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Extending the universal one-loop effective action by regularization scheme translating operators

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Abstract: We extend the universal one-loop effective action (UOLEA) by operators which translate between dimensional reduction (DRED) and dimensional regularization (DREG). These regularization scheme translating operators allow for an application of the UOLEA to supersymmetric high-scale models matched to non-supersymmetric effective theories. The operators are presented in a generic, model independent form, suitable for implementation into generic spectrum generators.

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Contents

1 Introduction

With the discovery of the Higgs boson at the Large Hadron Collider (LHC) [1, 2] it became clear that the Standard Model of Particle Physics (SM) is a good description of the physics at and below the electroweak scale. However, it is also clear that the SM does not provide a complete description of nature, as it fails to describe phenomena such as dark matter and does not incorporate gravity, for example. Besides these weaknesses, there are also many open questions, e.g., whether the electroweak vacuum is stable up to the Planck scale or whether there is a hierarchy problem and how it may be avoided. Many models beyond the SM (BSM) have been proposed to address the open questions and the drawbacks of the SM. One of the promising SM extensions is supersymmetry (SUSY), which can provide a solution to the hierarchy problem, explain the deviation of the anomalous magnetic moment of the muon and the stability of the electroweak vacuum. However, no supersymmetric particles with masses below the TeV scale have been discovered so far, which means that if supersymmetry is realized in nature, the SUSY particles may be heavier than the TeV scale. This finding is supported by the measured value of the Higgs boson mass of $M_h = 125.09 \pm 0.32$ GeV [3]: SUSY models often predict the mass of the SM-like Higgs boson to be of the order of the Z boson mass, $M_Z = 91.1876$ GeV, at tree-level. In order to raise the predicted Higgs mass to its measured value, large loop corrections are required, which can be achieved by the presence of multi-TeV colored SUSY particles. Large loop corrections, on the other hand, spoil the convergence of the perturbation series, leading to large uncertainties in fixed-order calculations.

Effective field theories (EFTs) are a well suited approach to obtain precise low-energy predictions of BSM models with heavy particles. However, depending on the mass hierarchy of the studied high-scale model, many different EFTs must be considered. In order to avoid repetition in the derivation of all possible EFT Lagrangians, the universal one-loop effective action (UOLEA) has been developed [4–6]. It provides generic expressions for the Wilson coefficients of the operators of the effective Lagrangian up to 1-loop level and dimension six. These generic expressions are well suited to be implemented into generic spectrum generators such as `FeynRules` [7–10], `FlexibleSUSY` [11, 12] or `SARAH` [13–16] to calculate precise predictions in all possible low-energy EFTs in a fully automated way.

The currently known effective operators of the UOLEA [4–6] are renormalized in the $\overline{\text{MS}}$ scheme. Although this scheme is well suited to renormalize non-supersymmetric models, it is cumbersome to apply it to supersymmetric models, because the underlying dimensional regularization (DREG) [17] explicitly breaks supersymmetry [18]. To nevertheless perform loop calculations in an $\overline{\text{MS}}$ renormalized SUSY model one would have to restore supersymmetry, for example by introducing supersymmetry-restoring counter terms, as discussed for example in [19–21]. In supersymmetric models regularization by dimensional reduction (DRED) [22] is currently known to not break supersymmetry up to the 3-loop level [23–25] and is therefore widely adopted in SUSY loop calculations. In order to apply the UOLEA to a scenario, where heavy particles of a supersymmetric model (renormalized in the $\overline{\text{DR}}$ scheme) are integrated out at a high scale and a non-supersymmetric EFT (renormalized in the $\overline{\text{MS}}$ scheme) results at low energies, the change of the regularization scheme from DRED to DREG must be accounted for by shifting the running parameters by finite terms. For general renormalizable softly broken supersymmetric gauge theories these parameter shifts have been known at the 1-loop level for a long time [19]. However, in the formalism of the UOLEA the generic effective operators that correspond to such a regularization scheme change are currently unknown and reconstructing them from the results of ref. [19] is difficult due to the presence of finite field renormalizations.

In this paper we present all 1-loop effective operators that appear in the effective Lagrangian when changing the regularization scheme from DRED to DREG, assuming that the (not necessarily supersymmetric) UV model is renormalizable. We perform the calculation in the formalism of effective field theories by making use of the fact that the difference between DRED and DREG can be expressed by the presence/absence of so-called ϵ-scalars [23]. The ϵ-scalars are integrated out from the DRED-regularized UV model and the resulting effective operators are formulated in the language of the UOLEA. Our generic results complement the currently known generic expressions of the UOLEA and allow for its application to supersymmetric high-scale models and its implementation into generic spectrum generators. Finally, we show that our results are in agreement with the known generic parameter conversion terms of ref. [19].

In section 2 we briefly review the formalism of ϵ-scalars in DRED and give projection relations and Lagrangian terms necessary for the calculation of the regularization scheme translating operators, which we derive in section 3. We apply our derived effective Lagrangian in section 4 to the general supersymmetric model of ref. [19] to show that our results reproduce the parameter relations derived in that reference. We conclude in section 5.

2 Epsilon scalars in dimensional reduction

In the following we briefly review the relation between DRED and DREG, relevant to the derivation of the effective Lagrangian in section 3. In DRED an infinite dimensional space is introduced, which has the characteristics of a 4-dimensional space, denoted as $Q4S$. This quasi-4-dimensional space is decomposed as $Q4S = QdS \oplus Q\epsilon S$, where QdS is an infinite dimensional space that is formally d-dimensional and $Q\epsilon S$ is its complement, formally of dimension $\epsilon = 4 - d$ [24]. The metrics of the spaces $Q4S$, QdS and $Q\epsilon S$ are denoted by g^μ_ν, $\hat{g}^\mu_\nu$ and $\tilde{g}^\mu_\nu$, respectively, and satisfy

$$g^\mu_\nu = \hat{g}^\mu_\nu + \tilde{g}^\mu_\nu, \qquad g^\mu_\mu = 4, \tag{2.1}$$
$$g^{\mu\nu}\tilde{g}^\rho_\nu = \tilde{g}^{\mu\rho}, \qquad \tilde{g}^\mu_\mu = \epsilon, \tag{2.2}$$
$$g^{\mu\nu}\hat{g}^\rho_\nu = \hat{g}^{\mu\rho}, \qquad \hat{g}^\mu_\mu = d, \tag{2.3}$$
$$\hat{g}^{\mu\nu}\tilde{g}^\rho_\nu = 0. \tag{2.4}$$

The signature of the metric of $Q\epsilon S$ is $(-, -, \dots)$. In DRED momenta are taken to be d-dimensional, whereas gauge fields and γ-matrices are taken to be 4-dimensional. We use the convention of a totally anti-commuting γ^5. Due to the decomposition of $Q4S$ it is convenient to split the gauge field $A^a_\mu \in Q4S$ into two parts, $A^a_\mu = \hat{A}^a_\mu + \epsilon^a_\mu$, with $\hat{A}^a_\mu \in QdS$ and $\epsilon^a_\mu \in Q\epsilon S$. The ϵ-dimensional field ϵ^a_μ is a scalar under d-dimensional Lorentz transformations and is referred to as ϵ-scalar [23]. With respect to the gauge group associated with A^a_μ the ϵ-scalar transforms in the adjoint representation. After the gauge field has been split in this way, the Lagrangian may contain the following additional terms with ϵ-scalars,

$$\mathcal{L} = \mathcal{L}_\phi + \mathcal{L}_\psi + \mathcal{L}_\epsilon, \tag{2.5}$$
$$\mathcal{L}_\phi = \epsilon^a_\mu \epsilon^\mu_b F^a_b[\phi_1, \phi_2, \dots \phi_n], \tag{2.6}$$
$$\mathcal{L}_\psi = \epsilon^a_\mu \bar{\psi}_i \tilde{\gamma}^\mu \Gamma T^a_{ij} \psi_j, \tag{2.7}$$
$$\mathcal{L}_\epsilon = -\frac{1}{2}(D^\mu \epsilon_\nu)^a (D_\mu \epsilon^\nu)_a + \frac{1}{2} m^2_\epsilon \epsilon^a_\mu \epsilon^\mu_a - \frac{1}{4} g^2 f^{abc} f^{ade} \epsilon^\mu_b \epsilon^d_\mu \epsilon^\nu_c \epsilon^e_\nu, \tag{2.8}$$

where ϕ_i and ψ_i denote scalars and fermions, respectively. In eq. (2.6) F^a_b is a function of the scalar fields and may contain linear and quadratic terms. The symbol $\tilde{\gamma}^\mu$ denotes a γ-matrix projected onto $Q\epsilon S$, $\tilde{\gamma}^\mu = \tilde{g}^\mu_\nu \gamma^\nu$, and Γ is some 4×4 matrix that contains products of $\{\mathbf{1}, \gamma^\mu, \gamma^5\}$. In the following we denote any projection of a Lorentz tensor $T^{\sigma\rho\cdots}$ onto $Q\epsilon S$ by $\tilde{T}^{\mu\nu\cdots} = \tilde{g}^\mu_\sigma \tilde{g}^\nu_\rho \cdots T^{\sigma\rho\cdots}$. Similarly, tensors projected onto QdS are denoted as $\hat{T}^{\mu\nu\cdots}$. The m^2_ϵ-dependent term in eq. (2.8) can be removed by shifting the mass terms of the scalar fields ϕ_i as described in ref. [26], i.e. by changing the renormalization scheme from $\overline{\text{DR}}$ to $\overline{\text{DR}}'$. Nevertheless, due to the remaining extra ϵ^a_μ-dependent terms in the Lagrangian (2.5), the difference between DRED and DREG manifests in the presence of extra Feynman diagrams with ϵ-scalars, which contribute additional finite terms to divergent loop amplitudes due to the contraction relation (2.2).

In the following section we integrate out the ϵ-scalars using the language of effective field theories. In the limit $\epsilon \to 0$ this effectively results in a change of the regularization scheme from DRED to DREG. The resulting additional finite 1-loop operators that appear in the "effective" Lagrangian can be absorbed by a re-definition of the fields and the running parameters, leading to the same parameter relations given in ref. [19].

