












































































































































Search for the double-charmonium state with $\eta_c J/\psi$ at Belle



The Belle collaboration

<https://belle.kek.jp/>

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ABSTRACT: We measure the cross section of $e^+e^- \rightarrow \eta_c J/\psi$ at the $\Upsilon(nS)$ ($n = 1-5$) on-resonance and 10.52 GeV off-resonance energy points using the full data sample collected by the Belle detector with an integrated luminosity of 955 fb^{-1} . We also search for double charmonium production in $e^+e^- \rightarrow \eta_c J/\psi$ via initial state radiation near the $\eta_c J/\psi$ threshold. No evident signal of the double charmonium state is found, but evidence for the $e^+e^- \rightarrow \eta_c J/\psi$ process is found with a statistical significance greater than 3.3σ near the $\eta_c J/\psi$ threshold. The average cross section near the threshold is measured and upper limits of cross sections are set for other regions.

KEYWORDS: e^+e^- Experiments, Exotics, QCD, Spectroscopy

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1 Introduction

Early in this century, a number of exotic states, the so called “ XYZ ” particles, have been discovered [1] via their decays into two heavy-flavor mesons and/or a quarkonium and one or two light hadrons. Vector states with $J^{PC} = 1^{--}$, such as the $\psi(4260)$ [2], $\psi(4360)$ [3, 4], and $\psi(4660)$ [3], are alternatively called Y states. The $\psi(4260)$ state is observed for the first time by the BABAR experiment with a mass of $(4259 \pm 8_{-6}^{+2})$ MeV/ c^2 using the initial state radiation (ISR) events $e^+e^- \rightarrow \gamma_{\text{ISR}}\pi^+\pi^-J/\psi$ [2]. The observation was later confirmed by CLEO [5] and Belle [6]. In ref. [7], the authors calculated the mass of $\psi(4230)$ as 4238 ± 31 MeV/ c^2 using lattice quantum chromodynamics (QCD) by treating this state as a molecule. Moreover, the authors predict two additional exotic states with quark compositions of $c\bar{s}\bar{c}$ and $cc\bar{c}$ with masses of (4450 ± 100) MeV/ c^2 and (6400 ± 50) MeV/ c^2 , respectively.

In 2017, a dedicated analysis performed by the BESIII experiment revealed that the so-called $\psi(4260)$ state is not simply one resonance but two [8]. The first, called the $\psi(4230)$ state, has a lower mass and a much narrower width, while the second at around 4.32 GeV/ c^2

is observed for the first time with a significance greater than 7.6σ . The lower-mass resonance was also observed in $e^+e^- \rightarrow \pi^+\pi^-h_c$ [9, 10], $e^+e^- \rightarrow \omega\chi_{c0}$ [11] and $\pi\bar{D}D^* + c.c.$ events [12]. The $\psi(4230)$ state is also observed to have a relative large decay rate to lower charmonium states via η transition; *viz* $\psi(4230) \rightarrow \eta J/\psi$ [13], $\psi(4230) \rightarrow \eta' J/\psi$ [14].

Recently, BESIII reported the cross section measurements of $e^+e^- \rightarrow K^+K^-J/\psi$ [15, 16]. A structure is observed with $M = 4487.7 \pm 13.3 \pm 24.1$ MeV/ c^2 , which is very close to the above prediction of 4450 MeV/ c^2 for $cs\bar{c}\bar{s}$. Belle also reported a structure with a mass around 4620 MeV/ c^2 in the cross section measurements of $e^+e^- \rightarrow D_s^+D_{s1}(2536)^- + c.c.$ [17] and $D_s^+D_{s2}^*(2573)^- + c.c.$ [18]. Additionally, LHCb reported the observation of a possible $cs\bar{c}\bar{s}$ state in $B^+ \rightarrow K^+\phi J/\psi$ decays [19, 20]. Furthermore, LHCb reported pronounced structures in the invariant mass spectrum of J/ψ pairs [21]. An enhancement in the near-double- J/ψ threshold region from 6.2 to 6.8 GeV is seen, followed by another narrow peak around 6.9 GeV, dubbed $X(6900)$. The interaction between the two charmonia may not be strong enough to form a tight bound state, many theoretical studies adopt the compact tetraquark picture [22]. The compact diquark anti-diquark structure $[QQ][\bar{Q}\bar{Q}]$ is the most popular one, but the mass predictions are quite model dependent [23–35]. More experimental and theoretical investigations are crucial to understand them.

The lowest mass combination of charmonia to which a vector $cc\bar{c}\bar{c}$ could decay is $\eta_c J/\psi$, and this process may have a relative large branching fraction. We present the results of a search for such a vector $cc\bar{c}\bar{c}$ state, hereinafter designated Y_{cc} , with the Belle detector [36]¹ at the KEKB asymmetric-energy e^+e^- collider [38, 39].² The integrated luminosity is 980 fb⁻¹, about 70% of which were collected at the $\Upsilon(4S)$ resonance; the rest were taken at other $\Upsilon(1, 2, 3, 5S)$ states or center-of-mass (c.m.) energies just below the $\Upsilon(4S)$ or the $\Upsilon(nS)$ peaks by tens of MeV, as well as various c.m. energies between 10.63 GeV and 11.02 GeV. Initial state radiation (ISR) allows us to search for the double-charmonium state in the near-threshold region. We first measure the cross section of $e^+e^- \rightarrow \eta_c J/\psi$ on the on-resonance $\Upsilon(nS)$ energy point, providing validation for our method as well as a solid check for the next-to-next-to-leading-order calculation in the nonrelativistic QCD approach [41]. We then search for the possible $\eta_c J/\psi$ and Y_{cc} signals in the near-threshold region. We extrapolate the measured cross section from the on-resonance points to the near-threshold region to check whether the possible $\eta_c J/\psi$ signals here are from continuum production.

2 Belle detector and data samples

The Belle detector [36, 37] is a large-solid-angle magnetic spectrometer that consists of a silicon vertex detector (SVD), a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter (ECL) consisting of CsI(Tl) crystals. All these detector components are located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux-return located outside of the coil is instrumented with resistive plate chambers to detect K_L^0 mesons and to identify muons.

¹Also see detector section in ref. [37].

