

Strange-quark mass effects in the B_s meson's light-cone distribution amplitude

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ABSTRACT: We investigate the differences between the light-cone distribution amplitudes (LCDAs) of B_s mesons and B_q mesons (with $q = u, d$) induced by a non-vanishing strange-quark mass $m_s \neq 0$ (compared to $m_q \simeq 0$). To this end, we consider the so-called “radiative tail” which is related to the short-distance expansion of the relevant light-ray operators in heavy-quark effective theory. We extend the calculation of the according matching coefficients, including operators linear in m_s for both the leading and sub-leading 2-particle LCDAs. Based on a generic parameterization for the leading LCDA, we discuss the effect on its shape on a quantitative level, and compare our findings with recent results on the inverse moments of the B_q and B_s LCDAs from QCD sum rules.

KEYWORDS: Bottom Quarks, Properties of Hadrons

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1 Introduction

The light-cone distribution amplitudes (LCDAs) of the B_s meson enter as fundamental hadronic input functions for exclusive B_s -meson decays into light energetic particles in the QCD factorization approach. Important examples are non-leptonic 2-body decays, see e.g. refs. [1–4], and rare radiative and semileptonic decays, see e.g. refs. [5, 6]. Comprehensive reviews about the phenomenology of these decays and more references can be found e.g. in refs. [7, 8].

In the past, the LCDAs for B_u^\pm and B_d^0 mesons have been studied intensively, which has led to a number of phenomenological and theoretical constraints. Most importantly, the radiative leptonic decay $B^- \rightarrow \gamma \ell^- \bar{\nu}$ at large recoil energy is sensitive to the inverse moment of the leading B -meson LCDA, denoted as $\phi_B^+(\omega)$ in the following, where ω denotes the light-cone projection of the momentum of the light quark in the heavy meson. While an analogous observable does not exist for B_s mesons, the effect of a non-vanishing strange-quark mass on the inverse moment has recently been estimated from a QCD sum-rule

analysis [9]. This non-perturbative information essentially constrains the difference between the B_s and B_q LCDAs at small and intermediate values of the light-cone momentum ω . On the other hand, at large values $\omega \gg \Lambda_{\text{QCD}}$ the so-called “radiative tail” of the LCDAs can be computed in fixed-order perturbation theory. The 1-loop result for the B_q meson has been derived from the analysis of the cut-off dependence of the positive moments in momentum space [10] and from a short-distance operator-product expansion in position space [11]. For phenomenological studies in general, and in order to interpolate between the behavior of the LCDAs at small and large values of ω , in particular, one needs explicit parameterizations of the B_q and B_s -meson’s LCDAs. A generic parameterization based on a systematic expansion in terms of associated Laguerre polynomials has recently been proposed in ref. [12]. The expansion coefficients in this parameterization fulfill integral bounds that control the systematic uncertainties from the truncation of the expansion.¹

In this letter, we derive the strange-quark mass effects on the radiative tail by including the linear terms in m_s in the short-distance expansion of the relevant light-ray operator. The result can be inferred in a straightforward manner from the perturbative calculation of the LCDA for the B_c meson, which has been derived in a non-relativistic setup in ref. [15]. We compute the constraints on the expansion coefficients in the parameterization [12] from the radiative tail, and compare the resulting estimate for the inverse moments of the B_s and B_q -meson LCDAs with the expectations from QCD sum rules. The outline of the article is as follows. In the next section, we provide the basic definitions of the B -meson 2-particle LCDAs, together with the main features of the parameterization we are going to use. We also introduce the short-distance expansion of the relevant light-ray operator, including the known results for the matching coefficients in the case of vanishing spectator mass. Section 3 is devoted to the derivation of the matching coefficient for the dimension-4 operator proportional to the strange-quark mass. By a detailed comparison we show how the result can be inferred from the calculation of LCDAs in a non-relativistic setup [15]. In particular, we show how the matching coefficients in position space can easily be obtained from momentum-space Feynman integrals in dimensional regularization, by performing simple subtractions on the level of the Fourier transformation prior to the expansion in $D - 4$ dimensions. In section 4 we work out the constraints from the short-distance expansion on the LCDAs, using the generic parameterization of the B -meson LCDA advocated for in ref. [12]. Truncating the expansion after three terms, we give numerical results for the expansion coefficients and the resulting inverse moments of the LCDAs for the massive and massless case and compare with recent results from QCD sum rules [9]. We end the article with our conclusions. In appendix A we provide a generic derivation of the short-distance expansion of a 2-particle light-ray operator with arbitrary Dirac structure from which we can also read off the matching coefficients for the subleading 2-particle LCDA of the B meson. In appendix B we briefly discuss the extrapolation of our results to the case of LCDAs for B_c -mesons. In appendix C we study the impact of the dimension-5 operators on the LCDA parameterization.

¹This parameterization has already been used [13] in the context of QED corrections to $B_s \rightarrow \mu^+ \mu^-$ decays, when internal photons resolve the hadronic structure of the B_s meson [14].

2 Preliminaries

2.1 Definition of 2-particle LCDAs

The LCDA that appears at leading power in QCD factorization theorems for exclusive B -meson decays can be defined as the hadronic matrix element of a 2-particle light-ray operator in heavy-quark effective theory (HQET) [16],

$$im_B f_B^{\text{HQET}}(\mu) \phi_B^+(\omega; \mu) = \int \frac{d\tau}{2\pi} e^{i\omega\tau} \langle 0 | \bar{q}(\tau n) [\tau n, 0] \not{n} \gamma_5 h_v(0) | \bar{B}(v) \rangle, \quad (2.1)$$

where v^μ with $v^2 = 1$ is the four-velocity of a heavy meson, $B = B_q, B_s$, and $f_B^{\text{HQET}}(\mu)$ is the decay constant in the static limit. Furthermore, n^μ with $n^2 = 0$ is a light-like Lorentz vector, and $\omega = n \cdot k$ can be viewed as the light-cone projection of the light spectator-quark momentum k^μ . Here, for simplicity, we are considering a frame where $v \cdot n = 1$. The support properties of the matrix element in eq. (2.1) are

$$\phi_B^+(\omega, \mu) \neq 0 \quad \text{for } \omega \in [0, \infty). \quad (2.2)$$

An analogous definition for the sub-leading 2-particle LCDA reads

$$im_B f_B^{\text{HQET}}(\mu) \phi_B^-(\omega; \mu) = \int \frac{d\tau}{2\pi} e^{i\omega\tau} \langle 0 | \bar{q}(\tau n) [\tau n, 0] \not{\bar{n}} \gamma_5 h_v(0) | \bar{B}(v) \rangle, \quad (2.3)$$

where $\bar{n}^\mu = 2v^\mu - n^\mu$. In the following, we concentrate on the leading LCDA $\phi_B^+(\omega)$. In appendix A we repeat the calculation in a generalised way and give analogous results for $\phi_B^-(\omega)$.

2.2 A generic parameterization for $\phi_B^+(\omega)$

In this letter we use the following generic parameterization for the leading B -meson LCDA [12],

$$\phi_B^+(\omega, \mu_0) = \frac{\omega e^{-\omega/\omega_0}}{\omega_0^2} \sum_{k=0}^K \frac{a_k(\mu_0)}{1+k} L_k^{(1)}(2\omega/\omega_0), \quad (2.4)$$

at a low renormalization scale $\mu_0 \sim 1$ GeV. Here $L_k^{(1)}(z)$ are associated Laguerre polynomials, and $\omega_0 = \mathcal{O}(\Lambda_{\text{QCD}})$ is an auxiliary reference momentum. The expansion coefficients a_k fulfill the integral bound,

$$\sum_{k=0}^{\infty} |a_k(\mu_0)|^2 = 2\omega_0 \int_0^\infty d\omega \left(\left| \phi_B^+(\omega, \mu_0) \right|^2 + \omega_0^2 \left| \frac{d\phi_B^+(\omega, \mu_0)}{d\omega} \right|^2 \right) < \infty. \quad (2.5)$$

Note that the coefficients a_k are dimensionless quantities that implicitly depend on ω_0 . In particular, the expansion of the inverse moment in this parameterization reads

$$\lambda_B^{-1} \equiv \int_0^\infty \frac{d\omega}{\omega} \phi_B^+(\omega, \mu_0) = \frac{1}{\omega_0} \sum_{k=0}^K \frac{1 + (-1)^k}{2} \frac{a_k(\mu_0)}{1+k} = \frac{a_0 + a_2/3 + \dots}{\omega_0}, \quad (2.6)$$

while the short-distance behavior in position space can be obtained from the Fourier transformed LCDA,

$$\begin{aligned}\tilde{\phi}_B^+(\tau, \mu_0) &= \int_0^\infty d\omega e^{-i\omega\tau} \phi_B^+(\omega, \mu_0) \\ &= \frac{1}{(1+i\omega_0\tau)^2} \sum_{k=0}^K a_k(\mu_0) \left(\frac{i\omega_0\tau-1}{i\omega_0\tau+1}\right)^k \\ &= \sum_{k=0}^K (-1)^k a_k(\mu_0) \left[1 - 2(k+1)i\omega_0\tau - (3+4k+2k^2)\omega_0^2\tau^2 + \mathcal{O}(i\omega_0\tau)^3\right].\end{aligned}\quad (2.7)$$

We also note that this parameterization allows for a straightforward implementation of the 1-loop renormalization-group evolution, using the eigenfunctions of the Lange-Neubert kernel [17] as derived in ref. [18] and ref. [19].

