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Superradiance and stability of the regularized 4D charged Einstein-Gauss-Bonnet black hole

Cheng-Yong Zhang,^a Shao-Jun Zhang,^{b,c} Peng-Cheng Li^{d,e} and Minyong Guo^{d,1}

^aDepartment of Physics and Siyuan Laboratory, Jinan University,
Guangzhou 510632, China

^bInstitute for Theoretical Physics and Cosmology, Zhejiang University of Technology,
Hangzhou 310023, China

^cUnited Center for Gravitational Wave Physics, Zhejiang University of Technology,
Hangzhou 310032, China

^dCenter for High Energy Physics, Peking University,
No.5 Yiheyuan Rd, Beijing 100871, P. R. China

^eDepartment of Physics and State Key Laboratory of Nuclear Physics and Technology,
Peking University,
No.5 Yiheyuan Rd, Beijing 100871, P.R. China

E-mail: zhangcy@email.jnu.edu.cn, sjzhang84@hotmail.com,
lipch2019@pku.edu.cn, minyongguo@pku.edu.cn

ABSTRACT: We investigated the superradiance and stability of the regularized 4D charged Einstein-Gauss-Bonnet black hole which is recently inspired by Glavan and Lin [Phys. Rev. Lett. 124, 081301 (2020)]. We found that the positive Gauss-Bonnet coupling constant α enhances the superradiance, while the negative α suppresses it. The condition for superradiant instability is proved. We also worked out the quasinormal modes (QNMs) of the charged Einstein-Gauss-Bonnet black hole and found that the real part of all the QNMs does not satisfy the superradiance condition and the imaginary parts are all negative. Therefore this black hole is stable. When α makes the black hole extremal, there are normal modes.

KEYWORDS: Black Holes, Models of Quantum Gravity

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¹Corresponding author.

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1 Introduction

As elementary particles, black holes play a central role in gravity including the general relativity and other modified theories of gravity. Numerous studies of past have proven that black holes enjoy many extremely nontrivial effects. One of the most interesting effects is the Penrose progress, which is found to be a new mechanism that energy can be extracted from the Kerr black hole [1]. The most essential reason is the existence of ergoregions, where timelike particles can have negative energies. And superradiant effects soon entered people's view as the wave counterpart of the Penrose progress [2–4]. Penrose progress and superradiance both require dissipations which can be provided by the ergoregion for an uncharged, stationary and axisymmetric spacetime. But unlike Penrose progress, superradiance can also occur in a nonrotating charged black hole geometry [5, 6], which is fundamentally different from the first situation. Since a spacetime containing a nonrotating charged black hole is believed to be a effectively dissipative environment for charged fields.

Along this line of superradiance, lots of studies have been investigated in many aspects. On the side of a nonrotating charged black hole, superradiance from RN black holes has been done in [7] at linearized level by considering a charged scalar field propagating on a RN background with the help of frequency-domain method. The time-domain method was applied to extract amplification factors in [8]. Also, an analytical treatment was proposed to calculate the amplification factors when the frequency is small [9]. These results

agree well and support the existence of superradiation in RN black hole geometry mutually. Furthermore, some other investigations including superradiance in nonasymptotically flat spacetimes, analogue black hole geometries or higher dimensional spacetimes, superradiance beyond GR have attracted a lot of attention as well. One is suggested to refer to the comprehensive review [7] to overlook the whole picture. Therein, superradiance from black holes in alternative theories of gravity has been studied only in a few cases [10–15]. Whether superradiance can be stronger in modified theories of gravity still remains an open question [7].

As we know, the Einstein-Gauss-Bonnet (EGB) gravity is accounted as one of the most promising candidates for modified gravity [16]. And recently, a regularized 4D EGB gravity was proposed in [17], where the authors rescale the Gauss-Bonnet coupling constant $\alpha \rightarrow \alpha/(D-4)$ in the limit $D \rightarrow 4$ and found the corresponding static black hole solution, see also [18–20]. This new finding has stimulated a lot of attentions on many fronts [21–53]. However, it's worth noting that the regularized 4D EGB gravity was found inconsistent based on the theory proposed in [17]. Fortunately, this problem was cured by [54, 55], where a well-defined theory was formulated.

