{Q-P} + \frac{7}{P_{\rm eff}}\left(1 - \frac{2}{3(Q-P)}\right)\,. \tag{4.70}$$

If we tune the leading contribution to the vacuum energy and set $Q-P=3$ we obtain

$$P_{\rm eff} \approx 63.5\,,\quad \frac{\phi_0^2}{V_X} \approx 0.75\,. \tag{4.71}$$

Recalling the factor of two in the definition of the meson field (3.28) we note that along the D-flat direction, the bilinears of the canonically normalized charged matter fields that appear in the original Kahler potential have a somewhat smaller vev

$$< Q_c Q_c^\dagger > = < \tilde{Q}_c \tilde{Q}_c^\dagger > = < Q_c \tilde{Q}_c > \approx 0.37\, m_{pl}^2\,, \tag{4.72}$$

which makes it a bit easier to justify the truncation of the higher order terms in the Kahler potential for hidden sector matter.

We find numerically that for the minimum value $Q-P=3$, the tuning of the cosmological constant by varying the constants A_1 and A_2 inside the superpotential results in fixing the value of $P_{\rm eff}$ at

$$P_{\rm eff} \approx 61.648\,, \tag{4.73}$$

while the canonically normalized meson vev squared is stabilized at

$$\phi_c^2 = \frac{\phi_0^2}{V_X} \approx 0.746\,, \tag{4.74}$$

thus confirming the analytical results above. For example, we obtain the values in (4.73) and (4.74) by minimizing the scalar potential numerically for the following toy example with two moduli

$$P = 27\,,\; Q = 30\,,\; A_1 = 27\,,\; A_2 = 2.1544\,,\; N_1 = N_2 = 1\,, V_X = s_1^{\frac{7}{6}} s_2^{\frac{7}{6}} + \frac{1}{3} s_1^{1} s_2^{\frac{4}{3}} + \frac{1}{2} s_1^{\frac{1}{3}} s_2^{2}$$
$$\Rightarrow s_1 \approx 34.52\,,\;\; s_2 \approx 63.13\,.$$

It is instructive to compare the moduli vevs obtained above numerically with the values obtained by using the analytic expression (4.54). However, before we can apply (4.54) we need to determine the values of a_i at the minimum. This can be done by plugging the values of N_i, n_i^k and c_k into the system (4.31) and solving it numerically. To compute s_i from (4.54) we use $P_{\rm eff}$ from (4.73) in order to get a better accuracy. As a result, we obtain:

$$a_1 \approx 0.825\,, \quad a_2 \approx 1.51\,, \quad s_1 \approx 34.7\,, \quad s_2 \approx 63.4\,,$$

which confirms explicitly that the analytic expression (4.54) for the moduli vevs at the minimum is indeed very accurate and reliable. Here we also verified that the Hessian of the volume has Lorentzian signature.

Although the value in (4.74) is not much smaller than one, the combination $\frac{\phi_0^2}{3V_X}$ inside the inverse Kahler metric (4.36) has a value

$$\frac{\phi_0^2}{3V_X} \approx 0.25\,, \tag{4.75}$$

which is small enough to make the quartic and higher order terms which we kept inside the inverse Kahler metric much smaller.

As we will see in the computations that follow, the value of $P_{\rm eff}$ will enter into many quantities relevant for particle physics, such as tree-level gaugino masses, etc. Here we note that small changes in $P_{\rm eff}$ do not affect the supersymmetry breaking masses much, but do change the cosmological constant significantly. For instance, while changing the value of $P_{\rm eff}$ in the range $61 \leq P_{\rm eff} \leq 62$ hardly affects the values of the soft breaking terms, as will be evident from the corresponding explicit formulas, such small changes in $P_{\rm eff}$ result in vastly different values of the vacuum energy:

$$61 \leq P_{\rm eff} \leq 62 \quad \Rightarrow \quad -\left(m_{3/2}m_{pl}\right)^2 \times 10^{-3} \lesssim V_0 \lesssim +\left(m_{3/2}m_{pl}\right)^2 \times 10^{-3}\,. \tag{4.76}$$

Therefore, once we coarsely tune $P_{\rm eff}$ to its approximate value, the cosmological constant problem becomes completely decoupled from the rest of particle physics. Even though this should be the case, it is satisfying to see it explicitly in a complete example of moduli coupled to matter.

Note also that in the original paper [1, 2] we obtained $P_{\rm eff} \approx 83$. This is due to the different matter Kahler potential considered there. As we will see, this numerical difference will result in slightly different values for the soft breaking terms if compared to those obtained in [1–3].

Recall that for a stable minimum to exist it is necessary that $Q - P \geq 3$. We have seen that when $Q - P = 3$ and the minimum of the potential is approximately tuned to zero, the value of $P_{\rm eff} \approx 60$ which ensures that the moduli (4.54) can be reliably fixed at values large enough to satisfy the supergravity approximation. On the other hand, when $Q - P = 4$, from (4.68) we get $P_{\rm eff} \approx 20$, in which case if Q is fixed, the moduli vevs become smaller by about a factor of four. Thus, unless the ranks of hidden sector gauge groups are incredibly large, situations when $Q - P > 3$ may put our solutions well outside of the supergravity approximation. Therefore, from now on we will only consider the case when $Q - P = 3$ to ensure the validity of the regime where our construction is reliable.

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5 Masses and soft supersymmetry breaking terms

5.1 Gravitino mass

In supergravity the bare gravitino mass is defined as

$$m_{3/2} = m_{pl} e^{K/2} |W| = e^{K/2} |x| A_2 e^{-b_2 V_Q} \,, \tag{5.1}$$

and can now be computed since we stabilized V_Q explicitly. It is given by

$$m_{3/2} = m_{pl} \frac{e^{\frac{\phi_0^2}{2V_X}}}{8\sqrt{\pi} V_X^{3/2}} |P - Q| \frac{A_2}{Q} e^{-\frac{P_{\rm eff}}{Q-P}} \,. \tag{5.2}$$

When the cosmological constant is tuned such that (4.73) is satisfied, for $Q - P = 3$ we obtain

$$m_{3/2} \approx 9 \times 10^5 (\text{TeV}) \frac{C_2}{V_X^{3/2}} \,, \tag{5.3}$$

where $C_2 \equiv A_2/Q$ was defined in (3.27). Calculating C_2 goes beyond the scope of this paper. Here we will treat C_2 as a phenomenological parameter with values $C_2 \sim \mathcal{O}(0.1-1)$ since it may experience a mild exponential suppression as in (3.27).

On the other hand, the actual value at which the volume V_X must be stabilized can be almost uniquely determined from the scale of Grand Unification. In particular, we can use equation (4.12) in [14, 15] to express

$$G_N = \frac{1}{8\pi m_p^2} = \frac{\alpha_{GUT}^3 V_Q^{7/3} L(\mathcal{Q})^{2/3}}{32\pi^2 M_{GUT}^2 V_X} \,, \tag{5.4}$$

where the factor $L(\mathcal{Q})$ is due to the threshold corrections from the Kaluza-Klein modes and is given by

$$L(\mathcal{Q}) = 4q \sin^2 (5\pi w/q) \,, \tag{5.5}$$

such that $5w$ is not divisible by q. For typical values

$$\alpha_{GUT} = \frac{1}{V_Q} = \frac{1}{25} \,, \quad M_{GUT} = 2 \times 10^{16}\, \text{GeV} \,, \tag{5.6}$$

we obtain

$$V_X = 137.4 \times L(\mathcal{Q})^{2/3} \,. \tag{5.7}$$

In table 1 we list a few typical benchmark values for the volume and the resulting gravitino mass up to the overall factor C_2.

Interestingly, the gravitino mass scale naturally turns out to be constrained to $m_{3/2} \sim \mathcal{O}(10)$TeV. While this is presumably large enough to alleviate the gravitino problem, it is also small enough to give *some* of the superpartners masses which can be easily accessible at the LHC energies. As we will see below, this is possible because of the significant suppression of the tree-level gaugino masses relative to $m_{3/2}$.

	Point 1	Point 2	Point 3	Point 4	Point 5	Point 6	Point 7
q	2	3	4	4	6	6	6
w	1	1, 2	1, 3	2	1, 5	2, 4	3
V_X	549.6	594.5	549.6	872.4	453.7	943.7	1143.2
$m_{3/2}/C_2$	70 TeV	62 TeV	70 TeV	35 TeV	93.0 TeV	31 TeV	23 TeV

Table 1. Typical values of V_X and $m_{3/2}$ divided by C_2 for different values of q and w.