2.3 Short-distance expansion

The short-distance expansion (OPE) of the light-cone operator in the definition of the B-meson LCDA reads² (see also ref. [11] for the massless case)

$$\begin{aligned}\mathcal{O}_+(\tau) &= \bar{q}(\tau n) [\tau n, 0] \not{n} \gamma_5 h_v(0) = \sum_{n=3}^\infty \sum_{k=1}^{K_n} c_k^{(n)}(\tau) \mathcal{O}_k^{(n)}(0) \\ &= c_1^{(3)}(\tau) \bar{q} \not{n} \gamma_5 h_v \\ &\quad + c_1^{(4)}(\tau) \bar{q} (i n \cdot \overleftarrow{D}) \not{n} \gamma_5 h_v + c_2^{(4)}(\tau) \bar{q} (i v \cdot \overleftarrow{D}) \not{n} \gamma_5 h_v + c_3^{(4)}(\tau) m \bar{q} \not{n} \gamma_5 h_v \\ &\quad + \dots\end{aligned}\quad (2.8)$$

This translates into the following expression for the Fourier transformed LCDA,

$$\tilde{\phi}_+(\tau) = c_1^{(3)}(\tau) + \bar{\Lambda} \left(\frac{4}{3} c_1^{(4)}(\tau) + c_2^{(4)}(\tau)\right) - m \left(c_3^{(4)}(\tau) + \frac{1}{3} c_1^{(4)}(\tau)\right) + \mathcal{O}(\tau^2), \quad (2.9)$$

where we have used the hadronic matrix elements of the local HQET operators [15, 16],

$$\langle 0 | \mathcal{O}_1^{(3)} | \bar{B}(v) \rangle = i m_B f_B^{\text{HQET}}, \quad (2.10)$$

and

$$\frac{\langle 0 | \mathcal{O}_1^{(4)} | \bar{B}(v) \rangle}{\langle 0 | \mathcal{O}_1^{(3)} | \bar{B}(v) \rangle} = \frac{4\bar{\Lambda} - m}{3}, \quad \frac{\langle 0 | \mathcal{O}_2^{(4)} | \bar{B}(v) \rangle}{\langle 0 | \mathcal{O}_1^{(3)} | \bar{B}(v) \rangle} = \bar{\Lambda}, \quad \frac{\langle 0 | \mathcal{O}_3^{(4)} | \bar{B}(v) \rangle}{\langle 0 | \mathcal{O}_1^{(3)} | \bar{B}(v) \rangle} = -m. \quad (2.11)$$

Here $\bar{\Lambda} = m_B - m_b|_{\text{OS}}$ refers to the HQET residual mass parameter in the on-shell scheme. In the following, we will renormalize the above operators in the $\overline{\text{MS}}$ scheme, and the light quark mass $m = \bar{m}(\mu)$ is to be understood in the same scheme, accordingly.

²Due to heavy-quark spin symmetry, the generic Dirac structure for the OPE of non-local currents reads $\bar{q}(\tau n) [\tau n, 0] \Gamma h_v(0) = \sum \bar{q}(0) A(n, v, iD) \Gamma h_v(0)$. Here the matrix $A(n, v, iD)$ will contain a string of an even number of γ matrices in the massless case [10]. Linear terms in the spectator mass m , on the other hand, come with an *odd* number of γ matrices, as a consequence of the HQET Feynman rules, see also appendix A.

The 1-loop contributions to the OPE coefficients for the massless case have already been calculated in ref. [11], with the result³

$$\begin{aligned}
 c_1^{(3)}(\tau) &= 1 - \frac{\alpha_s C_F}{4\pi} \left(2L^2 + 2L + \frac{5\pi^2}{12} \right) + \mathcal{O}(\alpha_s^2), \\
 c_1^{(4)}(\tau) &= -i\tau \left[1 - \frac{\alpha_s C_F}{4\pi} \left(2L^2 + L + \frac{5\pi^2}{12} \right) + \mathcal{O}(\alpha_s^2) \right], \\
 c_2^{(4)}(\tau) &= -i\tau \left[-\frac{\alpha_s C_F}{4\pi} (4L - 3) + \mathcal{O}(\alpha_s^2) \right],
 \end{aligned}
 \tag{2.12}$$

where $L = \ln(i\tau\mu e^{\gamma_E})$ and $C_F = 4/3$. The operator $\mathcal{O}_3^{(4)}$ does not appear in the massless limit. Its coefficient can be obtained in fixed-order perturbation theory by matching suitable on-shell matrix elements of the left- and right-hand side of the OPE. A simple way is to consider partonic amplitudes between an incoming heavy quark with velocity v^μ and a light anti-quark with momentum $k^\mu = mv^\mu$ into the vacuum. This essentially corresponds to the setup in ref. [15], where the light-quark mass m fulfills $m_b \gg m \gg \Lambda_{\text{QCD}}$ and to first approximation the B -meson is described as a non-relativistic bound state. In this way one can infer the information on $c_3^{(4)}(\tau)$ by taking the Fourier transform of the 1-loop expression for the non-relativistic LCDAs and expanding to first order in the light quark mass m . Notice that in the non-relativistic limit the HQET parameter is identified with the light spectator mass,

$$\bar{\Lambda} = m (1 + \mathcal{O}(\alpha_s)).$$

Therefore, the non-relativistic LCDAs only fix a linear combination of the dimension-4 contributions. Using the results for $c_1^{(3)}(\tau)$ and $c_{1,2}^{(4)}(\tau)$ from ref. [11], we can determine the remaining coefficient $c_3^{(4)}(\tau)$ unambiguously. As a by-product, we can also compute the $\mathcal{O}(\alpha_s)$ corrections to the relation $\bar{\Lambda} \simeq m$ in the non-relativistic set-up. In appendix A, we will also present an independent derivation of the individual matching coefficients for an arbitrary Dirac structure, which includes the results for the short-distance expansion of $\mathcal{O}_+(\tau)$ as a special case.

3 Matching coefficients for the massive case

We derive the matching relation by taking on-shell matrix elements of the left- and right-hand side of the OPE in eq. (2.8), and, for simplicity, projecting onto a particular spin configuration, such that for $k^\mu = mv^\mu$ the product of heavy- and light-quark spinors can be replaced by

$$u(v, s) \bar{v}(k, s') \longrightarrow -\frac{1 + \not{v}}{2} \gamma_5,$$

up to an irrelevant normalization constant, which amounts to setting

$$\bar{v}(k, s') \not{v} \gamma_5 u(v, s) \longrightarrow 2, \quad \bar{v}(k, s') \not{v} \not{v} \gamma_5 u(v, s) \longrightarrow -2.$$

³In the same reference one can also find the 1-loop matching coefficients for the dimension-5 operators in the massless case. A brief study of the effect of dimension-5 terms can be found in appendix C.

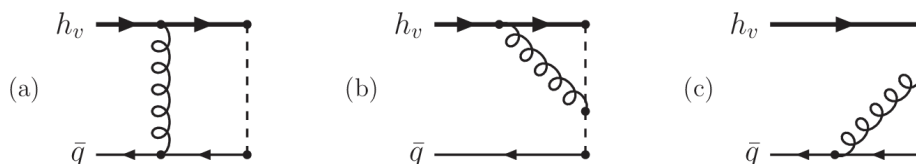


Figure 1. The three 1-loop Feynman diagrams contributing to the matching calculation. (The dashed line indicates the Wilson line.)

The calculation then corresponds to the non-relativistic setup in ref. [15], where the LCDAs of a B meson with a massive light quark, $m_b \gg m \gg \Lambda_{\text{QCD}}$, can be calculated in fixed-order perturbation theory. With this one finds

$$\begin{aligned}
 & 1 - im\tau + \frac{\alpha_s C_F}{4\pi} \tilde{I}^+(\tau) + \mathcal{O}(\tau^2, \alpha_s^2) \\
 &= c_1^{(3)}(\tau) \left(1 + \frac{\alpha_s C_F}{4\pi} \tilde{I}_1^{(3)} \right) + m c_1^{(4)}(\tau) \left(1 + \frac{\alpha_s C_F}{4\pi} \tilde{I}_1^{(4)} \right) \\
 & \quad + m c_2^{(4)}(\tau) - m c_3^{(4)}(\tau) + \mathcal{O}(\tau^2, \alpha_s^2).
 \end{aligned} \tag{3.1}$$

Here, we have already taken into account that the coefficients $c_{2,3}^{(4)}$ only start at order α_s . The function $\tilde{I}^+(\tau)$ on the left-hand side is the Fourier transform of the integral $I^+(\omega)$ which determines the $\mathcal{O}(\alpha_s)$ corrections to the non-relativistic LCDA as defined in ref. [15]. On the right-hand side, in the limit $\tau \rightarrow 0$, the 1-loop corrections are encoded in the momentum-space integrals

$$\tilde{I}_1^{(3)} = \int_0^\infty d\omega I^+(\omega) \quad \text{and} \quad \tilde{I}_1^{(4)} = \frac{1}{m} \int_0^\infty d\omega \omega I^+(\omega). \tag{3.2}$$

The contribution of these diagrams is thus related to those of the non-local operator as given above, but we have to keep in mind that the $\overline{\text{MS}}$ subtraction has to be performed *after* the ω integration, in order to account for the renormalization of *local* operators. To the considered order, the matching relation can thus be rewritten as

$$\begin{aligned}
 & \frac{\alpha_s C_F}{4\pi} \left(\tilde{I}^+(\tau) - \tilde{I}_1^{(3)} + im\tau \tilde{I}_1^{(4)} \right) \\
 &= \left(c_1^{(3)}(\tau) - 1 \right) + m \left(i\tau + c_1^{(4)}(\tau) + c_2^{(4)}(\tau) - c_3^{(4)}(\tau) \right) + \mathcal{O}(\tau^2, \alpha_s^2).
 \end{aligned} \tag{3.3}$$

3.1 Analysis of the individual diagrams in Feynman gauge

It is instructive to consider the individual contributing Feynman diagrams (in Feynman gauge), as illustrated in figure 1.

