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Holographic RG flow in a new $\mathrm{SO}(3) \times \mathrm{SO}(3)$ sector of ω-deformed $\mathrm{SO}(8)$ gauged $\mathcal{N} = 8$ supergravity

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Abstract: We consider a certain $\mathcal{N} = 1$ supersymmetric, $\mathrm{SO}(3) \times \mathrm{SO}(3)$ invariant, subsector of the ω-deformed family of $\mathrm{SO}(8)$-gauged $\mathcal{N} = 8$ four-dimensional supergravities. The theory contains two scalar fields and two pseudoscalar fields. We look for stationary points of the scalar potential, corresponding to AdS vacua in the theory. One of these, which breaks all supersymmetries but is nonetheless stable, is new. It exists only when $\omega \neq 0$. We construct supersymmetric domain wall solutions in the truncated theory, and we give a detailed analysis of their holographic dual interpretations using the AdS/CFT correspondence. Domain walls where the pseudoscalars vanish were studied previously, but those with non-vanishing pseudoscalars, which we analyse numerically, are new. The pseudoscalars are associated with supersymmetric mass deformations in the CFT duals. When ω is zero, the solutions can be lifted to M-theory, where they approach the Coulomb-branch flows of dielectric M5-branes wrapped on S^3 in the deep IR.

Keywords: Supergravity Models, AdS-CFT Correspondence

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Contents

1 Introduction

For thirty years after its construction in 1982 [1], the four-dimensional SO(8) gauged maximally supersymmetric $\mathcal{N}=8$ supergravity was widely considered to be a unique theory. Interestingly, using the embedding tensor formulation [2], it was recently realized that there exists a family of deformations of the theory, characterised by a single parameter commonly called ω, associated with a mixing of the electric and magnetic vector fields employed in the SO(8) gauging [3, 4]. Inequivalent $\mathcal{N}=8$ theories are parameterised by values of ω in the interval $0 \leq \omega \leq \pi/8$. This development has raised numerous interesting questions, such as its possible higher-dimensional string/M-theory origin and the consequences of the ω deformation for the holographic dual theory.

The potential for the 70 scalar fields in the $\mathcal{N}=8$ theory depends non-trivially on the ω parameter, and the structure of the stationary points, which is already rich in the original undeformed theory, becomes even more involved in the deformed theories. As in the undeformed case, the investigation of the stationary points in the complete theory is extremely complicated, and in order to render the problem tractable, one has to consider consistent truncations in which only subsets of the scalar fields are retained. There have been a number of studies in which truncations of the new ω-deformed maximal supergravity have been performed, typically with the focus being on finding scalar-field truncations in which the scalar potential still has a non-trivial dependence on the parameter ω, leading to a richer structure of anti-de Sitter (AdS) stationary points, with the nature of the vacuum

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states now being dependent on ω. One can also then look for domain-wall solutions that approach the AdS stationary points asymptotically at infinity.

The truncations are achieved by setting to zero all the fields that transform non-trivially under some subgroup of the SO(8) symmetry of the original theory, thus ensuring the consistency of the truncation. Cases that have been studied involve retaining the subset of scalar fields invariant under an SO(7), G_2, SU(3) or SO(3) $\times$ SO(3) subgroup [3, 5–9], or else the seven scalars parameterising the diagonal elements of the $\mathrm{SL}(8,\mathbf{R})/\mathrm{SO}(8)$ coset associated with the 35 self-dual scalars [10]. These various truncations are parallel to the consistent scalar-field truncations performed for the the original de Wit-Nicolai theory [11–19]. Consistent truncations retaining U(1) gauge fields have also been considered, giving rise to an ω-deformed version of the STU supergravity [20], and a one-parameter extension [21] of an Einstein-Maxwell-scalar system [22] previously obtained via a reduction from eleven dimensions on a seven-dimensional Sasaki-Einstein manifold. The latter has been used in a study of holographic condensed matter systems.

In this paper, we consider a new consistent truncation of SO(8) gauged $\mathcal{N} = 8$ supergravity, by keeping the fields invariant under a different SO(3) $\times$ SO(3) subgroup of SO(8), which we denote by $\mathrm{SO}(3)_D \times \mathrm{SO}(3)_R$. This subgroup, and its associated invariant tensors, is defined in appendix A. One way to characterise it is by starting from $\mathrm{SO}(3)_1 \times \mathrm{SO}(3)_2 \times \mathrm{SO}(3)_3 \times \mathrm{SO}(3)_4 \subset \mathrm{SO}(8)$. The factor $\mathrm{SO}(3)_D$ is then the diagonal in $\mathrm{SO}(3)_1 \times \mathrm{SO}(3)_2 \times \mathrm{SO}(3)_3$, and the factor $\mathrm{SO}(3)_R$ is $\mathrm{SO}(3)_4$. The truncated theory preserves $\mathcal{N} = 1$ supersymmetry, and it encompasses the scalar sectors invariant under SO(7), G_2 and SO(4) as special cases.

Within the de Wit-Nicolai theory, this sector does not yield new critical points. However, it does provide two new critical points in the ω-deformed theories. (These are absent in the undeformed theory because the value of the scalar potential at these points goes to infinity in the limit when ω goes to zero.) By construction, the two new critical points preserve $\mathrm{SO}(3)_D \times \mathrm{SO}(3)_R$ global symmetry. Moreover, one of them also preserves $\mathcal{N} = 3$ supersymmetry in the full $\mathcal{N} = 8$ theory [23], while the other one, which had not been found previously, is non-supersymmetric but nonetheless stable.

Via the AdS/CFT correspondence, stable AdS solutions in supergravity theories correspond to local conformal field theories (CFT) living on the boundary of AdS. Two AdS critical points may be connected by a domain-wall solution, which is interpreted as the holographic description of an RG flow from one CFT in the ultra-violet (UV) to another CFT in the infra-red (IR). There are also interesting classes of holographic flows starting from AdS in the UV and flowing to a non-AdS spacetime in the deep IR. In such solutions, the scalar fields flow to infinite values at the IR end of the flow, thus rendering the IR geometry singular. In fact, most of the known domain-wall solutions belong to this class. It seems natural to interpret these solutions as RG flows to non-conformal IR quantum field theories. A proper understanding of the nature of the IR singularities of the geometry and the corresponding QFT in the IR requires embedding the lower-dimensional solution into the UV-complete string or M-theory. From the higher-dimensional perspective, the singularities are physically allowable if they are associated with branes of positive tension. Examples are holographic Coulomb-branch flows, such as those studied in [19, 24, 25]. In some other ex-

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amples, involving brane polarization [26], the singularities are placed at the locus of the dielectric branes [27]. A very recent paper suggests that a singular lower-dimensional solution can lift to a smooth higher-dimensional solution [28]. Properties of the IR QFT are also revealed by the study of the higher-dimensional brane configuration. Depending on the nature of the sources triggering the flow, the structure of the IR theories can take diverse forms.

In the context of the $\mathrm{AdS}_4/\mathrm{CFT}_3$ correspondence and its uplift to M-theory, supersymmetric domain-wall solutions flowing to non-AdS spacetimes in the IR have not been well studied and only very few examples are known. This is a consequence of the complexity of the $\mathcal{N}=8$ supergravity theory, which contains 70 scalar fields . The main purpose of this paper is to explore new domain-wall solutions captured by the consistently-truncated $\mathrm{SO}(3)_R \times \mathrm{SO}(3)_D$-invariant sector of the SO(8) gauged $\mathcal{N}=8$ supergravities. We begin with the study of such supersymmetric domain-wall solutions in the original undeformed de Wit-Nicolai theory, since in this case embedding within M-theory is known, and furthermore the dual CFT is known to be the ABJM theory. A proper interpretation in terms of a UV-complete framework can therefore be achieved. Near the boundary AdS, the leading fall-off coefficients of the scalars and pseudoscalars in the truncated theory are interpreted as the vacuum expectation values (VEVs) of dimension-1 primary operators and the supersymmetric mass terms in the dual CFT respectively. When the pseudoscalars are turned off, the supersymmetric domain-wall solutions were found analytically, describing the Coulomb-branch flows on M2-branes spreading out into six possible distributions in the transverse space. When the pseudoscalars are turned on, the complexity of the flow equations is such that we are only able to obtain the solutions numerically, by integrating the flow equation from the IR to the UV. The solutions we find correspond to flows driven by both the VEV and the mass terms. The competition between the VEV and mass terms leads to a variety of possible IR singularities in the geometry. The physical solutions approach the Coulomb branch flow of dielectric M5-branes wrapping on S^3 in the deep IR.

We then turn to the supersymmetric domain-wall solutions in the ω-deformed theories, within the same truncated scalar sector. We are interested in supersymmetric holographic RG flows, and for these it now turns out that the pseudoscalars are necessarily active. The singular IR behaviors of the solutions are similar to those arising in the $\omega=0$ case. However, since the higher-dimensional origin of the ω-deformed theories is currently unknown, we must necessarily postpone for now any attempt to give a complete interpretation of the ω-deformed supersymmetric domain-wall solutions. In this regard, we note that a recent paper [29] contains a no-go theorem showing that the ω-deformed gauged supergravities cannot be realised via a compactification that is locally described by ten or eleven dimensional supergravity.

The plan of the paper is as follows. In section 2 we discuss the consistent truncation of the ω-deformed $\mathcal{N}=8$ supergravities to the $\mathrm{SO}(3)_D \times \mathrm{SO}(3)_R$ invariant sector, focusing in particular on the four scalar fields, and their scalar potential. We obtain the first-order equations implied by imposing the requirement of $\mathcal{N}=1$ supersymmetry on the domain-wall solutions, and we show how the scalar potential may be written in terms of a superpotential. In section 3 we study the critical points of the scalar potential. These include a variety of critical points that were found previously in truncations with larger

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invariant symmetry groups containing $SO(3)_D \times SO(3)_R$, in addition to the new non-supersymmetric critical point that arises in our truncation when $\omega \neq 0$. In section 4 we discuss in detail the $\mathcal{N}=1$ supersymmetric domain-wall solutions that are asymptotic to the maximally-symmetric $\mathcal{N}=8$ AdS solution in the UV, in the case when $\omega=0$ so that we can lift the solutions to M-theory and thus give a holographic dual interpretation via the ABJM model. We discuss this both for the case of vanishing pseudoscalars, for which the domain-wall solutions had been found analytically in earlier work, and also when the pseudoscalars are non-vanishing, in which case we have to resort to numerical analysis. In section 5 we extend our discussion to the case where the ω deformation parameter is non-zero. Supersymmetric domain walls must now necessarily have non-vanishing pseudoscalar fields, and hence all our discussion in this section is based on the numerical analysis of the solutions. After presenting our conclusions in section 6, we include two appendices in which we give details of the embedding of $SO(3)_D \times SO(3)_R$ in SO(8), and some of the conventions for gamma matrices and uplift formulae that we employ in the paper.