3 Regularization scheme translating operators in the UOLEA

To derive the operators that translate between DRED and DREG we consider a general renormalizable gauge theory with the gauge group G and the Lagrangian $\mathcal{L}$, which contains real scalar fields ϕ_i, Dirac fermions ψ_i and a set of four-component Majorana fermions λ_i.[1] We furthermore assume that the theory is regularized in DRED. The gauge field A^a_μ is split into a d- and an ϵ-dimensional component, as described in section 2, and we distinguish the ϵ-scalars from the scalars ϕ_i.

To calculate the effective action up to the 1-loop level, we first split all fields $\omega_i \in \{\phi_i, \psi_i, \lambda_i, \hat{A}^a_\mu, \epsilon^a_\mu\}$ into a background part $\omega_{B,i}$, satisfying the classical equations of motion, and a corresponding fluctuation $\delta\omega_i$. The calculation is going to be performed using a covariant derivative expansion [4, 27, 28] in order to obtain a manifestly gauge invariant result. This means in particular that the operator[2] $\hat{P}_\mu \equiv i\hat{D}_\mu = i\hat{\partial}_\mu + g\hat{A}^a_{B,\mu}T^a$, where $\hat{A}^a_{B,\mu}$ is the background gauge field, should be kept as a whole in the calculation and not be split into $\hat{\partial}_\mu$ and $\hat{A}^a_{B,\mu}$. Furthermore, to obtain an action which is gauge invariant under transformations of $\hat{A}^a_{B,\mu}$ we only fix the gauge of the fluctuation $\delta\hat{A}^a_\mu$ [29]. We choose a gauge fixing Lagrangian of the form [4]

$$\mathcal{L}_{\text{g.f.}} = -\frac{1}{2\xi}\left[\xi(m_A)_{ab}\eta^b + \hat{D}^\mu\delta\hat{A}^a_\mu\right]^2, \tag{3.1}$$

where the fields η^a are the Goldstone bosons corresponding to the spontaneously broken generators of the gauge group and m_A is the diagonal mass matrix of the gauge bosons. The part of the Lagrangian containing the ghost fields is given by

$$\mathcal{L}_{\text{ghost}} = \bar{c}^a(-\hat{D}^2 - \xi m_A^2)_{ab}c^b. \tag{3.2}$$

In the following the Goldstone bosons are not treated separately, but are regarded as part of the vector of scalar fields ϕ_i. Moreover, for the purpose of this calculation the fluctuation $\delta\hat{A}^a_\mu$ can be treated as a scalar field transforming in the adjoint representation under background gauge transformations [4]. Similarly, the ghosts can be regarded as usual fermions in the adjoint representation of the gauge group. In the following calculation the

[1] The formulation of the Lagrangian in terms of Dirac and Majorana fermions has been chosen in order to diagonalize the operator $(i\hat{\not{D}} - m)$.

[2] Note that whereas this notation suggests that we are treating a simple gauge group we are not restricted to this case. The notation is to be understood with a sum over all factors of the gauge group with their respective gauge couplings.

path integral over the ghosts can be performed directly and is independent of ϵ-scalars. The ghosts will therefore not be considered further in this paper. The second variation of the action around the background fields then reads

$$\begin{aligned}\delta^2\mathcal{L} = {}& \delta\bar{\psi}\Delta_\psi\delta\psi + \delta\bar{\lambda}\Delta_\lambda\delta\lambda - \frac{1}{2}\delta\epsilon_\mu\tilde{\Delta}^{\mu\nu}_\epsilon\delta\epsilon_\nu - \frac{1}{2}\delta\Phi\Delta_\Phi\delta\Phi \\ & - \delta\bar{\psi}\tilde{X}^\mu_{\bar{\psi}\epsilon}\delta\epsilon_\mu - \delta\bar{\psi}X_{\bar{\psi}\Phi}\delta\Phi - \delta\bar{\lambda}\tilde{X}^\mu_{\bar{\lambda}\epsilon}\delta\epsilon_\mu - \delta\bar{\lambda}X_{\bar{\lambda}\phi}\delta\Phi \\ & + \delta\epsilon_\mu\tilde{X}^\mu_{\epsilon\psi}\delta\psi + \delta\Phi X_{\Phi\psi}\delta\psi + \delta\epsilon_\mu\tilde{X}^\mu_{\epsilon\lambda}\delta\lambda + \delta\Phi X_{\Phi\lambda}\delta\lambda \\ & + \delta\bar{\psi}X_{\bar{\psi}\lambda}\delta\lambda + \delta\bar{\lambda}X_{\bar{\lambda}\psi}\delta\psi - \frac{1}{2}\delta\epsilon_\mu\tilde{X}^\mu_{\epsilon\Phi}\delta\Phi - \frac{1}{2}\delta\Phi\tilde{X}^\mu_{\Phi\epsilon}\delta\epsilon_\mu,\end{aligned} \tag{3.3}$$

where

$$\delta\Phi = \begin{pmatrix}\delta\phi \\ \delta\hat{A}_\mu\end{pmatrix}, \tag{3.4}$$

$$X_{\Phi\omega} = \begin{pmatrix}X_{\phi\omega} \\ \hat{X}^\mu_{\hat{A}\omega}\end{pmatrix}, \tag{3.5}$$

$$X_{\omega\Phi} = \begin{pmatrix}X_{\omega\phi} & \hat{X}^\mu_{\omega\hat{A}}\end{pmatrix}, \tag{3.6}$$

and

$$X_{\omega\sigma} \equiv -\left.\frac{\delta^2\mathcal{L}_{\text{int}}}{\delta\omega\delta\sigma}\right| \tag{3.7}$$

denotes the derivative of the interaction Lagrangian, $\mathcal{L}_{\text{int}}$, with respect to the fields ω and σ, evaluated at the background field configuration. Furthermore we have introduced the abbreviations

$$\Delta_\Phi \equiv \begin{pmatrix}\Delta_\phi & \hat{X}^\mu_{\phi\hat{A}} \\ \hat{X}^\mu_{\hat{A}\phi} & \hat{\Delta}^{\mu\nu}_{\hat{A}}\end{pmatrix}, \tag{3.8}$$

$$\Delta_\psi \equiv \not{P} - m_\psi + X_{\bar{\psi}\psi}, \tag{3.9}$$

$$\Delta_\lambda \equiv \frac{1}{2}\not{P} - \frac{1}{2}m_\lambda + X_{\bar{\lambda}\lambda}, \tag{3.10}$$

$$\Delta_\phi \equiv -P^2 + m_\phi^2 + X_{\phi\phi}, \tag{3.11}$$

$$\hat{\Delta}^{\mu\nu}_{\hat{A}} \equiv P^2\hat{g}^{\mu\nu} - 2\hat{P}^\nu\hat{P}^\mu + \hat{P}^\mu\hat{P}^\nu\left(1 + \frac{1}{\xi}\right) + m_A^2\hat{g}^{\mu\nu}, \tag{3.12}$$

$$\tilde{\Delta}^{\mu\nu}_\epsilon \equiv \tilde{g}^{\mu\nu}(P^2 - m_\epsilon^2) + \tilde{X}^{\mu\nu}_{\epsilon\epsilon}. \tag{3.13}$$

In any product that contains Φ the Lorentz indices are fully contracted, for example

$$\delta\Phi X_{\Phi\psi}\delta\psi = \begin{pmatrix}\delta\phi & \delta\hat{A}_\mu\end{pmatrix}\begin{pmatrix}X_{\phi\psi} \\ \hat{X}^\mu_{\hat{A}\psi}\end{pmatrix}\delta\psi. \tag{3.14}$$

In addition, in eq. (3.3) all indices, except for the Lorentz indices of $Q\epsilon S$, have been suppressed for brevity. Eq. (3.3) can be simplified further due to the constraints on the possible couplings of ϵ-scalars to other fields as given in eqs. (2.5)–(2.8): we can solve the classical equations of motion in a perturbation expansion in couplings. The leading term is proportional to an operator of the form $\bar{\psi}\tilde{\gamma}^\mu\psi$ and thus every term in the series will contain this operator. In the limit $\epsilon \to 0$ this operator vanishes, which means that the background fields of the ϵ-scalars can be set to zero from the start. This property can be used to simplify eq. (3.3), because from eqs. (2.6) and (2.8) it follows that $\tilde{X}^\mu_{\Phi\epsilon} = \tilde{X}^\mu_{\epsilon\Phi} = 0$ for vanishing ϵ-scalar background fields.

To perform the path integral, we shift the Dirac and Majorana fermions to eliminate terms with mixed fermionic and bosonic fluctuations as described in ref. [30]. We first shift the Majorana fermions by

$$\delta\lambda' = \delta\lambda - \Delta_\lambda^{-1}\left[\tilde{X}^\nu_{\bar{\lambda}\epsilon}\delta\epsilon_\nu + X_{\bar{\lambda}\Phi}\delta\Phi - X_{\bar{\lambda}\psi}\delta\psi\right], \tag{3.15}$$

$$\delta\bar{\lambda}' = \delta\bar{\lambda} + \left[\delta\epsilon_\mu\tilde{X}^\mu_{\epsilon\lambda} + \delta\Phi X_{\Phi\lambda} + \delta\bar{\psi}X_{\bar{\psi}\lambda}\right]\Delta_\lambda^{-1}, \tag{3.16}$$

and afterwards the Dirac fermions by

$$\delta\psi' = \delta\psi - \Lambda_\psi^{-1}\left[\tilde{\Xi}^\nu_{\bar{\psi}\epsilon}\delta\epsilon_\nu + \Xi_{\bar{\psi}\Phi}\delta\Phi\right], \tag{3.17}$$

$$\delta\bar{\psi}' = \delta\bar{\psi} + \left[\delta\epsilon_\mu\tilde{\Xi}^\mu_{\epsilon\psi} + \delta\Phi\Xi_{\Phi\psi}\right]\Lambda_\psi^{-1}, \tag{3.18}$$

and introduce the following abbreviations

$$\tilde{\Lambda}^{\mu\nu}_\epsilon = \tilde{\Delta}^{\mu\nu}_\epsilon - 2\tilde{X}^\mu_{\epsilon\lambda}\Delta_\lambda^{-1}\tilde{X}^\nu_{\bar{\lambda}\epsilon}, \tag{3.19}$$

$$\Lambda_\Phi = \Delta_\Phi - 2X_{\Phi\lambda}\Delta_\lambda^{-1}X_{\bar{\lambda}\Phi}, \tag{3.20}$$

