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Causality constraints on black holes beyond GR

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Abstract: We derive causality constraints on the simplest scalar-tensor theories in which black holes differ from what General Relativity predicts, a scalar coupled to the Gauss-Bonnet or the Chern-Simons terms. Demanding that time advances are unobservable within the regime of validity of these effective field theories, we find their cutoff must be parametrically of the same size as the inverse Schwarzschild radius of the black holes for which the non-standard effects are of order one. For astrophysical black holes within the range of current gravitational wave detectors, this means a cutoff length of the order of kilometers. We further explore the leading additional higher-dimensional operators potentially associated with the scale of UV completion and discuss their phenomenological implications for gravitational wave science.

Keywords: Black Holes, Effective Field Theories, Scattering Amplitudes

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Contents

1 Introduction

The detection of gravitational waves from black hole mergers has opened a new window into the nature of gravitational interactions. In particular, the possibility to study gravity in the strong field regime for the first time has motivated a surge of interest in field theories that allow for black hole solutions different from the ones predicted by General Relativity (GR).

In the absence of a compelling guiding principle, the intrinsic complexity of the merger process has encouraged the study of simple models where deviations from GR could be order one. This is the case of scalar-tensor theories featuring the lowest-dimensional non-minimal couplings of a scalar field to gravity, capable of sourcing detectable scalar hair around black holes: a massless (shift-symmetric) scalar coupled to the Gauss-Bonnet (GB) invariant [1, 2] or to the Chern-Simons (CS) term, a.k.a. Pontryagin invariant, [3],

$$S = \int d^4x\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\nabla_\mu\phi)^2 + M_{\rm Pl}\alpha\phi\mathcal{R}^2_{\rm GB} + M_{\rm Pl}\tilde{\alpha}\phi R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}\right)\,, \tag{1.1}$$

where $\mathcal{R}^2_{\rm GB} \equiv R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2$ and $\tilde{R}^{\mu\nu\rho\sigma} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}R_{\alpha\beta}{}^{\rho\sigma}$. α and $\tilde{\alpha}$ are the length-scales (squared) parametrizing the strength of the non-minimal scalar couplings. For shift-symmetric scalar theories a no-hair theorem [4] basically selects these two interactions as the only ones leading to black hole hair [5]. See [6] for other types of hair in black hole geometries.

While very interesting from the phenomenological point of view, it is crucial to understand how much one can learn about the fundamental properties of gravity via the study of these models in the context of gravitational wave observations. To answer this important question, the very first step is to assess the consistency of such extensions of GR with what we already know: that GR provides a good description of gravitational interactions down to μm scales [7] and, at the most basic level, that the principles of unitarity, locality, and causality hold there.

Based on simple causality arguments we will find that the cutoff of the effective field theory (EFT) in eq. (1.1) is bounded from above as $\Lambda \lesssim 1/|\hat{\alpha}|^{1/2}$, where $\hat{\alpha} = \alpha + i\tilde{\alpha}$. Since the effects beyond GR (BGR), associated with the scalar hair of black holes, are observable when $|\hat{\alpha}|/r_s^2 = O(1)$, where r_s is the Schwarzschild radius, we find that for phenomenological applications, i.e. for black holes of astrophysical size, $\Lambda \lesssim 1/\,\mathrm{km}$. Therefore, the observability of black holes with scalar hair comes at the high price of a very limited regime of validity of these models. In fact, we will argue that the observation of $O(|\hat{\alpha}|/\,\mathrm{km}^2)$ non-standard effects due to the scalar hair of astrophysical black holes is likely at odds with standard gravity at distances shorter than $|\hat{\alpha}|^{1/2}$, or, from a more dramatic perspective, it would point to the violation of fundamental principles below that scale.

Our causality bound is a generalization of the well-known fact that effective field theories exhibiting non-minimal 3-graviton or 2-photon plus 1-graviton interactions, if extrapolated beyond their regime of validity, display time advances when in a gravitational background, in conflict with causality [8, 9]. In section 2 we show that the scalar-graviton mixing induced by the non-minimal couplings in eq. (1.1) leads as well to a macroscopic violation of causality unless $\Lambda \lesssim 1/|\hat{\alpha}|^{1/2}$, in which case the time advance is never observable within the EFT regime of validity. Based on this bound as well as those found in [9], along with the theoretical constraints on gravitational EFTs recently derived using dispersion relations [10, 11], we will extract generic lessons on the power counting of gravitational EFT operators, of relevance for gravitational wave science.

While our bound renders the EFT in eq. (1.1) at the verge of its regime of validity for the physical systems of interest, there is a small range of scales where it could remain interesting. What are the effects one can expect from such a low cutoff? In section 3.1 we investigate this question by means of dispersion relations, which connect observables at low energies, i.e. EFT coefficients, with the high-energy dynamics that underlies them, on the basis of the unitarity, locality and causality of the scattering amplitudes. Due to the weakness of the non-minimal gravitational interactions compared to GR, as enforced by causality, we find that our one-loop positivity conditions are not powerful enough to extract a robust answer. Nevertheless, given that setting $|\hat{\alpha}| \sim 1/\Lambda^2$, so as to maximize the BGR effects within the EFT regime, fixes the power counting of the EFT, we are able to identify in section 3.2 the leading higher-dimensional operators that should generically (yet not generally) become large. The reader not interested in the more formal discussion of gravitational positivity bounds is invited to directly go to this latter subsection, which is the starting point of our phenomenological analysis.

In section 4 we explore the phenomenological consequences of the additional EFT operators. The main generic lesson we extract is that it would be of great significance to extend the black hole solutions and numerical studies of their merger, obtained so far in the literature for the scalar-GB and dynamical-CS gravity theories ($\tilde{\alpha} = 0$ and $\alpha = 0$ respectively), to include these operators. This conclusion holds insofar there exist a UV completion in which gravity remains well described by GR at scales lower than $|\hat{\alpha}|^{1/2} \sim$ km, an important caveat that we chose to be agnostic about and leave for future investigation. We present our outlook and conclusions in section 5.

In appendix A we discuss how our arguments could be extended to place theoretical constraints on the idea of spontaneous scalarization around black holes [12, 13].

2 Time advance bounds

In this section we compute the time delay that the two graviton polarizations and the massless scalar experience when scattering against a very heavy (classical) gravitational source in the eikonal regime, following [9, 14] to include the effects of the non-minimal couplings in eq. (1.1). These interactions lead to a non-diagonal transition amplitude between graviton and scalar, such that one of the propagation eigenmodes travels faster than what is allowed by the causal structure of the asymptotic spacetime, thus violating asymptotic causality [15]. This is analogous to the case of gravitons and photons discussed in [9, 14], where non-minimal gravitational 3-point interactions, encoded by the operators $R_{\mu\nu\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}R^{\alpha\beta\mu\nu}$ and $F_{\mu\nu}F_{\rho\sigma}R^{\mu\nu\rho\sigma}$, give rise to a mixing between the two graviton or the two photon helicities, respectively, and which results in a net macroscopic time advance for one of the propagating eigenmodes.[1] Since this happens for scattering at sufficiently small impact parameters, avoiding causality violation sets an upper bound on the cutoff of the EFT, Λ, where dynamics that is not captured by the EFT must become relevant. For recent works discussing the notion of causality in the gravitational context, we point the reader to e.g. [17–22].

Let us then consider the scattering of graviton and scalar with an spectator of mass m, within the so-called eikonal limit, $s \gg t$, where s is the center of mass energy of the collision and $t = -|\vec{q}|^2 \equiv q^2$, where $\vec{q}$ is the exchanged momentum. We take the spectator to be very heavy and nearly at rest, acting as a gravitational source against which the massless probe particle, of energy ω, scatters. In such a kinematical configuration, $m \gg \omega \gg q$, the leading contribution to the gravitational amplitude for $r_s\omega > 1$, with $r_s = m/(4\pi M_{\rm Pl}^2)$ the Schwarzschild radius of the target, is obtained after summing over ladder diagrams from single graviton exchange, see figure 1. The S-matrix takes an exponential form, $S = e^{i\delta(\omega,b)}$, where

$$\delta(\omega, \vec{b}) = \frac{1}{4m\omega}\int \frac{d^{D-2}q}{(2\pi)^{D-2}} e^{i\vec{q}\cdot\vec{b}}\mathcal{M}(\omega, \vec{q})\,, \tag{2.1}$$

is the eikonal phase shift and $\vec{b}$ the impact parameter [23, 24]. As we show below, the phase shift is in general a matrix in helicity space, from which, after diagonalization, one can extract the classical time delay for the propagation eigenmodes simply as $\Delta t = \partial_\omega\delta$.[2]

[1] Similar ideas have been considered for quadratic gravity in [16].

[2] One could consider as well, as done in [9], the sub-planckian scattering against a coherent state of a large number $N \gg 4\pi M_{\rm Pl}^2/s$ of relativistic particles, a.k.a. shock waves.

Let us briefly go over the time delay for a probe particle minimally coupled to gravity, that is the Shapiro time delay. The tree-level amplitude is helicity-preserving and universal,

$$\mathcal{M}^{\text{GR}}_{\text{tree}} \simeq \frac{1}{M_{\text{Pl}}^2}\frac{(2m\omega)^2}{q^2}\,. \tag{2.2}$$

We can compute the associated phase shift by performing the integral eq. (2.1) in $D-2$ dimensions, where $D = 4-2\epsilon$ is used as a regularization,

$$\delta^{\text{GR}} = \frac{m\omega}{4\pi M_{\text{Pl}}^2}\Gamma\left(\frac{D-4}{2}\right)\frac{1}{b^{(D-4)/2}} = 2\omega r_s\left(-\frac{1}{2\epsilon}-\frac{\gamma_E}{2}-\log b\right)+O(\epsilon)\,, \tag{2.3}$$

where $b \equiv |\vec{b}|$. Subtracting the time delay measured at a reference impact parameter $b_0 \gg b$, we obtain the result,

$$\Delta t_{\text{GR}} = 2r_s \log(b_0/b)\,. \tag{2.4}$$

This is the Shapiro time delay for a signal travelling at an impact parameter b from a source with Schwarzschild radius r_s, as measured by an observer at an impact parameter $b_0 \gg b$.

Within GR, the leading corrections to the phase shift are of order r_s/b, associated to amplitude terms in momentum space of order q/ω, arising from the eikonal expansion as well as non-linear gravitational interactions [25]. Note that when these corrections become large, that is when $r_s \sim b$, the deflection angle of the probe, $\theta = -\omega^{-1}\partial_b\delta$, is no longer small. Let us point out as well that as long as $r_s\omega > 1$, the Shapiro time delay is larger than the quantum-mechanical uncertainty associated with the probe wave, i.e. $\Delta t_{\text{GR}} > 1/\omega$.

