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Tackling the SDC in AdS with CFTs

Florent Baume^{a,b} and José Calderón Infante^a

^a*Instituto de Física Teórica UAM-CSIC,
Cantoblanco, 28049 Madrid, Spain*

^b*Department of Physics and Astronomy, University of Pennsylvania
Philadelphia, PA 19104, U.S.A.*

E-mail: fbaume@sas.upenn.edu, j.calderon.infante@csic.es

ABSTRACT: We study the Swampland Distance Conjecture for supersymmetric theories with AdS_5 backgrounds and fixed radius through their $\mathcal{N} = 2$ SCFT holographic duals. By the Maldacena-Zhiboedov theorem, around a large class of infinite-distance points there must exist a tower of exponentially massless higher-spin fields in the bulk, for which we find bounds on the decay rate in terms of the conformal data. We discuss the origin of this tower in the gravity side for type IIB compactification on S^5 and its orbifolds, and comment about more general cases.

KEYWORDS: Conformal Field Theory, Superstring Vacua, AdS-CFT Correspondence

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1 Introduction

Quantum gravity constraints on effective field theories have recently come under renewed scrutiny, and several proposals have emerged under the umbrella of the Swampland Programme [1, 2], see [3, 4] for reviews. Among them, the Swampland Distance Conjecture (SDC) [2] is arguably — with the Weak Gravity Conjecture [5] — one of the most studied facets of this endeavour. It posits that at points in moduli space located at infinite geodesic field distance from an arbitrary reference point, there exists a tower of infinitely many states becoming exponentially light with the geodesic distance,

$$\frac{m}{M_{\text{Pl}}} \sim e^{-\alpha_G \cdot \text{dist}_G}, \quad \text{as } \text{dist}_G \rightarrow \infty, \quad (1.1)$$

leading to a breakdown of the effective theory. The distance is computed with respect to the metric of the moduli space, G , and the associated exponential decay rate, $\alpha_G > 0$, is moreover expected to be of order one in Planck units. For type IIB string theory on a Calabi-Yau three-fold, it has been possible to put specific lower bounds on α_G by connecting the tower to BPS states [6]. Similar lower bounds have also been proposed in conjunction to the Transplanckian Censorship Conjecture [7–9], and with the RG flow of BPS strings in 4d $\mathcal{N} = 1$ EFTs [10].

The SDC has been mainly studied in the context of four- [11–18] or six-dimensional [19] Minkowski theories with eight or more supercharges obtained by dimensional reduction of type II string theories, or their lifts to strong coupling. Using the beautifully-intricate web

of dualities of string theory, it was proposed that the tower of massless states corresponds to either a decompactification limit or a tensionless weakly-coupled fundamental string in disguise [14–16], although it may be required to take quantum corrections into account to make them manifest [16, 20–22]. Note that, in the latter case, the tower of states generically contains arbitrarily large higher-spin fields. See [23] for implications in (quasi-)dS spaces.

A variation of this framework is the inclusion of a potential [16, 24–36], which may lift the flat spacetime geometry to an AdS space. In the limit of large AdS radius in Planck units, $LM_{\text{Pl}} \rightarrow \infty$, a similar behaviour is expected, with an infinite tower of states also becoming massless, behaving as $m/M_{\text{Pl}} \sim (LM_{\text{Pl}})^{-\alpha}$, $\alpha > 0$ [37]. In supersymmetric cases a strong version of the conjecture suggests $\alpha = \frac{1}{2}$, usually interpreted as a consequence of the no-scale-separation condition between the internal manifold and the AdS radius. In string-theoretic realisations of these AdS geometries, the tower is often identified with a sector of Kaluza-Klein modes. Part of the internal manifold and the AdS space are stabilised by the same fluxes and, as a consequence, the AdS radius and a breathing mode of the compact space are linked together. The limit of large radius will then also lead to a decompactification. For recent works, see [38–42].

This proposal is somewhat different from what one would naively call the Swampland Distance Conjecture for moduli spaces of AdS vacua. Even though it is exploring the possible AdS vacua of the theory, it is not about the continuously-connected part parametrised by massless scalars, which we refer to in this work as the moduli space. It is rather about the different branches of vacua parametrised by massive scalars. In string theory constructions, the presence of fluxes will give masses to the scalars controlling these limits and can therefore no longer be considered as moduli in the usual sense. Typically one considers different branches of vacua in this setup by changing the flux quanta.

This raises the question of whether the SDC extends to moduli spaces of AdS vacua in the sense described above, and what kind of towers of states can be expected to appear. In those setups, the AdS scale in Planck units, LM_{Pl} , remains fixed throughout all the moduli space. This is the kind of trajectories we want to tackle in this work.

In this context, an open question is whether it is possible to consider decompactification limits. Such trajectories would imply the possibility of tuning the size of an internal dimension without changing the AdS radius at all. Current models featuring a separation of scales always link the AdS radius and the internal dimensions in some way, while the limits we are interested in would require them to be independent. Although inconclusive, current understanding of AdS vacua seems to disfavour such trajectories and leads to the intriguing possibility that equi-dimensional and non-equi-dimensional limits in AdS are distinguished, being called SDC and ADC directions, respectively.

In the case of equi-dimensional limits it is reasonable to expect the appearance of tensionless strings. However, an immediate challenge one faces is that those points are out of the regime of parametric control of the usual supergravity description of these vacua. Indeed, the tension of these strings will eventually fall below the AdS scale, leading to the notoriously difficult problem of quantising strings in highly-curved backgrounds. To retain control over the theory one conversely assumes a weakly-curved background, corresponding to a semi-classical approximation. A possible way to go around the issue is to make use of

the AdS/CFT correspondence [43]. Our aim here is to analyse a possible extension of the SDC by studying the evolution of physical quantities through their CFT duals.

Using holography as a tool to study the Swampland programme has already bore fruits. Proofs of no-go theorems applied to global symmetries as well as the Weak Gravity Conjectures in AdS were established by relating black hole quantities to conformal data [44–46]. More recently, positivity bounds were related to moduli stabilisation constraints in AdS₄/CFT₃, as well as possible connections to the SDC [47]. Closer to our setup, the classical moduli space has further been shown to be a coset in the case of AdS₅ gauged supergravity with sixteen (real) supercharges [48].

Moduli and marginal deformations. In AdS/CFT, each field of mass m in the bulk is associated on the boundary to a conformal operator of dimension Δ . The dictionary between scalars and ℓ -symmetric traceless tensors is given in AdS₅/CFT₄ by:

$$m^2 L^2 = \Delta(\Delta - 4), \quad (\text{scalars}); \quad (1.2)$$

$$m^2 L^2 = (\Delta + \ell - 2)(\Delta - \ell - 2), \quad (\ell\text{-symmetric traceless tensors}). \quad (1.3)$$

The variation of any mass as a function of the moduli in the bulk can thus be controlled by tuning parameters of the CFT. We note that, as we demand the AdS radius, L , to be fixed in Planck units, we can use these expressions to evaluate the mass of a given state in Planck units up to some numerical coefficients which are irrelevant to our analysis.