In addition to the gravitino mass it is instructive to compute the scale of gaugino condensation. Using (4.73) the volume of the hidden sector associative cycle for $Q - P = 3$ is given by

$$V_Q = \frac{QP_{\text{eff}}}{2\pi(Q-P)} \approx \frac{10Q}{\pi}\,, \tag{5.8}$$

From (3.22) the scale of gaugino condensation in the second hidden sector is

$$\Lambda \sim m_{pl}\frac{e^{-\frac{2\pi}{3Q}V_Q}}{2\pi^{1/6}V_X^{1/2}} \approx m_{pl}\frac{e^{-20/3}}{2\pi^{1/6}V_X^{1/2}} \approx \frac{1.1\times 10^{14}\text{GeV}}{L(Q)^{1/3}}\,. \tag{5.9}$$

5.2 Moduli masses

In order to compute the masses of the moduli we first need to evaluate the matrix V_{mn} with $m, n = \overline{1, N+1}$, with components given by

$$V_{ij} = \frac{\partial^2 V}{\partial s_i \partial s_j}\,, \quad V_{i\,N+1} = \frac{\partial^2 V}{\partial s_i \partial \phi_0}\,, \quad V_{N+1\,N+1} = \frac{\partial^2 V}{\partial \phi_0 \partial \phi_0}\,, \tag{5.10}$$

at the minimum of the potential. However, because the Kahler metric in (4.34) is not diagonal, we also need to find a unitary transformation U which diagonalizes the Kahler metric. We denote all the components of the Kahler metric as $K_{m\bar{n}}$. Then, by diagonalizing $K_{m\bar{n}}$ we obtain

$$K_k \delta_{k\bar{l}} = U^\dagger_{km} K_{m\bar{n}} U_{\bar{n}\bar{l}}\,. \tag{5.11}$$

After that, we need to rescale the fluctuations of the moduli around the minimum by the corresponding $1/\sqrt{2K_k}$ factors so that the new *real* scalar fields have canonical kinetic terms. At the end, finding the moduli mass squared eigenvalues boils down to diagonalizing the following matrix

$$M^2_{kl} = \frac{1}{2}\frac{1}{\sqrt{K_k K_l}} U^\dagger_{km} V_{mn} U_{nl}\,. \tag{5.12}$$

Unlike most of the other masses, the detailed form of the moduli mass matrix does depend upon the detailed form of V_X. Therefore we have resorted to numerical analyses in this case and found that there is one heavy modulus whose mass mainly depends on Q and for $Q = 30$

$$M \sim O(200-300) \times m_{3/2}\,, \tag{5.13}$$

and N lighter moduli with masses

$$m_i \sim O(1) \times m_{3/2}\,, \quad i = \overline{1, N}\,. \tag{5.14}$$

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The heavy modulus arises from the fluctuation which deforms the volume of the three-cycle $V_{\mathcal{Q}}$, while N-1 light moduli originate from the fluctuations approximately preserving the volume and tangential to the hyperplane defined by

$$\vec{N} \cdot \vec{s} - V_{\mathcal{Q}} = 0\,. \tag{5.15}$$

The remaining light modulus represents the fluctuations of the hidden sector meson ϕ mixed with the geometric moduli.

5.3 Gaugino masses

The universal tree-level contribution to the gaugino masses can be computed from the standard supergravity formula [35, 36]

$$m_{1/2}^{\text{tree}} = \frac{e^{K/2} F^i \partial_i f_{vis}}{2i \text{Im} f_{vis}}\,, \tag{5.16}$$

where the visible sector gauge kinetic function is another integer combination of the moduli

$$f_{vis} = \sum_{i=1}^{N} N_i^{vis} z_i\,. \tag{5.17}$$

Note that, since the dominant F-term is that of the meson field, the gaugino masses at tree level will be suppressed wrt the gravitino mass. Since the scalar masses typically get contributions of order $m_{3/2}$ the expectation is to have light gauginos and heavier scalars, as we will indeed verify shortly.

Plugging the solution for α (4.51) into (4.63) while using the definitions (4.38) we obtain

$$e^{K/2} F^i \approx -i \frac{2 s_i}{P_{\text{eff}}} \left(1 + \frac{2 V_X}{(Q-P)\phi_0^2}\right) m_{3/2}\,, \tag{5.18}$$

where we dropped the overall phase factor $e^{-i\gamma_W}$. It is now straightforward to compute the tree-level gaugino mass

$$m_{1/2}^{\text{tree}} \approx -\frac{1}{P_{\text{eff}}} \left(1 + \frac{2 V_X}{(Q-P)\phi_0^2} + \mathcal{O}\left(\frac{1}{P_{\text{eff}}}\right)\right) m_{3/2}\,. \tag{5.19}$$

It is interesting to note that this formula is identical to the leading order expression previously obtained in [1, 2] when one replaces the combination ϕ_0^2/V_X by the canonically normalized meson field. Here, again the suppression coefficient is completely independent of the number of moduli N as well as the integers N_i (N_i^{vis}) appearing inside either the hidden sector (4.3) or the visible sector (5.17) gauge kinetic functions. Moreover, all the detailed dependence on the individual moduli is completely buried inside the volume V_X and the gravitino mass $m_{3/2}$ (which also depends on V_X) and therefore expression (5.19) is universally valid for any G_2 manifold that yields positive solutions of the system of equations in (4.23). Hence, despite the presence of a huge number of unknown microscopic parameters, the tree-level gaugino masses in (5.19) depend on very few of them. Moreover, when the cosmological constant is tuned to a small value and $Q - P = 3$, the gaugino mass suppression coefficient becomes completely fixed! Indeed, using (4.73) and (4.74) for $Q - P = 3$ we obtain

$$m_{1/2}^{\text{tree}} \approx -0.0307 \times m_{3/2}\,. \tag{5.20}$$

This result gets slightly corrected by the threshold corrections to the gauge kinetic function from the Kaluza-Klein modes computed in [14, 15]

$$\alpha_{GUT}^{-1} = f_{vis} + \frac{5}{2\pi}\mathcal{T}_\omega\,. \tag{5.21}$$

In the above formula, $\mathcal{T}_\omega$ is a topological invariant (Ray-Singer torsion)

$$\mathcal{T}_\omega = \ln\left(4\sin^2(5\pi w/q)\right)\,, \tag{5.22}$$

where w and q are integers such that $5w$ is not divisible by q. In this case, the tree-level gaugino mass is given by

$$m_{1/2}^{\text{tree}} \approx -0.0307\,\eta \times m_{3/2}\,, \tag{5.23}$$

where

$$\eta = 1 - \frac{5\,g_{GUT}^2}{8\pi^2}\mathcal{T}_\omega\,. \tag{5.24}$$

5.4 Anomaly mediated contribution to the gaugino masses

Because of the substantial suppression of the universal tree-level gaugino mass, it makes sense to take into account the anomaly mediated contributions which appear at one-loop. The anomaly mediated contributions are given by the following general expression [37, 38]

$$\begin{aligned} m_a^{AM} = -\frac{g_a^2}{16\pi^2}\Bigg[&-\left(3C_a - \sum_\alpha C_a^\alpha\right)e^{K/2}\overline{W} \\ &+\left(C_a - \sum_\alpha C_a^\alpha\right)e^{K/2}F^nK_n + 2\sum_\alpha\left(C_a^\alpha e^{K/2}F^n\partial_n\ln\tilde{K}_\alpha\right)\Bigg]\,, \end{aligned} \tag{5.25}$$

where C_a and $\sum_\alpha C_a^\alpha$ are the quadratic Casimirs of the a-th gauge group and $\tilde{K}_\alpha$ are eigenvalues of the Kahler metric for the visible sector fields (3.43). Assuming the MSSM particle content, we have the following values for the Casimirs

$$\begin{aligned} \text{U}(1):\ \ C_a = 0 \qquad & \sum_\alpha C_a^\alpha = \frac{33}{5} \\ \text{SU}(2):\ \ C_a = 2 \qquad & \sum_\alpha C_a^\alpha = 7 \\ \text{SU}(3):\ \ C_a = 3 \qquad & \sum_\alpha C_a^\alpha = 6\,. \end{aligned} \tag{5.26}$$

Plugging the solution for α (4.51) into (4.64) while using the definitions (4.38) we obtain

$$e^{K/2}F^\phi \approx \phi\left(1 - \frac{7}{3P_{\text{eff}}}\right)\left(1 + \frac{2V_X}{(Q-P)\phi_0^2}\right)m_{3/2}\,, \tag{5.27}$$

where we dropped the overall phase factor $e^{-i\gamma_W}$. Combining (5.18), (5.27) and using (4.5), (3.43), (3.44) and (3.32) we now compute the contributions

$$e^{K/2}F^n K_n = e^{K/2}F^i K_i + e^{K/2}F^\phi K_\phi = \left(\frac{\phi_0^2}{V_X} + \frac{7}{P_{\rm eff}}\right)\left(1 + \frac{2V_X}{(Q-P)\phi_0^2}\right) m_{3/2}$$
$$e^{K/2}F^n \partial_n \ln \tilde{K}_\alpha = \frac{1}{3}\left(1 + \frac{2V_X}{(Q-P)\phi_0^2}\right)\left(c(s_i)\frac{\phi_0^2}{V_X} + \frac{7-3\lambda}{P_{\rm eff}}\right) m_{3/2}\,. \tag{5.28}$$

In the above we also used (4.10) and (4.11) together with the definition of a_i in (2.16) as well as its contraction property (2.17). We also dropped unknown subleading contributions proportional to $e^{K/2}F^i\partial_i c(s_i) \sim m_{1/2}^{\rm tree} s_i \partial_i c(s_i)$.