$$\Lambda_\psi = \Delta_\psi - X_{\bar{\psi}\lambda}\Delta_\lambda^{-1}X_{\bar{\lambda}\psi}, \tag{3.21}$$

$$\tilde{\Xi}^\mu_{\bar{\psi}\epsilon} = \tilde{X}^\mu_{\bar{\psi}\epsilon} - X_{\bar{\psi}\lambda}\Delta_\lambda^{-1}\tilde{X}^\mu_{\bar{\lambda}\epsilon}, \tag{3.22}$$

$$\Xi_{\bar{\psi}\Phi} = X_{\bar{\psi}\Phi} - X_{\bar{\psi}\lambda}\Delta_\lambda^{-1}X_{\bar{\lambda}\Phi}, \tag{3.23}$$

$$\tilde{\Xi}^\mu_{\epsilon\psi} = \tilde{X}^\mu_{\epsilon\psi} - \tilde{X}^\mu_{\epsilon\lambda}\Delta_\lambda^{-1}X_{\bar{\lambda}\psi}, \tag{3.24}$$

$$\Xi_{\Phi\psi} = X_{\Phi\psi} - X_{\Phi\lambda}\Delta_\lambda^{-1}X_{\bar{\lambda}\psi}, \tag{3.25}$$

$$\tilde{\Xi}^\mu_{\epsilon\Phi} = -2\tilde{X}^\mu_{\epsilon\lambda}\Delta_\lambda^{-1}X_{\bar{\lambda}\Phi}, \tag{3.26}$$

$$\tilde{\Xi}^\mu_{\Phi\epsilon} = -2X_{\Phi\lambda}\Delta_\lambda^{-1}\tilde{X}^\mu_{\bar{\lambda}\epsilon}. \tag{3.27}$$

For Dirac fermions the shifts (3.17)–(3.18) can be performed independently. For Majorana fermions λ and $\bar{\lambda}$ are not independent and it is necessary that $\delta\lambda'\gamma^0 = \delta\bar{\lambda}'$ for the shifted fields $\delta\lambda'$ and $\delta\bar{\lambda}'$ defined in (3.15)–(3.16), respectively. That this is indeed the case is shown in appendix A. After shifting the fermions in this way, the variation takes the form

$$\begin{aligned}\delta^2\mathcal{L} = {} & \delta\bar{\psi}'\Lambda_\psi\delta\psi' + \delta\bar{\lambda}'\Delta_\lambda\delta\lambda' - \frac{1}{2}\delta\Phi(\Lambda_\Phi - 2\Xi_{\Phi\psi}\Lambda_\psi^{-1}\Xi_{\bar{\psi}\Phi})\delta\Phi \\ & - \frac{1}{2}\delta\epsilon_\mu(\tilde{\Lambda}^{\mu\nu}_\epsilon - 2\tilde{\Xi}^\mu_{\epsilon\psi}\Lambda_\psi^{-1}\tilde{\Xi}^\nu_{\bar{\psi}\epsilon})\delta\epsilon_\nu - \frac{1}{2}\delta\epsilon_\mu(\tilde{\Xi}^\mu_{\epsilon\Phi} - 2\tilde{\Xi}^\mu_{\epsilon\psi}\Lambda_\psi^{-1}\Xi_{\bar{\psi}\Phi})\delta\Phi \\ & - \frac{1}{2}\delta\Phi(\tilde{\Xi}^\mu_{\Phi\epsilon} - 2\Xi_{\Phi\psi}\Lambda_\psi^{-1}\tilde{\Xi}^\mu_{\bar{\psi}\epsilon})\delta\epsilon_\mu.\end{aligned} \tag{3.28}$$

In eq. (3.28) the fermionic and bosonic fluctuations are now completely decoupled and the part which depends on the ϵ-scalars can be written as

$$\delta^2\mathcal{L}_{\Phi\epsilon} = -\frac{1}{2}\begin{pmatrix}\delta\epsilon_\mu & \delta\Phi\end{pmatrix}\begin{pmatrix}\tilde{\Omega}^{\mu\nu}_\epsilon & \tilde{X}^\mu_{\epsilon\mathrm{Ph}} \\ \tilde{X}^\nu_{\mathrm{Ph}\epsilon} & \Delta_{\mathrm{Ph}}\end{pmatrix}\begin{pmatrix}\delta\epsilon_\nu \\ \delta\Phi\end{pmatrix}, \tag{3.29}$$

where

$$\tilde{\Omega}^{\mu\nu}_\epsilon = \tilde{\Lambda}^{\mu\nu}_\epsilon - 2\tilde{\Xi}^\mu_{\epsilon\psi}\Lambda^{-1}_\psi\tilde{\Xi}^\nu_{\bar{\psi}\epsilon}, \tag{3.30}$$

$$\tilde{X}^\mu_{\epsilon\mathrm{Ph}} = \tilde{\Xi}^\mu_{\epsilon\Phi} - 2\tilde{\Xi}^\mu_{\epsilon\psi}\Lambda^{-1}_\psi\Xi_{\bar{\psi}\Phi}, \tag{3.31}$$

$$\tilde{X}^\mu_{\mathrm{Ph}\epsilon} = \tilde{\Xi}^\mu_{\Phi\epsilon} - 2\Xi_{\Phi\psi}\Lambda^{-1}_\psi\tilde{\Xi}^\mu_{\bar{\psi}\epsilon}, \tag{3.32}$$

$$\Delta_{\mathrm{Ph}} = \Lambda_\Phi - 2\Xi_{\Phi\psi}\Lambda^{-1}_\psi\Xi_{\bar{\psi}\Phi}. \tag{3.33}$$

The term Δ_{Ph} does not depend on the ϵ-scalars. Performing the path integral over the ϵ-scalars and the scalars Φ_i we find the effective action

$$\Gamma = \frac{i}{2}\log\det\begin{pmatrix}\tilde{\Omega}^{\mu\nu}_\epsilon & \tilde{X}^\mu_{\epsilon\mathrm{Ph}} \\ \tilde{X}^\nu_{\mathrm{Ph}\epsilon} & \Delta_{\mathrm{Ph}}\end{pmatrix} \equiv \frac{i}{2}\log\det Q. \tag{3.34}$$

The matrix Q can be brought into a diagonal form by inserting U and V to the left and to the right of Q and by choosing

$$U = \begin{pmatrix}\mathbb{1} & -\tilde{X}_{\epsilon\mathrm{Ph}}\Delta^{-1}_{\mathrm{Ph}} \\ 0 & \mathbb{1}\end{pmatrix}, \tag{3.35}$$

$$V = \begin{pmatrix}\mathbb{1} & 0 \\ -\Delta^{-1}_{\mathrm{Ph}}\tilde{X}_{\mathrm{Ph}\epsilon} & \mathbb{1}\end{pmatrix}. \tag{3.36}$$

The resulting effective action reads

$$\Gamma = \frac{i}{2}\log\det\left(\tilde{\Omega}^{\mu\nu}_\epsilon - \tilde{X}^\mu_{\epsilon\mathrm{Ph}}\Delta^{-1}_{\mathrm{Ph}}\tilde{X}^\nu_{\mathrm{Ph}\epsilon}\right) + \frac{i}{2}\log\det\Delta_{\mathrm{Ph}}, \tag{3.37}$$

where only the first term depends on the ϵ-scalars. Substituting the expressions for $\tilde{\Omega}^{\mu\nu}_\epsilon$, $\tilde{X}^\mu_{\epsilon\mathrm{Ph}}$, $\Delta^{-1}_{\mathrm{Ph}}$ and $\tilde{X}^\nu_{\mathrm{Ph}\epsilon}$ into the first term we find the ϵ-dependent part

$$\Gamma = \frac{i}{2}\log\det\left(\tilde{\Lambda}^{\mu\nu}_\epsilon - 2\tilde{\Xi}^\mu_{\epsilon\psi}\Lambda^{-1}_\psi\tilde{\Xi}^\nu_{\bar{\psi}\epsilon} - \tilde{W}^{\mu\nu}\right) + \cdots, \tag{3.38}$$

$$\tilde{W}^{\mu\nu} = \left(\tilde{\Xi}^\mu_{\epsilon\Phi} - 2\tilde{\Xi}^\mu_{\epsilon\psi}\Lambda^{-1}_\psi\Xi_{\bar{\psi}\Phi}\right)\left(\Lambda_\Phi - 2\Xi_{\Phi\psi}\Lambda^{-1}_\psi\Xi_{\bar{\psi}\Phi}\right)^{-1}\left(\tilde{\Xi}^\nu_{\Phi\epsilon} - 2\Xi_{\Phi\psi}\Lambda^{-1}_\psi\tilde{\Xi}^\nu_{\bar{\psi}\epsilon}\right). \tag{3.39}$$

In a standard EFT calculation eq. (3.38) is written as a trace in momentum space and must be expanded in powers of p/M to obtain a local action, where M is the mass of the heavy particle to be integrated out. In our calculation, however, all 1-loop integrals get multiplied by ϵ, so only the divergent parts give a non-zero contribution to Γ in the limit $\epsilon \to 0$. Since the divergences in a renormalizable gauge theory are local [31, 32], we obtain

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a local action. By performing a power counting we find that the only terms that yield divergent momentum integrals are

$$\Gamma_{\text{div}} = \frac{i}{2} \log\det\left(\tilde{\Delta}^{\mu\nu}_{\epsilon} - 2\tilde{Y}^{\mu\nu}_{\lambda} - 2\tilde{Y}^{\mu\nu}_{\psi} + 2\tilde{Z}^{\mu\nu}_{\lambda\psi} + 2\tilde{Z}^{\mu\nu}_{\psi\lambda}\right), \tag{3.40}$$

$$\tilde{Y}^{\mu\nu}_{\omega} = \tilde{X}^{\mu}_{\epsilon\omega}\Delta^{-1}_{\omega}\tilde{X}^{\nu}_{\bar{\omega}\epsilon}, \tag{3.41}$$

$$\tilde{Z}^{\mu\nu}_{\omega\sigma} = \tilde{X}^{\mu}_{\epsilon\omega}\Delta^{-1}_{\omega}X_{\bar{\omega}\sigma}\Delta^{-1}_{\sigma}\tilde{X}^{\nu}_{\bar{\sigma}\epsilon}. \tag{3.42}$$