²See also following articles up to ref. [40].

The signal Monte Carlo (MC) samples of the ISR processes and the decays of η_c and J/ψ are simulated with the PHOKHARA [42] and EVTGEN [43] event generators, respectively, with the decay branching fractions and resonance parameters of η_c and J/ψ taken from ref. [44]. These events are processed by a detector simulation based on GEANT3 [45]. The generic MC samples, corresponding to six times the integrated luminosity of the data, of $e^+e^- \rightarrow \Upsilon(nS)$ events with subsequent $\Upsilon(nS)$ decays and $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) events are used to check the backgrounds. A tool named TOPOANA [46] is used to visualize the MC event types after event selection. A series of signal MC samples is generated with different $m(Y_{cc})$ assumptions to estimate the signal resolutions as well the c.m. energy-dependent efficiency.

3 Event selection

Two distinct reconstruction methods are implemented in this analysis. One is exclusive reconstruction of $\eta_c J/\psi$, and the other is inclusive reconstruction using J/ψ or $J/\psi\gamma_{\text{ISR}}$ for $\Upsilon(nS)$ on/off resonance or near-threshold data.

Charged particle tracks are required to have impact parameters perpendicular to and along the beam direction with respect to the interaction point of less than 1.0 and 4.0 cm, respectively. The transverse momentum of each track is required to be greater than 100 MeV/c. For particle identification (PID), except for tracks from K_S^0 , information from different detector subsystems is combined to form the likelihood \mathcal{L}_i for species i , where $i = e, \mu, \pi, K, \text{ or } p$ [47]. Particles with $\mathcal{L}_K/(\mathcal{L}_K + \mathcal{L}_\pi) > 0.6$ are regarded as kaons, while those with a ratio below 0.4 are identified as pions. A track with $\mathcal{L}_p/(\mathcal{L}_p + \mathcal{L}_\pi) > 0.6$ and $\mathcal{L}_p/(\mathcal{L}_p + \mathcal{L}_K) > 0.6$ is identified as a proton. Distinct likelihoods are used for muon [48] and electron identification [49]: we require $\mathcal{L}_\mu/(\mathcal{L}_\mu + \mathcal{L}_K + \mathcal{L}_\pi) > 0.1$ for muon candidates and $\mathcal{L}_e/(\mathcal{L}_e + \mathcal{L}_{\text{non-}e}) > 0.01$ for electron candidates. These PID requirements provide a relatively high selection efficiency and a low misidentification rate, i.e., about 80% and 7%, respectively, for pions.

The photon with the largest energy in the c.m. frame in an event is taken as the ISR photon; its energy is required to be greater than 1 GeV. K_S^0 candidates are reconstructed by combining two tracks of opposite charge reconstructed using the pion hypothesis and consistent with originating from a displaced vertex. Combinatorial background is suppressed using a neural network [50]. Neutral pion candidates are reconstructed from pairs of photons, each photon having deposited energy of at least 50 MeV in the barrel region of the ECL (polar angle within interval $[32.2^\circ, 128.7^\circ]$), or at least 100 MeV in the end-caps (polar angle within interval $[12.4^\circ, 31.4^\circ] \cup [130.7^\circ, 155.1^\circ]$). The invariant mass of the π^0 candidate is required to be within the interval $[0.115, 0.155]$ GeV/c², which encompasses a 3σ mass window around the nominal mass. A mass-constrained fit is performed to each surviving π^0 candidate to improve its momentum resolution.

Lepton pairs, e^+e^- or $\mu^+\mu^-$, are used to reconstruct J/ψ . To reduce the effects of bremsstrahlung and final-state radiation, all photons within a 50 mrad cone of the initial electron or positron direction are included in the calculation of the candidate's four-momentum. The mass window of J/ψ is optimized to be $M(e^+e^-) \in [3.000, 3.120]$ GeV/c²

and $M(\mu^+\mu^-) \in [3.075, 3.125]$ GeV/ c^2 using the figure of merit $FOM = N_S/\sqrt{N_B + 0.5}$, where N_S is the number of events in signal MC (depending on different mass window requirements, and N_B is the number of events in generic MC). In the exclusive reconstruction, six hadronic channels are used to reconstruct η_c : $p\bar{p}$, $p\bar{p}\pi^0$, $K_S^0 K^\pm \pi^\mp$, $K^+ K^- \pi^0$, $K^+ K^- K^+ K^-$, $2(\pi^+ \pi^- \pi^0)$. The number of extra charged tracks found in the detector must be less than three. The optimized mass window is $M(\eta_c) \in [2.78, 3.08]$ GeV/ c^2 . The candidate with the smallest value of the mass recoiling against $\eta_c J/\psi$, $M_{\text{recoil}}^2 \equiv |p_{e^+e^-} - p(\eta_c) - p(J/\psi)|^2/c^2$ is chosen as the best candidate; here, p is the four-momentum of the specified particle in lab frame. In the inclusive reconstruction, the number of charged tracks is required to be greater than four to suppress the QED backgrounds.

4 Data analysis

4.1 Double charmonium production at $\Upsilon(nS)$ on-resonance and $\Upsilon(4S)$ off-resonance energies

In this part, we only analyze the datasets at the $\Upsilon(nS)$ resonances as well as the $\Upsilon(4S)$ off-resonance sample, corresponding to a luminosity of 955 fb $^{-1}$. In double charmonium production at $\Upsilon(nS)$ on-resonance and $\Upsilon(4S)$ off-resonance energies, the square of the missing mass in the exclusive reconstruction is required to be within the interval $[-0.05, 0.08]$ GeV $^2/c^4$ to increase the signal purity. The invariant mass distributions of $\eta_c J/\psi$ in exclusive reconstruction are shown in figure 1 for data from the selected energy points. The main background are due to combinatorial η_c and J/ψ candidates. From a study of sideband events in the data, no peaking background is expected. Unbinned extended maximum likelihood fits are performed to the $\eta_c J/\psi$ invariant mass distributions except for the $\Upsilon(3S)$ dataset. Signal components are described using shapes derived from MC simulation, and smoothed using kernel estimation [51]. The background components are described with a first order polynomial. Solely for the $\Upsilon(3S)$ dataset, the upper limit at the 90% confidence level (C.L.) on the signal yield is estimated.³ Cross sections are calculated with the formula

$$\sigma = \frac{N_{\text{sig}}}{\epsilon \mathcal{L} \mathcal{B}(J/\psi \rightarrow \ell^+ \ell^-) \mathcal{B}(\eta_c \rightarrow 6 \text{ channels})}, \quad (4.1)$$

where N_{sig} is the number of signal events, ϵ is the reconstruction efficiency, \mathcal{L} is the integrated luminosity, and \mathcal{B} includes the corresponding branching fractions. The calculated cross sections are shown in table 1.

