Inflaxion dark matter

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ABSTRACT: A new mechanism for producing axion dark matter is proposed. By invoking low-scale inflation and a kinetic mixing between the axion and the inflaton, it is shown that the axion is driven to a field point slightly displaced from the potential minimum, which can give rise to the observed dark matter abundance. In this framework, different combinations of the axion and inflaton fields play various cosmological roles, including generating the cosmological perturbations, reheating the universe, and serving as dark matter. The kinetic mixing also relates the dark matter lifetime with the reheating temperature. The mechanism tames axions that would otherwise overdominate the universe, and thus opens up new windows in the axion parameter space, including decay constants at the GUT scale and higher.

Contents

1 Introduction

The remarkable agreement between measurements of the cosmic microwave background (CMB) and the Lambda Cold Dark Matter (ΛCDM) model [1] gives us confidence that we understand well several aspects of the early history of the universe. The existing data supports the picture that the Hot Big Bang cosmology started with inflation, a period of exponentially accelerated expansion [2–4]. The simplest models of inflation involve a scalar field slowly rolling down a flat potential. We know neither at what scale the process took place, nor the temperature at which the plasma was reheated at the end of it. We are fairly certain that the reheat temperature must be above the MeV scale for the Big Bang Nucleosynthesis (BBN) to successfully take place. Most of the known mechanisms for generating the matter-antimatter asymmetry require even higher reheating temperatures, however there are also baryogenesis models that can operate at scales not much above BBN [5]. While (very) low-scale inflation remains as a possibility, the improved observational limit on primordial gravitational waves has tightened the bound on the inflation scale from above, now disfavoring simple high-scale inflation models composed of single monomial terms [6].

Another feature essential to our understanding of the universe is the existence of cold dark matter. Its actual nature remains a mystery, but several candidates have been proposed. Among them, axions provide a viable and well studied possibility; these include the QCD axion which solves the strong CP problem [7–9], as well as axion-like fields that can arise for instance from string theory compactifications [10–12]. Axions can have a wide range of masses and couplings to matter, depending on the theoretical framework in which they are embedded. Two properties of axions are of particular importance in this study: (i) that they are pseudo Nambu-Goldstone Bosons (pNGBs) of a spontaneously broken global U(1) symmetry, and as such obey a shift symmetry, (ii) that an initial displacement of the axion field from its potential minimum gives rise to a relic abundance of axion dark matter in the later universe.

The most well-studied mechanism that gives rise to the axion displacement is the vacuum misalignment scenario, in which the symmetry breaking happens before or during the inflationary epoch, and thus inflation renders the axion field homogeneous within the observable universe. The axion stays frozen at this initial field value while its mass is smaller than the Hubble expansion rate of the early universe, then eventually starts to oscillate and behaves as cold dark matter. For an axion-like field whose mass stays constant since the inflationary epoch, the vacuum misalignment gives rise to a present-day abundance of

$$\Omega_\sigma \sim 0.1 \times \left(\frac{\theta_\star f}{10^{17}\,\mathrm{GeV}} \right)^2 \left(\frac{m_\sigma}{10^{-22}\,\mathrm{eV}} \right)^{1/2}. \tag{1.1}$$

Here m_σ is the axion mass, f is the decay constant, and $\theta_\star$ is the misalignment angle which parametrizes the initial field displacement as $\sigma_\star = \theta_\star f$; considering the axion potential to have a periodicity $\sigma \to \sigma + 2\pi f$, the angle lies within the range $-\pi < \theta_\star < \pi$. An axion with $|\theta_\star| \sim 1$ makes up the observed dark matter abundance if, for instance, the decay constant is $f \sim 10^{17}\,\mathrm{GeV}$ and the mass $m_\sigma \sim 10^{-22}\,\mathrm{eV}$. This also implies that if $f \gtrsim 10^{17}\,\mathrm{GeV}$, the axion would overdominate the universe and give rise to the so-called cosmological moduli problem unless $\theta_\star$ is sufficiently smaller than unity and/or the mass satisfies $m_\sigma \lesssim 10^{-22}\,\mathrm{eV}$. An ultralight axion dark matter with $m_\sigma \sim 10^{-22}\,\mathrm{eV}$ has been a subject of active investigation over the years as it leaves distinct signatures in the small-scale structures of the universe [13] (see e.g. [12, 14] for reviews). However it was recently pointed out that these are in tension with observational data on the Lyman-α forest [15–17] and galaxy rotation curves [18]. In the vacuum misalignment scenario, the relevant dynamics for the axion field takes place after inflation is over, and the abundance is derived by solving the equation of motion of the single axion field, assuming that couplings to other fields can be neglected.

In this paper we explore the possibility that the axion couples to the inflaton via a kinetic mixing term. From the point of view of an effective field theory, with the axion and the inflaton being the relevant degrees of freedom, there is no *a priori* reason to forbid such a mixing as it respects the axion shift symmetry. The main goal of this work is to demonstrate that this kinetic mixing gives rise to a new mechanism for producing axion dark matter. We find the most interesting scenario to be when the Hubble scale of inflation, H_{inf}, is smaller than the axion mass, so that the axion possesses a potential

that noticeably affects its dynamics during inflation. In this case, without a kinetic mixing the axion would simply be subject to damped oscillations, would relax to the minimum of its potential where its energy density vanishes, and thus would not contribute to the dark matter abundance. We will show that the kinetic mixing keeps the axion displaced from its minimum during inflation, thus providing a source for misalignment. This opens up new windows of the parameter space of axion dark matter, relevant to some experimental searches. In the diagonal basis, the oscillating field that serves as dark matter is a linear combination of the inflaton and axion. Hence, we dub it the *inflaxion*.

Kinetic mixings among multiple axions have been considered in previous literature [19–23], but to our knowledge not in the context of inflation. Our inflaton can also be axion-like, but this is not a requirement necessary to the end of our proposed mechanism.

The axion, being a pNGB, is expected to be light. As in this work we focus on inflationary Hubble scales lower than m_σ, we are led to consider low-scale models of inflation. In refs. [24, 25] the authors considered the QCD axion, without any direct coupling to the inflaton, and studied the regime $m_\sigma < H_{\rm inf} < \Lambda_{\rm QCD}$. In such a case the axion already has a potential during inflation, but is not oscillating; classically the field slowly rolls toward its minimum, but its quantum fluctuations tend to keep it displaced from the minimum. Given a long enough period of inflation, the two competing effects lead to an equilibrium configuration which results in a probabilistic distribution for the misalignment angle, typically peaked at values $|\theta_\star| \ll 1$. In turn, this opens up the large f window. However for this mechanism to be predictive, it is important to reach equilibrium; this requires an extraordinarily long period of inflation of 10^{20} *e*-folds or more, which could pose a challenge for inflationary model building.[1] In our scenario, on the other hand, we consider even lower inflation scales of $H_{\rm inf} < m_\sigma$. The field evolution will be dominated by purely classical dynamics, and there will be no strict requirement on the number of inflationary *e*-folds.

Our mechanism can be applied to generic axions. We discuss both the QCD axion and axion-like particles in section 2 where we present the basic construction of our mechanism. Then in section 3 we describe how it can produce dark matter and give rise to a consistent cosmological history. We explore the parameter space in section 4; here, with the aim of concentrating on the salient features of the dynamics while avoiding complications, we mainly focus on axion-like particles whose mass stays constant throughout the cosmic evolution. Then we conclude with a discussion of directions for further research in section 5. Some calculational details and possibilities with the QCD axion are left to the appendices. In appendix A we introduce a field basis that diagonalizes the kinetic and mass terms of the inflaton-axion system. In appendix B we explain axion misalignment and slow-roll inflation in the diagonal basis. In appendix C we comment on the implications for the QCD axion.

