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Lagrangian description of the partially massless higher spin $N = 1$ supermultiplets in AdS_4 space

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Abstract: In the recent paper [1] the classification of non-unitary representations of the three dimensional superconformal group has been constructed. From AdS/CFT they must correspond to $N = 1$ supermultiplets containing partially massless fields in AdS_4. Moreover, the simplest example of such supermultiplets which contains a partially massless spin-2 was explicitly constructed. In this paper we extend this result and develop explicit Lagrangian construction of general $N = 1$ supermultiplets containing partially massless fields with arbitrary superspin. We use the frame-like gauge invariant description of partially massless higher spin bosonic and fermionic fields. For the two types of the supermultiplets (with integer and half-integer superspins) each one containing two partially massless bosonic and two partially massless fermionic fields we derive the supertransformations leaving the sum of their free Lagrangians invariant such that the AdS_4 superalgebra is closed on-shell.

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Contents

1 Introduction

In the recent paper [1] the classification of non-unitary representations of the three dimensional superconformal group has been constructed. From AdS/CFT-correspondence (i.e. from the fact that the very same superalgebra plays the role of the super-AdS_4 algebra in the bulk and of the superconformal one on the boundary) there must exist their analogues in four-dimensional Anti de Sitter space (AdS_4) as well.[1] By the structure of the supermultiplets they constructed, the authors of [1] suggested that they correspond to the supermultiplets with the partially massless fields which are also non-unitary in AdS_4. Moreover, the simplest example of such supermultiplets which contain partially massless spin-2, massless spin-1, massless spin-3/2 and a massive spin-3/2 was explicitly constructed. The dynamical description of the arbitrary supermultiplets was not studied. In this paper we fill this gap and construct explicit Lagrangian realization of all $N=1$ supermultiplets containing partially massless fields with arbitrary integer and half-integer superspins.

The partially massless fields [2–6] (non-unitary in AdS) of integer s or half-integer $s+1/2$ spins are labelled by the depth $t \in \{0, 1, 2, \ldots, (s-1), s\}$. Two boundary values $t=0$ and $t=s$ correspond to massless and massive cases respectively. For other values of

[1] It is well known hat in de Sitter space the N=1 supersymmetry is inconsistent, yielding imaginary spin 3/2 mass parameter and non-real Lagrangian in local theory. Some argument concerning the existence of N=1 supergravity in dS is presented in [13] however, these aspects are outside the scope of this article.

t we have pure partially massless field which propagates $2(t+1)$ degrees of freedom. As it was shown in [1] the general partially massless $N=1$ supermultiplets are described by the diagrams

$$
\begin{tikzcd}[column sep=small]
 & {[s+\frac{1}{2}]_t} \arrow[dl, dash] \arrow[dr, dash] & \\
{[s]_t} \arrow[dr, dash] & Y=s & {[s]_{t-1}} \arrow[dl, dash] \\
 & {[s-\frac{1}{2}]_{t-1}} &
\end{tikzcd}
\qquad
\begin{tikzcd}[column sep=small]
 & {[s-\frac{1}{2}]_t} \arrow[dl, dash] \arrow[dr, dash] & \\
{[s]_t} \arrow[dr, dash] & Y=s-\frac{1}{2} & {[s-1]_{t-1}} \arrow[dl, dash] \\
 & {[s-\frac{1}{2}]_{t-1}} &
\end{tikzcd}
\tag{1.1}
$$

Here integers s and t are the spin and the depth of the partially massless fields. As in the massive case $N=1$ partially massless supermultiplets contain a pair of the bosonic fields and a pair of the fermionic ones. For instance, left diagram describes partially massless supermultiplet with superspin $Y=s$ which contains two partially massless bosonic spin-s fields of depth t and $(t-1)$, partially massless fermionic spin-$(s+1/2)$ field of depth t and partially massless fermionic spin-$(s-1/2)$ field of depth $(t-1)$. Taking into account that depth t partially massless fields propagate $2(t+1)$ degrees of freedom it is easy to check that the number of bosonic and fermionic degrees of freedom matches and equals $4t$.

For the description of the individual partially massless higher spin bosonic and fermionic fields we use the frame-like gauge invariant description similar to the massive case [7–9]. In such formalism partially massless spin $s(s+1/2)$ of the depth t is described by a set of massless fields with spins $s, s-1, \ldots, s-t$ combined together into one system.[2] To combine partially massless fields into supermultiplets (1.1) we follow the strategy of our recent paper [9] where massive higher spin supermultiplets were constructed. For the Lagrangian we just take the sum of four free Lagrangians for the two partially massless bosonic and the two partially massless fermionic fields entering the supermultiplet. Then for each pair of bosonic and fermionic fields (we call it superblock in what follows) we find the supertransformations leaving the sum of their two Lagrangians invariant. After that we combine all four possible superblocks and adjust their parameters so that the algebra of the supertransformations be closed on-shell.

The paper is organized as follows. In section 2 we give non-unitary frame-like gauge invariant formulation for free partially massless arbitrary integer and half-inter spins. In section 3 we consider superblocks containing one partially massless bosonic and one partially massless fermionic fields and find corresponding supertransformations. In section 4 we combine the constructed partially massless superblocks into the partially massless supermultiplets.

[2] As in the massive case the fields with spins $s-1, \ldots, s-t$ are auxiliary and play a role of the Stuckelberg fields. In works [11, 12] it was shown that in the metric-like formalism they can be derived from a log radial dimensional reduction of the massless theory. In the case of the frame-like formalism we used, in general the reduction produces more field components than it is necessary so one has to exclude the unnecessary ones by solving their equations and/or gauge fixing. This is even more true for the supermultiplets because starting with $N=1$ supersymmetry in higher dimensions one usually ends with the $N=2$ supersymmetry and again has to truncate somehow to go back to $N=1$.

JHEP08(2019)116

2 Partially massless higher spin fields

In this section we provide frame-like gauge invariant formulation for (non-unitary) partially massless fields with arbitrary integer and half-inter spins in AdS_4 space.

2.1 Partially massless bosons

The gauge invariant formulation for the partially massless fields can be easily obtained from the general massive case just by adjusting the value of mass parameter. But unitarity requires that the sign of the cosmological term be positive so that naturally the partially massless fields live in de Sitter space. In this work we use the gauge invariant formulation for the partially massless fields in AdS_4 space where half the number of components have wrong signs of the kinetic terms. Such a description is explicitly non-unitary but the Lagrangian is hermitian and all coefficients are real.

In such approach a partially massless integer spin-s field of depth $t = (s-l-1)$ is formulated in terms of massless fields with spins $(l+1) \leq k \leq s$. Each massless bosonic fields with spin $k \geq 2$ (the case of the partially massless bosonic fields of the last depth $t = (s-1)$ requires introduction of the spin-1 component and has to be considered separately) described by the physical one-form $f^{\alpha(k-1)\dot{\alpha}(k-1)}$ and the auxiliary one-forms $\Omega^{\alpha(k)\dot{\alpha}(k-2)}, \Omega^{\alpha(k-2)\dot{\alpha}(k)}$. They are two-component multispinors symmetric on its local dotted and undotted spinorial indices separately. These fields satisfy the following reality condition

$$(f^{\alpha(k-1)\dot{\alpha}(k-1)})^\dagger = f^{\alpha(k-1)\dot{\alpha}(k-1)}, \qquad (\Omega^{\alpha(k)\dot{\alpha}(k-2)})^\dagger = \Omega^{\alpha(k-2)\dot{\alpha}(k)}. \tag{2.1}$$