(a) Vertex correction. Let us first consider the vertex correction, i.e. the gluon exchange between the light and heavy quark. Before $\overline{\text{MS}}$ subtraction, the result for the left-hand side of the OPE in momentum space reads [15]

$$I_a^+(\omega) = 2\omega \Gamma(1 + \epsilon) \left(\frac{\mu^2 e^{\gamma_E}}{(m - \omega)^2} \right)^\epsilon \left\{ \frac{2}{(m - \omega)^2} - \frac{\theta(m - \omega)}{m(m - \omega)} - \frac{\theta(\omega - m)}{\omega(\omega - m)} \right\}. \tag{3.4}$$

From this we calculate the Fourier transform as

$$\begin{aligned} \tilde{I}_a^+(\tau) &= \int_0^\infty d\omega e^{-i\omega\tau} I_a^+(\omega) \\ &= e^{-im\tau} \left(\frac{2}{\epsilon} + 4L - 6 \operatorname{Ei}(im\tau) + 4im\tau \operatorname{Ei}(im\tau) \right) - 4 + \frac{2 - 2e^{-im\tau}}{im\tau} + \mathcal{O}(\epsilon) \\ &= \frac{2}{\epsilon} - 2L + 3 \ln \frac{\mu^2}{m^2} - 2 - i\tau m \left(\frac{2}{\epsilon} - 6L + 5 \ln \frac{\mu^2}{m^2} + 7 \right) + \mathcal{O}(m^2, \epsilon), \end{aligned} \quad (3.5)$$

where L is defined as above and $\operatorname{Ei}(z)$ is the exponential integral function. Similarly, the vertex correction to the local operator $\mathcal{O}_1^{(3)}$ is obtained as [15]

$$\tilde{I}_{1,a}^{(3)} = \int_0^\infty d\omega I_a^+(\omega) = \frac{3}{\epsilon} + 3 \ln \frac{\mu^2}{m^2} - 2 + \mathcal{O}(\epsilon). \quad (3.6)$$

The vertex correction to the local operator $\mathcal{O}_1^{(4)}$ translates into the ω moment of the integral $I_a^+(\omega)$,

$$\tilde{I}_{1,a}^{(4)} = \frac{1}{m} \int_0^\infty d\omega \omega I_a^+(\omega) = \frac{5}{\epsilon} + 5 \ln \frac{\mu^2}{m^2} + 3 + \mathcal{O}(\epsilon). \quad (3.7)$$

With this the contribution of the vertex correction to the matching relation is obtained as

$$\begin{aligned} &\tilde{I}_a^+(\tau) - \tilde{I}_{1,a}^{(3)} + im\tau \tilde{I}_{1,a}^{(4)} \\ &= \int_0^\infty d\omega \left(e^{-i\omega\tau} - 1 + i\omega\tau \right) I_a^+(\omega) \\ &= -\frac{1}{\epsilon} - 2L - i\tau m \left(-\frac{3}{\epsilon} - 6L + 4 \right) + \mathcal{O}(m^2, \epsilon). \end{aligned} \quad (3.8)$$

Notice that the dependence on the IR logarithms $\ln \frac{\mu^2}{m^2}$ has dropped out in the difference of the three terms, such that the matching coefficients only depend on the UV logarithms L , as they should. Furthermore, note that the above relation holds for the *bare* Feynman integral, and the final (and finite) result stems from the fact that the short-distance expansion of the exponential $e^{-i\omega\tau}$ and the expansion in ϵ of the dimensionally regularized integrals do not commute. On the other hand, the subtraction terms generated by the expansion of the Fourier factor,

$$e^{-i\omega\tau} \rightarrow e^{-i\omega\tau} - \sum_n \frac{(-i\omega\tau)^n}{n!},$$

allow us to expand the remaining integrand $I_a^+(\omega)$ in momentum space with respect to the small mass parameter m *prior* to the integration,

$$I_a^+(\omega) \simeq \Gamma(1 + \epsilon) \left(\frac{\mu^2 e^{\gamma_E}}{\omega^2} \right)^\epsilon \left(\frac{2}{\omega} + (6 + 4\epsilon) \frac{m}{\omega^2} + \mathcal{O}(m^2/\omega^3) \right), \quad (3.9)$$

which reflects the contribution to the radiative tail of the LCDA in momentum space. We have used this strategy to compute the matching contribution of the vertex diagram for a generic strange-quark momentum in appendix A.

(b) Wilson line with heavy quark. Next, we consider the coupling of the Wilson-line gluon to the heavy quark which yields [15]

$$I_b^+(\omega) = 2\Gamma(\epsilon) \int_0^\infty dk \left(\frac{\mu^2 e^{\gamma_E}}{k^2} \right)^\epsilon \frac{\delta(\omega - m - k) - \delta(\omega - m)}{k}. \quad (3.10)$$

The Fourier transform reads

$$\begin{aligned} \tilde{I}_b^+(\tau) &= \int_0^\infty d\omega e^{-i\omega\tau} I_b^+(\omega) \\ &= e^{-i\tau m} \left(-\frac{1}{\epsilon^2} - \frac{2L}{\epsilon} - 2L^2 - \frac{5\pi^2}{12} \right) + \mathcal{O}(\epsilon). \end{aligned} \quad (3.11)$$

In this case, there are no local subtraction integrals,

$$\int_0^\infty d\omega (1 - i\omega\tau) I_b^+(\omega) = 2\Gamma(\epsilon) \int_0^\infty dk \left(\frac{\mu^2 e^{\gamma_E}}{k^2} \right)^\epsilon (-i\tau) = 0, \quad (3.12)$$

because any term from the Taylor expansion of the exponential results in scaleless integrals in dimensional regularization. As a consequence, the short-distance expansion for $\tilde{I}_b^+(\tau)$ in eq. (3.11) factorizes into the trivial expansion of the Fourier factor and a *universal* 1-loop factor which includes the double-logarithmic dependence on τ and affects all tree-level matching coefficients in the same way.

(c) Wilson line with light quark. Finally, the diagram with the Wilson-line gluon coupling to the light quark reads [15]

$$I_c^+(\omega) = 2\Gamma(\epsilon) \int_0^m dk \frac{m-k}{m} \left(\frac{\mu^2 e^{\gamma_E}}{k^2} \right)^\epsilon \frac{\delta(k-m+\omega) - \delta(\omega-m)}{k}, \quad (3.13)$$

where the Fourier transform is given by

$$\begin{aligned} \tilde{I}_c^+(\tau) &= \int_0^\infty d\omega e^{-i\omega\tau} I_c^+(\omega) \\ &= i\tau m \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{m^2} + 3 \right) + \mathcal{O}((i\tau m)^2, \epsilon). \end{aligned} \quad (3.14)$$

The analogous contribution from the local operator yields $\tilde{I}_{1,c}^{(3)} = 0$ and

$$\begin{aligned} \tilde{I}_{1,c}^{(4)} &= -\frac{2\Gamma(\epsilon)}{m} \int_0^m dk \frac{m-k}{m} \left(\frac{\mu^2 e^{\gamma_E}}{k^2} \right)^\epsilon \\ &= \left(\frac{\mu^2 e^{\gamma_E}}{m^2} \right)^\epsilon \frac{\Gamma(\epsilon-1)}{2\epsilon-1} = -\left(\frac{1}{\epsilon} + 3 + \ln \frac{\mu^2}{m^2} \right) + \mathcal{O}(\epsilon). \end{aligned} \quad (3.15)$$

As the integral $I_c^+(\omega)$ only involves the low-momentum region, $\omega < m$, the short-distance expansion of the Fourier exponential and dimensional regularization commute, and therefore the net contribution to the matching from diagram (c) is zero,

$$\tilde{I}_c^+(\tau) - \tilde{I}_{1,c}^{(3)} + im\tau \tilde{I}_{1,c}^{(4)} = \int_0^\infty d\omega (e^{-i\omega\tau} - 1 + i\omega\tau) I_c^+(\omega) = 0. \quad (3.16)$$

3.2 1-loop result for the matching coefficient $c_3^{(4)}(\tau)$

Inserting the above results for the 1-loop integrals and the known Wilson coefficients from the massless case into the matching relation (3.3), we obtain the following result for the remaining Wilson coefficient $c_3^{(4)}(\tau)$ after $\overline{\text{MS}}$ renormalization:

$$c_3^{(4)}(\tau) = -i\tau \left[\frac{\alpha_s C_F}{4\pi} (L - 1) + \mathcal{O}(\alpha_s^2) \right], \quad (3.17)$$

which represents one of the new theoretical results of our analysis.

3.3 1-loop result for $\bar{\Lambda}$ in the non-relativistic limit

As a cross-check and by-product, we can insert the result for the Wilson coefficients into the OPE result for the LCDA in eq. (2.9), and obtain

$$\begin{aligned} \tilde{\phi}_+(\tau) = & \left[1 - i\tau \frac{4\bar{\Lambda} - m}{3} \right] \left[1 - \frac{\alpha_s C_F}{4\pi} \left(2L^2 + 2L + \frac{5\pi^2}{12} \right) \right] \\ & + i\tau \bar{\Lambda} \frac{\alpha_s C_F}{4\pi} \left(\frac{8}{3} L - 3 \right) + i\tau m \frac{\alpha_s C_F}{4\pi} \left(\frac{4}{3} L - 1 \right) + \mathcal{O}(\alpha_s^2, \tau^2). \end{aligned} \quad (3.18)$$

On the other hand, the direct computation of the 1-loop corrections to the non-relativistic LCDA in position space results in (see also ref. [15] for the momentum-space computation)

$$\tilde{\phi}_+(\tau) \Big|_{\text{NR}} = 1 - i\tau m + \frac{\alpha_s C_F}{4\pi} \left(\tilde{I}_a^+(\tau) + \tilde{I}_b^+(\tau) + \tilde{I}_c^+(\tau) - \tilde{I}_{1,a}^{(3)} \right)_{\overline{\text{MS}}} + \dots \quad (3.19)$$

Comparing the two expressions, we can read off the relation between the HQET parameter $\bar{\Lambda}$ and the light quark mass in the non-relativistic limit,

$$\bar{\Lambda} \Big|_{\text{NR}} = m \left[1 + 3 \frac{\alpha_s C_F}{4\pi} \ln \frac{\mu^2}{m^2} + \mathcal{O}(\alpha_s^2) \right]. \quad (3.20)$$

With $m = \bar{m}(\mu)$ in the $\overline{\text{MS}}$ scheme, and $\bar{\Lambda} = \bar{\Lambda}_{\text{pol}}$ in the pole-mass scheme, one indeed finds $d\bar{\Lambda}/d\ln\mu = 0$ to the considered order in α_s .