Using the definition (5.25) we then obtain the following expression for the anomaly mediated contributions to the gaugino masses

$$m_a^{AM} \approx \frac{\alpha_{GUT}}{4\pi}\left[\left(3C_a - \sum_\alpha C_a^\alpha\right)\left(1 - \frac{1}{3}\left(1 + \frac{2V_X}{(Q-P)\phi_0^2}\right)\left(\frac{\phi_0^2}{V_X} + \frac{7}{P_{\rm eff}}\right)\right)\right. \tag{5.29}$$
$$\left. + \frac{2}{3}\left(1 + \frac{2V_X}{(Q-P)\phi_0^2}\right)\left[(1-c(s_i))\frac{\phi_0^2}{V_X} + \frac{3\lambda}{P_{\rm eff}}\right]\sum_\alpha C_a^\alpha\right] \times m_{3/2}\,,$$

where we have explicitly separated the conformal anomaly contribution from the Konishi anomaly term using (3.44).

Notice the appearance of the function $c(s_i)$ which controls the size of the higher order corrections to the matter Kahler potential. As expected, when $\lambda = 0$ the Konishi anomaly vanishes in the exactly sequestered case [39], i.e. when $c(s_i) = 1$. Again, in the leading order in $1/P_{\rm eff}$, when $c(s_i) = 0$ the result obtained above is almost the same as the one in [1, 2]. Just like in the case with tree-level gaugino masses, the above result is completely independent of the detailed moduli dependence of the volume V_X and therefore is completely general. In what follows, we will regard the value of the function $c(s_i)$ at the minimum of the scalar potential as a phenomenological parameter

$$c \equiv c(s_i)\,. \tag{5.30}$$

When we set $\lambda = 0$ and $Q - P = 3$, tune the leading contribution to the vacuum energy by imposing the constraint (4.73), use (4.74) and combine the above formula with the tree-level contribution (5.23), we obtain the following expression for the total gaugino masses

$$M_a \approx \left[-0.0307\,\eta + \alpha_{GUT}\left(0.0364\left(3C_a - \sum_\alpha C_a^\alpha\right) + 0.0749\,(1-c)\sum_\alpha C_a^\alpha\right)\right] \times m_{3/2}\,. \tag{5.31}$$

Note that as was previously pointed out in [39], in the limit when $c \to 1$ we obtain a particular type of a mirage pattern for gaugino masses [40–44]. However, as we will see below, in this limit the scalars become tachyonic and therefore, the exact mirage pattern is disfavored. An exact numerical computation confirms the above result giving

$$M_a \approx \left[-0.03156\,\eta + \alpha_{GUT}\left(0.034086\left(3C_a - \sum_\alpha C_a^\alpha\right) + 0.07926\,(1-c)\sum_\alpha C_a^\alpha\right)\right] \times m_{3/2}\,. \tag{5.32}$$

Substituting the MSSM Casimirs (5.26) into (5.32) we then obtain

$$\begin{aligned} M_1 &\approx \left(-0.03156\,\eta + \alpha_{GUT}\left(-0.22497 + 0.52313\,(1-c)\right)\right) \times m_{3/2} \\ M_2 &\approx \left(-0.03156\,\eta + \alpha_{GUT}\left(-0.03409 + 0.55483\,(1-c)\right)\right) \times m_{3/2} \\ M_3 &\approx \left(-0.03156\,\eta + \alpha_{GUT}\left(0.10226 + 0.47557\,(1-c)\right)\right) \times m_{3/2}\,. \end{aligned} \tag{5.33}$$

The form of (5.33) allows us to see explicitly that for $c = 0$ the Konishi anomaly contribution is larger than the contribution from the conformal anomaly by a factor of a few, which is what made the gaugino mass spectrum in [3] very different from other known patterns. However, as we will see below, suppressing the scalar masses relative to the gravitino mass by tuning the coefficient c will automatically result in a large suppression of the Konishi anomaly.

5.5 Scalars

The masses of the unnormalized scalars can be computed from the following general expression [35, 36]

$$m'^2_{\alpha\bar{\beta}} = \left(m^2_{3/2} + V_0\right)\tilde{K}_{\alpha\bar{\beta}} - e^K F^n \bar{F}^{\bar{m}}\left(\partial_{\bar{m}}\partial_n \tilde{K}_{\alpha\bar{\beta}} - \partial_{\bar{m}}\tilde{K}_{\alpha\bar{\gamma}}\tilde{K}^{\bar{\gamma}\delta}\partial_n\tilde{K}_{\delta\bar{\beta}}\right)\,. \tag{5.34}$$

Since the vacuum energy is tuned to zero we set $V_0 = 0$ in the above. Using (3.39), (4.5), (4.10), (4.11), the contraction properties (2.8), (2.9), (3.32) and the F-terms (5.18), (5.27) we obtain from (5.34) in the leading order

$$m'^2_{\alpha\bar{\beta}} \approx (1 - c(s_i))\left(m^2_{3/2} - \frac{7}{3}\left(m^{\rm tree}_{1/2}\right)^2\right)\tilde{K}_{\alpha\bar{\beta}} + \lambda\left(m^{\rm tree}_{1/2}\right)^2\tilde{K}_{\alpha\bar{\beta}}\,, \tag{5.35}$$

where for consistency reasons we only kept contributions linear in $c(s_i)$ and dropped unknown subleading terms proportional to the derivatives of $c(s_i)$, such as e.g. $e^K F^i \bar{F}^{\bar{j}}\partial_i\partial_{\bar{j}}c(s_i) \sim \left(m^{\rm tree}_{1/2}\right)^2 s_i s_j \partial_i\partial_j c(s_i)$. In the above derivation we also used the following properties

$$e^K F^i \bar{F}^{\bar{j}}\partial_i\partial_{\bar{j}}\hat{K} = 7\left(m^{\rm tree}_{1/2}\right)^2 \;\;\Rightarrow\;\; e^K F^n \bar{F}^{\bar{m}}\partial_n\partial_{\bar{m}}\frac{\phi\bar{\phi}}{3V_X} = m^2_{3/2} - \frac{7}{3}\left(m^{\rm tree}_{1/2}\right)^2\,. \tag{5.36}$$

Notice that despite the presence of the derivatives of the Kahler metric $\tilde{K}_{\alpha\bar{\beta}}$ in the definition (5.34), the final expression (5.35) contains $\tilde{K}_{\alpha\bar{\beta}}$ only as an overall multiplicative factor. This happened because the moduli F-terms (5.18) up to a phase are essentially given by $e^{K/2}F^i = 2s_i \times m^{\rm tree}_{1/2}$ and the matrix $\kappa_{\alpha\bar{\beta}}(s_i)$ is a homogeneous function satisfying (3.32). Therefore, diagonalization and canonical normalization of the corresponding kinetic terms automatically results in universal masses for the canonically normalized scalars

$$m^2_\alpha \approx (1 - c(s_i))\left(m^2_{3/2} - \frac{7}{3}\left(m^{\rm tree}_{1/2}\right)^2\right) + \lambda\left(m^{\rm tree}_{1/2}\right)^2\,. \tag{5.37}$$

After setting $\lambda = 0$, we tune the leading contribution to the vacuum energy by imposing the constraint (4.73) and use (5.20) to obtain from (5.37)

$$m_\alpha \approx (1-c)^{1/2}0.999\,m_{3/2}\,, \tag{5.38}$$

where we again treat the value of the function $c(s_i)$ for a given vacuum as a phenomenological parameter (5.30). A numerical computation in this case gives excellent agreement

$$m_\alpha \approx (1-c)^{1/2} 0.998\, m_{3/2} \approx (1-c)^{1/2}\, m_{3/2}\,. \tag{5.39}$$

Again, for $c=0$ we recover the old result in [1, 2] where all the scalars have a flavor-universal mass equal to the gravitino mass.