2.1 Non-minimal scalar-tensor trilinear interactions

The (pseudo)scalar-graviton 3-point interactions associated with the $\phi\mathcal{R}^2_{\text{GB}}$ and $\phi R\tilde{R}$ operators in eq. (1.1) give rise to an eikonal phase shift that is not diagonal with respect to the helicity of the probe particle. This, along with the energy dependence of the interaction, results in time advances at energies where the EFT is still weakly coupled.

In order to compute the phase shift, we consider 4-point scattering amplitudes associated with tree-level graviton exchange between a scalar or graviton and a heavy spectator, which we take to be a scalar, S, without loss of generality. The corresponding Feynman diagrams are shown in figure 1.

Using spinor-helicity variables and taking all the particles (with complex momenta) as incoming, the relevant 3-point amplitudes read as follows:

$$\mathcal{M}^{\text{GB/CS}}_{1_\phi 2_{h^{++}} 3_{h^{++}}} = \frac{2\hat{\alpha}}{M_{\text{Pl}}}[23]^4\,, \quad \mathcal{M}^{\text{GB/CS}}_{1_\phi 2_{h^{--}} 3_{h^{--}}} = \frac{2\hat{\alpha}^*}{M_{\text{Pl}}}\langle 23\rangle^4\,, \tag{2.5}$$

where we recall that $\hat{\alpha} = \alpha + i\tilde{\alpha}$ and that $\hat{\alpha} = 0$ and $\alpha = 0$ correspond to the scalar Gauss-Bonnet and dynamical Chern-Simons gravity theories, respectively. In the center of mass frame, we parametrize the spinors for the external gravitons in the regime $m \gg \omega \gg q$ as

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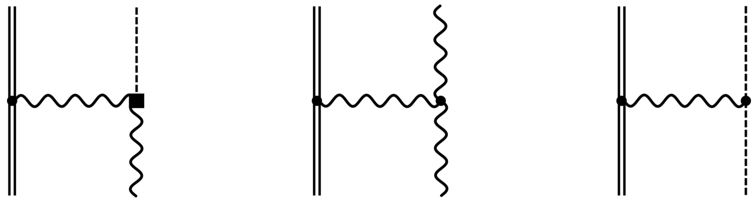

Figure 1. Leading tree-level diagrams for the eikonal scattering of graviton and scalar against a heavy target. Wiggly lines represent gravitons, dashed lines the massless scalar, and double lines the massive source. The square vertex corresponds to the ϕRR helicity-changing interaction.

in [14]

$$
\begin{aligned}
3] &\simeq i\sqrt{2|\vec{p}|}\begin{pmatrix} -\frac{q_1-iq_2}{4|\vec{p}|} \\ 1 \end{pmatrix}, & \langle 3 &\simeq i\sqrt{2|\vec{p}|}\left(-\frac{q_1+iq_2}{4|\vec{p}|}\,,\ \ 1\right), \\
4] &\simeq \sqrt{2|\vec{p}|}\begin{pmatrix} \frac{q_1-iq_2}{4|\vec{p}|} \\ 1 \end{pmatrix}, & \langle 4 &\simeq \sqrt{2|\vec{p}|}\left(\frac{q_1+iq_2}{4|\vec{p}|}\,,\ \ 1\right),
\end{aligned}
\tag{2.6}
$$

where $\vec{p} = |\vec{p}|\hat{z}$, $|\vec{p}| = \sqrt{\omega^2 - \vec{q}^{\,2}/4}$, is the momentum of the probe, orthogonal to the exchanged momentum $\vec{q}$ with components q_1, q_2. With these, we find the 4-point scattering amplitudes

$$
\begin{aligned}
\mathcal{M}^{\text{GB/CS}}_{1_S 2_S 3_{h^{++}} 4_\phi} = \mathcal{M}^{\text{GB/CS}}_{1_S 2_S 3_\phi 4_{h^{++}}} &\simeq -\frac{2\hat{\alpha}}{M_{\text{Pl}}^2}\frac{(q_1+iq_2)^2}{q^2}(2m\omega)^2\,, \\
\mathcal{M}^{\text{GB/CS}}_{1_S 2_S 3_{h^{--}} 4_\phi} = \mathcal{M}^{\text{GB/CS}}_{1_S 2_S 3_\phi 4_{h^{--}}} &\simeq -\frac{2\hat{\alpha}^*}{M_{\text{Pl}}^2}\frac{(q_1-iq_2)^2}{q^2}(2m\omega)^2\,.
\end{aligned}
\tag{2.7}
$$

Defining $b_\pm = (b_1 \pm ib_2)/2$, we have $\vec{b}\cdot\vec{q} = b_+(q_1 - iq_2) + b_-(q_1 + iq_2)$, and as before the eikonal phase shift matrix is obtained by taking the impact-parameter transform of the amplitudes,

$$
\begin{aligned}
\delta^{\text{GB/CS}}_{1_S 2_S 3_{h^{++}} 4_\phi} = \delta^{\text{GB/CS}}_{1_S 2_S 3_\phi 4_{h^{++}}} &= -2\omega r_s\,\frac{\hat{\alpha}}{b_-^2}\,, \\
\delta^{\text{GB/CS}}_{1_S 2_S 3_{h^{--}} 4_\phi} = \delta^{\text{GB/CS}}_{1_S 2_S 3_\phi 4_{h^{--}}} &= -2\omega r_s\,\frac{\hat{\alpha}^*}{b_+^2}\,.
\end{aligned}
\tag{2.8}
$$

These helicity-changing contributions add up to the helicity-preserving ones from minimal coupling, to yield the phase shift matrix

$$
\delta^{\text{GR+GB/CS}} \simeq 2\omega r_s \begin{pmatrix} D & 0 & A \\ 0 & D & A^* \\ A^* & A & D \end{pmatrix}, \tag{2.9}
$$

with rows $(h^{++},\, h^{--}\,, \phi)$ and

$$
D = -\frac{1}{2\epsilon} - \frac{\gamma_E}{2} - \log b\,, \quad A = -\frac{\hat{\alpha}}{b_-^2}\,. \tag{2.10}
$$

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After diagonalizing, we find the eigenvalues

$$\delta_0 = 2\omega r_s\left(-\frac{1}{2\epsilon}-\frac{\gamma_E}{2}-\log b\right)\,,\quad \delta_\pm = 2\omega r_s\left(-\frac{1}{2\epsilon}-\frac{\gamma_E}{2}-\log b\pm\sqrt{2}\frac{|\hat{\alpha}|}{b^2}\right)\,,\tag{2.11}$$

where the first corresponds to a pure graviton state, while the other two to a scalar-graviton mixed state. The time delay that the latter propagating eigenmodes acquire are

$$\Delta t_\pm = 2r_s\left(\log\frac{b_0}{b}\pm\sqrt{2}\frac{|\hat{\alpha}|}{b^2}\right)\,.\tag{2.12}$$

At small enough impact parameters, Δt_- becomes negative, that is a time advance, signalling a potential violation of causality. Phrased in another way, for a given impact parameter there is a time advance if the GB/CS coefficient is large enough, $|\hat{\alpha}|\gtrsim b^2\log(b_0/b)$. To avoid acausality at low energies, the EFT computation must therefore break down at distances such that this condition cannot be satisfied.[3] This implies the GB/CS coupling is parametrically bounded by the cutoff of the EFT as

$$|\hat{\alpha}|\lesssim\frac{\log(b_0\Lambda)}{\Lambda^2}\,.\tag{2.13}$$

Several comments are in order. For the violation of causality to potentially be resolvable and thus become problematic, the time advance should be larger than the quantum uncertainty of the wave-packet, $|\Delta t_-|>1/\omega$. For impact parameters where the BGR contribution is assumed to dominate, this condition reads[4]

$$|\Delta t_-|\sim r_s\frac{|\hat{\alpha}|}{b^2}>\frac{1}{\omega}\,,\tag{2.14}$$

which for impact parameters down to the minimum cutoff length implied by eq. (2.13), i.e. $b\sim|\hat{\alpha}|^{1/2}$ (neglecting the log), just requires $r_s\omega>1$. Equivalently, eq. (2.14) defines an impact parameter below which the would-be time advance is resolvable, $b_r=(r_s\omega|\hat{\alpha}|)^{1/2}$. This is larger than the minimum cutoff length within the EFT regime of validity, i.e. $b_r>|\hat{\alpha}|^{1/2}$, as long as $r_s\omega>1$. Therefore, even if potentially resolvable, as long as eq. (2.13) is satisfied there is never an actual time advance. Alternatively, one could also argue that the time advance is actually not resolvable at $b\sim|\hat{\alpha}|^{1/2}$ because the UV completion precludes $r_s\omega>1$, which in practice means a cutoff such that $\omega\lesssim\Lambda\lesssim 1/r_s$. In this case, the condition $|\Delta t_-|<1/\omega$ for $\omega\sim\Lambda$ and $b\sim r_s\sim 1/\Lambda$ implies $\Lambda\lesssim|\hat{\alpha}|^{1/2}$, just like in eq. (2.13) up to $O(1)$ factors, which in any case we are oblivious about.

The bound eq. (2.13) depends on the logarithm of an unspecified scale $b_0\gg b$, because of the infrared (IR) divergent nature of gravity in four dimensions. While the identification of IR-finite scattering observables in gravity remains an open and interesting problem (see

[3] That is, at a distance $1/\Lambda>b_*$, where the largest impact parameter at which a time advance is found, b_*, is given by $|\hat{\alpha}|\sim b_*^2\log(b_0/b_*)$. Note that the dynamics needed to restore causality on time scales of order b_* should only involve momentum transfers $q_*\sim 1/b_*$, meaning that new physics must appear at scales $1/b_*$ or smaller, a priori regardless of ω and r_s.

[4] This condition can also be interpreted as the one for which the beyond-GR contribution to the time delay is resolvable on its own.

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e.g. [26] for a recent attempt in the context of causality constraints), we do not regard this divergence as a serious drawback that invalidates our bounds. In fact, note that even if we consider an IR scale of the order of the size of the observable universe, we merely find $\log(\Lambda/H_0) \sim 50$.

Finally, let us point out that $\Lambda \lesssim |\hat{\alpha}|^{-1/2}$ implies that the EFT description must break down at energies much lower than the strong coupling scale associated with the GB/CS interactions. Indeed, the scale where the trilinear scalar-tensor coupling becomes large, indicated by e.g. the 4-graviton amplitude mediated by the scalar becoming strong, $\mathcal{M} \sim (\hat{\alpha}/M_{\rm Pl})^2 E^6 \sim 1$, is

$$\Lambda_\alpha = \left(\frac{M_{\rm Pl}}{|\hat{\alpha}|}\right)^{1/3}, \tag{2.15}$$

much larger than the actual cutoff of the EFT, unless $|\hat{\alpha}| \sim 1/M_{\rm Pl}^2$.

