# Naturalness of the relaxion mechanism

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ABSTRACT: The relaxion mechanism is a novel solution to the hierarchy problem. In this first statistical analysis of the relaxion mechanism, we quantify the relative plausibility of a QCD and a non-QCD relaxion model versus the Standard Model with Bayesian statistics, which includes an automatic penalty for fine-tuning. We find that in light of the hierarchy between the weak and Planck scales, relaxion models are favoured by colossal Bayes-factors. Constraints upon e.g., the vacuum energy during relaxation, however, shrink the Bayes-factors such that relaxion models are only slightly favoured. Including the bounds on $|\theta_{\rm QCD}|$ shatters the plausibility of the QCD relaxion model as it typically yields $|\theta_{\rm QCD}| \gg 0$. Finally, we augment our models with scalar-field inflation and consider measurements of inflationary observables from BICEP/Planck. We find that, all told, the Standard Model is favoured by huge Bayes-factors as the relaxion models require fine-tuning such that the Hubble parameter is less than the height of the periodic barriers. Thus, whilst we confirm that relaxion models could solve the hierarchy problem, we find that their unconventional cosmology is at odds with their plausibility.

# Contents

## 1   Introduction

Graham et al. [1] recently proposed a relaxation mechanism [2–4] that solves the hierarchy problem [5–8] by utilising the dynamics of an axion-like field, dubbed the relaxion. In the Standard Model (SM), the hierarchy problem originates from quadratic corrections to the weak scale. Whereas supersymmetry cancels them with new quadratic corrections involving supersymmetric particles [9], the relaxion mechanism cancels them with the vacuum expectation value (VEV) of a relaxion field.

     The ingenuity of the relaxion mechanism is that the dynamics of the relaxion field ensure a precise cancellation without patent fine-tuning of parameters or initial conditions. Within the relaxion paradigm, interactions between a complex Higgs doublet, $h$, and an axion-like field, $\phi$, govern the weak scale via the scalar potential [1]

$$V = \left(\mu^2 - \kappa\langle a\rangle\phi\right)h^2 - m_b^3\langle h\rangle\cos\left(\frac{\phi}{f}\right) - m^2\langle a\rangle\phi + \lambda h^4, \tag{1.1}$$

where, because of quadratic corrections, we expect that the masses should be close to the cut-off $\Lambda$, i.e., $\mu^2 \sim m^2 \sim \Lambda^2$, $m_b$ and $f$ are coupling constants of dimension mass, $\langle a\rangle$ is

the VEV of a spurion field that breaks a shift symmetry $\phi \to \phi + 2\pi f$, $\kappa$ is a dimensionless coupling, and $\langle h \rangle$ is the VEV of the Higgs field, which is a function of the relaxion field $\phi$.

Let us label the co-efficient of $h^2$ in the relaxion potential

$$m_h^2(\phi) \equiv \mu^2 - \kappa \langle a \rangle \phi, \tag{1.2}$$

for convenience, such that the Higgs VEV may be written

$$\langle h \rangle = \begin{cases} \sqrt{\frac{-m_h^2(\phi)}{2\lambda}} & m_h^2(\phi) < 0, \\ 0 & \text{otherwise.} \end{cases} \tag{1.3}$$

If the Higgs VEV is non-zero, the cosine term provides a periodic barrier for the relaxion field with barriers separated by $2\pi f$. In the unbroken phase in which $\langle h \rangle = 0$, the barrier is down and the relaxion field slowly rolls down a linear potential. Once $m_h^2(\phi) < 0$, however, the potential is such that the Higgs field acquires a VEV, $\langle h \rangle \neq 0$, breaking electroweak symmetry (EWSB) and raising the periodic barrier. The now-raised periodic barrier traps the relaxion field in a minimum. If the relaxion field cannot roll past a local minimum, it results in a weak scale of about

$$\langle h \rangle \gtrsim f \frac{m^2 \langle a \rangle}{m_b^3}. \tag{1.4}$$

Thus this mechanism could result in $\langle h \rangle \ll M_{\rm P}$.

We require, inter alia, that the relaxion field dissipates energy as it rolls or else it would have sufficient kinetic energy to surmount the periodic barriers. In the relaxion paradigm, this is ensured by Hubble friction — a term analogous to a friction term in the Euler-Lagrange equation for the relaxion field originating from the expansion of the Universe see e.g. ref. [10]:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0, \tag{1.5}$$

where $H$ is the Hubble parameter. If Hubble friction is substantial, the relaxion field could be in a slow-roll regime in which the acceleration $\ddot{\phi}$ can be neglected.

Ostensibly, the relaxion mechanism ameliorates fine-tuning associated with the weak scale, but Raidal et al. [11] stress that it could require a fine-tuned inflationary sector if the relaxion is the QCD axion because of constraints upon the Hubble parameter during inflation. Unfortunately, there is no consensus in high-energy physics on the appropriate measure of fine-tuning or about the logical foundations of fine-tuning arguments, despite their prominence. In earlier work to judge fine-tuning in relaxion models, Jaeckel et al. [12] developed a new formalism based on their intuition, whereas Raidal et al. [11] utilised common Barbieri-Giudice style measures [13, 14]. In section 2, we critique Jaeckel's approach and instead advocate a Bayesian methodology, discussed numerous times over the last decade in the context of fine-tuning in supersymmetric models [15–22]. In this methodology, in light of experimental data about the weak scale and inflation, we update our belief in a model with a Bayesian evidence. We further analyse the relaxion potential in section 3. We describe our models — minimal relaxion models and the SM augmented by scalar-field inflation — in section 4 and calculate their Bayesian evidences in section 5. This is the

first statistical analysis of a relaxion model. We close in section 6 with a brief discussion of our findings.

## 2  Bayesian fine-tuning

Bayesian statistics provides a logical framework for updating beliefs in scientific theories in light of data see e.g. ref. [23–25]. This methodology is becoming increasingly common in high-energy physics see e.g. ref. [26–69] and cosmology see e.g. ref. [70–72], and arguably captures the essence of the hierarchy problem [15–22] and the principle of Occam's razor see e.g. ref. [73]. We briefly recapitulate the essential details.

The Bayesian framework enables one to assign numerical measures to degrees of belief. To assess two models, $M_a$ and $M_b$, one begins by quantifying one's relative degree of belief in the models, prior to considering any experimental data. This is known as the prior odds,

$$\text{Prior odds} \equiv \frac{P(M_a)}{P(M_b)}, \tag{2.1}$$

where $P(M)$ is one's prior belief in a model $M$. From the prior odds, we can calculate the posterior odds — one's relative degree of belief in the models updated with experimental data,

$$\text{Posterior odds} \equiv \frac{P(M_a \,|\, D)}{P(M_b \,|\, D)}, \tag{2.2}$$

where $D$ represents experimental data e.g., in this work data from BICEP/Planck. The prior odds and the posterior odds are related by a so-called Bayes-factor:

$$\text{Posterior odds} = \text{Bayes-factor} \times \text{Prior odds}. \tag{2.3}$$

By applying Bayes' theorem, it can be readily shown that the Bayes-factor is a ratio of probability densities,

$$\text{Bayes-factor} \equiv \frac{p(D \,|\, M_a)}{p(D \,|\, M_b)}, \tag{2.4}$$

where the probability densities in question are known as Bayesian evidences or just evidences. The evidence for a model $M$ can be calculated by Bayes' theorem and marginalisation,

$$\mathcal{Z} \equiv p(D \,|\, M) = \int p(D \,|\, M, \boldsymbol{p}) \cdot p(\boldsymbol{p} \,|\, M) \prod \mathrm{d}\boldsymbol{p} \tag{2.5}$$

where $\boldsymbol{p}$ are the model's parameters, $p(D \,|\, M, \boldsymbol{p})$ is a so-called likelihood function — the probability density of our observed data given parameters $\boldsymbol{p}$ — and $p(\boldsymbol{p} \,|\, M)$ is our prior density for the model's parameters $\boldsymbol{p}$.