2 Truncated $\mathcal{N}=1$ SUGRA Lagrangian

The scalar potential of the ω-deformed family of SO(8) gauged $\mathcal{N}=8$ supergravities can be described conveniently in the symmetric gauge, where the $E_7/SU(8)$ scalar coset representative is parameterized as

$$\mathcal{V} = \exp\begin{pmatrix} 0 & -\frac{1}{2\sqrt{2}}\phi_{IJKL} \\ -\frac{1}{2\sqrt{2}}\phi^{MNPQ} & 0 \end{pmatrix}. \tag{2.1}$$

Here $\phi^{ijk\ell}$ are complex scalar fields, totally antisymmetric in the rigid SU(8) indices, and obeying the complex self-duality constraint

$$\phi_{IJKL} = \frac{1}{4!}\,\varepsilon_{IJKLMNPQ}\,\phi^{MNPQ}. \tag{2.2}$$

Note that in the symmetric gauge SU(8) and SO(8) indices are identified. Introducing coordinates x^I on $\mathbb{R}^8$ (where I is an $\mathbf{8}_s$ index), the 35 complex scalar fields can be written as

$$\Phi = \frac{1}{4!}\,\phi_{IJKL}dx^I \wedge dx^J \wedge dx^K \wedge dx^L. \tag{2.3}$$

The $SO(3)_D \times SO(3)_R$ invariant subset that we shall be considering in this paper are given by

$$\begin{aligned}
\Psi_1 &= \psi_1 dx^1\wedge dx^2\wedge dx^3\wedge dx^8 + \bar{\psi}_1 dx^4\wedge dx^5\wedge dx^6\wedge dx^7,\\
\Psi_2 &= \psi_2(-dx^1\wedge dx^2\wedge dx^4\wedge dx^7 + dx^1\wedge dx^2\wedge dx^5\wedge dx^6 + dx^1\wedge dx^3\wedge dx^4\wedge dx^6\\
&\quad +dx^1\wedge dx^3\wedge dx^5\wedge dx^7 - dx^2\wedge dx^3\wedge dx^4\wedge dx^5 + dx^2\wedge dx^3\wedge dx^6\wedge dx^7)\\
&\quad +\bar{\psi}_2(dx^1\wedge dx^4\wedge dx^5\wedge dx^8 - dx^1\wedge dx^6\wedge dx^7\wedge dx^8 + dx^2\wedge dx^4\wedge dx^6\wedge dx^8\\
&\quad +dx^2\wedge dx^5\wedge dx^7\wedge dx^8 + dx^3\wedge dx^4\wedge dx^7\wedge dx^8 - dx^3\wedge dx^5\wedge dx^6\wedge dx^8).
\end{aligned} \tag{2.4}$$

(A detailed derivation of the invariant 4-forms can be found in appendix A.) Here Ψ_1 and Ψ_2 parameterise an $\frac{SL(2,R)}{SO(2)} \times \frac{SL(2,R)}{SO(2)}$ coset. Having obtained the form of the scalar 56-bein $\mathcal{V}$ for

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the consistent truncation we are considering, it is a mechanical, if somewhat involved, procedure to substitute it into the expressions given in [4] for the various terms in the Lagrangian of the ω-deformed $\mathcal{N}=8$ gauged supergravity. Introducing four real scalar fields by writing

$$\psi_1 = \frac{1}{2}\phi_1\, e^{\mathrm{i}\sigma_1}, \qquad \psi_2 = \frac{1}{2}\phi_2\, e^{\mathrm{i}\sigma_2}, \tag{2.5}$$

the Einstein and scalar sectors of the $\mathcal{N}=1$ truncation are described by the Lagrangian

$$e^{-1}\mathcal{L} = R - \frac{1}{2}\Big((\partial\phi_1)^2 + \sinh^2\phi_1(\partial\sigma_1)^2\Big) - 3\Big((\partial\phi_2)^2 + \sinh^2\phi_2(\partial\sigma_2)^2\Big) - V. \tag{2.6}$$

The potential V can be written as $V = g^2\widetilde{V}$, where g is the gauge coupling constant and

$$\begin{aligned} 64\widetilde{V} \;=\; & -256\cosh^4\phi_2 + 2\cosh\phi_1\cosh^2\phi_2(-57 - 20\cosh 2\phi_2 + 13\cosh 4\phi_2 + 24\sinh^4\phi_2\cos 4\sigma_2) \\ & +8\sinh\phi_1\sinh^3 2\phi_2\,(\cos(\sigma_1 - 3\sigma_2) + 3\cos(\sigma_1+\sigma_2)) \\ & +4\sinh^3\phi_2\Big\{16(\cos(2\omega - 3\sigma_2) + 3\cos(2\omega+\sigma_2))(1 - \cosh\phi_1\cosh^3\phi_2) \\ & +\sinh\phi_1\sinh^3\phi_2\,(6\sin(2\omega+\sigma_1)\sin 2\sigma_2 - 2\cos(2\omega+\sigma_1 - 6\sigma_2) \\ & -8\sinh\phi_1(3\sinh\phi_2 + \sinh 3\phi_2)\,\cos(2\omega - \sigma_1) \\ & -\frac{3}{2}\sinh\phi_1(17\sinh\phi_2 + 5\sinh 3\phi_2)\,\cos(2\omega+\sigma_1)\cos 2\sigma_2\Big\}. \end{aligned} \tag{2.7}$$

Note that the potential is invariant under the transformations

$$\omega \to \omega + \pi/4, \qquad \sigma_1 \to \sigma_1 - \pi/2, \qquad \sigma_2 \to \sigma_2 + \pi/2 \tag{2.8}$$

$$\omega \to -\omega, \qquad \sigma_1 \to -\sigma_1, \qquad \sigma_2 \to -\sigma_2, \tag{2.9}$$

and so inequivalent theories are characterised by the parameter ω lying in the interval $[0,\pi/8]$.

The potential can be expressed in terms of a superpotential W, with

$$\widetilde{V} = 2\left(4|\frac{\partial W}{\partial\phi_1}|^2 + \frac{2}{3}|\frac{\partial W}{\partial\phi_2}|^2 - 3|W|^2\right), \tag{2.10}$$

and

$$W = -e^{-\mathrm{i}\omega}(1-|\zeta_1|^2)^{-\frac{1}{2}}(1-|\zeta_2|^2)^{-3}\Big[4\zeta_2^3 e^{2\mathrm{i}\omega} - 3\zeta_2^4 - 1 + \zeta_1\zeta_2^6 e^{2\mathrm{i}\omega} + 3\zeta_1\zeta_2^2 e^{2\mathrm{i}\omega} - 4\zeta_1\zeta_2^3\Big], \tag{2.11}$$

where we have defined

$$\zeta_1 = \tanh\frac{1}{2}\phi_1\, e^{-\mathrm{i}\sigma_1}, \qquad \zeta_2 = \tanh\frac{1}{2}\phi_2\, e^{\mathrm{i}\sigma_2}. \tag{2.12}$$

We are interested in $\mathcal{N}=1$ supersymmetric domain-wall solutions, of the form

$$ds^2 = d\rho^2 + e^{2A(\rho)}\eta_{\mu\nu}\,dx^\mu dx^\nu. \tag{2.13}$$

The existence of a Killing spinor requires that the first-order equations

$$\phi_1' - \mathrm{i}\sinh\phi_1\sigma_1' + 4ge^{2\mathrm{i}\alpha}\frac{\partial\overline{W}}{\partial\phi_1} = 0,$$

$$\phi_2' + \mathrm{i}\sinh\phi_2\sigma_2' + \frac{2}{3}ge^{2\mathrm{i}\alpha}\frac{\partial\overline{W}}{\partial\phi_2} = 0,$$
$$A' - g|W| = 0, \quad \partial_\rho|\epsilon| - \frac{1}{2}g|W| = 0 \tag{2.14}$$

should be satisfied, where a prime denotes a derivative with respect to ρ and we use the notation $W = |W|\, e^{2\mathrm{i}\alpha}$.

The solutions of these first-order equations also obey the second-order equations of motion that follow from the Lagrangian (2.6). This can be seen easily as follows. The action (including the Gibbons-Hawking term) evaluated on the domain-wall ansatz is given by

$$\mathcal{S} = \int d\rho\, e^{3A}\Big[3(A' - g|W|)^2 - \frac{1}{2}|\phi_1' - \mathrm{i}\sinh\phi_1\sigma_1' + 4g\frac{\partial\bar{W}}{\partial\phi_1}e^{2\mathrm{i}\alpha}|^2$$
$$-3|\phi_2' + \mathrm{i}\sinh\phi_2\sigma_2' + \frac{2}{3}g\frac{\partial\bar{W}}{\partial\phi_2}e^{2\mathrm{i}\alpha}|^2\Big] - \Big[2g\, e^{3A}\,|W|\Big]_{-\infty}^{\infty}, \tag{2.15}$$

and this is clearly extremised by the solutions of (2.14). (Here we adopt the same notation as [13].) Using $\gamma_2\,\epsilon_8 = (\epsilon^8)^*$, and

$$\partial_{\sigma_1}|W| = |W|\sinh\phi_1\partial_{\phi_1}\arg W, \qquad \partial_{\sigma_2}|W| = -|W|\sinh\phi_2\partial_{\phi_2}\arg W, \tag{2.16}$$

the solutions to the Killing spinor equation are given by

$$\epsilon_8 = e^{\frac{1}{2}A(\rho)+\mathrm{i}\alpha}\begin{pmatrix}\varepsilon_1\\ \varepsilon_2\\ 0\\ 0\end{pmatrix}, \tag{2.17}$$

where ε_1 and ε_1 are two real constants. Utilizing (2.16), eqs. (2.14) can be rewritten as

$$\phi^{I\prime} = -2g\,\mathcal{K}^{IJ}\,\frac{\partial|W|}{\partial\phi^J}, \qquad A' = g\,|W|, \tag{2.18}$$

where ϕ^I denotes all four real scalars, and $\mathcal{K}^{IJ}$ is the inverse metric for the kinetic terms in the scalar coset. Eq. (2.18) implies that the BPS equations in the bulk describe a gradient flow in the scalar coset manifold, with $-g|W|$ being the "potential" whose gradient drives the flow. When the solutions to (2.18) are asymptotically-AdS domain walls and therefore correspond to an RG flow in the dual CFT, eq. (2.18) also implies a holographic strong a-theorem[1]

$$A'' = -2g^2\,\frac{\partial|W|}{\partial\phi^I}\,\mathcal{K}^{IJ}\,\frac{\partial|W|}{\partial\phi^J} \leq 0. \tag{2.19}$$

[1]Recently, gradient flows and a strong a-theorem were studied in three dimensions [30], demonstrating the existence of a candidate a-function for renormalisable Chern-Simons matter theories at two-loop order. The monotonic behavior of the a-function along renormalisation group flows is related to the β-function via a gradient flow equation involving a positive-definite metric similar to that in our holographic discussion. The function A defined in [30] is equivalent to our $|W|$ here.

3 Critical points of the scalar potential

The symmetry group $\mathrm{SO}(3)_D \times \mathrm{SO}(3)_R$ can be embedded in SO(8) through the chain

$$\mathrm{SO}(3)_D \times \mathrm{SO}(3)_R \subset G_2 \subset \mathrm{SO}(7) \subset \mathrm{SO}(8). \tag{3.1}$$

On the other hand the SU(3) invariant sector of $\mathcal{N} = 8$ supergravity has been thoroughly studied both in the original de Wit and Nicolai theory [11, 12, 18] and in the ω-deformed case [6], with the group embedding

$$\mathrm{SU}(3) \subset G_2 \subset \mathrm{SO}(7) \subset \mathrm{SO}(8). \tag{3.2}$$

Using the Newton-Raphson method, with the potential (2.7), we scanned for its critical points. We found all the previously-known critical points with G_2 or SO(7) symmetry, and also two critical points with $\mathrm{SO}(3)_D{\times}\mathrm{SO}(3)_R$ symmetry. One of them, preserving $\mathcal{N} = 3$ supersymmetry, was first discovered in [23]. For this critical point, the dependence of the two complex scalars, and the associated cosmological constant, on ω are displayed in figure 1. This was the first example of an $\mathcal{N} = 3$ supersymmetric vacuum in SO(8) gauged $\mathcal{N} = 8$ supergravity. The mass spectrum of the fluctuations around this vacuum is given by [23]:

$$\begin{aligned} m^2 L_0^2 :\quad & 1 \times (3(1+\sqrt{3})); \quad 6 \times (1+\sqrt{3}); \quad 1 \times (3(1-\sqrt{3})); \quad 6 \times (1-\sqrt{3}); \\ & 4 \times \left(-\frac{9}{4}\right); \quad 18 \times (-2); \quad 12 \times \left(-\frac{5}{4}\right); \quad 22 \times 0. \end{aligned} \tag{3.3}$$

(The integer to the left of the multiplication sign indicates the degeneracy of the mass eigenvalue, while the number to the right indicates the corresponding mass-squared.) Owing to the supersymmetry, the Breitenlohner-Freedman bound $m^2\, L_0^2 \geq -\frac{9}{4}$ [31] is necessarily respected.