In these notations the gauge invariant Lagrangian for the partially massless bosonic field can be written as follows:

$$\begin{aligned}(-1)^\sigma \frac{1}{i}\mathcal{L} = \sum_{k=l+1}^{s} \Big[& k\Omega^{\alpha(k-1)\beta\dot{\alpha}(k-2)} E_\beta{}^\gamma \Omega_{\alpha(k-1)\gamma\dot{\alpha}(k-2)} \\ & -(k-2)\Omega^{\alpha(k)\dot{\alpha}(k-3)\dot{\beta}} E_{\dot{\beta}}{}^{\dot{\gamma}} \Omega_{\alpha(k)\dot{\alpha}(k-3)\dot{\gamma}} \\ & +2\Omega^{\alpha(k-1)\beta\dot{\alpha}(k-2)} e_\beta{}^{\dot{\beta}} D f_{\alpha(k-1)\dot{\alpha}(k-2)\dot{\beta}} - \text{h.c.}\Big] \\ + \sum_{k=l+2}^{s} a_k \Big[& E_{\beta(2)} \Omega^{\alpha(k-2)\beta(2)\dot{\alpha}(k-2)} f_{\alpha(k-2)\dot{\alpha}(k-2)} \\ & + \frac{(k-2)}{k} E_{\beta(2)} f^{\alpha(k-3)\beta(2)\dot{\alpha}(k-1)} \Omega_{\alpha(k-3)\dot{\alpha}(k-1)} - \text{h.c.}\Big] \\ + \sum_{k=l+1}^{s} b_k [& f^{\alpha(k-2)\beta\dot{\alpha}(k-1)} E_\beta{}^\gamma f_{\alpha(k-2)\gamma\dot{\alpha}(k-1)} - \text{h.c.}].\end{aligned} \tag{2.2}$$

Here the even/odd parameter σ determines the common sign of the Lagrangian that will be important for the construction of the supermultiplets. The Lagrangian (2.2) is invariant under the following gauge transformations:

$$\begin{aligned}\delta f^{\alpha(k-1)\dot{\alpha}(k-1)} = {} & D\xi^{\alpha(k-1)\dot{\alpha}(k-1)} + e_\beta{}^{\dot{\alpha}} \eta^{\alpha(k-1)\beta\dot{\alpha}(k-2)} + e^\alpha{}_{\dot{\beta}} \eta^{\alpha(k-2)\dot{\alpha}(k-1)\dot{\beta}} \\ & - \frac{(k-1)a_{k+1}}{2(k+1)} e_{\beta\dot{\beta}} \xi^{\alpha(k-1)\beta\dot{\alpha}(k-1)\dot{\beta}} + \frac{a_k}{2k(k-1)} e^{\alpha\dot{\alpha}} \xi^{\alpha(k-2)\dot{\alpha}(k-2)}\end{aligned}$$

JHEP08(2019)116

$$\delta\Omega^{\alpha(k)\dot\alpha(k-2)} = D\eta^{\alpha(k),\dot\alpha(k-2)} - \frac{a_{k+1}}{2} e_{\beta\dot\beta}\eta^{\alpha(k)\beta\dot\alpha(k-2)\dot\beta} + \frac{a_k}{2k(k+1)} e^{\alpha\dot\alpha}\eta^{\alpha(k-1)\dot\alpha(k-3)} + \frac{b_k}{2k} e^{\alpha}{}_{\dot\beta}\xi^{\alpha(k-1)\dot\alpha(k-2)\dot\beta}, \tag{2.3}$$

provided

$$\begin{aligned} b_k &= \frac{2s(s+1)l(l+1)}{k(k-1)(k+1)}\lambda^2 \\ {a_k}^2 &= \frac{4(s-k+1)(s+k)(k-l-1)(k+l)}{(k-2)(k-1)}\lambda^2. \end{aligned} \tag{2.4}$$

In what follows we assume that all parameters a_k are positive. It is also worth to note that Lagrangian (2.2) is parity invariant that is invariant under spatial reflections. These transformations can be defined by operator P as follows

$$\begin{aligned} Pf^{\alpha(k-1)\dot\alpha(k-1)} &= f^{\alpha(k-1)\dot\alpha(k-1)}, & P\Omega^{\alpha(k)\dot\alpha(k-2)} &= \Omega^{\alpha(k-2)\dot\alpha(k)}, \\ Pe^{\alpha\dot\alpha} &= e^{\alpha\dot\alpha}, & PE^{\alpha\beta} &= E^{\dot\alpha\dot\beta}. \end{aligned} \tag{2.5}$$

Using the fact that Lagrangian in four dimensions is differential 4-form which is proportional to antisymmetric tensor $\varepsilon_{\mu\nu\rho\sigma}$ and $P\varepsilon_{\mu\nu\rho\sigma} = -\varepsilon_{\mu\nu\rho\sigma}$ we can see that the Lagrangian (2.2) is P-invariant. Moreover, due to the Lagrangian is quadratic in fields, it describes both parity-even boson defined by (2.5) and parity-odd one defined by

$$Pf^{\alpha(k-1)\dot\alpha(k-1)} = -f^{\alpha(k-1)\dot\alpha(k-1)}, \quad P\Omega^{\alpha(k)\dot\alpha(k-2)} = -\Omega^{\alpha(k-2)\dot\alpha(k)}. \tag{2.6}$$

In the gauge invariant formalism we use, for each field (physical or auxiliary) there exist a corresponding gauge invariant object ("torsion" or "curvature"). Their form is completely determined by the structure of the gauge transformations (2.3):[3]

$$\begin{aligned} \mathcal{T}^{\alpha(k-1)\dot\alpha(k-1)} &= Df^{\alpha(k-1)\dot\alpha(k-1)} + e_\beta{}^{\dot\alpha}\Omega^{\alpha(k-1)\beta\dot\alpha(k-2)} + e^\alpha{}_{\dot\beta}\Omega^{\alpha(k-2)\dot\alpha(k-1)\dot\beta} \\ &\quad - \frac{(k-1)a_{k+1}}{2(k+1)} e_{\beta\dot\beta} f^{\alpha(k-1)\beta\dot\alpha(k-1)\dot\beta} + \frac{a_k}{2k(k-1)} e^{\alpha\dot\alpha} f^{\alpha(k-2)\dot\alpha(k-2)}, \\ \mathcal{R}^{\alpha(k)\dot\alpha(k-2)} &= D\Omega^{\alpha(k),\dot\alpha(k-2)} - \frac{a_{k+1}}{2} e_{\beta\dot\beta}\Omega^{\alpha(k)\beta\dot\alpha(k-2)\dot\beta} \\ &\quad + \frac{a_k}{2k(k+1)} e^{\alpha\dot\alpha}\Omega^{\alpha(k-1)\dot\alpha(k-3)} + \frac{b_k}{2k} e^\alpha{}_{\dot\beta} f^{\alpha(k-1)\dot\alpha(k-2)\dot\beta}. \end{aligned} \tag{2.7}$$

In this work we use a formalism analogous to the so-called 1 and 1/2 order formalism, very well known in supergravity. Namely, we do not introduce any supertransformations for the auxiliary fields, instead all calculations are done using the "zero torsion conditions":

$$\mathcal{T}^{\alpha(k-1)\dot\alpha(k-1)} \approx 0 \Rightarrow e_\beta{}^{\dot\alpha}\mathcal{R}^{\alpha(k-1)\beta\dot\alpha(k-2)} + e^\alpha{}_{\dot\beta}\mathcal{R}^{\alpha(k-2)\dot\alpha(k-1)\dot\beta} \approx 0. \tag{2.8}$$

As for the supertransformations for the physical fields, the variation of the Lagrangian can be compactly written using the gauge invariant curvatures given above:

$$\delta\mathcal{L} = -(-1)^\sigma 2i \sum_{k=l+1}^{s} \mathcal{R}^{\alpha(k-1)\beta\dot\alpha(k-2)} e_\beta{}^{\dot\beta}\delta f_{\alpha(k-1)\dot\alpha(k-2)\dot\beta} - \text{h.c.} \tag{2.9}$$

[3]Note that to construct a full set of gauge invariant objects one has to introduce a number of so-called extra fields. But these fields do no enter the free Lagrangian so in what follows we omit them.