In this paper, we will explore the superradiance and stability of the regularized 4D charged Einstein-Gauss-Bonnet black hole geometry [28]. Firstly, we determined the constraints on α and the charge of the black hole Q maintaining the event horizon. Then, by considering a charged massless scalar perturbation of the background we obtain the amplification factor of the superradiance and give a detailed analysis of the effects of α under different other parameters. We also pay attention on stability. We use the asymptotic iteration method (AIM) [56, 57] to solve the quasinormal modes (QNMs) of the charged scalar perturbation numerically by considering the system as a scattering process. We find there's no instability for the charged 4D EGB black hole under perturbations no matter α is positive or negative in the corresponding allowed region. And very interestingly, we find normal modes are survived for the extremal black hole which is worthy of further studies.

The paper is organized as follows. In section 2, we shortly revisit the regularized 4D EGB gravity and determine the constraints on the GB coupling constant and the charge of the black hole to insure the spacetime contains a static charged black hole. In section 3, we discuss the scalar field perturbation. And we move to the amplification in section 4. The stability of the regularized 4D charged EGB black hole is investigated in section 5. We summarize our conclusions in section 6.

2 The spherically symmetric 4D charged EGB black hole

The action of the EGB gravity with electromagnetic field in D -dimensional spacetime has the form

$$S = \frac{1}{16\pi} \int d^D x \sqrt{-g} \left[R + \frac{\alpha}{D-4} \mathcal{G}^2 - F_{\mu\nu} F^{\mu\nu} \right]. \quad (2.1)$$

Here we have rescaled the coupling constant α by a factor $\frac{1}{D-4}$. The Gauss-Bonnet term reads

$$\mathcal{G}^2 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} = \frac{1}{4}\delta^{\mu\nu\alpha\beta}_{\rho\sigma\gamma\delta} R^{\rho\sigma}_{\mu\nu} R^{\gamma\delta}_{\alpha\beta}. \quad (2.2)$$

The Maxwell tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, in which A_μ is the gauge potential. Varying the action with respect to the metric, one gets the equation of motion

$$G_{\mu\nu} + \frac{\alpha}{D-4} H_{\mu\nu} = T_{\mu\nu}. \quad (2.3)$$

Here $G_{\mu\nu}$ is the Einstein tensor and

$$H_{\mu\nu} = 2(RR_{\mu\nu} - 2R_{\mu\sigma}R^\sigma_\nu - 2R_{\mu\sigma\nu\rho}R^{\sigma\rho} - R_{\mu\sigma\rho\beta}R^{\sigma\rho\beta}_\nu) - \frac{1}{2}g_{\mu\nu}\mathcal{G}^2. \quad (2.4)$$

The energy-momentum tensor of the Maxwell field takes in this form

$$T_{\mu\nu} = \frac{1}{4} \left(F_{\mu\sigma}F^\sigma_\nu - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} \right). \quad (2.5)$$

The term $H_{\mu\nu}$ comes from the variation of the Gauss-Bonnet term, which is a topological invariant term in four dimension. Therefore it does not contribute to the dynamics in four dimension in general. It can be checked that $H_{\mu\nu}$ always contains a factor $D-4$ and thus disappears when $D=4$. However, by rescaling the coupling constant α , the factor $D-4$ is canceled in (2.3). Then the Gauss-Bonnet term gives rise to non-trivial dynamics and regularized black hole solutions were discovered recently [17, 28].

The spherically symmetric charged black hole solution of (2.3) in four dimension has the form

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (2.6)$$

where

$$f(r) = 1 + \frac{r^2}{2\alpha} \left(1 \pm \sqrt{1 + 4\alpha \left(\frac{2M}{r^3} - \frac{Q^2}{r^4} \right)} \right) \quad (2.7)$$

The gauge potential is

$$A = -\frac{Q}{r}dt. \quad (2.8)$$

Here M is the black hole mass parameter, Q is the charge of the black hole. In the vanishing limit of α and in the far region, only the negative branch recovers the Reissner-Nordström (RN) black hole. Thus we will only study the negative branch in this paper.

The solution has at most two horizons in appropriate parameter region,

$$r_\pm = M \pm \sqrt{M^2 - Q^2 - \alpha}. \quad (2.9)$$

We see that Q can be greater than M when α is negative. However, α can not be too negative.¹ It must be ensured that the metric function is well defined when $r > r_+$. We show the allowed parameter region in figure 1. Hereafter, we fix $M=1$ for convenience. In region A, there is only one horizon r_+ . In region B, there are two horizons $r_\pm$. The allowed region for α is

$$\begin{cases} Q^2 - 4 - 2\sqrt{4 - 2Q^2} < \alpha < 1 - Q^2, & \text{when } 0 < Q < \sqrt{3/2}, \\ Q^2 - 4 - 2\sqrt{4 - 2Q^2} < \alpha < Q^2 - 4 + 2\sqrt{4 - 2Q^2}, & \text{when } \sqrt{3/2} < Q < \sqrt{2}. \end{cases} \quad (2.10)$$

¹Actually when α is negative, the metric function may not be real inside the event horizon. However, since we focus on the region outside the event horizon, we allow α to be negative in this work. See the details in [26, 27].