The other $\mathrm{SO}(3)_D \times \mathrm{SO}(3)_R$ critical point that we found is non-supersymmetric. It is, however, stable against fluctuations. The mass spectrum of the perturbations around this vacuum depends on the value of ω. For example, for $\omega = \pi/8$ the spectrum is given by

$$\begin{aligned} m^2 L_0^2 :\quad & 1 \times (6.72079); \quad 1 \times (5.29013); \quad 4 \times (-1.96647); \quad 9 \times (-1.73861); \\ & 9 \times (-1.60284); \quad 1 \times (-1.59124); \quad 8 \times (-1.18046); \quad 5 \times (-0.98076); \\ & 4 \times (-0.73134); \quad 5 \times (0.61746); \quad 1 \times (0.58185); \quad 22 \times 0. \end{aligned} \tag{3.4}$$

Note that even though it is not supersymmetric, the Breitenlohner-Freedman bound is not violated. For this critical point, we show in the left-hand plot of figure 2 the evolution of the values of the scalars as ω varies from 0 to $\pi/4$. The right-hand plot shows the dependence of the cosmological constant on ω. It should be noted that for each of the $\mathrm{SO}(3)_D \times \mathrm{SO}(3)_R$ vacua, the critical point disappears when ω goes to zero, since the value of the cosmological constant then diverges in each case. For completeness, in table 1, we list all the critical points contained in the $\mathrm{SO}(3)_D \times \mathrm{SO}(3)_R$ invariant sector.

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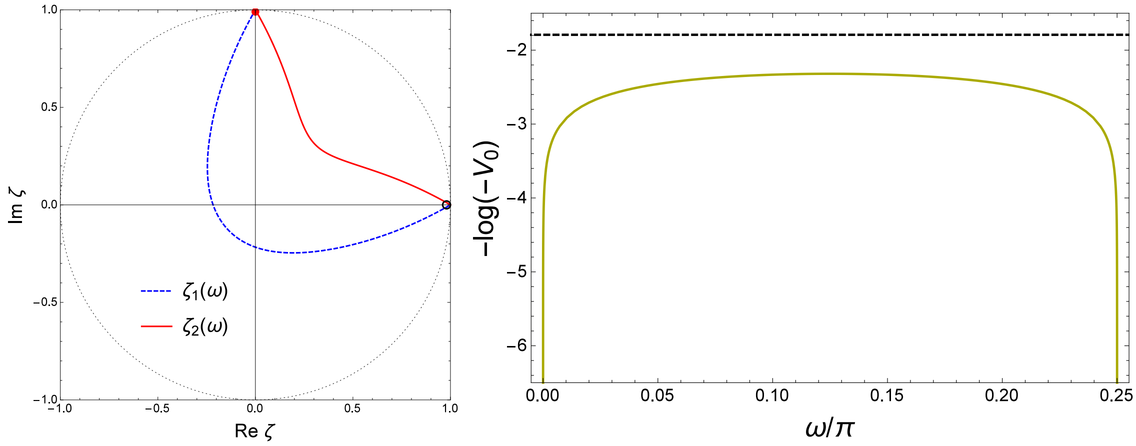

Figure 1. ω dependence of $\mathcal{N} = 3$ $\mathrm{SO}(3)_D \times \mathrm{SO}(3)_R$ critical point. The black circle (on the right of the plot) is $\omega = 0$; the red dot (at the top) is $\omega = \pi/4$. The dashed line in the right-hand plot corresponds to $V_0 = -6$.

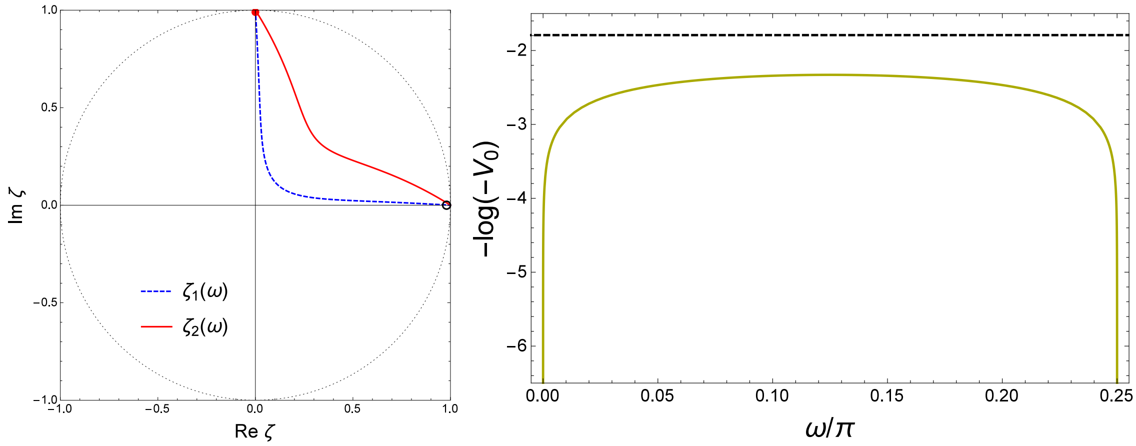

Figure 2. ω dependence of the $\mathcal{N} = 0$ $\mathrm{SO}(3)_D \times \mathrm{SO}(3)_R$ critical point. The black circle (on the right of the plot) is $\omega = 0$; the red dot (at the top) is $\omega = \pi/4$. The dashed line in the right-hand plot corresponds to $V_0 = -6$.

Three remarks are in order:

a) The two transformations (2.8) and (2.9) combine into a symmetry in the case that $\omega = \pi/8$:

$$\sigma_1 \to -\sigma_1 - \pi/2, \quad \sigma_2 \to -\sigma_2 + \pi/2. \tag{3.5}$$

This can be seen in the table.

b) Points related by

$$\phi_i \to -\phi_i, \quad \sigma_i \to \sigma_i + \pi$$

have the same location in the complex plane, and hence are equivalent.

symmetry	ϕ_1	σ_1	ϕ_2	σ_2	$V(g=1)$	stability
$\mathcal{N}=8$						
SO(8)	0	–	0	–	-6	$\surd$
$\mathcal{N}=0$						
$\mathrm{SO}(7)_-$	0.4195	$\frac{\pi}{2}$	0.4195	$-\frac{\pi}{2}$	-6.7482	$\times$
$\mathcal{N}=0$						
$\mathrm{SO}(7)_-$	0.6406	$-\frac{\pi}{2}$	0.6406	$\frac{\pi}{2}$	-7.7705	$\times$
$\mathcal{N}=0$						
$\mathrm{SO}(7)_+$	0.4195	π	0.4195	π	-6.7482	$\times$
$^*\mathcal{N}=0$						
$\mathrm{SO}(7)_+$	0.6406	0	0.6406	0	-7.7705	$\times$
$\mathcal{N}=1$						
G_2	0.4840	$\frac{3\pi}{4}$	0.4195	$\frac{3\pi}{4}$	-7.0397	$\surd$
$\mathcal{N}=1$						
G_2	0.6579	-1.9693	0.6579	1.9693	-7.9430	$\surd$
$^*\mathcal{N}=1$						
G_2	0.6579	0.3985	0.6579	-0.3985	-7.9430	$\surd$
$^*\mathcal{N}=0$						
G_2	$\log\vartheta$	$-\frac{\pi}{4}$	$\log\vartheta$	$\frac{\pi}{4}$	-4ϑ	$\surd$
$^*\mathcal{N}=3$						
$\mathrm{SO}(3)_D\times\mathrm{SO}(3)_R$	$\log\vartheta/\sqrt{3}$	$\frac{3\pi}{4}$	$\log\vartheta$	$\frac{\pi}{4}$	-4ϑ	$\surd$
$^*\mathcal{N}=0$						
$\mathrm{SO}(3)_D\times\mathrm{SO}(3)_R$	0.3114	$-\frac{\pi}{4}$	0.9914	$\frac{\pi}{4}$	-10.271	$\surd$

Table 1. Critical points in the $\omega=\pi/8$ theory. We use ϑ to denote the number $\sqrt{3+2\sqrt{3}}$. The mass spectra of fluctuations around the critical points are independent of ω, except for the last one. Points marked with "*" disappear when $\omega\to 0$, while the two points with $\mathrm{SO}(7)_-$ symmetry become degenerate in energy.

c) It is interesting to see that there are two critical points with the same cosmological constant, but with different residual symmetry; one with G_2 and the other with $\mathrm{SO}(3)_D\times\mathrm{SO}(3)_R$ invariance.

4 Holographic $\mathcal{N}=1$ RG flows on M2-branes

In this section, we study the domain-wall solutions to eq. (2.14). In particular, we are interested in solutions approaching the trivial $\mathcal{N}=8$ AdS vacuum in the UV. We shall first restrict our discussion to the $\omega=0$ case, in which the supergravity is just the original de Wit-Nicolai theory. The boundary CFT corresponding to the trivial $\mathcal{N}=8$ vacuum is the ABJM theory [32] with Chern-Simons level $k=1$ or $k=2$, for which the supersymmetry

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is enhanced from $\mathcal{N} = 6$ to $\mathcal{N} = 8$ [33–35]. Therefore, our domain-wall solutions describe the RG flows on M2-branes driven by $\mathcal{N} = 1$ deformations.

Owing to the fact that SO(8) is manifest in the gravity theory but is not manifest in the ABJM theory under the large N limit, it is not straightforward to map the bulk scalars to the boundary primary operators. However, we recall that the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ ABJM theory is equivalent to the BLG [36–39] theory, which *is* manifestly SO(8) invariant. Thus, to infer the form of the primary operators dual to the $\mathrm{SO}(3)_D \times \mathrm{SO}(3)_R$ invariant bulk scalar fields, we make the reasonable assumption that the structures of the primary operators characterised by representations of the R-symmetry group do not depend explicitly on the rank of the gauge group. Based on this assumption, we identify the dual primary operators in the large-N ABJM theory by first mapping the bulk scalars to primary operators in the BLG theory, and then recasting these operators in terms of the fields in the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ ABJM theory.

We divide the 70 bulk scalars into real (self-dual) and imaginary (anti-self dual) parts, by writing

$$\phi_{ijkl} = S_{ijkl} + \mathrm{i}P_{ijkl}. \tag{4.1}$$

The 70 scalars can be mapped into two 35-dimensional symmetric traceless tensors as follows:

$$\mathcal{O}^{(1)}_{IJ} = (\Gamma^{ijkl})_{IJ}\, S_{ijkl}, \qquad \mathcal{O}^{(2)}_{\alpha\beta} = (\Gamma^{ijkl})_{\alpha\beta}\, P_{ijkl}, \tag{4.2}$$

where (i, I, α) label the $(\mathbf{8}_c, \mathbf{8}_v, \mathbf{8}_s)$ representations of SO(8) respectively. The SO(8) gamma matrices are expressible in terms of triality rotation matrices. In our conventions, the SO(8) gamma matrices take the form

$$\Gamma^i = \begin{pmatrix} 0 & \hat{\Gamma}^i_{I\alpha} \\ (\hat{\Gamma}^i)^{\mathrm{T}}_{\alpha I} & 0 \end{pmatrix}. \tag{4.3}$$

The details of the SO(8) gamma matrices can be found in appendix B.

From now on, for simplicity of discussion, we shall without loss of generality choose the gauge coupling constant to be $g = 1$. Consequently, the trivial $\mathcal{N} = 8$ vacuum is the standard AdS spacetime with unit radius, and the perturbations of the 70 scalars around this vacuum have mass squared $m^2 = -2$. In terms of the coordinate $z = e^{-\rho}$, the boundary of AdS is defined at $z \to 0$. Near the boundary of the $\mathcal{N} = 8$ supersymmetric AdS vacuum, the scalars ϕ_{ijkl} behave as $\phi_{ijkl} \sim z\, \phi^{(1)}_{ijkl} + z^2\, \phi^{(2)}_{ijkl}$ in which the two modes are both renormalizable [40]. Boundary conditions preserving $\mathcal{N} = 8$ supersymmetry require [41, 42]

$$S_{ijkl} \sim z\, S^{(1)}_{ijkl}, \qquad P_{ijkl} \sim z^2\, P^{(2)}_{ijkl}. \tag{4.4}$$

This set of boundary conditions amounts, in the dual holographic picture, to $S^{(1)}_{ijkl}$ and $P^{(2)}_{ijkl}$ being the VEVs of 35 dimension-1 scalar operators and 35 dimension-2 pseudoscalar operators in the dual SCFT respectively. In terms of the fields in the BLG theory, these operators take the form

$$\mathbf{35}_v : \mathrm{Tr}(\phi_I\phi_J) - \frac{1}{8}\delta_{IJ}\mathrm{Tr}(\phi_K\phi_K), \qquad \mathbf{35}_s : \mathrm{Tr}(\psi_\alpha\psi_\beta) - \frac{1}{8}\delta_{\alpha\beta}\mathrm{Tr}(\psi_\lambda\psi_\lambda). \tag{4.5}$$

The expansion coefficients $S_{ijkl}^{(2)}$ and $P_{ijkl}^{(1)}$ then correspond to the sources for these operators, according to the standard AdS/CFT dictionary. However, we shall see below that besides the source terms, the coefficients $S_{ijkl}^{(2)}$ can depend on terms quadratic in the VEVs of the $\mathbf{35}_v$ operators. This phenomenon has been seen previously, in the study of a continuous distribution of branes (see [43] for a review). As far as we are aware, there is no clear way to related these terms to the VEVs of higher-dimension operators. Also, we emphasize that when the dual SCFT is deformed by some operators, the operators in (4.5) will receive corrections.