JHEP08(2019)116

2.2 Partially massless fermions

To construct a gauge invariant Lagrangian for the fermionic fields one only needs physical fields. So to describe partially massless spin s+1/2 field of depth $t = (s-l-1)$ we introduce a set of one-forms $\Phi^{\alpha(k)\dot\alpha(k-1)}$, $\Phi^{\alpha(k-1)\dot\alpha(k)}$, $l+1 \le k \le s$ which are symmetric on their dotted and undotted spinorial indices separately and satisfying a reality condition

$$(\Phi^{\alpha(k)\dot\alpha(k-1)})^\dagger = \Phi^{\alpha(k-1)\dot\alpha(k)}.$$

The Lagrangian for the partially massless fields in AdS_4 has the form

$$\begin{aligned}
(-1)^\tau \mathcal{L} = & \sum_{k=l+1}^{s} \Phi_{\alpha(k-1)\beta\dot\alpha(k-1)} e^\beta{}_{\dot\beta} D\Phi^{\alpha(k-1)\dot\alpha(k-1)\dot\beta} \\
& + \sum_{k=l+2}^{s} c_k [E^{\beta(2)}\Phi_{\alpha(k-2)\beta(2)\dot\alpha(k-1)}\Phi^{\alpha(k-2)\dot\alpha(k-1)} + \text{h.c.}] \\
& + \sum_{k=l+1}^{s} d_k \Big[(k+1)\Phi_{\alpha(k-1)\beta\dot\alpha(k-1)} E^\beta{}_\gamma \Phi^{\alpha(k-1)\gamma\dot\alpha(k-1)} \\
& \qquad -(k-1)\Phi_{\alpha(k)\dot\alpha(k-2)\dot\beta} E^{\dot\beta}{}_{\dot\gamma} \Phi^{\alpha(k)\dot\alpha(k-2)\dot\gamma} + \text{h.c.}\Big].
\end{aligned} \tag{2.10}$$

As in the bosonic case half the number of components have wrong signs of the kinetic terms. Such a description is explicitly non-unitary but the Lagrangian is hermitian and all coefficients are real. In what follows we assume that the parameters c_k are positive while τ (even/odd) in Lagrangian (2.10) parameterize the common sign of the Lagrangian.

This Lagrangian is invariant under the following gauge transformation:

$$\begin{aligned}
\delta\Phi^{\alpha(k)\dot\alpha(k-1)} = & D\xi^{\alpha(k)\dot\alpha(k-1)} + e_\beta{}^{\dot\alpha}\eta^{\alpha(k)\beta\dot\alpha(k-2)} + 2d_k e^\alpha{}_{\dot\beta}\xi^{\alpha(k-1)\dot\alpha(k-1)\dot\beta} \\
& -c_{k+1} e_{\beta\dot\beta}\xi^{\alpha(k)\beta\dot\alpha(k-1)\dot\beta} + \frac{c_k}{(k-1)(k+1)} e^{\alpha\dot\alpha}\xi^{\alpha(k-1)\dot\alpha(k-2)},
\end{aligned} \tag{2.11}$$

provided

$$\begin{aligned}
d_k = & \pm\frac{(s+1)(l+1)}{2k(k+1)}\lambda \\
{c_k}^2 = & \frac{(s-k+1)(s+k+1)(k-l-1)(k+l+1)}{k^2}\lambda^2.
\end{aligned} \tag{2.12}$$

We assume that all parameters c_k are positive. The sign of d_k (which is not fixed by the gauge invariance) plays an important role in the construction of the supermultiplets. As it will be seen below the pair of the fermions entering $N=1$ supermultiplet must have opposite signs of d_k forming in this way the Dirac mass-like term.

In the fermionic case for each field we also have a corresponding gauge invariant object (as in the bosonic case we omit any extra fields):

$$\begin{aligned}
\mathcal{F}^{\alpha(k)\dot\alpha(k-1)} = & D\Phi^{\alpha(k)\dot\alpha(k-1)} + 2d_k e^\alpha{}_{\dot\beta}\Phi^{\alpha(k-1)\dot\alpha(k-1)\dot\beta} \\
& -c_{k+1} e_{\beta\dot\beta}\Phi^{\alpha(k)\beta\dot\alpha(k-1)\dot\beta} + \frac{c_k}{(k-1)(k+1)} e^{\alpha\dot\alpha}\Phi^{\alpha(k-1)\dot\alpha(k-2)}.
\end{aligned} \tag{2.13}$$

JHEP08(2019)116

Using these curvatures, the variation of the Lagrangian (2.10) under the supertransformations can be compactly written as follows:

$$\delta\mathcal{L} = -(-1)^{\tau} \sum_{k=\tilde{l}+1}^{s} \mathcal{F}_{\alpha(k-1)\beta\dot\alpha(k-1)} e^{\beta}{}_{\dot\beta} \delta\Phi^{\alpha(k-1)\dot\alpha(k-1)\dot\beta} + \text{h.c.} \tag{2.14}$$

3 Partially massless superblocks

As it has been shown in [1], the partially massless supermultiplets in AdS_4, corresponding to non-unitary supersymmetric representations, similarly to the massive case contain two bosonic and two fermionic partially massless fields with the properly adjusted depths (see diagrams (1.1) in Introduction):

$$\begin{tikzcd}[arrows=-]
& \Phi_{[s+\frac{1}{2}]_t} \arrow[dl] \arrow[dr] & \\
f_{[s]_t} \arrow[dr] & & f'_{[s]_{t-1}} \arrow[dl] \\
& \Psi_{[s-\frac{1}{2}]_{t-1}} &
\end{tikzcd}
\qquad
\begin{tikzcd}[arrows=-]
& \Phi_{[s-\frac{1}{2}]_t} \arrow[dl] \arrow[dr] & \\
f_{[s]_t} \arrow[dr] & & f'_{[s-1]_{t-1}} \arrow[dl] \\
& \Psi_{[s-\frac{1}{2}]_{t-1}} &
\end{tikzcd}$$

Here integers s and t label spin and depth respectively. For a given s the depth t of bosonic (fermionic) partially massless field with spin $s(s+1/2)$ go from 1 to $(s-1)$. The authors of [1] also have found Lagrangian realization of the simplest supermultiplets containing partially massless spin-2, massive spin-3/2 and two massless fields with spin 3/2 and spin 1 (it arises from the right diagram at $s=2$ and $t=1$). They have studied it from the partially massless limit of the full massive supermultiplet. Such limit in AdS is non-unitary and lead to that norms of kinetic terms of spin-2 and spin-1 fields in Lagrangian are opposite, the same holds for the two spin-3/2 fields.

In this work we systematically study generic partially massless supermultiplets corresponding to the above diagrams for $s>2$ and $1 \le t < (s-1)$ working from the beginning with the partially massless fields. We follow the same strategy we used for the construction of the massive supermultiplets [9]. At first, we consider two possible pairs of the bosonic and fermionic partially massless fields (superblocks), namely $(s, s+1/2)$ and $(s-1/2, s)$, and find the supertransformations which leave the sum of their free Lagrangians invariant. Then we consider the whole system of four fields and choose the parameters in such a way that the algebra of the supertransformations is closed.