The moduli space, parametrised by the vacuum expectation value (vev) of the moduli, z^i , is then identified with the conformal manifold, the space of exactly marginal deformations, λ^i , of the CFT, see e.g. [49, 50]. This conformal manifold is endowed with the so-called Zamolodchikov metric, χ , that is the dual of the bulk moduli space metric, G :

$$(\mathcal{M}_{\text{mod}}, G_{ij}(z)) \longleftrightarrow (\mathcal{M}_{\text{CFT}}, \chi_{ij}(\lambda)). \quad (1.4)$$

More specifically, the Zamolodchikov metric is mapped to the metric in moduli space measured in AdS units up to a constant prefactor, such that in Planck units we have:

$$(LM_{\text{Pl}})^3 G_{ij}(z) \sim \chi_{ij}(\lambda). \quad (1.5)$$

The specific form of the metric can be computed as a series in large N , dual to the weakly-coupled quantum-gravity expansion in the bulk [51]. As the partition function of the bulk on the boundary is identified with the generating functional of correlation functions of the CFT, corrections on either side are guaranteed to match on the other. This means that we can use (1.5) to compute, at least in principle, the metric in moduli space from the Zamolodchikov metric in any regime of the theory.

As it is well known, unitarity furthermore imposes constraints on the CFT data. For instance the dimension is bounded from below, and an operator with spin ℓ must satisfy:

$$\begin{aligned} \Delta &\geq 1, & \ell &= 0; \\ \Delta &\geq \ell + 2, & \ell &> 0. \end{aligned} \quad (1.6)$$

For $\ell = 0$ the bound is saturated by a free scalar field and maps to tachyonic fields in the bulk, while the flavour currents and the energy-momentum tensor sit at the bound for $\ell = 1, 2$, respectively. For $\ell > 2$, the bound is associated to so-called higher-spin conserved currents. By the Maldacena-Zhiboedov theorem [52] and its extensions [53–57], having a single higher-spin conserved current implies the existence of an infinite number of them. Moreover this can only occur in the presence of (generalised) free fields. Such higher-spin currents will be a central part of this work.

With this framework, the study of the Swampland Distance Conjecture for AdS space-time can then be rephrased as an analysis of the possible infinite-distance points of conformal manifolds. We will focus on four-dimensional $\mathcal{N} = 2$ SCFTs, corresponding to supergravity theories with sixteen (real) supercharges on AdS_5 , although we will comment on implications in other dimensions. In those theories, $(\mathcal{N} = 2)$ -preserving exactly marginal deformations can only correspond to variations of (complexified) gauge couplings [58, 59], and it is then easy to give a physical interpretation to the towers of states in terms of gauge data.

In particular, we will be able to track how the dimension of certain operators behaves near a class of infinite-distance points corresponding to weakly-coupled gauge subsectors of the SCFT. Among them, we find the above-mentioned infinite tower of higher-spin currents that saturate the unitarity bound (1.3) in the presence of free fields. In the bulk, this leads to an infinite tower of higher-spin fields becoming massless and satisfying the condition (1.1). We will further estimate the decay rate, that will be shown to be *at least* of order one.

This work is structured as follows: in section 2 we describe how to use AdS/CFT to study the SDC in the bulk by working in the moduli space of $\text{AdS}_5 \times S^5$ vacua of type IIB string theory. The high degree of supersymmetry is enough to compute the metrics exactly and therefore constitutes the simplest example to study. In section 3 we review some important properties of the conformal manifolds associated to four-dimensional $\mathcal{N} = 2$ SCFTs and study the SDC on general grounds. In section 4 we make contact again with the bulk by examining a specific family of $\mathcal{N} = 2$ SCFTs with known bulk duals. We give our conclusions and discuss applications to other dimensions in section 5. Additionally, we briefly review unitary representations of the $\mathcal{N} = 2$ superconformal algebra in appendix A.

Note added. When finalising the preparation of this article, we became aware of an upcoming work by Perlmutter, Rastelli, Vafa, and Valenzuela [60], which explores similar ideas.

2 A warm-up: type IIB $\text{AdS}_5 \times S^5$ vacua

As a first example and to set our nomenclature and conventions, we consider the family of AdS_5 vacua obtained by compactifying Type IIB on S^5 with N units of F_5 -flux. It is the most celebrated example of the holographic principle, being dual to $\mathcal{N} = 4$ super-Yang-Mills theory with gauge group $\text{SU}(N)$ in four dimensions. Due to the high amount of supersymmetry, non-renormalisation theorems makes it possible to compute relevant quantities exactly.

This case will be useful to exemplify the inclusion of the moduli space into the scalars manifold, including the stabilised scalar fields. It will make a clear distinction between the limits we want to explore and the “ADC directions”, where the AdS scale, L , is allowed to vary [37]. It will also serve to illustrate the issues of parametric control that occur when approaching infinite-distance points in AdS moduli space and how the dual CFT picture allows one to circumvent them.

This family of solutions is parametrised by N and the complex axio-dilaton,

$$\tau = C_0 + i \frac{1}{g_s}, \quad (2.1)$$

made out of the string coupling, g_s , and the Type IIB axion, C_0 . The AdS radius, L , is forced to coincide with the radius of the five-sphere and is set to:

$$L^4 = 4\pi g_s N \alpha'^2 = \frac{1}{4\pi^3} g_s N M_s^{-4}. \quad (2.2)$$

In terms of the five-dimensional Planck mass, it is rewritten as

$$L M_{\text{Pl}} \sim N^{2/3}, \quad M_{\text{Pl}} \sim g_s^{-1/4} N^{5/12} M_s, \quad (2.3)$$

so that keeping the AdS scale fixed in Planck units corresponds to fixing N . Thus, the moduli space of AdS vacua is parametrised solely by the axio-dilaton.

From the perspective of compactification, this is understood as a stabilisation of the scalar associated to the breathing mode of the sphere through fluxes, such that it is no longer a modulus. Note that with the nomenclature established in the introduction, this stabilised scalar is associated to the ADC direction and not part of the moduli space of massless scalars.

Computing the moduli space metric usually requires one to perform the dimensional reduction of type IIB supergravity on S^5 , including kinetic terms for the axio-dilaton and the breathing mode, and substitute (2.2).¹ However, we can here take full advantage of type IIB S-duality which is preserved in this background, and constrains the metric to be:

$$ds^2 \sim \frac{d\tau d\bar{\tau}}{\text{Im}(\tau)^2}, \quad (2.4)$$

up to numerical factors that will be irrelevant for our purposes. It is then obvious that there are only two infinite-distance points: $\text{Im}\tau \rightarrow 0, \infty$, which are physically equivalent. We focus the rest of the discussion on the latter.

One could naively expect the SDC to work exactly as it does in flat space: using the metric (2.4) any geodesic approaching $\text{Im}\tau \rightarrow \infty$ is forced to move only along the $\text{Im}\tau$ -direction, and as a consequence the distance behaves logarithmically with g_s . The associated tower of states is then identified with string excitations controlled by the string scale, M_s , which falls polynomially to zero in Planck units. Putting the two together, we find the expected exponential behaviour.

¹According to the generalised SDC [37], this should also include the contribution to the distance due to the change of the AdS scale, which is irrelevant here as it is kept fixed.

However, for this argument to work a key point is to remain under parametric control along the trajectory. In particular the quantisation of the string with the usual methods requires one to be in the *weakly-curved* regime. This imposes the string scale to be above the AdS scale:

$$LM_s \sim (g_s N)^{1/4} \gg 1. \quad (2.5)$$

As the AdS scale is fixed along the trajectory, we are no longer under parametric control as $\text{Im}\tau \rightarrow \infty$. This extra condition does not arise when considering the moduli spaces of Minkowski vacua such as those considered in the usual compactification to flat backgrounds.