The likelihood function is uncontroversial as its form is dictated by the nature of an experiment and it is a critical ingredient in Bayesian and frequentist statistics. The role and form of the prior density, however, remain contentious issues. In as much as it is possible, we pick objective priors that reflect our knowledge or ignorance about a parameter and respect rational constraints from e.g., symmetries.

We calculate Bayes-factors for the SM augmented with scalar-field inflation $(\text{SM} + \sigma)$ versus relaxion models. The final step — that of updating one's prior odds with a Bayes-factor to find one's posterior odds — is left to the reader. That is not to say that a Bayes-factor is independent of any prior choices — it is in fact a functional of the priors for the parameters of the models in question.

Before closing, we briefly discuss attempts to quantify fine-tuning in a relaxion model by Jaeckel et al. [12] and by Raidal et al. [11]. Raidal et al. employed Barbieri-Giudice style measures of fine-tuning [13, 14]. Whilst intuitive, such measures lack a logical foundation, though emerge in intermediate steps in a calculation of the Bayesian evidence [15–22]. Jaeckel et al. developed a novel measure of electroweak fine-tuning, $F$, based on the fraction of a model's parameter space, $\boldsymbol{p}$, that predicts a weak scale less than that observed:

$$\frac{1}{F} \equiv \frac{V_{v(\boldsymbol{p}) \leq v}}{V} = \frac{\int \theta(v - v(\boldsymbol{p})) \prod d\boldsymbol{p}}{\int \prod d\boldsymbol{p}}. \tag{2.6}$$

This measure contrasts with Barbieri-Giudice measures in that it considers a model's entire parameter space rather than a single point in it. Jaeckel's measure, however, depends on one's choice of parameterisation or measure for the parameter space.[1]

Curiously, Jaeckel's measure in eq. (2.6) is reminiscent of the Bayesian evidence if one considers measurements of the weak scale, especially if one writes (unnecessary) normalisation factors for the priors,

$$\mathcal{Z} = \frac{\int p(v \mid M, \boldsymbol{p}) \cdot p(\boldsymbol{p} \mid M) \prod d\boldsymbol{p}}{\int p(\boldsymbol{p} \mid M) \prod d\boldsymbol{p}} \quad \text{vs.} \quad \frac{1}{F} = \frac{\int \theta(v - v(\boldsymbol{p})) \prod d\boldsymbol{p}}{\int \prod d\boldsymbol{p}}. \tag{2.7}$$

The differences are that Jaeckel et al. pick a step-function for the likelihood for the weak scale, $v$, and omit a measure for the volume of parameter space, i.e., a prior. In other words, by following their noses and attempting to formulate fine-tuning in a logical manner, Jaeckel et al. create an ersatz Bayesian evidence.

## 3 Analysis of relaxion potential

Let us further analyse the relaxion potential in eq. (1.1),

$$V = \left(\mu^2 - \kappa \langle a \rangle \phi\right) h^2 - m_b^3 \langle h \rangle \cos\left(\frac{\phi}{f}\right) - m^2 \langle a \rangle \phi + \lambda h^4.$$

As in ref. [74], for simplicity we consider only linear terms in the relaxion field $\phi$. The equations $\partial V / \partial \phi = 0$ and $\partial V / \partial h = 0$ result in a transcendental equation,

$$\sin(\phi/f) = \frac{f \kappa \langle a \rangle}{m_b^3} \left(\frac{m^2/\kappa + \langle h \rangle^2}{\langle h \rangle}\right). \tag{3.1}$$

By graphing as in figure 1, one finds that if there is a solution, it lies in the interval $\langle h \rangle_{\min} \leq \langle h \rangle \leq \langle h \rangle_{\max}$ where

$$\langle h \rangle_{\min} = \frac{m_b^3 - \sqrt{m_b^6 - 4\kappa m^2 \langle a \rangle^2 f^2}}{2\kappa \langle a \rangle f} \quad \text{and} \quad \langle h \rangle_{\max} = \sqrt{\frac{-\mu^2 + \kappa \langle a \rangle \langle \phi \rangle_{\max}}{2\lambda}}, \tag{3.2}$$

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[1]However, Jaeckel et al. [12] includes a general discussion of parameterisation in the context of fine-tuning.
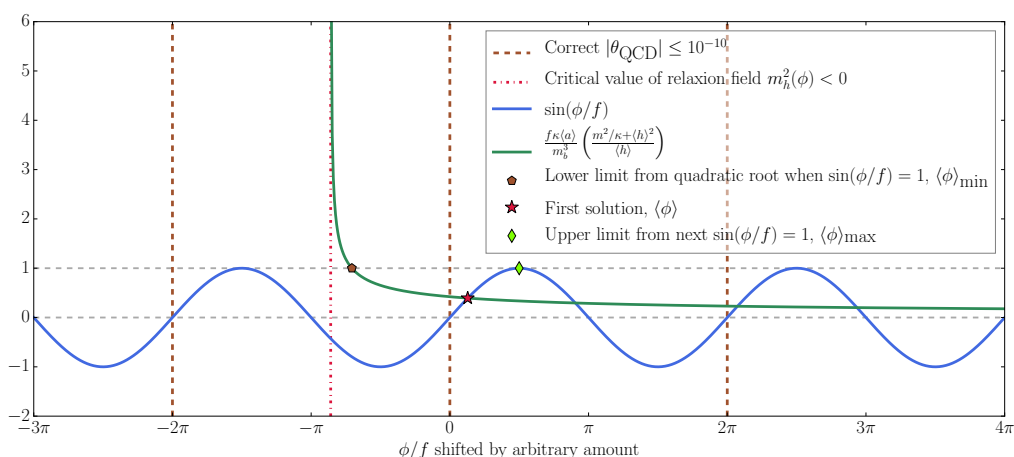
**Figure 1.** Graphing the left-hand side (blue line) and right-hand side (green line) of the transcendental equation in eq. (3.1). The solution (red star) lies in the interval in eq. (3.2), marked by a brown pentagon and a green diamond. In unusual cases, the second point at which the right-hand side equals plus one (not shown) may be a sharper bound. The $\phi/f$-axis is shifted such that correct $|\theta_{\rm QCD}|$ occurs at small multiples of $2\pi$ (vertical brown dashed lines) close to the solution. EWSB is broken once the critical value of the relaxion field is surpassed (vertical red dot-dashed line).

and

$$\langle\phi\rangle_{\rm min} = \frac{2\lambda\langle h\rangle^2_{\rm min} + \mu^2}{\kappa\langle a\rangle} \quad \text{and} \quad \langle\phi\rangle_{\rm max} = (2n + 1/2)\pi f \tag{3.3}$$

where $n$ is the smallest integer such that $\langle\phi\rangle_{\rm max} > \langle\phi\rangle_{\rm min}$. If the square-root is imaginary, there are no solutions, otherwise, there are zero to four solutions inside the interval, which must be identified numerically. The interval results from recognising that a solution must lie between the point at which the right-hand side of eq. (3.1) equals plus one, matching the maximum of the left-hand side, and the subsequent point at which the latter is again maximal. If required, one can improve this interval with piece-wise expressions by graphing. In some cases, the positive quadratic root, similar to that for $\langle h\rangle_{\rm min}$, is a sharper bound for $\langle h\rangle_{\rm max}$. If the barrier height is substantial, the lower bound reduces to the approximation for $\langle h\rangle$ in eq. (1.2), that is,

$$\langle h\rangle_{\rm min} \approx f\frac{m^2\langle a\rangle}{m_b^3} \quad \text{if} \quad \frac{4\kappa m^2\langle a\rangle^2 f^2}{m_b^6} \ll 1. \tag{3.4}$$