3.1 Ansatz for the supertransformations

We choose the following ansatz for the supertransformations for a pair of the partially massless bosonic and fermionic fields (superblock):

$$\begin{aligned}
\delta f^{\alpha(k-1)\dot\alpha(k-1)} &= \alpha_{k-1}\Phi^{\alpha(k-1)\beta\dot\alpha(k-1)}\zeta_{\beta} - \bar\alpha_{k-1}\Phi^{\alpha(k-1)\dot\alpha(k-1)\dot\beta}\zeta_{\dot\beta} \\
&\quad + \alpha'_{k-1}\Phi^{\alpha(k-1)\dot\alpha(k-2)}\zeta^{\dot\alpha} - \bar\alpha'_{k-1}\Phi^{\alpha(k-2)\dot\alpha(k-1)}\zeta^{\alpha},
\end{aligned}$$

$$
\begin{aligned}
\delta\Phi^{\alpha(k)\dot\alpha(k-1)} &= \beta_{k-1}\Omega^{\alpha(k)\dot\alpha(k-2)}\zeta^{\dot\alpha} + \gamma_{k-1}f^{\alpha(k-1)\dot\alpha(k-1)}\zeta^{\alpha} \\
&\quad +\beta'_k\Omega^{\alpha(k)\beta\dot\alpha(k-1)}\zeta_\beta + \gamma'_k f^{\alpha(k)\dot\alpha(k-1)\dot\beta}\zeta_{\dot\beta}, \\
\delta\Phi^{\alpha(k-1)\dot\alpha(k)} &= \bar\beta_{k-1}\Omega^{\alpha(k-2)\dot\alpha(k)}\zeta^{\alpha} + \bar\gamma_{k-1}f^{\alpha(k-1)\dot\alpha(k-1)}\zeta^{\dot\alpha} \\
&\quad +\bar\beta'_k\Omega^{\alpha(k-1)\dot\alpha(k)\dot\beta}\zeta_{\dot\beta} + \bar\gamma'_k f^{\alpha(k-1)\beta\dot\alpha(k)}\zeta_\beta.
\end{aligned}
\tag{3.1}
$$

where all coefficients are complex. Note that this is most general possible expression with respect to symmetrization and contraction of spinor indices.[4] As we will see below coefficients in the supertransformations can be pure real or pure imaginary. It depends on a parity of bosonic fields that is on how bosonic fields transform under spatial reflections. The parity is defined by operator P, acting on bosonic fields it gives

$$
Pf^{\alpha(k-1)\dot\alpha(k-1)} = \pm f^{\alpha(k-1)\dot\alpha(k-1)}, \quad P\Omega^{\alpha(k)\dot\alpha(k-2)} = \pm\Omega^{\alpha(k-2)\dot\alpha(k)}.
$$

The $+$, $-$ signs define parity-even and parity-odd bosonic fields respectively. Considering fermionic fields $\Phi^{\alpha(k),\dot\alpha(k-1)}$ and parameter of supertransformations ζ^α as parity-even, one can see that in the case of parity-even(odd) bosonic fields coefficients α_k, α'_k are imaginary(real) and $\beta_k, \beta'_k, \gamma_k, \gamma'_k$ are real(imaginary). As in the case of the massive supermultiplets, partially massless ones have to contain two bosonic fields with opposite parities since it arises from the massive one in partially massless limit. Hence we have to consider partially massless superblocks with parity-even bosonic field as well as parity-odd one. So to unify these two cases we begin with complex coefficients in supertransformations (3.1).

In the gauge invariant formulation that we use it is easy to see that not only spins but the depths of the superpartners must be related. Indeed, let us consider partially massless bosonic field $f_{[s]_t}$ of spin s and depth $t = (s-l)$, which involves the field variables $f^{\alpha(k-1)\dot\alpha(k-1)}$ with $l \le k \le s$, i.e. it has maximal helicity s and minimal one l. Then there are only four possible superpartners, namely, partially massless fermions with maximal helicities $s \pm 1/2$ and minimal ones $l \pm 1/2$. Denoting the partially massless fermionic field of spin $s + 1/2$ and depth $t = (s-l)$ as $\Phi_{[s+\frac{1}{2}]_t}$, which involves the field variables $\Phi^{\alpha(k)\dot\alpha(k-1)}$ with $l \le k \le s$, four possible superpartners of bosonic field $f_{[s]_t}$ can be represented by diagram

$$
\begin{tikzcd}
\Phi_{[s+\frac{1}{2}]_{t+1}} \arrow[dr, dash] & & \Phi_{[s+\frac{1}{2}]_t} \arrow[dl, dash] \\
& f_{[s]_t} \arrow[dl, dash] \arrow[dr, dash] & \\
\Phi_{[s-\frac{1}{2}]_t} & & \Phi_{[s-\frac{1}{2}]_{t-1}}
\end{tikzcd}
\tag{3.2}
$$

[4]Remind that we use a formalism analogous to the so-called 1 and 1/2 order formalisms, which is very well known in supergravity. Namely, we do not introduce any supertransformations for the auxiliary fields, instead all calculations are done using the "zero torsion conditions" (2.8). The main advantage of the multispinor formalism is that there are only two operations with spinor indices: contraction and symmetrization. Therefore, the ansatz for supertransformations (3.1) is indeed the most general one.

JHEP08(2019)116

One can see from the diagram given above there are just four superblocks that form partially massless supermultiplets. Note that the ansatz for supertransformations (3.1) is valid for all types of the superblocks which differ only by the boundary conditions.

Now let us consider a sum of the bosonic (2.9) and fermionic (2.14) variations under the supertransformations (3.1). Using the torsion zero conditions (2.8), it can be written in the form $\delta\mathcal{L}+\delta\mathcal{L}'$, where

$$\begin{aligned}
\delta\mathcal{L} = \sum_{k=2}^{s}\Big[&-(-1)^{\tau}(k-1)\bar{\beta}_{k-1}\mathcal{F}_{\alpha(k-1)\beta\dot{\alpha}(k-1)}e^{\beta}{}_{\dot{\beta}}\Omega^{\alpha(k-2)\dot{\alpha}(k-1)\dot{\beta}}\zeta^{\alpha} \\
&+(-1)^{\sigma}4i\alpha_{k-1}\Phi_{\alpha(k-2)\beta\gamma\dot{\alpha}(k-1)}e^{\gamma}{}_{\dot{\gamma}}\mathcal{R}^{\alpha(k-2)\dot{\alpha}(k-1)\dot{\gamma}}\zeta^{\beta} \\
&-(-1)^{\tau}\bar{\gamma}_{k-1}(\mathcal{F}_{\alpha(k-1)\beta\dot{\alpha}(k-1)}e^{\beta}{}_{\dot{\beta}}f^{\alpha(k-1)\dot{\alpha}(k-1)}\zeta^{\dot{\beta}} \\
&+(k-1)\mathcal{F}_{\alpha(k-1)\beta\dot{\alpha}(k-1)}e^{\beta}{}_{\dot{\beta}}f^{\alpha(k-1)\dot{\alpha}(k-2)\dot{\beta}}\zeta^{\dot{\alpha}})\Big] + \text{h.c.}, \qquad (3.3) \\
\delta\mathcal{L}' = \sum_{k=2}^{s+1}\Big[&-(-1)^{\tau}\bar{\beta}'_{k-1}\mathcal{F}_{\alpha(k-2)\gamma\dot{\alpha}(k-2)}e^{\gamma}{}_{\dot{\gamma}}\Omega^{\alpha(k-2)\dot{\alpha}(k-2)\dot{\gamma}\dot{\beta}}\zeta_{\dot{\beta}} \\
&-(-1)^{\sigma}i4(k-1)\alpha'_{k-1}\Phi_{\alpha(k-2)\beta\dot{\alpha}(k-2)}e^{\beta}{}_{\dot{\beta}}\mathcal{R}^{\alpha(k-2)\dot{\alpha}(k-2)\dot{\beta}\dot{\gamma}}\zeta_{\dot{\gamma}} \\
&-(-1)^{\tau}\bar{\gamma}'_{k-1}\mathcal{F}_{\alpha(k-2)\gamma\dot{\alpha}(k-2)}e^{\gamma}{}_{\dot{\gamma}}f^{\alpha(k-2)\beta\dot{\alpha}(k-2)\dot{\gamma}}\zeta_{\beta}\Big] + \text{h.c.}
\end{aligned}$$