Using the results of ref. [6] and the methods described in ref. [33] we find the following effective Lagrangian containing all contributions from integrating out the ϵ-scalars,

$$\begin{aligned}16\pi^2\epsilon\mathcal{L}_{\text{reg}} = &-\sum_i (m^2_\epsilon)_i(\tilde{X}^{\mu}_{\epsilon\epsilon\mu})_{ii} + \frac{1}{2}\sum_{ij}(\tilde{X}^{\mu}_{\epsilon\epsilon\nu})_{ij}(\tilde{X}^{\nu}_{\epsilon\epsilon\mu})_{ji} \\ &+ \sum_{ij} 2^{c_{F_j}}\left[2m_{\psi j}(\tilde{X}^{\mu}_{\epsilon\psi})_{ij}(\tilde{X}_{\bar{\psi}\epsilon\mu})_{ji} + (\tilde{X}^{\mu}_{\epsilon\psi})_{ij} i\hat{D}_\nu\hat{\gamma}^\nu(\tilde{X}_{\bar{\psi}\epsilon\mu})_{ji}\right] \\ &- \sum_{ijk} 2^{c_{F_j}+c_{F_k}-1}(\tilde{X}^{\mu}_{\epsilon\psi})_{ij}\hat{\gamma}^\nu(X_{\bar{\psi}\psi})_{jk}\hat{\gamma}_\nu(\tilde{X}_{\bar{\psi}\epsilon\mu})_{ki} \\ &+ \frac{\epsilon}{12}\,\text{tr}\left[\hat{G}'_{\mu\nu}\hat{G}'^{\mu\nu}\right],\end{aligned} \tag{3.43}$$

where $\hat{G}'_{\mu\nu} = -ig\hat{G}^a_{\mu\nu}t^a$, $\hat{G}^a_{\mu\nu} = \hat{\partial}_\mu\hat{A}^a_\nu - \hat{\partial}_\nu\hat{A}^a_\mu + gf^{abc}\hat{A}^b_\mu\hat{A}^c_\nu$ and $c_F = 0$ for Dirac fermions and $c_F = 1$ for Majorana fermions. All quantities with Lorentz indices appearing in eq. (3.43) are still projected onto either QdS or $Q\epsilon S$. After inserting the respective functional derivatives into this equation each term on the right hand side will contain a factor ϵ. One can then divide the equation by ϵ and take the limit $\epsilon \to 0$. After this limit has been taken there is no difference between d-dimensional and 4-dimensional quantities anymore and the hats can be removed. It should be pointed out that in eq. (3.43) the Latin indices contain all indices (generation, gauge, ...), except for the Lorentz indices of the ϵ-scalars. Thus, the sums are to be interpreted as a trace over all indices with the coefficient given by eq. (3.43). Also, we consider the Majorana spinors λ and $\bar{\lambda}$ to be independent. This convention has to be followed when calculating quantities like $X_{\bar{\lambda}\lambda}$ from the Lagrangian of the full model. Furthermore, we stress that the order of $\hat{D}_\nu$ and $\hat{\gamma}^\nu$ in the second line matters, whenever $\hat{D}_\nu$ contains chiral projectors.

In the next section we apply eq. (3.43) to reproduce the general parameter relations given in ref. [19] for a supersymmetric Lagrangian. However, we'd like to remark that eq. (3.43) is a generalization of the results of ref. [19], because it contains terms that correspond to field renormalizations and tadpoles and can be applied also to non-supersymmetric models regularized in DRED.

4 Applications

In this section we apply eq. (3.43) to reproduce the parameter relations given in ref. [19] for a supersymmetric Lagrangian with the gauge coupling g corresponding to a simple gauge group, a gaugino mass parameter M, a gaugino-fermion-scalar coupling g_λ, a Yukawa coupling Y^{ijk} and a quartic scalar coupling λ^{ij}_{kl}.

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4.1 Gauge coupling

The relation between the DRED and the DREG gauge coupling can be obtained from the last term in eq. (3.43), where the limit $\epsilon \to 0$ can be taken immediately,

$$\mathcal{L}_{\text{reg,gauge}} = \frac{1}{12(16\pi^2)} \operatorname{tr}\left[G'_{\mu\nu} G'^{\mu\nu}\right] = -\frac{g^2}{12(16\pi^2)} C(G) G^a_{\mu\nu} G^{\mu\nu}_a, \tag{4.1}$$

where $G'_{\mu\nu} = -ig G^a_{\mu\nu} t^a$ and t^a are the generators in the adjoint representation of the gauge group, $(t^a)^{bc} = -if^{abc}$, and $C(G)\delta^{ab} = \operatorname{tr}[t^a t^b] = f^{acd} f^{bcd}$. The term in eq. (4.1) can be absorbed into a finite field renormalization of the gauge field and a shift in the gauge coupling,

$$(A^a_\mu)^{\text{DRED}} \to \left(1 - \frac{1}{2}\delta Z_A\right) (A^a_\mu)^{\text{DREG}}, \tag{4.2}$$

$$g^{\text{DRED}} \to g^{\text{DREG}} - \delta g, \tag{4.3}$$

with

$$\delta Z_A = \frac{g^2}{3(16\pi^2)} C(G), \tag{4.4}$$

$$\delta g = -\frac{1}{2} g \delta Z_A. \tag{4.5}$$

From eq. (4.5) one obtains

$$g^{\text{DREG}} = g^{\text{DRED}} \left(1 - \frac{g^2}{6(16\pi^2)} C(G)\right), \tag{4.6}$$

which agrees with the result of ref. [19].

4.2 Gaugino mass parameter

We assume that the supersymmetric Lagrangian (regularized in DRED) contains the kinetic and the soft-breaking gaugino mass term

$$\mathcal{L}_\lambda = \frac{1}{2}\bar{\lambda}^a \left(i\hat{\not{\partial}} - M^{\text{DRED}}\right) \lambda^a, \tag{4.7}$$

where λ^a denotes the gaugino Majorana spinor, transforming in the adjoint representation of the gauge group. In addition, there is an interaction term between the gauginos and the ϵ-scalars,

$$\mathcal{L}_{\epsilon\lambda} = \frac{g}{2}\bar{\lambda}^b \tilde{\gamma}^\mu \epsilon^a_\mu (t^a)_{bc} \lambda^c, \tag{4.8}$$

with $t^a_{bc} = -if^{abc}$. When the ϵ-scalars are integrated out, the following two terms from the second line of eq. (3.43) contribute to the relation between M^{DRED} and M^{DREG}:

$$16\pi^2 \epsilon \mathcal{L}_{\text{reg},\lambda} = 2\left(2M \tilde{X}^\mu_{\epsilon\lambda} \tilde{X}_{\bar{\lambda}\epsilon\mu} + \tilde{X}^\mu_{\epsilon\lambda} i\hat{D}_\nu \hat{\gamma}^\nu \tilde{X}_{\bar{\lambda}\epsilon\mu}\right). \tag{4.9}$$

The derivatives $\tilde{X}^{\mu}_{\epsilon\lambda}$ and $\tilde{X}^{\mu}_{\bar{\lambda}\epsilon}$ are obtained from $\mathcal{L}_{\epsilon\lambda}$ and read

$$(\tilde{X}^{\mu}_{\epsilon\lambda})^a_b = \frac{g}{2}\bar{\lambda}_c\tilde{\gamma}^{\mu}(t^a)_{cb}, \tag{4.10}$$

$$(\tilde{X}^{\mu}_{\bar{\lambda}\epsilon})^a_b = -\frac{g}{2}\tilde{\gamma}^{\mu}(t^a)_{bc}\lambda_c, \tag{4.11}$$

which yields

$$16\pi^2\epsilon\mathcal{L}_{\text{reg},\lambda} = \frac{g^2}{2}\bar{\lambda}^a\left(-\tilde{\gamma}^{\mu}i\hat{\partial\!\!\!/}\tilde{\gamma}_{\mu} - 2M\tilde{\gamma}^{\mu}\tilde{\gamma}_{\mu}\right)(t^b)_{ac}(t^b)_{cd}\lambda^d, \tag{4.12}$$

$$= \frac{g^2}{2}\epsilon\bar{\lambda}^a\left(i\hat{\partial\!\!\!/} - 2M\right)C(G)\lambda^a, \tag{4.13}$$

with $(t^b)_{ac}(t^b)_{cd} = f^{abc}f^{dbc} = C(G)\delta^{ad}$ and $\tilde{\gamma}^{\mu}\tilde{\gamma}_{\mu} = \epsilon$. After dividing by ϵ and taking the limit $\epsilon \to 0$ the terms in eq. (4.13) can be absorbed by the finite field and parameter re-definitions

$$(\lambda^a)^{\text{DRED}} \to \left(1 - \frac{1}{2}\delta Z_{\lambda}\right)(\lambda^a)^{\text{DREG}}, \tag{4.14}$$

$$M^{\text{DRED}} \to M^{\text{DREG}} - \delta M, \tag{4.15}$$

with

$$\delta Z_{\lambda} = \frac{g^2}{16\pi^2}C(G), \tag{4.16}$$

$$\delta M = M\left(2\frac{g^2}{16\pi^2}C(G) - \delta Z_{\lambda}\right). \tag{4.17}$$

Thus, the relation between the gaugino mass parameter in DRED and DREG reads

$$M^{\text{DREG}} = M^{\text{DRED}} + \delta M = M^{\text{DRED}}\left(1 + \frac{g^2}{16\pi^2}C(G)\right), \tag{4.18}$$

which is in agreement with the result of ref. [19].