Furthermore, the anomaly contributions to the scalar mass squareds are suppressed relative to the gravitino mass and since we wish to consider generic $\mathcal{O}(1)$ values of $(1-c)$ we will neglect such contributions. Concretely we are going to consider only those values of $0<c<1$ which give

$$\frac{1}{16\pi^2} \ll \frac{m_\alpha}{m_{3/2}}\,, \tag{5.40}$$

such that the anomaly mediated contributions to the scalar masses can be safely neglected. However, one can certainly extend our model and include such contributions. Once again, the result above is completely independent of the details of V_X and therefore holds for any G_2 manifold that solves the system (4.23) with $a_i>0$ such that the Kahler metric at the minimum is positive definite.

5.6 Trilinear couplings

The unnormalized trilinear couplings for the visible sector fields can be computed from the following general expression [35, 36]

$$A'_{\alpha\beta\gamma} = \frac{\overline{W}}{|W|} e^{K/2} F^m \left[K_m Y'_{\alpha\beta\gamma} + \partial_m Y'_{\alpha\beta\gamma} - \left(\tilde{K}^{\delta\bar{\rho}} \partial_m \tilde{K}_{\bar{\rho}\alpha} Y'_{\delta\beta\gamma} + (\alpha \leftrightarrow \beta) + (\alpha \leftrightarrow \gamma) \right) \right]\,, \tag{5.41}$$

where $\{\alpha\,,\beta\,,\gamma\}$ label visible sector matter fields and $Y'_{\alpha\beta\gamma}$ are the unnormalized Yukawas that appear in the superpotential. Recall that the Yukawa couplings $Y'_{\alpha\beta\gamma}$ arise from the membrane instantons wrapping associative cycles $Q^{\alpha\beta\gamma}$, which connect isolated singularities supporting the corresponding matter multiplets. They are given by

$$Y'_{\alpha\beta\gamma} = C_{\alpha\beta\gamma} e^{i2\pi \sum_i m_i^{\alpha\beta\gamma} z_i}\,. \tag{5.42}$$

The integer combination of the moduli $V_{Q^{\alpha\beta\gamma}} = \sum_i m_i^{\alpha\beta\gamma} s_i$ gives the volume of the associative cycle $Q^{\alpha\beta\gamma}$ connecting co-dimension seven singularities α, β and γ where the chiral multiplets are localized. The coefficients $C_{\alpha\beta\gamma}$ are constants. The relation between the physical and unnormalized Yukawa couplings is given by

$$Y_{\alpha\beta\gamma} = \frac{\overline{W}}{|W|} e^{K/2} Y'_{\alpha\beta\gamma} \left(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma \right)^{-1/2}\,. \tag{5.43}$$

Using (5.18) and (5.42) we can compute the contribution

$$e^{K/2} F^m \partial_m Y'_{\alpha\beta\gamma} = Y'_{\alpha\beta\gamma} \frac{4\pi}{P_{\rm eff}} \left(1 + \frac{2V_X}{(Q-P)\phi_0^2} \right) V_{Q^{\alpha\beta\gamma}} m_{3/2}\,. \tag{5.44}$$

Similarly, using (3.39), (4.5), (4.10), (4.11), the contraction properties (2.8), (2.9), (3.32) and the F-terms (5.18), (5.27) we find

$$e^{K/2}F^m\tilde{K}^{\delta\bar{\rho}}\partial_m\tilde{K}_{\bar{\rho}\alpha} = \frac{\delta^\delta_\alpha}{3}\left(1+\frac{2V_X}{(Q-P)\phi_0^2}\right)\left(c(s_i)\frac{\phi_0^2}{V_X}+\frac{7-3\lambda}{P_{\rm eff}}\right)m_{3/2}\,, \tag{5.45}$$

where for consistency reasons we only retained contributions linear in $c(s_i)$ and dropped unknown subleading terms proportional to $e^{K/2}F^i\partial_i c(s_i) \sim m_{1/2}^{\rm tree}s_i\partial_i c(s_i)$. Again, in the above expressions we did not display the overall phase factor $e^{-i\gamma_W}$. Using the definition (5.41) along with (5.28), (5.44) and (5.45) we obtain the following expression for the physical (normalized) trilinear couplings at tree-level

$$A_{\alpha\beta\gamma} = Y_{\alpha\beta\gamma}\left(1+\frac{2V_X}{(Q-P)\phi_0^2}\right)\left((1-c(s_i))\frac{\phi_0^2}{V_X}+\frac{3\lambda+4\pi V_{Q^{\alpha\beta\gamma}}}{P_{\rm eff}}\right)m_{3/2}\,, \tag{5.46}$$

which gets reduced to the result in [1, 2] when $c=0$ and $\lambda=0$. Once again, the detailed structure of the volume V_X played absolutely no role in our ability to obtain the above expression for the tree-level trilinear couplings. The actual volumes of three-cycles $V_{Q^{\alpha\beta\gamma}}$ do depend on the microscopic properties of G_2 manifolds and in our general framework these parameters remain undetermined. However, below we will present a good argument for dropping such volume contributions completely when the third generation trilinear couplings are computed.

Setting $\lambda=0$ and $Q-P=3$, when the leading contribution to the vacuum energy is tuned we obtain for the reduced trilinears (the physical trilinears divided by the physical Yukawa couplings)

$$\tilde{A}_{\alpha\beta\gamma} \equiv \frac{A_{\alpha\beta\gamma}}{Y_{\alpha\beta\gamma}} = \left(1.41(1-c)+0.386\times V_{Q^{\alpha\beta\gamma}}\right)m_{3/2}\,. \tag{5.47}$$

From the corresponding numerical calculation we obtain the following result

$$\tilde{A}_{\alpha\beta\gamma} = \left(1.494(1-c)+0.3966\times V_{Q^{\alpha\beta\gamma}}\right)m_{3/2}\,. \tag{5.48}$$

Since the physical Yukawa couplings for the third generation fermions are much larger than the first two generation Yukawas, one can typically neglect the trilinears for the first and second generations. Moreover, the large size of the third generation Yukawas implies that the volumes of the three-cycles of the corresponding membrane instantons are very small. In fact, because the top Yukawa is of order one, one can assume that the point p_1 supporting the up-type Higgs $\mathbf{5}$ of SU(5) coincides with the point p_2 supporting the third generation $\mathbf{10}$, so that the coupling $H_u\mathbf{10_3}\mathbf{10_3}$ has no exponential suppression [7, 14, 15]. At the same time the point p_3 supporting the down-type $\bar{\mathbf{5}}$ Higgs and the point p_4 supporting the third generation matter $\bar{\mathbf{5}}$ are distinct but still close to $p_2\,, p_1$ so that the coupling of $H_d\bar{\mathbf{5}}_\mathbf{3}\mathbf{10_3}$ which accounts for the bottom(tau) Yukawa is slightly smaller than the top Yukawa at the GUT scale. These considerations completely justify dropping the corresponding $V_{Q^{\alpha\beta\gamma}}$ terms for the third generation trilinears which then become simplified

$$\tilde{A}_t = \tilde{A}_b = \tilde{A}_\tau \approx 1.494(1-c)\,m_{3/2}\,. \tag{5.49}$$