We are therefore leaving the phase of the moduli space where the supergravity description is valid, making a qualitative assesement of the SDC impossible, as the two infinite-distance points, $\tau = 0, i\infty$, are both inaccessible in that regime. However, we are conversely entering a phase where a weakly-coupled description in terms of the conformal theory is appropriate. There, the bulk axio-dilaton is identified with the complexified gauge coupling,

$$\tau = \tau_{\text{YM}} = \frac{\theta}{2\pi} + i \frac{4\pi}{g_{\text{YM}}^2}, \quad (2.6)$$

and parametrises the only possible ($\mathcal{N} = 4$)-preserving marginal deformation. Due to the amount of supersymmetry, the Zamolodchikov metric, χ , is found to be quantum exact and can be computed through usual diagrammatic methods, or by localisation techniques reviewed next section:

$$\chi_{\tau\bar{\tau}} \sim \frac{N^2 - 1}{\text{Im}(\tau)^2}. \quad (2.7)$$

The numerator is set by the dimension of the gauge group $\mathcal{G} = \text{SU}(N)$, and we once again ignored irrelevant order one prefactors. As expected from the bulk, there are also two physically-equivalent infinite-distance points related by S-duality, and the bulk limit $\text{Im}\tau \rightarrow \infty$ corresponds to a free theory, $g_{\text{YM}} = 0$, on the CFT side.

The operators of the CFT are gauge-invariant composite operators made out of fields in the $\mathcal{N} = 4$ vector multiplet.² Their conformal dimensions are given by the sum of the free value and their anomalous dimension, γ :

$$\Delta = \Delta_{\text{free}} + \gamma(\tau). \quad (2.8)$$

In the free limit, the conformal dimension is obtained by naive dimensional analysis. For instance, the lowest-lying operators is given by $\text{Tr}\phi^2$ and is of dimension $\Delta = 2$ in the limit $\text{Im}\tau \rightarrow \infty$. Using the dictionary (1.2), it therefore corresponds to a field at the Breitenlohner-Freedman (BF) bound in the bulk.

From the SDC one expects a tower of states becoming massless exponentially with the distance at the infinite-distance point. To see what happens on the CFT side, we can use perturbation theory to write the leading contribution in the $g_{\text{YM}} \rightarrow 0$ limit as

$$\Delta = \Delta_{\text{free}} + \eta g_{\text{YM}}^\beta + \mathcal{O}(g_{\text{YM}}^{\beta+1}) = \Delta_{\text{free}} + \eta e^{-\alpha_\chi \text{dist}_\chi(\tau)}, \quad (2.9)$$

²For simplicity, we do not take into consideration the R-symmetry structure of these fields, and generically denote any of the scalars transforming in the **6** of $\text{SU}(4)_R$, or later any other scalar, by ϕ . For our purpose, it will be irrelevant.

where α_χ, β, η are coefficients depending on the type of operator considered. In the second equality we have used the expression for the distance with respect to the Zamolodchikov metric in terms of the Yang-Mills coupling. We can easily see that — with the exception of operators whose dimensions are protected by a selection rule — the conformal dimension falls exponentially fast to its free value. An important class of such operators are spin- ℓ operators of the form:

$$J_{\mu_1 \dots \mu_\ell} = \bar{\phi} \overleftrightarrow{\partial}_{(\mu_1} \dots \overleftrightarrow{\partial}_{\mu_\ell)} \phi - (\text{traces}) . \quad (2.10)$$

These operators have an anomalous dimension at a generic point of the conformal manifold but — using the equations of motion — become conserved in the free limit and saturates the unitarity bound (1.6). The presence of these higher-spin conserved currents in a CFT in fact implies that the theory is free by the Maldacena-Zhiboedov theorem [52].

In the bulk they are identified with higher-spin fields that become massless exponentially fast:

$$M_\ell^2 L^2 = (\Delta + \ell - 2)(\Delta - (\ell + 2)) \sim e^{-\alpha_G \text{dist}_G(\tau)} . \quad (2.11)$$

We therefore indeed have a tower of massless modes in the bulk when going to the infinite-distance point, behaving according to the Swampland Distance Conjecture.

However, the unitarity bound (1.6) implies that fields dual to scalar operators of the CFT — e.g. single-trace operators, $\mathcal{O} \sim \text{Tr} \phi^n$ — remain massive in the bulk and are regularly spaced for sufficiently large n :

$$M_{\text{scal.}}^2 L^2 \sim n^2 + \mathcal{O}(e^{-\text{dist}(\tau)}) . \quad (2.12)$$

This is a striking difference with respect to the usual results of the SDC for Minkowski backgrounds: in this case the tower is formed by higher-spin modes, which are in principle interacting,³ but there are only a small number of massless scalar fields. In flat space, the tower always contains an infinite number of massless scalars. These residual masses in our setup are likely related to the presence of curvature and fluxes.

The origin of the tower is however clear: as $g_s \rightarrow 0$, the higher-spin fields are those expected from a tensionless fundamental string. The density of the tower is moreover linear, $M_\ell^2 \sim \ell$, which agrees with the flat space expectation, while that of a Kaluza-Klein tower is $M_k^2 \sim k^2$ for sufficiently large k [62]. As we elaborate in the following section, this is a very generic behaviour when a tower of higher-spin conserved currents appears in the CFT, and lends credence to the expectation that infinite-distance points at fixed AdS radius should not be decompactification limits.

One can also ask about the order of magnitude of α_G in equation (2.11). From (2.7) we see that α_χ in equation (2.10) is, up to order one factors, given by $\alpha_\chi \sim \dim(\text{SU}(N))^{-1/2}$. However, we recall that the relation between the moduli space metric and that of the conformal manifold (1.5) introduces a dependence on (LM_{Pl}) , which in turn depends on N (2.3). This factor enters in the relation between α_χ and α_G , which is nothing but taking

³For a review of the higher-spin/CFT duality, see e.g. [61].

into account that they are measured in AdS and Planck units. All in all, we find that the exponential rate is order one in Planck units:

$$\alpha_G \sim (LM_{\text{Pl}})^{3/2} \alpha_\chi \sim \mathcal{O}(1). \quad (2.13)$$

Before closing this section, let us come back to the issue of a well-defined supergravity description. Parametric control is lost when $LM_s < 1$, the scale at which the tower of higher-spin modes falls below the one set by the AdS radius. To obtain an effective description in that regime, it would be required to integrate out these fields in a consistent way, and the new cut-off of the theory would be below the AdS scale. This does not seem to be a meaningful description of the physics in AdS. This suggests that an infrared description of quantum gravity in terms of AdS_5 supergravity is not appropriate to describe an infinite-distance limit, and such a point has to be located at the boundary of the quantum moduli space. One is instead forced to go to the CFT dual to probe such a limit, where the theory is free.

While $\mathcal{N} = 4$ super-Yang-Mills and type IIB $\text{AdS}_5 \times S^5$ vacua are very constrained by symmetries and their respective metrics can be understood throughout the entire moduli space, they illustrate a behaviour that is quite universal: when a subsector of the theory becomes free, an infinite number of higher-spin conserved currents always appear at that point in the conformal manifold. In addition, single-trace operators, as the ones we used in equation (2.12), are omnipresent in conformal gauge theories. It is also general that there can only be a small number of scalar fields sitting at the BF bound, as the dual operators must take the form $\text{Tr}(\phi^2)$.

3 $\mathcal{N} = 2$ conformal manifolds in four dimensions

Strengthened by the observations made in the previous section, we would now like to extend these arguments to theories with less supersymmetry. We will focus on theories with sixteen real supercharges in the bulk, in particular those obtained by compactifying on an orbifold of S^5 . In the SCFT dual, half of the five-dimensional supercharges are mapped to superconformal generators, and one obtains four-dimensional $\mathcal{N} = 2$ SCFTs. Before studying the infinite-distance points in both description in more details, let us review some well-established facts about $\mathcal{N} = 2$ theories.