4.3 Gaugino coupling

We consider a supersymmetric and gauge invariant Lagrangian with a gaugino λ^a, charged scalars ϕ_i and Dirac fermions ψ_i. The left- and right-handed components of the ψ_i are assumed to originate from superfields transforming in the (generally reducible) representation R and its conjugate representation $\bar{R}$, respectively. The mass eigenstates are obtained from a diagonalization of a mass matrix by two unitary matrices L and R. The scalar fields ϕ_{1i} and ϕ_{2i} originate from the same superfields as the left- and right-handed components of the Dirac fermions, respectively, and the scalar mass eigenstates are obtained from the diagonalization of the mass matrix with the unitary matrix U. The Lagrangian, formulated in terms of the mass eigenstate fields ψ_i, ϕ_{1i} and ϕ_{2i} then contains the coupling term

$$\mathcal{L}_{g_\lambda} = \sqrt{2}g_{\lambda}\Big(\phi^*_{1i}U^{\dagger}_{ij}\bar{\lambda}^a(t^a_R)_{jk}L_{km}P_L\psi_m - \bar{\psi}_iR^T_{ij}P_L(t^a_R)_{jk}\lambda^aU^*_{km}\phi^*_{2m} + \text{h.c.}\Big), \tag{4.19}$$

where t^a_R denotes the generator in the representation of the Dirac fields ψ_i. Here the Latin indices run over both flavor indices and gauge group indices. In DRED supersymmetry ensures that $g^{\rm DRED}_\lambda = g$, where g denotes the gauge coupling. In DREG supersymmetry is explicitly violated and one has $g^{\rm DREG}_\lambda \neq g$. To the relation between $g^{\rm DRED}_\lambda$ and $g^{\rm DREG}_\lambda$ the third line of eq. (3.43) contributes, which reads for the considered case

$$16\pi^2\epsilon\mathcal{L}_{{\rm reg},g_\lambda} = -\tilde{X}^\mu_{\epsilon\lambda}\hat{\gamma}^\nu X_{\bar{\lambda}\psi}\hat{\gamma}_\nu\tilde{X}_{\bar{\psi}\epsilon\mu} - \tilde{X}^\mu_{\epsilon\psi}\hat{\gamma}^\nu X_{\bar{\psi}\lambda}\hat{\gamma}_\nu\tilde{X}_{\bar{\lambda}\epsilon\mu}. \tag{4.20}$$

The derivatives $\tilde{X}^\mu_{\epsilon\lambda}$ and $\tilde{X}^\mu_{\bar{\lambda}\epsilon}$ have been calculated in section 4.2 already. The derivatives $\tilde{X}^\mu_{\epsilon\psi}$ and $\tilde{X}^\mu_{\bar{\psi}\epsilon}$ can be obtained from the Dirac fermion–ϵ-scalar coupling of the DRED Lagrangian

$$\mathcal{L}_{\epsilon\bar{\psi}\psi} = g\epsilon^a_\mu\bar{\psi}_i\tilde{\gamma}^\mu\left(R^T_{ij}(t^a_R)_{jk}R^*_{kl}P_R + L^\dagger_{ij}(t^a_R)_{jk}L_{kl}P_L\right)\psi_l, \tag{4.21}$$

which yields

$$(\tilde{X}^\mu_{\epsilon\psi})^a_l = g\bar{\psi}_i\tilde{\gamma}^\mu(T^a_R)_{il}, \tag{4.22}$$

$$(\tilde{X}^\mu_{\bar{\psi}\epsilon})^a_i = -g\tilde{\gamma}^\mu(T^a_R)_{il}\psi_l, \tag{4.23}$$

where we have introduced the abbreviation

$$(T^a_R)_{il} = R^T_{ij}(t^a_R)_{jk}R^*_{kl}P_R + L^\dagger_{ij}(t^a_R)_{jk}L_{kl}P_L. \tag{4.24}$$

The derivatives $X_{\bar{\lambda}\psi}$ and $X_{\bar{\psi}\lambda}$ can be read off eq. (4.19) and read

$$(X_{\bar{\lambda}\psi})^a_j = \sqrt{2}g_\lambda\left[\phi^*_{1i}(U^\dagger t^a_R L)_{ij}P_L - \phi_{2i}(U^T t^a_R R^*)_{ij}P_R\right] \equiv \sqrt{2}g_\lambda A^a_j, \tag{4.25}$$

$$(X_{\bar{\psi}\lambda})^a_i = \sqrt{2}g_\lambda\left[(L^\dagger t^a_R U)_{im}\phi_{1m}P_R - (R^T t^a_R U^*)_{im}P_L\phi^*_{2m}\right] \equiv \sqrt{2}g_\lambda B^a_i. \tag{4.26}$$

Inserting all derivatives into eq. (4.20) yields

$$\begin{aligned}16\pi^2\epsilon\mathcal{L}_{{\rm reg},g_\lambda} &= \frac{g_\lambda g^2}{\sqrt{2}}\left[\bar{\lambda}^i\tilde{\gamma}^\mu(t^a_G)_{ij}\hat{\gamma}^\nu A^j_k\hat{\gamma}_\nu\tilde{\gamma}_\mu(T^a_R)_{kl}\psi_l + \bar{\psi}_i\tilde{\gamma}^\mu(T^a_R)_{ij}\hat{\gamma}^\nu B^k_j\hat{\gamma}_\nu\tilde{\gamma}_\mu(t^a_G)_{kl}\lambda_l\right]\\ &= \frac{d}{4}\sqrt{2}g_\lambda g^2\epsilon C(G)\left(\bar{\lambda}^i A^i_l\psi_l + \bar{\psi}_i B^l_i\lambda^l\right),\end{aligned} \tag{4.27}$$

where we used that

$$\tilde{\gamma}^\mu\tilde{\gamma}_\mu = \epsilon, \tag{4.28}$$

$$(t^a_G)_{ij}A^j_k(T^a_R)_{kl} = \frac{1}{2}C(G)A^i_l, \tag{4.29}$$

$$(t^a_G)_{kl}(T^a_R)_{ij}\gamma^\mu B^k_j = \frac{1}{2}\gamma^\mu C(G)B^l_i, \tag{4.30}$$

and $(t^a_G)_{bc} = -if^{abc}$ being the generators in the adjoint representation of the gauge group. In addition to the term on the r.h.s. of eq. (4.27) finite field renormalizations of the Dirac fermions and of the gaugino contribute to the difference between $g^{\rm DRED}_\lambda$ and $g^{\rm DREG}_\lambda$. The field renormalization of the gaugino has already been calculated in section 4.2. The field

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renormalization of the Dirac fermion follows from the second term in the second line of eq. (3.43), which reads

$$16\pi^2 \epsilon \mathcal{L}_{\text{reg},\bar{\psi}\psi} = -g^2 \bar{\psi}_i \tilde{\gamma}^\mu (T_R^a)_{il} i\hat{\not\partial} \tilde{\gamma}_\mu (T_R^a)_{lk} \psi_k. \tag{4.31}$$

After taking the limit $\epsilon \to 0$ the terms on the r.h.s. of eqs. (4.31) and (4.27) can be absorbed by the finite field and parameter re-definitions

$$\psi_i^{\text{DRED}} \to \left(\delta_{ij} - \frac{1}{2}(\delta Z_\psi)_{ij}\right) \psi_j^{\text{DREG}}, \tag{4.32}$$

$$g_\lambda^{\text{DRED}} \to (1 - \delta g_\lambda)\, g_\lambda^{\text{DREG}}, \tag{4.33}$$

where

$$(\delta Z_\psi)_{ij} = \frac{g^2}{16\pi^2} C(r_i) \delta_{ij}, \tag{4.34}$$

$$\delta g_\lambda = \frac{g^2}{32\pi^2} C(G) - \frac{1}{2} \delta Z_\psi, \tag{4.35}$$

and we used $(T_R^a)_{il}(T_R^a)_{lk} = C(r_i)\delta_{ik}$. Here, the r_i are the irreducible components of the representation R and the index i is not summed over. From eq. (4.35) one obtains the relation

$$g_\lambda^{\text{DREG}} = g^{\text{DRED}} (1 + \delta g_\lambda) = g^{\text{DRED}} \left(1 + \frac{g^2}{32\pi^2} \left[C(G) - C(r_i)\right]\right), \tag{4.36}$$

which depends on the irreducible representation in which the chiral superfield transforms. The relation (4.36) agrees with the result of ref. [19].

4.4 Yukawa coupling

We consider a supersymmetric and gauge invariant Lagrangian with the superpotential

$$\mathcal{W} = \frac{1}{6} Y_{ijk} \Phi_i \Phi_j \Phi_k, \tag{4.37}$$

where Φ_i are chiral superfields and Y_{ijk} is the Yukawa coupling. Furthermore we assume that the Weyl fermionic components of the superfields can be arranged into Dirac fermions ψ_i. The left- and right-handed components of these Dirac fermions are assumed to originate from the diagonalization of a mass matrix using the unitary matrices R and L. The scalar fields ϕ_i may originate from the diagonalization of the scalar mass matrix with the unitary matrix U. The Lagrangian, formulated in terms of the mass eigenstate fields ψ_i and ϕ_i, then contains the Yukawa coupling term

$$\mathcal{L}_y = \frac{1}{6} Y_{ijk} U_{il} \phi_l L_{jm} R_{kn} \bar{\psi}_n P_L \psi_m + \text{h.c.} \equiv \frac{1}{6} \Upsilon_{lmn} \phi_l \bar{\psi}_n P_L \psi_m + \text{h.c.}, \tag{4.38}$$

where $\Upsilon_{lmn} = Y_{ijk} U_{il} L_{jm} R_{kn}$. The DRED to DREG parameter conversion for the Yukawa coupling receives field renormalization contributions from ψ_i, which originate from the second term in the second line of eq. (3.43). In addition, the term in the third line of eq. (3.43) (and its hermitian conjugate) give an explicit contribution to the Yukawa coupling, which reads

$$16\pi^2\epsilon\mathcal{L}_{\mathrm{reg},y} = -\frac{1}{2}\tilde{X}^{\mu}_{\epsilon\psi}\hat{\gamma}^{\nu}X_{\bar{\psi}\psi}\hat{\gamma}_{\nu}\tilde{X}_{\bar{\psi}\epsilon\mu}. \tag{4.39}$$

The appearing derivatives read

$$X_{\bar{\psi}_n\psi_m} = -\frac{1}{6}\Upsilon_{lmn}\phi_l P_L, \tag{4.40}$$

$$(\tilde{X}^{\mu}_{\epsilon\psi})^a_l = g\bar{\psi}_i\tilde{\gamma}^{\mu}(T^a_{R_\psi})_{il}, \tag{4.41}$$

$$(\tilde{X}^{\mu}_{\bar{\psi}\epsilon})^a_i = -g\tilde{\gamma}^{\mu}(T^a_{R_\psi})_{il}\psi_l, \tag{4.42}$$

and one obtains

$$16\pi^2\epsilon\mathcal{L}_{\mathrm{reg},y} = -\frac{d}{12}g^2\epsilon\bar{\psi}_i(T^a_{R_\psi})^F_{ij}(\Upsilon_{lkj}\phi_l P_L)(T^a_{R_\psi})_{km}\psi_m, \tag{4.43}$$

where we have used that $\hat{\gamma}^{\nu}\hat{\gamma}_{\nu} = d$, $\tilde{\gamma}^{\mu}\tilde{\gamma}_{\mu} = \epsilon$ and we have defined

$$(T^a_{R_\psi})_{il} = R^T_{ij}(t^a_{R_\psi})_{jk}R^*_{kl}P_R + L^\dagger_{ij}(t^a_{R_\psi})_{jk}L_{kl}P_L, \tag{4.44}$$

$$(T^a_{R_\psi})^F_{il} = R^T_{ij}(t^a_{R_\psi})_{jk}R^*_{kl}P_L + L^\dagger_{ij}(t^a_{R_\psi})_{jk}L_{kl}P_R. \tag{4.45}$$