2 Inflaton-axion mixing

2.1 Axions coupled to a strong sector

Let us start by considering an axion that is coupled to some strong sector. As an example of such a field, we focus on the QCD axion. Then our mechanism is described by the

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[1]For other interesting ideas on axion dark matter with low-scale inflation, see e.g. [26, 27].

following simple model where the axion σ is derivatively coupled to another real field ϕ,

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma + \frac{g_s^2}{32\pi^2}\frac{\sigma}{f}G_{A\mu\nu}\tilde{G}_A^{\mu\nu} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \alpha\,g^{\mu\nu}\partial_\mu\phi\partial_\nu\sigma, \quad (2.1)$$

where f is the axion decay constant, $G_{A\mu\nu}$ the gluon field strength. We refer to ϕ as the inflaton, since we will be supposing that its effective potential $V(\phi)$ has a shape capable of inducing cosmic inflation as well as providing a graceful exit from it. The axion field, odd under parity, respects a discrete shift symmetry $\sigma \to \sigma + 2\pi f$, which is not violated by the kinetic mixing term. By requiring the eigenvalues of the kinetic matrix to be positive (otherwise one of the physical degrees of freedom would be non-dynamical or a ghost), the dimensionless coupling α of the kinetic mixing is restricted to lie within the range

$$-1 < \alpha < 1. \quad (2.2)$$

The kinetic terms can be diagonalized by redefining the fields, for instance, as

$$\sigma = \frac{1}{\sqrt{1-\alpha^2}}\Sigma, \quad \phi = \Phi - \frac{\alpha}{\sqrt{1-\alpha^2}}\Sigma, \quad (2.3)$$

for which the Lagrangian takes the form

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\Sigma\partial_\nu\Sigma + \frac{g_s^2}{32\pi^2}\frac{\Sigma}{F}G\tilde{G} - \frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - V\left(\Phi - \frac{\alpha}{\sqrt{1-\alpha^2}}\Sigma\right). \quad (2.4)$$

Ignoring for a moment the potential V, then Σ is a standard canonical axion with a decay constant $F = \sqrt{1-\alpha^2}\,f$. The coupling to the gluons yields an effective potential for Σ with periodicity $2\pi F$ which can roughly be approximated by a cosine,

$$U(\Sigma, T) = m_\Sigma^2(T)\,F^2\left\{1 - \cos\left(\frac{\Sigma}{F}\right)\right\}. \quad (2.5)$$

The potential vanishes at temperatures much higher than the QCD scale $\Lambda_{\rm QCD} \approx 200\,{\rm MeV}$, while at temperatures below $\Lambda_{\rm QCD}$, its mass at the minimum is [28]

$$m_\Sigma \approx 6 \times 10^{-6}\,{\rm eV}\left(\frac{10^{12}\,{\rm GeV}}{F}\right) \quad \text{for } T \ll \Lambda_{\rm QCD}. \quad (2.6)$$

Going back to the (σ, ϕ) basis, σ similarly obtains an effective potential, but with a periodicity $2\pi f$; rewriting as $(\Sigma, F) \to (\sigma, f)$ in (2.5) and (2.6) gives the potential and mass for σ.

Now, let us consider the effects of V. The shift symmetry of σ, which was manifest in (2.1), is realized in (2.4) as a symmetry under a shift of both Σ and Φ that keeps the combination $\Phi - (\alpha/\sqrt{1-\alpha^2})\Sigma$ fixed. If we assume that ϕ is also parity-odd and that the minimum of V preserves parity, then our theory in the vacuum respects P and CP. (The assumption of CP conservation is not necessary for the dark matter production mechanism we describe later, but is useful for illustrating the nature of the vacuum.) Setting the minimum of $V(\phi)$ as $\phi_{\rm min} = 0$, then the global minima of the full scalar potential at zero temperature, $U(\Sigma, T=0) + V$, are at the CP conserving points

$$\Sigma_{\rm min} = 2\pi kF, \quad \Phi_{\rm min} = \frac{\alpha}{\sqrt{1-\alpha^2}}2\pi kF, \quad (2.7)$$

with k an integer. In Minkowski spacetime, displacing the fields from a minimum results in coupled oscillations. The fields' values averaged over a period longer than the timescale of the oscillations still correspond to Σ_{min} and Φ_{min}. However in an expanding universe, given that V is sufficiently flat, the Hubble friction can keep the field Φ in slow-roll motion while it approaches a minimum. When Φ is effectively frozen at a point away from the minima, the axion Σ would also get displaced from its CP-conserving minima. The same effect is seen in the original basis (2.1) as V rendering the field ϕ spacetime-dependent and thus sourcing $\Box\phi$; this in turn generates an effective linear potential for σ via the kinetic mixing. We discuss these dynamics in detail in section 3.

We should also remark that from an effective field theory point of view, there can further be potential couplings of the form $I(\phi)\cos(\sigma/f)$ which do not violate the axion's discrete shift symmetry. Throughout this paper we assume such terms to be negligible. However even if they were not, our mechanism of axion production can work, but in parameter regions different from what we show in the following discussions.

2.2 General axions

In the following, we will also be discussing axion-like fields which are not necessarily coupled to any strong sector. Hence hereafter we consider σ to be a pNGB of some global U(1) symmetry that is explicitly broken (for instance by quantum gravitational effects [29, 30]), and analyze a model of the form

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \alpha\,g^{\mu\nu}\partial_\mu\phi\partial_\nu\sigma + L_c[\sigma,\phi,\Psi]. \quad (2.8)$$

The mass term of the axion is understood to arise from expanding the periodic potential around one of the minima, and the axion mass m_σ may or may not depend on the temperature. L_c represents the couplings of σ and ϕ to other matter fields which we have collectively denoted as Ψ; its details will later be discussed.

We will see in the following sections that different combinations of ϕ and σ — the inflaxions — will play various cosmological roles such as a canonical inflaton producing the cosmological perturbations, a reheaton for heating up the universe, and dark matter.

3 Inflaxion mechanism

Let us now show how the model (2.8) produces dark matter, and gives rise to a consistent cosmological history. We consider a flat FRW universe with metric

$$ds^2 = -dt^2 + a(t)^2 d\boldsymbol{x}^2, \quad (3.1)$$

in which the homogeneous equations of motion of the fields read

$$(1-\alpha^2)(\ddot{\sigma} + 3H\dot{\sigma}) = \alpha V'(\phi) - m_\sigma^2\sigma, \quad (3.2)$$

$$(1-\alpha^2)(\ddot{\phi} + 3H\dot{\phi}) = -V'(\phi) + \alpha\, m_\sigma^2\sigma. \quad (3.3)$$

Here we have supposed the axion mass m_σ to be a constant, and ignored the matter coupling L_c. An overdot represents a t-derivative, $H = \dot{a}/a$ is the Hubble rate, and $V'(\phi) = dV(\phi)/d\phi$.

3.1 Low-scale inflation

The inflaton potential $V(\phi)$ is assumed to give rise to a sufficiently long period of cosmic inflation, produce curvature perturbations in good agreement with observation, and provide a graceful exit from inflation.[2] Our mechanism is insensitive to the detailed form of the inflaton potential, however, we assume the energy scale of inflation to be sufficiently low such that the U(1) symmetry is broken before inflation, and also that the axion potential exists during inflation. We identify the symmetry breaking scale with the axion decay constant f, and in cases where the axion potential arises only below some energy scale, we denote this by Λ (which is $\Lambda_{\rm QCD}$ for a QCD axion). Then our requirements translate into a bound on the de Sitter temperature during inflation,

$$\frac{H_{\rm inf}}{2\pi} < f, \Lambda, \tag{3.4}$$

where $H_{\rm inf}$ represents the typical value of the Hubble rate during inflation.

Furthermore, we assume the inflationary scale to be even lower than the axion mass,

$$H_{\rm inf} < m_\sigma. \tag{3.5}$$

This forces the axion to undergo a damped oscillation along its potential, and thus eventually be stabilized at the minimum.[3] However, with the kinetic mixing with the inflaton, the axion's effective potential minimum is shifted from the origin. This is seen by setting the left hand side of (3.2) to zero to obtain the quasi-static solution,

$$\sigma \simeq \frac{\alpha V'(\phi)}{m_\sigma^2} \simeq -\frac{3\alpha H\dot\phi}{m_\sigma^2}. \tag{3.6}$$

Upon moving to the far right hand side, we have assumed the inflaton potential to satisfy the slow-roll conditions and used $3H\dot\phi \simeq -V'$. A more detailed derivation of this solution, together with the verification of the slow-roll approximation for the non-canonical ϕ, are given in appendix B. Because the axion settles down to the field value (3.6) during inflation, the initial axion misalignment for the post-inflationary evolution is uniquely fixed by the Lagrangian parameters, unlike the case for the vacuum misalignment scenario.

Let us set the minimum of $V(\phi)$ where the inflaton settles down to after the end of inflation as $\phi = 0$. It is seen from (3.6) that the field displacements during inflation take hierarchical values,

$$\left| \frac{\sigma}{\alpha\phi} \right| \simeq \left| -3\frac{H^2}{m_\sigma^2} \frac{\dot\phi}{H\phi} \right| \ll 1, \tag{3.7}$$

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[2]If the third and higher order derivatives of $V(\phi)$ are negligible, then one can diagonalize the fields such that they do not interact with each other. Our assumption for $V(\phi)$ can thus be rephrased as the requirement that one of the diagonal fields possesses a flat enough potential to drive inflation. See appendix B for detailed discussions.

[3]When the axion mass is not much larger but only comparable to the Hubble scale, the axion would not oscillate but instead rapidly roll towards the minimum [31, 32]. Hence also in this case the axion approaches the solution (3.6).
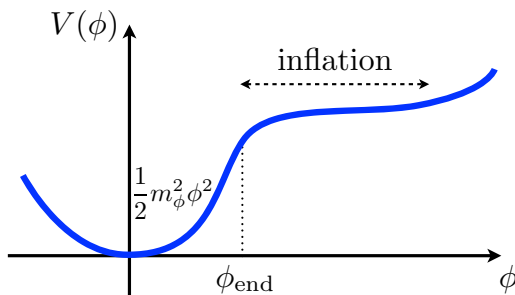
Figure 1. Schematic of inflaton potential for small-field inflation.

where the inequality follows from (3.5) and $|\dot{\phi}/H\phi| \ll 1$ (otherwise inflation would quickly end within a Hubble time).