This implies that $\kappa\langle a\rangle \ll m_b^3/(4f\langle h\rangle)$. A necessary (though not sufficient) condition for solutions to the transcendental equation is that the root in eq. (3.2) must be real,

$$\frac{4\kappa m^2\langle a\rangle^2 f^2}{m_b^6} \le 1. \tag{3.5}$$

In other words, the relaxion mechanism ensures that the weak scale is independent of quadratic corrections to the Higgs mass from a cut-off or unknown high-scale physics,

solving the hierarchy problem. In fact, the Higgs VEV is bounded by an expression that is independent of the Higgs mass,

$$\langle h \rangle_{\text{min}} \leq \langle h \rangle \leq \langle h \rangle_{\text{max}} \leq \sqrt{\langle h \rangle_{\text{min}}^2 + \kappa \langle a \rangle f \pi / \lambda}. \tag{3.6}$$

The Higgs mass $\mu^2$ and any corrections affect the position of $\langle h \rangle$ inside this interval, but not the interval itself. The width of this interval is typically small such that numerically solving for the VEV of the Higgs field inside the interval may be unnecessary.

Unfortunately, if the relaxion is the QCD axion, we expect that barrier height $m_b^3$ is connected to QCD, such that $m_b \sim \Lambda_{\text{QCD}}$, resulting in

$$\langle a \rangle \lesssim \langle h \rangle \times 10^{-18} \simeq 10^{-16} \, \text{GeV}, \tag{3.7}$$

where we impose an experimental limit on the QCD decay constant, $f \gtrsim 10^9 \, \text{GeV}$, and pick $m \simeq 1 \, \text{TeV}$ in eq. (3.4). Thus achieving a small weak scale requires a tiny spurion VEV, $\langle a \rangle \lll M_{\text{P}}$. Such a small coupling may be natural as it breaks a shift-symmetry see e.g. ref. [75]; however, there may be issues due to the gauge symmetry at the basis of the construction [76, 77].

## 3.1 $|\theta_{\text{QCD}}|$ in relaxion models

Let us investigate whether a relaxion model might resolve the strong-CP problem [78] by explaining $|\theta_{\text{QCD}}| \lesssim 10^{-10}$ [79]. Prima facie, the expression for $|\theta_{\text{QCD}}|$ is simple see e.g. ref. [80],

$$|\theta_{\text{QCD}}| = |\langle \phi \rangle / f \quad \text{on} \; -\pi \; \text{to} \; \pi|. \tag{3.8}$$

Numerically, however, this cannot be used for calculating $|\theta_{\text{QCD}}|$ — as $\langle \phi \rangle / f$ is substantial, there is a breakdown in numerical precision in expressions such as $\langle \phi \rangle / f \mod 2\pi$. Instead, we find the principal solution for $|\theta_{\text{QCD}}|$,

$$|\theta_{\text{QCD}}| = \arcsin \left| \frac{f \kappa \langle a \rangle}{m_b^3} \left( \frac{m^2/\kappa + \langle h \rangle^2}{\langle h \rangle} \right) \right|, \tag{3.9}$$

by utilising eq. (3.1).

The minimum $|\theta_{\text{QCD}}|$ obtainable occurs at the minimum of the right-hand side of eq. (3.1), such that, if there is a solution at that point, $\sin(\phi/f)$ is as close to zero as possible. Thus we find that

$$\min |\theta_{\text{QCD}}| = \begin{cases} \arcsin \left| \frac{2 \langle a \rangle f m \sqrt{\kappa}}{m_b^3} \right| & \text{if } \langle h \rangle_{\text{minima}} \leq \langle h \rangle_{\text{max}}, \\ \arcsin \left| 1 - \frac{\kappa \langle a \rangle f \pi}{2 \lambda \langle h \rangle_{\text{min}}^2} + \cdots \right| = \pi/2 - \sqrt{2\epsilon} + \cdots & \text{otherwise,} \end{cases} \tag{3.10}$$

where $\langle h \rangle_{\text{minima}} = m/\sqrt{\kappa}$ which minimises the right-hand side of eq. (3.1) and the second line is never less than the first line. The terms represented by the ellipses are higher powers of $\eta$ and $\epsilon$,

$$\langle h \rangle_{\text{max}} \approx \langle h \rangle_{\text{min}}(1 + \epsilon) \quad \text{where} \quad \epsilon \equiv \frac{\kappa \langle a \rangle f \pi}{2 \lambda \langle h \rangle_{\text{min}}^2} \ll 1 \quad \text{and} \quad \eta \equiv \frac{\kappa \langle h \rangle_{\text{min}}^2}{m^2} \ll 1. \tag{3.11}$$

We expand to first order in $\epsilon$ and neglect all powers of $\eta$. Eq. (3.10) originates from considering that the minimum possible $|\theta_{\text{QCD}}|$ would occur at the minimum of the right-hand side of eq. (3.1) if that minima occurred at $\phi/f \approx 2n\pi$ and not $\phi/f \approx (2n+1)\pi$, such that $|\theta_{\text{QCD}}| \approx 0$ and not $|\theta_{\text{QCD}}| \approx \pi$. If that minima occurs, however, outside the interval for the possible solutions for $\langle h \rangle$, it is impossible. In that case, the right-hand side of eq. (3.1) is monotonic inside the interval for the possible solutions for $\langle h \rangle$ and we consider the right-hand side evaluated at $\langle h \rangle_{\text{max}}$ from eq. (3.6). As the general expression is rather complicated, we apply the approximations in eq. (3.11), which are reasonable for phenomenologically viable points. In fact, phenomenologically viable points are always in the second regime in which $\langle h \rangle_{\text{minima}} = m/\sqrt{\kappa} \gg \langle h \rangle_{\text{max}}$, such that $\min |\theta_{\text{QCD}}| \approx \arcsin 1 = \pi/2$. This is confirmed in our numerical analysis.

## 3.2 Finite-temperature effects

In this paper and in the literature so far, the relaxion mechanism was analysed at zero temperature.[2] Finite-temperature effects could, however, non-trivially affect the relaxion potential in eq. (1.1):

- Non-perturbative effects responsible for the induced effective potential of the relaxion are temperature dependent [82]. However, since this affects only the heights of the barriers and not their spacing and since Hubble friction during inflation is typically substantial, it is unclear whether finite-temperature effects would impact the viability of the relaxion mechanism.

- Finite-temperature corrections to the effective potential would alter the shape of the potential (increasing the gradients of the slopes), possibly delaying the onset of EWSB [83]. If EWSB is delayed until a late time (corresponding to a lower temperature after reheating), it could constrain when inflation must start through the requirement that it lasts at least 50 $e$-folds after EWSB. Furthermore, the flat regions of the zero-temperature inflaton potential are strongly modified by finite-temperature effects.

- We find that the reheating temperature in our relaxion models is typically of order $10^{10}$ GeV. At such a high temperature, electroweak symmetry could be easily restored, with the effect of further hindering the viability of the model.

Clearly all the mentioned finite-temperature effects have the potential to impose further constraints on the relaxion model parameter space, to an extent that will be quantified in following projects.