4 Constraints on the generic LCDA parameterization

In this section we will work out the theoretical constraints on the B -meson LCDAs that follow from the perturbative results for the radiative tail, together with the generic parameterization in eq. (2.4), following the procedure outlined in ref. [12]. The main purpose of this analysis is to determine whether the theoretical information resulting from the radiative tail is compatible with complementary studies of the inverse moments of the B -meson LCDAs from QCD sum rules. Indeed, we find very good agreement, both on a qualitative and quantitative level (within the uncertainties). We stress that the results from the perturbative tail *alone* cannot provide precise predictions for the LCDAs. Rather, the conclusion to be drawn is that the constraints could and should be used — together with other independent theoretical information from sum rules or lattice — in future phenomenological analyses of B_s decays in the framework of QCD factorization or QCD light-cone sum rules.

4.1 Determination of the expansion parameters

Following ref. [12], the short-distance expansion for the LCDA $\tilde{\phi}_+(\tau)$ in eq. (3.18) translates into constraints onto the expansion coefficients a_k in eq. (2.7). To this end, we take an imaginary-valued reference point $\tau = \tau_0$ such that

$$x_0 \equiv i\tau_0\mu_0 e^{\gamma_E} = \mathcal{O}(1),$$

for a given reference scale, which we will fix as $\mu_0 = 1$ GeV. This ensures that the logarithms $L = \log x_0$ in the matching coefficients of the OPE are not large. In the numerical analysis below, we fix $x_0 \equiv 1$ for simplicity. In order to compare with our parameterization, we have to expand $\tilde{\phi}_B^+(\tau)$ in powers of τ_0 , and therefore for this expansion to converge we further have to require that the auxiliary reference momentum ω_0 in the parameterization of the B_s or B_q LCDA satisfies

$$n_0 \equiv i\tau_0\omega_0 \ll 1.$$

Finally, as becomes apparent below, we have to require $\omega_0 \gtrsim \bar{\Lambda}_a, m_a$ to avoid large enhancement factors in the resulting expressions for the expansion parameters a_k . In the numerical analysis, we use $n_0 \equiv 1/3$ which satisfies these requirements.

In the following, we consider the parameterization of the LCDA truncated at $K = 2$. The OPE results for the LCDA and its first derivative at the point τ_0 provide two independent conditions that determine the parameters a_0 and a_1 , while a_2 remains unconstrained. In this way, we find

$$a_0 = 2 + a_2 - \frac{4\bar{\Lambda} - m}{6\omega_0} + \frac{\alpha_s C_F}{4\pi} \left(-\frac{1}{x_0} \frac{\mu_0 e^{\gamma_E}}{\omega_0} (1 + 2 \ln x_0) + \dots \right), \quad (4.1)$$

$$a_1 = 1 + 2a_2 - \frac{4\bar{\Lambda} - m}{6\omega_0} + \frac{\alpha_s C_F}{4\pi} \left(-\frac{1}{x_0} \frac{\mu_0 e^{\gamma_E}}{\omega_0} (1 + 2 \ln x_0) + \dots \right), \quad (4.2)$$

where only the α_s corrections that are enhanced by μ_0/ω_0 are shown for the moment. As those are independent of the light quark mass, it is convenient to absorb them by the same redefinition as in ref. [12],

$$\bar{\Lambda} \equiv \bar{\Lambda}_a(\mu_0, x_0) \left[1 + \frac{\alpha_s C_F}{4\pi} \left(10 \ln x_0 + \frac{15}{4} \right) \right] - \frac{\alpha_s C_F}{4\pi} \frac{3\mu_0 e^{\gamma_E}}{2x_0} (1 + 2 \ln x_0). \quad (4.3)$$

With this definition, the 1-loop result for the expansion parameters with $K = 2$ reads

$$\begin{aligned} a_0 &= Z(x_0) \left(2 - \frac{4\bar{\Lambda}_a(\mu_0, x_0) - m_a(x_0)}{6\omega_0} - 2r(x_0) \right) + a_2, \\ a_1 &= Z(x_0) \left(1 - \frac{4\bar{\Lambda}_a(\mu_0, x_0) - m_a(x_0)}{6\omega_0} - r(x_0) \right) + 2a_2, \end{aligned} \quad (4.4)$$

which holds to order $\mathcal{O}(\alpha_s)$. Here we introduce

$$r(x_0) \equiv \frac{\alpha_s C_F}{6\pi} \frac{8\bar{\Lambda}_a x_0(1 + \ln x_0) + m_a x_0(1 - 2 \ln x_0)}{\mu_0 e^{\gamma_E}} = \mathcal{O}(\alpha_s n_0), \quad (4.5)$$

and

$$Z(x_0) \equiv 1 + \frac{\alpha_s C_F}{4\pi} \left(-2 \ln^2 x_0 + 2 \ln x_0 + 2 - \frac{5\pi^2}{12} \right), \quad (4.6)$$

and

$$m_a(x_0) \equiv m \left(1 - \frac{\alpha_s C_F}{4\pi} (3 + 4 \ln x_0) \right), \quad (4.7)$$

as a short-hand notation. Our definitions of $\bar{\Lambda}_a$, m_a and Z have been chosen such that the parameterization for the position-space LCDA with finite truncation K satisfies

$$\begin{aligned} \tilde{\phi}_+(0)|_K &= \sum_{k=0}^K (-1)^k a_k = Z(x_0) - r(x_0) + \mathcal{O}(\alpha_s^2, n_0^2), \\ \tilde{\phi}'_+(0)|_K &= -2i\omega_0 \sum_{k=0}^K (-1)^k (1+k) a_k = -Z(x_0) \frac{4i\bar{\Lambda}_a - im_a}{3} + \mathcal{O}(\alpha_s^2, \bar{\Lambda}_a n_0), \end{aligned} \quad (4.8)$$

which generalizes the corresponding relations in ref. [12] to the case of a massive light spectator quark.

4.2 Numerical results

As stressed in the previous subsection, the aim of the numerical analysis is *not* to provide precise predictions for the B_q and B_s -meson LCDAs, but rather to figure out to what extent the theoretical constraints from the radiative tail can be used in future phenomenological analyses. For that reason, our focus will be on the inverse moments λ_{B_q} and λ_{B_s} , which play a dominant role in applications of QCD factorization to exclusive B decays. The plots that we show below are aimed to illustrate our findings on a semi-quantitative level. For that reason, most of the input parameters are simply set to their central values. Nevertheless, we quote the expected uncertainties on the input parameters, where possible.

As already stated, we take $x_0 \equiv 1$ and $n_0 = 1/3$ for the dimensionless combinations of τ_0 , μ_0 and ω_0 as defined in the previous section. The renormalization scale is fixed to $\mu_0 = 1$ GeV, which results in $\omega_0 \simeq 594$ MeV. The corresponding value of the strong coupling is taken as $\alpha_s(\mu_0) = 0.5$. With this the value of the HQET parameter for the B_q meson in the scheme defined above is [12]

$$\bar{\Lambda}_a^{(q)}(\mu_0, x_0 = 1) \simeq 367 \text{ MeV}.$$

The corresponding value for B_s mesons is obtained by taking the hadronic mass differences from $M_{B_s} = 5.367$ GeV, $M_{B_q} = 5.279$ GeV [20], together with eq. (4.3), leading to

$$\bar{\Lambda}_a^{(s)}(\mu_0, 1) \simeq \bar{\Lambda}_a^{(q)}(\mu_0, 1) + \left(1 - \frac{\alpha_s C_F}{4\pi} \frac{15}{4} \right) (M_{B_s} - M_{B_q}) \simeq 437 \text{ MeV}.$$

For the strange-quark mass in the $\overline{\text{MS}}$ scheme we adopt $m_s(\mu_0) = 126_{-7}^{+15}$ MeV [20], which via eq. (4.7) translates to

$$m_a^{(s)}(\mu_0) \simeq (106 \pm 10) \text{ MeV}.$$

Inverse moment of the LCDA for B_q meson. Inserting the above values into the matching relations for the expansion coefficients eq. (4.4), we find for the B_q meson

$$\begin{aligned} a_0^{(q)} &\simeq 1.78 - 0.47 \frac{\bar{\Lambda}_a^{(q)}}{367 \text{ MeV}} + a_2^{(q)} \simeq 1.31 + a_2^{(q)}, \\ a_1^{(q)} &\simeq 0.89 - 0.42 \frac{\bar{\Lambda}_a^{(q)}}{367 \text{ MeV}} + 2a_2^{(q)} \simeq 0.47 + 2a_2^{(q)}. \end{aligned} \quad (4.9)$$

To establish upper and lower limits for $a_2^{(q)}$, we adopt the ad-hoc criterion suggested in ref. [12],

$$\frac{|a_1^{(q)}|^2}{|a_0^{(q)}|^2 + |a_1^{(q)}|^2} < 0.25, \quad \frac{|a_2^{(q)}|^2}{|a_0^{(q)}|^2 + |a_1^{(q)}|^2 + |a_2^{(q)}|^2} < 0.1, \quad (4.10)$$

which should ensure a reasonable convergence of the integral bound in eq. (2.5). Numerically, this translates to

$$-0.33 < a_2^{(q)} < 0.20. \quad (4.11)$$

The estimate for the inverse moment from eq. (2.6),

$$\lambda_{B_q} \simeq \frac{445 \text{ MeV}}{1.33 - 0.35 \frac{\bar{\Lambda}_a^{(q)}}{367 \text{ MeV}} + a_2^{(q)}} \simeq \frac{445 \text{ MeV}}{0.98 + a_2^{(q)}}, \quad (4.12)$$

is depicted in figure 2 in the given interval for the expansion parameter $a_2^{(q)}$. We compare this estimate with the latest sum-rule result $\lambda_{B_q} = 383 \pm 153 \text{ MeV}$ [9] and observe that, for a large region of the considered $a_2^{(q)}$ interval, both estimates are in good agreement, with a slight preference for positive values of $a_2^{(q)}$. The value of $a_2^{(q)}$ can also be roughly estimated by adhering to $K = 2$ and including the constraints from the dimension-5 operators which is studied in appendix C and yields compatible values with the above analysis within the uncertainties. We also notice that the value for the inverse moment λ_{B_q} is positively correlated with the value of the HQET parameter $\bar{\Lambda}_a^{(q)}$, but the two hadronic quantities are not simply proportional to each other.