With the condition (3.5), inflation is effectively a single-field model. (The canonical inflaton field whose fluctuation sources curvature perturbations is given by a linear combination of σ and ϕ; this is shown as φ_{L} in (A.8).) Therefore no isocurvature fluctuations of dark matter is produced. We also remark that due to the low inflation scale, the inflationary mechanism is envisaged to be a small-field model with tiny variation of the Hubble rate during inflation.[4]

3.2 End of inflation

Slow-roll inflation comes to an end when the ϕ field approaches a point along the potential where it is no longer flat. The field then rolls rapidly towards its potential minimum at $\phi = 0$, and starts to oscillate. We suppose that the potential around the origin is approximately quadratic, up to the field value ϕ_{end} where inflation ends:

$$V(\phi) \simeq \frac{1}{2}m_\phi^2\phi^2 \quad \text{for} \quad |\phi| \lesssim |\phi_{\mathrm{end}}|. \tag{3.8}$$

A schematic of the inflaton potential we are envisaging can be seen in figure 1. Then since the end of inflation corresponds to the time when $V \simeq 2M_p^2 H^2$ is satisfied (cf. (B.10)), the inflaton field value at this point is obtained as

$$\phi_{\mathrm{end}}^2 \simeq \left(\frac{2M_p H_{\mathrm{end}}}{m_\phi}\right)^2, \tag{3.9}$$

with H_{end} denoting the Hubble rate at the end of inflation.

During inflation, the inflaton's effective mass is smaller than the Hubble rate, and hence also smaller than the axion mass, $|V''| \ll H_{\mathrm{inf}}^2 < m_\sigma^2$. However the mass m_ϕ at the vacuum can be much larger than $|V''|^{1/2}$ during inflation; this is particularly so for small-field inflation models in which various potential terms conspire to realize a flat potential in some field range. It will turn out that, in order for the inflaton-axion system to provide

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[4]However, in the presence of spectator fields that predominantly produce the curvature perturbations (such as curvatons), large-field models can also account for low inflation scales, see e.g. [33].

a dark matter candidate, the inflaton mass after inflation needs to be larger than that of the axion, i.e.,

$$m_\phi^2 \gg m_\sigma^2. \tag{3.10}$$

Let us thus impose this condition hereafter.

In order to analyze the field dynamics in the post-inflationary epoch, let us redefine the fields such that the kinetic and mass terms in the Lagrangian are diagonalized,

$$\frac{\mathcal{L}}{\sqrt{-g}} = \sum_{i=\mathrm{DM,RH}} \left(-\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi_i \, \partial_\nu \varphi_i - \frac{1}{2} m_i^2 \varphi_i^2 \right) + L_c[\sigma, \phi, \Psi]. \tag{3.11}$$

The exact expressions for the diagonal fields and their masses are given in appendix A. Here, under the mass hierarchy (3.10), the expressions at the leading order in m_σ^2/m_ϕ^2 take a simple form of

$$\varphi_{\mathrm{DM}} \simeq \alpha \, \phi + \sigma, \quad \varphi_{\mathrm{RH}} \simeq \sqrt{1-\alpha^2} \left(\phi - \alpha \frac{m_\sigma^2}{m_\phi^2} \, \sigma \right), \tag{3.12}$$

$$m_{\mathrm{DM}}^2 \simeq m_\sigma^2, \qquad m_{\mathrm{RH}}^2 \simeq \frac{m_\phi^2}{1-\alpha^2}, \tag{3.13}$$

where in (3.12) we have displayed each of the coefficients of ϕ and σ at the leading order. Each of the fields φ_{DM} and φ_{RH} undergoes harmonic oscillations about the origin, and thus behaves as pressureless matter in the post-inflationary epoch. φ_{DM} has approximately the same mass as the axion, and will serve as a dark matter candidate, as its subscript indicates. φ_{RH}, on the other hand, is much heavier (note that from (2.2) and (3.10) follows $m_{\mathrm{DM}}^2 \ll m_{\mathrm{RH}}^2$), and this field will reheat the universe via its decay.

We defer the discussions on decay to the next section, and here we estimate the energy densities of the oscillating fields. As we saw in (3.7), the field values of ϕ and σ obey a hierarchy $|\alpha\phi| \gg |\sigma|$ during inflation. This can break down towards the end of inflation, depending on the details of the exit from inflation. However let us suppose that at the end of inflation the inequality is still marginally satisfied, i.e., $|\alpha\phi_{\mathrm{end}}| \gtrsim |\sigma_{\mathrm{end}}|$, and write the field displacement of φ_{DM} using (3.9) as

$$\varphi_{\mathrm{DM\,end}}^2 = \left(\frac{C\alpha M_p H_{\mathrm{end}}}{m_\phi} \right)^2. \tag{3.14}$$

Here C is a model-dependent dimensionless parameter that is typically larger than unity due to the factor 2 in (3.9) and the contribution from σ_{end}, but not much larger, say, $C \sim 10$. Considering φ_{DM} to start oscillating with this initial amplitude at the end of inflation, the ratio between its energy density which is initially dominated by the potential energy, $\rho_{\mathrm{DM}} \simeq m_{\mathrm{DM}}^2 \varphi_{\mathrm{DM}}^2 / 2$, and the total density of the universe $\rho_{\mathrm{tot}} = \rho_{\mathrm{DM}} + \rho_{\mathrm{RH}} = 3 M_p^2 H^2$ is obtained as

$$\frac{\rho_{\mathrm{DM}}}{\rho_{\mathrm{tot}}} \simeq \frac{C^2 \alpha^2}{6} \frac{m_\sigma^2}{m_\phi^2}. \tag{3.15}$$
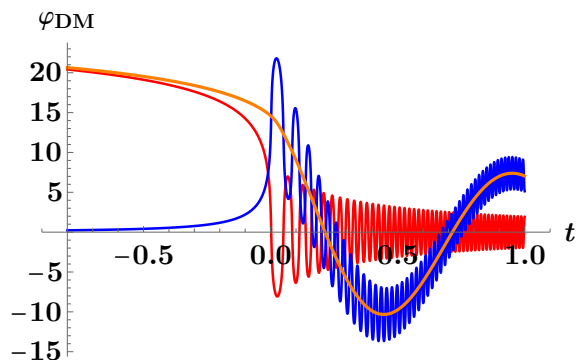
Figure 2. Field evolution at the end of inflation for $\varphi_{\rm DM}$ (orange), σ (blue), and $\alpha\phi$ (red). Field values are normalized by $\alpha M_p H_{\rm end}/m_\phi$. Time is in units of $2\pi/m_\sigma$, with $t = 0$ denoting when $-\dot{H}/H^2 = 1$. Shown is the case of $V(\phi) \propto 1 - {\rm sech}(\phi/\mu)$, with $m_\sigma \approx 10 H_{\rm inf}$ and $m_\phi \approx 10^3 H_{\rm inf}$.

This density ratio stays constant while the fields oscillate harmonically,[5] since both $\rho_{\rm DM}$ and $\rho_{\rm RH}$ redshift as $\propto a^{-3}$. One sees that the density fraction of $\varphi_{\rm DM}$ is suppressed by the mass hierarchy (3.10), and thus the universe is dominated by $\varphi_{\rm RH}$, i.e., $\rho_{\rm DM} \ll \rho_{\rm RH} \simeq \rho_{\rm tot}$.

The actual field evolutions at around the end of inflation can be seen in figure 2. For this plot we numerically solved the equations of motion (3.2) and (3.3) together with the Friedmann equation, for a case where the inflaton potential is of the form[6]

$$V(\phi) = m_\phi^2 \mu^2 \left(1 - \frac{2}{e^{\phi/\mu} + e^{-\phi/\mu}} \right). \tag{3.16}$$

Shown are time evolutions of $\varphi_{\rm DM} = \alpha\phi + \sigma$ (orange curve), σ (blue), and $\alpha\phi$ (red), with the field values normalized by $\alpha M_p H_{\rm end}/m_\phi$. The horizontal axis shows the dimensionless time in units of $2\pi/m_\sigma$ (i.e. the oscillation period of $\varphi_{\rm DM}$), and $t = 0$ is set to the end of inflation when $-\dot{H}/H^2$ becomes unity. The parameters are chosen as $\mu = 4 \times 10^{15}\,{\rm GeV}$, $m_\phi = 10^2 m_\sigma$, and $|\alpha| \ll 1$. These conditions are enough to specify the field dynamics in terms of the normalized quantities; in particular, the plot is independent of the explicit values of m_ϕ and α. The inflation scale for the potential (3.16) is $H_{\rm inf} \simeq m_\phi \mu/\sqrt{3} M_p$, hence our choice of μ gives $m_\phi \approx 10^3 H_{\rm inf}$. One clearly sees through this example that the field $\varphi_{\rm DM}$ starts oscillating at around the end of inflation[7] with a frequency $\sim m_\sigma$, and an initial amplitude (3.14) whose numerical coefficient C is of order 10. The non-diagonal fields σ and ϕ further carry an oscillatory mode with frequency $\sim m_\phi/\sqrt{1 - \alpha^2}$, which corresponds to the $\varphi_{\rm RH}$ component.