## 3.3 Baryon asymmetry

We observe a significant baryon asymmetry in our Universe. Sakharov [84] demonstrated that generating this asymmetry — baryogenesis — would require a departure from thermal

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[2]We note, however, that ref. [81] considers finite-temperature effects in an alternative relaxion mechanism.

equilibrium, $\mathcal{C}$ and $\mathcal{CP}$ violation, and baryon number violation. In the relaxion paradigm, however, the final 50 or so $e$-folds of inflation occur during or immediately after EWSB and inevitably wash-out any potential net baryon number generated in this process see e.g. ref. [85]. Novel mechanisms that invoke multi-step phase transitions are also ruled out since the fields must be in the final SM vacuum at the end of inflation.[3] Scenarios in which the inflaton itself could generate the required baryon asymmetry see e.g. ref. [86, 87] also appear to be incompatible with the relaxion mechanism because of the further constraints implied by the already heavily constrained dynamics of the inflaton. Finally weak sphalerons are also exponentially VEV suppressed [88] after the electroweak phase transition which means that any subsequent baryogenesis scenario would have to rely on a different source of baryon and lepton number violation. As we shall see, these difficulties would strengthen our conclusions about the viability of inflation in the present framework.

## 4 Description of models

We apply Bayesian model comparison to three models: the SM augmented with single-field scalar-field inflation (SM $+ \sigma$), a QCD relaxion model and a general relaxion model. For other relaxion models, see e.g., ref. [74, 89–96]. Ultimately, we wish to find whether the relaxion mechanism ensures that a relaxion model is favoured by the Bayesian evidence versus the SM. In each model, all scalar-fields receive quadratic corrections to their masses from a cut-off, $\Lambda$, which lies close to the Planck scale.

### 4.1 The Standard Model with scalar-field inflation

The SM Higgs sector is described by two bare Lagrangian parameters — $\mu^2$ and $\lambda$ — in the SM Higgs potential,

$$V_h = \mu^2|h|^2 + \lambda|h|^4, \tag{4.1}$$

and a cut-off at which the bare parameters are specified, $\Lambda$. We augment the SM with mixed inflation, a canonical model of scalar-field inflation see e.g. ref. [97]. Mixed inflation is described by an inflaton mass, $m_\sigma^2$, and quartic coupling, $\lambda_\sigma$, in a potential

$$V_\sigma = \frac{1}{2}m_\sigma^2\sigma^2 + \lambda_\sigma\sigma^4 \tag{4.2}$$

and the number of $e$-folds, $N_{e\text{-fold}}$. We denote this model by SM $+ \sigma$.

Note that in the SM $+ \sigma$ model, the evidence approximately factorises into a factor for the weak scale and a factor for the inflationary observables, $r$, $n_s$ and $A_s$,

$$\begin{aligned}
\mathcal{Z} &= p(M_Z, r, n_s, A_s \,|\, \text{SM} + \sigma) \\
&\approx p(M_Z \,|\, \text{SM} + \sigma) \cdot p(r, n_s, A_s \,|\, \text{SM} + \sigma) \\
&= p(M_Z \,|\, \text{SM}) \cdot p(r, n_s, A_s \,|\, \sigma)
\end{aligned} \tag{4.3}$$

as the measurements are independent and model parameters that affect inflationary observables do not affect the weak scale and vice-versa, with the exception of the cut-off, $\Lambda$, which results in quadratic corrections to the inflaton mass and the Higgs mass.

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[3]One could in principle have a multi-step phase transition that departed from the SM vacuum for baryogenesis and later returned to it but this somewhat undermines the motivation for relaxion models.

### 4.1.1 Calculation of observables

For comparison with measurements from Planck in our statistical analysis, we calculate inflationary observables via the so-called slow-roll parameters [98]:

$$\epsilon(\sigma) \equiv \frac{M_{\mathrm{P}}^2}{16\pi}\left(\frac{V'(\sigma)}{V(\sigma)}\right)^2 \quad \text{and} \quad \eta(\sigma) \equiv \frac{M_{\mathrm{P}}^2}{8\pi}\frac{V''(\sigma)}{V(\sigma)} \tag{4.4}$$

where a prime indicates a derivative with respect to the inflaton field $\sigma$. Inflation finishes once the inflaton field reaches a value $\sigma_f$ such that $\epsilon(\sigma_f) = 1$. The number of $e$-folds desired before inflation ends (and in the case of the relaxion, after EWSB),

$$N_{e\text{-fold}} \simeq \frac{-8\pi}{M_{\mathrm{P}}^2} \int_{\sigma_i}^{\sigma_f} \frac{V(\sigma)}{V'(\sigma)}\,\mathrm{d}\sigma, \tag{4.5}$$

determines the inflaton field at the beginning of inflation, $\sigma_i$. The number of $e$-folds desired should be $N_{e\text{-fold}} \gtrsim 50$. The spectral index, $n_s$, and the ratio of scalar to tensor perturbations, $r$, may be written to first order in the slow-roll parameters as see e.g. ref. [99]

$$n_s = 1 - 6\epsilon(\sigma_i) + 2\eta(\sigma_i) \quad \text{and} \quad r = 4\pi\epsilon(\sigma_i). \tag{4.6}$$

The normalisation of the potential governs the amplitude of scalar perturbations and the Hubble parameter,

$$A_s = \frac{1}{M_{\mathrm{P}}^6}\frac{128\pi}{3}\frac{V(\sigma_i)^3}{|V'(\sigma_i)|^2}, \tag{4.7}$$

$$H = \sqrt{\frac{V(\sigma_i)}{3M_{\mathrm{P}}^2}}, \tag{4.8}$$

but cannot affect $r$ or $n_s$. The normalisation of the scalar perturbations is arbitrary and varies in the literature. For comparison with Planck data, we pick that of the Planck experiment see e.g. ref. [100]. We include a quadratic correction to the inflaton mass — to include a dominant quantum contribution to fine-tuning — but otherwise our formulas are tree-level. We solve for the inflaton field at the beginning and end of inflation, $\sigma_i$ and $\sigma_f$, with numerical methods.

The mass of the $Z$ boson — which represents the weak scale — is calculated in the usual manner,

$$M_Z^2 = \frac{-g^2}{2\lambda}\left(\mu^2 + \beta\Lambda^2\right), \tag{4.9}$$

where $\beta$ is a loop factor. The QCD phase is an input parameter.

### 4.2 Relaxion models

We consider two relaxion models described by the potential in eq. (1.1). In the first model, we do not identify the relaxion with the Peccei-Quinn axion that solves the strong CP problem [78], whereas in the second model, the relaxion is indeed the Peccei-Quinn axion.

For a necessary epoch of low-scale inflation after the relaxion mechanism, we extend the relaxion potential in eq. (1.1) by the most general renormalisable single-field inflaton potential see e.g. ref. [11] with an inflaton field $\sigma$,

$$V = m_3^3\sigma + \frac{1}{2}m_2^2\sigma^2 + \frac{1}{3}m_1\sigma^3 + \frac{1}{4}\lambda_\sigma\sigma^4. \tag{4.10}$$

We suppose that pre-inflation multi-field dynamics dictate that inflation begins at the origin, $\sigma = 0$, as in Raidal et al. [11]. This introduces only four parameters: four couplings in the potential — the desired number of $e$-folds, $N_{e\text{-fold}}$, is not an input parameter. We, furthermore, tune a dressed vacuum energy, $\rho$, such that the cosmological constant vanishes in the vacuum, i.e., $V(\langle\sigma\rangle,\ldots)+\rho = 0$. Thus, low-scale inflation with $H \lll M_P$ is achieved provided $V(\sigma = 0) = \rho \lll M_P^4$. This implies that the potential must be fine-tuned such that $|V(\langle\sigma\rangle)| \lll M_P^4$.