Figure 1. The parameter region where allows the event horizon r_+ . Here we have fixed $M = 1$. Region B corresponds to $Q^2 - 4 - 2\sqrt{4 - 2Q^2} < \alpha < 1 - Q^2$ and $0 < Q < \sqrt{3/2}$. Region A corresponds to $Q^2 - 4 - 2\sqrt{4 - 2Q^2} < \alpha < Q^2 - 4 + 2\sqrt{4 - 2Q^2}$ and $0 < Q < \sqrt{2}$.

Note that when $Q > 1$, the solution has no RN black hole limit since α cannot tend to 0 now. It was shown that α should be small enough for the gravitational perturbations to be stable [21]. However, it is necessary and important to study the stability of black holes under various kinds of perturbation since black holes usually behave differently for different kinds of perturbation.

3 The charged scalar perturbation

We consider a charged massless scalar perturbation of the background (2.6). It is known that fluctuations of order $\mathcal{O}(\epsilon)$ in the scalar field in a given background induce changes in the spacetime geometry of order $\mathcal{O}(\epsilon^2)$ [7]. Therefore to leading order we can study the perturbations on a fixed background geometry. The massless charged scalar field ψ has the perturbation equation as

$$0 = D^\mu D_\mu \psi = g^{\mu\nu} (\nabla_\mu - iqA_\mu) (\nabla_\nu - iqA_\nu) \psi. \quad (3.1)$$

In spherically symmetric background, one can decompose the scalar field function as

$$\psi = \sum_{lm} \int d\omega e^{-i\omega t} \Psi(r) Y_{lm}(\theta, \phi). \quad (3.2)$$

Here $Y_{lm}(\theta, \phi)$ is the spherical harmonics on the two sphere S^2 . What we are interested in is the radial part of the equation.

$$0 = \frac{1}{r^2} \partial_r (r^2 f \partial_r \Psi) + \left(\frac{1}{f} \left(\omega - \frac{qQ}{r} \right)^2 - \frac{l(l+1)}{r^2} \right) \Psi. \quad (3.3)$$

By introducing the tortoise coordinate r_* and a new variable Ψ_0 as

$$dr = f dr_*, \quad \Psi = \frac{\Psi_0(r)}{r}, \quad (3.4)$$

the radial equation can be written as the Schrödinger-like form

$$0 = \frac{\partial^2 \Psi_0}{\partial r_*^2} + \left(\left(\omega - \frac{qQ}{r} \right)^2 - V_{\text{eff}} \right) \Psi_0, \quad (3.5)$$

where the effective potential reads

$$V_{\text{eff}} = f \left(\frac{l(l+1)}{r^2} + \frac{\partial_r f}{r} \right). \quad (3.6)$$

The tortoise coordinate r_* ranges from $-\infty$ to ∞ as r runs from the event horizon r_+ to infinity. The effective potential vanishes as $r_* \rightarrow \pm\infty$ and has a potential barrier in the intermediate region. The asymptotic solution of (3.5) can be worked out as

$$\Psi_0 \rightarrow \begin{cases} \mathcal{T} e^{-i\left(\omega - \frac{qQ}{r_+}\right)r_*}, & r \rightarrow r_+, \\ \mathcal{R} e^{i\omega r_*} + \mathcal{I} e^{-i\omega r_*}, & r \rightarrow \infty, \end{cases} \quad (3.7)$$

where $\mathcal{I}$ corresponds to the incident amplitude at the infinity, $\mathcal{T}, \mathcal{R}$ are the reflected and transmitted amplitudes, respectively. Thus we take this problem as a scattering process. Note that there is no outgoing waves near the event horizon. Since the background is stationary, the field equation is invariant under the transformations $t \rightarrow -t$ and $\omega \rightarrow -\omega$. There is another solution which satisfies the complex conjugate boundary conditions. From (3.5), we see that Ψ_0^* (the complex conjugate of Ψ_0) also satisfy the radial equation. Ψ_0 and Ψ_0^* are linearly independent. Their Wronskian $W = \Psi_0 \frac{\partial \Psi_0^*}{\partial r_*} - \Psi_0^* \frac{\partial \Psi_0}{\partial r_*}$ is a constant and independent of r_* . Evaluating W near the event horizon and at the infinity, we get a relation

$$|\mathcal{R}|^2 = |\mathcal{I}|^2 - \frac{1}{\omega} \left(\omega - \frac{qQ}{r_+} \right) |\mathcal{T}|^2. \quad (3.8)$$

