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Timelike entanglement entropy in dS_3/CFT_2

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Abstract: In the context of dS_3/CFT_2, we propose a timelike entanglement entropy defined by the renormalization group flow. This timelike entanglement entropy is calculated in CFT by using the Callan-Symanzik equation. We find an exact match between this entanglement entropy and the length of a timelike geodesic connecting two different spacelike surfaces in dS_3. The counterpart of this entanglement entropy in AdS_3 is a spacelike one, also induced by RG flow and extends all the way into the bulk of AdS_3. As a result, in both AdS_3/CFT_2 and dS_3/CFT_2, there exist exactly three entanglement entropies, providing precisely sufficient information to reconstruct the three-dimensional bulk geometry.

Keywords: AdS-CFT Correspondence, Holography and Hydrodynamics, de Sitter space

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Contents

1 Introduction

The gauge/gravity correspondence [1–5] states that a $(d+1)$-dimensional gravitational theory is equivalent to a d-dimensional conformal field theory (CFT), which provides a computational holographic realization. One of the central concerns in the duality is to extract the bulk geometry, kinematics and even dynamics, from the CFT quantities. Since Ryu and Takayanagi identified the holographic entanglement entropy (EE) of CFTs with the lengths of the geodesics anchored on the conformal boundary of the bulk geometry in [6, 7], it is believed that EE should play a major role in constructing the bulk geometry.

However, recent work has suggested that the traditional *spacelike* EE does not fully capture the entangling properties of CFTs. In ref. [8], we observed that there exists a severe inconsistency in the corrected EE of the $T\bar{T}$ deformed version of the AdS_3/CFT_2 correspondence. To resolve the inconsistency, we proposed that, in addition to the traditional spacelike EE, a *timelike* EE must be introduced. Remarkably, such a timelike EE has been explicitly addressed in the AdS/CFT context [9–14] recently. Unlike the traditional spacelike EE, which measures entangling between spacelike subsystems, the timelike EE reflects the entangling between timelike intervals. Since the $2d$ conformal boundary of AdS_3 is Minkovski, it looks natural that both spacelike and timelike EEs exist in the AdS_3/CFT_2.

The timelike EE in the context of the dS_3/CFT_2 correspondence [4, 15–17] is perhaps more intriguing. Naively, it appears that dS_3/CFT_2 leaves no room for a timelike EE. To see this clearly, let us consider the dS_3 in the planar coordinate,

$$ds^2_{\mathrm{dS}} = -dt^2 + e^{2t/\ell_{\mathrm{dS}}}\left(dx^2 + dy^2\right), \tag{1.1}$$

where $t \in (-\infty, \infty)$ and ℓ_{dS} is the dS radius. The future (or equivalently the past) boundary $\mathcal{I}^+$ as $t \to +\infty$, where the dual CFT_2 lives, is obviously a Euclidean plane that has two spatial directions. On the other hand, the AdS_3 in the planar coordinate reads,

$$ds^2_{\mathrm{AdS}} = d\xi^2 + e^{2\xi/\ell_{\mathrm{AdS}}}\left(-dt^2 + dx^2\right), \tag{1.2}$$