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[5]Depending on the inflaton potential at around the end of inflation, the fields may exchange energy through a parametric resonance; then the dark matter fraction might be modified from the estimate of (3.15).

[6]This potential is adopted as a toy model for describing the late stage of inflation and the subsequent oscillatory phase, hence we do not worry whether it produces observationally viable curvature perturbations on scales that have exited the Hubble horizon long before the end of inflation.

[7]Although the field $\varphi_{\rm DM} = \alpha\phi + \sigma$ can be defined at all times, one should keep in mind that this forms a diagonal basis together with $\varphi_{\rm RH}$ only after inflation when the potential (3.16) has become approximately quadratic.

As we will show in the next section, the couplings of ϕ and σ with other matter fields provide decay channels for both diagonal fields, and induce decay rates that are typically larger for $\varphi_{\rm RH}$ than $\varphi_{\rm DM}$. This allows $\varphi_{\rm RH}$ to decay and start the Hot Big Bang cosmology, while $\varphi_{\rm DM}$ being long-lived. For this scenario, it is essential that the condition (3.10) holds so that the mass hierarchy between the inflaton and the axion flips at the end of inflation, i.e.,

$$|V''| \ll m_\sigma^2 \quad \text{(during inflation)} \quad \longrightarrow \quad m_\phi^2 \gg m_\sigma^2 \quad \text{(after inflation)}. \qquad (3.17)$$

Consequently the diagonal basis rotates in the ϕ-σ plane (from (A.8) to (3.12)), giving most of the energy density in the post-inflation universe to the heavier diagonal field.

If (3.10) does not hold and the axion mass remains larger, then the heavier diagonal field (i.e. $\varphi_{\rm H}$ in (A.7)) would continue to be placed close to 0 after inflation, carrying only a tiny fraction of the total energy density. Reheating would then complete with the decay of the lighter field, which is typically also the longer-lived one, and thus neither of the fields could survive until today to serve as dark matter.

Before ending this section we should remark that, although we have been treating the axion potential as a quadratic, it is actually periodic in σ with a periodicity of order the decay constant. The quadratic approximation is valid in the vicinity of the potential minima, and thus we have been implicitly assuming the axion displacement to be of $|\sigma| < f$. Let us rewrite this condition at the end of inflation in a more conservative form of

$$|\varphi_{\rm DM\,end}| < f, \qquad (3.18)$$

and impose this as a constraint on the model parameters. Violation of this condition implies that the axion may rotate along its periodic potential multiple times during inflation; the resulting random axion misalignment angle at the end of inflation may also be able to explain the dark matter abundance, however, we will not consider this possibility in the following discussions.

3.3 Reheating and dark matter stability

The axion σ can couple to other matter fields through shift-symmetric operators. Together with ϕ's coupling terms, they provide decay channels[8] for both $\varphi_{\rm RH}$ and $\varphi_{\rm DM}$. This is easily seen from inverting (3.12), which at the leading order in m_σ^2/m_ϕ^2 gives

$$\sigma \simeq \varphi_{\rm DM} - \frac{\alpha}{\sqrt{1-\alpha^2}}\,\varphi_{\rm RH}, \quad \phi \simeq \alpha\frac{m_\sigma^2}{m_\phi^2}\,\varphi_{\rm DM} + \frac{1}{\sqrt{1-\alpha^2}}\,\varphi_{\rm RH}. \qquad (3.19)$$

Here one particularly sees that the couplings the dark matter field $\varphi_{\rm DM}$ obtains through ϕ is suppressed by $(m_\sigma/m_\phi)^2$.

For concreteness, let us for example study the following matter couplings:

$$L_c[\sigma, \phi, \Psi] = \frac{G_{\sigma\gamma\gamma}}{4}\sigma F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{G_{\phi\gamma\gamma}}{4}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} + g_{\phi ff}\,\phi\bar{\psi}i\gamma^5\psi, \qquad (3.20)$$

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[8]The phenomenology of kinetically mixed axions interacting with other matter fields was also studied in e.g. [21, 23].

where $F_{\mu\nu}$ is the field strength of either photons or hidden photons, ψ is a Dirac fermion, and $G_{\phi(\sigma)\gamma\gamma}$, $g_{\phi ff}$ are coupling constants. Here we have considered ϕ to be a pseudoscalar so that its kinetic mixing term with the axion conserves parity. As we want to reheat the visible sector above the BBN temperature, the obvious choice is to couple the scalar fields to the visible photons and/or the Standard Model (SM) fermions. In this case, given the SM gauge symmetry, the coupling $g_{\phi ff}$ would contain the suppression m_ψ/M, with M some cutoff scale. If instead we reheated into a hidden sector, then we should address how to subsequently reheat the visible one. That requires more model building, which is beyond the scope of this paper. Here we only consider perturbative reheating, for the sake of keeping the argument as concise and clear as possible. Another option is to study preheating, which could be very efficient and would be of particular interest in the case of the QCD inflaxion. We leave such a study to future work.

The decay rates of the dark matter and reheaton fields to photons via $\sigma F\tilde{F}$ are (here we neglect the possibility that the contributions to the photon couplings through σ and ϕ cancel each other)

$$\Gamma(\varphi_{\mathrm{DM}} \to \gamma\gamma) \simeq \frac{G_{\sigma\gamma\gamma}^2}{64\pi} m_\sigma^3, \qquad \Gamma(\varphi_{\mathrm{RH}} \to \gamma\gamma) \simeq \frac{\alpha^2}{(1-\alpha^2)^{5/2}} \frac{G_{\sigma\gamma\gamma}^2}{64\pi} m_\phi^3, \tag{3.21}$$

where we have written the diagonal masses in terms of m_σ and m_ϕ using (3.13). Here, the ratio between the decay rates of φ_{DM} and φ_{RH} is suppressed by the mass hierarchy as $\Gamma_{\mathrm{DM}}/\Gamma_{\mathrm{RH}} \propto (m_\sigma/m_\phi)^3$. On the other hand, the two-photon decay rates induced by $\phi F\tilde{F}$ are

$$\Gamma(\varphi_{\mathrm{DM}} \to \gamma\gamma) \simeq \alpha^2 \frac{G_{\phi\gamma\gamma}^2}{64\pi} \frac{m_\sigma^7}{m_\phi^4}, \qquad \Gamma(\varphi_{\mathrm{RH}} \to \gamma\gamma) \simeq \frac{1}{(1-\alpha^2)^{5/2}} \frac{G_{\phi\gamma\gamma}^2}{64\pi} m_\phi^3, \tag{3.22}$$

with ratio $\Gamma_{\mathrm{DM}}/\Gamma_{\mathrm{RH}} \propto (m_\sigma/m_\phi)^7$. The decay rates to fermions, if they are much lighter than both scalars ($m_\psi \ll m_\sigma \ll m_\phi$) are

$$\Gamma(\varphi_{\mathrm{DM}} \to f\bar{f}) \simeq \alpha^2 \frac{g_{\phi ff}^2}{8\pi} \frac{m_\sigma^5}{m_\phi^4}, \qquad \Gamma(\varphi_{\mathrm{RH}} \to f\bar{f}) \simeq \frac{1}{(1-\alpha^2)^{3/2}} \frac{g_{\phi ff}^2}{8\pi} m_\phi, \tag{3.23}$$

with ratio $\Gamma_{\mathrm{DM}}/\Gamma_{\mathrm{RH}} \propto (m_\sigma/m_\phi)^5$. It should be noted that for the channels (3.22) and (3.23) induced by the inflaton ϕ, the decay rate of φ_{DM} is suppressed compared to that of φ_{RH} not only due to kinematics, but further because of the suppression of the effective coupling as seen in (3.19). One can also envisage a situation where the coupled fermions are lighter than the inflaton but heavier than the axion, in which case the decay of φ_{DM} would be kinematically forbidden.