This relation is independent of the details of the effective potential barrier. Note that when

$$0 < \omega < \frac{qQ}{r_+}, \quad (3.9)$$

the reflected amplitude $|\mathcal{R}|^2$ can be larger than the incident amplitude $|\mathcal{I}|^2$. The wave is amplified. This phenomenon is called as superradiance. The amplification factor is defined as

$$Z = |\mathcal{R}|^2 / |\mathcal{I}|^2 - 1. \quad (3.10)$$

In the next section, we will study the amplification factor in detail.

4 The amplification factor of the superradiance

To work out the amplification factor, one should solve the radial equation firstly. However, the radial equation is hard to solve analytically in general. Most of the analytical studies were done with some approximation such as the frequency tends to zero or the black hole is very small [7, 58–63]. In this paper, we solve the radial equations numerically to study the whole parameter region. The numerical method adopted here is described in following.

The solution near the event horizon and the infinity can be written respectively as

$$\Psi_0 = \begin{cases} (r - r_+)^{-\frac{i}{2\kappa}(\omega - \frac{qQ}{r_+})} \sum_{j=0}^n a_j (r - r_+)^j, & r \rightarrow r_+, \\ e^{i\omega r} \sum_{j=0}^m \frac{b_j}{r^j} + e^{-i\omega r} \sum_{j=0}^m \frac{c_j}{r^j}, & r \rightarrow \infty. \end{cases} \quad (4.1)$$

Here κ is the surface gravity on the event horizon r_+ . n, m are the expansion orders. Coefficients a_j, b_j, c_j depend on frequency ω . They can be determined by plugging the above expansions into the radial equation (3.5) and comparing the corresponding coefficients at each order. It can be found that all $a_{j>0}$ are proportional to a_0 , and $b_{j>0}$ to b_0 , $c_{j>0}$ to c_0 . We can set $a_0 = 1$ since the radial equation is linear. Given a frequency ω , the only remaining unknown coefficients are b_0 and c_0 . Using the boundary condition near the event horizon, the radial equation can be integrated numerically outwards. By comparing the numerical solution with the asymptotic solution (4.1) at infinity, we can get the coefficients b_0 and c_0 . Note that b_0 corresponds to $\mathcal{R}$ and c_0 to $\mathcal{I}$ in (3.7). Then the amplification factor can be calculated as $Z = |b_0/c_0|^2 - 1$.

Since we have fixed $M = 1$ in this paper, the free parameters are Q, q, α and l . We analyze their effects on the amplification factor in detail in this section.

4.1 The effects of α at different l and Q when $q = 1$

The amplification factors when $q = 1$ are shown in figure 2. Let us consider the case of $l = 0$ (the solid lines) first. We see that the amplification factor is positive just in the region $0 < \omega < \frac{qQ}{r_+}$. This implies that it is indeed the superradiance. The superradiance region is enlarged by positive α and shrunk by negative α , due to the fact that the event horizon r_+ decreases for positive α and increases for negative α , as can be seen from (2.9). In each panel, we see that the amplification factor is enhanced by the positive α and suppressed by the negative α . From left panel to right panel, we also see that the superradiance is enhanced by the black hole charge Q . In the middle panel when $Q = 0.8$, there are plateaus of the amplification factor when α tends to make the black hole extremal. The amplification factor acts as a step-like function of ω . This implies all modes with frequency satisfying (3.9) are amplified almost equally. Beyond the superradiant condition, the reflected wave amplitude decreases so sharply that the amplification factor falls to -1 .