2.2 Causality bounds on power counting

In this section we reinterpret the causality constraints in terms of bounds on the power counting of gravitational EFTs. With this aim, let us consider the generic form of a scalar theory coupled to gravity, in which the heavy degrees of freedom, of mass Λ or higher (i.e. Λ is the EFT cutoff), have been integrated out

$$\mathcal{L} = \frac{1}{2}\hat{M}_{\rm Pl}^2 R + \frac{\Lambda^4}{g^2} L^{(0)}\left(\frac{\nabla_\mu}{\Lambda}, \frac{\zeta R_{\mu\nu\rho\sigma}}{\Lambda^2}, \frac{g\phi}{\Lambda}\right) + \dots . \tag{2.16}$$

In the spirit of naive dimensional analysis (NDA) each covariant derivative ∇_μ is weighted by $1/\Lambda$, and each (scalar) field ϕ by g/Λ. The coupling g parametrizes the strength with which the heavy states couple to the light degrees of freedom, with $g \sim 4\pi$ the usual non-perturbative coupling limit. Note that instead of considering the Riemann tensor, $R_{\mu\nu\rho\sigma} \sim \partial_\mu\partial_\nu h_{\rho\sigma}$, simply as a two-derivative object thus weighted by $1/\Lambda^2$, we introduce a dimensionless parameter ζ to allow for the possibility that gravitational interactions beyond GR's minimal coupling are enhanced w.r.t. standard NDA.[5] We will elaborate on such a generalized power counting below. Each ϕ interaction comes with a decay constant f, identified with (or defined as)

$$f = \frac{\Lambda}{g}. \tag{2.17}$$

At this point we can already distinguish the two interesting scenarios, for which it is enough to consider the standard power counting $\zeta = 1$ and to realize that the EH action receives a contribution from both terms in eq. (2.16). When $\hat{M}_{\rm Pl}^2 \gg f^2$, the EH action is dominated by the first term and the effective Planck scale is $M_{\rm Pl} \sim \hat{M}_{\rm Pl}$. Gravity is external to the ultraviolet (UV) dynamics giving rise to $L^{(0)}$, a.k.a. "elementary". Instead, when $\hat{M}_{\rm Pl}^2 \ll f^2$, we have $M_{\rm Pl} \sim f$ and the heavy dynamics constitutes a bona fide UV completion of gravity. Phrasing it in terms of the coupling g, the minimum coupling $g \sim \Lambda/M_{\rm Pl}$ corresponds to the "composite" limit of gravity. This is the case of string theory

[5]As usual we work with a dimensionless graviton field, whose interactions are eventually weighted by $1/M_{\rm Pl}$ once its kinetic term is canonically normalized, following the normalization of the Einstein-Hilbert (EH) term.

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(or more generally, potential tree-level UV completions with infinitely many higher-spins particles, see e.g. [9, 27]), where $\Lambda \sim M_s$ the string scale, as well as of loop-level completions based on a large number of species, $N \sim (4\pi M_{\rm Pl}/\Lambda)^2$, where $g \sim 4\pi/\sqrt{N}$ [28, 29]. Note that in this limit one finds the largest coefficients for gravitational EFT operators with none or a single matter field, since they scale as $1/g^2$ or $1/g$ respectively. The fact that $g \gtrsim \Lambda/M_{\rm Pl}$ is reminiscent of the weak gravity conjecture [30].

In terms of scattering amplitudes, the two scenarios are distinguished by the maximal size of e.g. 2-to-2 graviton processes within the EFT regime of validity, i.e. $E \lesssim \Lambda$. From minimal coupling we have $\mathcal{M}^{\rm GR} \sim (E/M_{\rm Pl})^2 \lesssim (\Lambda/M_{\rm Pl})^2$. Instead, an effective operator like $R^3_{\mu\nu\rho\sigma}$ leads to an amplitude $\mathcal{M}^{\rm BGR}_{(\zeta=1)} \sim E^6/(g^2\Lambda^2 M_{\rm Pl}^4) \lesssim f^2\Lambda^2/M_{\rm Pl}^4$, smaller than GR except in the limit $M_{\rm Pl} \sim f$, in which case the two amplitudes are of the same size at the cutoff. The same analysis can be reproduced if instead of amplitudes one considers other (classical) gravitational observables and distances, rather than energies, within EFT control, i.e. $r \gtrsim 1/\Lambda$.

For the generalized power counting $\zeta > 1$, the discussion is very much analogous, except for the important difference that now the BGR effects can become larger than the GR prediction for energies well described by the EFT. The composite case corresponds to $\hat{M}_{\rm Pl}^2 \ll \zeta f^2$, for which we have $M_{\rm Pl} \sim \sqrt{\zeta} f$. Therefore, $\zeta \ll (M_{\rm Pl}/f)^2$ corresponds to the case where gravity is external to the UV dynamics. Elementary or composite, we find that non-standard gravitational interactions, in the form of $R^3_{\mu\nu\rho\sigma}$, give rise to enhanced 4-graviton amplitudes

$$\mathcal{M}^{\rm BGR}_{1_h 2_h 3_h 4_h} \sim \zeta^3 \frac{E^6 f^2}{\Lambda^4 M_{\rm Pl}^4} \lesssim \zeta^3 \frac{\Lambda^2 f^2}{M_{\rm Pl}^4} \lesssim \frac{\Lambda^2 M_{\rm Pl}^2}{f^4} = \left(\frac{g M_{\rm Pl}}{f}\right)^2 , \tag{2.18}$$

where the first inequality follows from $E \lesssim \Lambda$ and the second from $\zeta \lesssim (M_{\rm Pl}/f)^2$. Note that for $\zeta \gtrsim (M_{\rm Pl}/f)^{2/3}$ the amplitude is larger than in GR, and it becomes non-perturbatively strong, i.e. $\mathcal{M} \sim (4\pi)^2$, for EFT cutoffs well below the maximal gravity cutoff given by $4\pi M_{\rm Pl}$. As we discuss in the following, it is precisely this possibility that causality constraints forbid.[6]

Let us start by recalling that each specific UV theory within the class of theories described in the IR by eq. (2.16) comes with $O(1)$ factors not captured by the power counting. Even more importantly, the presence of symmetries can enforce some EFT operators to have vanishing coefficients, for instance if ϕ is Nambu-Goldstone boson with a shift symmetry $\phi \to \phi + c$ (as the scalar field that concerns us in this work), any potential term for ϕ vanishes. However, beyond the well-known selection rules from symmetries, there are further requirements that an EFT must satisfy if it is to be consistent with the

[6]It is perhaps instructive to compare with EFTs for spin-1 (abelian or non-abelian) gauge fields,

$$\mathcal{L} = \frac{1}{4\hat{e}^2} F_{\mu\nu}^2 + \frac{\Lambda^4}{g^2} L^{(0)}\left(\frac{D_\mu}{\Lambda}, \frac{\zeta F_{\mu\nu}}{\Lambda^2}, \frac{g\phi}{\Lambda}\right) ,$$

with the elementary and composite limits given respectively by $\hat{e} \ll g/\zeta$ (and effective gauge coupling $e \sim \hat{e}$) and $\hat{e} \gg g/\zeta$ ($e \sim g/\zeta$). As discussed in [31], in the strongly coupled gauge field scenario $\zeta \sim g/e \gtrsim 1$ one finds 4-point amplitudes (from e.g. $F^4_{\mu\nu}$ operators) $\mathcal{M} \sim g^2 (E/\Lambda)^4$, which can be larger than the amplitude from minimal coupling, $\mathcal{M} \sim e^2$, for energies within the EFT.

fundamental principles of unitarity, locality, and causality (and if it is to arise from UV dynamics that abides by such principles). Indeed, it was found in [9] that causality, in the form of absence of a (resolvable) time advance, leads to a constraint on the size of corrections to the cubic graviton coupling, arising from an operator $\alpha_3 M_{\rm Pl}^2 R^3_{\mu\nu\rho\sigma}$, given by $\alpha_3 \lesssim 1/\Lambda^4$. In terms of the power counting eq. (2.16), $\alpha_3 \sim \zeta^3/(gM_{\rm Pl}\Lambda)^2$, such a bound implies $\zeta \lesssim (M_{\rm Pl}/f)^{2/3}$, precisely such that the BGR effects never get to dominate over GR, see below eq. (2.18). This conclusion seems to be generic. The similar bound we have derived in section 2.1 on the GB/CS non-minimal coupling of gravitons to a scalar, $|\hat\alpha| \lesssim 1/\Lambda^2$, when interpreted in terms of our power counting, $|\hat\alpha| \sim \zeta^2/(gM_{\rm Pl}\Lambda)$, implies $\zeta \lesssim (M_{\rm Pl}/f)^{1/2}$. Once again, this forbids the 4-graviton amplitude mediated by the scalar from getting larger than in GR if restricted to energies within the EFT, $E \lesssim \Lambda$,

$$\mathcal{M}^{\rm GB/CS}_{1_{h^{++}}2_{h^{++}}3_{h^{--}}4_{h^{--}}} \sim |\hat\alpha|^2 \frac{E^6}{M_{\rm Pl}^2} \lesssim \frac{\Lambda^2}{M_{\rm Pl}^2}\,. \tag{2.19}$$

It is illuminating to realize that in the case of a standard power counting $\zeta = 1$, these causality constraints robustly imply that $g \gtrsim \Lambda/M_{\rm Pl}$ (or equivalently $f \lesssim M_{\rm Pl}$), as we expected from the simple NDA considerations on the elementary vs composite nature of gravity. In turn, if one is interested in genuine UV completions of gravity, i.e. $g \sim \Lambda/M_{\rm Pl}$, these bounds imply that $\zeta \lesssim 1$ and therefore that the EFTs in which non-minimal interactions are enhanced beyond standard NDA have no gravitational completions consistent with fundamental principles.