For generic values of c the trilinears are of the same order as the gravitino mass. In the limit $c \to 1$, the reduced trilinear couplings at tree-level become suppressed relative to the gravitino mass. Note that as c approaches one, the suppression of the trilinear couplings above is much stronger than that of the scalars. In this case, the anomaly-mediated contributions may become comparable to the tree-level ones and therefore must be taken into account. General expressions given in [45, 46] can be simplified in the nearly sequestered limit as

$$\tilde{A}_a^{AM} = -\frac{1}{16\pi^2}\gamma_a\left(e^{K/2}\overline{W} - \frac{1}{3}e^{K/2}F^n K_n\right) + \frac{(1-c)}{16\pi^2}X_a\, m_{3/2}\,, \tag{5.50}$$

where the last term denotes the unknown contributions vanishing in the sequestered limit. Note that such terms are suppressed compared to the tree-level piece (5.49) due to the loop factor. As long as $(1-c)$ is small enough, they become subleading and we will drop them in further analysis. Using (5.28) and substituting the corresponding MSSM expressions for γ_as, where we set $g_1 = g_2 = g_3 = g_{GUT}$, we obtain the following expressions for the anomaly mediated contributions to the reduced trilinear couplings

$$\begin{aligned}
\tilde{A}_t^{AM} &\approx -\frac{1}{16\pi^2}\left(-\frac{46}{5}g_{GUT}^2+6Y_t^2+Y_b^2\right)\left(1-\frac{1}{3}\left(1+\frac{2V_X}{(Q-P)\phi_0^2}\right)\left(\frac{\phi_0^2}{V_X}+\frac{7}{P_{\rm eff}}\right)\right)m_{3/2}\\
\tilde{A}_b^{AM} &\approx -\frac{1}{16\pi^2}\left(-\frac{44}{5}g_{GUT}^2+Y_t^2+6Y_b^2+Y_\tau^2\right)\left(1-\frac{1}{3}\left(1+\frac{2V_X}{(Q-P)\phi_0^2}\right)\left(\frac{\phi_0^2}{V_X}+\frac{7}{P_{\rm eff}}\right)\right)m_{3/2}\\
\tilde{A}_\tau^{AM} &\approx -\frac{1}{16\pi^2}\left(-\frac{24}{5}g_{GUT}^2+3Y_b^2+4Y_\tau^2\right)\left(1-\frac{1}{3}\left(1+\frac{2V_X}{(Q-P)\phi_0^2}\right)\left(\frac{\phi_0^2}{V_X}+\frac{7}{P_{\rm eff}}\right)\right)m_{3/2}\,.
\end{aligned} \tag{5.51}$$

When we set $Q-P=3$, tune the tree-level vacuum energy by imposing the constraint (4.73), use (4.74) and combine the above formula with the tree-level contribution (5.48), we obtain

$$\begin{aligned}
\tilde{A}_t &\approx 1.41(1-c)m_{3/2} - 0.0029\left(-\frac{46}{5}g_{GUT}^2+6Y_t^2+Y_b^2\right)m_{3/2}\\
\tilde{A}_b &\approx 1.41(1-c)m_{3/2} - 0.0029\left(-\frac{44}{5}g_{GUT}^2+Y_t^2+6Y_b^2+Y_\tau^2\right)m_{3/2}\\
\tilde{A}_\tau &\approx 1.41(1-c)m_{3/2} - 0.0029\left(-\frac{24}{5}g_{GUT}^2+3Y_b^2+4Y_\tau^2\right)m_{3/2}\,.
\end{aligned} \tag{5.52}$$

Numerical computations give the following expressions for the total reduced trilinears

$$\begin{aligned}
\tilde{A}_t &\approx 1.494(1-c)m_{3/2} - 0.0027\left(-\frac{46}{5}g_{GUT}^2+6Y_t^2+Y_b^2\right)m_{3/2}\\
\tilde{A}_b &\approx 1.494(1-c)m_{3/2} - 0.0027\left(-\frac{44}{5}g_{GUT}^2+Y_t^2+6Y_b^2+Y_\tau^2\right)m_{3/2}\\
\tilde{A}_\tau &\approx 1.494(1-c)m_{3/2} - 0.0027\left(-\frac{24}{5}g_{GUT}^2+3Y_b^2+4Y_\tau^2\right)m_{3/2}\,,
\end{aligned} \tag{5.53}$$

which demonstrate a fairly high accuracy of the analytically derived result in (5.52).

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5.7 μ and $B\mu$ -terms

The full hidden sector plus visible sector Kahler potential and superpotential can be written in the following general form

$$\begin{aligned} K_{\text{total}} &= K\left(s_i, \phi, \bar{\phi}\right) + \tilde{K}_{\alpha\bar{\beta}}(s_i, \phi, \bar{\phi}) Q^\alpha Q^{\dagger\bar{\beta}} + Z_{\alpha\beta}(s_i, \phi, \bar{\phi}) Q^\alpha Q^\beta + h.c. \\ \hat{W} &= W_{np} + \mu' Q^\alpha Q^\beta + Y'_{\alpha\beta\gamma} Q^\alpha Q^\beta Q^\gamma + \ldots. \end{aligned} \tag{5.54}$$

Here, ϕ denote the hidden sector matter fields while Q^α are visible sector chiral matter fields where $\tilde{K}_{\alpha\bar{\beta}}(s_i, \phi, \bar{\phi})$ is the visible sector Kahler metric and $Y'_{\alpha\beta\gamma}$ are the corresponding unnormalized Yukawa couplings. It can be shown that the supersymmetric mass parameter μ' can be forbidden by requiring certain discrete symmetries which are also used in order to solve the problem of doublet-triplet splitting [47]. Hence, in our analysis we will rely on the Giudice-Masiero mechanism [48] in generating effective μ and $B\mu$ terms where the bilinear coefficient $Z_{\alpha\beta}(s_i, \phi, \bar{\phi})$ in (5.54) plays a key role. The general expressions for the normalized μ and $B\mu$ are given by [35, 36]

$$\begin{aligned} \mu &= \left(\frac{\overline{W}_{np}}{|W_{np}|} e^{K/2} \mu' + m_{3/2} Z - e^{K/2} F^{\bar{m}} \partial_{\bar{m}} Z\right) (\tilde{K}_{H_u} \tilde{K}_{H_d})^{-1/2} \\ B\mu &= \left[\frac{\overline{W}_{np}}{|W_{np}|} e^{K/2} \mu' (e^{K/2} F^m \left[K_m + \partial_m \ln \mu' - \partial_m \ln(\tilde{K}_{H_u} \tilde{K}_{H_d})\right] - m_{3/2}) \right. \\ &\quad + (2m_{3/2}^2 + V_0)\, Z - m_{3/2} e^{K/2} F^{\bar{m}} \partial_{\bar{m}} Z + m_{3/2} e^{K/2} F^m (\partial_m Z - Z \partial_m \ln(\tilde{K}_{H_u} \tilde{K}_{H_d})) \\ &\quad \left. - e^K F^{\bar{m}} F^n (\partial_{\bar{m}} \partial_n Z - \partial_{\bar{m}} Z \partial_n \ln(\tilde{K}_{H_u} \tilde{K}_{H_d}))\right] (\tilde{K}_{H_u} \tilde{K}_{H_d})^{-1/2}. \end{aligned} \tag{5.55}$$

where we can set $\mu' = 0$. Unfortunately, at this point we do not have a reliable way to compute the Higgs bilinear $Z_{\alpha\beta}(s_i, \phi, \bar{\phi})$ for G_2 compactifications. Therefore, in our analysis we will parameterize the μ and $B\mu$ terms as follows

$$\begin{aligned} \mu &= Z^1_{\text{eff}}\, m_{3/2} \\ B\mu &= Z^2_{\text{eff}}\, m^2_{3/2}, \end{aligned} \tag{5.56}$$

and treat Z^1_{eff} and Z^2_{eff} as phenomenological parameters. Naturally, we expect that ${Z^{1,2}_{\text{eff}}} \sim \mathcal{O}(1)$ and, as we will see in the next section, tuning μ parameter in order to get the correct value of the Z - boson mass boils down to tuning the values of ${Z^{1,2}_{\text{eff}}}$.

6 Generalization to the case when $\kappa(s_i)$ is a non-trivial homogeneous function

Recall that the above results have been obtained assuming that the factor $\kappa(s_i)$ appearing in the Kahler potential (4.5) of the hidden sector matter is a pure constant. In this section we briefly outline the main results for the case when $\kappa(s_i)$ is a general homogeneous function of the moduli of degree zero, satisfying the property (3.2). Here we will not give any explicit analytic derivations (these are quite tedious and would mostly resemble the computations in the preceding sections) and instead present numerical evidence that most

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of results obtained for the simplified case $\kappa(s_i) = 1$ can be directly extended to the more general scenario.