Schematically, the structure of the variations has the form "curvature $\times$ field". The fact, that both the Lagrangians and their variations are defined only up to a total derivative, leads to a number of non-trivial identities on such terms. The general form of these identities were given in appendix A of [9] and they are applicable to the case at hands, the only difference being in the explicit expressions for the coefficients a_k, b_k, c_k and d_k. Using these identities one can express the parameters α and γ in terms of β:

$$\alpha_k = (-1)^{\sigma+\tau}i\frac{k}{4}\bar{\beta}_k, \qquad \alpha'_k = -(-1)^{\sigma+\tau}\frac{i}{4k}\bar{\beta}'_{k-1}, \qquad (3.4)$$

$$\gamma_k = 2d_{k+1}\bar{\beta}_k, \qquad \gamma'_k = 2d_k\bar{\beta}'_k. \qquad (3.5)$$

Also we obtain recurrence relations on the parameters β_k and β'_k:

$$2(k+1)\beta_{k-1}c_{k+1} = k\beta_k a_{k+1}, \qquad \beta'_{k-1}a_{k+1} = 2\beta'_k c_k. \qquad (3.6)$$

Last but not least, we obtain four independent equations which relate β and β' as well as the bosonic and fermionic depth parameters:

$$0 = \frac{\beta'_{k-1}c_k}{(k-1)} - \frac{\beta'_k a_{k+1}}{2(k+1)} + \lambda\beta_{k-1} - \gamma_{k-1}, \qquad (3.7)$$

$$0 = (k-1)\beta_{k-1}c_k - \frac{(k-2)}{2}\beta_{k-2}a_k + \lambda\beta'_{k-1} - \gamma'_{k-1}, \qquad (3.8)$$

$$0 = \frac{(k-1)}{2k}\bar{\beta}_{k-1}b_k - 2kd_k\gamma_{k-1} + \lambda\bar{\gamma}_{k-1} - \frac{\bar{\gamma}'_k a_{k+1}}{2k(k+1)}, \qquad (3.9)$$

$$0 = \frac{\bar{\beta}'_{k-1}b_k}{2} - 2(k-1)d_{k-1}\gamma'_{k-1} - \lambda\bar{\gamma}'_{k-1} - \bar{\gamma}_{k-1}c_k. \qquad (3.10)$$

In the next two subsections we find the solutions of these equations for the two possible partially massless superblocks.

JHEP08(2019)116

3.2 Solution for the superblock $(s+1/2, s)$

Let us consider a superblock containing a partially massless boson with spin s and depth $t = (s-l-1)$ and a partially massless fermion with spin $s+1/2$ and depth $\tilde{t} = (s-\tilde{l}-1$. The explicit expressions for the bosonic coefficients have the form:

$$\begin{aligned} b_k &= \frac{2s(s+1)l(l+1)}{k(k-1)(k+1)}\lambda^2, \\ {a_k}^2 &= \frac{4(s-k+1)(s+k)(k-l-1)(k+l)}{(k-2)(k-1)}\lambda^2, \end{aligned}$$

while the fermionic ones look like:

$$\begin{aligned} d_k &= \pm\frac{(s+1)(\tilde{l}+1)}{2k(k+1)}\lambda, \\ {c_k}^2 &= \frac{(s-k+1)(s+k+1)(k-\tilde{l}-1)(k+\tilde{l}+1)}{k^2}\lambda^2. \end{aligned}$$

Recall that the parameters α_k and γ_k are determined in terms of β by (3.4) and (3.5). Now let us consider equation (3.9) at $k=s$. This gives

$$[l(l+1) - (\tilde{l}+1)^2]\bar{\beta}_{s-1} = \mp(\tilde{l}+1)\beta_{s-1},$$

where the sign corresponds to that of d_k, and provides us with the relation on the bosonic and fermionic depths:

$$\begin{aligned} \tilde{l} &= l, & \bar{\beta}_{s-1} &= \pm\beta_{s-1}, \\ \tilde{l} &= l-1, & \bar{\beta}_{s-1} &= \mp\beta_{s-1}. \end{aligned}$$

Remind that a real (imaginary) values of β_{s-1} corresponds to a parity-even (parity-odd) bosonic field. Now we proceed with the solution of all remaining equations and obtain, for $\tilde{l} = l$

$$\beta_k = \sqrt{\frac{(s+k+2)(k+l+2)}{k}}\beta, \quad \beta'_k = -\sqrt{k(s-k)(k-l)}\beta, \tag{3.11}$$

and for $\tilde{l} = l-1$

$$\beta_k = \sqrt{\frac{(s+k+2)(k-l+1)}{k}}\beta, \quad \beta'_k = -\sqrt{k(s-k)(k+l+1)}\beta, \tag{3.12}$$

where $\beta = \rho$ ($\beta = i\rho$) takes pure real (imaginary) value. These four solutions correspond to upper line in diagram (3.2) and schematically can be presented as

$$\begin{array}{ccc} [s+\frac{1}{2}]_t^{\tau,\pm} & \qquad & [s+\frac{1}{2}]_{t+1}^{\tau,\mp} \\ \rho\Big| & & \rho\Big| \\ [s]_t^{\sigma,\pm} & & [s]_t^{\sigma,\pm} \end{array} \tag{3.13}$$

with following clarifying features. $\pm$ signs for the bosons correspond to their parity, while for the fermions to the sign of d_k. The σ, τ parameterize norm of kinetic terms in Lagrangian for bosons (2.2) and fermions (2.10) respectively, they are still unfixed at this stage. The real parameter ρ corresponds to one free parameter in supertransformations. Each superblock must have its own parameter ρ.

3.3 Solution for the superblock $(s, s-1/2)$

Now let us turn to the second superblock which contain a partially massless boson with spin s and depth $t = (s-l-1)$ and a partially massless fermion with spin $s-1/2$ and depth $\tilde{t} = (s-\tilde{l}-2)$. Thus for the bosonic field we still have the same expressions for the coefficients a_k and b_k as in the previous subsection, while for the fermion we obtain

$$\begin{aligned} d_k &= \pm\frac{s(\tilde{l}+1)}{2k(k+1)}\lambda, \\ {c_k}^2 &= \frac{(s-k)(s+k)(k-\tilde{l}-1)(k+\tilde{l}+1)}{k^2}\lambda^2. \end{aligned}$$

First of all, let us consider equation (3.8) at $k = s$. This gives us:

$$[l(l+1) - (\tilde{l}+1)^2]\beta'_{s-1} = \pm(\tilde{l}+1)\bar{\beta}'_{s-1}.$$

where the sign corresponds to that of d_k. Thus in this case we again have four possible solutions:

$$\begin{aligned} &\tilde{l} = l, && \bar{\beta}_{s-1} = \mp\beta_{s-1}, \\ &\tilde{l} = l-1, && \bar{\beta}_{s-1} = \pm\beta_{s-1}. \end{aligned}$$

Then the solution of the remaining equations gives, for $\tilde{l} = l$

$$\beta_k = \sqrt{\frac{(s-k-1)(k+l+2)}{k}}\beta, \quad \beta'_k = \sqrt{k(s+k+1)(k-l)}\beta, \tag{3.14}$$

and for $\tilde{l} = l-1$

$$\beta_k = \sqrt{\frac{(s-k-1)(k-l+1)}{k}}\beta, \quad \beta'_k = \sqrt{k(s+k+1)(k+l+1)}\beta, \tag{3.15}$$

where again $\beta = \rho$ $(\beta = i\rho)$ takes pure real (imaginary) value. These four solutions corresponds to lower line in diagram (3.2) and schematically can be presented as

$$\begin{array}{ccc} [s]_t^{\sigma,\pm} & \qquad & [s]_t^{\sigma,\pm} \\ \rho\,\Big| & & \rho\,\Big| \\ [s-\frac{1}{2}]_t^{\tau,\pm} & & [s-\frac{1}{2}]_{t-1}^{\tau,\mp} \end{array} \tag{3.16}$$

Here all additional notations are the same as in previous case (3.13).