We consider the domain-wall solutions asymptotic to the trivial $\mathcal{N}=8$ vacuum in the UV at $z=0$. Denoting $\zeta_1(z)=S(z)+\mathrm{i}P(z)$ and $\zeta_2(z)=\widetilde{S}(z)+\mathrm{i}\widetilde{P}(z)$, we find that near the UV boundary, the solutions to eq. (2.14) take the form

$$\begin{aligned} S(z) &= \alpha_1\, z + \alpha_2\, z^2 + \ldots, \\ P(z) &= \beta_1\, z + \beta_2\, z^2 + \ldots, \\ \widetilde{S}(z) &= \widetilde{\alpha}_1\, z + \widetilde{\alpha}_2\, z^2 + \ldots, \\ \widetilde{P}(z) &= \widetilde{\beta}_1\, z + \widetilde{\beta}_2\, z^2 + \ldots, \end{aligned} \tag{4.6}$$

where

$$\alpha_2 = -3\widetilde{\alpha}_1^2 + 3\widetilde{\beta}_1^2, \qquad \widetilde{\alpha}_2 = -\alpha_1\widetilde{\alpha}_1 - 2\widetilde{\alpha}_1^2 + \beta_1\widetilde{\beta}_1 + 2\widetilde{\beta}_1^2, \tag{4.7}$$

and

$$\beta_2 = 6\widetilde{\alpha}_1\widetilde{\beta}_1, \qquad \widetilde{\beta}_2 = \widetilde{\alpha}_1\beta_1 + \alpha_1\widetilde{\beta}_1 + 4\widetilde{\alpha}_1\widetilde{\beta}_1. \tag{4.8}$$

(Recall that ζ_1 and ζ_2 parameterise the 4-form (2.4).) Using (4.2), the 4-form is mapped to

$$\begin{aligned} \mathcal{O}_{IJ}^{(1)} &= S\,\mathrm{diag}(-1,-1,-1,-1,1,1,1,1) + \widetilde{S}\,\mathrm{diag}(0,0,0,0,-2,-2,-2,6), \\ \mathcal{O}_{\alpha\beta}^{(2)} &= P\,\mathrm{diag}(-1,-1,-1,-1,1,1,1,1) + \widetilde{P}\,\mathrm{diag}(0,0,0,0,-2,-2,-2,6). \end{aligned} \tag{4.9}$$

In general, $\mathcal{O}^{(1)}$ and $\mathcal{O}^{(2)}$ will differ from each other by a similarity transformation generated by $\Gamma^8_{I\alpha}$, which is the identity matrix in our conventions. The expressions (4.9) suggest that the $\mathbf{35}_v$ operators receive VEVs proportional to $\mathcal{O}^{(1)}$, and that also a fermion mass term of the form $\mathcal{O}^{(2)}$ is turned on in the dual SCFT. The supersymmetric completion of the fermion mass term must include a bosonic mass term whose coefficient should be quadratic in the fermionic mass parameter. This implies that the quadratic β terms in (4.7) are related to the boson mass parameter, since β_1 and β_2 correspond to the fermion mass parameters. We can check the relation between the boson and fermion mass parameters explicitly, by utilizing the $\mathcal{N}=1$ formulation of the BLG theory [44]. As mentioned previously, the gauge group of the BLG theory is $\mathrm{SU}(2)\times\mathrm{SU}(2)$, and therefore one cannot take a large-N limit of the theory with N being the rank of the gauge group. Nonetheless, we shall see that a naive application of the BLG theory gives rise to a result matching with the gravity dual.

The $\mathcal{N}=1$ formulation of the BLG theory is expressed in terms of the superfield

$$\Phi_I = \phi_I + \theta\,\hat{\Gamma}^8_{I\alpha}\,\psi^\alpha - \theta^2\,F_I, \tag{4.10}$$

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where the gauge group indices and the $\mathrm{SO}(1,2)$ spinor indices are suppressed. In order to have the fermion mass term of the form $\mathcal{O}^{(2)}$ in (4.9), the BLG action must be deformed by

$$\Delta\mathcal{L}_{\mathrm{BLG}} = \mathcal{O}^{(2)}_{IJ}\,\mathrm{Tr}(\Phi_I\Phi_J), \tag{4.11}$$

since in our conventions $\hat{\Gamma}^8_{I\alpha} = \delta_{I\alpha}$. In the component language, this deformation contains terms of the form $\mathcal{O}^{(2)}_{IJ}\,\mathrm{Tr}(\phi_I F_J)$. The supersymmetric kinetic term contains a contribution $\frac{1}{2}\mathrm{Tr}(F_I F_I)$ quadratic in the auxiliary field F_I. Integrating out F_I generates a positive-definite mass term $\frac{1}{2}(\mathcal{O}^{(2)})^2_{IJ}\,\phi_I\phi_J$ for ϕ_I. Decomposing the mass matrix into its traceless part and its trace, we find that the traceless part is given by

$$\frac{1}{2}(\mathcal{O}^{(2)})^2_{\{IJ\}} = 3\widetilde{\beta}_1^2\,\mathrm{diag}(-1,-1,-1,-1,1,1,1,1) + (\beta_1\widetilde{\beta}_1 + 2\widetilde{\beta}_1^2)\,\mathrm{diag}(0,0,0,0,-2,-2,-2,6), \tag{4.12}$$

whilst the trace part takes the form

$$\frac{1}{2}(\mathcal{O}^{(2)})^2_{IJ}\,\delta^{IJ} = \frac{1}{2}(\beta_1^2 + 6\widetilde{\beta}_1^2). \tag{4.13}$$

From eq. (4.12), we see the familiar 8×8 matrices associated with the $\mathrm{SO}(3)_{\mathrm{D}}\times\mathrm{SO}(3)_{\mathrm{R}}$ invariant scalars. Miraculously, the coefficients in front of these matrices are exactly the same as the quadratic β terms in the sub-leading expansion coefficients of the self-dual scalars S and $\widetilde{S}$. In the standard AdS/CFT dictionary, the trace part of the boson mass matrix is not captured by supergravity fields. The associated operator $\mathrm{Tr}(\phi_I\phi_I)$ belongs to the Konishi multiplet which is dual to the shortest stringy mode. However, as we have seen, the non-chiral operator $\mathrm{Tr}(\phi_I\phi_I)$ appears in the $\mathcal{N}=1$ mass terms. In fact, there is no contradiction. As pointed out in [45], when the CFT is deformed the form of the chiral operators changes, and they mix with other operators. In the case of the supersymmetric mass deformation, the chiral operator (4.5) mixes with the non-chiral operator $\mathrm{Tr}(\phi_I\phi_I)$, giving the scalars a positive-definite mass.

In the following, we briefly discuss how to reformulate the $\mathbf{35}_v$ and $\mathbf{35}_s$ operators in the framework of the ABJM theory. The global symmetry of the ABJM theory is $\mathrm{SU}(4)\times\mathrm{U}(1)_{\mathrm{b}}$, rather than $\mathrm{SO}(8)$. Under this $\mathrm{SU}(4)\times\mathrm{U}(1)_{\mathrm{b}}$ subgroup, the $\mathbf{8}_c$ of $\mathrm{SO}(8)$ branches into

$$\mathbf{8}_c \to \mathbf{6}_0 + \mathbf{1}_2 + \mathbf{1}_{-2}, \tag{4.14}$$

which implies

$$\begin{aligned} &\mathbf{8}_v \to \mathbf{4}_1 + \bar{\mathbf{4}}_{-1}, && \mathbf{8}_s \to \mathbf{4}_1 + \bar{\mathbf{4}}_{-1},\\ &\mathbf{35}_v \to \mathbf{10}_2 + \overline{\mathbf{10}}_{-2} + \mathbf{15}_0, && \mathbf{35}_s \to \mathbf{10}_{-2} + \overline{\mathbf{10}}_2 + \mathbf{15}_0. \end{aligned} \tag{4.15}$$

Group theoretically, (4.15) means that the $\mathbf{35}_v$ operators in (4.5) should be replaced by three sets of operators in the ABJM theory, namely

$$\begin{aligned} \mathbf{15}_0 :&\quad \mathrm{Tr}\left(Z^\dagger_A Z^B - \frac{1}{4}\delta^B_A Z^\dagger_C Z^C\right),\\ \mathbf{10}_2 :&\quad \mathrm{Tr}\left(Z^A Z^B \mathcal{M}^{-2}\right), \end{aligned}$$

$$\mathbf{10}_{-2}: \qquad \mathrm{Tr}\left(Z_A^\dagger Z_B^\dagger \mathcal{M}^2\right), \tag{4.16}$$

where A and B label the $\mathbf{4}$ of SU(4), while $\mathcal{M}^{-2}$ and $\mathcal{M}^2$ are the monopole operators in the proper representations of the gauge group needed for gauge invariance of the operators [46]. When the gauge group is SU(2) $\times$ SU(2), the bifundamental scalars of the ABJM theory are related to the original BLG variables with SO(4) indices through

$$Z^A = \phi^A + \mathrm{i}\phi^{A+4}, \qquad \phi^I = \frac{1}{2}(\phi_4^I \mathbb{1} + \mathrm{i}\phi_i^I \sigma^i), \tag{4.17}$$

where the SO(8) $\mathbf{8}_v$ indices are raised using δ^{IJ}. Similarly, one can reformulate the $\mathbf{35}_s$ operators of SO(8) in the framework of ABJM.

4.1 Uplift to eleven dimensions

The $\mathbf{35}_v$ dilatons parameterise the coset SL$(8,\mathbf{R})$/SO(8), and one can use the local SO(8) symmetry to diagonalise the coset, so that the scalar Lagrangian can be written in terms of seven dilatons $\vec{\varphi}$

$$\mathcal{L} = R - \frac{1}{2}(\partial\vec{\varphi})^2 - V, \tag{4.18}$$

where

$$V = -\frac{1}{8}g^2\left(\left(\sum_{i=1}^{8} X_i\right)^2 - 2\sum_{i=1}^{8} X_i^2\right). \tag{4.19}$$

The eight X_i, subject to the constraint

$$\prod_{i=1}^{8} X_i = 1, \tag{4.20}$$

are parameterized by the seven dilatons as

$$X_i = e^{-\frac{1}{2}\vec{b}_i\cdot\vec{\varphi}}, \tag{4.21}$$

where $\vec{b}_i$ are the weight vectors of the fundamental representation of SL$(8,\mathbf{R})$, satisfying

$$\vec{b}_i\cdot\vec{b}_j = 8\delta_{ij} - 1, \qquad \sum_i \vec{b}_i = 0, \qquad \sum_i (\vec{u}\cdot\vec{b}_i)\,\vec{b}_i = 8\vec{u}, \tag{4.22}$$

and $\vec{u}$ is an arbitrary vector. Setting

$$X_1 = X_2 = X_3 = X_4 = e^{\frac{1}{2}\phi_1}, \quad X_5 = X_6 = X_7 = e^{-\frac{1}{2}(\phi_1 - 2\phi_2)}, \quad X_8 = e^{-\frac{1}{2}(\phi_1 + 6\phi_2)}, \tag{4.23}$$

the Lagrangian (4.18) becomes equivalent to our Lagrangian (2.6) in the case that $\sigma_1 = \sigma_2 = 0$.