The main difference from the previous case is in the form of the moduli vevs at the minimum of the scalar potential. These are now given by

$$s_i \approx \frac{1}{N_i}\left(a_i + c_i\frac{\phi_c^2}{3}r\right)\frac{3QP_{\rm eff}}{14\pi(Q-P)}\,, \tag{6.1}$$

where $r \approx 3/2$ when $Q - P = 3$. Note that the vev ϕ_c^2 of the canonically normalized effective meson field at the minimum is given by the same expression as in the case when $\kappa(s_i) = 1$:

$$\phi_c^2 \equiv \kappa(s_i)\frac{\phi_0^2}{V_X} \approx \frac{2}{Q-P} + \frac{7}{P_{\rm eff}}\left(1 - \frac{2}{3(Q-P)}\right)\,, \tag{6.2}$$

where $P_{\rm eff}$ is exactly the same as in (4.68). Keep in mind that the analytic expression (6.2) is only valid when the leading contribution to the vacuum energy is tuned to zero. Parameters c_i are defined as

$$c_i \equiv s_i\frac{\partial \ln\kappa(s_i)}{\partial s_i}\,, \quad \text{no sum over } i\,, \tag{6.3}$$

and satisfy

$$\sum_{i=1}^{N} c_i = 0\,, \tag{6.4}$$

because $\kappa(s_i)$ is a homogeneous function of degree zero. At the minimum of the potential parameters a_i and c_i can be determined by solving a system of $2N$ coupled transcendental equations

$$\begin{aligned} s_i\frac{\partial \hat{K}}{\partial s_i}\Big|_{s_i=\frac{1}{N_i}\left(a_i+c_i\frac{\phi_c^2}{3}r\right)} + 3a_i &= 0\,, \\ s_i\frac{\partial \ln\kappa(s_i)}{\partial s_i}\Big|_{s_i=\frac{1}{N_i}\left(a_i+c_i\frac{\phi_c^2}{3}r\right)} - c_i &= 0\,. \end{aligned} \tag{6.5}$$

Once again, the structure of the soft supersymmetry breaking terms remains virtually unchanged in the leading order in $1/P_{\rm eff}$ expansion and does not depend on the precise details of the function $\kappa(s_i)$. In fact, the only modification compared to the previously derived expressions is the replacement of the following combination

$$\frac{\phi_0^2}{V_X} \to \kappa(s_i)\frac{\phi_0^2}{V_X}\,, \tag{6.6}$$

whose vev at the minimum is given by (6.2) and is exactly the same as before! However, it turns out that the soft supersymmetry breaking terms now become slightly sensitive to the compactification details via the subleading corrections. The most sensitive parameter is the tree-level gaugino mass. Up to an overall phase it is given by

$$m_{1/2}^{\rm tree} \approx -\frac{m_{3/2}}{P_{\rm eff}}\left(1+\frac{2V_X}{(Q-P)\kappa(s_i)\phi_0^2}+\mathcal{O}\left(\frac{1}{P_{\rm eff}}\right)\right) = -\frac{m_{3/2}}{P_{\rm eff}}\left(1+\frac{2}{(Q-P)\phi_c^2}+\frac{\delta}{P_{\rm eff}}\right)\,, \tag{6.7}$$

where we introduced a phenomenological quantity $\delta \sim \mathcal{O}(1-10)$ in order to parameterize the additional correction. In the numerical toy examples we studied, we obtain a $\mathcal{O}(1-10)\%$ variation in the value of the tree-level gaugino mass, while the other soft terms vary by less than 1%. For the sake of completeness we shall list the expressions for the remaining soft breaking terms

$$\begin{aligned}
m_a^{AM} &\approx \frac{\alpha_{GUT}}{4\pi}\left(\left(3C_a - \sum_\alpha C_a^\alpha\right)K_1 + \frac{2}{3}K_2\sum_\alpha C_a^\alpha\right) \times m_{3/2}\,, \\
\tilde{A}^{\rm tree}_{\alpha\beta\gamma} &\approx \left(K_2 + \frac{4\pi}{P_{\rm eff}}\left(1+\frac{2V_X}{(Q-P)\kappa(s_i)\phi_0^2}\right)V_{Q^{\alpha\beta\gamma}}\right)\times m_{3/2}\,, \\
\tilde{A}_a^{AM} &\approx -\frac{1}{16\pi^2}\gamma_a K_1 \times m_{3/2}\,, \\
m_\alpha^2 &\approx (1-c(s_i))\left(m_{3/2}^2 - \frac{7}{3}\left(m_{1/2}^{\rm tree}\right)^2\right) + \lambda\left(m_{1/2}^{\rm tree}\right)^2\,,
\end{aligned} \tag{6.8}$$

where we defined

$$\begin{aligned}
K_1 &\equiv 1 - \frac{1}{3}\left(1+\frac{2V_X}{(Q-P)\kappa(s_i)\phi_0^2}\right)\left(\kappa(s_i)\frac{\phi_0^2}{V_X} + \frac{7}{P_{\rm eff}}\right)\,, \\
K_2 &\equiv \left(1+\frac{2V_X}{(Q-P)\kappa(s_i)\phi_0^2}\right)\left((1-c(s_i))\,\kappa(s_i)\frac{\phi_0^2}{V_X} + \frac{3\lambda}{P_{\rm eff}}\right)\,.
\end{aligned} \tag{6.9}$$

To illustrate the high accuracy of the analytical results presented here we present a simple toy example with two moduli.

$$V_X = s_1 s_2^{4/3}\,, \quad \kappa(s_i) = 1 + \frac{s_1^2}{s_2^2}\,. \tag{6.10}$$

For the following choice of the parameters in the superpotential

$$A_1 = 27\,,\ A_2 \approx 2.638\,,\ P = 27\,,\ Q = 30\,,\ N_1 = 1\,,\ N_2 = 2\,, \tag{6.11}$$

where A_2 was tuned to cancel the leading contribution to the vacuum energy, we obtain numerically

$$s_1 \approx 71.67\,,\ s_2 \approx 13.02\,,\ \phi_0 \approx 7.24\,,\ \Rightarrow\ \phi_c^2 \approx 0.746\,,\ P_{\rm eff} \approx 61.68\,. \tag{6.12}$$

Notice that the above values of ϕ_c^2 and $P_{\rm eff}$ are extremely close to those in (4.74) and (4.73), obtained numerically for the case when $\kappa(s_i) = 1$. These values are also in very good agreement with the corresponding analytical results. By solving the system (6.5) for the above example we obtain

$$a_1 = 1\,,\ a_2 = \frac{4}{3}\,,\ c_1 = -c_2 \approx 1.939\,, \tag{6.13}$$

and using the above values in the analytic expression (6.1) with $P_{\rm eff} \approx 61.68$ we find

$$s_1 \approx 72.5\,,\ s_2 \approx 12.8\,, \tag{6.14}$$

which agree well with the numerically obtained values in (6.12). To verify the tree-level gaugino mass formula numerically we need to know the integers $N_1^{\rm vis}$ and $N_2^{\rm vis}$ for the visible sector gauge kinetic function. Here we list three representative examples where we varied $N_1^{\rm vis}$ and $N_2^{\rm vis}$, while keeping everything else fixed

$$\begin{aligned} N_1^{\rm vis} &= 1\,,\ N_2^{\rm vis} = 0\,,\ \Rightarrow\ m_{1/2}^{\rm tree} \approx -0.029 \times m_{3/2}\,, \\ N_1^{\rm vis} &= 1\,,\ N_2^{\rm vis} = 3\,,\ \Rightarrow\ m_{1/2}^{\rm tree} \approx -0.032 \times m_{3/2}\,, \\ N_1^{\rm vis} &= 0\,,\ N_2^{\rm vis} = 1\,,\ \Rightarrow\ m_{1/2}^{\rm tree} \approx -0.037 \times m_{3/2}\,, \end{aligned} \tag{6.15}$$

which demonstrate a mild dependence of the tree-level gaugino mass on the compactification-specific details. Using $\phi_c^2 \approx 0.746$ together with $P_{\rm eff} \approx 61.68$ in the analytic formula (6.7) for $Q - P = 3$ we obtain

$$m_{1/2}^{\rm tree} \approx -\left(0.031 + 0.00026 \times \delta\right) \times m_{3/2}\,, \tag{6.16}$$

which is in fairly good agreement with the numerical results. The numerical results for the remaining soft terms are given by the following expressions

$$\begin{aligned} m_a^{AM} &\approx \alpha_{GUT}\left(0.0337\left(3C_a - \sum_\alpha C_a^\alpha\right) + 0.0792\,(1-c)\sum_\alpha C_a^\alpha\right) \times m_{3/2}\,, \\ \tilde{A}_a &\approx 1.49(1-c)m_{3/2} - \gamma_a \times 0.0027 \times m_{3/2} \\ m_\alpha &\approx (1-c)^{1/2}\, m_{3/2}\,, \\ K_1 &\approx 0.424\,, \qquad K_2 \approx 1.494\,(1-c)\ , \end{aligned} \tag{6.17}$$

and are virtually unchanged compared to the numerical results in (5.32), (5.53) and (5.39), computed for the case when $\kappa(s_i) = 1$. The values obtained from the corresponding analytic expressions are in good agreement with the numerical values above and give essentially the same results as in the case when $\kappa(s_i) = 1$ because the values of $P_{\rm eff}$ and ϕ_c^2 barely changed. Thus, the effect of including a non-trivial function $\kappa(s_i)$ in the Kahler potential for the hidden sector matter fields can be reliably described by a single parameter δ that appears in the subleading contributions to the tree-level gaugino mass, while the remaining soft terms stay essentially unaffected.