The cosmological constant poses an infamous fine-tuning problem see e.g. ref. [101]. In almost all known models, agreement with measurements of the cosmological constant requires extreme fine-tuning between a bare cosmological constant in the Lagrangian, $\rho_0$, quantum corrections and contributions from spontaneous symmetry breaking i.e., $V(\langle\sigma\rangle,\ldots)$. Because all models that we consider suffer from this fine-tuning problem, fine-tuning penalties from the cosmological constant would approximately cancel in ratios of Bayesian evidences. We ensure that the second epoch of inflation cannot spoil the relaxion mechanism by applying conditions on the Hubble parameter during inflation.

### 4.2.1   Relaxion physicality conditions

There are parameter points for which the back-reaction to EWSB fails to trap the relaxion field in a minimum. If that were the case, the relaxion mechanism would fail and the point would be in severe disagreement with observations. Graham et al. list conditions required for a successful relaxion mechanism [1]:

$$H^2 M_P^2 > \frac{\mu^2 m^2}{\kappa} \qquad\qquad \text{(vacuum energy)} \tag{4.11}$$

$$H < m_b \qquad\qquad \text{(barriers form)} \tag{4.12}$$

$$H^3 < m^2\langle a\rangle \qquad\qquad \text{(classical beats quantum)} \tag{4.13}$$

We assign zero likelihood to a point that violates the resulting condition,

$$\sqrt{\frac{\mu^2 m^2}{\kappa}} < M_P \min(m_b, m^{2/3}\langle a\rangle^{1/3}). \tag{4.14}$$

Graham et al. also list the conditions

$$N_{e\text{-fold}} \gtrsim \frac{H^2}{\kappa\langle a\rangle^2} \qquad\qquad \text{(inflation lasts long enough)} \tag{4.15}$$

$$\kappa\langle a\rangle\mu^2 f \sim m_b^3\langle h\rangle \qquad\qquad \text{(barrier heights)} \tag{4.16}$$

We assume that a first epoch of inflation is provided by the slow-rolling relaxion fields, and cosmological constant later cancelled when the Higgs and relaxion fields acquire VEVs, and

that this epoch provides an acceptable Hubble parameter, as described in ref. [11]. The latter condition is unnecessary as we solve the potential with numerical methods, checking whether a solution exists. To avoid destroying the periodic barriers, the second epoch of inflation must, however, satisfy,

$$H < m_b \tag{4.17}$$

where, as before, $m_b$ is related to the height of the periodic barriers. The barriers result from a phase-transition to a QCD condensate (or a condensate associated with similar non-QCD dynamics). A Hubble parameter $H > m_b$ would reverse that transition, destroying the barriers see e.g. ref. [1].

### 4.2.2   Calculation of (electroweak and QCD) observables

We calculated the VEVs of the Higgs and relaxion fields with numerical methods based on bisecting the interval in eq. (3.2), from which we calculated the mass of the $Z$-boson,

$$M_Z = g\langle h \rangle \tag{4.18}$$

and $|\theta_{\rm QCD}|$ (see section 3.1). In the non-QCD relaxion model, $|\theta_{\rm QCD}|$ is an input parameter. The calculations for the inflationary observables were identical to those in the SM+$\sigma$ model.

## 5   Bayesian analysis

We calculated Bayesian evidences for our SM $+\sigma$ model and relaxion models with (Py)-MultiNest [102–105], which utilises the nested sampling algorithm [106, 107] for Monte-Carlo integration in Bayesian evidences in eq. (2.5) (though delta-functions were first integrated by hand).[4] This requires two ingredients: a likelihood function and a set of priors. Our likelihood function, summarised in table 1, was a product of at most five factors:

- weak-scale: a likelihood function for measurements of the mass of the $Z$-boson [79]. In the SM, this is approximated by a delta-function and integrated by hand. In a relaxion model, this is impossible, as there is no analytic expression for the $Z$-boson mass as a function of the Lagrangian parameters.

- conditions: if a relaxion model (i.e., a point in a relaxion model's parameter space) violates physicality conditions in section 4.2.1, we assign a likelihood of zero, since it would be in stark disagreement with observations.

- decay: a likelihood function for the experimental lower-limit on $f_a$, the axion decay constant, approximated by a step-function see e.g. ref. [108].

- theta: a likelihood function for the experimental upper-limit on $|\theta_{\rm QCD}|$, approximated by a step-function [79].

---

[4]We utilised importance sampling, picked 1000 live points and a stopping criteria of 0.01 in MultiNest.

| Parameter | Measurement | Likelihood function |
|---|---|---|
| `weak-scale` | | |
| $M_Z$ | $91.1876 \pm 0.0021\,\text{GeV}$ [79] | Dirac in SM, Gaussian in relaxion |
| `decay` | | |
| $f_a$ | $f_a \gtrsim 10^9\,\text{GeV}$ [108] | Step-function |
| `theta` | | |
| $|\theta_{\text{QCD}}|$ | $|\theta_{\text{QCD}}| \lesssim 10^{-10}$ [79] | Step-function |
| `inflation` | | |
| $r$ | $r < 0.12$ at $95\%$ [109] | Step-function |
| $n_s$ | $0.9645 \pm 0.0049$ [110] | Gaussian |
| $\ln(10^{10} A_s)$ | $3.094 \pm 0.034$ [110] | Gaussian |

**Table 1.** Likelihoods included in our Bayesian evidences for the scale of electroweak symmetry breaking, the axion decay constant, $|\theta_{\text{QCD}}|$ and BICEP/Planck measurements of inflationary observables. Note that we neglect statistical correlations in Planck measurements of inflationary observables.

- **inflation**: a likelihood for the spectral index, $n_s$, the ratio of scalar to tensor perturbations, $r$, and the amplitude of scalar perturbations, $A_s$, from Planck and BICEP measurements [109, 110]. For simplicity, we neglect correlations amongst Planck measurements and impose an upper-limit for the scalar-to-tensor ratio.

We applied the likelihoods incrementally in five calculations per model: only `weak-scale`; adding `conditions`; adding a lower-bound on the axion decay constant, `decay`; adding an upper bound on $|\theta_{\text{QCD}}|$, `theta`; and finally adding BICEP/Planck measurements in `inflation`. This enabled us to assess the individual impacts of the constraints.

We picked uninformative scale-invariant priors for the dimensionful Lagrangian parameters and cut-off because we are ignorant of their scale, a linear prior for $|\theta_{\text{QCD}}|$, reflecting a shift-symmetry, and a linear prior for $N_{e\text{-fold}}$ because the number of $e$-folds is already a logarithmic quantity. Our prior ranges are summarised in table 2. All massive parameters — $\mu^2$ and $m^2$ and inflaton masses — receive quadratic corrections from a cut-off, such that we expect that without fine-tuning $\mu^2 \sim m^2 \sim \Lambda^2$. The main difference between the priors for our QCD relaxion model and general relaxion model is that in the former, the barrier height is related to the QCD scale, whilst in the latter, it is no greater than about the weak scale. Furthermore, we assigned zero prior weight to corners of parameter space in which a relaxion Lagrangian parameter with dimension mass exceeded the cut-off scale, $\Lambda$, as in an effective field theory such corners may be considered inconsistent or implausible. This eliminated approximately 18% of the prior volume in table 2.