In [19], a set of domain-wall solutions was found, given by

$$ds^2 = \left(\frac{1}{2}gr\right)^4\left(\prod_i H_i\right)^{1/4} dx^\mu dx_\mu + \left(\prod_i H_i\right)^{-1/4}\frac{4dr^2}{g^2r^2},$$

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$$X_i = H_i^{-1}\Big(\prod_{j=1}^{8} H_j\Big)^{1/8}, \tag{4.24}$$

where

$$H_i = 1 + \frac{\ell_i^2}{r^2}.$$

The transformation

$$r^2 \to r^2 + \eta, \quad \ell_i^2 \to \ell_i^2 - \eta, \tag{4.25}$$

is a diffeomorphism, and so the inequivalent solutions are parameterized by only seven, rather than eight, of the constants ℓ_i^2. In particular, η can be chosen so that the smallest of the ℓ_i^2 is set to zero, while keeping the remaining ones non-negative.

The domain-wall solutions are singular in the IR, and the nature of the singularity depends on the number of ℓ_i^2 that can be set to zero by means of the shift symmetry (4.25). In terms of the new coordinates, where the smallest of the ℓ_i^2 have been shifted to zero, the singular IR behavior of the metric is given by

$$ds_4^2 \simeq R^{\frac{8-k}{2}} dx^\mu dx_\mu + R^{\frac{k-4}{2}} dR^2 = \rho^{\frac{2(8-k)}{k}} dx^\mu dx_\mu + d\rho^2, \tag{4.26}$$

where k is the number of ℓ_i^2 that are set to zero by the shift. To compare with (4.6), we expand $S \equiv \tanh \frac{1}{2}\phi_1$ and $\widetilde{S} \equiv \tanh \frac{1}{2}\phi_2$ in terms of z, which is related to r by

$$\frac{dr}{dz} = -\frac{r}{2z}\Big(\prod_i H_i\Big)^{1/8}. \tag{4.27}$$

We find that

$$\begin{aligned} S(z) &= \frac{1}{8}(-4a+3b+c)z + \frac{1}{16}(-28a^2+15b^2+12bc+c^2)z^2 + \cdots\,, \\ \widetilde{S}(z) &= \frac{1}{8}(c-b)z + \frac{1}{16}(-8ab-5b^2+8ac+4bc+c^2)z^2 + \cdots\,, \end{aligned}$$

where

$$\ell_1^2 = \ell_2^2 = \ell_3^2 = \ell_4^2 = a, \qquad \ell_5^2 = \ell_6^2 = \ell_7^2 = b\,, \qquad \ell_8^2 = c. \tag{4.28}$$

By setting

$$a = -\alpha_1, \quad b = \alpha_1 - 2\tilde{\alpha}_1, \quad c = \alpha_1 + 6\tilde{\alpha}_1, \tag{4.29}$$

we see that (4.29) reproduces (4.6). From (4.28), one can see that the leading coefficients are invariant under the shift (4.25), and that they therefore have an invariant physical meaning. Below, we shall show that in fact they are related to the VEVs of the $\mathbf{35}_v$ operators.

The solution in (4.24) can be uplifted to eleven dimensions, where it describes a continuous distribution of M2-branes [19]. The uplifted solution is given by

$$d\hat{s}_{11}^2 = H^{-2/3}(-dt^2 + d\vec{x}\cdot d\vec{x}) + H^{1/3} ds_8^2, \qquad \hat{F} = dt\wedge dx^1 \wedge dx^2 \wedge dH^{-1}, \tag{4.30}$$

where

$$H = \frac{1}{(g/2)^6\, r^6\, \Delta} \qquad \Delta = (H_1\cdots H_8)^{1/2}\sum_{i=1}^{8}\frac{\mu_i^2}{H_i}. \tag{4.31}$$

The transverse-space metric ds_8^2 is given by

$$ds_8^2 = \frac{\Delta\, dr^2}{\sqrt{H_1 \cdots H_8}} + r^2 \sum_{i=1}^{8} H_i d\mu_i^2, \tag{4.32}$$

where $\sum_i \mu_i^2 = 1$ defines a unit S^7 in $\mathbb{R}^8$. The metric (4.32) can be expressed as a flat Euclidean 8-metric $ds_8^2 = dy^m dy^m$ by making the coordinate transformation

$$y_i = r\sqrt{H_i}\,\mu_i. \tag{4.33}$$

In terms of these Euclidean coordinates, the harmonic function H takes the form

$$H = (g/2)^{-6} \int \frac{\sigma(\vec{y}\,')d^8y'}{|\vec{y} - \vec{y}\,'|^6}, \tag{4.34}$$

where σ is the normalized distribution function of the M2-branes.

Besides the trivial coincident branes case described by a delta-function distribution, solutions in our truncated theory correspond to six possible distributions of M2-branes, depending on the relative magnitudes of the constants ℓ_i^2, which are given in (4.28).

- ℓ_8^2 is the smallest among the ℓ_i^2. In this case, using the shift symmetry (4.25), ℓ_8^2 can be set to zero, and the M2-branes are distributed in a 7-ellipsoid. The explicit form of the distribution function can be found in [19], which suggests the existence of branes with negative tension. Thus, solutions in this class are unphysical.

- $\ell_5^2 = \ell_6^2 = \ell_7^2$ are the smallest. In this case, $\ell_5^2 = \ell_6^2 = \ell_7^2$ can be set to zero by the shift symmetry, and the resulting geometry describes M2-branes distributed in a 5-ellipsoid. The distribution function is positive definite.

- $\ell_1^2 = \ell_2^2 = \ell_3^2 = \ell_4^2$ are the smallest. In this case, $\ell_1^2 = \ell_2^2 = \ell_3^2 = \ell_4^2$ can be set to zero by shift symmetry. Accordingly, we find M2-branes with positive tension, distributed in a 4-ellipsoid.

- $\ell_5^2 = \ell_6^2 = \ell_7^2 = \ell_8^2$ are the smallest. In fact, this case corresponds to the SO(4) truncation of $\mathcal{N} = 8$ supergravity [47]. The domain-wall solution is sourced by M2-branes distributed in a 4-ball with positive tension.

- $\ell_1^2 = \ell_2^2 = \ell_3^2 = \ell_4^2 = \ell_8^2$ are the smallest. This case describes positive-tension M2-branes distributed in a 3-ball.

- $\ell_1^2 = \ell_2^2 = \ell_3^2 = \ell_4^2 = \ell_5^2 = \ell_6^2 = \ell_7^2$ are the smallest. This case is contained in the G_2 truncation of $\mathcal{N} = 8$ supergravity [14]. The domain-wall solution is sourced by positive-tension M2-branes distributed in a segment.

To summarize, except for the case where the M2-branes are distributed in a 7-ellipsoid, the domain-wall solutions are all sourced by M2-branes with positive tension, and therefore they may be considered to be physical.

The harmonic function H has a Taylor expansion at large $|\vec{y}\,|$ given by

$$H = \frac{1}{(g/2)^6 |\vec{y}\,|^6}\left(1 + \sum_{n=2}^{\infty} \frac{2^n (n+1)(n+2) d^{(n)}_{i_1 i_2 \ldots i_n} \hat{y}^{i_1} \hat{y}^{i_2} \ldots \hat{y}^{i_n}}{|\vec{y}\,|^n}\right), \tag{4.35}$$

where $\hat{y}^i \equiv y^i/|\vec{y}\,|$, and the partial-wave expansion coefficients $d^{(n)}_{i_1 i_2 \ldots i_n}$ are totally symmetric and traceless, transforming in the $(n, 0, 0, 0)$ representation under SO(8). The scaling dimension assigned to $d^{(n)}_{i_1 i_2 \cdots n}$ is $\Delta = n/2$ rather than $\Delta = n$, since at large $\vec{y}$, $|\vec{y}\,|^2$ is asymptotic to the standard radial coordinate of AdS. In general, the higher-order coefficients $d^{(n)}_{i_1 \cdots i_n}$ with $n > 2$ are present, which means there are VEVs for higher-dimension operators as well as for those in the $\mathbf{35}_v$. Therefore, although the consistent truncation keeps only a finite number of fields, the profile of the Coulomb branch flow captures infinitely many VEVs. For our case, from (4.33), we find

$$r = |\vec{y}\,|\left(1 - \frac{a(\hat{y}_1^2 + \hat{y}_2^2 + \hat{y}_3^2 + \hat{y}_4^2) + b(\hat{y}_5^2 + \hat{y}_6^2 + \hat{y}_7^2) + c\hat{y}_8^2}{2|\vec{y}\,|^2} + \cdots\right). \tag{4.36}$$

Using the equation above, we obtain the partial-wave expansion of the harmonic function H up to $\mathcal{O}(1/|\vec{y}\,|^8)$, in which the coefficients $d^{(2)}_{i_1 i_2}$ are given by

$$\begin{aligned} d^{(2)}_{i_1 i_2} &= \frac{1}{48}\,\mathrm{diag}\Big((4a - 3b - c) \times \mathbb{1}_4, (5b - 4a - c) \times \mathbb{1}_3, 7c - 4a - 3b\Big), \\ &= \frac{1}{6}\,\mathrm{diag}\Big(-\alpha_1 \times \mathbb{1}_4, (\alpha_1 - 2\widetilde{\alpha}_1) \times \mathbb{1}_3, -(\alpha_1 + 6\widetilde{\alpha}_1)\Big). \end{aligned} \tag{4.37}$$

4.2 Holographic RG flow with $\mathcal{N} = 1$ mass deformations

In this section, we study the solutions to eq. (2.14) when the pseudoscalars are turned on. In this case, besides the non-trivial AdS$_4$ G_2 critical point found in [11], the other interesting solutions are domain walls asymptotic to AdS$_4$ in the ultraviolet. Specifically, for the domain-wall solutions we shall consider,[2] the pseudoscalars P and $\widetilde{P}$ behave like $P(z) \sim \beta_1 z$ and $\widetilde{P} \sim \widetilde{\beta}_1 z$ near the AdS$_4$ boundary. These solutions are dual to RG flows on M2-branes driven by $\mathcal{N} = 1$ mass deformations. An exact solution of this type was found in [27], in the SO(4) gauged $\mathcal{N} = 4$ supergravity. As we shall see, domain-wall solutions with nontrivial pseudoscalar profiles are generically singular at a certain IR cutoff $z_{\rm IR}$. It turns out that the metrics of these solutions share similar singular behaviors with the 4D Coulomb branch flow metrics.

In general, the IR cutoff is attained when the moduli of the two complex scalars ψ_1 and ψ_2 defined in (2.5) approach infinity. To visualize these flows, we associate each complex scalar with a complex plane. It turns out that the degree of the IR singularity depends on the direction in which the complex scalar approaches infinity in the complex plane. There are also some special flows in which only one of the complex scalar fields blows up at the IR cutoff, while the other field tends to a finite value. Owing to the complexity of the flow equations (2.14), we are not able to classify all possible solutions. Instead, we shall

[2]Domain-wall solutions interpolating between two supersymmetric AdS vacua have been studied in [14, 17].

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present several representative examples of solutions that exhibit different types of singular IR behaviors.

We shall use the coordinates defined in (2.13), in which the IR cutoff $z_{\rm IR}$ corresponds to $\rho = \rho_{\rm IR}$. Under the diffeomorphism $\rho \to \rho + \lambda$, the IR cutoff $\rho_{\rm IR}$ and Fefferman-Graham expansion coefficients are changed to

$$\rho_{\rm IR} \to \rho_{\rm IR} - \lambda, \qquad (\alpha_1, \beta_1, \widetilde{\alpha}_1, \widetilde{\beta}_1) \to e^{-\lambda}\,(\alpha_1, \beta_1, \widetilde{\alpha}_1, \widetilde{\beta}_1). \tag{4.38}$$

Therefore, the combinations $e^{-\rho_{\rm IR}}(\alpha_1, \beta_1, \widetilde{\alpha}_1, \widetilde{\beta}_1)$ are invariant under the shift of ρ, and they characterise the solution. By shifting ρ one can choose a special coordinate in which $\rho_{\rm IR} = 0$, and the Fefferman-Graham coefficients $(\alpha_1, \beta_1, \widetilde{\alpha}_1, \widetilde{\beta}_1)$ are then equal to the shift-invariant quantities. We shall present the domain-wall solutions using this particular choice of coordinate. For technical convenience, we work with the redefined the scalar fields

$$\zeta_1 = \tanh \frac{1}{2}\phi_1\, e^{-{\rm i}\sigma_1}, \quad \zeta_2 = \tanh \frac{1}{2}\phi_2\, e^{{\rm i}\sigma_2}. \tag{4.39}$$

Infinity in the complex ψ plane is mapped into the unit circle in the complex ζ plane.