7 Electroweak scale spectrum

In order to obtain the corresponding MSSM spectrum at the electroweak scale we need to RG-evolve all the masses and couplings from the GUT scale down to the electroweak scale. This procedure was described in great detail in [3]. Here we will only highlight a few important points and give the final results.

As we have seen in the previous section, at the GUT scale, the gaugino masses are non-universal and highly suppressed relative to the gravitino mass. On the other hand, unless c is very close to one, the scalars, trilinear couplings and the μ-term are all of order $m_{3/2}$. Hence, we can define a scale m_s at which all the heavy states decouple and the effective theory below that scale is the Standard Model plus gauginos. More specifically, we can choose the decoupling scale m_s to be the geometric mean of the stop masses

$$m_s = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}\,. \tag{7.1}$$

This is okay as long as the mass differences between the lightest stop and the other heavy states is not too large. Then, the running can be done at one loop in two stages with tree-level matching at the scale m_s. This method, however, does not capture the two-loop effects, which may give significant contributions to the running. Thus, in what follows we will utilize the SOFTSUSY package [49] and perform the running at two-loops with the full the MSSM spectrum and account for the effects from the heavy scalars via threshold corrections.

7.1 Gauginos

As one notices from (6.17), due to the anomaly mediated contribution, the gaugino masses are sensitive to the value of α_{GUT}. However, the value of α_{GUT} is only determined once we know the exact spectrum and run the gauge coupling up to the GUT scale. Therefore, there is a feedback mechanism, which allows us to completely fix the gaugino masses by imposing the gauge coupling unification. In practice, we first pick an initial value of $\alpha_{GUT} \sim 1/25$, compute the gaugino masses, scalar masses, trilinears, etc. at the GUT scale and run them down to the electroweak scale where we compute the spectrum. We then run the gauge couplings up using two-loop RGEs to check if they unify at the same value of α_{GUT} as we chose to compute the gaugino masses. If there is disagreement, we change the value of α_{GUT} by a small increment and repeat the steps until there is a match. In addition, parameter η which appears inside the gaugino masses and was defined in (5.24) can be safely set to one. This is because as one varies the integers w and q inside (5.22) over a reasoble range, the torsion, unless specifically tuned, is so small that that the KK threshold corrections can be neglected.

Since $M_{Higgsino} \sim \mu \sim \mathcal{O}(m_{3/2})$, there is a substantial threshold contribution from the Higgs-Higgsino loops which has to be taken into account when computing bino (M_1) and wino (M_2) masses [3, 50, 50–52]:

$$\Delta M_{1,2} \approx -\frac{\alpha_{1,2}}{4\pi}\frac{\mu\sin(2\beta)}{\left(1-\frac{\mu^2}{m_A^2}\right)}\ln\frac{\mu^2}{m_A^2} \approx \frac{\alpha_{1,2}}{4\pi}\mu = \frac{\alpha_{1,2}}{4\pi}Z^1_{\text{eff}}m_{3/2}\,. \tag{7.2}$$

In the above expression we expanded the logarithm using $\frac{\mu^2}{m_A^2} \sim 1$ and used $\tan\beta \sim \mathcal{O}(1)$. The latter is especially true when $2\,Z^1_{\text{eff}} \approx Z^2_{\text{eff}}$. We also relied on the fact that the supersymmetric μ-term almost does not change with the RG evolution so one can use (5.56). Since $m_{3/2} \sim \mathcal{O}(10)$TeV, the above correction to M_2 can be as large as a few hundred GeV. It turns out that this contribution is intimately related to the value of parameter c that directly affects the scalar masses and indirectly forces the value of μ to get smaller, as the scalars get lighter. For $0 \le c \lesssim 0.05$ and $0.8 \lesssim \mu/m_{3/2}$ the contribution (7.2) is actually large enough to completely alter the nature of the LSP depending on the sign of μ. In this

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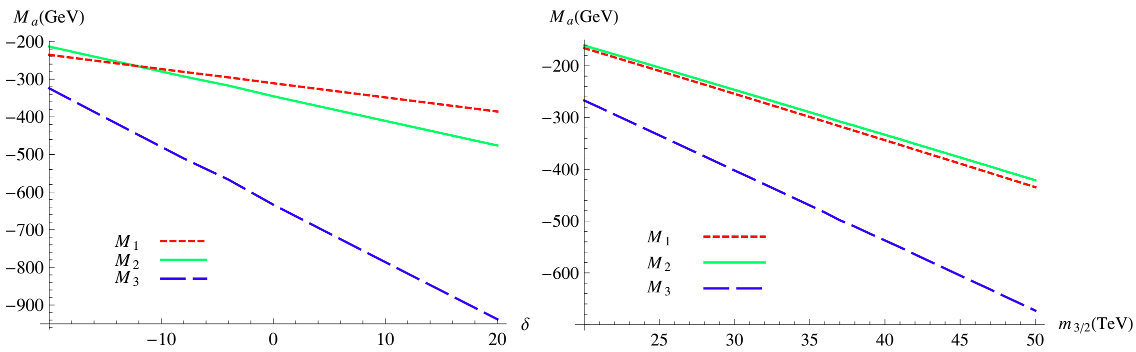

Figure 1. Left panel: Gaugino mass parameters at the electroweak scale as functions of δ. In the above computation, we used SOFTSUSY with the high scale input $m_{3/2} = 30\,\text{TeV}$, $\mu < 0$, $\tan\beta = 2.5$, $c = 0$. We have verified by examining the neutralino mixing matrix that at $\delta \approx -12$ the LSP type changes from Wino to Bino as δ is increased. Right panel: Gaugino mass parameters at the electroweak scale as functions of $m_{3/2}$. For each data point we checked the neutralino mixing matrix to confirm that the LSP is Wino-like. The plot was generated using SOFTSUSY with $\mu < 0$, $c = 0$, $\delta = -15$, $\tan\beta = 2.5$. The KK threshold correction to the visible sector gauge kinetic function was neglected in both plots.

respect, the sign of δ that parameterizes the subleading corrections to the tree-level gaugino mass (6.16) also plays an important role. In particular, from the left plot in figure 1 where we picked $c = 0$ there is a region where $\delta \lesssim -12$ such that the LSP is Wino-like, while for $\delta \gtrsim -12$ the LSP becomes mostly Bino. Furthermore, as one can see from the plot, there exists a small range of values where M_1 and M_2 become nearly degenerate. This is certainly an intriguing possibility, which may provide for a well-tempered neutralino candidate [51]. Note that in the Wino-like LSP case, the lightest chargino and neutralino are degenerate at tree-level, i.e. $\widetilde{\chi}_1^0 = \widetilde{\chi}_1^\pm = M_2$. However as we take into account the 1-loop contribution from the gauge bosons [52], this degeneracy is removed, as is seen from the corresponding entries in table 2. Such splitting was discussed in detail for the pure anomaly mediation scenario in [53, 54] and is given by

$$\Delta M_{1-loop} = \frac{\alpha_2 M_2}{4\pi}\left(f\left(\frac{m_W}{M_2}\right) - c_W^2 f\left(\frac{m_Z}{M_2}\right) - s_W^2 f(0)\right), \tag{7.3}$$

where $f(a) \equiv \int_0^1 dx(2+2x)\ln[x^2+(1+x)a^2]$. Typically we obtain $160\,\text{MeV} < \Delta M_{1-loop} < 200\,\text{MeV}$. Because of this, the lightest charginos are quasi-stable and decay into LSP plus soft pions or soft leptons. In the collider context such decays would take place well inside the detector leaving short charged tracks.

In addition to (7.2), the EW threshold corrections from gaugino-gauge-boson loops must also be included, especially for the gluino

$$\Delta M_3^{\text{rad}} = \frac{3\alpha_3}{4\pi}\left(3\ln\left(\frac{M_{\text{EW}}^2}{M_3^2}\right) + 5\right) M_3\,. \tag{7.4}$$

Unfortunately, we have no technical handle on the size of the parameter c, though we expect it to be small and hence a Wino LSP is quite generic when $c \approx 0$. As we increase the value of parameter c, the LSP quickly becomes Bino-like. There are two reasons for this effect. First, the ratio M_2/M_1 at the GUT scale grows as c is increased from zero to one. At the same time, the scalars and higgsinos become lighter relative to the gravitino mass. In particular, for a fixed $m_{3/2}$ the lower bound on the Higgs mass forces us to consider somewhat larger values of $\tan\beta$ which in turn leads to smaller values of μ thus significantly reducing the contributions (7.2) from higgsinos. Recall that it was primarily due to this contribution from heavy higgsinos for $\mu < 0$ that the LSP could become Wino-like. Thus, due to the increase of M_2/M_1 at the GUT scale and the decrease in the higgsino mass, the Wino-like LSP case becomes rapidly excluded as we increase the value of c. Benchmarks 4 and 5 in table 2 demonstrate that for generic values of $c > 0$ the LSP is always Bino-like.