These behaviors can be understood intuitively from the effective potential which were plotted in figure 3. Note that the reflected amplitude comes from two sides. The one from the extracted energy near the horizon which should cross over the potential barrier to escape to the infinity, we denote as $\mathcal{R}_0$. The one reflected by the effective potential barrier of the incident wave, we denote as $\mathcal{R}_1$. We see that as α increases in each panel,

Figure 2. Effects of α on the amplification factor when $q = 1$. Solid lines for $l = 0$, dashed lines for $l = 1$. We take $Q = 0.2, 0.8$ and 1.4 as examples to exhibit our results. When $Q = 0.2$, the range of α is $(-7.92, 0.96)$. When $Q = 0.8$, the range of α is $(-6.66, 0.36)$. When $Q = 1.4$, the range of α is $(-2.61, -1.47)$. We vary α in the reasonable region. The lines with $\alpha = 0$ correspond to the RN black hole.

Figure 3. The effective potential corresponding to those with the same parameters in figure 2. Solid lines for $l = 0$, dashed lines for $l = 1$.

the effective potential barrier decreases (the solid lines). The extracted energy from the near horizon can escape to the infinity easier and leads to a larger reflected amplitude $\mathcal{R}_0$. Therefore the amplification factor increases as α increases. However, when the frequency ω is large enough, the superradiance ceases. Now $\mathcal{R}_0 \rightarrow 0$ and $\mathcal{R}_1$ dominates. The lower effective potential barrier, the smaller reflected amplitude $\mathcal{R}_1$. The total factor Z should decrease as α increases now. This phenomenon can be seen indeed in the far left panel of figure 2.

Now we turn to the cases of $l = 1$ (the dashed lines in figure 2 and figure 3). In each panel of figure 2, we see that compared to the cases of $l = 0$, the amplification factor is much suppressed in the region (3.9). But the reflected amplitude is nonzero due to the superradiance. It is in fact almost equal to the incident amplitude due to the superradiance. This phenomenon is very different from the neutral cases, where the reflected amplitude is suppressed heavily in the whole frequency region for larger l . Beyond the superradiance region (3.9), $\mathcal{R}_0 \rightarrow 0$ and $\mathcal{R}_1$ dominates. The factor Z of $l = 1$ is negative and larger than that of $l = 0$. This can also be understood from the effective potential, as shown in figure 3. The effective potential barrier increases with l . The waves with lower l are more likely to cross over the potential barrier and be absorbed by the black hole and thus leading to smaller $\mathcal{R}_1$, while those with higher l is more likely to be reflected by the higher barrier and thus leading to larger $\mathcal{R}_1$. So the factor Z is larger for larger l .

Figure 4. Effects of α on the amplification factor Z when $q = 5$. The other parameters are the same as those in figure 2. The lines with $\alpha = 0$ correspond to the cases of the RN black hole. The small wiggles for small ω are caused by numerical error.

4.2 The effects of α at different l and Q when $q = 5$

We take the same parameters as those in the last subsection, except by setting $q = 5$, to study the effect of q on the amplification factor. Unlike to Q , the parameter q is unlimited in principle. The amplification factors are shown in figure 4. In each panel, we see the similar behaviors as those in figure 2. But now the amplification factor is much enhanced in the whole frequency region by q . Even the higher l modes can have significant amplification factors. The superradiance region $0 < \omega < \frac{qQ}{r_+}$ is also enlarged by q . Interestingly, there appears a peak in the amplification factor before it falls. When $Q = 0.2$ and $\alpha = 0.95$ (the nearly extremal black hole), this peak rises to nearly 100% above the amplification factor of the lower frequency. When $Q = 0.8$, the step-like behavior of the amplification factor becomes more obvious. Note that though the superradiance region is changed by α , the amplification factor is almost unchanged with α . All the modes satisfying the condition (3.9) are amplified equally.

4.3 The condition for instability

We have shown the superradiance of the 4D charged EGB black hole under the charged scalar perturbation. Since the incident wave can be amplified when its frequency satisfies (3.9), one may suspect that the system is unstable. To clarify this, here we show the condition for the instability.

Multiplying (3.5) by the complex conjugate Ψ_0^* on both sides and integrating it, we get

$$\Psi_0^* \frac{\partial \Psi_0}{\partial r_*} \Big|_{-\infty}^{\infty} + \int_{-\infty}^{\infty} \left(\omega - \frac{qQ}{r} \right)^2 |\Psi_0|^2 dr_* = \int_{-\infty}^{\infty} V_{\text{eff}} |\Psi_0|^2 dr_* + \int_{-\infty}^{\infty} \left| \frac{\partial \Psi_0}{\partial r_*} \right|^2 dr_*. \quad (4.2)$$