We first discuss the solutions in which $|\zeta_1|$ and $|\zeta_2|$ both approach 1 in the IR at $\rho = 0$. Near the unit circle, we may perform a perturbative expansion for ζ_1 and ζ_2, writing

$$\zeta_1 = (1 - \delta r_1)e^{-i(\sigma_1 + \delta\sigma_1)}, \qquad \zeta_2 = (1 - \delta r_2)e^{i(\sigma_2 + \delta\sigma_2)}. \tag{4.40}$$

When ζ_1 and ζ_2 approach the unit circle from generic angles specified by $\sigma_{1\rm IR}$ and $\sigma_{2\rm IR}$, the perturbations at leading order satisfy the equations

$$\begin{aligned}
&\frac{d(\delta r_1)}{d\rho} = f_1(\sigma_{1\rm IR}, \sigma_{2\rm IR})\frac{\delta r_1^{1/2}}{\delta r_2^3} + \cdots\,, \qquad \frac{d(\delta r_2)}{d\rho} = f_1(\sigma_{1\rm IR}, \sigma_{2\rm IR})\frac{1}{\delta r_1^{1/2}\delta r_2^2} + \cdots\,,\\
&\frac{d(\delta \sigma_1)}{d\rho} = -f_2(\sigma_{1\rm IR}, \sigma_{2\rm IR})\frac{\delta r_1^{3/2}}{\delta r_2^3} + \cdots\,, \quad \frac{d(\delta \sigma_2)}{d\rho} = f_3(\sigma_{1\rm IR}, \sigma_{2\rm IR})\frac{1}{\delta r_1^{1/2}\delta r_2} + \cdots\,,
\end{aligned} \tag{4.41}$$

where the non-vanishing coefficients f_1, f_2 and f_3 are given by

$$\begin{aligned}
f_1(\sigma_1, \sigma_2) &= \sqrt{2}\sin^2\frac{\sigma_2}{2}|f_0|, \quad f_0 = 3\sin\frac{\sigma_1}{2} + 2\sin\left(\frac{\sigma_1}{2} - \sigma_2\right) + \sin\left(\frac{\sigma_1}{2} - 2\sigma_2\right),\\
f_2(\sigma_1, \sigma_2) &= {\rm sign}(f_0)\sqrt{2}\sin^2\frac{\sigma_2}{2}\left(3\cos\frac{\sigma_1}{2} + 2\cos\left(\frac{\sigma_1}{2} - \sigma_2\right) + \cos\left(\frac{\sigma_1}{2} - 2\sigma_2\right)\right),\\
f_3(\sigma_1, \sigma_2) &= {\rm sign}(f_0)\sqrt{2}\sin\frac{\sigma_2}{2}\sin\left(\frac{\sigma_1}{2} - \sigma_2\right)\left(\cos\frac{\sigma_2}{2} + \cos\frac{3\sigma_2}{2}\right).
\end{aligned} \tag{4.42}$$

An example of a solution in this class is given in figure 3, where $\sigma_{1\rm IR} = \frac{11}{8}\pi$ and $\sigma_{2\rm IR} = \frac{13}{8}\pi$. Near the IR region $\rho = 0$, solutions for the perturbations take the form

$$\begin{aligned}
\delta r_1 &= 0.25\,\rho^{2/7}, & \delta r_2 &= 1.508507\,\rho^{2/7},\\
\delta \sigma_1 &= 0.106352\,\rho^{4/7}, & \delta \sigma_2 &= -0.226322\,\rho^{4/7},
\end{aligned} \tag{4.43}$$

from which we can obtain the IR behavior of the scale factor $e^{2A(\rho)}$, and therefore the IR behavior of the metric. It turns out that the metric shares a similar singular behavior with

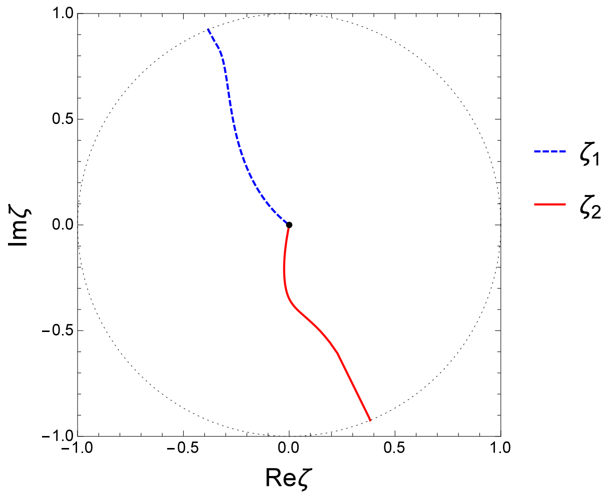

Figure 3. An example of the most common flow with both ϕ_1 and ϕ_2 blowing up in the IR. This flow is obtained by choosing $\sigma_{1\mathrm{IR}} = \frac{11}{8}\pi$ and $\sigma_{2\mathrm{IR}} = \frac{13}{8}\pi$.

the 4D Coulomb branch flow metric whose 11D uplift describes a continuous distribution of M2-branes on a 7-ellipsoid, namely,

$$\rho \to 0, \quad ds_4^2 \simeq d\rho^2 + \rho^{2/7} dx^\mu dx_\mu. \tag{4.44}$$

By integrating the equation from the IR to the UV, we can read off the Fefferman-Graham coefficients, finding

$$\{\alpha_1, \beta_1, \tilde{\alpha}_1, \tilde{\beta}_1\} = \{-1.282820, 0.971868, -0.191382, -0.959326\}. \tag{4.45}$$

As mentioned before, since we are using a radial coordinates where $\rho_{\mathrm{IR}} = 0$, the Fefferman-Graham coefficients are equal to the ρ-shift-invariant combinations.

If ζ_1 and ζ_2 approach the unit circle in some special directions such that $f_1(\sigma_{1\mathrm{IR}}, \sigma_{2\mathrm{IR}}) = 0$, the singular behavior of the solution shows different features. Having $f_1(\sigma_{1\mathrm{IR}}, \sigma_{2\mathrm{IR}}) = 0$ implies either $\sigma_{2\mathrm{IR}} = 0$ or $f_0 = 0$. In the former case, the equations of motion of the perturbations near $\rho = 0$ are given by

$$\begin{aligned} \frac{d(\delta r_1)}{d\rho} &= \frac{3\sqrt{2}|\sin(\sigma_{1\mathrm{IR}}/2)|\delta r_1^{1/2}}{2\delta r_2} + \cdots, & \frac{d(\delta r_2)}{d\rho} &= \frac{\sqrt{2}|\sin(\sigma_{1\mathrm{IR}}/2)|}{2\delta r_1^{1/2}} + \ldots, \\ \frac{d(\delta\sigma_1)}{d\rho} &= -\frac{3\sqrt{2}|\cos(\sigma_{1\mathrm{IR}}/2)|\delta r_1^{3/2}}{2\delta r_2} + \cdots, & \frac{d(\delta\sigma_2)}{d\rho} &= \frac{\sqrt{2}|\cos(\sigma_{1\mathrm{IR}}/2)|\delta r_2}{3\delta r_1^{1/2}} + \cdots. \end{aligned} \tag{4.46}$$

An explicit example of a solution for the $\sigma_{1\mathrm{IR}} = 5\pi/4$ case is given in figure 4. Near the IR cutoff, the perturbations take the form

$$\begin{aligned} \delta r_1 &= 0.01\,\rho^{6/5}, & \delta r_2 &= 16.3320\,\rho^{2/5}, \\ \delta\sigma_1 &= 2.07107 \times 10^{-5}\,\rho^{12/5}, & \delta\sigma_2 &= -88.9118\,\rho^{4/5}, \end{aligned} \tag{4.47}$$

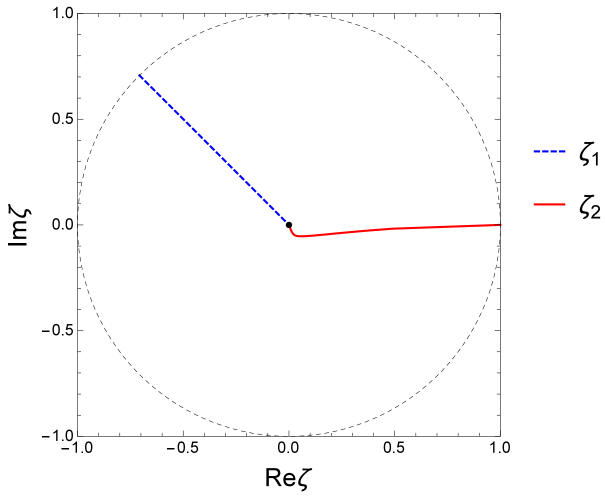

Figure 4. A typical flow obtained by choosing $\sigma_{1\text{IR}} = 5\pi/4$ and $\sigma_{2\text{IR}} = 0$.

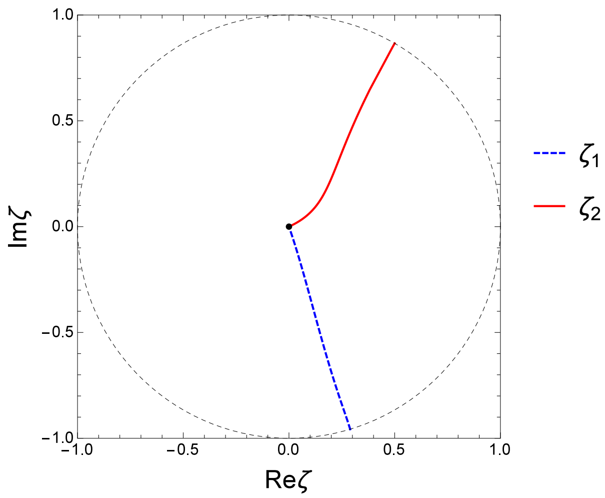

Figure 5. A typical flow in $f_0 = 0$ case given by $\sigma_{1\text{IR}} = 1.27712$, $\sigma_{2\text{IR}} = \pi/3$.

which we leads to the following singular behavior of the metric near the IR cutoff:

$$ds_4^2 \simeq d\rho^2 + \rho^{6/5} dx^\mu dx_\mu. \tag{4.48}$$

This IR singularity is similar to the one appearing in the 4D Coulomb-branch flow metric whose 11D uplift describes a continuous distribution of M2-branes on a 5-ellipsoid. In this example, the Fefferman-Graham expansion coefficients of the two complex scalars are given by

$$\{\alpha_1, \beta_1, \tilde{\alpha}_1, \tilde{\beta}_1\} = \{-1.419990, 1.418240, 0.049763, -0.139465\}. \tag{4.49}$$

An example for the $f_0 = 0$ case is given by $\sigma_{1\text{IR}} = 1.27712$ and $\sigma_{2\text{IR}} = \pi/3$. The numerical solutions for the two complex scalars are exhibited in figure 5. In this solution,

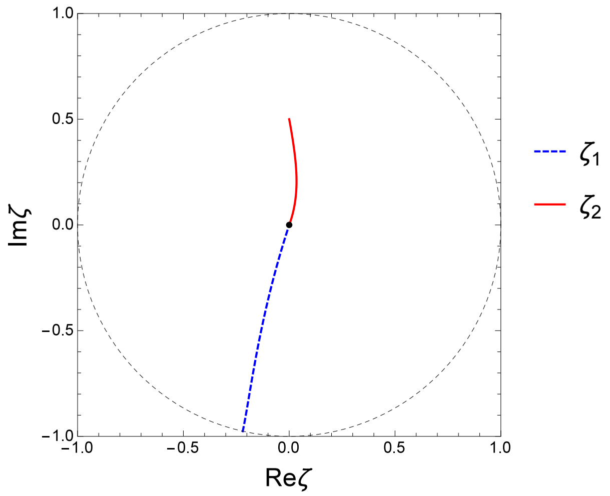

Figure 6. A typical flow with $\zeta_{2\text{IR}} = 0.5$, $\sigma_{1\text{IR}} = 1.792110$ and $\sigma_{2\text{IR}} = \pi/2$. Unlike previous examples, ϕ_2 remains finite in the IR limit.