Of course, pure Bino LSP is almost certainly excluded by the standard cosmological considerations [51]. Namely, because binos do not annihilate efficiently, the dark matter relic density becomes unacceptably large. However, this problem can be avoided when the higgsinos, which annihilate efficiently, are light enough to mix with gauginos. If the higgsino component of the LSP is significant, it can easily reduce the relic density to acceptable levels by increasing the annihilation crossection of the LSPs. It turns out that for generic values of c, $0 \leq c < 1$, the higgsinos are always much heavier than the gauginos. This is because at the decoupling scale m_s, the μ^2-term must be of the same order of magnitude as $m^2_{H_u}$ to give a correct value of the Z-boson mass, and since for typical values of c we get $|m^2_{H_u}| \gg M^2_{1,\,2}$, the higgsinos do not mix with gauginos.

parameter	Point 1	Point 2	Point 3	Point 4	Point 5	Point 6	Point 7
$m_{3/2}$	20000	20000	20000	20000	30000	50000	30000
δ	-15	-12	0	-15	15	-15	-15
c	0	0	0	0.1	0.5	0	0
$\tan\beta$	3	2.65	2.65	3	3	2.5	3
μ	-11943	-13377	-13537	-10969	-10490	-34019	+17486
LSP type	Wino	Wino	Bino	Bino	Bino	Wino	Bino
M_1	165	173	203	181	484	434	252
M_2	158	173	225	189	662	421	242
M_3	262	297	423	328	1328	673	395
$m_{\tilde{g}}$	401	449	622	492	1784	1001	596.8
$m_{\widetilde{\chi}_1^0}$	145.1	155.6	189	170	473	373.4	271
$m_{\widetilde{\chi}_2^0}$	153	159	214.3	181.5	702.4	397	334.2
$m_{\widetilde{\chi}_3^0}$	11905	13321	13479	10938	10486	33886	17441
$m_{\widetilde{\chi}_4^0}$	11906	13322	13479	10939	10487	33886	17442
$m_{\widetilde{\chi}_1^\pm}$	145.2	155.8	214.5	181.7	702.6	373.6	334.2
$m_{\widetilde{\chi}_2^\pm}$	11970	13383	13540	11001	10560	34044	17540
$m_{\tilde{d}_L}, m_{\tilde{s}_L}$	19799	19803	19809	18785	21052	49524	29727
$m_{\tilde{u}_L}, m_{\tilde{c}_L}$	19801	19812	19818	18784	21034	49600	29725
$m_{\tilde{b}_1}$	15342	15250	15224	14635	16783	38473	23236
$m_{\tilde{t}_1}$	9130	8779	8662	8928	11151	22887	14264
$m_{\tilde{e}_L}, m_{\tilde{\mu}_L}$	19948	19948	19951	18926	21164	49889	29930
$m_{\tilde{\nu}_{e_L}}, m_{\tilde{\nu}_{\mu_L}}$	19950	19954	19952	18927	21168	49903	29934
$m_{\tilde{\tau}_1}$	19934	19941	19940	18914	21156	49874	29909
$m_{\tilde{\nu}_{\tau_L}}$	19936	19944	19942	18916	21158	49876	29913
$m_{\tilde{d}_R}$	19848	19851	19845	18832	21096	49694	29794
$m_{\tilde{u}_R}, m_{\tilde{c}_R}$	19850	19853	19858	18832	21094	49700	29792
$m_{\tilde{s}_R}$	19849	19851	19856	18832	21096	49695	29767
$m_{\tilde{b}_2}$	19829	19833	19838	18810	21075	49669	29758
$m_{\tilde{t}_2}$	15342	15251	15224	14635	16783	38470	23235
$m_{\tilde{e}_R}, m_{\tilde{\mu}_R}$	19978	19977	19977	18953	21196	49948	29966
$m_{\tilde{\tau}_2}$	19948	19957	19955	18930	21174	49904	29928
m_{h_0}	116.4	114.3	114.6	116.0	115.9	115.1	114.6
$m_{H_0}, m_{A_0}, m_{H^\pm}$	24614	25846	25943	23158	25029	65690	36623
$\tilde{A}_t$	12159	11539	11445	10898	9626	30139	18812
$\tilde{A}_b$	27381	27321	27427	24744	21850	68441	41148
$\tilde{A}_\tau$	30068	30092	30124	27109	23022	75221	45099

Table 2. Low scale spectra for seven benchmark G_2-MSSM models generated by SOFTSUSY package. All masses are in GeV. The top mass was taken to be $m_t = 171.3\,\text{GeV}$. Here we only give absolute values of the gaugino masses and suppress the relative phases. The spectra are largely determined by four parameters $m_{3/2}$, δ, c and $\tan\beta$. The Kaluza-Klein threshold corrections to the gaugino masses have been neglected. For the above spectra, the gauge couplings unify at the value of $\alpha_{GUT}^{-1} \approx 26$ at the scale $M_{GUT} \approx 2 \times 10^{16}$ GeV.

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7.2 Squarks and sleptons

Recall that at the GUT scale all the squarks and sleptons have a universal mass (5.39), which for generic values of c ($0 \leq c < 1$) is smaller but nevertheless typically of the same order of magnitude as the gravitino mass. However, as we evolve these down to the electroweak scale, the third generation scalars become significantly lighter whereas the first and second generation scalars experience a very mild change in their masses. Indeed, because the third generation Yukawa couplings are large, the stops, sbottoms, and staus are affected through the corresponding trilinear couplings (5.53), which are of $\mathcal{O}(m_{3/2})$. As one can see from table 2, this effect is especially dramatic for the lightest stop $\tilde{t}_1$. Yet, it is still much heavier than the gauginos and is effectively decoupled from the spectrum at the electroweak scale.

However, since gluinos can be pair produced at the LHC via gluon fusion, the gluinos (which have to decay via a quark-squark pair) have a sizeable branching fraction into top-stop — precisely because the stop is the lightest squark. This leads to events containing up to four top quarks at the LHC [55].

7.3 Radiative electroweak symmetry breaking

The existence of the electroweak symmetry breaking (EWSB) in the effective theory below the decoupling scale m_s is determined by whether there exists a negative eigenvalue in the Higgs mass matrix

$$\begin{pmatrix} m^2_{H_u} + \mu^2 & -B\mu \\ -B\mu & m^2_{H_d} + \mu^2 \end{pmatrix} \tag{7.5}$$

at the scale where the scalars decouple [56]. Recall that at the GUT scale, $m^2_{H_u} = m^2_{H_d} = (1-c)\, m^2_{3/2}$ whereas $\mu = Z^1_{\rm eff} m_{3/2}$ and $B\mu = Z^2_{\rm eff} m^2_{3/2}$. It is well known that the positive contribution into the running of the up Higgs mass parameter squared from the stop is crucial for radiative EWSB as it drives $m^2_{H_u}$ negative. It turns out that for a fixed value of the gravitino mass $m_{3/2}$, as we vary parameter c there exists a narrow range of values $Z^{1,2}_{\rm eff}$ for which the matrix (7.5) has a negative eigenvalue above the decoupling scale m_s defined in (7.1).

However, unless we force $Z^2_{\rm eff} \ll 2\, Z^1_{\rm eff}$, all the entries in the above matrix are $\mathcal{O}(m^2_{3/2})$. Therefore, both the lightest Higgs mass and the Z-boson mass naturally come out to be of $\mathcal{O}(m_{3/2})$. For the same reason, there is little mixing between the two Higgs doublets and naturally $\tan\beta \sim \mathcal{O}(1)$ is predicted in our framework. In practice, parameters $Z^{1,2}_{\rm eff}$ must be tuned in such a way that the corresponding eigenvalue turns negative right at the decoupling scale [56] so that $m_Z \approx 91\,\text{GeV}$. This fine tuning is a manifestation of the so-called *little hierarchy problem* - the hierarchy between the electroweak scale $M_{EW} \sim \mathcal{O}(100)\, GeV$ and the scale where the scalars decouple $m_s \sim \mathcal{O}(10)\,\text{TeV}$. Once M_Z is tuned, the Standard Model Higgs mass turns out to be $m_h < 130\,\text{GeV}$.

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