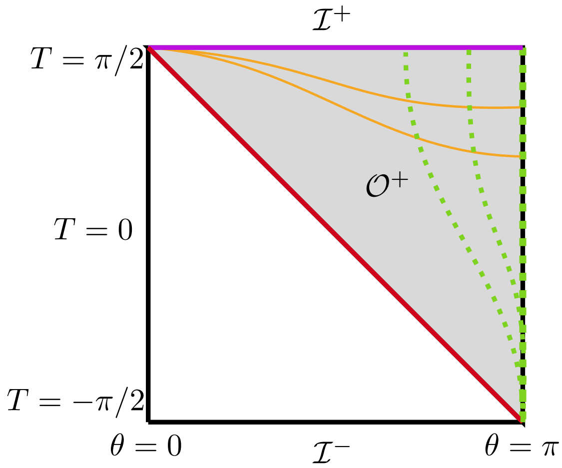

Figure 1. Penrose diagram of de Sitter spacetime. $\mathcal{I}^{\pm}$ is the *global* past and future spheres. The two vertical boundaries $\theta = 0, \pi$ are the north pole and south pole respectively. Each point in the interior represents an $\mathbb{S}^1$. A horizontal slice is an $\mathbb{S}^2$. The planar coordinate (1.1) covers the shadow region $\mathcal{O}^+$, comprising the causal future of the south pole. The green dashed lines are constant $r = \sqrt{x^2 + y^2}$. Orange lines of constant t are shown. The violet line $\mathcal{I}^+$ denotes the future boundary $t \to \infty$. The red line indicates the past horizon $t \to -\infty$.

which is related to the familiar Poincare coordinate with $\xi/\ell_{\text{AdS}} \to -\log[z/\ell_{\text{AdS}}]$. It is then easy to see that, the double Wick rotation,

$$\xi \to it, \quad t \to iy \quad \ell_{\text{AdS}} \to i\ell_{\text{dS}}, \quad x \to x, \tag{1.3}$$

a usual operation from AdS_3 to dS_3, transforms both the spacelike and timelike EEs in $\text{AdS}_3/\text{CFT}_2$ to spacelike EEs along the two directions in dS_3/CFT_2.

Remarkably, the above double Wick rotation already gives some hints about the existence of a timelike EE in dS_3/CFT_2. The purpose of this paper is to introduce a nontrivial timelike EE in the dS_3/CFT_2 context, which might be interpreted as the entanglement between the future and the past. To this end, it is of help to draw the Penrose diagram of dS_3 in figure (1) by using the conformal coordinate,

$$ds^2_{\text{dS}} = \frac{1}{\cos^2 T}(-dT^2 + d\theta^2 + \sin^2\theta d\phi^2), \tag{1.4}$$

with $-\pi/2 < T < \pi/2$. In the figure, $\mathcal{I}^{\pm}$ are the *global* past and future spheres. The two vertical boundaries $\theta = 0, \pi$ are the north pole and south pole respectively. Each point in the interior represents an $\mathbb{S}^1$. A horizontal slice is an $\mathbb{S}^2$. The planar coordinate (1.1) covers the shadow region $\mathcal{O}^+$, comprising the causal future of the south pole. The green dashed lines are constant $r = \sqrt{x^2 + y^2}$. Orange lines of constant t are shown. The violet line $\mathcal{I}^+$ denotes the future boundary $t \to \infty$. The red line indicates the past horizon $t \to -\infty$.

In dS_3/CFT_2, it is sufficient to consider the inflation patch $\mathcal{O}^+$, which consists of a collection of flat spacelike slices. Ref. [4] suggested that time evolution along these slices in dS_3 is equivalent to scale transformations of the dual CFT_2. In other words, time evolution is the inverse renormalization group (RG) flow [18], and the temporal dimension in the bulk can be interpreted as the renormalization scale μ of the CFT, where the future boundary $\mathcal{I}^+$ corresponds to the ultraviolet (UV) region of the CFT and the past horizon (red line with $t \to -\infty$) corresponds to the infrared (IR) region of the CFT. Guided by these facts, we will show that in dS_3/CFT_2, a timelike EE can be defined through the RG flow equation, which turns out to be $S_A = -\frac{\mathrm{i}\, c_{\rm dS}}{6} \log \frac{\xi}{\epsilon}$, where $c_{\rm dS}$ is the central charge, ϵ is the UV cutoff and ξ is an IR-like cutoff (correlation length). Since this RG flow induced EE involves both IR and UV cutoffs, it is convenient to refer to it as IR EE. The usual EE involving only UV cutoff is then called UV EE to distinguish between the two. Based on the hypothesis that the RG flow of the non-unitary CFT is dual to cosmological time evolution in dS, we will demonstrate that the IR EE is dual to a timelike geodesic connecting spacelike surfaces at different times in the bulk of dS_3. Our derivations do not require unitarity of the CFT, which is consistent with non-unitary CFT duals of de Sitter space [4, 15–17].