By picking scale-invariant priors for the Lagrangian parameters, we implicitly acknowledge our lack of information about a possible UV theory that might generate them. If there were sufficient reason to believe that a particular mechanism might generate them, imprint-

| Parameter | Prior | |
|---|---|---|
| **SM $+\ \sigma$** | | |
| $\mu^2$ | Log | $10^{-40}$, 1 |
| $\lambda$ | Log | $10^{-4}$, $4\pi$ |
| $\Lambda^2$ | Log | $10^{-4}$, 1 |
| $m_\sigma^2$ | Log | $10^{-40}$, 1 |
| $\lambda_\sigma$ | Log | $10^{-20}$, $4\pi$ |
| $N_{e\text{-fold}}$ | Linear | 50, 500 |
| $|\theta_{\mathrm{QCD}}|$ | Linear | $0, \pi$ |
| **QCD relaxion** | | |
| $\mu^2$ | Log | $10^{-40}$, 1 |
| $\lambda$ | Log | $10^{-4}$, $4\pi$ |
| $\Lambda^2$ | Log | $10^{-4}$, 1 |
| $\kappa$ | Log | $10^{-4}$, $4\pi$ |
| $\langle a \rangle$ | Log | $10^{-50}$, 1 |
| $m_b$ | Log | $10^{-1}\Lambda_{\mathrm{QCD}}$, $10\Lambda_{\mathrm{QCD}}$ |
| $m^2$ | Log | $10^{-40}$, 1 |
| $f$ | Log | $10^{-50}$, 1 |
| $m_1$ | Log | $10^{-100}$, 1 |
| $m_2^2$ | Log | $10^{-100}$, 1 |
| $m_3^3$ | Log | $10^{-100}$, 1 |
| $\lambda_\sigma$ | Log | $10^{-40}$, $4\pi$ |
| **Non-QCD relaxion, as for QCD relaxion except** | | |
| $m_b$ | Log | $10^{-6}\langle h \rangle$, $10^{-1}\langle h \rangle$ |
| $|\theta_{\mathrm{QCD}}|$ | Linear | $0, \pi$ |

**Table 2.** Priors for parameters in SM augmented with scalar-field inflation (SM $+\ \sigma$) and relaxion models. Masses are in Planck units. Furthermore, regions in parameter space in which a dimensionful relaxion Lagrangian parameter exceeded a cut-off were assigned zero prior weight.

ing distinguishing correlations between parameters, it should be reflected in one's choice of priors. In supersymmetric models, for instance, the knowledge of particular breaking mechanisms such as minimal supergravity (mSUGRA) results in relations between Lagrangian parameters at the GUT scale that must be acknowledged in the choice of suitable priors. Likewise, the identification of a UV-completion of the relaxion model could, in principle, ameliorate the fine-tuning of the scenario as the Bayes-factor is a functional of the priors, however, to-date we know of no compelling reason to pick anything other than scale-invariant priors.

## 5.1 Evidences

The evidences and Bayes-factors for the SM $+ \sigma$ and relaxion models are summarised in table 3. We find that, considering only a measurement of the weak scale (i.e., `weak-scale`), relaxion models are favoured by colossal Bayes-factors of about $10^{30}$. This is similar to findings for the constrained minimal supersymmetric SM versus the SM [17], and was expected, as the SM with a Planck-scale cut-off makes an egregious generic prediction for the weak scale.

The physicality conditions (`conditions` in section 4.2.1) dramatically impact the preference for relaxion models. The conditions wipe-out a fraction of the relaxion models' parameter spaces and shrink the Bayes-factors by about $10^{-28}$. The preference for relaxion models versus the SM almost entirely disappears. In other words, despite their success in solving the hierarchy problem, relaxion models are hamstrung by severe fine-tuning associated with their physicality conditions.

The preference for the QCD relaxion model is further damaged by measurements of the axion decay constant, `decay`, and the $|\theta_{\rm QCD}|$, `theta`. The latter results in approximately zero preference for the QCD relaxion model as it predicts that $|\theta_{\rm QCD}| \approx \pi/2$ (see eq. (3.10)). The preference of about $10^2$ for a non-QCD relaxion model versus the SM is unaffected by `decay` and `theta`.

The final data-set of inflationary observables (`inflation`) also impairs the credibility of the considered relaxion models. Low-scale inflation, required in the relaxion paradigm, suffers severe fine-tuning as it requires a light scalar, and thus results in partial Bayes-factors of about $10^{-30}$ for relaxion models versus the SM. Thus, all data considered, the SM $+ \sigma$ model is favoured by a Bayes-factor of at least about $10^{25}$. We note that eq. (4.14) results in an approximate limit of $\mu^2 \sim m^2 \lesssim (10^8\,{\rm GeV})^2$, such that by chance

$$\frac{\mu^2}{\beta M_{\rm P}^2} \frac{m^2}{\beta M_{\rm P}^2} \sim \frac{M_Z^2}{\beta M_{\rm P}^2}. \tag{5.1}$$

The factors are in fact approximately the fractions of parameter space in which a scalar mass is fine-tuned to be so light versus a cut-off, $M_{\rm P}$. Thus tuning two scalar masses — $\mu^2$ and $m^2$ — in a relaxion model to be $\mu^2 \sim m^2 \sim (10^8\,{\rm GeV})^2$ results in a similar fine-tuning penalty as tuning a single scalar mass such that $M_Z \sim 100\,{\rm GeV}$. This, in essence, explains why the evidence for the SM and relaxion models are similar, if one considers only `weak + conditions`. Note that lowering the quadratic corrections by supersymmetrizing the SM and relaxion models see e.g. ref. [94] could favour relaxion models, as from eq. (5.1) a Bayes-factor might scale as the cut-off squared. Lowering the Planck mass, on the other hand, might help slightly less, as it would lower the bounds on scalar masses from eq. (4.14).

To further investigate this issue, we relaxed the Planck-scale cut-off, plotting evidence as a function of the cut-off in the SM and our QCD relaxion model in figure 2. By doing so, we wish to confirm that our QCD relaxion model would be favoured versus the SM, were the cut-off much lower than the Planck scale. We find in figure 2 that, although we previously found that the relaxion model was not favoured versus the SM with a Planck-scale cut-off, if the cut-off were lowered in each model to about $10^8\,{\rm GeV}$, the relaxion
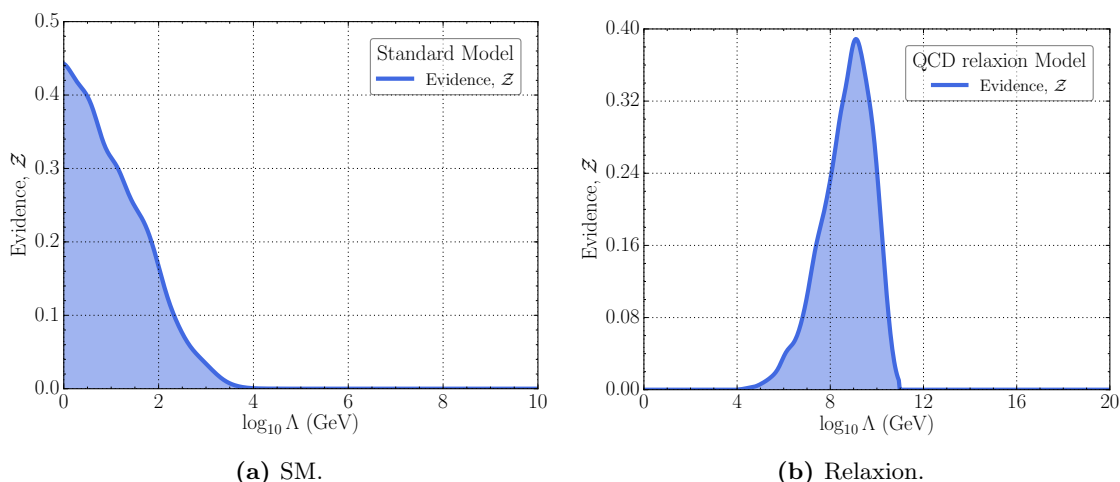
**Figure 2.** The evidence as a function of the cut-off, $\Lambda$, in (a) the SM and (b) a relaxion model. The evidence includes `weak-scale` and `conditions`. This illustrates that a relaxion model could be significantly favoured if the cut-off were lowered from the Planck scale to about $10^8$ GeV e.g., by supersymmetrizing the SM and relaxion model. The evidences are plotted in arbitrary units.

model could be significantly favoured. In other words, the relaxion mechanism may solve the little-hierarchy problem in a supersymmetric model with soft-breaking masses at about $10^8$ GeV, but not the hierarchy problem by itself. Hence, in isolation, our QCD relaxion model cannot improve fine-tuning compared to the SM.