The gauge invariance of eq. (4.38) implies

$$Y_{lnj}(t^a_{R_\psi})_{mj} = Y_{jnm}(t^a_{R_\phi})_{jl} + Y_{ljm}(t^a_{R_\psi})_{jn}, \tag{4.46}$$

where $(t^a_{R_\phi})$ are the generators of the representation under which the scalar fields transform. Using this relation one can simplify the r.h.s. of eq. (4.43) by writing

$$(T^a_{R_\psi})^F_{ij}(\Upsilon_{lkj}\phi_l P_L)(T^a_{R_\psi})_{km} = \frac{1}{2}\Upsilon_{lmi}\phi_l P_L\big[C(r_{\psi,m}) + C(r_{\psi,i}) - C(r_{\phi,l})\big], \tag{4.47}$$

which yields

$$16\pi^2\epsilon\mathcal{L}_{\mathrm{reg},y} = -\frac{d}{24}g^2\epsilon\bar{\psi}_i\Upsilon_{lmi}\phi_l P_L\big[C(r_{\psi,m}) + C(r_{\psi,i}) - C(r_{\phi,l})\big]\psi_m, \tag{4.48}$$

for the irreducible representations $r_{\psi,m}$, $r_{\psi,i}$ and $r_{\phi,l}$. This term and the appearing terms bilinear in the fields ψ_i can be absorbed by the finite field and parameter re-definitions

$$\psi_i^{\mathrm{DRED}} \to \left(1 - \frac{1}{2}\delta Z_{\psi,i}\right)\psi_i^{\mathrm{DREG}}, \tag{4.49}$$

$$Y^{\mathrm{DRED}} \to (1 - \delta Y)\, Y^{\mathrm{DREG}}, \tag{4.50}$$

with

$$\delta Z_{\psi,i} = \frac{g^2}{16\pi^2}C(r_{\psi,i}), \tag{4.51}$$

$$\delta Y_{lmn} = \frac{g^2}{16\pi^2}\big[C(r_{\psi,m}) + C(r_{\psi,n}) - C(r_{\phi,l})\big] - \frac{1}{2}\left(\delta Z_{\psi,m} + \delta Z_{\psi,n}\right), \tag{4.52}$$

where the limit $\epsilon \to 0$ has been taken. From eq. (4.52) one obtains the relation

$$Y^{\mathrm{DREG}}_{lmn} = Y^{\mathrm{DRED}}_{lmn}\left(1 + \delta Y\right), \tag{4.53}$$

$$= Y^{\mathrm{DRED}}_{lmn}\left\{1 + \frac{g^2}{32\pi^2}\big[C(r_{\psi,m}) + C(r_{\psi,n}) - 2C(r_{\phi,l})\big]\right\}, \tag{4.54}$$

which agrees with the result of ref. [19].

4.5 Quartic scalar coupling

Here we reproduce the known result for the relation between quartic scalar couplings in DRED and DREG. We consider a general gauge invariant (not necessarily supersymmetric) Lagrangian with the quartic scalar coupling term

$$\mathcal{L}_\lambda = -\frac{1}{4}\lambda_{ijkl}\varphi_i^*\varphi_j^*\varphi_k\varphi_l. \tag{4.55}$$

We assume that the gauge eigenstate fields φ_i are rotated into mass eigenstates ϕ_i with a unitary matrix U. The only contribution to the relation between λ^{DRED} and λ^{DREG} originates from the second term in the first line of eq. (3.43),

$$16\pi^2\epsilon\mathcal{L}_{\text{reg},\lambda} = \frac{1}{2}\sum_{ij}(\tilde{X}^\mu_{\epsilon\epsilon\nu})_{ij}(\tilde{X}^\nu_{\epsilon\epsilon\mu})_{ji}\,. \tag{4.56}$$

The derivative $(\tilde{X}^\nu_{\epsilon\epsilon\mu})_{ji}$ can be obtained from the kinetic term of the scalar fields, $(D_\mu\phi)_i^\dagger(D^\mu\phi)_i$, which contains the coupling to ϵ-scalars. These couplings are of the form

$$\mathcal{L}_{\epsilon\phi} = g^2\phi_i^*(T^a)_{ij}(T^b)_{jk}\phi_k\epsilon^a_\mu\epsilon^b_\nu\tilde{g}^{\mu\nu}, \tag{4.57}$$

where $(T^a) = U^\dagger t^a U$. From this coupling we find

$$\tilde{X}^{\mu\nu}_{\epsilon\epsilon} = g^2\tilde{g}^{\mu\nu}\phi_i^*\{T^a,T^b\}_{ij}\phi_j \tag{4.58}$$

and the contribution to the effective Lagrangian is

$$16\pi^2\epsilon\mathcal{L}_{\text{reg},\lambda} = \frac{g^4}{2}\tilde{g}^\mu_\nu\phi_i^*\{T^a,T^b\}_{ij}\phi_j\tilde{g}^\nu_\mu\phi_k^*\{T^b,T^a\}_{kl}\phi_l \tag{4.59}$$

$$= \frac{g^4}{2}\epsilon\varphi_i^*\varphi_k^*\varphi_j\varphi_l\{t^a,t^b\}_{ij}\{t^b,t^a\}_{kl}, \tag{4.60}$$

where we have used $\tilde{g}^\mu_\nu\tilde{g}^\nu_\mu = \tilde{g}^\mu_\mu = \epsilon$. The term on the r.h.s. of eq. (4.60) can be absorbed by the parameter re-definition

$$\lambda^{\text{DRED}} = \lambda^{\text{DREG}} - \delta\lambda \tag{4.61}$$

with

$$\lambda^{\text{DREG}}_{ijkl} = \lambda^{\text{DRED}}_{ijkl} + \delta\lambda, \tag{4.62}$$

$$= \lambda^{\text{DRED}}_{ijkl} - \frac{g^4}{16\pi^2}\Big(\{t^a,t^b\}_{ik}\{t^b,t^a\}_{jl} + \{t^a,t^b\}_{il}\{t^b,t^a\}_{jk}\Big). \tag{4.63}$$

The relation (4.63) agrees with the result of ref. [19].

4.6 Trilinear, quadratic and tadpole couplings

In a supersymmetric gauge theory, renormalized in the $\overline{\text{DR}}'$ scheme, without spontaneous symmetry breaking the quartic scalar coupling λ_{ijkl} is the only coupling from the scalar potential which receives a non-zero contribution from the ϵ-scalars [19]. In a spontaneously broken gauge theory, however, there may be additional non-zero contributions to the trilinear, quadratic and tadpole scalar couplings from the ϵ-scalars. These non-zero contributions originate from replacing the scalar fields ϕ_i by non-zero vacuum expectation values v_i (VEVs) and corresponding perturbations η_i, as $\phi_i = v_i + \eta_i$. Therefore it is expected that the contribution to the other scalar couplings from the ϵ-scalars is proportional to the VEVs. In this section we calculate the relation of the trilinear, quadratic and tadpole scalar couplings between DRED and DREG in a general spontaneously broken gauge theory using the result of eq. (3.43). We consider a theory with a simple gauge group G that is spontaneously broken by the VEVs of some real scalar fields ϕ_i. The scalar potential in such a general renormalizable gauge theory reads

$$-V(\phi) = \xi_i\phi_i + \frac{1}{2}m^2_{ij}\phi_i\phi_j + \frac{1}{3}h_{ijk}\phi_i\phi_j\phi_k + \frac{1}{4}\lambda_{ijkl}\phi_i\phi_j\phi_k\phi_l, \tag{4.64}$$

where all couplings are totally symmetric. Expanding the scalar fields around their VEVs as $\phi_i = v_i + \eta_i$ yields the potential

$$\begin{aligned}-V(\eta) = \xi_i v_i &+ \frac{1}{2}m^2_{ij}v_iv_j + \frac{1}{3}h_{ijk}v_iv_jv_k + \frac{1}{4}\lambda_{ijkl}v_iv_jv_kv_l \\ &+ \left(\xi_i + m^2_{ij}v_j + h_{ijk}v_jv_k + \lambda_{ijkl}v_jv_kv_l\right)\eta_i \\ &+ \left(\frac{1}{2}m^2_{ij} + h_{ijk}v_k + \frac{3}{2}\lambda_{ijkl}v_kv_l\right)\eta_i\eta_j \\ &+ \left(\frac{1}{3}h_{ijk} + \lambda_{ijkl}v_l\right)\eta_i\eta_j\eta_k + \frac{1}{4}\lambda_{ijkl}\eta_i\eta_j\eta_k\eta_l\,.\end{aligned} \tag{4.65}$$

In order for a minimum to be attained at $\eta_i = 0\,\forall i$ the following conditions must be satisfied

$$\xi_i + m^2_{ij}v_j + h_{ijk}v_jv_k + \lambda_{ijkl}v_jv_kv_l = 0 \qquad \forall i. \tag{4.66}$$

When integrating out the ϵ-scalars one obtains corrections to the potential (4.65) from the first line of eq. (3.43),

$$16\pi^2\epsilon\mathcal{L}_{\mathrm{reg},\eta} = -\sum_a (m^2_\varepsilon)_a(\tilde{X}^\mu_{\varepsilon\varepsilon\mu})_{aa} + \frac{1}{2}\sum_{ab}(\tilde{X}^\mu_{\varepsilon\varepsilon\nu})_{ab}(\tilde{X}^\nu_{\varepsilon\varepsilon\mu})_{ba}\,. \tag{4.67}$$