the IR expansion of the perturbations takes the form

$$\begin{aligned} \delta r_1 &= 1.0\,\rho^{6/11}, & \delta r_2 &= 1.15686\,\rho^{4/11}, \\ \delta\sigma_1 &= 0.545275\,\rho^{12/11}, & \delta\sigma_2 &= -0.193171\,\rho^{12/11}, \end{aligned} \tag{4.50}$$

which leads to a novel singular behavior of the metric which has not been observed in the Coulomb-branch flow metric, with

$$ds_4^2 \simeq d\rho^2 + \rho^{6/11} dx^\mu dx_\mu. \tag{4.51}$$

The UV expansion is characterised by the coefficients

$$\{\alpha_1, \beta_1, \tilde{\alpha}_1, \tilde{\beta}_1\} = \{0.487525, -1.402800, 0.564112, 0.257166\}. \tag{4.52}$$

Having shown several examples in which both ζ_1 and ζ_2 approach the unit circle in the IR, we now present an example in which ζ_2 limits to a point inside the unit circle; in other words, ϕ_2 stays finite at the IR cutoff. In general, this imposes a very complicated functional relation among $|\zeta_2|$, $\sigma_{1\text{IR}}$ and $\sigma_{2\text{IR}}$. However, we find that if $\sigma_{2\text{IR}} = \pi/2$, this expression reduces to the rather simple form

$$\sigma_{1\text{IR}} = 2\arctan\left(\frac{1 + |\zeta_{2\text{IR}}|^2}{2|\zeta_{2\text{IR}}|}\right). \tag{4.53}$$

For $|\zeta_{2\text{IR}}| = 0.5$, the numerical solution is given in figure 6. The IR expansion of the solution takes the form

$$\begin{aligned} \delta r_1 &= 0.846883\,\rho^2, & \delta\sigma_1 &= -1.02003\,\rho^4, \\ \delta\text{Re}\zeta_2 &= 0.39589\,\rho^{1.47386}, & \delta\text{Im}\zeta_2 &= -1.96043\,\rho^{1.47386}, \end{aligned} \tag{4.54}$$

for which the singular IR behavior of the metric takes the form

$$ds_4^2 \simeq d\rho^2 + \rho^2 dx^\mu dx_\mu. \tag{4.55}$$

In this case, it should be noted that the convenient variables to study the perturbation of ζ_2 are $(\delta\mathrm{Re}\zeta_2,\, \delta\mathrm{Im}\zeta_2)$ rather than $(\delta r_2,\, \delta\sigma_2)$. This IR singularity is similar to the one appearing in the 4D Coulomb-branch flow metric whose 11D uplift describes a continuous distribution of M2-branes on a 4-ellipsoid. The UV expansion is specified by the coefficients

$$\{\alpha_1, \beta_1, \tilde{\alpha}_1, \tilde{\beta}_1\} = \{-0.453355, -1.947710, 0.210519, 0.285747\}. \tag{4.56}$$

Now we turn to the higher-dimensional interpretation of the solutions with non-trivial pseudoscalars, which correspond to turning on mass deformations on M2 branes. Pseudoscalars in the 4D supergravity appear both in the 11D metric and the 3-form potential. Non-vanishing pseudoscalars generally lead to non-trivial internal components of the 3-form potential. Thus it is conceivable that the uplift of the lower-dimensional solution with non-vanishing pseudoscalars implies the presence of M5-branes in the bulk geometry. Previous studies [27, 48, 49] provide evidence for this intuition, with M2-branes growing into dielectric M5-branes wrapped on S^3 via the Myers effect [26].

When pseudoscalars are turned on, we have found four classes of solutions exhibiting different IR singularities. In order to see whether these singular solutions are physically allowable as brane configurations, we shall embed our solutions into 11D supergravity. The metric ansatz for lifting $\mathcal{N} = 8$ gauged supergravity to $D = 11$ is given in [50], with (in the absence of the four-dimensional Yang-Mills fields)

$$d\hat{s}_{11}^2 = \Delta^{-1}\, ds_4^2 + g_{mn}(x, y)\, dy^m dy^n, \tag{4.57}$$

where the inverse of the internal seven-dimensional metric is given by

$$\Delta^{-1} g^{mn}(x, y) = \frac{1}{4} \mathring{K}^{m\,IJ}\, \mathring{K}^{n\,KL}\, (u_{ij}{}^{IJ} + v_{ijIJ})(u^{ij}{}_{KL} + v^{ijKL}). \tag{4.58}$$

In the above formula, the warp factor is given by

$$\Delta = \sqrt{\frac{\det g_{mn}(x, y)}{\det \mathring{g}_{mn}(y)}}, \tag{4.59}$$

where $\mathring{g}^{mn}(y)$ is the metric on the unit 7-sphere and the 28 Killing vectors $\mathring{K}^{m\,IJ}$ are those of the unit 7-sphere. These may be described in terms of the coordinates X^A on an 8-dimensional Euclidean space, subject to the constraint $X^A X^A = 1$, as

$$\mathring{K}^{IJ} = \frac{1}{2} (\Sigma_{IJ})_{AB} \left(X^A \frac{\partial}{\partial X^B} - X^B \frac{\partial}{\partial X^A} \right). \tag{4.60}$$

Here Σ_{IJ} are the 8×8 chiral projection of the SO(8) 2-Gamma matrices Γ_{IJ}. (AB are spinor indices.) The explicit form of Γ_{IJ} and the adapted coordinates X^A can be found in appendix B.

Using the above formulae, we have computed the uplifted metric for the $\mathrm{SO}(3)_D \times \mathrm{SO}(3)_R$ invariant sector of SO(8) gauged $\mathcal{N}=8$ gauged supergravity. The resulting expressions for each component of the 11D metric are long, and seem to be unsuitable for presentation. In spite of the complexity of the 11D metric, we can still extract its behavior near the location of the singularity, say at $\rho = 0$, using numerical techniques. This encodes information about the brane configuration. Plugging in specific values for the angles on the deformed S^7, we can read off the singular behavior of the 11D metric. The robustness of this singular behavior can be checked by varying the values of the angles. By this means, we find that the uplift of the solution given in figure 3 can be brought to the form, near $\rho = 0$,

$$d\hat{s}_{11}^2 \simeq H^{-2/3} ds_{3\|}^2 + H^{1/3} ds_{8\perp}^2, \qquad H = r, \quad r = \rho^{7/4}. \tag{4.61}$$

This singular behavior is the same as that appearing in the metric describing M2-branes distributed in a 7-ellipsoid, which involves M2-branes with negative tension. Thus, the class of flow solutions represented by the example in figure 3 is not physically acceptable.

For M5-branes distributed in an n-ellipsoid, near the IR singularity the metric behaves like

$$d\hat{s}_{11}^2 \simeq H^{-1/3} ds_{6\|}^2 + H^{2/3} ds_{5\perp}^2, \quad H = r^{n-3}. \tag{4.62}$$

We find that after being uplifted to 11D, the metrics corresponding to the solutions given in figure 4 and figure 6 can be brought to the above forms with $n = 2$ and $n = 1$ respectively, by identifying ρ as the appropriate power of r. The six directions on the world-volume of the M5-branes consist of three directions on the world-volume of the M2-branes, and three directions from the 7-sphere. Since there are only three flat directions in the world-volume of the M5-branes, they must wrap on the remaining three directions. We are comparing such a configuration of M5-branes with one in which the M5-branes are flat and infinitely large. This comparison is valid, as long as we focus on the region infinitesimally close to brane. From the results given in [19], it can be deduced that for $n \leq 3$, the geometry (4.62) is sourced by M5-branes with positive tension. Thus solutions possessing similar singular behaviors to the ones shown in figure 4 and figure 6 are physically allowed, with the singularity being balanced by normal positive-tension brane sources.

We also computed the uplift for the class of solutions represented by figure 5, for which the warp factor is given by

$$\Delta \sim \rho^{-10/11}. \tag{4.63}$$

The 11D metric for this solution does not approximate any Coulomb-branch metric of M-branes given in [19] in the near horizon region, indicating that a new supersymmetric solution should exist in 11D supergravity.

5 Holographic $\mathcal{N} = 1$ RG flows in the ω-deformed theory

In the ω-deformed theory, it is straightforward to verify that in the second-order equations of motion, σ_1 and σ_2 can be consistently to set zero. Moreover, when $\sigma_1 = \sigma_2 = 0$, there exists a set of first-order equations whose solutions also obey the remaining second-order equations. This first-order set resembles the one governing the holographic Coulomb-branch flow on M2-branes. It is given by

$$\phi_1' = -4g\frac{\partial |W|}{\partial \phi_1}\Big|_{\sigma_1=\sigma_2=0}, \quad \phi_2' = -\frac{2}{3}g\frac{\partial |W|}{\partial \phi_2}\Big|_{\sigma_1=\sigma_2=0}, \quad A' = g|W|. \tag{5.1}$$

However, one also can show that

$$\frac{\partial |W|}{\partial \sigma_1}\Big|_{\sigma_1=\sigma_2=0} \neq 0, \qquad \frac{\partial |W|}{\partial \sigma_2}\Big|_{\sigma_1=\sigma_2=0} \neq 0, \tag{5.2}$$

and so solutions to the first-order equations (5.1) do not preserve any supersymmetry in the ω-deformed theory. By contrast, the solutions to the analogous equations in the undeformed theory do preserve the $\mathcal{N}=4$ supersymmetry of the bulk supergravity theory. This is therefore a sharp distinction between the ω-deformed theory and the undeformed theory. Since we are interested in the supersymmetric holographic RG flows, it follows that pseudoscalars will always play a non-trivial role in the domain-wall solutions. A supersymmetric domain-wall solution interpolating between the SO(8) point and the G_2 point in the ω-deformed theory has been studied in [8].

Owing to the lack of any known higher-dimensional origin for the ω-deformed SO(8) gauged $\mathcal{N}=8$ supergravity, it is unclear which 3D CFT should be its holographic dual. However, if we assume that such a dual CFT exists, and that it belongs to a certain type of Chern-Simons matter theory, the bulk scalar fields should still be dual to scalar operators that are bilinear in the boundary scalars or fermions, as was discussed in section 3 for the undeformed case.

This assumption leads us to a preferred redefinition of the bulk scalar fields, for the following reason. Using the current definition of the scalar fields, near the boundary of AdS_4 the functional relations amongst the Fefferman-Graham coefficients will ostensibly depend on the ω parameter. However, as in the $\omega=0$ case, via the AdS/CFT correspondence these functional relations should match with those relating the bosonic mass parameters to the fermionic ones. These are determined by the boundary $\mathcal{N}=1$ supersymmetry, and should therefore be independent of ω. This suggests that the ostensible ω-dependence of the functional relations amongst the Fefferman-Graham coefficients should be a technical artifact that can be removed by redefining the bulk scalar fields. Indeed, in terms of the new complex scalar fields

$$\zeta_1' = e^{\frac{2}{3}\mathrm{i}\omega}\zeta_1, \qquad \zeta_2' = e^{\frac{2}{3}\mathrm{i}\omega}\zeta_2, \tag{5.3}$$

the equalities (4.7) are maintained. As a consistency check, if we write

$$\zeta_1' = S' + \mathrm{i}P', \qquad \zeta_2' = \widetilde{S}' + \mathrm{i}\widetilde{P}', \tag{5.4}$$

then in order that S', $\widetilde{S}'$ and P', $\widetilde{P}'$ should be dual to $\Delta=1$ and $\Delta=2$ primary operators respectively, the boundary conditions

$$S' \sim z\, S^{(1)'}, \qquad \widetilde{S}' \sim z\, \widetilde{S}^{(1)'}, \qquad P' \sim z^2\, P^{(2)'}, \qquad \widetilde{P}' \sim z^2\, \widetilde{P}^{(2)'}, \tag{5.5}$$

should preserve the $\mathcal{N}=1$ supersymmetry of ω-deformed supergravity in the bulk. In terms of the original fields, the above boundary conditions amount to

$$\begin{aligned}
&\cos\frac{2}{3}\omega\, S_1^{(2)} - \sin\frac{2}{3}\omega\, P_1^{(2)} = 0, && \cos\frac{2}{3}\omega\, S_2^{(2)} - \sin\frac{2}{3}\omega\, P_2^{(2)} = 0, \\
&\cos\frac{2}{3}\omega\, P_1^{(1)} + \sin\frac{2}{3}\omega\, S_1^{(1)} = 0, && \cos\frac{2}{3}\omega\, P_2^{(1)} + \sin\frac{2}{3}\omega\, S_2^{(1)} = 0.
\end{aligned} \tag{5.6}$$

These are consistent with the $\mathcal{N}=1$ boundary conditions given in [42] for the ω-deformed SO(8) gauged $\mathcal{N}=8$ supergravity.