Inverse moment of the LCDA for B_s meson. Similarly, the coefficients for the B_s meson are calculated as

$$\begin{aligned} a_0^{(s)} &\simeq 1.78 - 0.56 \frac{\bar{\Lambda}_a^{(s)}}{437 \text{ MeV}} + 0.023 \frac{m_a^{(s)}}{105 \text{ MeV}} + a_2^{(s)} \simeq 1.24 + a_2^{(s)}, \\ a_1^{(s)} &\simeq 0.89 - 0.50 \frac{\bar{\Lambda}_a^{(s)}}{437 \text{ MeV}} + 0.025 \frac{m_a^{(s)}}{106 \text{ MeV}} + 2a_2^{(s)} \simeq 0.42 + 2a_2^{(s)}. \end{aligned} \quad (4.13)$$

Compared to the B_q meson, the resulting flavor-symmetry breaking effect for the coefficients $a_{0,1}^{(s)}$ is of the expected size (10–15%). It is thus reasonable to expect that this remains true for the (yet) undetermined coefficients $a_2^{(q,s)}$ as well. Moreover, we can use the analogous convergence criterion in eq. (4.10) as for the B_q meson to constrain the interval to consider for the parameter $a_2^{(s)}$, leading to

$$-0.32 < a_2^{(s)} < 0.21. \quad (4.14)$$

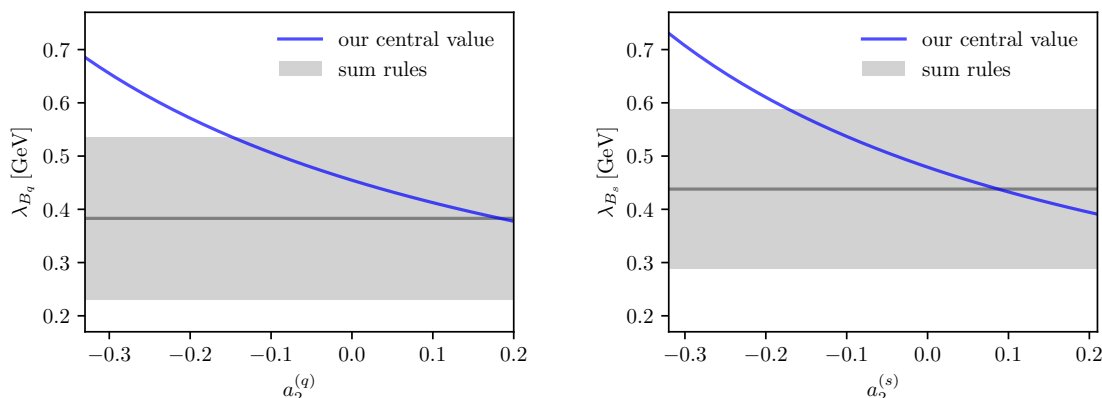


Figure 2. Estimates for the inverse moment λ_B of the B -meson LCDA (Left: for B_q . Right: for B_s). Blue solid line: central value following from the constraint of the radiative tail on the truncated parameterization, as a function of the expansion coefficient a_2 . Gray band: estimate from the latest sum-rule analysis [9].

Considering now the inverse moment of the B_s -meson LCDA, we find

$$\lambda_{B_s} \simeq \frac{455 \text{ MeV}}{1.33 - 0.42 \frac{\bar{\Lambda}_a^{(s)}}{437 \text{ MeV}} + 0.017 \frac{m_a^{(s)}}{106 \text{ MeV}} + a_2^{(s)}} \simeq \frac{455 \text{ MeV}}{0.93 + a_2^{(s)}}. \quad (4.15)$$

The aforementioned correlation between λ_B and $\bar{\Lambda}_a$ remains, while the explicit effect of the strange-quark mass turns out to be marginal. On the right-hand side of figure 2 we show our result as a function of $a_2^{(s)}$ in comparison with the value $\lambda_{B_s} = 438 \pm 150 \text{ MeV}$, which was determined from QCD sum rules [9]. We find again that our approach to implement the constraints from the radiative tail is well compatible with the sum-rule estimates.

The ratio $\lambda_{B_s}/\lambda_{B_q}$. Considering the ratio of inverse moments, we find

$$\frac{\lambda_{B_s}}{\lambda_{B_q}} \simeq \frac{1.33 - 0.35 \frac{\bar{\Lambda}_a^{(q)}}{367 \text{ MeV}} + a_2^{(q)}}{1.33 - 0.42 \frac{\bar{\Lambda}_a^{(s)}}{437 \text{ MeV}} + 0.017 \frac{m_a^{(s)}}{106 \text{ MeV}} + a_2^{(s)}} \simeq \frac{0.98 + a_2^{(q)}}{0.93 + a_2^{(s)}}. \quad (4.16)$$

As already discussed, the main flavor-symmetry breaking effect stems from the difference between the HQET parameters for B_q and B_s mesons, while the explicit effect of the strange-quark mass is small. This leaves the dependence on the undetermined coefficients $a_2^{(q,s)}$ which are varied on a compact parameter space, constrained by the convergence of the integral bound and flavor symmetry. Given that the maximal values for $a_2^{(q,s)}$ allowed by our convergence criterion are about 0.3, and we do not expect flavor-symmetry corrections to be larger than 30%, we consider $|\delta a_2| < 0.1$ to be a conservative bound. In figure 3 we illustrate our numerical result for eq. (4.16) in two different ways: on the left-hand side we plot the ratio as a function of the difference $\delta a_2 = a_2^{(s)} - a_2^{(q)}$, with the value of $a_2^{(q)}$ varied within the interval constrained by eq. (4.10). On the right-hand side, we plot the ratio as a function of $a_2^{(q)}$, assuming different ranges for $|\delta a_2|$. Again, in both cases we observe good agreement with the sum-rule estimate 1.19 ± 0.14 for that ratio from ref. [9] (which has

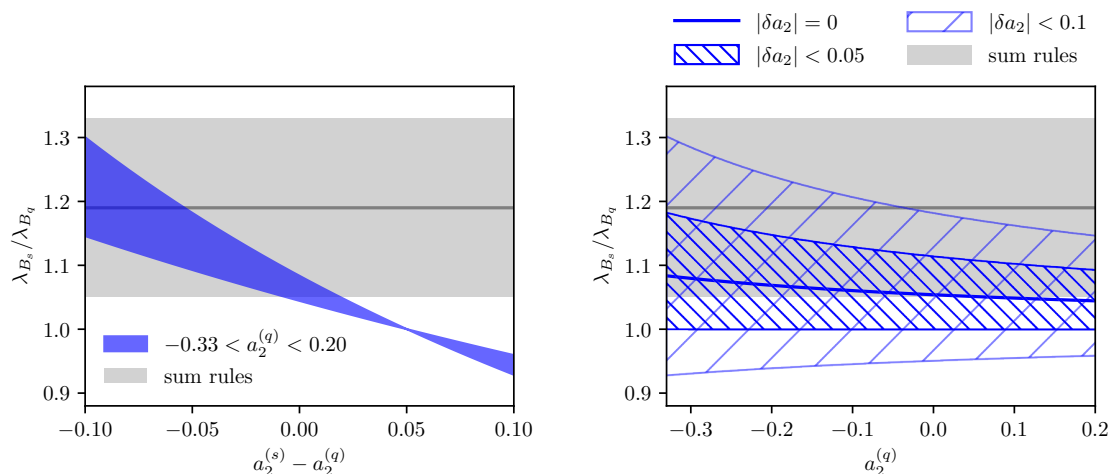


Figure 3. Estimates for the ratio of inverse moments $\lambda_{B_s}/\lambda_{B_q}$ of the $B_{s,q}$ -meson LCDAs (Left: as a function $a_2^{(s)} - a_2^{(q)} \equiv \delta a_2$. Right: as a function of $a_2^{(q)}$). Blue (hatched) bands: value ranges following from the constraint of the radiative tail on the parameterization, under the given additional constraint. Gray band: estimate from the latest sum-rule analysis [9].

smaller uncertainty than the estimates for the individual inverse moments). While $\delta a_2 = 0$ is allowed, the comparison shows a slight preference for $a_2^{(s)} - a_2^{(q)} < 0$, which would repeat the trend seen in $a_{0,1}^{(s)} - a_{0,1}^{(q)}$, and is also in line with the constraints from the dimension-5 operators provided in appendix C.

In conclusion, our numerical study reveals that the perturbative constraints from the radiative tail of the LCDAs, together with our generic parameterization, yield inverse moments that are consistent with independent studies, even at a low truncation order. Our formalism shows a clear correlation between the inverse moments of the LCDAs and the HQET parameter $\bar{\Lambda}$, where in most parts of the allowed parameter space we have $\lambda_{B_s} > \lambda_{B_q}$. We also find that the explicit effect of the strange-quark mass in the short-distance expansion gives only a minor effect on our result. For that reason, we have also extrapolated our analysis to the case of the B_c meson in HQET (assuming $m_c \sim \mu_0 \ll m_b$), which is briefly summarized in appendix B. Our findings suggest that the rescaled LCDAs, $\lambda_B \phi_B^+(\omega)$ as a function of the variable $x = \omega/\lambda_B$, are almost independent of the spectator-quark flavor, even in the case of charmed B mesons.