The J/ψ recoil-mass distributions in the inclusive reconstruction are shown in figure 2, where the J/ψ recoil mass is defined as $M_{\text{recoil}}(J/\psi) \equiv \sqrt{|p_{e^+e^-} - p_{J/\psi}|^2/c}$ and p is the four-momentum. To improve the resolution on the recoil mass, we replace the recoil mass with $M_{\text{recoil}}(J/\psi) + M(J/\psi) - m(J/\psi)$, where $M(J/\psi)$ is the reconstructed J/ψ mass and

³The upper limit is calculated by using a frequentist method with unbounded profile likelihood treatment of systematic uncertainties, which is implemented by a C++ class TROLKE in the ROOT framework [52]. The number of the observed events is assumed to follow a Poisson distribution, the number of background events and the efficiency are assumed to follow Gaussian distributions.

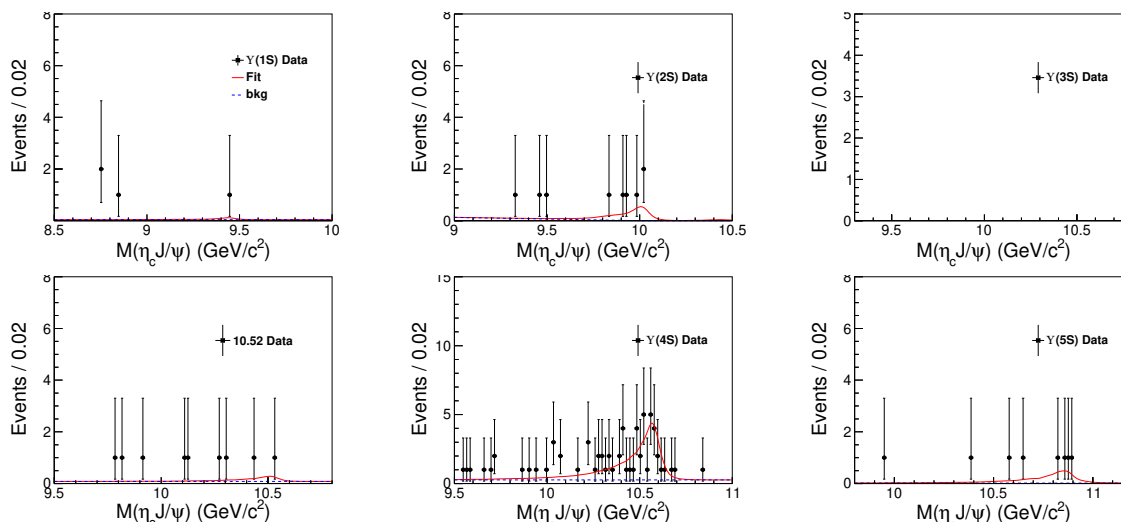


Figure 1. Fit to the invariant mass of $\eta_c J/\psi$ at selected energy points. Dots with error bars show the experimental data, red curves the fit results, and blue dashed curves the background.

	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(3S)$	10.52 GeV	$\Upsilon(4S)$	$\Upsilon(5S)$
\mathcal{L} [fb $^{-1}$]	5.7	24.9	2.9	89.4	711.0	121.4
N^{exc}	$0.7^{+1.5}_{-0.9}$	$6.2^{+3.1}_{-2.3}$	< 1.9	$2.6^{+3.5}_{-2.5}$	$45.0^{+8.9}_{-8.2}$	$6.5^{+3.4}_{-2.7}$
ϵ^{exc}	8.3%	6.9%	5.7%	5.6%	5.6%	5.4%
σ^{exc} [fb]	$57^{+122}_{-73} \pm 6$	$140^{+70}_{-52} \pm 14$	< 442	$20^{+27}_{-19} \pm 6$	$44^{+9}_{-8} \pm 5$	$39^{+20}_{-14} \pm 7$
N^{inc}	23.7 ± 12.3	62.0 ± 17.9	8.5 ± 5.2	94.7 ± 23.8	1116.2 ± 62.9	91.1 ± 21.5
ϵ^{inc}	38.6%	29.6%	26.4%	26.1%	25.4%	24.7%
σ^{inc} [fb]	$89.1 \pm 46.2 \pm 20.5$	$70.1 \pm 20.2 \pm 8.9$	$91.8 \pm 56.2 \pm 52.3$	$33.8 \pm 8.5 \pm 2.8$	$52.1 \pm 2.9 \pm 5.0$	$25.4 \pm 6.0 \pm 2.8$
σ^{comb} [fb]	$78.3^{+47.5}_{-43.0}$	80.2 ± 20.4	$87.0^{+71.0}_{-59.0}$	32.5 ± 8.5	50.2 ± 5.0	27.5 ± 6.1

Table 1. Signal yields and the measured cross sections for different channels at selected c.m. energy points. Here \mathcal{L} is the integrated luminosity; ϵ is the reconstruction efficiency; N and σ are, respectively, the signal yields and measured cross section, where the superscript “exc/inc” indicates the exclusive/inclusive analysis; and σ^{comb} is the cross section combining the two reconstruction analyses. The first uncertainties in $\sigma^{\text{exc/inc}}$ are statistical and the second are systematic. The σ^{comb} total uncertainties are given including the statistical and systematic.

$m(J/\psi)$ is the nominal mass from ref. [44]. Clear η_c , χ_{c0} , and $\eta_c(2S)$ signals are found in the recoil-mass spectra, as in previous Belle measurements [53, 54]. Unbinned extended maximum likelihood fits are performed to the recoil-mass spectra. The signals are described using shapes derived from MC simulation smeared with a Gaussian. Parameters of the Gaussian functions are free for the fit to the $\Upsilon(4S)$ on-resonance data sample, but fixed to the values obtained in the $\Upsilon(4S)$ fit result when fitting to other energy points. Background is described with a third-order-polynomial. Signal yields from the fits are listed in table 1 together with the cross sections. The cross sections here are calculated using a formula similar to eq. (4.1) but omitting the η_c branching fractions.