In the dS/CFT correspondence, we further clarify that timelike and spacelike EEs are intrinsically different. It is not surprising to find that, via an analytical continuation, the timelike IR EE in dS/CFT rotates to a spacelike IR EE in AdS/CFT, which obviously is associated with the emergent radial direction of AdS!

It is then illuminating to note that, in both dS_3/CFT_2 and AdS_3/CFT_2, there are exactly three EEs, which fits precisely to reconstruct the three-dimensional bulk geometry. This is just what has been studied in refs. [8, 19, 20].

The remainder of this paper is outlined as follows. In section 2, we derive the IR EE by using the Callan-Symanzik equation in CFT. In section 3, we calculate the length of a timelike geodesic which connects two distinct spacelike boundaries and find it matches the IR EE perfectly. Section 4 is for conclusion and discussions.

2 RG flow induced entanglement entropy in QFT

In this section, using the Callan-Symanzik equation, we present a universal derivation of the RG flow induced EE, which we refer to as the IR EE, for a generic CFT. When applied to the context of dS/CFT, the IR EE is timelike.

The dS/CFT correspondence is not as well understood as the AdS/CFT correspondence. There are only limited examples of CFTs that are dual to dS. In addition to a four dimensional higher spin gravity example [21], a recent remarkable construction has been given for the dS_3/CFT_2 correspondence [16, 17].

In the canonical formalism for gravity, the quantum state residing on a static compact slice Σ_t can be described by the Hartle-Hawking wavefunction $\Psi_{\rm dS}[\gamma]$, where γ is the metric on Σ_t. The dS/CFT could be defined through the dictionary [15],

$$\Psi_{\rm dS}[\gamma] = Z_{\rm CFT}[\gamma], \quad t \to \infty \tag{2.1}$$

where $Z_{\rm CFT}$ is the partition function of the CFT_2 living on Σ_∞. Since the CFTs dual to dS are non-unitary [4, 15–17], another universal quantity is needed to measure the entanglement. To this end, parallel to the EE in unitary CFT, a complex-valued quantity known as the pseudoentropy is introduced in refs. [9, 22–29]. Dividing the total system

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into two subsystems A and B, the pseudoentropy is defined by the von Neumann entropy,

$$S_A = -\mathrm{Tr}\left[\tau_A \log \tau_A\right], \tag{2.2}$$

of the reduced transition matrix

$$\tau_A = \mathrm{Tr}_B \left[\frac{|\psi\rangle\langle\varphi|}{\langle\varphi \mid \psi\rangle}\right]. \tag{2.3}$$

Here, $|\psi\rangle$ and $|\varphi\rangle$ are two different quantum states in the total Hilbert space that is factorized as $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$. It should be emphasized that the pseudoentropy defined by eq. (2.2) is generic and does not depend on the details of a particular non-unitary CFT. As the usual EE, the pseudoentropy could also be captured by the replica method [30, 31] in path integral formalism. Denoting the manifold corresponding to $\langle\varphi \mid \psi\rangle$ as $\mathcal{M}_1$ and the manifold corresponding to $\mathrm{Tr}_A\left(\tau_A\right)^n$ as $\mathcal{M}_n$, the n-th pseudo Rényi entropy reads

$$S_A^{(n)} = \frac{1}{1-n} \log\left[\frac{Z_{\mathcal{M}_n}}{\left(Z_{\mathcal{M}_1}\right)^n}\right], \tag{2.4}$$

where $Z_{\mathcal{M}}$ is the partition function over the manifold $\mathcal{M}$. The n-sheeted Riemann surfaces $\mathcal{M}_n$ in dS is constructed in the same way as in AdS. Taking the limit $n \to 1$ yields the pseudoentropy

$$S_A = \lim_{n\to 1} \frac{1}{1-n} \log\left[\frac{Z_{\mathcal{M}_n}}{\left(Z_{\mathcal{M}_1}\right)^n}\right], \tag{2.5}$$

which can be regarded as a well-defined EE in the $\mathrm{dS}_3/\mathrm{CFT}_2$ context.