5 Summary

To summarize, in this work we have studied the 2-particle light-cone distribution amplitudes (LCDAs) of the B_s meson, focusing on the effect of the strange-quark mass on the radiative tail, i.e. the behavior of the LCDAs at large light-cone momentum fraction or small light-cone separations. To this end, we have reconsidered the short-distance expansion of 2-particle light-ray operators in HQET, and calculated the 1-loop matching coefficients, where our new results include the dimension-4 operator proportional to the strange-quark mass, see eq. (3.17), and the case of the subleading 2-particle B -meson LCDA, see eq. (A.12). We have shown in some detail how the matching procedure can be conveniently performed,

starting from the asymptotic behavior of momentum-space Feynman integrals for on-shell matrix elements, and performing the necessary local subtractions on the level of the Fourier transform to position space, prior to the $\overline{\text{MS}}$ subtractions, see the discussion after eq. (3.8). In this way, our approach is very similar to the calculation of the LCDAs for non-relativistic bound states in HQET [15].

On the basis of this result, we have studied the numerical effect of the short-distance constraints on the level of a generic parameterization of the LCDA, proposed by two of us in ref. [12]. The value and first derivative of the LCDA in position space at a suitably chosen small but non-zero light-cone separation are used to constrain two expansion coefficients. The size of the remaining coefficients is limited by an integral bound. We have used this approach to obtain numerical estimates for inverse moment of the leading B_q and B_s LCDAs, which turn out to be in very reasonable agreement with independent results from QCD sum rules [9], see figures 2 and 3. In particular, we find that the dominant flavor-symmetry breaking effect for the radiative tail is induced by the difference between the HQET mass parameters, $\bar{\Lambda}_{B_s} > \bar{\Lambda}_{B_q}$, while the explicit effect of the strange-quark mass only has a minor numerical effect. This suggests that our result can also be extrapolated to the case of LCDAs for the B_c -meson in the limit $m_c \ll m_b$. In this way we also obtain a numerical estimate for the latter which is briefly discussed in appendix B.

Our study shows that the generic parameterization of the leading B -meson LCDA in ref. [12], together with theoretical constraints from the short-distance behavior computed in this work, and estimates for the inverse moments from QCD sum rules in ref. [9], provide a consistent framework to be used for future phenomenological fits to exclusive B_s decays into energetic hadrons.

Acknowledgments

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A Radiative tail of $\tilde{\phi}_-(\tau)$ and general matching calculation

In this appendix, we provide the matching calculation for the OPE of a generic 2-particle light-ray operator in HQET, from which one can also read off the result for the radiative tail of the sub-leading 2-particle LCDA ϕ_B^- of the B meson. We define the short-distance expansion of a 2-particle HQET light-ray operator with arbitrary Dirac structure Γ as

$$\begin{aligned}
 \mathcal{O}_\Gamma(\tau) &= \bar{q}(\tau n) [\tau n, 0] \Gamma h_v(0) \\
 &= c_1^{(3)}(\tau) \bar{q}(0) \frac{\not{n}\not{\psi}}{2} \Gamma h_v(0) + d_1^{(3)}(\tau) \bar{q}(0) \frac{\psi\not{n}}{2} \Gamma h_v(0) \\
 &\quad + c_1^{(4)}(\tau) \bar{q}(0) (in \cdot \overleftarrow{D}) \frac{\not{n}\not{\psi}}{2} \Gamma h_v(0) + d_1^{(4)}(\tau) \bar{q}(0) (in \cdot \overleftarrow{D}) \frac{\psi\not{n}}{2} \Gamma h_v(0) \\
 &\quad + c_2^{(4)}(\tau) \bar{q}(0) (iv \cdot \overleftarrow{D}) \frac{\not{n}\not{\psi}}{2} \Gamma h_v(0) + d_2^{(4)}(\tau) \bar{q}(0) (iv \cdot \overleftarrow{D}) \frac{\psi\not{n}}{2} \Gamma h_v(0) \\
 &\quad + c_3^{(4)}(\tau) m \bar{q}(0) \frac{2\not{\psi} - \not{n}}{2} \Gamma h_v(0) + d_3^{(4)}(\tau) m \bar{q}(0) \frac{\not{n}}{2} \Gamma h_v(0) + \mathcal{O}(\tau^2), \quad (\text{A.1})
 \end{aligned}$$

which for $\Gamma = \not{n}\gamma_5$ reduces to the OPE for $\mathcal{O}_+(\tau)$ in eq. (2.8). Here we find it convenient to separate the individual structures by means of light-cone projectors,

$$P_+ = \frac{\not{n}\not{\psi}}{2} = \frac{\not{n}\not{\ell}}{4}, \quad P_- = \frac{\not{\psi}\not{\ell}}{2} = \frac{\not{\ell}\not{\psi}}{4}, \quad (\text{A.2})$$

with $P_\pm^2 = P_\pm$ and $P_+ + P_- = 1$. Taking, on the other hand, the Dirac matrix $\Gamma = \not{n}\gamma_5$, we obtain the corresponding OPE which determines the radiative tail of $\phi_B^-(\tau)$,

$$\begin{aligned} \mathcal{O}_-(\tau) &= \bar{q}(\tau n) [\tau n, 0] \not{n}\gamma_5 h_v(0) = \sum_{n=3}^{\infty} \sum_{k=1}^{K_n} d_k^{(n)}(\tau) \mathcal{O}_k^{\prime(n)}(0) \\ &= d_1^{(3)}(\tau) \bar{q} \not{n}\gamma_5 h_v \\ &\quad + d_1^{(4)}(\tau) \bar{q} (i n \cdot \overleftarrow{D}) \not{n}\gamma_5 h_v + d_2^{(4)}(\tau) \bar{q} (i v \cdot \overleftarrow{D}) \not{n}\gamma_5 h_v \\ &\quad + d_3^{(4)}(\tau) m \bar{q} \not{\psi} \not{n}\gamma_5 h_v + \dots \end{aligned} \quad (\text{A.3})$$

The radiative tail of the LCDA $\tilde{\phi}_B^-(\tau)$ is then given by

$$\tilde{\phi}_-(\tau) = d_1^{(3)}(\tau) + \bar{\Lambda} \left(\frac{2}{3} d_1^{(4)}(\tau) + d_2^{(4)}(\tau) \right) - m \left(d_3^{(4)}(\tau) - \frac{1}{3} d_1^{(4)}(\tau) \right) + \mathcal{O}(\tau^2). \quad (\text{A.4})$$

Here, we have used again the hadronic matrix elements of the local HQET operators,

$$\langle 0 | \mathcal{O}_1^{\prime(3)} | \bar{B}(v) \rangle = i m_B f_B^{\text{HQET}}, \quad (\text{A.5})$$

and

$$\frac{\langle 0 | \mathcal{O}_1^{\prime(4)} | \bar{B}(v) \rangle}{\langle 0 | \mathcal{O}_1^{\prime(3)} | \bar{B}(v) \rangle} = \frac{2\bar{\Lambda} + m}{3}, \quad \frac{\langle 0 | \mathcal{O}_2^{\prime(4)} | \bar{B}(v) \rangle}{\langle 0 | \mathcal{O}_1^{\prime(3)} | \bar{B}(v) \rangle} = \bar{\Lambda}, \quad \frac{\langle 0 | \mathcal{O}_3^{\prime(4)} | \bar{B}(v) \rangle}{\langle 0 | \mathcal{O}_1^{\prime(3)} | \bar{B}(v) \rangle} = -m. \quad (\text{A.6})$$

According to our detailed discussion in the main text, the contribution of the vertex diagram (a) to the matching coefficients can then be obtained from a Fourier integral with subtractions,

$$\int_0^\infty d\omega \left(e^{-i\omega\tau} - 1 + i\omega\tau + \dots \right) I_a^\Gamma(\omega, m, k), \quad (\text{A.7})$$

where in the integrand the function $I_a^\Gamma(\omega, m, k)$ is now calculated for general external on-shell quark states, where the light quark carries momentum k^μ :

$$I_a^\Gamma(\omega, m, k) = -i \int [d\ell] \delta(\omega - n \cdot (k - \ell)) \frac{\bar{v}(k) \not{\psi}(-\not{k} + \not{\ell} + m) \Gamma u(v)}{[(k - \ell)^2 - m^2 + i0][v \cdot \ell + i0][\ell^2 + i0]}. \quad (\text{A.8})$$

Expanding in the external momentum and mass, we find

$$\begin{aligned} I_a^\Gamma(\omega, m, k) &= \frac{\Gamma(1 + \epsilon)}{\omega} \left(\frac{\mu^2 e^{\gamma_E}}{\omega^2} \right)^\epsilon \\ &\quad \times \bar{v}(k) \left\{ \left(2 + (1 + 2\epsilon) \frac{n \cdot k}{\omega} + (4 + 2\epsilon) \frac{v \cdot k}{\omega} \right) \frac{\not{n}\not{\psi}}{2} \right. \\ &\quad \left. + \left(-2 - (1 + 2\epsilon) \frac{n \cdot k}{\omega} + (2 - 2\epsilon) \frac{v \cdot k}{\omega} \right) \frac{\not{\psi}\not{n}}{2} \right. \\ &\quad \left. - \frac{m}{\omega} \not{\psi} + \mathcal{O}(\omega^{-2}) \right\} \Gamma u(v). \end{aligned} \quad (\text{A.9})$$