We combine cross sections measured from the two reconstruction methods with the following approach. First, we extract the cross-section-dependent likelihood distributions from the two methods. For a given cross section in the certain data sample, the possibility that we observe the current number of signal events is estimated according to the fits. (For the exclusive reconstruction with $\Upsilon(3S)$ data, we assume the number of the observed events follows the Poisson distribution.) Consequently, a cross-section-dependent joint probability density function (PDF) is obtained. We smear the PDF with Gaussian functions whose widths model the systematic uncertainties that are discussed below. The peak of the PDF is taken as the nominal result, and the positions bounding 68% of the total integrated area under the PDF are taken as the uncertainties. The final cross section results are listed in table 1 and plotted in figure 3.

We fit the cross sections to extrapolate to the $\eta_c J/\psi$ threshold region to estimate the continuum contribution if any signal is found there. There should be two sources of $\eta_c J/\psi$ production: one is $e^+e^- \rightarrow \gamma^* \rightarrow \eta_c J/\psi$, which is so called continuum production, and the other is $e^+e^- \rightarrow \Upsilon(nS) \rightarrow \gamma^* \rightarrow \eta_c J/\psi$. This mechanism should be similar to the production of $e^+e^- \rightarrow \mu^+\mu^-$. Thus, we estimate the fractions produced from the continuum relative to $e^+e^- \rightarrow \mu^+\mu^-$ total cross sections at $\Upsilon(nS)$ energy point in ref. [55], where the corresponding fractions of the continuum production are about 5/6 and 4.5/4.75 for $\Upsilon(1S)$ and $\Upsilon(2S)$, respectively. Since we do not find a signal in the $\Upsilon(3S)$ dataset, and the uncertainty at this energy point is very large, we use a fraction of one at this energy point. For the $\Upsilon(4S)$ and $\Upsilon(5S)$, we assume that signals originate exclusively from continuum production. We fit the cross sections of $e^+e^- \rightarrow \eta_c J/\psi$ from the continuum production at $\Upsilon(1/2S)$ along with the $e^+e^- \rightarrow \eta_c J/\psi$ at other energy points with the empirical function

$$\sigma = A \frac{\sqrt{2\mu\Delta M}}{\left(\frac{s}{s_0}\right)^n}, \tag{4.2}$$

where $\mu = \frac{m(\eta_c)m(J/\psi)}{m(\eta_c)+m(J/\psi)}$ is the reduced mass, $\Delta M = \sqrt{s} - m(\eta_c) - m(J/\psi)$ is the mass difference, $m(\eta_c)$ and $m(J/\psi)$ are the η_c and J/ψ nominal masses [44]; $s_0 \equiv (10.58 \text{ GeV}/c^2)^2$ is the c.m. energy at the $\Upsilon(4S)$ resonance, A and n are free parameters whose best-fit values are $A = (11.4 \pm 0.9) \text{ c}^2 \cdot \text{fb}/\text{GeV}$, and $n = 4.5 \pm 1.3$.

4.2 $e^+e^- \rightarrow \eta_c J/\psi$ near threshold

In this part, we use the entire 980 fb^{-1} Belle dataset. For exclusive reconstruction of $e^+e^- \rightarrow \eta_c J/\psi$ near-threshold events, the squared recoil mass of the $\eta_c J/\psi$ system ($|p_{e^+e^-} - p_{\eta_c J/\psi}|^2/c^2$) is required to be within the interval $[-1, 2] \text{ GeV}^2/c^4$ to improve the signal purity. Here, the mass window is larger than in the previous section because of the possibility of a second ISR photon. To suppress the possible background from $\Upsilon(4S) \rightarrow B\bar{B}$, the ratio of the second to the zeroth order Fox-Wolfram moments⁴ is required to be greater than 0.13. To improve the resolution of the Y_{cc} signal in the inclusive reconstruction, we use the corrected recoil mass $M_{\text{recoil}}^{\text{corr}}(\gamma_{\text{ISR}}) \equiv M_{\text{recoil}}(\gamma_{\text{ISR}}) - M_{\text{recoil}}(\gamma_{\text{ISR}} J/\psi) + m(\eta_c)$,

⁴The Fox-Wolfram moments were introduced in ref. [56]. The modified Fox-Wolfram moments used in this paper are described in ref. [57].

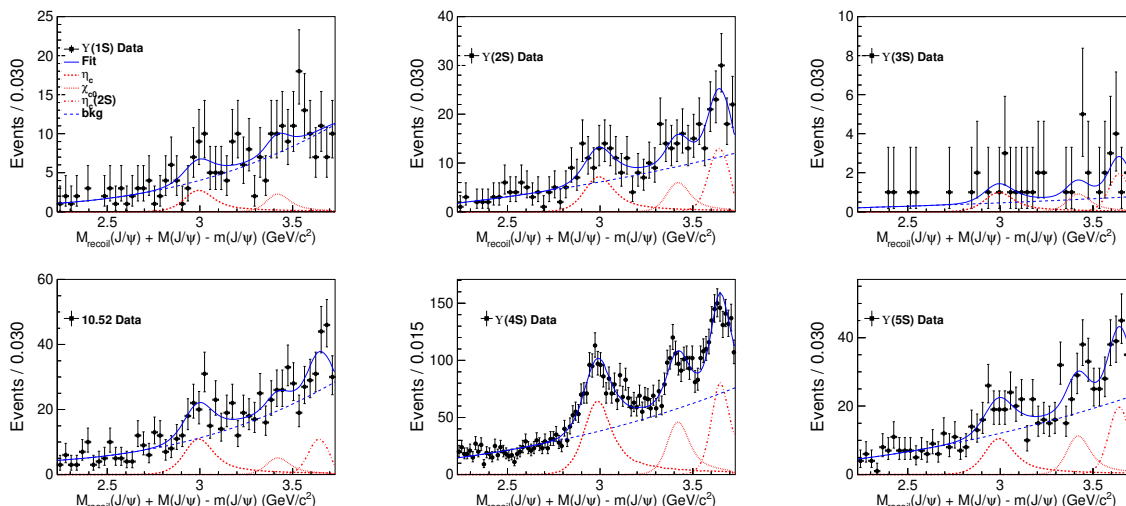


Figure 2. Fit to the mass recoiling against J/ψ at the selected energy points. Dots with error bars show the experimental data, blue curves the fit results, and blue long dashed curves the background. The red dashed, dotted, and dashed-dotted curves show the signal for η_c , χ_{c0} , and $\eta_c(2S)$, respectively.