As mentioned above, to be able to define a notion of distance on the conformal manifold, $\mathcal{M}_{\text{CFT}}$, the relevant object is the so-called Zamolodchikov metric, χ . Denoting the set of all exactly marginal operators by $\mathcal{O}_i$ and their associated coupling constants by τ_i , it is defined as the coefficient of the two-point correlators of marginal operators:

$$\langle \mathcal{O}_i(x) \mathcal{O}_j^\dagger(y) \rangle = \frac{\chi_{i\bar{j}}(\tau)}{|x-y|^8}. \quad (3.1)$$

Supersymmetry as well as the number of spacetime dimensions constrain the structure of the superconformal multiplets, the possible marginal deformations, and the properties of the metric. For $\mathcal{N} = 2$, a relevant class of multiplets are the chiral (resp. anti-chiral)

multiplets, denoted $\mathcal{E}_r$ (resp. $\bar{\mathcal{E}}_{-r}$), with r their $U(1)_R$ charge. They have the property of being annihilated by four of the supercharges:⁴

$$[\bar{Q}_{\dot{a}a}, \mathcal{E}_r] = 0, \quad \Delta = r. \quad (3.2)$$

Our convention for the quantum numbers and the relevant notions pertaining to superconformal multiplets and their primaries are reviewed briefly in appendix A.

These multiplets form a ring under the operator product expansion, and can be used to probe a host of properties of a given SCFT. For us, their importance comes from the fact that the chiral ring contains the only possible marginal operator preserving eight supercharges.

To be able to define the $\mathcal{N} = 2$ Zamolodchikov metric (3.1), an operator, $\mathcal{O}$, must satisfy the following properties: be exactly marginal, $\Delta_{\mathcal{O}} = 4$; be a singlet under the R-symmetry group, $SU(2) \times U(1)$, namely $(R, r) = (0, 0)$; and annihilated by all supercharges, $Q, \bar{Q}$ (up to total derivatives). Note that such an operator need not be a *superconformal* primary, but simply a conformal primary. It turns out that in the case of four-dimensional $\mathcal{N} = 2$ theories the only such operator is the bottom component of $\mathcal{E}_2$ (or its conjugate) [58, 59]. This operator is indeed by definition annihilated by all anti-chiral supercharges, $\bar{Q}$, and is reached from the superconformal primary by successive applications of the four remaining supercharges, Q . As each application of a supercharge increases the conformal dimension by $1/2$, it is also exactly marginal, $\Delta = 4$. One can further verify that all these marginal operators are then proportional to θ - or gauge kinetic terms,

$$\mathcal{O} = Q^4 \mathcal{E}_2 \sim \text{Tr}(F \wedge *F + iF \wedge F). \quad (3.3)$$

When a Lagrangian description is available, the deformation term is obtained by integrating the multiplet over superspace and corresponds to an F-term:

$$\delta \mathcal{L} = \tau^i \int d^4 \theta (\mathcal{E}_2)_i + c.c. \quad (3.4)$$

As such, the only possible marginal deformations preserving $\mathcal{N} = 2$ correspond to a modification of Yang-Mills couplings.

3.1 The Zamolodchikov metric

From the discussion above, the conformal manifold metric is therefore related to the two-point functions of the superconformal primaries of the chiral multiplets, $\langle \mathcal{E}_{2,i} \bar{\mathcal{E}}_{-2,j} \rangle$. The structure of the conformal manifold of four-dimensional $\mathcal{N} = 2$ SCFTs is extremely constrained. It was indeed shown that superconformal symmetry imposes the conformal manifold to be Hodge-Kähler, and that its Kähler potential, K , is related to the partition function on the four-sphere [64–66]:

$$\chi_{i\bar{j}} = 192 \partial_i \bar{\partial}_{\bar{j}} K, \quad K = 12 \log(Z_{S^4}). \quad (3.5)$$

⁴We use the nomenclature of $\mathcal{N} = 2$ superconformal multiplets of [63] and, by abuse of notation, also denote their superconformal primaries by $\mathcal{E}_r$.

This is a very powerful statement, as the four-sphere partition function of such theories can then be computed via localisation techniques [67]. Indeed, if the SCFT has a Lagrangian description anywhere in the conformal manifold, the partition function can be written as an integral over the Cartan subalgebra, $\mathfrak{h}$, of the gauge group:

$$Z_{S^4}(\tau_i, \bar{\tau}_i) = \int_{\mathfrak{h}} da \, \Delta(a) |Z_{\Omega}(a, \tau_i)|^2, \quad (3.6)$$

where $\Delta(a)$ is the Vandermonde determinant, and the integrand factorises as

$$Z_{\Omega} = Z_{\Omega, \text{cl}}(a, \tau_i) \cdot Z_{\Omega, \text{loop}}(a) \cdot Z_{\Omega, \text{inst}}(a, \tau_i). \quad (3.7)$$

The classical contribution is universal,

$$|Z_{\Omega, \text{cl}}(a)|^2 = \exp\left(2\pi \text{Im}(\tau) \text{Tr} a^2\right), \quad (3.8)$$

while the one-loop and instanton contributions depend on the spectrum of the theory under consideration. For the special subset of $\mathcal{N} = 4$ super-Yang-Mills theories, there are no one-loop or instanton contributions, and the computation is reduced to performing a Gaussian integral:

$$Z_{S^4}^{\mathcal{N}=4}(\tau, \bar{\tau}) \sim (\text{Im}\tau)^{-\dim(\mathcal{G})/2}. \quad (3.9)$$

Taking derivatives, one arrives again to the result advertised in equation (2.7).

For a generic $\mathcal{N} = 2$ theory one can obtain the metric as a formal power series by performing an expansion with respect to marginal couplings. This technique has been used to find the perturbative expansion of the Zamolodchikov metric to high order in SQCD [65, 68] and the large- N limit of necklace theories [69].

3.2 The SDC and weakly-gauged points

It is easy to see that at any point of the manifold for which a subset $\{\tau_a\}$ of the marginal couplings go to the free limit, $\text{Im}\tau_a \rightarrow \infty$, the four-sphere partition function is dominated by the classical term (3.8). After performing a change of variable, the contribution from one-loop and instanton terms is negligible, and one recovers the same Gaussian integral obtained for $\mathcal{N} = 4$ (3.9) for each sector:

$$Z_{S^4}(\tau_i) \sim \prod_a Z_{S^4}^{\mathcal{N}=4}(\tau_a), \quad \text{as } \text{Im}\tau_a \rightarrow \infty. \quad (3.10)$$

We can now see that the behaviour we observed for $\mathcal{N} = 4$ is very generic in this limit. We can again construct infinite towers of composite operators out of all possible fields of the theory. In the $\mathcal{N} = 2$ case, we now have as many directions as there are gauge couplings — or equivalently chiral multiplets of R-charge two — setting the dimension of the conformal manifold. The relevant operators will be those made out of combinations of ℓ appropriately-symmetrised derivatives and n scalars coming from the vector multiplets.⁵

⁵Note that these operators must be *bona fide* conformal operators, i.e. eigen-operators of the dilatation generator, and there will in general be mixing between operators with the same quantum numbers. This point is irrelevant to our analysis, as we are only interested in the qualitative behaviour near the infinite-distance point.

Their conformal dimensions is then

$$\Delta_{\mathcal{O}_{n,\ell}} = n + \ell + \gamma(\tau_1, \dots, \tau_{\dim \mathcal{M}}). \quad (3.11)$$

A particularity of operators constructed out of scalars coming from vector multiplets is that any interaction term involving them will either come from gauged kinetic terms or from F-terms, and therefore always involve powers of the coupling. In the limit where $\text{Im}\tau_a \rightarrow \infty$, the anomalous dimensions will be proportional to the gauge couplings, and using the form of the Zamolodchikov metric (3.10) one finds:

$$\Delta_{\mathcal{O}_{n,\ell}} \sim n + \ell + \eta e^{-\alpha_X \text{dist}_X(\tau_a)}. \quad (3.12)$$

Note that, while for $\mathcal{N} = 4$ super-Yang-Mills the case-dependent coefficient η was a pure number, it now can depend on the other couplings that are not taken to the free limit.