4 Partially massless supermultiplets

As we have already mentioned, each partially massless supermultiplet contains two bosonic and two fermionic fields. As in the massive case, the two bosons must have opposite parities, and it turns out to be important that the two fermions have opposite signs of their mass

JHEP08(2019)116

terms. Moreover, the depths of the partial masslessness for each field must be properly adjusted. Schematically, the two possible supermultiplets (1.1) look like:

$$
\begin{array}{ccccc}
 & & [s+\frac{1}{2}]_t^{\tau_1,+} & & \\
 & \rho_1 \diagup & & \diagdown \rho_3 & \\
[s]_t^{\sigma_1,+} & & Y = s & & [s]_{t-1}^{\sigma_2,-} \\
 & \rho_2 \diagdown & & \diagup \rho_4 & \\
 & & [s-\frac{1}{2}]_{t-1}^{\tau_2,-} & &
\end{array}
\qquad
\begin{array}{ccccc}
 & & [s-\frac{1}{2}]_t^{\tau_1,+} & & \\
 & \rho_1 \diagup & & \diagdown \rho_3 & \\
[s]_t^{\sigma_1,+} & & Y = s-\frac{1}{2} & & [s-1]_{t-1}^{\sigma_2,-} \\
 & \rho_2 \diagdown & & \diagup \rho_4 & \\
 & & [s-\frac{1}{2}]_{t-1}^{\tau_2,-} & &
\end{array}
$$

with the same additional notations that was used in the construction of superblocks (3.13), (3.16). Since these supermultiplets are constructed as combination of superblocks we should take independent parameters σ_i, τ_i which normalize kinetic terms for bosons and fermions respectively and independent ρ_i which parameterize supertransformations for a given superblock.

Let us use the notations (f_+, Ω_+) and (f_-, Ω_-) for the parity-even/parity-odd bosons and Φ_+, Φ_- for fermions according to their sign of d_k. In these notations the supertransformations for the whole supermultiplets are the combination of four separate superblocks corresponding to the parameters $\rho_{1,2,3,4}$ shown above. Namely, we take for the bosons:

$$
\begin{aligned}
\delta f_+^{\alpha(k-1)\dot\alpha(k-1)} &= \alpha_{k-1}|_{\rho_1} \Phi_+^{\alpha(k-1)\beta\dot\alpha(k-1)} \zeta_\beta + \alpha'_{k-1}|_{\rho_1} \Phi_+^{\alpha(k-1)\dot\alpha(k-2)} \zeta^{\dot\alpha} \\
&\quad + \alpha_{k-1}|_{\rho_2} \Phi_-^{\alpha(k-1)\beta\dot\alpha(k-1)} \zeta_\beta + \alpha'_{k-1}|_{\rho_2} \Phi_-^{\alpha(k-1)\dot\alpha(k-2)} \zeta^{\dot\alpha} + \text{h.c.}
\end{aligned}
$$

and similarly for $\delta f_-^{\alpha(k-1)\dot\alpha(k-1)}$ with the replacement $\rho_1 \to \rho_3$ and $\rho_2 \to \rho_4$, while for the fermions we use

$$
\begin{aligned}
\delta \Phi_+^{\alpha(k)\dot\alpha(k-1)} &= \beta_{k-1}|_{\rho_1} \Omega_+^{\alpha(k)\dot\alpha(k-2)} \zeta^{\dot\alpha} + \gamma_{k-1}|_{\rho_1} f_+^{\alpha(k-1)\dot\alpha(k-1)} \zeta^\alpha \\
&\quad + \beta'_k|_{\rho_1} \Omega_+^{\alpha(k)\beta\dot\alpha(k-1)} \zeta_\beta + \gamma'_k|_{\rho_1} f_+^{\alpha(k)\dot\alpha(k-1)\dot\beta} \zeta_{\dot\beta} \\
&\quad + \beta_{k-1}|_{\rho_3} \Omega_-^{\alpha(k)\dot\alpha(k-2)} \zeta^{\dot\alpha} + \gamma_{k-1}|_{\rho_3} f_-^{\alpha(k-1)\dot\alpha(k-1)} \zeta^\alpha \\
&\quad + \beta'_k|_{\rho_3} \Omega_-^{\alpha(k)\beta\dot\alpha(k-1)} \zeta_\beta + \gamma'_k|_{\rho_3} f_-^{\alpha(k)\dot\alpha(k-1)\dot\beta} \zeta_{\dot\beta}
\end{aligned}
$$

and similarly for $\delta\Phi_-^{\alpha(k)\dot\alpha(k-1)}$ with the replacement $\rho_1 \to \rho_2$ and $\rho_3 \to \rho_4$.

So to construct a complete partially massless supermultiplet we have to adjust these four parameters $\rho_{1,2,3,4}$ so that the algebra of supertransformations be closed. It means that the commutator of the two supertransformations must produce a combination of translations and Lorentz transformations:

$$
\{Q_\alpha, Q_{\dot\alpha}\} \sim P_{\alpha\dot\alpha}, \quad \{Q_\alpha, Q_\alpha\} \sim \lambda M_{\alpha\alpha}, \quad \{Q_{\dot\alpha}, Q_{\dot\alpha}\} \sim \lambda M_{\dot\alpha\dot\alpha}. \tag{4.1}
$$

The structure of the mass-shell condition (2.8) shows that, for example, the commutator on the bosonic field ${f_+}^{\alpha(k-1)\dot\alpha(k-1)}$ must only contain ${\Omega_+}^{\alpha(k)\dot\alpha(k-2)}$, ${\Omega_+}^{\alpha(k-2)\dot\alpha(k)}$, $f_+^{\alpha(k)\dot\alpha(k)}$,

${f_+}^{\alpha(k-1)\dot\alpha(k-1)}$ and ${f_+}^{\alpha(k-2)\dot\alpha(k-2)}$. This requirement leads to the number of relations on the parameters:

$$
\begin{aligned}
\alpha_{k-1}|_{\rho_1}\beta'_k|_{\rho_1} + \alpha_{k-1}|_{\rho_2}\beta'_k|_{\rho_2} = 0, \qquad \alpha'_{k-1}|_{\rho_1}\beta_{k-2}|_{\rho_1} + \alpha'_{k-1}|_{\rho_2}\beta_{k-2}|_{\rho_2} &= 0,\\
\alpha_{k-1}|_{\rho_1}\beta'_k|_{\rho_3} + \alpha_{k-1}|_{\rho_2}\beta'_k|_{\rho_4} = 0, \qquad \alpha'_{k-1}|_{\rho_1}\beta_{k-2}|_{\rho_3} + \alpha'_{k-1}|_{\rho_2}\beta_{k-2}|_{\rho_4} &= 0,\\
\alpha_{k-1}|_{\rho_1}\beta_{k-1}|_{\rho_3} + \alpha'_{k-1}|_{\rho_1}\beta'_{k-1}|_{\rho_3} + \alpha_{k-1}|_{\rho_2}\beta_{k-1}|_{\rho_4} + \alpha'_{k-1}|_{\rho_2}\beta'_{k-1}|_{\rho_4} &= 0,\\
\alpha_{k-1}|_{\rho_1}\gamma_{k-1}|_{\rho_3} - \bar\alpha'_{k-1}|_{\rho_1}\bar\gamma'_{k-1}|_{\rho_3} + \alpha_{k-1}|_{\rho_2}\gamma_{k-1}|_{\rho_4} - \bar\alpha'_{k-1}|_{\rho_2}\bar\gamma'_{k-1}|_{\rho_4} &= 0,\\
\alpha_{k-1}|_{\rho_1}\gamma'_k|_{\rho_3} - \bar\alpha_{k-1}|_{\rho_1}\bar\gamma'_k|_{\rho_3} + \alpha_{k-1}|_{\rho_2}\gamma'_k|_{\rho_4} - \bar\alpha_{k-1}|_{\rho_2}\bar\gamma'_k|_{\rho_4} &= 0,\\
\alpha'_{k-1}|_{\rho_1}\gamma_{k-2}|_{\rho_3} - \bar\alpha'_{k-1}|_{\rho_1}\bar\gamma_{k-2}|_{\rho_3} + \alpha'_{k-1}|_{\rho_2}\gamma_{k-2}|_{\rho_4} - \bar\alpha'_{k-1}|_{\rho_2}\bar\gamma_{k-2}|_{\rho_4} &= 0.
\end{aligned}
$$