2.2.1 Bounds from dispersion relations

This conclusion is reinforced by recent progress on the derivation of theoretical constraints on gravitational EFTs that go beyond causality violation in classical observables and therefore beyond corrections to cubic gravitational interactions [10, 11, 26, 32–41]. Such bounds are instead obtained via dispersion relations [42], which connect the coefficients of the EFT operators to the dynamics of their UV completions. These UV/IR relations, which we will review in some detail in section 3.1, are very powerful because of their generality, relying only on the basic assumptions of unitarity, locality and causality (encoded as the analyticity, crossing symmetry and boundedness of the scattering amplitudes).[7] Of particular relevance for the physics of black holes are the results of [10], which derived a lower bound on $\alpha_4 M_{\rm Pl}^2 R^4_{\mu\nu\rho\sigma}$ given by $\alpha_4 \gtrsim \alpha_3^2\Lambda^2$ (recall $\alpha_3 M_{\rm Pl}^2 R^3_{\mu\nu\rho\sigma}$), and of [11], which derived the upper bound $\alpha_4 \lesssim 1/\Lambda^6$. Both constraints restrict the BGR contribution to gravitational observables to be smaller than the prediction of GR. In fact, we should stress that if similar bounds were to be derived on non-standard higher-point amplitudes (with $n \geqslant 5$ gravitons) from $R^n_{\mu\nu\rho\sigma}$ operators, we would be led to the conclusion that the power counting in eq. (2.16) with $\zeta > 1$ is inconsistent altogether, i.e. regardless of f and not only for $f \sim M_{\rm Pl}$. While this seems like a plausible expectation, a robust derivation of theoretical constraints on higher-point amplitudes remains an open problem at the time of writing this work (see e.g. [44] for recent progress in this direction). If indeed $\zeta > 1$ is forbidden by

[7]The link between dispersion relations and causality, expressed as the absence of superluminal propagation, was pointed out in [43], and its connection with time delay has been recently discussed in [11, 37].

fundamental principles, we would come to the sensible conclusion that in a gravitational EFT the largest effects for a fixed cutoff are found when $f \sim M_{\rm Pl}$, therefore when gravitational interactions should dramatically change above Λ. We will provide further insight into this fact in section 5.

Before concluding this section, let us make some additional comments on the implications of the causality bounds on the phenomenology of black holes beyond GR. The constraint $\alpha_4 \lesssim 1/\Lambda^6$ [11] places modifications of GR due to quartic terms in the curvature on the same footing as those due to cubic terms. This means that there is no strong reason to discard the effects of $R^3_{\mu\nu\rho\sigma}$ operators, leading in the derivative expansion, while keeping those of $R^4_{\mu\nu\rho\sigma}$ [45–47].

The constraint we have obtained in section 2.1 on BGR scalar-tensor cubic couplings have also been recently derived in [11] via dispersion relations. In this regard, it is important to point out that even though our bound is robust up to $O(1)$ factors, contrary to the more precise (yet still IR divergent) one from dispersion relations, we believe our derivation is very valuable because it comes from a simple physical setup in which causality violation is a classical, macroscopic effect, therefore it does not rely on a priori stronger assumptions on the analyticity and polynomial boundedness of scattering amplitudes associated with causality.

NDA expectations are also confirmed by dispersion relations involving operators with extra derivatives acting on the curvature, for instance of the form

$$\alpha_5 M_{\rm Pl}^2 R^2_{\mu\nu\rho\sigma}(\nabla_\eta R_{\mu\nu\rho\sigma})^2, \quad \alpha_6 M_{\rm Pl}^2 (\nabla_\eta R_{\mu\nu\rho\sigma})^4, \tag{2.20}$$

which contribute to 4-graviton amplitudes as $\mathcal{M} \sim \alpha_J E^{2J}$ [10, 11, 41]. In particular, there are an infinite number of linear constraints on the EFT coefficients that take the form of two-sided bounds such as

$$-\alpha_4 \leqslant \alpha_5 \Lambda^2 \leqslant \alpha_4 \quad \text{and} \quad 0 \leqslant \alpha_6 \Lambda^4 \leqslant \alpha_4 \,, \tag{2.21}$$

and similar bounds for higher J, respectively odd or even.

Their interpretation in terms of a power counting is clear, subleading operators in the derivative expansion ∇/Λ cannot be enhanced over the leading ones.

In our discussion we have focussed on cubic and quartic operators built out of the Riemann tensor, with no mention of terms quadratic in the curvature. This is because $R^2_{\mu\nu\rho\sigma}$ operators do not contribute to graviton scattering amplitudes, given that the GB term $\mathcal{R}^2_{\rm GB}$ is a topological invariant (in $D = 4$) and because field redefinitions can be performed to eliminate any EFT operator built out of R and $R_{\mu\nu}$ in favour of matter terms (T and $T_{\mu\nu}$), therefore giving rise to amplitudes involving ϕ fields. For this reason, one might find it more convenient (although not necessary) to use a basis of EFT operators directly linked to scattering amplitudes, such as the one systematically constructed in [48]. In this respect, note that the relevant object giving rise to processes with gravitons on-shell is the Weyl tensor, $C_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - (g_{\mu[\rho}R_{\sigma]\nu} - g_{\nu[\rho}R_{\sigma]\mu}) + \frac{1}{3} g_{\mu[\rho}g_{\sigma]\nu}R$. This means that the relevant parts of the GB/CS operators in eq. (1.1) are $M_{\rm Pl}\alpha\, \phi C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$ and $M_{\rm Pl}\tilde{\alpha}\, \phi C_{\mu\nu\rho\sigma}\tilde{C}^{\mu\nu\rho\sigma}$. These are also the terms behind the scalar hair of black holes, since black holes are Ricci-flat gravitational solutions ($R, R_{\mu\nu} = 0$) at zeroth order in α, $\tilde{\alpha}$.

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Finally, causality bounds on pure scalar operators are also relevant for the physics of hairy black holes, in particular

$$\frac{1}{4}c_2(\nabla_\mu\phi)^4\,, \tag{2.22}$$

i.e. the leading operator in the derivative expansion. Several recent works on dispersion relations that incorporate gravity have argued that $c_2 \gtrsim -1/(\Lambda^2 M_{\rm Pl}^2)$ (and likewise for the equivalent operator $F_{\mu\nu}^4$ in a theory of photons) [26, 33, 35, 36, 38, 49], bound that becomes a standard positivity constraint, $c_2 > 0$ [43], when gravity decouples, $M_{\rm Pl} \to \infty$. In particular, [38] has shown via dispersion relations at finite impact parameter that this is indeed the case up to a $\log(b_0\Lambda)$, as in eq. (2.13). Furthermore, the upper bound $c_2 \lesssim (4\pi)^2/\Lambda^4$ has been derived using similar techniques [50, 51]. These constraints on c_2 can be easily understood in terms of the power counting in eq. (2.16). Since $c_2 \sim g^2/\Lambda^4$, the upper and lower bounds correspond, respectively, to the maximum coupling in the spirit of NDA, $g \lesssim 4\pi$, and to the minimum coupling in a gravitational theory, $g \gtrsim \Lambda/M_{\rm Pl}$ (recall that power counting estimates are insensitive to the sign of the operators' coefficient).

3 Signs of UV completion

In the previous section we have argued that gravitational EFTs where black holes have scalar hair, eq. (1.1), must have a cutoff $\Lambda \lesssim |\hat{\alpha}|^{-1/2}$. In terms of the power counting eq. (2.16) with $\zeta = 1$, the maximum cutoff of these theories corresponds to the minimum NDA coupling $g \sim \Lambda/M_{\rm Pl}$, while EFTs with a larger coupling, or equivalently $f < M_{\rm Pl}$, must have a lower cutoff for the same value of $|\hat{\alpha}|$.

In this section we try to infer from an EFT point of view what additional low-energy effects are associated with a generic UV completion at the scale Λ, in particular one that is unitary, local and causal. We focus on the leading corrections in the derivative and field expansion [48], restricted to CP even operators

$$\Delta S = \int d^4x\sqrt{-g}\left[M_{\rm Pl}^2\left(\alpha_3\mathcal{I} + \alpha_4\mathcal{C}^2 + \alpha_4'\tilde{\mathcal{C}}^2\right) + \frac{c_2}{4}(\nabla_\mu\phi)^4 + \frac{d_1}{2}\mathcal{C}(\nabla_\mu\phi)^2\right]\,, \tag{3.1}$$

where $\mathcal{C} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$, $\tilde{\mathcal{C}} = R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}$ and $\mathcal{I} = R_{\mu\nu}{}^{\rho\sigma}R^{\mu\nu\alpha\beta}R_{\alpha\beta\rho\sigma}$. Note that the last operator is equivalent, by the leading-order scalar equation of motion, to the cubic Galileon term $(\nabla\phi)^2\Box\phi$. We first investigate the constraints on the coefficients above that arise from dispersion relations at one loop, which take the form of lower bounds that depend on $|\hat{\alpha}|$.[8] Precisely because of the upper bound $|\hat{\alpha}| \lesssim 1/\Lambda^2$, we find that such dispersion relations are in fact dominated by standard gravitational contributions, rendering the constraints on the operators in eq. (3.1) inapplicable and phenomenologically irrelevant. We therefore leave aside general bounds and turn to generic expectations based on power counting. We show how typical UV completions of the scalar-GB or dynamical-CS theories likely give rise to higher-curvature terms of the same parametric size, i.e. $\alpha\Lambda^2 \sim \alpha_3\Lambda^4 \sim \alpha_4\Lambda^6$, if these arise at the same loop order.

[8] There are no constraints of this form from dispersion relations at tree level. This is in contrast with the lower bound $\alpha_4 \gtrsim \alpha_3^2\Lambda^2$ [10].