We therefore obtain the same qualitative behaviour observed for $\mathcal{N} = 4$: in the bulk there is an infinite tower of scalar fields of mass $(m_n L)^2 \sim n^2$, but more importantly, and as required by Maldacena-Zhiboedov theorem [52], there is also an infinite number of higher-spin currents of the form (2.10) that become conserved. The latter class of operators are again mapped to the bulk as an infinite tower of higher-spin modes becoming exponentially light with the distance as in (2.11), as required by the SDC. Similarly, the density of the tower being linear with the spin, we can again expect that these limits do not correspond to a partial decompactification in the string theory description.

Since the tower of states satisfies the SDC, moving away from the “interior” of the moduli space towards an infinite-distance point, the mass of the higher-spin states will eventually fall below the constant AdS radius. Similarly to what happens in the case of the moduli space of $\text{AdS}_5 \times S^5$ discussed in the previous section, the supergravity regime will again break down, and the appropriate description will be that of a weakly-coupled CFT. As will be discussed shortly, superconformal representation theory severely restricts the possible infinite-distance points of the conformal manifold. This means that moduli spaces of consistent AdS_5 supergravity theories with sixteen supercharges does not contain any infinite-distance points — at least of the type considered here — where an effective description does not completely breaks down when taking into account quantum corrections.

This breakdown is in spirit similar to what happens in flat space when trying to reach the small-volume point of Calabi-Yau moduli spaces. As one tries to approach it, one leaves geometric phase of the moduli space, and the usual classical moduli are not appropriate quantum variables, leading to a quantum obstruction. Such examples have been studied in the context of the SDC in [21, 62].

Conversely, an obvious difference with $\mathcal{N} = 4$ is that many of the operators will not go to their free value, even when they contain fields charged under the gauge group that decouples. The anomalous dimension of such composite operators will generically not be proportional to the associated couplings and there can be mixing with fields of another sector, if for instance they are in the bifundamental representation of groups whose coupling does not go in the free limit.

Having found a tower of states compatible with the SDC, we can now inquire about the order of magnitude of the exponential rate, α_G . One can consider several sectors

decoupling at different paces, and this will be reflected in the value of α_G . Let us introduce a parameter, t , describing the fastest gauge couplings satisfying $\text{Im}\tau_a \rightarrow \infty$. Those going to the same limit, but slower, can be described similarly by introducing an exponent, p_a :

$$\text{Im}(\tau_a) = t^{p_a}, \quad 0 < p_a \leq 1. \quad (3.13)$$

Of course, $p_a = 1$ only for the parameter — or family of parameters — going to the free limit the fastest. We note that all these trajectories are geodesics, as can be seen by using flat coordinates $\Phi_a \sim \log(\text{Im}\tau_a)$ with respect to the Zamolodchikov metric derived from (3.10) and checking that they are straight lines.

Using (3.10) and (3.5) one can estimate the distance in the Zamolodchikov metric in terms of this parameter, and then translate it to a distance in the moduli space using (1.5). One proceeds in the same fashion as for $\mathcal{N} = 4$ to obtain the usual logarithmic behaviour, and a decay rate,

$$\alpha_G \sim \left(\frac{(LM_{\text{Pl}})^3}{\sum_a p_a^2 \dim(\mathcal{G}_a)} \right)^{\frac{1}{2}}. \quad (3.14)$$

When the theory admits a point in the moduli space where a supergravity description in terms of Einstein gravity is available, we can estimate the value of LM_{Pl} by computing the trace-anomaly coefficients of the CFT, a, c . Going through the usual holographic computation, one obtains that at leading order in N , $(LM_{\text{Pl}})^3 \sim a$. The coefficients further agree up to linear corrections in N , $24(a - c) = n_v - n_h = \mathcal{O}(N)$, where n_h, n_v correspond to the number of $\mathcal{N} = 2$ hyper- and vector multiplets, respectively. For large-enough values of N , the standard formulas therefore yield:

$$a = \frac{5n_v + n_h}{24} \sim \frac{n_v}{4} = \frac{1}{4} \dim(\mathcal{G}). \quad (3.15)$$

Further gravitational corrections in the bulk will modify the value of the trace-anomaly coefficients, but those will always be subleading in N and will not modify the overall scaling. For our purpose, we can therefore use them to estimate the scaling of $(LM_{\text{Pl}})^3$ in terms of the dimension of the total gauge group:

$$\alpha_G \sim \left(\frac{\dim(\mathcal{G})}{\sum_a p_a^2 \dim(\mathcal{G}_a)} \right)^{\frac{1}{2}}. \quad (3.16)$$

We see that the denominator is bounded between $\dim(\mathcal{G}_{\text{dec.}})$, the dimension of the gauge subgroup decoupling the fastest which by assumption has $p_a = 1$, and the dimension of the total gauge group. For any free limit and large-enough groups, we find the bounds:

$$\mathcal{O}(1) \lesssim \alpha_G \lesssim \left(\frac{\dim(\mathcal{G})}{\dim(\mathcal{G}_{\text{dec.}})} \right)^{\frac{1}{2}}. \quad (3.17)$$

This means that the exponential rate is always of order one in Planck units, or larger. It is thus very easy to engineer limits with large α_G . For example a theory with gauge group $\text{SU}(N)^K$ in the limit where a single $\text{SU}(N)$ becomes free leads to:

$$\alpha_G \sim \sqrt{K}. \quad (3.18)$$

Note that while we focussed on $\mathcal{N} = 2$ four-dimensional theories where the relations between the trace-anomaly coefficients and the gauge group data are simple, estimating LM_{PI} in terms of group theoretical data of the gauge theory can be adapted *mutatis mutandis* to studies in other dimensions, trading a, c for the appropriate quantities. We therefore expect similar bounds in more general cases whenever a sector of the CFT decouples.

3.3 Beyond free points

If the Swampland Distance Conjecture is true, we expect infinite-distance points to be associated with an infinite towers of massless states in the bulk. As we have seen, those associated to a weak-gauge-coupling limit on the boundary CFT will have an infinite number of higher-spin conserved currents, as required by the Maldacena-Zhiboedov theorem. One may then ask what type of behaviour one can expect beyond those where a sector becomes free, if any.

For instance, one can consider a limit in which a tower of scalars become massless in the bulk and whether it is at infinite distance. Via the dictionary (1.2) such a tower can only appear if there are points with additional marginal operators in the boundary, which by $\mathcal{N} = 2$ superconformal representation theory only exists when there is a gauge symmetry enhancements of the CFT. We can always move slightly away from the conformal manifold onto the Coulomb branch by giving a vacuum expectation value to scalar fields inside the vector multiplet. As the dimension of the Coulomb branch is an invariant of the theory, the total rank of the gauge group is fixed and an infinite tower of scalars is not possible. Beyond $\mathcal{N} = 2$, we are not aware of any CFT exhibiting loci in the conformal manifold where an infinite number of new marginal deformations appear. These towers would be ideal candidates for Kaluza-Klein towers in the bulk and their apparent absence again provides support to the expectation that the ADC and SDC directions in moduli space are separate limits.