The derivatives $(\tilde{X}^{\mu\nu}_{\varepsilon\varepsilon})_{ab}$ in eq. (4.67) are to be taken with respect to the ϵ-scalar mass eigenstates, denoted by ε_a, and $(m^2_\varepsilon)_a$ are the corresponding mass eigenvalues. The derivatives can be calculated from the interaction Lagrangian between the ϵ-scalars and the scalar fields η_i, which reads

$$\mathcal{L}_{\epsilon\eta} = \frac{g^2}{2}\epsilon^a_\mu\epsilon^b_\nu\tilde{g}^{\mu\nu}(T^a)_{ij}(T^b)_{ik}\left(v_jv_k + 2\eta_jv_k + \eta_j\eta_k\right), \tag{4.68}$$

where $T^a = -it^a$ are real, antisymmetric matrices and t^a are the generators of the representation under which the η_i transform. The first term in the parentheses of (4.68) contributes to the mass matrix of ϵ-scalars, which reads

$$(m^2_\varepsilon)_{ab} = m^2_\epsilon\delta_{ab} + g^2(T^a)_{ij}v_j(T^b)_{ik}v_k. \tag{4.69}$$

Since this matrix is symmetric it can be diagonalized by an orthogonal matrix O such that $O_{ab}(m_\varepsilon^2)_{bc}O_{dc} = (m_\varepsilon^2)_a\delta_{ad}$. The corresponding mass eigenstates ε_a are then given by $\varepsilon_a = O_{ab}\epsilon_b$ and the interaction Lagrangian in terms of ε-scalar mass eigenstates becomes

$$\mathcal{L}_{\varepsilon\eta} = \frac{g^2}{2}\varepsilon_\mu^a\varepsilon_\nu^b\tilde{g}^{\mu\nu}(T_O^a)_{ij}(T_O^b)_{ik}\,(2\eta_j v_k + \eta_j\eta_k)\,, \tag{4.70}$$

where $T_O^a = O_{ab}T^b$ and we have omitted the ε-scalar mass term. From eq. (4.70) one obtains the derivatives

$$(\tilde{X}_{\varepsilon\varepsilon}^{\mu\nu})^{ab} = -\frac{g^2}{2}\tilde{g}^{\mu\nu}\{T_O^a, T_O^b\}_{kj}\,(2v_k\eta_j + \eta_k\eta_j)\,, \tag{4.71}$$

$$(\tilde{X}_{\varepsilon\varepsilon\mu}^{\mu})^{aa} = -\frac{g^2}{2}\epsilon\{T^a, T^a\}_{kj}\,(2v_k\eta_j + \eta_k\eta_j)\,, \tag{4.72}$$

and the contribution from the ϵ-scalars becomes

$$\begin{aligned}16\pi^2\epsilon\mathcal{L}_{\text{reg},\eta} = {}&(m_\varepsilon^2)_a\frac{g^2}{2}\epsilon\{T^a, T^a\}_{kj}\,(2v_k\eta_j + \eta_k\eta_j)\\ &+\frac{g^4}{8}\epsilon\{T^a, T^b\}_{kj}\{T^a, T^b\}_{lm}\,(4v_kv_l\eta_j\eta_m + 4v_k\eta_l\eta_j\eta_m + \eta_k\eta_l\eta_j\eta_m)\,.\end{aligned} \tag{4.73}$$

From eq. (4.73) one can see that for $v_i = 0\ \forall i$ there would only be a contribution to the quadratic and to the quartic scalar coupling, as was pointed out in ref. [19]. However, when $v_i \neq 0$ these two contributions also get distributed to other terms in the scalar potential. The new scalar potential including the contribution from the ϵ-scalars becomes

$$\begin{aligned}-V(\eta) = {}&\xi_iv_i + \frac{1}{2}m_{ij}^2v_iv_j + \frac{1}{3}h_{ijk}v_iv_jv_k + \frac{1}{4}\lambda_{ijkl}v_iv_jv_kv_l\\ &+\left(\xi_i + m_{ij}^2v_j + h_{ijk}v_jv_k + \lambda_{ijkl}v_jv_kv_l + \frac{1}{16\pi^2}v_k\mathcal{A}_{ki}\right)\eta_i\\ &+\left(\frac{1}{2}m_{ij}^2 + h_{ijk}v_k + \frac{3}{2}\lambda_{ijkl}v_kv_l + \frac{1}{2(16\pi^2)}\mathcal{A}_{ij} + \frac{1}{2(16\pi^2)}v_lv_k\mathcal{B}_{kilj}\right)\eta_i\eta_j\\ &+\left(\frac{1}{3}h_{ijk} + \lambda_{ijkl}v_l + \frac{1}{2(16\pi^2)}v_l\mathcal{B}_{lijk}\right)\eta_i\eta_j\eta_k\\ &+\left(\frac{1}{4}\lambda_{ijkl} + \frac{1}{8(16\pi^2)}\mathcal{B}_{ijkl}\right)\eta_i\eta_j\eta_k\eta_l,\end{aligned} \tag{4.74}$$

where we have introduced the abbreviations

$$\mathcal{A}_{ij} \equiv g^2(m_\varepsilon^2)_a\{T^a, T^a\}_{ij} = 2g^2(m_\varepsilon^2)_a(T^aT^a)_{ij}, \tag{4.75}$$

$$\mathcal{B}_{ijkl} \equiv g^4\{T^a, T^b\}_{ij}\{T^a, T^b\}_{kl}, \tag{4.76}$$

and all repeated indices are summed over. In eq. (4.74) all parameters are still defined in DRED. The 1-loop terms on the r.h.s. of eq. (4.74) can be absorbed by the parameter re-definitions

$$p^{\text{DRED}} = p^{\text{DREG}} - \delta p, \tag{4.77}$$

where $p \in \{\xi, m^2, h, \lambda, v\}$. Note, that $\eta^{\text{DRED}} = \eta^{\text{DREG}}$, because there is no contribution to the field renormalization of scalar fields from eq. (3.43). By demanding that the potential written in terms of DREG parameters takes the same form as in eq. (4.65) we obtain the following set of equations relating the shifts to the finite loop corrections from the ϵ-scalars

$$\delta\lambda_{ijkl} = \frac{1}{2(16\pi^2)}\mathcal{B}_{(ijkl)}, \tag{4.78}$$

$$\frac{1}{3}\delta h_{ijk} + \delta\lambda_{ijkl} v_l + \lambda_{ijkl}\delta v_l = \frac{1}{2(16\pi^2)} v_l \mathcal{B}_{l(ijk)}, \tag{4.79}$$

$$\frac{1}{2}\delta m^2_{ij} + \delta h_{ijk} v_k + h_{ijk}\delta v_k + \frac{3}{2}\delta\lambda_{ijkl} v_k v_l + 3\lambda_{ijkl}\delta v_k v_l = \frac{\mathcal{A}_{(ij)}}{2(16\pi^2)} + \frac{v_k v_l \mathcal{B}_{k(i|l|j)}}{2(16\pi^2)}, \tag{4.80}$$

$$\delta\xi_i + \delta m^2_{ij} v_j + m^2_{ij}\delta v_j + \delta h_{ijk} v_j v_k + 2h_{ijk}\delta v_j v_k + \delta\lambda_{ijkl} v_j v_l v_k + 3\lambda_{ijkl}\delta v_j v_k v_l = \frac{v_k \mathcal{A}_{ki}}{16\pi^2}, \tag{4.81}$$

where $T_{(i_1 i_2 \ldots i_n)} \equiv \frac{1}{n!}\sum_{\sigma\in S_n} T_{\sigma(i_1)\sigma(i_2)\ldots\sigma(i_n)}$ and S_n is the symmetric group over n symbols and $\mathcal{B}_{k(i|l|j)} = \frac{1}{2}\sum_{\sigma\in S_2} B_{k\sigma(i)l\sigma(j)}$. This set of equations is equivalent to

$$\delta\lambda_{ijkl} = \frac{1}{2(16\pi^2)}\mathcal{B}_{(ijkl)}, \tag{4.82}$$

$$\frac{1}{3}\delta h_{ijk} + \lambda_{ijkl}\delta v_l = \frac{v_l}{2(16\pi^2)}\left[\mathcal{B}_{l(ijk)} - \mathcal{B}_{(ijkl)}\right], \tag{4.83}$$

$$\frac{1}{2}\delta m^2_{ij} + h_{ijk}\delta v_k = \frac{\mathcal{A}_{ij}}{2(16\pi^2)} + \frac{v_k v_l}{2(16\pi^2)}\left[\mathcal{B}_{k(i|l|j)} - 3\mathcal{B}_{l(ijk)} + \frac{3}{2}\mathcal{B}_{(ijkl)}\right], \tag{4.84}$$

$$\delta\xi_i + m^2_{ij}\delta v_j = \frac{v_j v_k v_l}{(16\pi^2)}\left[-\mathcal{B}_{k(i|l|j)} + \frac{3}{2}\mathcal{B}_{l(ijk)} - \frac{1}{2}\mathcal{B}_{(ijkl)}\right], \tag{4.85}$$

where we have used that $\mathcal{A}_{(ij)} = \mathcal{A}_{ij}$, because $\mathcal{A}_{ij} = \mathcal{A}_{ji}$. Eq. (4.82) is equivalent to the result obtained in section 4.5 for complex scalar fields. The eqs. (4.83)–(4.85) can be simplified further by using the fact that the shifts of the vacuum expectation values δv_i can be related to the shifts δZ_η of the scalar fields and corresponding (auxiliary) background fields $\delta\hat{Z}_\eta$ as [34, 35]

$$\delta v_i = \frac{1}{2}\left(\delta Z_\eta + \delta\hat{Z}_\eta\right)_{ij} v_j\,. \tag{4.86}$$

As pointed out in refs. [34, 35], neither δZ_η nor $\delta\hat{Z}_\eta$ receive contributions from ϵ-scalars, which implies

$$\delta v_i = 0 \qquad \Leftrightarrow \qquad v_i^{\text{DREG}} = v_i^{\text{DRED}}\,. \tag{4.87}$$