3.1 Beyond positivity constraints

There exists an extensive literature on dispersion relations, in particular on non-gravitational theories, with many new results and applications found in recent years. We refer the reader to e.g. [37, 43, 52–58] and references therein for many of the details we will not present here. Dispersion relations are typically constructed by evaluating a 2-to-2 scattering amplitude $\mathcal{M}(s,t)$ over a closed circular contour in the complex s-plane,[9]

$$\Sigma_n(s,t) = \frac{1}{2\pi i}\oint_{\Gamma_s} ds' \frac{\mathcal{M}(s',t)}{(s'+\frac{t}{2})^{n+1}} \,. \tag{3.2}$$

Let us start with the scattering of the GB/CS scalar ϕ at low energies, $s \ll \Lambda^2$, neglecting for the time being GR's minimal gravitational coupling. The 4-scalar EFT interaction in eq. (3.1) leads to an amplitude,

$$\mathcal{M}^{\Delta S}_{1_\phi 2_\phi 3_\phi 4_\phi}(s,t) = \frac{c_2}{2}(s^2+t^2+u^2)\,, \tag{3.3}$$

which grows like s^2 for fixed t. Therefore, considering a small contour Γ_0 around $s' = -t/2$, the integral of the twice-subtracted amplitude, i.e. $n = 2$ in eq. (3.2), yields $\Sigma_2(0,t) = c_2$ At this point, unitarity and causality allow one to deform the contour away from the origin (for $0 \leqslant t \leqslant 4m^2$) in a controlled way. First, they imply that $\mathcal{M}(s,t)$ is analytic everywhere except in the real axis, where one finds singularities in the form of simple poles and branch cuts. The former correspond to particles exchanged at tree level going on shell, which in the case at hand belong only to the UV completion, either in the s-channel at $s \geqslant \Lambda^2$ or in the u-channel $s \leqslant -\Lambda^2 - t$. The branch cuts are associated with logarithms arising from loops and correspond to multi-particle production. Besides the loops of heavy states at and above the cutoff, there is an s-channel branch cut starting at $s = 4m^2$ from (one) loop diagrams of the IR degrees of freedom, and its $s \leftrightarrow u$ crossing symmetric counterpart. Because of real analyticity, $\mathcal{M}^*(s,t) = \mathcal{M}(s^*,t)$, these discontinuities are proportional to $\mathrm{Im}\mathcal{M}(s,t)$, which is positive for elastic scattering around $t = 0$. In particular, for the zeroth-order term in an expansion around the forward limit, the optical theorem fixes $\mathrm{Im}\mathcal{M}(s,0) = s\sqrt{1-4m^2/s}\,\sigma_\mathrm{T}(s)$, where σ_T is the total cross section from $1\,2 \to$ everything. In addition, unitarity and causality in theories with a mass gap imply that amplitudes are polynomially bounded as $\mathcal{M}(s,t)/s^2 \to 0$ for $|s| \to \infty$ as a result of the Froissart-Jin-Martin bound. Even though here we are interested in theories with a massless graviton, it has been argued from different perspectives that a growth smaller than s^2 holds as well with dynamical gravity, see e.g. [9, 37, 38, 59, 60]. Note then that when this is the case, the integral eq. (3.2) over a contour Γ_∞ at $|s| \to \infty$ vanishes for $n \geqslant 2$, i.e. $\Sigma_{n\geqslant 2}(\infty,t) = 0$. A dispersion relation is then finally derived by using Cauchy's theorem to deform the original contour Γ_0 to Γ_∞, leaving Σ_n as an integral over the aforementioned singularities. In the forward limit $t = 0$

[9] In a slight abuse of notation, we denote the amplitude as a function of the $s = -(p_1+p_2)^2$ and $t = -(p_1+p_3)^2$ Mandelstam variables as $\mathcal{M}$, like in section 2 where instead was a function of ω and $\vec{q}$. Besides, we work with all momenta incoming and recall $u = -(p_1+p_4)^2 = -s-t+4m^2$, where m is now the mass of the scattered states, that we will eventually take to zero.

of 4-scalar scattering, one then finds for $n = 2$

$$c_2 = \sum\nolimits_X \frac{2}{\pi} \int_0^\infty \frac{ds}{s^2} \sigma_{1_\phi 2_\phi \to X}(s) > 0\,. \quad \text{(w/o GR's minimal coupling)} \tag{3.4}$$

This positivity constraint can be improved by noticing that, while the cross sections for production of the heavy states associated with the UV completion are by construction not computable within the EFT, those for production of the low-energy states are, as long as we restrict them to energies $s \leqslant \Lambda^2$ [54, 61–64]. Since we are neglecting the minimal coupling of gravitons, the process with the largest cross section in the scalar EFT eq. (1.1) is the production of a pair of gravitons via the GB/CS coupling. The corresponding amplitude is

$$\mathcal{M}^{\text{GB/CS}}_{1_\phi 2_\phi 3_{h^{++}} 4_{h^{--}}} = \left(\frac{2|\hat{\alpha}|}{M_{\text{Pl}}}\right)^2 \langle 4|1|3]^4 \left(\frac{1}{t} + \frac{1}{u}\right)\,. \tag{3.5}$$

Explicitly including this contribution in the twice-subtracted dispersion relation eq. (3.4), we arrive at

$$c_2 > \frac{2}{\pi} \int_0^{\Lambda^2} \frac{ds}{s^2} \sigma^{\text{GB/CS}}_{\phi\phi \to h^{--} h^{++}} = \frac{1}{60\pi^2} \left(\frac{|\hat{\alpha}|\Lambda^2}{M_{\text{Pl}}}\right)^4\,. \quad \text{(w/o GR's minimal coupling)} \tag{3.6}$$

If one could ignore GR's contributions to the dispersion relation, as we have done this far, such a beyond-positivity bound would imply that the GB/CS scalar-tensor theories in eq. (1.1) are inconsistent with unitarity and causality unless they are supplemented with the $(\nabla\phi)^4$ operator. In particular, note that the larger the regime of validity of the EFT, i.e. the larger the cutoff Λ, the larger its coefficient c_2 would have to be.[10] However, neglecting GR's interactions would require in practice the existence of a consistent decoupling limit in which $M_{\text{Pl}} \to \infty$ yet the lower bound on c_2 remains non-zero. Expressing eq. (3.6) in terms of the strong coupling scale eq. (2.15), $c_2 \gtrsim \frac{1}{16\pi^2}(\Lambda/\Lambda_\alpha)^8 \Lambda_\alpha^{-4}$, this would require keeping Λ_α as well as Λ fixed. However, precisely because of the causality bound we derived in section 2, $\Lambda \lesssim 1/|\hat{\alpha}|^{1/2}$ for $\log(b_0\Lambda) \sim 1$ (or equivalently $\Lambda \lesssim (\Lambda_\alpha^3/M_{\text{Pl}})^{1/2}$), such a limit is not possible: if $M_{\text{Pl}} \to \infty$, then either $\Lambda_\alpha \to \infty$ or $\Lambda \to 0$, rendering the EFT invalid. In fact, even if one saturates the upper bound on $|\hat{\alpha}|$, the beyond-positivity contribution to c_2 in eq. (3.6) is only as large as a quantum correction in GR at one loop, i.e. $c_2 \gtrsim \frac{1}{16\pi^2} M_{\text{Pl}}^{-4}$.

We can explicitly check that one cannot ignore GR's minimal coupling if the upper bound $\Lambda \lesssim 1/|\hat{\alpha}|^{1/2}$ holds by retaking the steps above keeping $t \neq 0$ and with the low-energy contour now enclosing the graviton pole, at $s = 0$ and $s = -t$ ($u = 0$), of the 4-scalar amplitude in GR,

$$\mathcal{M}^{\text{GR}}_{1_\phi 2_\phi 3_\phi 4_\phi} = -\frac{1}{2M_{\text{Pl}}^2} \left(\frac{t^2 + u^2}{s} + \frac{s^2 + u^2}{t} + \frac{t^2 + s^2}{u}\right)\,. \tag{3.7}$$

[10] In section 4 we discuss the effects of the leading additional operators in eq. (3.1) on black holes with scalar hair. From that analysis one can arrive at the conclusion that for astrophysical black holes where $r_s \sim |\hat{\alpha}|^{1/2} \sim$ km, the effects of $(\nabla\phi)^4$ with c_2 fixed by eq. (3.6) would become $O(1)$, thus as important as the GB/CS term, for $\Lambda \gtrsim \mu\text{m}^{-1}$, precisely of the same order as the smallest scales where gravity has been experimentally tested [7] and at least up to which one would want any BGR theory to hold.

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Note that the forward limit is ill-defined because of the graviton t-channel exchange. The $n = 2$ dispersion relation then reads

$$-\frac{1}{M_{\rm Pl}^2 t} + c_2 + \beta_2^{(t)} \log\frac{t}{t_0} + O(t) = \frac{2}{\pi}\int_0^\infty ds \frac{{\rm Im}\mathcal{M}(s',t)}{\left(s+\frac{t}{2}\right)^3}\,. \tag{3.8}$$

We have included the one-loop UV divergence of the s^2 term of the 4-scalar amplitude arising from t-channel cuts, with β-function given by $\beta_2^{(t)} = +(13/160\pi^2)M_{\rm Pl}^{-4}$. This is of the same loop order as the r.h.s. of eq. (3.6). Indeed, as discussed in [58], the beyond-positivity contributions to the dispersion relation are equivalent to including the running of the coefficients of the EFT in the forward limit, associated with the UV divergences from s- and u-channel cuts. These cuts and the corresponding gravitational β-functions can be easily computed following the on-shell amplitude techniques presented in [65]. The $O(t)$ term in eq. (3.8) encodes the subleading terms in the forward limit, arising from e.g. higher-order EFT operators in the derivative expansion. For instance, the first such correction comes from the Galileon-like term $(\nabla\phi)^2(\nabla\nabla\phi)^2$, which gives rise to an stu term in the amplitude. Most importantly, as advanced at the end of section 2.2, the $1/t$ term in eq. (3.8) precludes setting a positive lower bound on c_2 as the one in eq. (3.6), unless the beyond-positivity contributions are larger than $-(M_{\rm Pl}^2 t)^{-1} \gtrsim (M_{\rm Pl}\Lambda)^{-2}$ [38]. As we discussed above, this is not the case because of the causality bound $|\hat\alpha| \lesssim 1/\Lambda^2$.

While it might naively seem from the discussion above that the main obstruction for the derivation of meaningful beyond-positivity bounds in gravitational EFTs is the t-channel graviton pole, the real reason for their ineffectiveness is the fact that BGR amplitudes larger than in GR are not consistent with causality. To show this, let us consider a dispersion relation for the 4-graviton amplitude with two positive and two negative helicities.[11] The amplitude in GR plus the leading BGR correction in the energy expansion from eq. (3.1) is given by

$$\mathcal{M}^{{\rm GR}+\Delta S}_{1_{h^{++}}2_{h^{--}}3_{h^{--}}4_{h^{++}}}(s,t) = \frac{\langle 23\rangle^4[14]^4}{M_{\rm Pl}^2} f(s,t)\,,\quad f(s,t) = \frac{1}{stu} + 8(\alpha_4+\alpha_4')\,. \tag{3.9}$$

Similarly to the scalar case, one can construct dispersion relations from the contour integral (see e.g. [10, 11, 32, 37] for more details)

$$\frac{1}{2\pi i}\oint_{\Gamma_s} ds' \frac{f(s',t)}{\left(s'+\frac{t}{2}\right)^{n+1}}\,. \tag{3.10}$$

In particular, for $n = 0$ one arrives at

$$\frac{8(\alpha_4+\alpha_4')}{M_{\rm Pl}^2} + \frac{\gamma_4}{\Lambda^2 t}\log\frac{-t}{\mu^2} + O(t) > \frac{2}{\pi}\int_0^{\Lambda^2}\frac{ds}{s^4}\sigma^{\rm GB/CS}_{h^{++}h^{--}\to\phi\phi,\,h^{--}h^{++}} \sim \frac{1}{16\pi^2}\left(\frac{|\hat\alpha|\Lambda}{M_{\rm Pl}}\right)^4\,. \tag{3.11}$$

On the r.h.s. we have explicitly included the beyond-positivity contribution from a scalar as well as a graviton loop via the GB/CS coupling, computed in a dispersive way from the

[11] Dispersion relations for a different choice of graviton helicities lead to constraints on the coefficients of other higher-dimensional operators, see e.g. [11].

corresponding cross sections. The corresponding amplitudes are given by eq. (3.5) and by

$$\mathcal{M}^{\mathrm{GB/CS}}_{1_{h++}2_{h--}3_{h++}4_{h--}} = -\left(\frac{2|\hat{\alpha}|}{M_{\mathrm{Pl}}}\right)^2 \frac{\langle 24\rangle^4[13]^4}{t}\,, \tag{3.12}$$

which proceeds via scalar exchange. While there is no contribution from tree-level graviton exchange in eq. (3.11), further forward limit singularities are generated at one loop in GR, with $\gamma_4 \sim +(1/16\pi^2)M_{\mathrm{Pl}}^{-4}$ [10]. Therefore, since the time-delay constraint $|\hat{\alpha}| \lesssim 1/\Lambda^2$ sets an upper bound on the r.h.s. $\lesssim \frac{1}{16\pi^2}(\Lambda M_{\mathrm{Pl}})^{-4}$, the beyond-positivity contribution is no larger than the one of GR, rendering the former immaterial to bound the quartic curvature operators.