Another possibility is an enhancement of the flavour group of the CFT. This requires a would-be flavour current to be part of a long multiplet that becomes short. As shown in [70, 71], an analysis of the recombination rules of long multiplets at threshold reveal the only such possibility to be a superconformal multiplet of type $\hat{\mathcal{C}}_{0,(\frac{1}{2},\frac{1}{2})}$. This multiplet contains a higher-spin conserved current, implying that there is again a sector of the SCFT that will decouple. It in turn means that the associated gauge enhancements in the bulk are at infinite distance.

There are also strongly-coupled points in the conformal manifold that are at infinite distance. These points are often free points in disguise, as there exists a duality transformation to a frame where there is a weakly-coupled sector. Such examples are plentiful in class S constructions, and we will consider specific cases in the next section.

While we are not able to show that there are no infinite-distance point that do not correspond to a decoupling limit of a $\mathcal{N} = 2$ SCFT, we are not aware of such a case. Using localisation techniques, it is in principle feasible to compute the Zamolodchikov metric in a non-perturbative regime by taking into account all loop and instanton corrections in (3.7).

Finally, there cannot be compact smooth conformal manifolds with $\mathcal{N} = 2$ supersymmetry [66], thereby excluding cases that do not admit any free limit at all. There is

furthermore a conjecture stating that any n -dimensional $\mathcal{N} = 2$ conformal manifold can be obtained by gauging n simple factors of the flavour symmetry associated to SCFTs with no marginal deformations [71]. In that sense, all the infinite-distance points studied in this work correspond to reversing (partially or completely) the process by returning to a flavour symmetry.

We close this section by noting that the results we have obtained carry to cases with lower dimensions and supersymmetry. Whenever a sector of the CFT becomes free there will always be a tower of massless higher-spin fields in the bulk. However, this does not mean that sending any marginal coupling to zero will involve a tower of the form (2.10). Indeed let us imagine an $\mathcal{N} = 1$ SCFT depending on two marginal parameters. As marginal operators need not be gauge deformations in that case, sending one of the parameters to zero does not imply that the anomalous dimensions of would-be conserved currents also vanish. It might still depend non-trivially on the other parameter, depending on the structure of the CFT, and the decoupled point could be at finite distance. While there is a possibility that it may be at infinite distance and an SDC tower still exists, it requires a further analysis of $\mathcal{N} = 1$ superconformal representations, which we leave for future works.

4 Orbifolds and $\mathcal{N} = 2$ necklace quivers

In section 3 we have reviewed the machinery of four-dimensional $\mathcal{N} = 2$ SCFTs to learn about the possible behaviour of SDC towers in AdS_5 vacua with sixteen supercharges. To understand the mechanisms responsible for the associated infinite-distance points in the bulk, as well as exploring points that are a priori not free, we now turn to an explicit construction in string theory, namely the family of AdS_5 vacua obtained by type IIB compactification on an orbifold of the form S^5/Γ . In order to conserve sixteen supercharges we are forced to consider the orbifold action to be an ADE-type discrete subgroup, $\Gamma \subset SU(2)$. For simplicity we focus on the A-type series, that is, on backgrounds of the form $AdS_5 \times S^5/\mathbb{Z}_K$.⁶ Other cases can be generalised straightforwardly.

The dual four-dimensional $\mathcal{N} = 2$ SCFT are the well-known necklace quiver theories with gauge group $\mathcal{G} = SU(N)^K$ [72, 73], which are represented by the affine Dynkin diagram $\hat{A}_{K-1}$, as depicted in figure 1. In addition to vector multiplets associated to each gauge factor, there are also hypermultiplets transforming in bifundamental representations of each pair of adjacent gauge factors. At large N , their Zamolodchikov metric was studied in [69].

Necklace theories can be obtained by projecting out modes of $\mathcal{N} = 4$ super-Yang-Mills theory with gauge group $SU(KN)$ and it is natural that the complexified gauge coupling of each gauge sector, τ_i , is related to that of the parent theory, τ_0 [72]:

$$\tau_i = \frac{\tau_0}{K}, \quad \tau = \sum_{i=1}^K \tau_i. \quad (4.1)$$

Recalling that $\tau_0 = \tau$ is the holographic dual of the axio-dilaton, we see that it is controlling the overall complex gauge coupling in this setup. As each gauge coupling, τ_i , is

⁶Seen as a constant-radius hypersurface on $\mathbb{C}^3$, the precise action of $\mathbb{Z}_k$ on the S^5 is given by $(z_1, z_2, z_3) \rightarrow (e^{2\pi i/K} z_1, e^{-2\pi i/K} z_2, z_3)$.

Figure 1. Left: quiver representation for the $\hat{A}_{K-1}$ necklace theory, with each node corresponding to a $\mathcal{N} = 2$ vector multiplet and each line a hypermultiplet in the bifundamental of the adjacent groups. Right: the torus with K minimal punctures, T_K^2 , of the associated class S construction.

again associated to a marginal deformation, we expect $K - 1$ additional complex moduli in the bulk. To separate these from the axio-dilaton, we choose the following basis of marginal couplings:

$$\tau_i \in \{\tau_0, \tau_a\}, \quad a = 1, \dots, K - 1. \quad (4.2)$$

Since the axio-dilaton is the only moduli of the unorbifolded theory, the other moduli should come from the twisted sector. Indeed, $\mathbb{Z}_K \subset \text{SU}(2)$ does not act freely on the five-sphere, but leaves a circle $S^1 \subset S^5$ of singular fixed points after orbifolding. This leads to a twist sector related to the $K - 1$ blow-up 2-cycles resolving these singularities. One finds five scalars for each 2-cycle: two axions and three of geometric origin. It turns out that the latter are stabilised by the potential, and consequently are not part of the moduli space. Thus, as proposed in [72], the $K - 1$ complex moduli we are looking for are the axions coming from the periods of the RR and NSNS 2-forms on the 2-cycles, b_a and c_a respectively. We will assume them to be normalised to have period one. All in all, one finds the following dictionary between the moduli and the marginal couplings:

$$\tau_a = c_a + \tau_0 b_a, \quad (4.3)$$

or in terms of Yang-Mills gauge couplings and theta-angles,

$$\frac{\theta_a}{2\pi} = c_a + b_a C_0, \quad \frac{4\pi}{g_a^2} = \frac{b_a}{g_s}. \quad (4.4)$$

Additional details on the duality between axions in the twisted sector and marginal couplings can be found in [74–76].

Notice that tuning the vevs of these twist-sector moduli corresponds to a deformation of the theory away from the one obtained by orbifolding $\text{AdS}_5 \times S^5$. Indeed, (4.1) is only recovered when the vev of these axions correspond to the orbifold point [77],

$$c_a = 0, \quad b_a = \frac{1}{K}. \quad (4.5)$$

There are various ways to reach an infinite-distance point by tuning the different moduli, which as we will see will lead us away from the orbifold point to regions where the supergravity description breaks down. The situation is similar to that of four-dimensional

$\mathcal{N} = 2$ theories in flat space, as we can tune one or more moduli at the same time, leading to different behaviours. We now analyse various limits and try to identify the associated infinite towers of states, both in the bulk and the CFT.

4.1 Overall free limit

From the definition of the moduli τ_i it is straightforward to see that the simplest limits are the ones for which all the gauge sectors are becoming free at the same rate. In the bulk, this is controlled by the limit

$$\text{Im}(\tau) \rightarrow \infty, \quad (4.6)$$

while b_a , c_a and C_0 are fixed. From the expression of the Zamolodchikov metric (3.10), we see that this point is indeed at infinite distance, and we are in a situation similar to that of section 2, where the fundamental Type IIB string becomes tensionless and weakly coupled. In this particular case, we can also make use of type IIB $\text{SL}(2, \mathbb{Z})$ duality to argue that the same behaviour occurs when reaching the overall strong coupling limit, $\text{Im}\tau \rightarrow 0$ with all the other moduli constant.