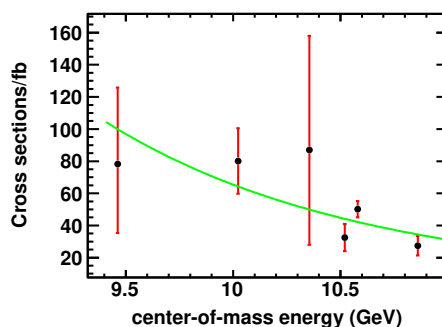


Figure 3. Combined results of the measured cross sections of $e^+e^- \rightarrow \eta_c J/\psi$ at the $\Upsilon(nS)$ on-resonance and $\Upsilon(4S)$ off-resonance energy points. The green curve is the fit result using eq. (4.2).

where $M_{\text{recoil}}(\gamma_{\text{ISR}})$ and $M_{\text{recoil}}(\gamma_{\text{ISR}}J/\psi)$ are the recoil masses of γ_{ISR} and $\gamma_{\text{ISR}}J/\psi$, respectively, and $m(\eta_c)$ is the η_c nominal mass [44].

The invariant mass of $\eta_c J/\psi$ and the recoil mass of γ_{ISR} are shown in figure 4. Events in common between the exclusive and inclusive samples are removed from the inclusive reconstruction to avoid double counting. The number of events increases near threshold in the mass spectrum of $\eta_c J/\psi$, but no similar enhancement is seen in the recoil mass of γ_{ISR} . A simultaneous unbinned maximum likelihood fit for the $\eta_c J/\psi$ invariant mass and γ_{ISR} recoil mass is performed. The signal-yield fractions from the two reconstruction methods are fixed to the corresponding branching fractions and reconstruction efficiencies. The background shapes are parameterized with the ARGUS function, whose parameters are obtained from the fit to the η_c and J/ψ sideband events. The signals are described with

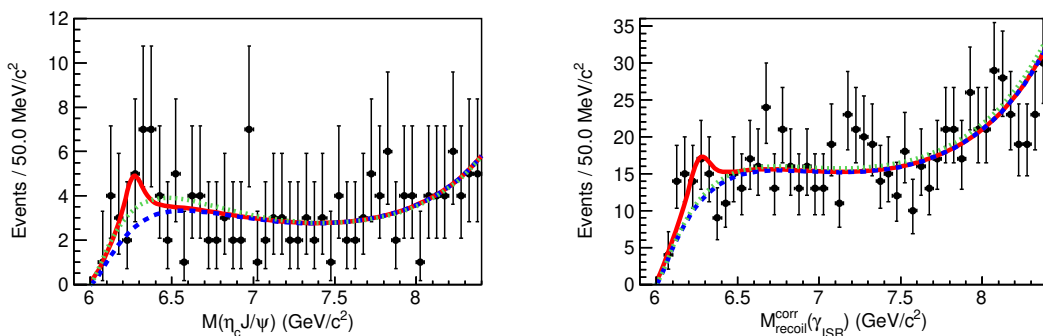


Figure 4. Simultaneous fit result to the invariant mass of $\eta_c J/\psi$ (left) and the γ recoil mass (right). In each panel, dots with error bars are from data, the red solid curve is the best fit result, the blue dashed curve the background component from the best fit, and the green dotted curve is the fit result without the signal components.

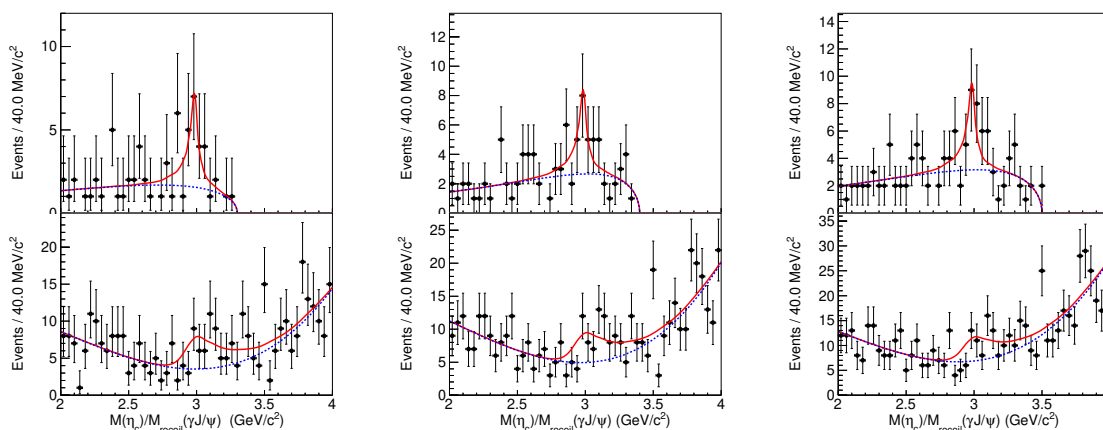


Figure 5. Simultaneous fit result to the invariant mass of reconstructed η_c (top) and the $\gamma_{\text{ISR}} J/\psi$ recoil mass (bottom). From left to right are events with $M(\eta_c J/\psi)$ and $M_{\text{recoil}}(\gamma) \in [6.0, 6.4]$, $[6.0, 6.5]$, and $[6.0, 6.6]$ GeV/c^2 . Dots with error bars are from data, the red solid curve is the best fit result, and the blue dashed curve is the background component from the best fit.

a Breit-Wigner function with free mass and width convolved with the Gaussian functions from the resolution study. The fit results are shown in figure 4. The significance of the Breit-Wigner peak component is 2.1σ , with mass and width of $(6267 \pm 43) \text{ MeV}/c^2$ and $(121 \pm 72) \text{ MeV}$, respectively. The signal yields are 9 ± 4 and 23 ± 11 from the exclusive and inclusive methods, respectively.