Since the definitions (2.4) and (2.5) are identical for both usual EE and pseudoentropy, the following derivations of IR EE are applicable to both unitary and non-unitary CFTs. Consider a $2d$ generic CFT_2 living on a curved surface $\mathcal{M}$ with the metric $ds^2 = \gamma_{ab} dx^a dx^b$. It is known that [32], for a classically scale-invariant theory where only dimensionless couplings are present, the Callan-Symanzik equation is

$$\left[\mu \frac{\partial}{\partial \mu} + 2 \int d^2x \, \gamma^{ab} \frac{\delta}{\delta \gamma^{ab}}\right] \log Z_{\mathrm{CFT}} = 0, \tag{2.6}$$

with the renormalization scale μ. The n-th Rényi entropy (2.4) of the subsystem A thus satisfies

$$\left[\mu \frac{\partial}{\partial \mu} + 2 \int d^2x \, \gamma^{ab} \frac{\delta}{\delta \gamma^{ab}}\right] S_A^{(n)} = 0. \tag{2.7}$$

Note that the expectation value of the stress tensor is given by

$$\langle T_a^a \rangle = -2 \frac{\gamma^{ab}}{\sqrt{\gamma}} \frac{\delta}{\delta \gamma^{ab}} \log Z_{\mathrm{CFT}}. \tag{2.8}$$

So the Callan-Symanzik equation of the n-th Rényi entropy can be rewritten as

$$\mu \frac{\partial}{\partial \mu} S_A^{(n)} = -\frac{\int_{\mathcal{M}_n} \langle T_a^a \rangle_{\mathcal{M}_n} - n \int_{\mathcal{M}_1} \langle T_a^a \rangle_{\mathcal{M}_1}}{1-n}. \tag{2.9}$$

The central charge c of the $2d$ CFT has a clear definition due to the presence of the Weyl anomaly

$$\langle T_a^a \rangle = +\frac{1}{2\pi} \frac{c}{12} \mathcal{R}, \tag{2.10}$$

with $\mathcal{R}$ the scalar curvature, and one has

$$\mu\frac{\partial}{\partial\mu}S_A^{(n)}=-\frac{c\left(\int_{\mathcal{M}_n}\mathcal{R}^{(n)}-n\int_{\mathcal{M}_1}\mathcal{R}\right)}{24\pi\left(1-n\right)}. \tag{2.11}$$

For the n-sheeted Riemannian surface in the presence of conical singularities, ref. [33] has shown that

$$\int_{\mathcal{M}_n}\mathcal{R}^{(n)}=n\int_{\mathcal{M}_1}\mathcal{R}+4\pi\left(1-n\right)\int_{\Sigma}1, \tag{2.12}$$

where Σ is the entangling surface. In our case here, $(\int_{\Sigma=\partial A}1)=\mathcal{A}$, which is the number of the boundary points of the subsystem A. Therefore, the Callan-Symanzik equation of the n-th Rényi entropy could be simplified as

$$\mu\frac{\partial}{\partial\mu}S_A^{(n)}=-\mathcal{A}\cdot\frac{c}{6}. \tag{2.13}$$

It is quite interesting to note that this RG flow induced n-th Rényi entropy $S_A^{(n)}$ is independent of n and therefore simply equals the RG flow induced IR EE! After replacing $\mu_{\rm UV}^{-1}/\mu_{\rm IR}^{-1}$ by the UV/IR cutoff ϵ/ξ, the RG flow induced IR EE is given by

$$S_A^{\rm IR}=S_A^{(n)}=-\int_{\mu_{\rm UV}}^{\mu_{\rm IR}}\mathcal{A}\cdot\frac{c}{6}\frac{d\mu}{\mu}=\mathcal{A}\cdot\frac{c}{6}\log\frac{\xi}{\epsilon}, \tag{2.14}$$

which is independent of the metric γ.