This allows us to derive the following relations for the trilinear, quadratic and tadpole scalar couplings between DREG and DRED,

$$h_{ijk}^{\text{DREG}} = h_{ijk}^{\text{DRED}} + \frac{3v_l}{2(16\pi^2)}\left[\mathcal{B}_{l(ijk)} - \mathcal{B}_{(ijkl)}\right], \tag{4.88}$$

$$(m^2_{ij})^{\text{DREG}} = (m^2_{ij})^{\text{DRED}} + \frac{\mathcal{A}_{ij}}{(16\pi^2)} + \frac{v_k v_l}{(16\pi^2)}\left[\mathcal{B}_{k(i|l|j)} - 3\mathcal{B}_{l(ijk)} + \frac{3}{2}\mathcal{B}_{(ijkl)}\right], \tag{4.89}$$

$$\xi_i^{\text{DREG}} = \xi_i^{\text{DRED}} + \frac{v_j v_k v_l}{(16\pi^2)}\left[-\mathcal{B}_{k(i|l|j)} + \frac{3}{2}\mathcal{B}_{l(ijk)} - \frac{1}{2}\mathcal{B}_{(ijkl)}\right]. \tag{4.90}$$

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The relations (4.87)–(4.90) represent a generalization of the known results of ref. [19] for a spontaneously broken gauge theory with non-zero VEVs. In the limit $v_i \to 0$, which was used in ref. [19], one obtains

$$h_{ijk}^{\text{DREG}} = h_{ijk}^{\text{DRED}}\,, \tag{4.91}$$

$$(m_{ij}^2)^{\text{DREG}} = (m_{ij}^2)^{\text{DRED}} - \frac{2g^2}{16\pi^2} m_\epsilon^2 C(r_i)\delta_{ij}\,, \tag{4.92}$$

$$\xi_i^{\text{DREG}} = \xi_i^{\text{DRED}}\,, \tag{4.93}$$

with $(T^a T^a)_{ij} = i^2 (t^a t^a)_{ij} = -C(r_i)\delta_{ij}$. The m_ϵ^2-dependence in eq. (4.92) can be removed by shifting the m_{ij}^2 parameters as described in ref. [26], which is equivalent to transforming from the $\overline{\text{DR}}$ into the $\overline{\text{DR}}'$ scheme as

$$(m_{ij}^2)^{\text{DRED}} = (m_{ij}^2)^{\text{DRED}'} + \frac{2g^2}{16\pi^2} m_\epsilon^2 C(r_i)\delta_{ij}\,. \tag{4.94}$$

In the $\overline{\text{DR}}'$ scheme one therefore obtains in the limit $v_i \to 0$,

$$h_{ijk}^{\text{DREG}} = h_{ijk}^{\text{DRED}'}\,, \qquad (m_{ij}^2)^{\text{DREG}} = (m_{ij}^2)^{\text{DRED}'}\,, \qquad \xi_i^{\text{DREG}} = \xi_i^{\text{DRED}'}\,, \tag{4.95}$$

which is the known result from ref. [19].

5 Conclusions

The universal one-loop effective action (UOLEA) is a very elegant tool to fully automate the derivation of the large set of effective Lagrangians of a given UV model with heavy particles. To date, however, only part of the UOLEA is known and only in dimensional regularization (DREG). Due to this restriction, the known part cannot be applied to supersymmetric UV models, regularized in dimensional reduction (DRED), with non-supersymmetric effective theories that are regularized in DREG.

In this paper we have extended the UOLEA by generic 1-loop operators which represent a translation between DRED and DREG. These operators allow for an application of the UOLEA to supersymmetric UV models with non-supersymmetric EFTs. As the UOLEA itself, our derived generic operators are well suited to be implemented into generic spectrum generators to fully automate the derivation of non-supersymmetric EFTs.

We have performed the calculation of the effective operators in the language of effective field theories, close to the formulation of the UOLEA. In our case the field to be integrated out is the unphysical ϵ-scalar, which occurs in DRED when the 4-dimensional gauge field is split into a d- and an ϵ-dimensional part. The resulting effective operators can be absorbed by a re-definition of the fields and parameters, leading to the well-known parameter translations in supersymmetric models of ref. [19].

In our calculation we have assumed that the UV theory is renormalizable and gauge invariant, but not necessarily supersymmetric. Within these restrictions our result is generic and contains even terms corresponding to finite field renormalizations. Furthermore, our result has an explicit m_ϵ^2 dependence, which can be removed by shifting the squared mass parameters of the scalar fields appropriately as described in ref. [26].

Finally we have applied our derived effective operators to various UV theories for illustration and to prove that the known results of ref. [19] can be reproduced. Furthermore, we have derived relations for scalar cubic, quadratic and tadpole couplings in a general renormalizable gauge theory with spontaneous symmetry breaking, complementing the results of ref. [19].

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A Consistency of shifts for Majorana fermions

We here show that the shifts (3.15) and (3.16) can be performed consistently for Majorana fermions. For Majorana fermions there is only one degree of freedom and so it is necessary that the shifted fields $\delta\bar{\lambda}'$ and $\delta\lambda'$ are related as $\delta\bar{\lambda}' = \delta\lambda'^{\dagger}\gamma^0$. That this is in fact the case follows from the Hermiticity of the Lagrangian. Consider the terms containing Majorana fermions in the original Lagrangian

$$\mathcal{L}_\lambda = \bar{\lambda}F\lambda + \epsilon^a_\mu\bar{\lambda}\tilde{G}^\mu_a\lambda + \phi\bar{\lambda}H\psi + \phi\bar{\psi}I\lambda + \phi\bar{\lambda}J\lambda, \tag{A.1}$$

where F, G, H, I and J are independent of the fields. Taking the Hermitian conjugate and using that $\mathcal{L}^\dagger_\lambda = \mathcal{L}_\lambda$ we find

$$\mathcal{L}_\lambda = \bar{\lambda}\gamma^0F^\dagger\gamma^0\lambda + \epsilon^a_\mu\bar{\lambda}\gamma^0(\tilde{G}^\mu_a)^\dagger\gamma^0\lambda + \phi\bar{\lambda}\gamma^0I^\dagger\gamma^0\psi + \phi\bar{\psi}\gamma^0H^\dagger\gamma^0\lambda, \tag{A.2}$$

which yields the relations

$$F = \gamma^0F^\dagger\gamma^0, \tag{A.3}$$
$$\tilde{G}^\mu_a = \gamma^0(\tilde{G}^\mu_a)^\dagger\gamma^0, \tag{A.4}$$
$$H = \gamma^0I^\dagger\gamma^0, \tag{A.5}$$
$$I = \gamma^0H^\dagger\gamma^0, \tag{A.6}$$
$$J = \gamma^0J^\dagger\gamma^0. \tag{A.7}$$

We can also relate F, G, H, I and J to the quantities appearing in the second variation of the Lagrangian by noting that

$$\begin{aligned}\delta^2\mathcal{L}_\lambda = {} & \delta\bar{\lambda}(F + \epsilon^a_\mu\tilde{G}^\mu_a + \phi J)\delta\lambda + \delta\bar{\lambda}\tilde{G}^\mu_a\lambda\delta\epsilon^a_\mu + \delta\epsilon^a_\mu\bar{\lambda}\tilde{G}^\mu_a\delta\lambda + \delta\bar{\lambda}(H\psi + J\lambda)\delta\phi \\ & + \delta\bar{\lambda}H\phi\delta\psi + \delta\bar{\psi}I\phi\delta\lambda + \delta\phi(\bar{\lambda}J + \bar{\psi}I)\delta\lambda + \cdots,\end{aligned} \tag{A.8}$$

where the extra terms indicated by the ellipsis do not include any variation of λ or $\bar{\lambda}$. Comparing this to (3.3) one obtains

$$\Delta_\lambda = (F + \epsilon^a_\mu \tilde{G}^\mu_a + \phi J), \tag{A.9}$$
$$\tilde{X}^\mu_{\epsilon\lambda} = \bar{\lambda}\tilde{G}^\mu_a, \tag{A.10}$$
$$\tilde{X}^\mu_{\bar{\lambda}\epsilon} = -\tilde{G}^\mu_a \lambda, \tag{A.11}$$
$$X_{\phi\lambda} = \bar{\lambda}J + \bar{\psi}I, \tag{A.12}$$
$$X_{\bar{\lambda}\phi} = -(H\psi + J\lambda), \tag{A.13}$$
$$X_{\bar{\psi}\lambda} = I\phi, \tag{A.14}$$
$$X_{\bar{\lambda}\psi} = H\phi. \tag{A.15}$$

From these relations and (A.3)–(A.7) it follows that

$$(\Delta^\dagger_\lambda)^{-1} = \gamma^0 \Delta^{-1}_\lambda \gamma^0, \tag{A.16}$$
$$(\tilde{X}^\mu_{\bar{\lambda}\epsilon})^\dagger = -\tilde{X}^\mu_{\epsilon\lambda}\gamma^0, \tag{A.17}$$
$$X^\dagger_{\bar{\lambda}\phi} = -X_{\phi\lambda}\gamma^0, \tag{A.18}$$
$$X^\dagger_{\bar{\lambda}\psi} = \gamma^0 X_{\bar{\psi}\lambda}\gamma^0. \tag{A.19}$$

Calculating the Dirac adjoint of the shift (3.15) we obtain

$$\begin{aligned}
\delta\lambda'^\dagger\gamma^0 &= \delta\bar{\lambda} - \left(\tilde{X}^{\mu\dagger}\delta\epsilon_\mu + X^\dagger_{\bar{\lambda}\phi}\delta\phi - \delta\psi^\dagger X^\dagger_{\bar{\lambda}\psi}\right)(\Delta^{-1}_\lambda)^\dagger\gamma^0 \\
&= \delta\bar{\lambda} - \left(-\tilde{X}^\mu_{\epsilon\lambda}\gamma^0\delta\epsilon_\mu - X_{\phi\lambda}\gamma^0\delta\phi - \delta\psi^\dagger\gamma^0 X_{\bar{\psi}\lambda}\gamma^0\right)\gamma^0(\Delta^{-1}_\lambda)\gamma^0\gamma^0 \\
&= \delta\bar{\lambda} + \left(\tilde{X}^\mu_{\epsilon\lambda}\delta\epsilon_\mu + X_{\phi\lambda}\delta\phi + \delta\bar{\psi}X_{\bar{\psi}\lambda}\right)\Delta^{-1}_\lambda \\
&= \delta\bar{\lambda}',
\end{aligned} \tag{A.20}$$

where we have used eqs. (A.16)–(A.19) in the second line and the definition (3.16) in the last line. We conclude that the shifts (3.15)–(3.16) are consistent with the required property for Majorana fermions, $\delta\bar{\lambda}' = \delta\lambda'^\dagger\gamma^0$.

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