Thus we have seen through the examples that, unless α is tiny, φ_{DM} is generically more stable than φ_{RH}, which allows for the possibility of φ_{DM} surviving until today, while φ_{RH} quickly decaying and reheating the universe. The time of reheating can be estimated as when the total decay rate Γ_{RH} becomes comparable to the Hubble rate. If Γ_{RH} computed as above exceeds the Hubble rate at the end of inflation, then φ_{RH} would quickly decay

after inflation ends.[9] Thus the Hubble rate at reheating can be estimated as

$$H_{\mathrm{reh}} = \begin{cases} H_{\mathrm{end}} & \text{for } \Gamma_{\mathrm{RH}} > H_{\mathrm{end}}, \\ \Gamma_{\mathrm{RH}} & \text{for } \Gamma_{\mathrm{RH}} \leq H_{\mathrm{end}}. \end{cases} \tag{3.24}$$

The decay of φ_{RH} opens the radiation-dominated era, and assuming the decay products to quickly thermalize, the initial temperature upon reheating is computed via

$$3M_p^2 H_{\mathrm{reh}}^2 = \frac{\pi^2}{30} g_*(T_{\mathrm{reh}}) T_{\mathrm{reh}}^4. \tag{3.25}$$

This temperature is required from BBN to be at least of [34, 35]

$$T_{\mathrm{reh}} \gtrsim 4\,\mathrm{MeV}, \tag{3.26}$$

which in turn sets a lower bound on Γ_{RH}.

As for φ_{DM}, we require its lifetime to be longer than the age of the universe,

$$\Gamma_{\mathrm{DM}} < H_0, \tag{3.27}$$

where the Hubble constant in the current universe is $H_0 \approx 1 \times 10^{-33}\,\mathrm{eV}$ [1]. In order to compute its present-day abundance, let us make some simplifying assumptions about the reheating temperature. Firstly, we assume it not to exceed the symmetry breaking scale so that the symmetry stays broken and the axion exists throughout the post-inflationary epoch,

$$T_{\mathrm{reh}} < f. \tag{3.28}$$

Moreover, in cases where the axion mass arises only below some scale Λ, we assume the reheating temperature to be also lower than this,

$$T_{\mathrm{reh}} < \Lambda. \tag{3.29}$$

Under these simplifying assumptions, the energy density of φ_{DM} continues to redshift as $\rho_{\mathrm{DM}} \propto a^{-3}$ after reheating. Assuming the entropy of the universe to be conserved after reheating, the dark matter densities today and upon reheating can be related as $\rho_{\mathrm{DM}\,0} = \rho_{\mathrm{DM\,reh}}(s_0/s_{\mathrm{reh}})$ using the entropy density s, which is a function of the temperature as

$$s = \frac{2\pi^2}{45} g_{s*}(T) T^3. \tag{3.30}$$

Combining these with (3.15) and (3.25) yields the ratio of the dark matter density to the total density in the present universe,

$$\Omega_{\mathrm{DM}} \simeq \frac{C^2 \alpha^2}{6} \frac{m_\sigma^2}{m_\phi^2} \frac{H_{\mathrm{reh}}^2}{H_0^2} \frac{s_0}{s_{\mathrm{reh}}} \sim 10^{-2}\, \alpha^2 \frac{m_\sigma^2}{m_\phi^2} \left(\frac{H_{\mathrm{reh}}}{H_0} \right)^{1/2}, \tag{3.31}$$

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[9]One should remember that the inflaton mass during inflation was much smaller than m_ϕ, and that $(\varphi_{\mathrm{DM}}, \varphi_{\mathrm{RH}})$ form a diagonal basis only after inflation. Hence $\Gamma_{\mathrm{RH}} > H_{\mathrm{end}}$ does not necessary mean that inflaton fluctuations should have decayed during inflation. The stabilized field (i.e. φ_{H} in appendix B) could decay during inflation; this would further damp the field's oscillation and thus lead to a very quick realization of the quasi-static solution (3.6).

where in the far right hand side we have substituted $C \sim 10$. We also note that the detailed values of the number of relativistic degrees of freedom $g_{(s)*}$ upon reheating do not affect the order-of-magnitude estimate.

Before turning to an analysis of the inflaxion parameter space, let us mention that even if the conditions (3.28) and (3.29) are violated, we may still get dark matter. Violation of (3.28) implies there would be a second symmetry breaking after reheating; this would lead to generation of topological defects which may eventually emit inflaxion dark matter. On the other hand by violating (3.29), the axion mass temporarily disappears upon reheating until the cosmic temperature cools down again to Λ; it would be interesting to analyze whether the observed dark matter abundance can also be achieved in this case.

4 Parameter space

The conditions we have imposed on the inflaxion parameters are (3.4), (3.5), (3.10), (3.18), (3.26), (3.27), (3.28), and (3.29). Furthermore, for φ_{DM} to make up the dark matter of the universe, its abundance (3.31) should satisfy $\Omega_{\mathrm{DM}} \approx 0.3$ [1]. Although some of the conditions were introduced only to simplify the computation and thus not necessarily required for our mechanism to provide dark matter, below we explore the parameter space that satisfies all these conditions.

Requiring φ_{RH} to quickly decay while φ_{DM} to be long-lived imposes constraints on the sort of matter couplings of ϕ and σ. From (3.24) follows $\Gamma_{\mathrm{RH}} \geq H_{\mathrm{reh}}$, which combined with (3.27) yields

$$\frac{\Gamma_{\mathrm{DM}}}{\Gamma_{\mathrm{RH}}} < \frac{H_0}{H_{\mathrm{reh}}}. \tag{4.1}$$

Here, if the ratio of the decay rates is determined by the mass ratio as

$$\frac{\Gamma_{\mathrm{DM}}}{\Gamma_{\mathrm{RH}}} \sim \left(\frac{m_\sigma}{m_\phi}\right)^n \tag{4.2}$$

with a positive power $n > 0$, then (4.1) translates into

$$\frac{m_\sigma}{m_\phi} \lesssim \left(\frac{H_0}{H_{\mathrm{reh}}}\right)^{\frac{1}{n}}. \tag{4.3}$$

This combined with $\alpha^2 < 1$ (cf. (2.2)) bounds the abundance (3.31) from above as

$$\Omega_{\mathrm{DM}} \lesssim 10^{-2} \left(\frac{H_{\mathrm{reh}}}{H_0}\right)^{\frac{n-4}{2n}}. \tag{4.4}$$

Since $H_{\mathrm{reh}}/H_0 \gg 1$, this bound shows that the power must satisfy $n > 4$ in order to reproduce the observed dark matter abundance. One particularly sees that with the axion-photon coupling $\sigma F \tilde{F}$ alone, which gives $n = 3$, the inflaxion cannot provide all of the dark matter.[10] A similar story holds for a derivative coupling of the axion to fermions of

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[10]Instead of using the simplifying assumption (4.2) where the dependencies on other parameters are dropped, one can also use the full expression (3.21) and find that, unless $|\alpha|$ is extremely close to unity, the coupling $\sigma F \tilde{F}$ alone does not allow inflaxion dark matter. However we should also note that if (3.29) is not satisfied, then the abundance is no longer given by (3.31) and then it might be possible to have dark matter with axionic couplings only.

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$\bar{\psi}\gamma^\mu\gamma^5\psi\,\partial_\mu\sigma$ which gives $n = 1$. On the other hand, the inflaton's couplings such as $\phi F\tilde{F}$ ($n = 7$) and $\phi\bar{\psi}i\gamma^5\psi$ ($n = 5$) fulfill the requirement.

In figure 3 we show the parameter space for an axion-like σ in the m_σ-$T_{\rm reh}$ plane. Here we assumed the axion mass to be constant throughout the cosmic evolution and thus ignored the conditions that involve Λ. The matter couplings are chosen in figure 3(a) as

$$L_c = \frac{\alpha_\gamma}{8\pi f}\sigma F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{G_{\phi\gamma\gamma}}{4}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{4.5}$$

and in figure 3(b) as

$$L_c = \frac{\alpha_\gamma}{8\pi f}\sigma F_{\mu\nu}\tilde{F}^{\mu\nu} + g_{\phi ff}\,\phi\bar{\psi}i\gamma^5\psi, \tag{4.6}$$

where the axion's coupling to the SM/hidden photons is taken to be inversely proportional to the decay constant f, with α_γ set equal to the electromagnetic fine structure constant $1/137$. To simplify the analysis, we have further imposed

$$\Gamma_{\rm RH} > H_{\rm end}, \tag{4.7}$$

so that $\varphi_{\rm RH}$ instantaneously decays at the end of inflation and reheats the universe, rendering $H_{\rm reh} = H_{\rm end}$ (see also discussions in Footnote 9). The other parameters are fixed to $f = 10^{17}\,{\rm GeV}$, $\alpha = 1/2$, $C = 10$, and the coupling constant in figure 3(a) as $G_{\phi\gamma\gamma} = 10^{-10}\,{\rm GeV}^{-1}$, and in figure 3(b) as $g_{\phi ff} = 10^{-3}$. The inflaton mass m_ϕ is fixed by requiring $\varphi_{\rm DM}$ to make up the entire dark matter abundance, i.e. $\Omega_{\rm DM} \approx 0.3$, and its value is shown by the black solid contours labeled with $\log_{10}(m_\phi/{\rm eV})$. For the chosen set of parameters, the conditions that most strongly restrict the parameter space are axion stabilization during inflation (3.5) which excludes the green regions in the plots, dark matter stability (3.27) excluding the red regions, instantaneous reheating (4.7) excluding the blue regions, as well as the BBN constraint (3.26) setting the lower bound of the displayed $T_{\rm reh}$-range.[11] The allowed windows are shown in white. We note that m_σ is approximately the same as the mass $m_{\rm DM}$ of the inflaxion dark matter field, and since we are supposing instantaneous reheating, $T_{\rm reh}$ is the same as the inflationary energy scale up to a factor that depends on g_*.