The reconstructed η_c mass and $\gamma_{\text{ISR}} J/\psi$ recoil mass spectra for events satisfying $M(\eta_c J/\psi)$ and $M_{\text{recoil}}(\gamma) \in [6.0, 6.4 \text{ or } 6.5 \text{ or } 6.6] \text{ GeV}/c^2$ are shown in figure 5. Fit results to other mass regions are shown in the appendix. No peaking backgrounds are expected according to the study of the generic MC sample. We perform a simultaneous unbinned maximum likelihood fit to the reconstructed η_c mass and $\gamma_{\text{ISR}} J/\psi$ recoil mass. The signal-yield fractions for the two reconstruction methods are fixed to the products of the corresponding branching fractions and reconstruction efficiencies obtained from simulated signal events.

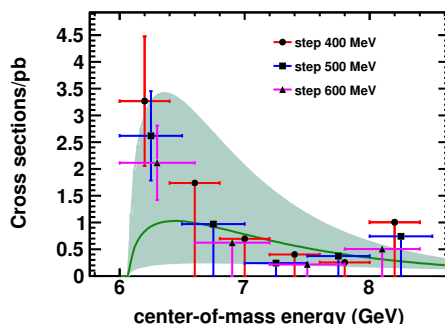


Figure 6. Measured cross sections of $e^+e^- \rightarrow \eta_c J/\psi$ near the threshold. From left to right are the cross sections measured in different step sizes (0.4, 0.5, 0.6 GeV/c^2). Green solid curve is the extrapolation from the fit to near-resonance energy points. The shadow area covers the $\pm 1\sigma$ range of the extrapolations.

regions (GeV/c^2)	$N_{\text{prod}} [\times 10^2]$	σ [pb]
[6.0, 6.4]	13.1 ± 3.6	$3.3 \pm 0.9 \pm 0.8$
[6.4, 6.8]	< 8.2	< 1.7
[6.8, 7.2]	< 3.9	< 0.7
[7.2, 7.6]	< 2.7	< 0.4
[7.6, 8.0]	< 2.1	< 0.3
[8.0, 8.4]	< 10.4	< 1.0
[6.0, 6.5]	13.4 ± 4.0	$2.7 \pm 0.8 \pm 0.2$
[6.5, 7.0]	< 6.1	< 1.0
[7.0, 7.5]	< 1.9	< 0.2
[7.5, 8.0]	< 3.8	< 0.4
[8.0, 8.5]	< 9.9	< 0.7
[6.0, 6.6]	13.3 ± 4.2	$2.1 \pm 0.7 \pm 0.2$
[6.6, 7.2]	< 5.0	< 0.6
[7.2, 7.8]	< 2.3	< 0.2
[7.8, 8.4]	< 7.4	< 0.5

Table 2. Measured cross sections for different $M(\eta_c J/\psi)$ and $M_{\text{recoil}}^{\text{corr}}(\gamma_{\text{ISR}})$ regions. Here, N_{prod} is the number of produced $\eta_c J/\psi$ signal events, and σ is the calculated cross section.

source	exclusive reconstruction	inclusive reconstruction
Tracking	1.4	0.7
Photon detection	0.0	2.0 (0.0)
PID	9.2	7.2
K_S selection	0.3	0.0
π^0 selection	3.5	0.0
η_c decays	0.9	0.0
J/ψ decays		0.5
Luminosity		1.4
Generator		1.0
Sum		8.1 (7.8)

Table 3. Summary of the systematic uncertainties (in %), excluding those for the fitting procedures. For both photon detection and total uncertainties, the main number shows the uncertainty for the Y_{cc} search, and the parentheses show the uncertainty for the cross section measurements at the selected on- (off-)resonance points.

The signal shapes are described using shapes derived from MC simulation and smoothed using kernel estimation [51], while the background is described with an ARGUS function [58] and third-order-polynomial for the exclusive and inclusive analyses, respectively.

The fit results are shown in figure 5. The significances of the η_c components are 3.9, 3.3, and 3.5σ for events from the three mass regions, respectively. The invariant mass of η_c and $\gamma_{\text{ISR}}J/\psi$ recoil mass for the other $M(\eta_c J/\psi)/M_{\text{recoil}}(\gamma_{\text{ISR}})$ mass regions are shown in the appendix, as well as the fit results. No evident signals are found in those distributions, and the upper limits of the number of produced events in different $\eta_c J/\psi$ mass regions are estimated at 90% C.L. with the following method. First, the profile likelihood distribution as a function of the number of produced events is extracted from the fit. Then this likelihood distribution is smeared with a Gaussian function whose width models the systematic uncertainty. This smeared distribution is then integrated from zero to infinity. The point at which the integral reaches 90% of the total value is taken as the upper limit. The number of produced events and the upper limits from different $M(\eta_c J/\psi)$ and $M_{\text{recoil}}(\gamma_{\text{ISR}})$ mass regions are listed in table 2.

The effective luminosity in each mass region is calculated according to ref. [59]. Using this, we calculate the cross sections near $\eta_c J/\psi$ mass threshold with an equation analogous to eq. (4.1); these are plotted in figure 6 as points with errors. We extrapolate the lineshape of the measured cross sections near $\Upsilon(nS)$ resonances according to eq. (4.2), and plot it in figure 6 as the solid curve for comparison with the measurements. We vary the parameters of the extrapolations based on the uncertainties and correlations. The range of the extrapolations are also shown on the plot. The measured cross sections near $\eta_c J/\psi$ mass threshold are consistent with the extrapolations from the $\Upsilon(nS)$ energy region according to their uncertainty.

5 Systematic uncertainty

Possible sources of systematic uncertainty include tracking, ISR photon detection, PID, K_S^0 reconstruction, π^0 reconstruction, the fitting procedure, integrated luminosity, and the η_c and J/ψ branching fractions, as listed in table 3.