The right hand side is real. Using the boundary condition (3.7) and taking the imaginary part of both sides, we get a relation

$$(a^2 + b^2)^{1/4} \cos \left(\frac{1}{2} \arctan \frac{b}{a} \right) + \omega_R - \frac{qQ}{r_+} + \int_{-\infty}^{\infty} 2\omega_I \left(\omega_R - \frac{qQ}{r_+} \right) \Psi_0^* \Psi_0 dr_* = 0. \quad (4.3)$$

Here $\omega = \omega_R + i\omega_I$ and $a = \omega_R^2 - \omega_I^2$, $b = 2\omega_R\omega_I$. Since $\arctan \frac{b}{a} \in (-\frac{\pi}{2}, \frac{\pi}{2})$, the first term is always positive.

The instability implies $\omega_I > 0$. When $\omega_I > 0$, there must be $\omega_R < \frac{qQ}{r_+}$ to keep the above formula hold. But when $\omega_R < \frac{qQ}{r_+}$, the sign of ω_I can not be determined. This means that the superradiance is the necessary but not sufficient condition for instability. To confirm the instability, one must ensure that the imaginary part of the frequency is positive.

In the following section, we will show numerically that the eigenfrequencies of the system have always negative imaginary part. Therefore, the system is stable, though it has superradiance. In fact, to trigger the instability, there should be an effective potential well outside the event horizon to trap the reflected wave from the near horizon region. However, it is easy to show that no potential well outside the event horizon and thus no instability of this background under charged scalar perturbations.

5 The stability of the regularized 4D charged EGB black hole

We studied the amplification factor by considering the system as a scattering process. To get the eigenfrequencies of the charged scalar perturbation, we should take the system as an eigenvalue problem. The boundary condition now was chosen as

$$\Psi_0 \rightarrow \begin{cases} e^{-i\left(\omega - \frac{qQ}{r_+}\right)r_*} \sim (r - r_+)^{-\frac{i}{2\kappa}\left(\omega - \frac{qQ}{r_+}\right)}, & r \rightarrow r_+, \\ e^{i\omega r_*} \sim r^{i\omega r_+} e^{i\omega r}, & r \rightarrow \infty, \end{cases} \quad (5.1)$$

where we used

$$r_* \rightarrow \begin{cases} \frac{1}{2\kappa} \ln(r - r_+), & r \rightarrow r_+. \\ r + r_+ \ln(r - r_+), & r \rightarrow \infty. \end{cases} \quad (5.2)$$

There is only ingoing waves near the event horizon and outgoing waves at the infinity. The system is dissipative and the frequency of the perturbations will be the composition of quasinormal modes (QNMs). Many numerical methods are developed to solve the quasinormal modes, such as the shooting method, the WKB approximation method, the Horowitz-Hubeny method and the continued fraction method (CFM) [64]. But here we will use the asymptotic iteration method (AIM) [56, 57].

5.1 The asymptotic iteration method

The AIM was used to solve the eigenvalue problem of the homogeneous second order differential equation [65, 66]. Let us introduce a new variable first

$$\xi = 1 - \frac{r_+}{r}. \quad (5.3)$$

It ranges from 0 to 1 as r runs from the event horizon to the infinity. The radial equation (3.5) becomes

$$0 = \frac{f(1-\xi)^2}{r_+} \left(\frac{\partial^2 \Psi_0}{\partial \xi^2} \frac{f(1-\xi)^2}{r_+} + \frac{\partial \Psi_0}{\partial \xi} \frac{(1-\xi)^2 \partial_\xi f - 2f(1-\xi)}{r_+} \right) + \left(\left(\omega - \frac{qQ}{r_+}(1-\xi) \right)^2 - \frac{(1-\xi)^2}{r_+^2} f(l(l+1) + (1-\xi)\partial_\xi f) \right) \Psi_0. \quad (5.4)$$

The solution satisfying the asymptotic behavior (5.1) in terms of ξ has the following form

$$\Psi_0 = \xi^{-\frac{i}{2\kappa}\left(\omega - \frac{qQ}{r_+}\right)} \left(\frac{1}{1-\xi}\right)^{i\omega r_+} e^{i\frac{\omega r_+}{1-\xi}} \chi(\xi), \quad (5.5)$$

in which $\chi(\xi)$ is a regular function of ξ in range $(0, 1)$. Function $\chi(\xi)$ obeys a homogeneous second order differential equation

$$\frac{\partial^2 \chi}{\partial \xi^2} = \lambda_0(\xi) \frac{\partial \chi}{\partial \xi} + s_0(\xi) \chi, \quad (5.6)$$