If these relations are fulfilled, the general form of the commutator looks like:

$$
\begin{aligned}
&[\delta_1,\delta_2]f_+^{\alpha(k-1)\dot\alpha(k-1)}\\
&\quad = (\alpha_{k-1}|_{\rho_1}\beta_{k-1}|_{\rho_1} + \alpha'_{k-1}|_{\rho_1}\beta'_{k-1}|_{\rho_1} + \alpha_{k-1}|_{\rho_2}\beta_{k-1}|_{\rho_2} + \alpha'_{k-1}|_{\rho_2}\beta'_{k-1}|_{\rho_2})\\
&\qquad\qquad\qquad\qquad \cdot[\xi^\alpha{}_{\dot\beta}\Omega_+^{\alpha(k-2)\dot\alpha(k-1)\dot\beta} + \xi_\beta{}^{\dot\alpha}\Omega_+^{\alpha(k-1)\beta\dot\alpha(k-2)}]\\
&\qquad +(\alpha_{k-1}|_{\rho_1}\gamma'_k|_{\rho_1} + \alpha_{k-1}|_{\rho_2}\gamma'_k|_{\rho_2})f_+^{\alpha(k-1)\beta\dot\alpha(k-1)\dot\beta}\xi_{\beta\dot\beta}\\
&\qquad +(\alpha'_{k-1}|_{\rho_1}\gamma_{k-2}|_{\rho_1} + \alpha'_{k-1}|_{\rho_2}\gamma_{k-2}|_{\rho_2})f_+^{\alpha(k-2)\dot\alpha(k-2)}\xi^{\alpha\dot\alpha}\\
&\qquad +(\alpha_{k-1}|_{\rho_1}\gamma_{k-1}|_{\rho_1} + \alpha'_{k-1}|_{\rho_1}\gamma'_{k-1}|_{\rho_1} + \alpha_{k-1}|_{\rho_2}\gamma_{k-1}|_{\rho_2} + \alpha'_{k-1}|_{\rho_2}\gamma'_{k-1}|_{\rho_2})\\
&\qquad\qquad\qquad\qquad \cdot[f_+^{\alpha(k-2)\beta\dot\alpha(k-1)}\eta^\alpha{}_\beta + f_+^{\alpha(k-1)\dot\alpha(k-2)\dot\beta}\eta^{\dot\alpha}{}_{\dot\beta}],
\end{aligned}
\tag{4.2}
$$

where

$$
\xi^{\alpha\dot\alpha} = \zeta_1^\alpha\zeta_2^{\dot\alpha} - \zeta_2^\alpha\zeta_1^{\dot\alpha}, \qquad \eta^{\alpha(2)} = \zeta_1^\alpha\zeta_2^\alpha - \zeta_2^\alpha\zeta_1^\alpha, \qquad \eta^{\dot\alpha(2)} = \zeta_1^{\dot\alpha}\zeta_2^{\dot\alpha} - \zeta_2^{\dot\alpha}\zeta_1^{\dot\alpha}.
$$

For the bosonic field ${f_-}^{\alpha(k-1)\dot\alpha(k-1)}$ the commutator has the same form with the replacements $\rho_1 \to \rho_3$ and $\rho_2 \to \rho_4$. Let us stress that all these bosonic components belong to the same supermultiplet, i.e. to the same irreducible representations. Thus all the expressions in round brackets in (4.2) must be k-independent. This gives additional restrictions on the parameters and also serves as quite a non-trivial test for our calculations.

4.1 Supermultiplets with half-integer superspin

The partially massless supermultiplet with the half-integer superspin $Y = (s-1/2)$ has the following structure:

$$
\begin{tikzcd}
 & {[s-\frac12]_t^{\tau_1,+}} \arrow[dl, dash, "\rho_1"'] \arrow[dr, dash, "\rho_3"] & \\
{[s]_t^{\sigma_1,+}} \arrow[dr, dash, "\rho_2"'] & Y = s-\frac12 & {[s-1]_{t-1}^{\sigma_2,-}} \arrow[dl, dash, "\rho_4"] \\
 & {[s-\frac12]_{t-1}^{\tau_2,-}} &
\end{tikzcd}
$$

JHEP08(2019)116

We have only a handful of free parameters in our disposal and a large number of equations to fulfill, however, the closure of the superalgebra is achieved at:

$$\sigma_2 = \sigma_1, \quad \tau_2 = \tau_1 + 1, \quad {\rho_1}^2 = {\rho_2}^2 = {\rho_3}^2 = {\rho_4}^2, \qquad \rho_1\rho_3 = \rho_2\rho_4. \tag{4.3}$$

Note that such relations between σ, τ parameters mean that two bosons must enter with opposite norms of kinetic terms and the same is for two fermions.[5] This is in agreement with [1] where the same result was obtained for the case of the supermultiplet with partially massless spin-2 field.

The final form for the commutators of the supertransformations on parity-even spin-s f_+ and parity-odd spin-$(s-1)$ f_- fields appears to be the same:

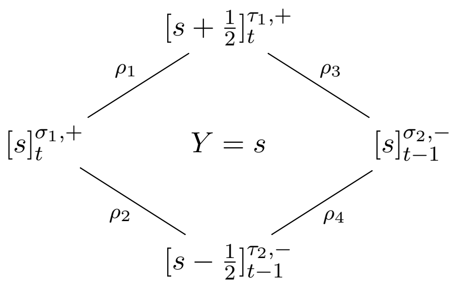
$$\begin{aligned}
\frac{1}{i{\rho_0}^2}[\delta_1, \delta_2] f_\pm^{\alpha(k-1)\dot\alpha(k-1)} &= \Omega_\pm^{\alpha(k-1)\beta\dot\alpha(k-2)}\xi_\beta{}^{\dot\alpha} + \Omega_\pm^{\alpha(k-2)\dot\alpha(k-1)\dot\beta}\xi^\alpha{}_{\dot\beta} \\
&\quad - \frac{(k-1)a_{k+1}}{2(k+1)} f_\pm^{\alpha(k-1)\beta\dot\alpha(k-1)\dot\beta}\xi_{\beta\dot\beta} + \frac{a_k}{2k(k-1)} f_\pm^{\alpha(k-2)\dot\alpha(k-2)}\xi^{\alpha\dot\alpha} \\
&\quad + \lambda[f_\pm^{\alpha(k-2)\beta\dot\alpha(k-1)}(\zeta_1^\alpha\eta^\alpha{}_\beta + f_\pm^{\alpha(k-1)\dot\alpha(k-2)\dot\beta}\eta^{\dot\alpha}{}_{\dot\beta}],
\end{aligned}$$

where a_k is given by (2.4) for spin s and spin $(s-1)$ respectively and

$${\rho_0}^2 = -(-1)^{\sigma_1+\tau_1}\frac{s(2l+1)}{2}{\rho_1}^2.$$

4.2 Supermultiplets with integer superspin

Now let us turn to the partially massless supermultiplet with integer superspin $Y = s$:



All the relations for the closure of the superalgebra are fulfilled provided:

$$\sigma_2 = \sigma_1 + 1, \quad \tau_2 = \tau_1, \quad {\rho_1}^2 = {\rho_2}^2 = {\rho_3}^2 = {\rho_4}^2, \quad \rho_1\rho_3 = \rho_2\rho_4. \tag{4.4}$$

Again we see that such relations between σ, τ parameters mean that two bosons must enter with opposite norm of kinetic terms and the same is for two fermions.[6]

[5] Let us recall that we work in metric signature $(+,-,-,-)$ which give overall $(-1)^s$ factor in norm of kinetic terms for given bosonic field with spin s. So the same factor for bosonic field with spin $(s-1)$ means opposite norm of kinetic terms $(-1)^s = -(-1)^{s-1}$. This explains unusual relation between σ_1, σ_2 in (4.3).