With a cut-off allowed to be as low as 10 TeV, considering `weak-scale`, `conditions` and `decay`, the Bayes-factor favours our QCD relaxion model by $10^6$ versus the SM and about $10^{31}$ versus the SM with Planck-scale quadratic corrections. Including low-scale inflation in e.g., a supersymmetrized relaxion model, however, might necessitate an inflaton mass $m_\sigma \ll M_{\rm SUSY}$. This little-hierarchy problem could scotch the Bayes-factor of $10^6$ in favour of the supersymmetrized relaxion model.

### 5.2 Observables

To illustrate the resolution of the hierarchy problem, in figure 3 we plot the priors for the $Z$-boson mass in the SM and our QCD relaxion model that result from the non-informative priors for Lagrangian parameters in table 2, that is,

$$p(\log M_Z \,|\, M) = \int \delta(\log M_Z - \log M_Z(\boldsymbol{p})) \, p(\boldsymbol{p} \,|\, M) \prod d\boldsymbol{p}. \tag{5.2}$$

This illustrates their generic predictions for the weak scale. This would be numerically equivalent to the Bayesian evidence if our data were $\log M_Z$ and we approximated our measurement with a Dirac function. Whereas the SM favours a weak scale close to the Planck scale, the relaxion model results in considerable probability mass at scales much less than the Planck scale, resolving the hierarchy problem. We find that if the relaxion is the QCD axion, the posterior probability that $|\theta_{\rm QCD}| \lesssim 10^{-10}$ is negligible, confirming our expectations.

| Data-set | weak-scale | +=conditions | +=decay | +=theta | +=inflation |
|---|---|---|---|---|---|
| $\mathcal{Z}\,(\mathrm{SM}+\sigma)\cdot\mathrm{GeV}$ | $10^{-34}$ | | | $10^{-45}$ | $10^{-53}$ |
| $\mathcal{Z}\,(\mathrm{relaxion})\cdot\mathrm{GeV}$ | $10^{-4}$ | $10^{-32}$ | | $10^{-42}$ | $10^{-77}$ |
| $\mathcal{Z}\,(\mathrm{QCD\ relaxion})\cdot\mathrm{GeV}$ | $10^{-4}$ | $10^{-34}$ | $10^{-39}$ | $\lll 10^{-39}$ | $\lll 10^{-81}$ |
| $B(\mathrm{relaxion}/\mathrm{SM}+\sigma)$ | $10^{30}$ | $10^{2}$ | | $10^{2}$ | $10^{-25}$ |
| $B(\mathrm{QCD\ relaxion}/\mathrm{SM}+\sigma)$ | $10^{30}$ | $1$ | $10^{-5}$ | $\lll 10^{5}$ | $\lll 10^{-28}$ |
| $B(\mathrm{QCD\ relaxion}/\mathrm{relaxion})$ | $1$ | $10^{-2}$ | $10^{-7}$ | $\lll 10^{3}$ | $\lll 10^{-3}$ |
| $P(\mathrm{relaxion}/\mathrm{SM}+\sigma)$ | | $10^{-28}$ | | $1$ | $10^{-27}$ |
| $P(\mathrm{QCD\ relaxion}/\mathrm{SM}+\sigma)$ | | $10^{-30}$ | $10^{-6}$ | $\lll 1$ | $10^{-33}$ |
| $P(\mathrm{QCD\ relaxion}/\mathrm{relaxion})$ | | $10^{-2}$ | $10^{-6}$ | $\lll 1$ | $10^{-6}$ |

**Table 3.** Evidences, $\mathcal{Z}$, Bayes-factors, $B$ and partial Bayes-factors, $P$, for the SM augmented with scalar-field inflation ($\mathrm{SM}+\sigma$), a relaxion toy-model and a QCD relaxion toy-model. We apply data incrementally in five data-sets: ($i$) the $Z$-boson mass (`weak-scale`), ($ii$) physicality conditions in relaxion models (`conditions`), ($iii$) constraints on the axion decay constant (`decay`), ($iv$) $|\theta_{\mathrm{QCD}}|$ (`theta`) and ($v$) BICEP/Planck measurements of inflationary observables (`inflation`). A Bayes-factor is a ratio of evidences, indicating the change in relative plausibility of two models in light of all data considered thus far. A partial Bayes-factor is a ratio of Bayes-factors, indicating the change in relative plausibility of two models in light of incrementing the data by a single data-set. A ratio of greater than one indicates that a relaxion toy-model is favoured. We highlight our most important findings in blue: that relaxion toy-models are favoured by about $10^{30}$ by the $Z$-boson mass, but that once all constaints are included, that preference is reversed to about $10^{-25}$ against relaxion toy-models.
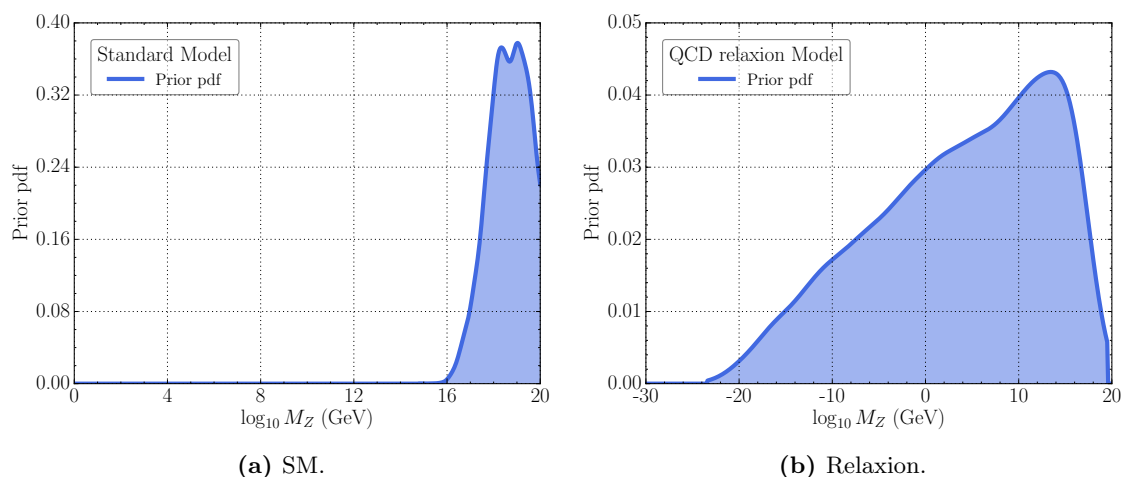