Performing the ω -integration, we end up with

$$\begin{aligned}
 & \int_0^\infty d\omega \left(e^{-i\omega\tau} - 1 + i\omega\tau + \dots \right) I_a^\Gamma(\omega, m, k) \\
 &= \bar{v}(k) \left\{ \left(-\frac{1}{\epsilon} - 2L + \left(\frac{1}{2\epsilon} + L \right) i\tau(n \cdot k) + \left(\frac{2}{\epsilon} + 4L - 3 \right) i\tau(v \cdot k) \right) \frac{\not{n}\not{\psi}}{2} \right. \\
 &\quad + \left(\frac{1}{\epsilon} + 2L - \left(\frac{1}{2\epsilon} + L \right) i\tau(n \cdot k) + \left(\frac{1}{\epsilon} + 2L - 3 \right) i\tau(v \cdot k) \right) \frac{\not{\psi}\not{n}}{2} \\
 &\quad \left. - \left(\frac{1}{2\epsilon} + L - 1 \right) i\tau m \not{\psi} + \mathcal{O}(\tau^2) \right\} \Gamma u(v). \tag{A.10}
 \end{aligned}$$

From this we can easily read off the contribution of the vertex diagram (a) to the individual Wilson coefficients. The contribution from diagram (b) (Wilson-line coupled to heavy quark) can be derived in a similar manner, with

$$\begin{aligned}
 & \int_0^\infty d\omega \left(e^{i\omega\tau} - 1 + i\omega\tau \right) I_b^\Gamma(\omega, k) \\
 &= \left(1 - i\tau(n \cdot k) + \mathcal{O}(\tau^2) \right) \left(-\frac{1}{\epsilon^2} - \frac{2L}{\epsilon} - 2L^2 - \frac{5\pi^2}{12} \right) \bar{v}(k) \frac{\not{n}\not{\psi} + \not{\psi}\not{n}}{2} \Gamma u(v). \tag{A.11}
 \end{aligned}$$

Finally, as discussed in the main part of the text, the diagram (c) with the Wilson line coupled to the light quark does not contribute to the matching. With this we confirm our results for the matching coefficients in the expansion of $\mathcal{O}_+(\tau)$. For the matching coefficients relevant to the LCDA ϕ_B^- we obtain after $\overline{\text{MS}}$ renormalization

$$\begin{aligned}
 d_1^{(3)}(\tau) &= 1 - \frac{\alpha_s C_F}{4\pi} \left(2L^2 - 2L + \frac{5\pi^2}{12} \right) + \mathcal{O}(\alpha_s^2), \\
 d_1^{(4)}(\tau) &= -i\tau \left[1 - \frac{\alpha_s C_F}{4\pi} \left(2L^2 - L + \frac{5\pi^2}{12} \right) + \mathcal{O}(\alpha_s^2) \right], \\
 d_2^{(4)}(\tau) &= -i\tau \left[-\frac{\alpha_s C_F}{4\pi} (2L - 3) + \mathcal{O}(\alpha_s^2) \right], \\
 d_3^{(4)}(\tau) &= -i\tau \left[\frac{\alpha_s C_F}{4\pi} (L - 1) + \mathcal{O}(\alpha_s^2) \right]. \tag{A.12}
 \end{aligned}$$

Plugging this result into the expression for the LCDA $\tilde{\phi}_B^-(\tau)$, we obtain

$$\begin{aligned}
 \tilde{\phi}_B^-(\tau) &= \left[1 - i\tau \frac{2\bar{\Lambda} + m}{3} \right] \left[1 - \frac{\alpha_s C_F}{4\pi} \left(2L^2 - 2L + \frac{5\pi^2}{12} \right) \right] \\
 &\quad + i\tau \bar{\Lambda} \frac{\alpha_s C_F}{4\pi} \left(\frac{8}{3} L - 3 \right) + i\tau m \frac{\alpha_s C_F}{4\pi} \left(\frac{4}{3} L - 1 \right) + \mathcal{O}(\alpha_s^2, \tau^2). \tag{A.13}
 \end{aligned}$$

We note that the terms in the second line are identical for $\tilde{\phi}_B^-(\tau)$ and $\tilde{\phi}_B^+(\tau)$.

B Numerical results for the LCDA of the B_c meson

We briefly give the analogous discussion to section 4 for the case of the B_c . Due to the higher mass of the charm quark, a larger renormalization scale is needed. We consider here

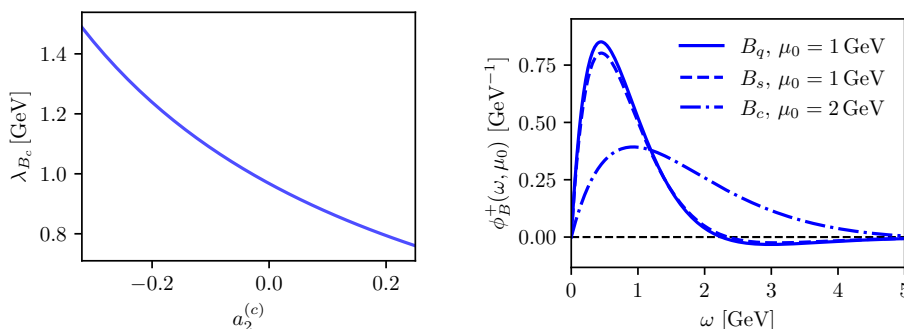


Figure 4. Left: estimate for the inverse moment λ_{B_c} of the B_c -meson LCDA as a function of the expansion coefficient a_2 . Here the blue solid line corresponds to the central value following from the constraint of the dimension-3 and dimension-4 operators on the radiative tail, based on the truncated parameterization. Right: comparison of the central values for our estimates of the B_q , B_s and B_c LCDAs. (Here the parameters $a_2^{(q,s,c)}$ are taken to be zero for simplicity.)

$\mu_0 = 2 \text{ GeV}$ such that $\alpha_s(\mu_0) \simeq 0.3$, and $\omega_0 = 1.18 \text{ GeV}$ (for $x_0 = 1$ and $n_0 = 1/3$). We take $M_{B_c} = 6274.47 \pm 0.32 \text{ MeV}$ and $m_c(m_c) = 1.27 \pm 0.2 \text{ GeV}$ [20], from which we calculate $m_c(\mu_0) \simeq 1.10 \text{ GeV}$ using the software RunDec [21]. This corresponds to

$$\bar{\Lambda}_a^{(c)}(\mu_0, x_0 = 1) \simeq 1244 \text{ MeV}, \quad m_a^{(c)}(\mu_0, x_0 = 1) \simeq 995 \text{ MeV}, \quad (\text{B.1})$$

which yields the coefficients

$$\begin{aligned} a_0^{(c)} &\simeq 1.87 - 0.77 \frac{\bar{\Lambda}_a^{(c)}}{1244 \text{ MeV}} + 0.12 \frac{m_a^{(c)}}{995 \text{ MeV}} + a_2^{(c)} = 1.22 + a_2^{(c)}, \\ a_1^{(c)} &\simeq 0.93 - 0.71 \frac{\bar{\Lambda}_a^{(c)}}{1244 \text{ MeV}} + 0.13 \frac{m_a^{(c)}}{995 \text{ MeV}} + 2a_2^{(c)} = 0.35 + 2a_2^{(c)}. \end{aligned} \quad (\text{B.2})$$

The convergence criterion then leads to

$$-0.32 < a_2^{(c)} < 0.25, \quad (\text{B.3})$$

and the inverse moment reads

$$\lambda_{B_c} \simeq \frac{885 \text{ MeV}}{1.40 - 0.57 \frac{\bar{\Lambda}_a^{(c)}}{1244 \text{ MeV}} + 0.090 \frac{m_a^{(c)}}{995 \text{ MeV}} + a_2^{(c)}} = \frac{885 \text{ MeV}}{0.91 + a_2^{(c)}}. \quad (\text{B.4})$$

Figure 4 shows the inverse moment as a function of $a_2^{(c)}$. Interestingly, the expansion parameters $a_k^{(c)}$ turn out to be not very different from the light-quark case. However, one should be aware that both the renormalization scale μ_0 and the reference momentum ω_0 are strikingly different for the two cases. As a consequence, the inverse moment λ_{B_c} comes out significantly larger than for light spectator quarks, close to the non-relativistic limit (for which $m_c \approx \bar{\Lambda}_{B_c} \approx \lambda_{B_c}$). For comparison, we have collected the relevant quantities for B_q , B_s , and B_c mesons in table 1. In addition, we have plotted the central values of our estimates for the LCDAs of B_q , B_s and B_c mesons in momentum space on the r.h.s. of figure 4. Here, for illustrative purposes, the expansion parameters a_2 have been set to

	B_q	B_s	B_c
μ_0	1 GeV	1 GeV	2 GeV
ω_0	594 MeV	594 MeV	1.18 GeV
$\bar{\Lambda}_a$	367 MeV	437 MeV	1.24 GeV
m_a	0	106 MeV	1.00 GeV
$a_0 - a_2$	1.31	1.24	1.22
$a_1 - 2a_2$	0.47	0.42	0.35
λ_B	(380, 690) MeV	(390, 730) MeV	(0.76, 1.49) GeV

Table 1. Comparison between key quantities characterizing the LCDAs of B_q , B_s , and B_c mesons (central values only).

zero for simplicity. As one would have intuitively expected, the effect of increasing the spectator mass is to lower the value and to increase the width of the peak at small light-cone momentum. On the other hand, if one rescaled the LCDAs by the corresponding estimate for the inverse moment, the three curves for the quantity $\lambda_B \phi_B^+(\omega/\lambda_B)$ would look almost indistinguishable.