There is again an infinite tower of higher-spin fields, dual to generalised currents, $J_{\mu_1 \dots \mu_\ell}$, becoming exponentially massless with the distance in Planck units:

$$\frac{M_\ell}{M_{\text{Pl}}} \sim e^{-\alpha_G \text{dist}_G(\tau)}. \quad (4.7)$$

These massless higher-spin currents have an obvious interpretation in the bulk: they originate from higher-spin excitations of tensionless fundamental strings. Moreover, being an overall free limit in the CFT with all gauge sectors decoupling at the same rate, we know from the discussion in section 3.1 that α_G is of order one.

Relaxing the condition $b_a = \text{const}$, it is possible to also explore limits in which different $\text{SU}(N)$ sectors decouple at different rates. For this more general case we know from (3.17) that α_G will be bounded between an order one number and $\sqrt{K}$, and can as a result be parametrically large.

As in $\mathcal{N} = 4$ SYM, but contrary to Minkowski backgrounds, only a small number of scalar fields become massless, while using (3.12) and the AdS/CFT dictionary, all other scalars have masses that are regularly spaced near the infinite-distance point:

$$\frac{M_n}{M_{\text{Pl}}} \sim n^2. \quad (4.8)$$

This case is however quite special, as everything is controlled by the value of the axio-dilaton and all fundamental fields of the CFT are free in that limit. We can reach a much richer network of infinite-distance point in moduli space by demanding that only a strict subset of the moduli change over the path, which in the CFT corresponds to taking only some of the gauge couplings to be free.

4.2 Strong-coupling points and dualities

Demanding the axio-dilaton to remain constant, tuning some — or all — of the $K - 1$ remaining moduli to zero, we can reach a variety of new points in the conformal manifold:

$$\tau = \text{const}, \quad b_a, c_a \rightarrow 0. \quad (4.9)$$

One could naively expect this path to lead to a finite-distance point, as we are moving in axionic directions. However, moving away from the orbifold point we cannot trust the usual supergravity description, but we can use the dual CFT to nonetheless learn about what happens in its neighbourhood.

Using the dictionary (4.4), sending the moduli to zero corresponds to keeping the overall complex coupling fixed while taking the remaining $K - 1$ gauge sectors to strong coupling:

$$\tau_0 = \text{const}, \quad g_a \rightarrow \infty, \quad \theta_a \rightarrow 0. \quad (4.10)$$

Note that this limit also demands the θ -angles to fall to zero. This setup was studied in details in [78] using Gaiotto's class S construction. In that framework, the necklace quivers are realised as a compactification of the six-dimensional $\mathcal{N} = (2, 0)$ SCFT of type A_{N-1} on a torus with K minimal regular punctures, T_K^2 , depicted in figure 1.

From this point of view, a trajectory in the conformal manifold in which the axio-dilaton is kept fixed corresponds to changing the relative position of the punctures, and the limit above brings them all together, possibly at different rates. A key point of the class S framework is that as one brings two punctures together, the torus develops a throat and there exist an S-duality frame in which a sector of the theory becomes a weakly-coupled gauge theory. Furthermore, this process is local, and does not depend on what happens in the rest of the surface [79].

In the case at hand, one obtains a sector consisting of a strongly-interacting SCFT, where the $\mathcal{N} = (2, 0)$ theory is compactified on a torus with a single puncture, connected to a weakly-coupled gauge theory with gauge group [78]:

$$\mathcal{G}_{\text{dec.}} \subset \hat{\mathcal{G}} = \text{SU}(2) \times \text{SU}(3) \times \cdots \times \text{SU}(K). \quad (4.11)$$

We are therefore once again in the situation described around equation (3.12): the conformal dimensions of operators built out of fields in the weakly-coupled vector multiplets have an anomalous dimension that is exponentially suppressed with the distance around the point (4.9) in this particular duality frame, which is at infinite distance. The Maldacena-Zhiboedov theorem again requires the presence of an infinite tower of higher-spin conserved currents.

In the bulk, we also have an infinite tower of higher-spin operators that become massless exponentially fast in Planck units, $M_\ell/M_{\text{Pl}} \sim e^{-\alpha_G \text{dist}_G}$. The decay rate can then be estimated using the bounds (3.17) in terms of the group theory data of the dual.

Let us comment on the stringy origin of these states. As b_a and c_a become small, so does the tension of D3-branes wrapped on the blow-up cycles:

$$T_{\text{D3}} \sim \int_{\Sigma_a} |C_2 + \tau B_2| \xrightarrow{b_a, c_a \rightarrow 0} 0. \quad (4.12)$$

One might be tempted to conclude that the massless higher-spin fields in the bulk come from such tensionless strings, particularly in the context of the Emergent String Conjecture [14]. However, little is known about the spectrum of these strings in AdS — in particular, in flat space they are non-critical and do not give rise to an infinite number of massless states —

and we are therefore unable to make such a claim. The origin of the tower of states remains elusive in these limits.

Intriguingly, it was proposed in [78] that there might be a dual description where there is no tower of higher-spin modes in the bulk. There, part of the SCFT is associated with a four-dimensional strongly-interacting gauge theory living on the boundary of AdS_5 , which is then coupled to the rest of the bulk, i.e. to the rest of the type IIB spectrum. At the point where the boundary SCFT becomes free, this gauge theory becomes weakly-coupled. Therefore the higher-spin conserved currents in the boundary are not mapped to massless higher-spin modes in the bulk, but to the higher-spin conserved currents of this four-dimensional gauge theory. This possibility however involves choosing non-standard boundary conditions, and the usual AdS/CFT dictionary does not apply. We leave an exploration of such a description and its relation to the SDC for future works.

Classically, the limits we have been considering would be at finite distance and we expect the infinite distance to be driven by quantum corrections.⁷ We note that, similarly to what happens in the case of $\text{AdS}_5 \times S^5$, as we move away from the orbifold point by tuning the axions, we leave the phase of the moduli space where the supergravity regime is valid, and enter a phase where the CFT description is more appropriate.

The behaviours described above generalise to a wide zoo of class S examples. Given a $\mathcal{N} = (2, 0)$ six-dimensional SCFT of ADE type, one can reach a four-dimensional $\mathcal{N} = 2$ SCFT by compactifying on a punctured compact Riemann surface [80], see [81] for a review. The surfaces can then be constructed as 3-punctured spheres glued by tubes, called “tinkertoys” [79, 82]. For SCFTs of type A_N , the allowed collisions of punctures leading to a weakly-coupled gauge sector have been studied in [83]. In some cases, one can construct a weakly-curved holographic gravity dual from M-theory on a background of the form $\text{AdS}_5 \times X_6$. As for $\text{AdS}_5 \times S^5$ and its orbifolds, the infinite-distance points will be on the boundary of the moduli space where there supergravity regime has broken down because the tower of state has reached a scale smaller than the AdS radius.

5 Conclusions

By studying the behaviour of states near a family of infinite-distance points in the moduli space of AdS vacua, we have taken a first step towards a possible extension of the Swampland Distance Conjecture to curved backgrounds. To that end, we have used the power of the dual four-dimensional $\mathcal{N} = 2$ superconformal symmetry, which allows one to reduce large classes of infinite-distance points to a case where a subsector of the SCFT becomes free. While we are unable to claim that all infinite-distance points correspond to free limits, we are not aware of possible counterexamples. In the bulk, there is then always an infinite number of higher-spin modes becoming exponentially massless playing the rôle of the SDC tower for AdS moduli spaces.

⁷This can be seen by considering the moduli space as a truncation of the theory obtained by placing the orbifold in flat space. For the latter, the moduli space is classically exact and our limits are known to be at finite distance [78].