The conditions on the decay rates obviously depend on the details of the matter couplings. As can be seen from the blue regions excluded by (4.7) taking different shapes in the two plots, the reheaton $\varphi_{\rm RH}$ decays predominantly through the inflaton's couplings in the allowed windows. For the dark matter $\varphi_{\rm DM}$, both of the inflaton and axion couplings can provide the main decay channel. The red dashed lines in the plots show where the condition (3.27) would be saturated if $\varphi_{\rm DM}$ were allowed to decay only through the axion's $\sigma F\tilde{F}$ coupling; one sees that at higher $T_{\rm reh}$ the decay of $\varphi_{\rm DM}$ is governed by this axion coupling, while at lower $T_{\rm reh}$ by the inflaton couplings. Different inflaton couplings open different windows inside the triangular area between the green region and the red dashed line. In particular in figure 3(b), if the fermion mass lies between those of the dark matter and reheaton as $m_{\rm DM} < 2m_\psi < m_{\rm RH}$, then the decay of the dark matter into fermions

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[11]In the plots we have ignored the time variation of the Hubble rate during inflation, thus $H_{\rm end}$ is written as $H_{\rm inf}$.
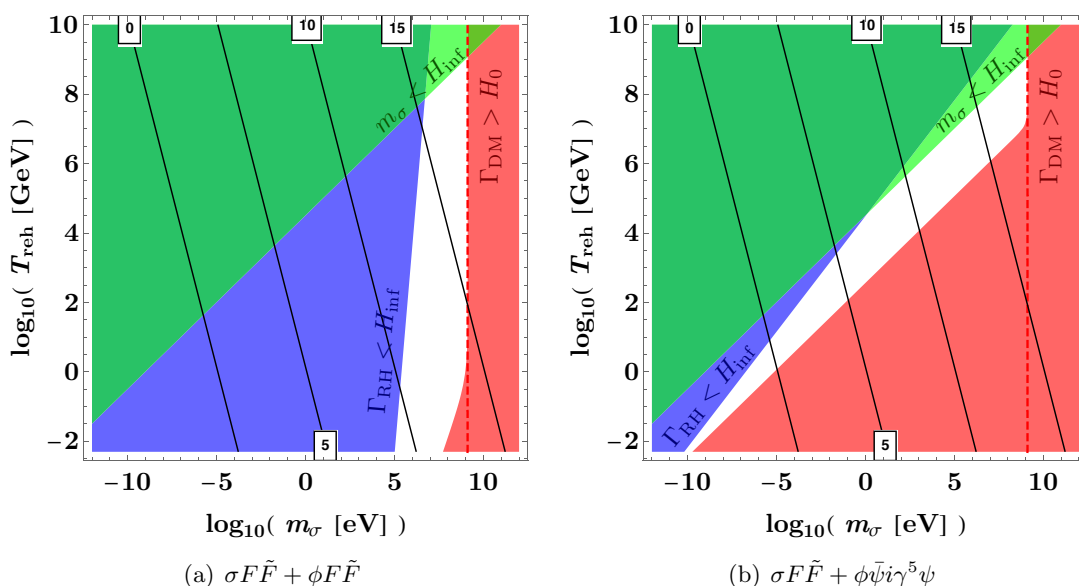
Figure 3. Parameter space of axion-like fields in terms of its mass m_σ and reheating temperature $T_{\rm reh}$. Regions where the inflaxion makes up the dark matter is shown in white. The colored regions are excluded from the requirements of axion stabilization during inflation (green), dark matter stability (red), and instantaneous reheating (blue). The lower end of the displayed $T_{\rm reh}$ region is set by BBN. The axion decay constant is fixed to $f = 10^{17}\,{\rm GeV}$, and the inflaton-axion mixing constant to $\alpha = 1/2$. In the left plot the inflaton is coupled to photons with $G_{\phi\gamma\gamma} = 10^{-10}\,{\rm GeV}^{-1}$, and in the right plot to fermions with $g_{\phi ff} = 10^{-3}$. Black solid contours show the inflaton mass in terms of $\log_{10}(m_\phi/{\rm eV})$. Red dashed lines show the edge of the dark matter stability exclusion region if the dark matter inflaxion were allowed to decay only through the $\sigma F\tilde{F}$ channel.

would be kinetically forbidden and hence the right edge of the allowed window would be extended to the red dashed line. In both plots, as the coupling constants $G_{\phi\gamma\gamma}$ and $g_{\phi ff}$ are increased, the regions sandwiched between the blue and red shift or expand to smaller m_σ values. We also note that the allowed windows become larger as f is increased, and as $|\alpha|$ is taken closer to unity.

With the conventional vacuum misalignment scenario, axions with masses and decay constants as shown here would overdominate the universe, unless the initial angle takes extremely tiny values. This can be readily checked from (1.1); given a vacuum misalignment angle of $|\theta_\star| \sim 1$, to make up the dark matter abundance with $f = 10^{17}\,{\rm GeV}$ would require an ultralight mass $m_\sigma \sim 10^{-22}\,{\rm eV}$, which on the other hand is in tension with the small-scale observations of the universe. However with the inflaxion mechanism, the observed dark matter abundance can be achieved with much larger axion masses, even under a number of conditions we have imposed only for the purpose of simplifying the analysis.

If it is the SM photon that the axion and inflaton are coupled to, it is natural to ask how the existing [36] and forecasted [23] experimental constraints on such couplings compare to those in our figure 3. With $f = 10^{17}\,{\rm GeV}$, the axion coupling is very small and hard to test, but in the mass region $m_\sigma \lesssim 10^{-7}\,{\rm eV}$ it could almost be within reach of the upcoming ABRACADABRA experiment [37, 38]. The inflaton coupling, $G_{\phi\gamma\gamma} = 10^{-10}\,{\rm GeV}^{-1}$, could

saturate the upper bounds from globular cluster stars [39] and the CAST helioscope [40], depending on the mass of φ_{RH}. However within the allowed window shown in figure 3(a), it is out of reach of current experiments [36]. The effective coupling for φ_{DM} obtained through the inflaton is much smaller than $G_{\phi\gamma\gamma}$ (cf. (3.19)).

In the plots we have considered an axion-like field whose mass stays constant during the cosmic evolution. When σ is the QCD axion, the condition (3.29) requires the reheating temperature to be below the QCD scale, which might pose a challenge for realizing an observationally viable cosmological scenario. (The same is true for axion-like fields lying in regions in the plots close to the lower edges.) Under this constraint, to open a QCD inflaxion window at $f \gtrsim 10^9\,\text{GeV}$ with the reheaton decaying into the SM photons as in (4.5) requires a very large coupling $G_{\phi\gamma\gamma}$, which would be experimentally excluded. On the other hand, a reheaton decaying into non-SM particles at such low reheating temperatures could also cause tension with observations, including the BBN and CMB constraints on extra radiation. As a proof of concept, we present the parameter space for QCD inflaxions with low-scale reheating in appendix C. However, since (3.29) was adopted only to simplify the analysis, it would be important to explore the possibilities beyond this restriction, which we leave for future work.

5   Conclusions

We have proposed a dynamical production mechanism for axion dark matter. The three key ingredients of the construction are low-scale inflation, a kinetic mixing between the axion and inflaton, and an inflaton mass after inflation that is larger than the axion mass. The axion during inflation is driven to a field point slightly displaced from the potential minimum, which enables the axion combined with the inflaton to make up the observed dark matter abundance. This opens up new parameter windows for the axion, enabling it to serve as dark matter with masses and decay constants that would lead to overproduction in the conventional vacuum misalignment scenario. We showed in particular that axion-like particles with decay constants of $f \sim 10^{17}\,\text{GeV}$ can make up dark matter without having extremely tiny initial misalignment angles, or ultralight masses that would spoil the small-scale structure of the universe. Although in this paper we did not investigate the full range of possibilities for the QCD axion, our mechanism can in principle also work for QCD axions with similarly large decay constants. The dark matter abundance produced by the inflaxion mechanism is uniquely determined by the Lagrangian parameters, unlike the case for the vacuum misalignment. Also due to the fact that inflaxion dark matter is free of isocurvature perturbations, it escapes from conventional cosmological constraints on axions.