**(a)** SM.  **(b)** Relaxion.

**Figure 3.** Prior distribution of $\log_{10}$ of the $Z$-boson mass in (a) the SM and (b) a QCD relaxion model including no data. The density at the correct weak scale in the relaxion model is much greater than that in the SM. This illustrates that the relaxion mechanism improves fine-tuning of the weak scale with respect to the SM. The densities are plotted in arbitrary units.
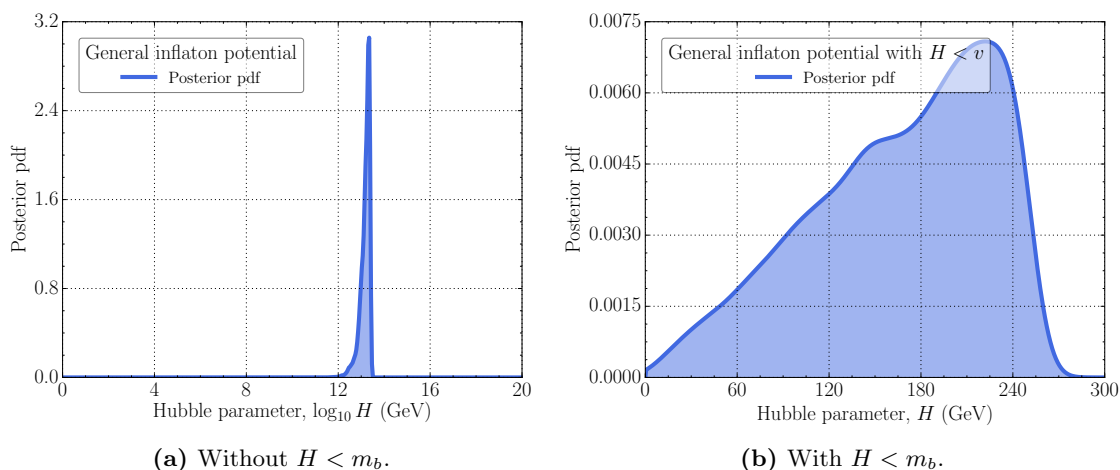
**(a)** Without $H < m_b$.        **(b)** With $H < m_b$.

**Figure 4.** Posterior distribution for the Hubble parameter, $H$, for a general renormalisable inflationary potential (eq. (4.10)). In (a), the constraint that the Hubble parameter is less than the height of the periodic barriers, $H < m_b$ (eq. (4.17)), is not applied; whereas in (b), it is applied. Both plots include Planck/BICEP measurements of $r$, $n_s$ and $A_s$.

To illustrate the fine-tuning required in the second epoch of inflation in a relaxion scenario, in figure 4 we plot the posterior distribution of the Hubble parameter $H$ with and without constraint that the Hubble parameter is less than the height of the periodic barriers, $H < m_b$ (eq. (4.17)), but with Planck/BICEP measurements of $r$, $n_s$ and $A_s$. In the unconstrained case in figure 4a, the posterior shows preference for $H \ggg m_b$, as expected, resulting in a tiny evidence once $H < m_b$ is applied. Consequently, the Bayesfactor significantly favours inflation in the SM + $\sigma$ model.

This can be understood analytically. As discussed in section 4.2, for $H \lll M_{\rm P}$, we must have that $V(\langle\sigma\rangle) \lll M_{\rm P}^4$, where the potential is an order-four polynomial. The mass-squared term is typically Planckian as it received a quadratic correction. The general solution involves roots of a cubic equation. For simplicity, instead consider that $m_3^3 = m_1 = 0$. By dimensional analysis, the result must be that $V(\langle\sigma\rangle) = f(\lambda)m_2^4 \sim M_P^4$. In fact, the result is that $V(\langle\sigma\rangle) = m_2^4/(4\lambda_\sigma)$. Other than by extreme fine-tuning of the mass-squared (which might not agree with Planck/BICEP measurements of $r$, $n_s$ and $A_s$), there is no chance that $H \lll m_b$. On the other hand, consider that all dimensional parameters are equal to the mass-squared term, $|m_1| = |m_2| = |m_3|$. Again, it must be that $V(\langle\sigma\rangle) = f(\lambda)m_2^4 \sim M_P^4$, though in this case the function $f$ is rather complicated. In any case, one must still require fine-tuning such that $V(\langle\sigma\rangle) \sim M_{\rm P}^4$ is avoided.

## 6    Discussion and conclusions

We constructed models that utilised a relaxation mechanism recently proposed by Graham et al. to solve the hierarchy problem. Unfortunately, finding the weak scale in relaxion models involves solving a transcendental equation with numerical methods. We presented

an analytic expression for an interval bounding the weak scale and an analytic expression for a lower bound on $|\theta_{\text{QCD}}|$, confirming that $|\theta_{\text{QCD}}| \approx \pi/2$ if the relaxion is the QCD axion.

We performed the first statistical analysis of a relaxion model by scanning relaxion models' parameter spaces with the nested sampling algorithm, considering data from measurements of the weak scale, the axion decay constant, $|\theta_{\text{QCD}}|$ and BICEP/Planck measurements of inflationary observables $r$, $n_s$ and $A_s$. This resulted in so-called Bayesian evidences for our relaxion models augmented with scalar-field inflation. In a similar manner, we calculated Bayesian evidences for the SM augmented with scalar-field inflation.

We found that the Bayes-factors — ratios of Bayesian evidences that indicate how one ought to update one's relative prior belief in two models — favoured relaxion models versus the SM by a colossal factor of about $10^{30}$ if one considered only the weak scale. Once we included physicality conditions upon inflation during relaxation, however, the Bayes-factors were decimated to about 100 for the non-QCD relaxion model and about 1 for the QCD relaxion model. The physicality condition on the energy density during the relaxion mechanism results, in fact, in a fine-tuning penalty similar to that for the hierarchy problem in the SM, as demonstrated in eq. (5.1). The resulting Bayes-factors are then of order unity.

Constraints upon the QCD relaxion decay constant and $|\theta_{\text{QCD}}|$ severely impair the plausibility of the QCD relaxion model, whereas inflationary observables measured by BICEP/Planck demolish that of the surviving, non-QCD relaxion model. In this regard, we find that the SM augmented with scalar-field inflation is favoured by a Bayes-factor of about $10^{25}$ with respect to the non-QCD relaxion model. This stems from a constraint upon the Hubble parameter during inflation: within the relaxion framework the Hubble parameter must be fine-tuned to $H \lll M_{\text{P}}$, in order to prevent inflation from destroying the periodic barriers in the relaxion potential. In contrast, the polynomial inflationary potentials supported by current observations generically predict that $H \lesssim M_{\text{P}}$, as explained in figure 4 and the accompanying discussion.

Thus, whilst the analysed relaxion models indeed solve the hierarchy problem leading to Bayes-factors of about $10^{30}$ in their favour, the same Bayes-factors are scotched by constraints upon parameters in the relaxion potential and the Hubble parameter during inflation, ultimately leading to a Bayes-factor of about $10^{25}$ in favour of the SM augmented with scalar-field inflation. We anticipate, furthermore, that detailed consideration of baryogenesis and thermal effects (including the disastrous possibility of reheating restoring electroweak symmetry) would further damage the plausibility of relaxion models and conclude that the required unconventional cosmology is the Achilles' heel of the relaxion mechanism.

To conclude, we also remark that our results stand in the absence of a clear UV-completion for the considered relaxion models and that the lack of information regarding the origins of the parameters in the relaxion Lagrangian was encoded in the adopted uninformative priors. If such a UV-completion were available, it would possibly dictate a different choice of priors that could re-establish the plausibility of the relaxion framework.

## Acknowledgments

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