The difference in tracking efficiency for tracks with momenta above 200 MeV/ c^2 between data and MC is $(-0.13 \pm 0.30 \pm 0.10)\%$ per track. The uncertainty of ISR photon detection efficiency, studied using radiative bhabha events, is 2.0%. This is only taken into account in the Y_{cc} search. We apply a reconstruction uncertainty of 0.35% per track in our analysis estimated by using partially reconstructed $D^{*+} \rightarrow D^0\pi^+$, $D^0 \rightarrow \pi^+\pi^-K_S^0$, and $K_S^0 \rightarrow \pi^+\pi^-$ events. According to the measurement of PID efficiency using the control sample $D^* \rightarrow D^0\pi$ with $D^0 \rightarrow K^-\pi^+$, we assign uncertainties of 1.1% for each kaon and 0.9% for each pion. For K_S^0 selection, we take 2.2% as the systematic uncertainty, following ref. [60]. For π^0 selection, the uncertainty is 2.3% according to a study of the $\tau \rightarrow \pi\pi^0\nu_\tau$ control sample [61].

Since we are using multiple channels to reconstruct η_c , the uncertainties of reconstruction efficiencies from these channels are combined using

$$\delta = \frac{\sum_i \delta_i \epsilon_i \mathcal{B}_i}{\sum_i \epsilon_i \mathcal{B}_i}, \quad (5.1)$$

where δ_i , ϵ_i , and \mathcal{B}_i are the systematic uncertainty, reconstruction efficiency, and branching fraction from the i -th η_c channel, respectively.

For the fitting procedure, we change the fitting range and the background function. The difference in signal yields from the nominal and alternate fits is taken as the systematic uncertainty, as shown in the appendix. For the fits with no significant signals, this systematic uncertainty is included in the upper limit estimation by taking the alternative fit result with the largest upper limit for the signal yield.

The uncertainty of the integrated luminosity is about 1.4%. The uncertainty of the branching fraction of $J/\psi \rightarrow \ell^+\ell^-$, 0.5%, is taken from ref. [44]. The uncertainty due to the branching fractions of η_c decays is studied with pseudo-experiments, in which we vary the branching fractions of η_c decays within 1σ , and calculate the overall reconstruction efficiency. After 1,000 trials, we obtain a distribution of reconstruction efficiencies, which is subsequently fit to a Gaussian. The width of this Gaussian is taken as the systematic uncertainty associated with the η_c -decay branching fractions.

We are using the PHOKHARA generator to simulate our signal events. The discrepancy of the energy of ISR photons between the simulation and the theoretical calculation [64] is less than 0.1%, with a statistical uncertainty of less than 1.0%; thus, we take the uncertainty of the generator PHOKHARA to be 1.0% as a conservative value.

The total systematic uncertainty is obtained by adding the individual components in quadrature. We use eq. (5.1) to combine the systematic uncertainties from the two methods used in the cross section measurements near threshold.

6 Summary

We perform the first search for a double-charmonium state with $e^+e^- \rightarrow \eta_c J/\psi$ near threshold via the ISR process. No significant signal of the double charmonium state is found in several bins of the invariant mass of $\eta_c J/\psi$ (for exclusive reconstruction) and the recoil mass of γ_{ISR} (for inclusive reconstruction). We measure the $\eta_c J/\psi$ production cross sections in several bins of the invariant mass of $\eta_c J/\psi$ and the recoil mass of γ_{ISR} . The cross sections for $e^+e^- \rightarrow \eta_c J/\psi$ nearest the threshold are significantly larger than in neighboring bins. Evidence with a statistical significance greater than 3.3σ is found for double charmonium production near the $\eta_c J/\psi$ threshold. The cross sections of double charmonium production at $\Upsilon(nS)$ on-resonance and $\Upsilon(4S)$ off-resonance data samples are also measured. The cross sections are fitted with a function $\sigma \propto 1/s^n$, and extrapolated to the lower $\eta_c J/\psi$ mass regions, where consistency within 1σ with our measurements in those regions is observed, albeit with relatively large measurement uncertainties. The search for double charmonium production at Belle II is expected to be revisited as SuperKEKB integrated luminosity reaches a few ab^{-1} or more.

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A Fit plots for $e^+e^- \rightarrow \eta_c J/\psi$ near threshold

Here we provide the fitting plots for different $M(\eta_c J/\psi)$ mass regions.