Two other field theoretic approaches to derive this IR EE (2.14) were given in ref. [30]. The first one parallels the proof of c-theorem. The second argument calculates a scalar field theory perturbed by a mass term. None of them requires unitarity. However, both approaches have to take the limit $n\to 1$ to get the IR EE from the n-th Rényi entropy, and the number of the boundary points $\mathcal{A}$ cannot be easily derived.

The IR EE (2.14) is universal and determined only by the central charge and the intrinsic correlation length of a specific CFT. In the dS/CFT context, it is known that the central charge of CFT_2 dual to dS_3 is imaginary-valued [15], and from the Brown-Henneaux's formula [34], we have

$$c=-\mathrm{i}\,c_{\rm dS}=-\mathrm{i}\,\frac{3\ell_{\rm dS}}{2G_N^{(3)}}. \tag{2.15}$$

Thus, the IR EE in the $\mathrm{dS}_3/\mathrm{CFT}_2$ correspondence is

$$S_A^{\rm IR}=-\frac{\mathrm{i}\,c_{\rm dS}}{6}\log\frac{\xi}{\epsilon}=-\mathrm{i}\,\frac{\ell_{\rm dS}}{4G_N^{(3)}}\log\frac{\xi}{\epsilon}, \tag{2.16}$$

where $\mathcal{A}=1$ is assumed.

It is crucial to understand that the IR EE should not be confused with the UV EE,

$$S_A^{\rm UV}=-\frac{\mathrm{i}\,c_{\rm dS}}{3}\log\frac{L}{\epsilon}+\frac{\pi c_{\rm dS}}{6}. \tag{2.17}$$

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The differences are quite distinct from the above expressions, since the IR EE does not have a real part but the UV EE does.[1] Additionally, the IR cutoff ξ used in the IR EE has a clear different interpretation from the entangling interval length $L = \Delta x$ used in the UV EE. The UV EE only holds as $L \ll \xi$.

As explained in the Introduction, in the dS/CFT context, the RG flow in the CFT corresponds the temporal direction of dS, thus the RG flow induced IR EE is nothing but a timelike EE in dS/CFT.

3 Holographic timelike entanglement entropy in dS_3/CFT_2

To exhibit the timelike feature of the IR EE explicitly, it is illuminating to study its corresponding dS bulk dual. For any two points (t_1, x_1, y_1) and (t_2, x_2, y_2) in the planar coordinate of dS_3 (1.1), the geodesic distance L is

$$\cos\left(\frac{L}{\ell_{\rm dS}}\right) = 1 + \frac{\left[\exp\left(-t_1/\ell_{\rm dS}\right) - \exp\left(-t_2/\ell_{\rm dS}\right)\right]^2 - \frac{1}{\ell_{\rm dS}^2}\left[(x_1-x_2)^2 + (y_1-y_2)^2\right]}{2\exp\left(-t_1/\ell_{\rm dS}\right)\exp\left(-t_2/\ell_{\rm dS}\right)}. \tag{3.1}$$

The length of a timelike geodesic between two points $(t_{+\infty} = \ell_{\rm dS}\log(\ell_{\rm dS}/\epsilon), x, y)$ and $(t_{-\infty} = \ell_{\rm dS}\log(\ell_{\rm dS}/\xi), x, y)$ is *exactly*

$$L\left(t_{+\infty}, t_{-\infty}\right) = \ell_{\rm dS} \arccos\left[\frac{\xi^2+\epsilon^2}{2\xi\epsilon}\right] = -\mathrm{i}\,\ell_{\rm dS}\log\left(\frac{\xi}{\epsilon}\right).$$

where the principal branch of the complex inverse cosine function is chosen. Assuming the Ryu-Takayanagi formula [6] also holds in dS/CFT, applying eq. (2.15), this timelike geodesic length gives the corresponding entropy

$$S_A = \frac{L}{4G_N^{(3)}} = -\frac{\mathrm{i}\,\ell_{\rm dS}}{4G_N^{(3)}}\log\frac{\xi}{\epsilon} = -\frac{\mathrm{i}\,c_{\rm dS}}{6}\log\frac{\xi}{\epsilon}, \tag{3.2}$$

which is *precisely* equal to the IR EE in eq. (2.16). So, we do find perfectly matched quantities in dS_3 and CFT_2. One is a timelike geodesic, another is the IR EE. Intriguingly, the match is exact and there is no need to take the UV or IR limits.