In summary, because causality demands that gravitational amplitudes within the EFT domain are dominated by GR, loop corrections from BGR interactions never lead to robust lower bounds on the coefficients of the EFT.

3.2 Power counting expectations

The results of the previous section can be understood in terms of NDA, when the power counting rules in eq. (2.16) are extended to include the possibility that EFT operators can be generated at one-loop order,

$$\mathcal{L} = \frac{1}{2}\hat{M}_{\mathrm{Pl}}^2 R + \frac{\Lambda^4}{g^2}\left[L^{(0)}\left(\frac{\nabla_\mu}{\Lambda}, \frac{R_{\mu\nu\rho\sigma}}{\Lambda^2}, \frac{g\phi}{\Lambda}\right) + \frac{g^2}{(4\pi)^2}L^{(1)}\left(\frac{\nabla_\mu}{\Lambda}, \frac{R_{\mu\nu\rho\sigma}}{\Lambda^2}, \frac{g\phi}{\Lambda}\right) + \cdots\right]. \tag{3.13}$$

This is because beyond-positivity contributions correspond to loop corrections within the EFT [37, 58, 65]. The one-loop NDA estimate for the $(\nabla\phi)^4$ operator is $(c_2)^{(1)} \sim \frac{1}{16\pi^2}g^4\Lambda^{-4}$, which for $g \sim \Lambda/M_{\mathrm{Pl}}$ matches the maximal value of the r.h.s. of eq. (3.6), i.e. for $|\hat{\alpha}| \sim 1/\Lambda^2$. Likewise, we can estimate the beyond-positivity contributions to quartic curvatures operators from $L^{(1)}$ in eq. (3.13), $(\alpha_4)^{(1)}/M_{\mathrm{Pl}}^2 \sim \frac{1}{16\pi^2}(\Lambda M_{\mathrm{Pl}})^{-4}$, which coincides with the r.h.s. of eq. (3.11) for the maximum value of the GB/CS coupling.

This discussion brings us to the important realization that, from the EFT standpoint, for UV completions where both the scalar-GB/CS term in eq. (1.1) and the operators in eq. (3.1) are generated at the same (tree-level) order, one should expect much larger coefficients for the latter than what discussed above. To see this, let us simply fix the power counting from the GB/CS term, assuming the maximal regime of validity of the EFT, $|\hat{\alpha}| \sim 1/\Lambda^2$ (a requirement, rather than a choice, if one is interested in phenomenological applications, see section 4). From $L^{(0)}$ in eq. (3.13), this sets $g \sim 1/|\hat{\alpha}|\Lambda M_{\mathrm{Pl}} \sim \Lambda/M_{\mathrm{Pl}}$, corresponding to a bona-fide UV completion of gravity, as discussed in section 2.2. Then, generic EFTs will feature

$$\alpha(\tilde{\alpha}) \sim \frac{1}{\Lambda^2}\,, \quad \alpha_3 \sim \frac{1}{\Lambda^4}\,, \quad \alpha_4, \alpha_4' \sim \frac{1}{\Lambda^6}\,, \quad c_2 \sim \frac{1}{\Lambda^2 M_{\mathrm{Pl}}^2}\,, \quad d_1 \sim \frac{1}{\Lambda^4}\,, \tag{3.14}$$

for the coefficients of the operators in eq. (3.1).

In the next section we investigate the phenomenological consequences of these estimates for the physics of black holes with scalar hair.

4 Phenomenological implications

In this section we discuss the main implications on the phenomenology of black holes of the upper bound on the GB/CS coupling $|\hat{\alpha}| \lesssim 1/\Lambda^2$, along with the implications associated with the additional EFT corrections that are expected from saturating such a bound, eq. (3.14). Our focus is on astrophysical black holes, in particular those detectable by LIGO-Virgo, which have sizes of a few solar masses, corresponding to Schwarzschild radii $r_s \gtrsim 10\,\mathrm{km}$.

We will discuss separately the scalar-GB and dynamical-CS gravity theories, reviewing in each case their imprints on the physics of black holes as well as the current experimental bounds on the couplings α and $\tilde{\alpha}$, respectively.

4.1 Black holes in scalar-GB gravity

From a perturbative point of view, one can argue that the metric of a hairy black hole still displays a horizon, characterized by a linear zero of the metric, much like in the Schwarzschild and Kerr cases. The effects due to the BGR dynamics rapidly vanish far away from the horizon ($r = r_s$), following the fall-off of the GB invariant, which sources both the scalar hair and the deviations from GR in the metric.

More in detail, in the static and spherically symmetric case, we have the metric

$$ds^2 = -h(r)dt^2 + f(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\,, \tag{4.1}$$

with

$$h(r), f(r) \sim \left(1 - \frac{r_s}{r}\right) \quad \text{for} \quad r \sim r_s\,. \tag{4.2}$$

At leading order in the dimensionless expansion parameter α/r^2, the GB invariant is the one of the Schwarzschild solution,

$$\mathcal{R}^2_{\rm GB} \equiv R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2 \sim \frac{r_s^2}{r^6}\,. \tag{4.3}$$

The scalar equation of motion reads

$$\Box\phi = M_{\rm Pl}\alpha\mathcal{R}^2_{\rm GB} \sim \frac{M_{\rm Pl}^2}{\Lambda_\alpha^3}\frac{r_s^2}{r^6}\,, \tag{4.4}$$

where in the last step we have traded the GB coupling α for the strong coupling scale Λ_α, given in eq. (2.15) ($\tilde{\alpha} = 0$). The scalar field profile is then completely determined by requiring that invariant quantities built out of it do not diverge for $r \geq r_s$ [66]. At asymptotically large distances, $r \to \infty$, the solution behaves as $\phi(r) \sim 1/r$. In addition, let us note that the largest value of the scalar radial derivative is estimated as,

$$\phi' \lesssim \frac{M_{\rm Pl}^2}{(\Lambda_\alpha r_s)^3}\,. \tag{4.5}$$

A first bound on the GB coupling comes from the requirement of the existence of real solutions for the scalar profile. In the simple EFT eq. (1.1), this condition requires that

$\alpha^2 < r_s^4/192$ [2]. The constraint is however dependent on additional EFT corrections from eq. (3.1) [67].

In order to estimate the impact of the scalar-GB operator on the background geometry, we can compute its ratio to GR, that is to $M_{\rm Pl}^2\mathcal{R}$ where $\mathcal{R} = \sqrt{R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}}$ is the typical curvature. Evaluating both terms on the background given by the Schwarzschild metric, $\mathcal{R} \sim r_s/r^3$, and the scalar solution from eq. (4.4), one finds [67]

$$\varepsilon_0(r) = \frac{M_{\rm Pl}\alpha\phi\mathcal{R}_{\rm GB}^2}{M_{\rm Pl}^2\mathcal{R}} \sim \left(\frac{\alpha}{r^2}\right)^2 . \tag{4.6}$$

Turning to perturbations, let us start by noting that the theory eq. (1.1) has no scalar self-interactions. Therefore, there is simply no possible screening effect associated to classical non-linearities. On the other hand, there is no direct coupling between the scalar field and matter, therefore no screening mechanism is required to have agreement with fifth-force constraints (if the theory is valid at the scales of those experiments).

Instead, the scalar-GB term gives rise to a kinetic mixing between scalar and graviton (the same leading to the causality bound of section 2.1), schematically of the form

$$M_{\rm Pl}\alpha\phi\mathcal{R}_{\rm GB}^2 \supset \varepsilon_{\rm mix}(r)\,\partial\phi\partial h\,, \tag{4.7}$$

where we are taking the fluctuations to be canonically normalized. This effect will be important when the mixing

$$\varepsilon_{\rm mix}(r) \sim \frac{\alpha r_s}{r^3}\,, \tag{4.8}$$

becomes of order one. Note that the two estimators of the BGR effects are related to each other, namely

$$\varepsilon_0 \sim \left(\frac{r}{r_s}\varepsilon_{\rm mix}\right)^2 . \tag{4.9}$$

In this scenario, a sizeable deviation of the quasi-normal mode (QNM) spectrum from the GR prediction is expected, strongly affecting the waveform during the ringdown phase of a merger.

Finally, let us turn to the phenomenology of a binary system of hairy black holes, each sourcing its own scalar profile as discussed before. The dynamical nature of the system implies that, just as it happens with gravitational waves, there will also be scalar wave emission. However, the latter is now dipolar instead of quadrupolar, therefore being much less suppressed than the former. This opens a new channel of power loss during the merger, which accelerates the rate of change in the orbital period. The effect accumulates during the inspiral phase, potentially producing an observable dephasing between the measured waveform and the one predicted by GR. To date, the absence of any observed effect of this type constitutes the most stringent experimental bound on the size of the scalar-GB coupling, $\alpha \lesssim (1.2\,\text{km})^2$ [68]. In terms of the strong coupling scale, this translates into $\Lambda_\alpha \gtrsim 10^{12}\,\text{km}^{-1}$. Let us add that such a bound strictly applies only to scalar-GB gravity with all other EFT corrections, in particular those in eq. (3.1), neglected or irrelevantly small.