A.1 Step size 400 MeV/c

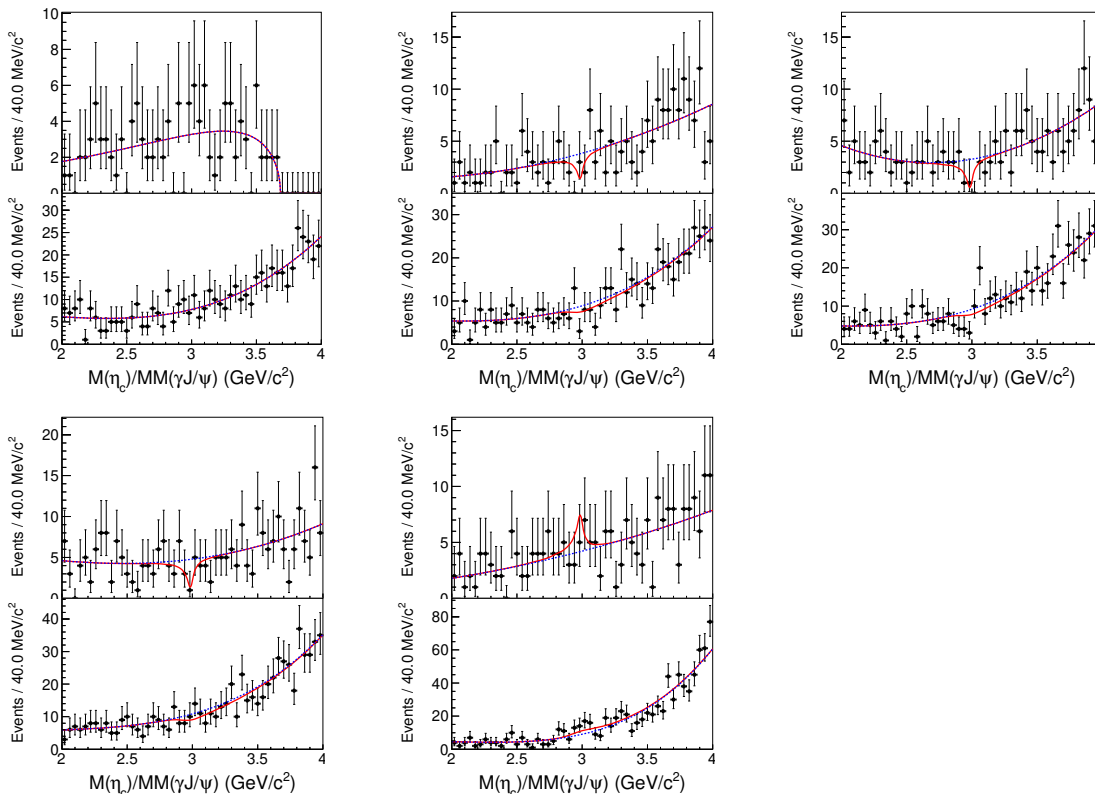


Figure 7. Simultaneous fit result to the invariant mass of reconstructed η_c (top) and the $\gamma_{\text{ISR}} J/\psi$ recoil mass (bottom). In the top row, from left to right are events with $M(\eta_c J/\psi)$ and $M_{\text{recoil}}(\gamma) \in [6.4, 6.8]$, $[6.8, 7.2]$, $[7.2, 7.6]$ GeV/c^2 , and in the bottom are $\in [7.6, 8.0]$, and $[8.0, 8.4]$ GeV/c^2 . Dots with error bars are from data, the red solid curve is the best fit result, and the blue dashed curve is the background component from the best fit.

A.2 Step size 500 MeV/c

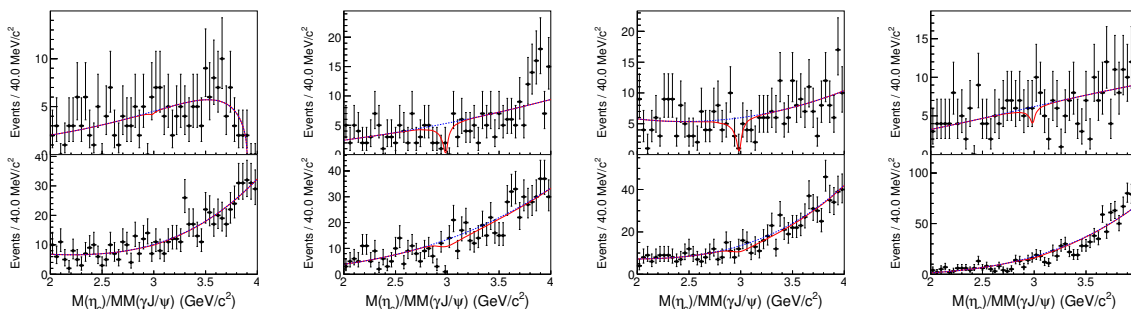


Figure 8. Simultaneous fit result to the invariant mass of reconstructed η_c (top) and the $\gamma J/\psi$ recoil mass (right). From left to right are events with $M(\eta_c J/\psi)$ and $M_{\text{recoil}}(\gamma) \in [6.5, 7.0]$, $[7.0, 7.5]$, $[7.5, 8.0]$, and $[8.0, 8.5]$ GeV/c^2 . Dots with error bars are from data, the red solid curve is the best fit result, and the blue dashed curve is the background component from the best fit.

A.3 Step size 600 MeV/c

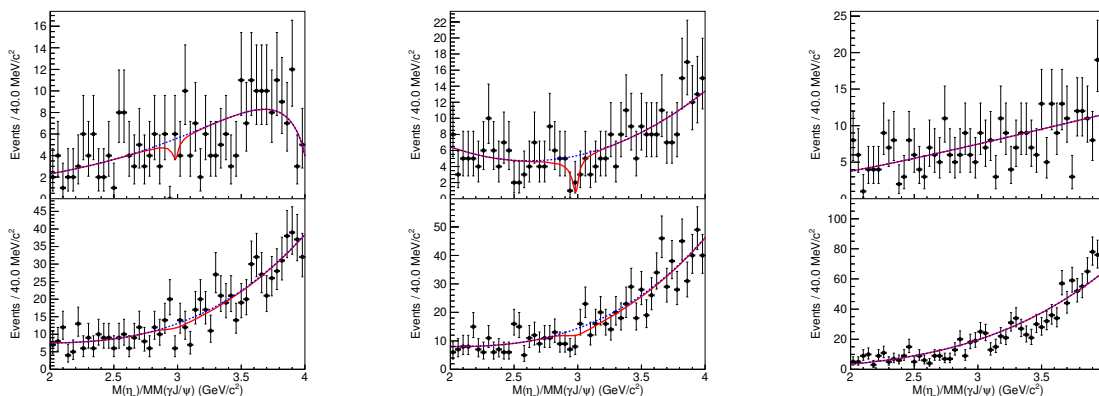


Figure 9. Simultaneous fit result to the invariant mass of reconstructed η_c (top) and the $\gamma J/\psi$ recoil mass (right). From left to right are events with $M(\eta_c J/\psi)$ and $M_{\text{recoil}}(\gamma) \in [6.6, 7.2]$, $[7.2, 7.8]$, and $[7.8, 8.4]$ GeV/c^2 . Dots with error bars are from data, the red solid curve is the best fit result, and the blue dashed curve is the background component from the best fit.

B Systematic uncertainties of fitting procedure

Here we provide the systematic uncertainties of fitting procedure from different fits.

dataset	inclusive	exclusive
$\Upsilon(1S)$	21.5	—
$\Upsilon(2S)$	9.8	2.2
$\Upsilon(3S)$	56.4	—
continuum	8.8	25.4
$\Upsilon(4S)$	3.4	4.0
$\Upsilon(5S)$	13.5	16.2

Table 4. Uncertainty of the cross sections for different datasets from fit procedure (in unit of %).

regions (GeV/c^2)	systematic uncertainty
[6.0, 6.4]	23.9
[6.0, 6.5]	6.0
[6.0, 6.6]	7.0

Table 5. Uncertainty of the cross sections measurement for different mass regions from fit procedure (in unit of %).

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