in which the coefficients

$$-\lambda_0(\xi) = -\frac{i}{\kappa\xi} \left(\omega - \frac{qQ}{r_+}\right) + \frac{f'(\xi)}{f(\xi)} + \frac{2(\xi - i\xi r_+ \omega + 2ir_+ \omega - 1)}{(\xi - 1)^2}, \quad (5.7)$$

$$\begin{aligned} -s_0(\xi) = & -\frac{1}{(\xi - 1)^4 \xi^2} \left(\frac{(\xi - 1)^2}{2\kappa} (\xi^2 (2r_+ \omega + i) - 4\xi r_+ \omega - i) \left(\omega - \frac{qQ}{r_+}\right) \right) \\ & - \frac{1}{(\xi - 1)^4 \xi^2} \left(\frac{(\xi - 1)^4}{4\kappa^2} \left(\omega - \frac{qQ}{r_+}\right)^2 + \xi^2 r_+ \omega (r_+ \omega (\xi - 2)^2 + i(\xi - 1)^2) \right) \\ & + i \frac{f'(\xi)}{(\xi - 1)^2 \xi f(\xi)} \left(\xi (2r_+ \omega + i) - \frac{(\xi - 1)^2}{2\kappa} \left(\omega - \frac{qQ}{r_+}\right) - \xi^2 (r_+ \omega + i) \right) \\ & - \frac{l(l+1)\xi}{(\xi - 1)^2 \xi f(\xi)} + \frac{1}{(\xi - 1)^4 f(\xi)^2} ((\xi - 1)qQ + r_+ \omega)^2. \end{aligned} \quad (5.8)$$

The coefficients $\lambda_0(\xi)$ and $s_0(\xi)$ are analytical functions in the interval $(0, 1)$. Now differentiating (5.6) with respect to ξ iteratively, we get an $(n+2)$ -th order differential equation

$$\chi^{(n+2)} = \lambda_n(\xi) \chi'(\xi) + s_n(\xi) \chi(x), \quad (5.9)$$

where the coefficients can be determined iteratively.

$$\begin{aligned} \lambda_n(\xi) &= \lambda'_{n-1}(\xi) + s_{n-1}(\xi) + \lambda_0(\xi) \lambda_{n-1}(\xi), \\ s_n(\xi) &= s'_{n-1}(\xi) + s_0(\xi) \lambda_{n-1}(\xi). \end{aligned} \quad (5.10)$$

In asymptotic iteration method, the cutoff of the iteration for large enough n is determined by

$$\frac{s_n(\xi)}{\lambda_n(\xi)} = \frac{s_{n-1}(\xi)}{\lambda_{n-1}(\xi)}. \quad (5.11)$$

Then the quasinormal modes ω can be worked out from this “quantization condition”. To improve the efficiency and accuracy, let us making an expansion around a regular point $\xi = \xi_0$,

$$\lambda_n(\xi) = \sum_{j=0}^{\infty} c_n^j (\xi - \xi_0)^j, \quad s_n(\xi) = \sum_{j=0}^{\infty} d_n^j (\xi - \xi_0)^j. \quad (5.12)$$

Figure 5. Effects of α on the fundamental quasinormal modes of the charged scalar perturbation when $q = 1$. Left panel for real part and right panel for imaginary part of the QNMs. We take $Q = 0.2, 0.5, 0.8$ and 1.2 as examples to exhibit our results. When $Q = 0.2$, the range of α is $(-7.92, 0.96)$. When $Q = 0.5$, the range of α is $(-7.49, 0.75)$. When $Q = 0.8$, the range of α is $(-6.66, 0.36)$. When $Q = 1.2$, the range of α is $(-4.68, -0.44)$. We vary α in the reasonable regions, respectively. The gray lines are corresponding threshold $\frac{qQ}{r_+}$ for superradiance since r_+ depends on α .

The expansion coefficients c_n^j and d_n^j are functions of frequency ω . Substituting the expansions into (5.10), we get iterative relations between these expansion coefficients.

$$c_n^j = (j+1)c_{n-1}^{j+1} + d_{n-1}^j + \sum_{k=0}^j c_0^k c_{n-1}^{j-k}, \quad (5.13)$$

$$d_n^j = (j+1)d_{n-1}^{j+1} + \sum_{k=0}^j d_0^k c_{n-1}^{j-k}. \quad (5.14)$$

In terms of the expansion coefficients, the “quantization condition” becomes

$$d_n^0 c_{n-1}^0 = d_{n-1}^0 c_n^0. \quad (5.15)$$

From this “quantization condition”, we can get the frequency ω of the perturbation. The efficiency and accuracy of AIM depends on the expansion point ξ_0 . We will ensure the reliability of the results by varying the expansion point ξ_0 and the iteration times n . We can reproduce the table I of [21] exactly when $Q \rightarrow 0$.