Considering a typical LIGO/Virgo black hole, with $r_s \sim 10\,\text{km}$, the above bound implies that the kinetic mixing is constrained to be $\varepsilon_{\rm mix} \lesssim 10^{-2}$. Furthermore, according to eq. (4.9) the effect on the background geometry is significantly suppressed, $\varepsilon_0 \lesssim 10^{-4}$. This conclusion justifies neglecting deviations from Schwarzschild background as we assumed initially.

4.2 EFT implications on scalar-GB black holes

The fact that the BGR effects on hairy black holes are relatively small, below the 10 % level, could have been anticipated from the requirement that the scalar-GB theory should be able to properly describe the black holes of interest. Indeed, as derived in section 2.1, causality sets an upper bound on the GB coupling and therefore on the size of the observable corrections relative to GR, which we denote generically with $\epsilon(r)$ — one instance being $\varepsilon_{\rm mix}$ in eq. (4.8). The BGR effects are largest near the horizon,

$$\epsilon(r_s) \sim \frac{\alpha}{r_s^2} \lesssim (\Lambda r_s)^{-2}\,, \tag{4.10}$$

where the inequality follows from causality, eq. (2.13) (with $\log(b_0\Lambda) \sim 1$ and $\tilde{\alpha} = 0$). Usefulness of the EFT requires a hierarchy between the cutoff and the relevant scales of the system. A sensible demand on the EFT is therefore that the black hole falls within the EFT regime of validity at least down to its Schwarzschild radius.[12] Therefore, BGR corrections can never become large, i.e. $\epsilon(r_s) \ll 1$. Furthermore, taking the current experimental upper bound on α as benchmark, the causality bound implies a very low maximal cutoff,

$$\Lambda \lesssim (1\,\text{km})^{-1}\,, \tag{4.11}$$

certainly much smaller than the strong coupling scale $\Lambda_\alpha \gtrsim (10^{-12}\,\text{km})^{-1}$.

Let us discuss now the additional BGR effects that could be expected from the UV completion in the form of higher-dimensional operators with coefficients fixed to eq. (3.14), where let us recall that such NDA estimates correspond to the maximal cutoff $\Lambda \sim \alpha^{-1/2} \sim (1\,\text{km})^{-1}$.

The operators in eq. (3.1) give rise to modifications of the geometry. We can estimate such modifications as in eq. (4.6) for the scalar-GB term, which we recall scales as $\varepsilon_0(r) \sim (\alpha/r^2)^2 \sim (\Lambda r)^{-4}$. Similarly, we find

$$\frac{\alpha_3\mathcal{I}}{\mathcal{R}} \sim \frac{r_s^2}{r^2}(\Lambda r)^{-4}\,, \quad \frac{\alpha_4\mathcal{C}^2}{\mathcal{R}} \sim \frac{r_s^3}{r^3}(\Lambda r)^{-6}\,, \quad \frac{c_2(\nabla\phi)^4}{M_{\rm Pl}^2\mathcal{R}} \sim \frac{r^5}{r_s^5}(\Lambda r)^{-10}\,, \quad \frac{d_1\mathcal{C}(\nabla\phi)^2}{M_{\rm Pl}^2\mathcal{R}} \sim \frac{r}{r_s}(\Lambda r)^{-8}\,. \tag{4.12}$$

While deviations introduced by the scalar-GB term are the largest, operators cubic in the Riemann tensor can become as important near the horizon. The deviations introduced by the rest of operators are subleading, being higher order in $(\Lambda r_s)^{-1}$, as expected from the derivative expansion and eq. (4.5). In addition, the operators built out of the scalar give rise to modifications of the scalar field profile, which we can estimate as

$$\frac{c_2(\nabla\phi)^4}{(\nabla\phi)^2} \sim \frac{r^2}{r_s^2}(\Lambda r)^{-6}\,, \quad \frac{d_1\mathcal{C}(\nabla\phi)^2}{(\nabla\phi)^2} \sim \frac{r_s^2}{r^2}(\Lambda r)^{-4}\,. \tag{4.13}$$

[12] As a matter of fact, one would like a gravitational EFT to be valid at least up to the scales where gravity has been experimentally tested, that is a cutoff $\Lambda \gtrsim \mu\text{m}^{-1}$ [7]. One is forced to give up on such a requirement if the scalar-GB theory is to be phenomenologically interesting for astrophysical black holes (see however the discussion in section 5).

Given the upper bound eq. (4.11), we conclude that the operators in eq. (3.1) should induce corrections on the metric and scalar of up to 0.01 % near the horizon of black holes with $r_s \sim 10\,\mathrm{km}$.

There are many other interesting signatures associated with the operators that we expect to be present in the scalar-GB EFT. The phenomenology of cubic and quartic curvature operators have been discussed in [69–73] and [45–47, 73], respectively. Besides modifications of the Schwarzschild (and Kerr) geometries, these include deviations from GR at the leading order in QNMs, quadrupole moments, non-vanishing Love numbers, and corrections to the gravitational-wave signals at relatively high post-Newtonian (PN) order. Since such corrections start at order $(\Lambda r)^{-4} \lesssim 10^{-4}$, they will not be easy to probe with the sensitivity of current experiments.

Operators involving the scalar field have received less attention in the literature. The impact of the cubic Galileon operator, which we have rewritten in eq. (3.1) as $d_1 R^2_{\mu\nu\rho\sigma}(\nabla_\eta\phi)^2$, has been discussed in [67]. Along with the operator $c_2(\nabla_\mu\phi)^4$, these EFT terms introduce modifications e.g. in the scalar and gravitational wave spectrum, as well as in the QNMs, predicted by the pure scalar-GB theory eq. (1.1), although a priori subleading due to the suppression by higher powers of $(\Lambda r)^{-2} \ll 1$. In particular, note that potential screening effects are not likely to be significant.

Let us recall once again that these conclusions appear to be a robust consequence of causality. Nevertheless, quantitative predictions for the gravitational observables, which typically require performing costly numerical simulations, are still interesting, if only to experimentally test the fundamental principles behind these expectations.

4.3 Black holes in dynamical-CS gravity

The pseudo-scalar-CS term in eq. (1.1) (with $\alpha = 0$) gives rise to a phenomenology of black holes similar to that discussed in the previous section, although with a few important differences. First, the Pontryagin invariant, $R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}$, vanishes in the Schwarzschild geometry, while it is non-zero for the Kerr geometry. Therefore, one needs to consider rotating black holes in order to have a non-vanishing scalar hair. To simplify the analysis, we follow [3] and treat the spin parameter of the black hole, a/r_s, perturbatively. We also work in an expansion in $\tilde{\alpha}/r_s^2$ since, similar to discussion for scalar-GB black holes in section 4.2, this is a consequence of causality, $\tilde{\alpha} \lesssim 1/\Lambda^2$, along with the requirement that the black holes of interest fall within the regime of validity of the EFT, i.e. $(\Lambda r_s)^{-2} \ll 1$.

At leading order, the equation of motion for the pseudo-scalar ϕ is given by

$$\Box\phi = M_{\rm Pl}\tilde{\alpha} R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma} \sim \frac{M_{\rm Pl}^2}{\Lambda_{\tilde{\alpha}}^3}\frac{r_s^2}{r^6}\frac{a}{r}\cos\theta\,, \tag{4.14}$$

where in the last step we have traded the CS coupling $\tilde{\alpha}$ for the strong coupling scale $\Lambda_{\tilde{\alpha}}$, given in eq. (2.15) ($\alpha = 0$ and changed subscript to avoid confusion). Subleading terms in the spin parameter scale as $(a/r_s)^3$. From the solution to this equation (see e.g. [3]), we can compute the scalar radial derivative, which roughly satisfies

$$\phi' \lesssim \frac{M_{\rm Pl}^2}{(\Lambda_{\tilde{\alpha}} r_s)^3}\frac{a}{r_s}\,. \tag{4.15}$$

Similarly to scalar-GB case, there is an approximate relation between corrections to the Kerr geometry, ε_0, and the kinetic mixing between pseudo-scalar and graviton, $\varepsilon_{\rm mix}$, given by

$$\varepsilon_0(r) \sim \left(\frac{a}{r_s}\varepsilon_{\rm mix}(r)\right)^2 . \tag{4.16}$$

A second difference w.r.t. the scalar-GB case is the nature of the most stringent experimental bounds on the CS coupling. Since the scalar background sourced around an isolated (spinning) black hole is dipolar, the emission of scalar waves from a black hole binary system starts from a quadrupole moment. Therefore, energy loss via scalar emission is further suppressed in the PN expansion compared to the scalar-GB case, such that no constraint can be derived on $\tilde{\alpha}$ given current sensitivities [74].

The strongest bound to date on the pseudo-scalar-CS coupling comes from independent measurements of the tidal deformability and of the moment of inertia in neutron stars [75]. The comparison between these measurements and the values predicted in dynamical-CS gravity yields the bound $\tilde{\alpha} \lesssim (8\,\text{km})^2$. Note this is weaker than the most stringent bound on the scalar-GB coupling, $\alpha \lesssim (1.2\,\text{km})^2$ by one order of magnitude. Nevertheless, if the pseudo-scalar-CS EFT is to be able to describe black holes with sizes down to $r_s \sim 10\,\text{km}$ (recall that the smallest black holes display the largest BGR effects, which in any case cannot become $O(1)$) with at least $10\,\%$ accuracy, it seems wise to consider a different benchmark for the coupling $\tilde{\alpha}$. Maximal testability compatible with causality then suggests to take $\tilde{\alpha} \sim 1/\Lambda^2 \sim (3\,\text{km})^2$.

In this case, the kinetic mixing between pseudo-scalar and graviton can induce stronger deviations in the QNM spectrum w.r.t. to GR [76–78], of order $\varepsilon_{\rm mix} \lesssim (\tilde{\alpha}/r_s^2) \sim 10^{-1}$ for black holes with $r_s \sim 10\,\text{km}$.

The discussion of the implications of the additional operators in eq. (3.1), present in generic UV completions of pseudo-scalar-CS theory, largely parallels that of section 4.2 and we do not repeat it here. Nevertheless, we wish to point out that to date much less work has been devoted to the study of these EFT effects for rotating black holes, see e.g. [71, 72].

Before closing the section, let us point out one last difference between the scalar-GB and dynamical-CS theories. While the scalar-GB operator leads to equations of motion of at most second order in (time) derivatives, the dynamical-CS operator gives rise to higher-derivative terms. These in principle could spoil the quantum stability of the theory. Considering perturbations of rotating black holes with pseudo-scalar-CS hair, for instance during the inspiral phase of a merger,[13] higher derivatives will become important at a mass scale $M_g^{-1} \sim \tilde{\alpha}\dot{\phi}_0/M_{\rm Pl}$, being $\dot{\phi}_0 \sim \omega\phi_0 \lesssim \phi_0/r_s$ the time derivative of the axionic field evaluated on the background. We can estimate this scale, at least in some appropriate regime, using the solution of eq. (4.14), finding

$$M_g \sim \frac{1}{a}\left(\frac{r_s^2}{\tilde{\alpha}}\right)^2 . \tag{4.17}$$

Since causality requires $\Lambda \lesssim \tilde{\alpha}^{-1/2}$, we find that the ghost's mass is above the EFT cutoff.