In flat space, the tower of light states indicates that the effective field theory description is no longer valid, and the SDC is parametrising how this breakdown occurs. By contrast, we find that in the bulk this always happens before reaching the infinite-distance point. For instance, this limit for $\text{AdS}_5 \times S^5$ vacua is located in the highly-curved regime. On general grounds, the SDC predicts that a tower of states will eventually fall below the AdS scale and an effective description would need a lower cut-off. Should this not be an appropriate description, it would mean that the landscape of AdS vacua in quantum gravity cannot admit an effective field theory description when getting close to infinite-distance points in moduli space, thereby strongly constraining the possible theories which can be coupled to quantum gravity. Note that how close to the infinite-distance point one can go with an effective theory depends on the AdS radius. In particular, it would be interesting to relate the lack of effective description to the species scale, as is done in flat space [84]. In this context, the species scale controls the effective gravity coupling, and thus the size of a typical quantum fluctuation around the background metric. If this logic applies to AdS, when the species scale becomes smaller than the AdS scale, they are large compared to the background, making a geometric description inconsistent.

For theories described by Einstein gravity at a point of the moduli space we have also been able to find bounds for the exponential decay constant. It must always be at least of order one in Planck units and is bounded from above by the ratio between the dimensions of the total gauge group and the decoupled sector. It is therefore possible to obtain a parametrically-large decay constant by engineering a small sector decoupling from a large gauge group.

We have applied this analysis to orbifolds of S^5 , with the associated CFT being described by necklace quivers. When all gauge nodes are decoupled, one finds a tensionless fundamental string in the bulk. Using the class-S description of that SCFT we were further able to relate the behaviour of the SDC for individual strong-coupling points to that of free limits via S-duality and found no other infinite-distance points. However, the stringy interpretation of the tower of states is less obvious in these cases: at these points, D3-branes wrapped on blow-up two-cycles become tensionless, but their flat-space avatars are non-critical strings and at finite distance in moduli space. Further, the effective description has broken down well before reaching that point. A relation with the Emergent String Conjecture [14] is therefore not conclusive, and calls for further analysis.

Many of the arguments we discussed here generalise to more arbitrary cases, with and without supersymmetry, that admit a CFT dual. The Maldacena-Zhiboedov theorem does not require supersymmetry and there will always be an infinite tower of higher-spin states in a limit leading to subsector of the CFT becoming free. If the marginal deformation is identified with a gauge coupling along the conformal manifold, it will be at infinite distance by looking at the Zamolodchikov metric and there will be exponentially light states accompanying it. However, the structure of conformal manifolds greatly depends on the spacetime dimension and number of supercharges. For instance, there are no supersymmetry-preserving marginal deformations in six dimensions, which in the bulk translates to all moduli being stabilised [85]. In lower dimensions however, there can be marginal deformations that go beyond changing gauge coupling constants. One would

therefore expect to have a far richer network of infinite-distance points in these cases. It might be very interesting to look for similar structures as the ones used in the context of Hodge theory, see e.g. [17, 18, 36, 86].

Furthermore, there also exists conformal manifolds which are compact, see e.g. [87]. While their holographic duals are not well understood, it would be interesting to see if the requirements needed to have a compact manifold can be related to swampland constraints in the bulk.

Unlike the Weak Gravity Conjecture, where black hole physics plays an important rôle, the current understanding of the SDC comes principally from string theory. Using the AdS/CFT correspondence therefore opens new avenues to explore this part of the Swampland programme. For instance, unitarity constraints and other features of superconformal symmetry might shed new light on the origin of the various conjectures and how they are related.

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A Superconformal representations of $SU(2, 2|2)$

We summarise in this appendix the basic notions of superconformal representations useful in this work. The $\mathcal{N} = 2$ superconformal group is $SU(2, 2|2)$, its bosonic subgroup being constituted of both the conformal and R-symmetry groups, $SO(2, 4) \times SU(2)_R \times U(1)_R$. The algebra contains the usual conformal generators, namely those of translations, rotations, and so-called special conformal transformations, $M_{\mu\nu}, P_\mu, K_\mu$, as well as the dilatation operator, $\mathcal{D}$. With the R-symmetry generators, it is supplemented by the super- and superconformal charges:

$$Q_{\alpha A}, \bar{Q}_{\dot{\alpha} A}, \quad S^{\alpha A}, \bar{S}^{\dot{\alpha} A} \quad \alpha, \dot{\alpha} = 1, 2, \quad A = 1, \dots, 4. \quad (\text{A.1})$$

One can then label any operator of the theory by the following quantum numbers:

$$[\Delta; j, \bar{j}; R; r]. \quad (\text{A.2})$$

In addition to the usual conformal dimension, Δ , which is the charge of the operator under dilatations, and the Lorentz Dynkin indices, $(j, \bar{j})$, of $\mathfrak{so}(1, 3) = \mathfrak{su}(2) \oplus \mathfrak{su}(2)$, we also define the R-charges, (R, r) , of $SU(2)_R \times U(1)_R$.

The spectrum of the theory organises itself into superconformal multiplets of $SU(2, 2|2)$ whose highest weights are called superconformal primaries. A superconformal primary, $\mathcal{O}$,

is then by definition annihilated by special conformal transformation generators — as for all usual conformal primaries — and superconformal charges:

$$[K_\mu, \mathcal{O}] = 0, \quad [S^{\alpha A}, \mathcal{O}] = 0 = [\bar{S}^{\dot{\alpha} A}, \mathcal{O}], \quad \alpha, \dot{\alpha} = 1, 2, \quad A = 1, \dots, 4 \quad (\text{A.3})$$

where the commutator or anti-commutator is used depending on whether $\mathcal{O}$ is fermionic or bosonic. Given a superconformal primary, the rest of the multiplet (called descendants), is then generated by successive applications of the translation operator, P_μ , and the regular supercharges, $Q_{\alpha A}, \bar{Q}_{\dot{\alpha} A}$. Note that due to their fermionic nature, the number of states generated by the supercharges is finite.

Moreover using the conformal algebra it possible to show that applying the shift of conformal dimension of descendants from its primary is (half-)quantised:

$$\Delta_{[Q, \mathcal{O}]} = \Delta_{\mathcal{O}} + \frac{1}{2}, \quad \Delta_{[P, \mathcal{O}]} = \Delta_{\mathcal{O}} + 1. \quad (\text{A.4})$$

Schematically for a bosonic superconformal primary, the descendants and their dimensions are found to be:

$$\begin{array}{ccccccc} \mathcal{O}, & [Q_{\alpha A}, \mathcal{O}], & [\bar{Q}_{\dot{\alpha} A}, \mathcal{O}], & \dots & [P_\mu, \mathcal{O}], & [P_\nu, [P_\mu, \mathcal{O}]], & \dots \\ \Delta, & \Delta + \frac{1}{2}, & \Delta + \frac{1}{2}, & \dots & \Delta + 1, & \Delta + 2, & \dots \end{array} \quad (\text{A.5})$$

Conversely, for superconformal charges and special conformal transformations, the sign of the shift is reversed.

A complete analysis of the representations then separates superconformal multiplets into two classes: long and short multiplets. The latter corresponds to cases where the superconformal primary is annihilated by a combination of the supercharges, in which case its dimension is set by the rest of the quantum numbers. In this work, we are mainly interested in a class of multiplet whose superconformal primary is annihilated by half of the supercharges, such as the chiral multiplets, $\mathcal{E}_r$. Long multiplets are unconstrained and their conformal dimensions are only bounded from below by unitarity.

Additional details and a complete classification of unitary superconformal multiplets can be found in e.g. [63, 88].

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