An important feature of the inflaxion mechanism is that the kinetic mixing, together with the rolling of the inflaton, rotates the diagonalized basis in the field space. This leads to different combinations of the inflaton and the axion fields, i.e. the inflaxions, to each generate the cosmological perturbations, reheat the universe, and serve as dark matter. The kinetic mixing also induces inflaxion dark matter to interact with other particles through both the axion couplings and the inflaton couplings. This in particular relates the reheating temperature of the universe with the lifetime of inflaxion dark matter. Moreover, the dark

matter lifetime can be rather short — not much longer than the age of the universe — even if the axion decay constant is large. These facts about the inflaxion couplings offer the possibility of producing smoking-gun signatures for distinguishing the inflaxion mechanism from other axion production scenarios such as the vacuum misalignment or emission from cosmic strings. For instance, the case where inflaxion dark matter decays with a lifetime close to the age of the universe can give rise to a number of observable signals, and may even relax the recent tensions in the measurements of the matter fluctuation amplitude of the universe [41]. Furthermore, parts of the axion window that our mechanism opens up can be probed by upcoming experiments such as ABRACADABRA. We also remark that the matter couplings the diagonal fields pick up through the axion could give rise to parity-violating signatures in the sky [42]. Moreover, if the duration of inflation is minimal and thus at the time when the CMB scales exit the horizon the axion is still oscillating with a non-negligible amplitude, then its mixing with the inflaton could give rise to oscillatory features in the curvature perturbation spectrum on observable scales.

The basic idea of the mechanism is to dilute away the axion density with low-scale inflation, while slightly exciting the axion from its post-inflationary vacuum state. The inflaxion achieves this through a derivative coupling with the slow-rolling inflaton. One may also construct variants with, e.g., a shift-symmetric Gauss-Bonnet coupling $\sigma \mathcal{G}_{GB}$ (which can also allow axions to induce baryogenesis [43]), or $\sigma F \tilde{F}$, given that there is a background of helical electromagnetic fields which can in principle be produced as in [44–46]. An alternative possibility would be to generate an additional small potential, possibly from some hidden gauge interactions that become strong only during inflation (see e.g. [47–49]).

We stress that some of the requirements for the inflaxions discussed in this paper, such as the axion potential to stay constant during reheating, were introduced merely for simplifying the analyses. In particular for the QCD axion, this implied a very low reheating temperature. However by lifting such simplifying conditions, other scenarios can also arise within the framework, such as a temporal vanishing of the axion potential upon reheating. Another possibility we did not study is a non-perturbative decay of the reheaton. It will be interesting to systematically study the range of possibilities to fully explore the axion parameter space capable of explaining the dark matter of our universe. Another important direction for further study is to realize inflaxions within an ultraviolet-complete framework. The inflaton-axion kinetic mixing can arise, for instance, in the low-energy limit of string compactifications via non-perturbative corrections to the Kähler potential in a similar fashion to [22]. The inflaxion mechanism could also provide a new cosmological picture of the axiverse, in which most axions settle down to the vacuum during low-scale inflation, except for those coupled to the inflaton and thence get excited to make up the dark matter. This picture may offer a way to avoid the cosmological moduli problem in the axiverse. We leave these topics for future investigation.

Acknowledgments

A Diagonal basis

In this appendix we diagonalize the kinetic and mass terms in the Lagrangian (2.8). Here we expand the effective potential $V(\phi)$ around some field value ϕ_* up to quadratic order,

$$V(\phi) = V_* + V_*'(\phi - \phi_*) + \frac{1}{2} V_*''(\phi - \phi_*)^2 + \mathcal{O}(\phi - \phi_*)^3, \qquad (A.1)$$

where $V_* = V(\phi_*)$, $V_*' = dV(\phi)/d\phi|_{\phi=\phi_*}$, etc., and we note in particular that the effective mass squared V_*'' can be either positive or negative. (In the main text the axion mass squared is considered to be $m_\sigma^2 > 0$, however the discussion in this appendix also applies to tachyonic masses, i.e. $m_\sigma^2 < 0$. Below we ignore any temperature dependence of the axion mass.) We can complete the square by the shift

$$\tilde{\phi} = \phi - \phi_* + \frac{V_*'}{V_*''}. \qquad (A.2)$$

The kinetic and potential terms up to quadratic order can then be diagonalized by the following field redefinitions,

$$\varphi_\pm = \left(\frac{X_\pm + \beta^2}{X_\pm^2 + \alpha^2 \beta^2} \right)^{1/2} \left(X_\pm \, \sigma + \alpha \, \tilde{\phi} \right), \qquad (A.3)$$

where

$$\beta^2 = \frac{m_\sigma^2}{V_*''}, \qquad X_\pm = \frac{1 - \beta^2 \pm \sqrt{(1 - \beta^2)^2 + 4\alpha^2 \beta^2}}{2}. \qquad (A.4)$$

One can check that the coefficients of σ and $\tilde{\phi}$ in (A.3) are real, hence $\varphi_\pm$ are guaranteed to be real fields. The Lagrangian (2.8) is rewritten in terms of these fields as

$$\frac{\mathcal{L}}{\sqrt{-g}} = \sum_{i=\pm} \left(-\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi_i \, \partial_\nu \varphi_i - \frac{1}{2} m_i^2 \varphi_i^2 \right) - V_* + \frac{(V_*')^2}{2V_*''} - \mathcal{O}(\phi - \phi_*)^3 + L_c[\sigma, \phi, \Psi], \quad (A.5)$$

with the mass squareds given by

$$m_\pm^2 = \frac{1 - X_\pm}{1 - \alpha^2} V_*''. \qquad (A.6)$$

Here, note that $m_\pm^2$ can be negative.

In the presence of a hierarchy between the masses such that

$$|m_\sigma^2| \gg |V_*''|, \qquad (A.7)$$

the expression (A.3) can be simplified. By expanding each of the coefficients of σ and $\tilde{\phi}$ to leading order in V_*''/m_σ^2, one obtains[12]

$$\varphi_L \simeq \alpha \sigma + \tilde{\phi}, \quad \varphi_H \simeq \sqrt{1 - \alpha^2} \left(\sigma - \frac{\alpha V_*''}{m_\sigma^2} \tilde{\phi} \right), \qquad (A.8)$$

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[12]We compute the leading order expressions for the coefficients, since σ and $\tilde{\phi}$ themselves can also take hierarchical field values that depend on their masses.

up to overall signs which do not affect the physics. Here $\varphi_{\rm L}$ and $\varphi_{\rm H}$ denote respectively the lighter and heavier of the two fields $\varphi_{\pm}$, whose masses at the leading order are

$$m_{\rm L}^2 \simeq V_*'', \quad m_{\rm H}^2 \simeq \frac{m_\sigma^2}{1-\alpha^2}. \tag{A.9}$$

The expressions in the opposite hierarchical regime $|m_\sigma^2| \ll |V_*''|$ can similarly be obtained by exchanging $(\sigma, m_\sigma^2) \leftrightarrow (\tilde{\phi}, V_*'')$.

B Misalignment of stabilized axion

B.1 Generic cosmological background

Let us analyze the field displacement of a stabilized axion σ using the diagonal basis introduced in appendix A. In the following discussions, the $\mathcal{O}(\phi-\phi_*)^3$ terms in the potential expansion (A.1) and the matter coupling $L_c[\sigma, \phi, \Psi]$ are neglected.

We consider a flat FRW universe (3.1), and suppose the axion mass to be larger than that of ϕ as well as the Hubble rate,

$$m_\sigma^2 \gg H^2, \, |V_*''|. \tag{B.1}$$

The approximate expressions for the diagonal fields for such a case are given in (A.8). From (A.9) combined with (2.2), one sees that the heavier field $\varphi_{\rm H}$ also possesses a super-Hubble mass, i.e. $m_{\rm H}^2 \gg H^2$. Hence $\varphi_{\rm H}$ undergoes a damped oscillation about its origin, and thus eventually gets stabilized at $\varphi_{\rm H} = 0$. In the (σ, ϕ) basis, this corresponds to

$$\sigma \simeq \frac{\alpha V_*''}{m_\sigma^2} \tilde{\phi} \simeq \frac{\alpha V'(\phi)}{m_\sigma^2}, \tag{B.2}$$

which shows that the axion field σ is displaced from its origin.