[6] Let us again recall that we work in metric signature $(+,-,-,-)$ which give overall $(-1)^s$ factor in norm of kinetic terms for given fermionic field with spin $s+1/2$. So the same factor for fermionic field with spin $s-1/2$ means opposite norm of kinetic terms $(-1)^s = -(-1)^{s-1}$. This explains unusual relation between τ_1, τ_2 in (4.4).

JHEP08(2019)116

The final form of the commutators of the supertransformations on parity-even and parity-odd bosonic spin-s fields have the same form as in the previous case:

$$\begin{aligned}\frac{1}{i{\rho_0}^2}[\delta_1,\delta_2]f_\pm^{\alpha(k-1)\dot\alpha(k-1)} &= \Omega_\pm^{\alpha(k-1)\beta\dot\alpha(k-2)}\xi_\beta{}^{\dot\alpha} + \Omega_\pm^{\alpha(k-2)\dot\alpha(k-1)\dot\beta}\xi^\alpha{}_{\dot\beta} \\ &-\frac{(k-1)a_{k+1}}{2(k+1)}f_\pm^{\alpha(k-1)\beta\dot\alpha(k-1)\dot\beta}\xi_{\beta\dot\beta} + \frac{a_k}{2k(k-1)}f_\pm^{\alpha(k-2)\dot\alpha(k-2)}\xi^{\alpha\dot\alpha} \\ &+\lambda[f_\pm^{\alpha(k-2)\beta\dot\alpha(k-1)}\eta^\alpha{}_\beta + f_\pm^{\alpha(k-1)\dot\alpha(k-2)\dot\beta}\eta^{\dot\alpha}{}_{\dot\beta}],\end{aligned}$$

where a_k is given by (2.4) for spin s and

$$\rho_0{}^2 = (-1)^{\sigma_1+\tau_1}\frac{(2s+1)(l+1)}{2}\rho_1{}^2.$$

5 Summary

In this paper we have presented the component Lagrangian description of partially massless higher spin on-shell arbitrary $N=1$ supermultiplets in four-dimensional AdS_4 space corresponding the classification given in [1].[7] The constructed supermultiplets are non-unitary and contain partially massless fields with appropriately chosen spins and depths. As in the massive case [9] we show that $N=1$ partially massless supermultiplets can be constructed as a combination of four partially massless superblocks containing one partially massless boson and one partially massless fermion. As a result we have derived both the supertransformations for the components of the supermultiplet and the corresponding invariant Lagrangian. Also we show that a closure of superalgebra requires that two bosons and two fermions must enter supermultiplets with opposite norm of kinetic terms. All our results are in agreement with the results of [1] and extend them. The constructed Lagrangian formulation describes a dynamics of arbitrary superspin partially massless supermultiplets in AdS_4 space.

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A Notations and conventions

We work in the frame-like multispinor formalism. It means that all objects are differential p-forms ($p=0,1,2,3,4$ in four dimensions) with multispinors as their local indices, i.e.

$$\Omega^{\alpha(m)\dot\alpha(n)} = dx^{\mu_1}\wedge\ldots\wedge dx^{\mu_p}\Omega_{\mu_1\ldots\mu_p}{}^{\alpha(m)\dot\alpha(n)}.$$

[7] In higher dimensions there exists much more rich spectrum of the partially massless fields including mixed symmetry ones. So, in principle, there may exists a whole zoo of the corresponding supermultiplets. However, as far as we know, such supersymmetric representations are not studied at present.

Here dx^μ are required to anti-commute $dx^\mu \wedge dx^\nu = -dx^\nu \wedge dx^\mu$ with respect to exterior product $\wedge$. World indices μ, ν are omitted everywhere; all expressions are completely antisymmetric on them. We use the condensed notations for local multispinor indices $\alpha, \dot\alpha$. Namely, all objects are totally symmetric on upper/low undotted/dotted indices $\alpha_1\alpha_2\cdots\alpha_k$, we denote them by the same letter with the number of indices in parentheses. For example:

$$\Omega^{(\alpha_1\alpha_2\ldots\alpha_m)(\dot\alpha_1\dot\alpha_2\ldots\dot\alpha_n)} = \Omega^{\alpha(m)\dot\alpha(n)}.$$

We also always assume if in expression spinor indices denoted by the same letters and placed on the same level are symmetrized, e.g.

$$\Omega^{\alpha(m)}\zeta^\alpha = \Omega^{(\alpha_1\ldots\alpha_m}\zeta^{\dot\alpha_{m+1})} = \Omega^{\alpha_1\ldots\alpha_m}\zeta^{\dot\alpha_{m+1}} + \text{permutations } (m \text{ terms}).$$

The spinor indices are raised and lowered with the antisymmetric tensors $\epsilon_{\alpha\beta}$ ($\epsilon_{\dot\alpha\dot\beta}$):

$$\epsilon_{\alpha\beta}\xi^\beta = -\xi_\alpha, \qquad \epsilon^{\alpha\beta}\xi_\beta = \xi^\alpha, \tag{A.1}$$

the same is true for dotted indices. Hence, all the symmetric multispinors are automatically traceless. Under the Hermitian conjugation, dotted and undotted indices are transformed one into another. For example:

$$\left(\Omega^{\alpha(m)\dot\alpha(n)}\right)^\dagger = \Omega^{\alpha(n)\dot\alpha(m)}.$$

The AdS_4 space is described by the background Lorentz connections $\omega^{\alpha(2)}$, $\omega^{\dot\alpha(2)}$, which enter implicitly through the Lorentz covariant derivative D, and the background frame $e^{\alpha\dot\alpha}$. We also use the basis elements for the two-, thee- and four-forms

$$e^a \sim e^{\alpha\dot\alpha}, \qquad E^{ab} \sim E^{\alpha(2)}, E^{\dot\alpha(2)}, \qquad E^{abc} \sim E^{\alpha\dot\alpha}, \qquad E^{abcd} \sim E, \tag{A.2}$$

defined as follows:

$$\begin{aligned} e^{\alpha\dot\alpha} \wedge e^{\beta\dot\beta} &= \varepsilon^{\alpha\beta}E^{\dot\alpha\dot\beta} + \varepsilon^{\dot\alpha\dot\beta}E^{\alpha\beta}, \\ E^{\alpha(2)} \wedge e^{\beta\dot\alpha} &= \varepsilon^{\alpha\beta}E^{\alpha\dot\alpha}, \\ E^{\alpha\dot\alpha} \wedge e^{\beta\dot\beta} &= \varepsilon^{\alpha\beta}\varepsilon^{\dot\alpha\dot\beta}E. \end{aligned} \tag{A.3}$$

The hermitian conjugation rules for the basis forms are:

$$\left(e^{\alpha\dot\alpha}\right)^\dagger = e^{\alpha\dot\alpha}, \qquad \left(E^{\alpha(2)}\right)^\dagger = E^{\dot\alpha(2)}, \qquad \left(E^{\alpha\dot\alpha}\right)^\dagger = -E^{\alpha\dot\alpha}, \qquad (E)^\dagger = -E. \tag{A.4}$$

The Lorentz covariant derivative is normalized so that

$$D \wedge D\Omega^{\alpha(m)\dot\alpha(n)} = -2\lambda^2(E^\alpha{}_\beta \wedge \Omega^{\alpha(m-1)\beta\dot\alpha(n)} + E^{\dot\alpha}{}_{\dot\beta} \wedge \Omega^{\alpha(m)\dot\alpha(n-1)\dot\beta}). \tag{A.5}$$

The parameter λ^2 is proportional to the curvature of the space-time. The AdS space has $\lambda^2 > 0$, while the dS space has $\lambda^2 < 0$. The case of $\lambda^2 = 0$ corresponds to the flat Minkowski space.

In the main text all the wedge product signs $\wedge$ are omitted.

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JHEP08(2019)116