5.2 The eigenfrequency of perturbation

We plot the fundamental quasinormal modes when $q = 1$ in figure 5. In the left panel, we see that the real part ω_R of the fundamental QNMs is a concave function of α . It also increases monotonically with Q . The most interesting point is that all ω_R live above the threshold $\frac{qQ}{r_+}$ of superradiance. For positive α or large Q , the real part ω_R is very close to the threshold. Therefore there should not be instability of the charged 4D EGB black hole under perturbations. We confirm this in the right panel by noticing that all the imaginary part ω_I of the QNMs are negative. ω_I increases with α at first and then keeps

Figure 6. The overtones of the QNMs when $Q = 0.8, l = 0$. α ranges from -1 to 0.35, where the black hole is nearly extremal. The left hand side is for $q = 1$, the right hand side for $q = 5$. The case of $\alpha = 0$ corresponds to RN black hole.

nearly unchanged before it increases with α again. Interestingly, for positive α , ω_I tends to 0 rapidly when the black hole becomes extremal. This implies the extremal 4D charged black hole may have normal modes. For negative α , the imaginary part ω_I can not reach 0 and rest on a finite negative value. These phenomenon also exists for higher l .

We show the overtones of the QNMs when $Q = 0.8$ and $l = 0$ in figure 6. The green lines corresponds to the cases of RN black hole. For positive α , we see that the imaginary part of the overtones increases with α . Note that when α makes the black hole nearly extremal, the imaginary part of the fundamental modes tends to zero for both $q = 1$ and $q = 5$. For negative α , the imaginary part of the frequency decreases. When q is small, the real part of the overtones changes very small. While for large q , the real part changes significantly.

6 Summary and discussion

We studied the parameter region of the regularized 4D charged EGB black hole where allows the event horizon. The black hole charge can be larger than the black hole mass due to the existence of negative GB coupling constant α .

With the help of appropriate boundary condition, the condition for superradiance was derived. We analyzed the effects of α on the amplification factor of the superradiance in detail. The positive α enhances the superradiance and the negative α suppresses it. Almost all the frequencies satisfying the superradiance condition region are amplified equally. Beyond this region, the reflected wave vanishes rapidly and the amplification factor falls like a step-like function. We analyzed this phenomenon from the viewpoint of effective potential.

To confirm whether the 4D charged EGB black hole has instability, we worked out the QNMs of the system using the asymptotic iteration method. We found that all the real part of the QNMs live beyond the superradiance condition (3.9). The imaginary part of QNMs are negative. Therefore here is no instability. We studied the effects of α on the QNMs in detail. For α that makes black hole extremal, we found that there may be normal modes. The normal modes were also found for the RN black hole under the massive charged scalar or the Proca field perturbations provided $\mu M = qQ$, where μ is the mass of the scalar or Proca field [67, 68]. The fields are marginally bound to the black hole and were called marginal clouds. Note that marginal clouds exist even if the black hole is not extremal. Their existence was attributed to the balance of the electromagnetic and gravitational forces there. One may suspect that the normal modes found here is also due to the balance of the repulsive GB term and the gravitational force. However, the normal modes found here exist only when the black hole is extremal and the charged scalar is massless. Further detailed study on this phenomenon is required. In fact, the stability of extremal black hole may be different qualitatively from that of non-extremal black hole. There is horizon instability universally [69].

We further proved the condition for instability of the 4D charged EGB black hole. Though there is superradiance, the system is stable under perturbations due to the absence of an effective potential well outside the event horizon to accumulate the energy and bounce them back to the black hole again. The potential well can be induced by an artificial mirror outside the horizon or by a massive scalar. It has been shown that asymptotically-flat charged black holes are stable against massive charged scalar perturbations [70, 71], but unstable for mirror boundary [72–75]. Since the potential well also appears in asymptotic dS or AdS spacetime, one can expect the superradiant instability of the 4D charged EGB-(A)dS black holes [76, 77]. In the following paper [78, 79], we will study them in detail.

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