B.2 Slow-roll inflation

We now consider the lighter diagonal field $\varphi_{\rm L}$ to be a slow-rolling inflaton. Let us combine its mass term and the constant offset in (A.5) into

$$U(\varphi_{\rm L}) = V_* - \frac{(V_*')^2}{2V_*''} + \frac{1}{2}m_{\rm L}^2 \varphi_{\rm L}^2. \tag{B.3}$$

Then one can check that once the axion is stabilized as (B.2), the effective potentials in the two bases $U(\varphi_{\rm L})$ and $V(\phi)$ agree as

$$U(\varphi_{\rm L}) \simeq V(\phi), \quad U'(\varphi_{\rm L}) \simeq V'(\phi), \quad U''(\varphi_{\rm L}) \simeq V''(\phi), \tag{B.4}$$

where the derivatives of U and V are understood to be in terms of $\varphi_{\rm L}$ and ϕ, respectively. Hence the values of the slow-roll parameters defined in the diagonal basis

$$\epsilon = \frac{M_p^2}{2} \left(\frac{U'(\varphi_{\rm L})}{U(\varphi_{\rm L})}\right)^2, \quad \eta = M_p^2 \frac{U''(\varphi_{\rm L})}{U(\varphi_{\rm L})}, \tag{B.5}$$

agree with those defined similarly in terms of $V(\phi)$. When these parameters are smaller than unity,[13] $\varphi_{\rm L}$ follows the slow-roll attractor,

$$3M_p^2 H^2 \simeq U(\varphi_{\rm L}), \quad 3H\dot{\varphi}_{\rm L} \simeq -U'(\varphi_{\rm L}). \tag{B.6}$$

One can further check that from these approximations follow

$$3M_p^2 H^2 \simeq V(\phi), \quad 3H\dot{\phi} \simeq -V'(\phi). \tag{B.7}$$

Hence we see that with the axion stabilized, the slow-roll conditions and approximations for ϕ take the same forms as those for a canonical single-field inflaton. Moreover, during slow-roll inflation, the axion displacement (B.2) can also be written as

$$\sigma \simeq -\frac{3\alpha H\dot{\phi}}{m_\sigma^2}. \tag{B.8}$$

The end of inflation, i.e. when the sign of $\ddot{a}$ changes from positive to negative, corresponds to the time when $-\dot{H}/H^2 = 1$. Given that $\varphi_{\rm H}$ continues to be stabilized at 0, this happens when the inflaton potential and kinetic energies are related by

$$U(\varphi_{\rm L}) = \dot{\varphi}_{\rm L}^2 = 2M_p^2 H^2. \tag{B.9}$$

Using (B.4), the end of inflation can also be described in terms of ϕ as the time of

$$V(\phi) \simeq 2M_p^2 H^2. \tag{B.10}$$

This relation, however, can receive corrections if for instance the higher order terms in the potential expansion (A.1) becomes important or the inflaton mass $|V''|$ becomes larger than m_σ^2 towards the end of inflation.

C QCD inflaxion

Before turning to the QCD inflaxion, we briefly review the conventional story of QCD axion dark matter. If the Peccei-Quinn [7] symmetry breaking happens before or during the inflationary epoch and is not restored afterwards, then the present-day abundance of the QCD axion arising from a vacuum misalignment is [50]

$$\Omega_\sigma \sim 0.1 \times \theta_\star^2 \left(\frac{f}{10^{12}\,{\rm GeV}} \right)^{7/6}. \tag{C.1}$$

An axion with an initial misalignment angle $|\theta_\star| \sim 1$ makes up the observed dark matter abundance if the decay constant is $f \sim 10^{12}\,{\rm GeV}$. However for $f \gg 10^{12}\,{\rm GeV}$, the axion would overdominate the universe unless the misalignment angle is tuned to lie near zero, i.e. $\theta_\star^2 \ll 1$ ("anthropic axion window" [51]). On the other hand for $f \ll 10^{12}\,{\rm GeV}$, the axion could make up the entire dark matter only if $\theta_\star \simeq \pm\pi$ which renders the axion abundance larger than the expression (C.1) due to anharmonic effects [50, 52]. However in this case the axion isocurvature perturbations would also be enhanced and thus such anharmonic axion dark matter is severely constrained by CMB measurements [53–55].

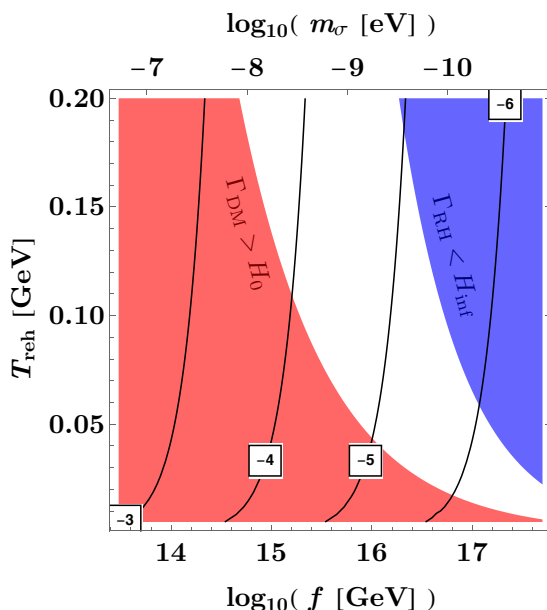[13]With $|\eta| \ll 1$, the mass hierarchy (B.1) becomes $m_\sigma^2 \gg H^2 \gg |V_*''|$.

Figure 4. Parameter space of the QCD axion in terms of its decay constant f, mass m_σ, and reheating temperature $T_{\rm reh}$. To simplify the analyses, $T_{\rm reh}$ is taken to be below the QCD scale; with such extremely low reheating temperatures, the axion window shown here should be considered as a proof of concept (see the main text for discussions). Regions where the QCD inflaxion can make up the dark matter is shown in white. The colored regions are excluded from the requirements of dark matter stability (red) and instantaneous reheating (blue). The lower end of the displayed $T_{\rm reh}$ region is set by BBN. The inflaton-axion mixing constant is fixed to $\alpha = 1/2$, and the inflaton is coupled to fermions with $g_{\phi ff} = 5 \times 10^{-3}$. Black solid contours show the inflaton mass in terms of $\log_{10}(m_\phi/\text{eV})$.

If, on the other hand, the symmetry breaks after inflation or is restored upon reheating, then axionic domain walls would form and overdominate the universe. The walls however can be made unstable by setting the number of degenerate vacua of the axion potential (the so-called domain wall number) to $N_{\rm W} = 1$ [56–59]; alternatively, one can have $N_{\rm W} > 1$ and gauge the $Z_{N_{\rm W}}$ symmetry so that all vacua are gauge equivalent [60], or introduce further breaking of the axion's shift symmetry to generate an energy bias between the vacua [61–64]. In these cases, axions are radiated from the axionic domain walls and cosmic strings and thus can in principle make up the dark matter. However the amount of axion dark matter produced is still uncertain; see [65, 66] for recent works on axion strings.

The inflaxion mechanism for the QCD axion is subject to conditions involving the QCD scale. In particular, the condition (3.29) adopted in the main text restricts the reheating temperature to be below the QCD scale, which poses a challenge for realizing an observationally viable cosmology with the inflaxions. Although (3.29) is merely introduced to simplify the analyses and not a necessary condition for the inflaxion framework, below we demonstrate as a proof of concept that all the conditions listed at the beginning of section 4 can in principle be met for the QCD axion. In the parameter region we will show in figure 4, the reheaton mass is much lighter than eV. Under this condition, one

example of the accompanying reheating scenario would be that the reheaton decays into very light non-SM fermions, which couple to SM particles and thus thermalize the visible sector above the BBN temperature. Here, in order to avoid the observational constraints on extra radiation [1], the non-SM fermion is assumed to have a time-varying mass, arising for instance from a coupling to a rolling scalar field, which quickly renders the fermion non-relativistic after reheating so that they do not contribute to ΔN_{eff}.

The parameter space is shown in figure 4 in terms of the axion decay constant f and the reheating temperature T_{reh}. The axion mass m_σ is a function of f as shown in (2.6). We used a linear scale for T_{reh}, and the lower end of the displayed T_{reh}-range is set by the BBN constraint (3.26), while the upper end is set to Λ_{QCD} from the simplifying condition (3.29). We adopted the parameter values $\alpha = 1/2$ and $C = 10$, required instantaneous reheating (4.7), and fixed the inflaton mass such that $\Omega_{\text{DM}} \approx 0.3$; the black solid contours are labeled with $\log_{10}(m_\phi/\text{eV})$. Moreover, we have considered couplings as in (4.6) between the axion and photons,[14] as well as the inflaton and fermions with $g_{\phi ff} = 5 \times 10^{-3}$; the latter interaction dominantly determines the decay rates of both φ_{RH} and φ_{DM} in the displayed parameter range. The conditions that most strongly restrict the parameter space besides (3.26) and (3.29) are the dark matter stability (3.27) excluding the red region, and instantaneous reheating (4.7) excluding the blue region. The white region denotes the allowed window. As the coupling $g_{\phi ff}$ is decreased, the position of the allowed window shifts towards smaller f values, hence larger axion and inflaton masses; the value of $g_{\phi ff}$ in the plot is chosen such that the allowed window is centered at $f \sim 10^{16}\,\text{GeV}$. We should also remark that the range of $3 \times 10^{17}\,\text{GeV} < f < 1 \times 10^{19}\,\text{GeV}$, which includes the region close to the right edge of the plot, is disfavored by black hole superradiance limits [67, 68].

If the inflaton is coupled instead to photons as in (4.5), then a very large coupling $G_{\phi ff}$ is required in order to open a QCD axion window at $f \gtrsim 10^9\,\text{GeV}$, which would be experimentally excluded. However this statement assumes a perturbative decay of the reheaton, and the picture may differ when one takes into account the possibility of non-perturbative decay (see e.g. [69, 70]). We leave a detailed study of the QCD inflaxion cosmology to future work.

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