C Possible impact of dimension-5 operators

In this appendix we study the possible impact of dimension-5 operators on the determination of the expansion coefficients a_k . At tree level, the following dimension-5 local operators have non-vanishing Wilson coefficients [16]

$$\bar{q} iG^{\mu\nu} h_\nu, \quad \frac{1}{2} \bar{q} \{i\overleftarrow{D}^\mu, i\overleftarrow{D}^\nu\} h_\nu.$$

The B -meson matrix element of the first operator does not have an explicit mass dependence. Translating the definition from ref. [16] into the covariant trace formalism, we have

$$\frac{\langle 0 | \bar{q}_\beta iG^{\mu\nu} (h_\nu)_\alpha | \bar{B}(v) \rangle}{\langle 0 | \mathcal{O}_1^{(3)} | \bar{B}(v) \rangle} = \frac{1}{4} \left[(1 + \psi) \left(\frac{\lambda_H^2 - \lambda_E^2}{3} (\gamma^\mu v^\nu - \gamma^\nu v^\mu) - \frac{\lambda_H^2}{3} i\sigma^{\mu\nu} \right) \gamma_5 \right]_{\alpha\beta}. \quad (\text{C.1})$$

The mass dependence for the second operator can be obtained by taking into account the quark mass in the Dirac equation for the light quark. In this way we find

$$\begin{aligned} & \frac{\frac{1}{2} \langle 0 | \bar{q}_\beta \{i\overleftarrow{D}^\mu, i\overleftarrow{D}^\nu\} (h_\nu)_\alpha | \bar{B}(v) \rangle}{\langle 0 | \mathcal{O}_1^{(3)} | \bar{B}(v) \rangle} \\ &= -\frac{1}{4} \left[(1 + \psi) \left(\frac{6\bar{\Lambda}^2 + 2\lambda_E^2 + \lambda_H^2 - 2m\bar{\Lambda} - m^2}{3} v^\mu v^\nu \right. \right. \\ & \quad \left. \left. - \frac{\bar{\Lambda}^2 + \lambda_E^2 + \lambda_H^2 - m^2}{3} g^{\mu\nu} - \frac{2\bar{\Lambda}^2 + \lambda_E^2 - 2m\bar{\Lambda}}{6} (\gamma^\mu v^\nu + \gamma^\nu v^\mu) \right) \gamma_5 \right]_{\alpha\beta}. \quad (\text{C.2}) \end{aligned}$$

With this the *tree level* expressions for the second Mellin moment of the momentum-space B -meson LCDAs follows as

$$\begin{aligned}
 \langle \omega^2 \rangle_+ &\equiv \int_0^\infty d\omega \omega^2 \phi_B^+(\omega) \Big|_{\text{tree}} = \frac{\langle 0 | \bar{q} (in \cdot \overleftarrow{D})^2 \not{v} \gamma_5 h_v | \bar{B}(v) \rangle}{\langle 0 | \mathcal{O}_1^{(3)} | \bar{B}(v) \rangle} \\
 &= \frac{6\bar{\Lambda}^2 + 2\lambda_E^2 + \lambda_H^2 - 2m\bar{\Lambda} - m^2}{3}, \\
 \langle \omega^2 \rangle_- &\equiv \int_0^\infty d\omega \omega^2 \phi_B^-(\omega) \Big|_{\text{tree}} = \frac{\langle 0 | \bar{q} (in \cdot \overleftarrow{D})^2 \not{v} \gamma_5 h_v | \bar{B}(v) \rangle}{\langle 0 | \mathcal{O}'_1^{(3)} | \bar{B}(v) \rangle} \\
 &= \frac{2\bar{\Lambda}^2 + \lambda_H^2 + 2m\bar{\Lambda} - m^2}{3}, \tag{C.3}
 \end{aligned}$$

generalizing the findings in ref. [16] to the massive case. In the non-relativistic limit, $\bar{\Lambda} \rightarrow m$, $\lambda_{E,H}^2 \rightarrow 0$, this consistently reduces to $\langle \omega^2 \rangle_\pm = m^2$.

Including the numerically dominant 1-loop corrections at dimension-5 level, making use of the results in eq. (7) in ref. [11], leads to

$$\begin{aligned}
 \tilde{\phi}_+(\tau) &= \left[1 - i\tau \langle \omega \rangle_+ - \tau^2 \frac{\langle \omega^2 \rangle_+}{2} \right] \left[1 - \frac{\alpha_s C_F}{4\pi} \left(2L^2 + 2L + \frac{5\pi^2}{12} \right) \right] \\
 &\quad + i\tau \bar{\Lambda} \frac{\alpha_s C_F}{4\pi} \left(\frac{8}{3} L - 3 \right) + i\tau m \frac{\alpha_s C_F}{4\pi} \left(\frac{4}{3} L - 1 \right) \\
 &\quad + \tau^2 \bar{\Lambda}^2 \frac{\alpha_s C_F}{4\pi} \left(\frac{10}{3} L - \frac{35}{9} + \mathcal{O}\left(\frac{m}{\bar{\Lambda}}\right) + \mathcal{O}\left(\frac{\lambda_{E,H}^2}{\bar{\Lambda}^2}\right) \right) \\
 &\quad + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\tau^3). \tag{C.4}
 \end{aligned}$$

Here, in the first line we took into account the universal double-logarithmic corrections proportional to $\langle \omega^2 \rangle_+$, while for the single-logarithmic corrections to the dimension-5 contributions we only included the terms proportional to $\bar{\Lambda}^2$. The missing α_s corrections at dimension-5 level are numerically suppressed by the small values for light quark masses and $\lambda_{E,H}^2$, as indicated.

Adjusting the parameters of the generic parameterization (2.7) for truncation level $K = 2$ and using the additional information on the second moment, the tree-level expressions for the expansion coefficients then follow as

$$\begin{aligned}
 a_0 &= 1 + \left(1 - \frac{\langle \omega \rangle_+}{2\omega_0} \right) + \frac{3}{4} \left(1 - \frac{\langle \omega \rangle_+}{\omega_0} + \frac{\langle \omega^2 \rangle_+}{6\omega_0^2} \right) + \dots, \\
 a_1 &= \left(1 - \frac{\langle \omega \rangle_+}{2\omega_0} \right) + \frac{3}{2} \left(1 - \frac{\langle \omega \rangle_+}{\omega_0} + \frac{\langle \omega^2 \rangle_+}{6\omega_0^2} \right) + \dots, \\
 a_2 &= \frac{3}{4} \left(1 - \frac{\langle \omega \rangle_+}{\omega_0} + \frac{\langle \omega^2 \rangle_+}{6\omega_0^2} \right) + \dots, \tag{C.5}
 \end{aligned}$$

where the first Mellin moment is given by $\langle \omega \rangle_+ = (4\bar{\Lambda} - m)/3$, see above. The numerical results are collected in the first three columns of table 2, where we also give the results

$n_0 = 1/3$	tree-level, pole-scheme			1-loop ^(*) , pole-scheme			1-loop, a -scheme
	dim-3	dim-4	dim-5	dim-3	dim-4	dim-5	dim-4
	$K = 0$	$K = 1$	$K = 2$	$K = 0$	$K = 1$	$K = 2$	$K = 2$
$a_0^{(q)}$	1	1.44	1.54	0.78	1.07	1.07	(0.98, 1.51)
$a_1^{(q)}$	—	0.44	0.65	—	0.26	0.23	(−0.19, 0.87)
$a_2^{(q)}$	—	—	0.11	—	—	−0.03	(−0.33, 0.20)
$a_0^{(s)}$	1	1.37	1.43	0.78	0.99	0.96	(0.92, 1.45)
$a_1^{(s)}$	—	0.37	0.49	—	0.20	0.10	(−0.22, 0.84)
$a_2^{(s)}$	—	—	0.06	—	—	−0.06	(−0.32, 0.21)

Table 2. Comparison of different estimates for the expansion parameters, depending on the order of perturbation theory, the order of the OPE, and the considered truncation level K . The last column refers to the default case discussed in the main body of the text. ^(*) Notice that part of the 1-loop corrections to the dimension-5 operators have been neglected, see the discussion around eq. (C.4).

for lower truncation level ($K = 0, 1$) which are obtained by dropping the corresponding columns in the formula above. From this we already get a rough estimate of the numerical convergence of our approach. Here, for the HQET parameters we now consider the pole-mass scheme, with the central value for the b -quark pole mass taken as $m_b \simeq 4.78$ GeV from ref. [20], which corresponds to

$$\bar{\Lambda}^{(q)} \simeq 500 \text{ MeV}, \quad \bar{\Lambda}^{(s)} \simeq 590 \text{ MeV},$$

together with $\lambda_E^2 = 0.01 \text{ GeV}^2$ and $\lambda_H^2 = 0.15 \text{ GeV}^2$ from ref. [22]. Notice that the corresponding 1-loop expressions for Λ_a would come out significantly larger than the values considered in section 4 (which are obtained from a 1-loop comparison with the DA-scheme defined in [10]). The difference between the two treatments of the HQET mass parameters may thus give some handle to estimate the associated scheme dependence of our results. In the same spirit, we now use the $\overline{\text{MS}}$ mass values for the light quarks (see above). For comparison, in the last column of table 2 we show the estimated ranges for the expansion coefficients as obtained from section 4, where the parameter a_2 is varied within the interval fixed by the ad-hoc convergence criterion, and the mass parameters are taken in the a -scheme. We observe that for the considered hadronic input values, the resulting central values for the expansion coefficients are compatible with the intervals obtained in section 4.

Based on eq. (C.4) we also determined the 1-loop result for the expansion coefficients at truncation level $K = 0, 1, 2$, whose central numerical values are again collected in table 2. We observe that the 1-loop corrections somewhat improve the convergence of the procedure compared to the tree-level case, while the central values for $a_{0,1,2}$ are entirely consistent with the ranges obtained in section 4. The central values also confirm the trend of decreasing values of $a_{1,2}$ for increasing spectator quark masses, as inferred from the comparison with the QCD sum rule results for λ_{B_q} and λ_{B_s} in section 4. Altogether, this further supports our conclusion that information from the OPE can be consistently implemented in global phenomenological analyses in the framework of QCD factorization, based on the parameterization proposed in ref. [12].

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