[13] This system allows us to consider non-vanishing time derivatives of the scalar background, which can lead to ghosts instabilities.

5 Summary and outlook

Gravitational wave science can potentially test deviations from General Relativity. While the EFT framework gives the best organizing principle to characterize physics beyond GR, the complexity of black hole merger events suggests that the theory space should first be reduced as much as possible before comparing data with predictions coming from different EFTs. A powerful and robust set of constraints on the space of consistent EFTs comes from the unitarity and causality of the (unknown) UV completion.

In this paper, we used causality arguments to derive phenomenologically interesting bounds on theories that, at low energies, comprise the graviton and a shift-symmetric scalar field. The presence of a coupling between the scalar and the Gauss-Bonnet or the Chern-Simons operators leads to black hole hair. If the couplings α and $\tilde{\alpha}$, in the notation of eq. (1.1), are large enough, $|\hat{\alpha}| \sim r_s^2$, hair can be measured in astrophysical black holes. The first consequence that we derived imposing the absence of a (resolvable) time advance is that the cutoff of the EFT cannot be parametrically larger than $|\hat{\alpha}|^{-1/2}$, i.e. $1/\,\mathrm{km}$ for solar mass BHs. This has implications for the structure of all the higher-dimensional operators in the theory. One could attempt to draw robust lower bounds on their coefficients using positivity constraints obtained via dispersion relations. However, the weakness of non-minimal gravitational interactions compared to GR, enforced by causality, implies that lower bounds from one-loop dispersion relations are phenomenologically irrelevant.

On the other hand, for such a low cutoff $\Lambda \sim 1/\,\mathrm{km}$, we used general power counting arguments to show that if both the scalar-GB/CS term and other operators are generated at the same (tree-level) order, the latter will also give sizable contributions in black hole dynamics.

The result that the UV cutoff of a phenomenologically interesting and causal EFT describing gravity must be lower than km^{-1} should not be considered only as a source of potentially large corrections to the effective description of astrophysical black holes. Instead, we should demand that this result is reconciled with our experimental knowledge of gravity. As a matter of fact, to date gravity has been probed in table-top experiments down to the scales of tens of microns in length, showing good agreement with Newtonian theory [7]. In light of this, one should at least prefer, if not demand, that GR is extended using a theory valid down to the μm, in such a way to describe the same observations as GR does.

In the scenarios studied above, being the cutoff at a much lower scale than $\mu\mathrm{m}^{-1}$, one needs to trust that the UV completion that gives rise either to the scalar-GB or to the dynamical-CS EFTs, does indeed reproduce the Newtonian potential at microscopic lengths. Similarly to what was argued in [45, 47] for quartic curvature terms, this might be the case of a “soft” UV completion that resolves the irrelevant operators in the EFT in such a way that interactions stop growing with the energy.

In addition to this requirement, we need the UV completion to contribute to the time delay in such a way that causality is preserved, in particular given the negative contributions (i.e. the time advance) from the low-energy operators.

This requirement is a substantial obstacle for both the scalar-GB and dynamical-CS EFTs.[14] Indeed, since the sizes of both the scalar-GB and of the axion-CS operators are chosen in such a way to follow the tree-level NDA, it appears difficult for loop-level effects in the UV physics to restore causality, unless the number of species scales as $(4\pi/g)^2 \sim (4\pi M_{\rm Pl}/\Lambda)^2$ [28, 29]. On the other hand, as it was argued in [9, 27], in order for causality to be restored by tree-level exchanges in the UV, one must introduce an infinite tower of higher-spin particles having a mass of order $M \sim \text{km}^{-1}$. In both these cases, the description of gravity below the km would be very different from what we know.

In light of this, there seem to be two reasonable attitudes. The first is to resign to the idea that both scalar-GB and pseudo-scalar-CS interactions cannot lead to testable modifications of GR. The second is trying to understand if there exists a UV completion of these models that restores causality without clashing with our knowledge of gravity at small distances. In any case, the detection of black hole hair would be revolutionary, telling us there is something fundamentally unexpected and so far unknown about gravitational dynamics.

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A Gauss-Bonnet scalarization

For scalar-tensor theories with no scalar shift-symmetry, one can consider non-linear couplings between ϕ and the GB invariant. Let us focus on the leading Z_2-symmetric ($\phi \to -\phi$) term in a field and derivative expansion,

$$S_{\rm GBization} = \int d^4x\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\nabla_\mu\phi)^2 + \lambda\phi^2\mathcal{R}_{\rm GB}^2\right), \tag{A.1}$$

where λ has dimensions of a length square. The sign of λ determines whether $\phi = 0$ is a stable solution or not around a gravitational source like a (static or spinning) black hole [79]. Spontaneous scalarization, i.e. a non-trivial scalar profile, generically develops for $|\lambda|/r_s^2 \gtrsim 1$, with $\lambda > 0$ provided the GB invariant is positive.[15]

From simple little group (helicity) selection rules, one can derive the 4-point interaction of two scalars and two gravitons associated with the non-minimal coupling to GB,

$$\mathcal{M}^{\rm GBization}_{1_\phi 2_\phi 3_{h^{++}} 4_{h^{++}}} = \frac{4\lambda}{M_{\rm Pl}^2}[34]^4\,, \quad \mathcal{M}^{\rm GBization}_{1_\phi 2_\phi 3_{h^{--}} 4_{h^{--}}} = \frac{4\lambda}{M_{\rm Pl}^2}\langle 34\rangle^4\,. \tag{A.2}$$

[14] As a matter of fact, consistency with unitarity and causality is an obstacle as well for any BGR deformation of phenomenological relevance above the μm, in particular if it involves higher-order terms in the curvature [11].

[15] This is the case e.g. on a static black hole background or away from the horizon on a rotating black hole background.

Note that this is associated with an inelastic $\phi h^{++} \to \phi h^{--}$ scattering amplitude, that vanishes in the forward limit $t \to 0$,

$$\mathcal{M}^{\text{GBization}}_{1_\phi 2_{h^{++}} \to 3_\phi 4_{h^{--}}} = \frac{4\lambda}{M_{\text{Pl}}^2} t^2 e^{4i\theta} \,, \tag{A.3}$$

where θ is just a phase (for physical momenta $[ij]^* = \langle ij \rangle = \sqrt{s_{ij}} e^{i\theta}$, with $s_{13} = t$).

While a dispersion relation for λ from the 2-scalar-2-graviton amplitude can be derived along the lines of section 3.1, the inelasticity of the amplitude precludes the derivation of a positivity bound $\lambda > 0$. One can actually come up with a simple (yet partial) tree-level UV completion that shows that the sign of λ is not fixed. This involves an additional massive scalar field Φ,

$$S^{\text{UV}}_{\text{GBization}} = \int d^4x \sqrt{-g} \left(\frac{1}{2} M_{\text{Pl}}^2 R - \frac{1}{2} (\nabla_\mu \phi)^2 - \frac{1}{2} (\nabla_\mu \Phi)^2 - \frac{1}{2} m_\Phi^2 \Phi^2 + M_{\text{Pl}} \alpha_\Phi \Phi \mathcal{R}^2_{\text{GB}} + g_\Phi \Phi \phi^2 \right) . \tag{A.4}$$

One can see that upon integrating out the massive scalar, one obtains

$$\lambda = \frac{M_{\text{Pl}} \alpha_\Phi g_\Phi}{m_\Phi^2} \,, \tag{A.5}$$

with no definite sign, since e.g. α_Φ can consistently be positive or negative. In addition, from this example one can infer that the size of λ is likely to be theoretically bounded. Indeed, since eq. (A.4) is itself an effective action, from the generic EFT perspective presented in section 2.2, the trilinear coupling is of order $g_\Phi \sim g\Lambda$ while $\alpha_\phi \sim 1/(g M_{\text{Pl}} \Lambda)$, where Λ is the cutoff and g a coupling. The mass of the heavy scalar can be at most of the order of the cutoff, i.e. $m_\Phi \sim \Lambda$, and should itself be identified with the cutoff of eq. (A.1). We are then led to the conjecture that the quadratic scalar-GB coupling should not be much larger than $\lambda \sim 1/\Lambda^2$ given the causality bound on α_Φ. Following similar arguments, one can start with the effective interaction $\lambda \phi^2 \mathcal{R}^2_{\text{GB}}$ and give the scalar a vacuum expectation value, which within the EFT can be at most $\langle \phi \rangle \sim \Lambda/g \lesssim M_{\text{Pl}}$. This gives rise to the scalar-GB term in eq. (1.1) with $M_{\text{Pl}} \alpha \sim \lambda \langle \phi \rangle$, which for $\lambda \lesssim 1/\Lambda^2$ is consistent with the causality bound we derived in section 2.1. Note that, at the end of the day, these arguments are just refined versions of the statement that, from the gravitational power counting discussed in section 2.2, we expect $\lambda \sim \zeta^2/\Lambda^2 \lesssim 1/\Lambda^2$ for $\zeta \lesssim 1$. However, in this case one cannot reinterpret such expectation as a consequence of the requirement that the BGR amplitude, eq. (A.2), should not become larger than GR's within the EFT, since the latter vanishes at tree level (and at one loop) [65].

Besides, one can construct a dispersion relation for the 4-scalar amplitude along the lines of section 3.1. In this case, the beyond positivity bound corresponding to eq. (A.1) is associated with a cross section for $\phi\phi \to h^{\pm\pm} h^{\pm\pm}$ which, similar to eq. (3.6), leads to $c_2 \gtrsim \frac{1}{16\pi^2} \lambda^2 (\Lambda/M_{\text{Pl}})^4$. If indeed $\lambda \lesssim 1/\Lambda^2$ regardless of the UV completion as long as this is unitary and causal, then the lower bound on c_2 is in fact inapplicable due to the graviton pole.

Finally, from the phenomenological point of view, scalarization of black holes turns out to be a dubious phenomenon, given that $\lambda/r_s^2 \lesssim (\Lambda r_s)^{-2} \lesssim 1$ if the EFT is to describe the black holes down to their horizon. However, let us recall that, differently from the cases discussed in the main text, for the theory of GB-scalarization we have not found solid evidence that causality forces the BGR effects to be subleading.

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