It is particularly evident that, under the double Wick rotation (1.3), the timelike IR EE in dS/CFT is transformed to a spacelike IR EE in AdS/CFT. In both dS and AdS, the geodesics that are dual to the IR EE extend all the way into the bulk, in sharp contrast to the UV EE whose endpoints are both attached to the boundary. As a result, the IR EE must provide indispensable information for the reconstruction of spacetime.

The various EEs in dS and AdS are connected via analytic continuations. In the planar or Poincare coordinates, they are transformed to each other through the double Wick rotation (1.3). However, this procedure is far from clear beyond the semiclassical limit and some more nontrivial calculation may be needed. We summarize the classifications of all the EEs in table 1.

[1] This is not the case in the AdS/CFT context, where both the IR EE and the traditional spacelike UV EE are real. Perhaps this is why IR EE is frequently overlooked as an independent EE, but mistakenly considered as only one half of the UV EE.

	dS/CFT	AdS/CFT
UV	Spacelike: $S_A = -\frac{\mathrm{i}\, c_{\mathrm{dS}}}{3}\log\left(\frac{X}{\epsilon}\right) + \frac{\pi c_{\mathrm{dS}}}{6}$	Spacelike: $S_A = \frac{c_{\mathrm{AdS}}}{3}\log\frac{X}{\epsilon}$
	Spacelike: $S_A = -\frac{\mathrm{i}\, c_{\mathrm{dS}}}{3}\log\left(\frac{Y}{\epsilon}\right) + \frac{\pi c_{\mathrm{dS}}}{6}$	Timelike: $S_A = \frac{c_{\mathrm{AdS}}}{3}\log\frac{T}{\epsilon} + \frac{\mathrm{i}\,\pi c_{\mathrm{AdS}}}{6}$
IR	Timelike: $S_A = -\frac{\mathrm{i}\, c_{\mathrm{dS}}}{6}\mathcal{A}\log\frac{\xi}{\epsilon}$	Spacelike: $S_A = \frac{c_{\mathrm{AdS}}}{6}\mathcal{A}\log\frac{\xi}{\epsilon}$

Table 1. Spacelike, timelike EEs and UV, IR EEs in dS/CFT and AdS/CFT. Two elements in each row are transformed to each other via an analytic continuation. Evidently, the IR EE and UV EE are completely distinct, especially from the perspective of dS/CFT. $\mathcal{A}$ is the number of the boundary points of the subsystem A.

4 Conclusions

In this paper, we introduced a timelike EE, in the context of dS/CFT correspondence. Since this RG flow induced EE is expressed by both IR and UV cutoffs, we called it as IR EE to distinguish it from the usual UV EE, which involves UV cutoff only. We demonstrated that this IR EE does perfectly match the length of a timelike geodesic connecting two distinct spacelike surfaces in dS_3. In AdS_3, the counterpart of this IR EE is spacelike. Our results reveal that there are three independent EEs in whether dS_3/CFT_2 or AdS_3/CFT_2, which provides just enough information to reconstruct the bulk geometry.

It is quite intriguing that, the match of this IR EE with the dual geodesic length is exact, working for any cutoffs, not restricted to the UV or IR limits.

While we considered the simplest pure dS in this paper, our findings could be extended to generic asymptotic dS(AdS) spacetimes whose time(radial) evolution corresponds to a nontrivial inverse RG flow.

It is difficult to ignore the potential role of the de Sitter cosmological event horizon as a natural entangling surface. It is very interesting that there is one candidate, namely the $T\bar{T}$ deformation, could realize this idea.

Refer to figure (1), the UV and the IR are connected at $r = \infty$, which leads us to speculate that the timelike EE, represented by the green dashed lines, may serve as a holographic screen for the inflation patch of de Sitter spacetime. This suggests that the inflation patch is in a mixed state and is entangled with another universe, or possibly even multiple universes.

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