Similarly to discussion we gave in the $\omega=0$ case, in the study of supersymmetric domain-wall solutions, when $\omega\neq 0$ we shall now work with the fields ζ_1' and ζ_2'. Infinity in the complex ψ' plane is mapped into the unit circle in the complex ζ' plane. From now on, for simplicity of presentation, we shall remove the primes from the fields; all the fields below should be interpreted as the ω-rotated ones defined in (5.3). Flow solutions in ω-deformed theory are driven by the ω-dependent "potential" $|W|$, and so the perturbative IR expansion of the equations of motion depends on ω. However, the singular IR behaviors of the solutions are similar to those arising in the $\omega=0$ case. For instance, for a generic solution in which both ζ_1 and ζ_2 attain the unit circle, the perturbative IR expansion is given by

$$\begin{aligned}
&\frac{d(\delta r_1)}{d\rho} = \tilde{f}_1(\sigma_{1\text{IR}},\sigma_{2\text{IR}})\frac{\delta r_1^{1/2}}{\delta r_2^3} + \cdots\,, && \frac{d(\delta r_2)}{d\rho} = \tilde{f}_1(\sigma_{1\text{IR}},\sigma_{2\text{IR}})\frac{1}{\delta r_1^{1/2}\delta r_2^2} + \cdots\,, \\
&\frac{d(\delta \sigma_1)}{d\rho} = -\tilde{f}_2(\sigma_{1\text{IR}},\sigma_{2\text{IR}})\frac{\delta r_1^{3/2}}{\delta r_2^3} + \cdots\,, && \frac{d(\delta \sigma_2)}{d\rho} = \tilde{f}_3(\sigma_{1\text{IR}},\sigma_{2\text{IR}})\frac{1}{\delta r_1^{1/2}\delta r_2} + \cdots\,,
\end{aligned} \tag{5.7}$$

where the non-vanishing ω-dependent coefficients $\tilde{f}_1$, $\tilde{f}_2$ and $\tilde{f}_3$ are given by

$$\begin{aligned}
\tilde{f}_1(\sigma_1,\sigma_2) &= \frac{1}{2\sqrt{2}}\left|\sin\left(\frac{\sigma_1}{2}-3\sigma_2+\frac{4\omega}{3}\right)+3\sin\left(\frac{\sigma_1}{2}+\sigma_2-\frac{4\omega}{3}\right)-4\sin\left(\frac{\sigma_1}{2}+\frac{4\omega}{3}\right)\right|, \\
\tilde{f}_2(\sigma_1,\sigma_2) &= \frac{\text{sign}(f_0)}{2\sqrt{2}}\cos\left(\frac{\sigma_1}{2}-3\sigma_2+\frac{4\omega}{3}\right)+3\cos\left(\frac{\sigma_1}{2}+\sigma_2-\frac{4\omega}{3}\right)-4\cos\left(\frac{\sigma_1}{2}+\frac{4\omega}{3}\right), \\
\tilde{f}_3(\sigma_1,\sigma_2) &= \frac{\text{sign}(f_0)}{2\sqrt{2}}\sin\left(\frac{\sigma_1}{2}-\sigma_2\right)\sin\left(2\sigma_2-\frac{4\omega}{3}\right).
\end{aligned} \tag{5.8}$$

An example of such flow solution with $\sigma_{1\text{IR}}=31\pi/24$ and $\sigma_{2\text{IR}}=41\pi/24$ in the $\omega=\pi/8$ theory is exhibited in figure 7, The solutions to the perturbation equation are given at the leading order by

$$\begin{aligned}
&\delta r_1 = 0.49\,\rho^{2/7}, && \delta r_2 = 1.34846\,\rho^{2/7} \\
&\delta \sigma_1 = 0.551838\,\rho^{4/7}, && \delta \sigma_2 = 0.180846\,\rho^{4/7}.
\end{aligned} \tag{5.9}$$

The metric is again of the form (4.44) and the UV expansion coefficients obtained by integrating the equations of motion from the IR to the UV are given by

$$\{\alpha_1,\beta_1,\tilde{\alpha}_1,\tilde{\beta}_1\} = \{-0.636249, 1.23047, 0.212564, -0.424938\}. \tag{5.10}$$

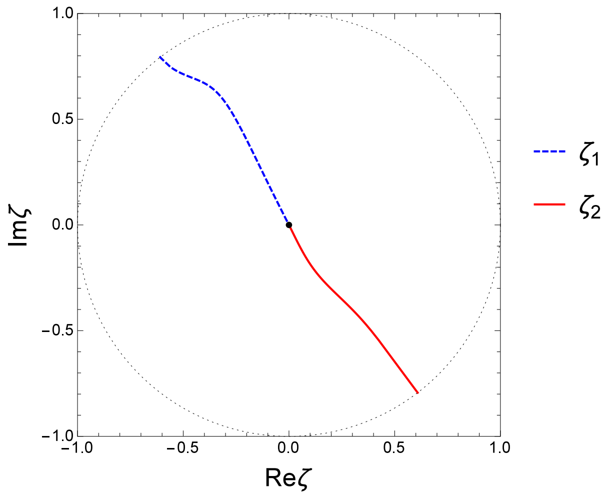

Figure 7. A typical flow with $\sigma_{1\mathrm{IR}} = \frac{31\pi}{24}$, $\sigma_{2\mathrm{IR}} = \frac{41\pi}{24}$ in the $\omega = \pi/8$ theory.

When $\tilde{f}_1$ vanishes, there are two branches of solutions. The first branch is obtained when $\sigma_{1\mathrm{IR}} = \pi - 2\omega/3$ and $\sigma_{1\mathrm{IR}} = 2\omega/3$. For each fixed non-zero ω, this branch consists of a one-parameter family of flow solutions, modulo the shift symmetry. However, when $\omega = 0$, as we discussed in the previous section, the solution can acquire an additional parameter and become a two-parameter solution. In the vicinity of the IR cutoff, the perturbations satisfy

$$\frac{d(\delta r_1)}{d\rho} = \frac{3\sqrt{2}\cos\omega\delta r_1^{1/2}}{2\delta r_2} + \cdots, \qquad \frac{d(\delta r_2)}{d\rho} = \frac{\sqrt{2}\cos\omega}{2\delta r_1^{1/2}} + \cdots,$$
$$\frac{d(\delta\sigma_1)}{d\rho} = -\frac{2\sqrt{2}\tan\omega\delta r_1^{3/2}}{\delta r_2^3} + \cdots, \qquad \frac{d(\delta\sigma_2)}{d\rho} = \frac{2\sqrt{2}\tan\omega\delta r_2}{6\delta r_1^{1/2}} + \cdots. \tag{5.11}$$

For $\omega = \pi/8$, a numerical solution belonging to this branch is plotted in figure (8). Near the IR cutoff, solutions to the perturbation equations are given as

$$\delta r_1 = 0.81\,\rho^{6/5}, \qquad \delta r_2 = 1.81467\,\rho^{2/5},$$
$$\delta\sigma_1 = -0.0893271\,\rho^{8/5}, \qquad \delta\sigma_2 = 0.492134\,\rho^{4/5}. \tag{5.12}$$

This leads to an IR singularity of the metric that takes the same form as (4.48). Near the UV boundary, the expansion coefficients are found to be

$$\{\alpha_1, \beta_1, \tilde{\alpha}_1, \tilde{\beta}_1\} = \{-1.18276, -0.453272, 0.19352, 0.176533\}. \tag{5.13}$$

When $(\sigma_{1\mathrm{IR}}, \sigma_{2\mathrm{IR}}) \neq (\pi - \frac{2\omega}{3}, \frac{2\omega}{3})$ but still with $\tilde{f}_1$ vanishing, one obtains the second branch of solutions, which is analogous to those in the $\omega = 0$ case with $f_0 = 0$ but $\sigma_{2\mathrm{IR}} \neq 0$. A solution belonging to this branch in the $\omega = \pi/8$ theory is found when $\sigma_{1\mathrm{IR}} = \frac{19\pi}{12}$ and $\sigma_{2\mathrm{IR}} = 0.319382\pi$. The numerical solution is shown in figure 9 with the IR expansion

$$\delta r_1 = \rho^{6/11}, \qquad \delta r_2 = 1.96456\,\rho^{4/11}$$
$$\delta\sigma_1 = 0.0511178\,\rho^{12/11}, \qquad \delta\sigma_2 = -0.0426812\,\rho^{12/11}. \tag{5.14}$$

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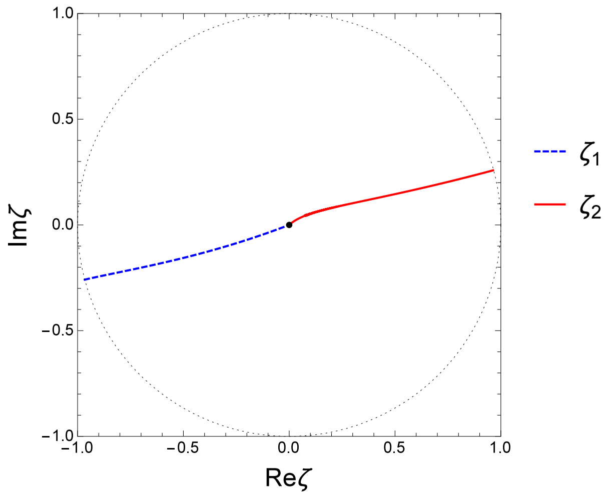

Figure 8. A flow solution with $\sigma_{1\mathrm{IR}} = \pi - \frac{2\omega}{3}, \sigma_{2\mathrm{IR}} = \frac{2\omega}{3}$ in the $\omega = \pi/8$ theory.

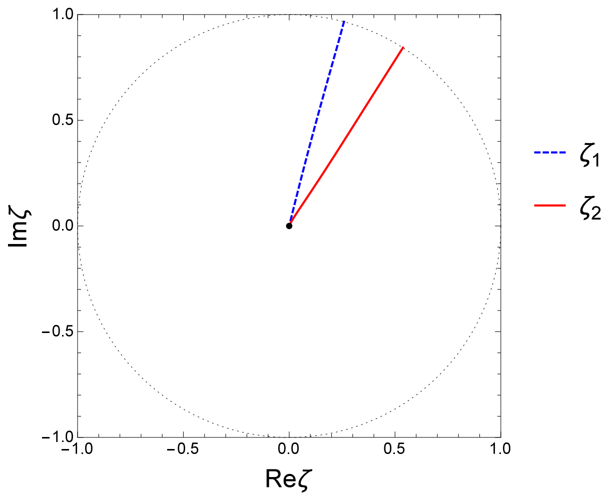

Figure 9. A typical flow obtained in the $\omega = \pi/8$ theory by choosing $\sigma_{1\mathrm{IR}} = \frac{19\pi}{12}$ and $\sigma_{2\mathrm{IR}} = 0.319382\pi$.

Accordingly, near the IR singularity, the metric is of the form given in (4.51). The UV expansion is characterised by the coefficients

$$\{\alpha_1, \beta_1, \tilde{\alpha}_1, \tilde{\beta}_1\} = \{0.195555, 0.801957, 0.0412681, 0.101782\}. \tag{5.15}$$

Similarly to the $\omega = 0$ case, the ω-deformed theory also admits solutions in which ζ_2 limits to a point inside the unit circle; in other words ϕ_2 stays finite at the IR cutoff. In general, this imposes a very complicated functional relation among $|\zeta_2|$, $\sigma_{1\mathrm{IR}}$ and $\sigma_{2\mathrm{IR}}$. However, we find that if $\sigma_{2\mathrm{IR}} = \frac{2\omega}{3}$, this expression becomes the simpler one

$$(|\zeta_{\mathrm{IR}}|^2 + 1) \sin\left(\sigma_{1\mathrm{IR}} - \frac{4\omega}{3}\right) + 2|\zeta_{\mathrm{IR}}| \cos\left(\sigma_{1\mathrm{IR}} - \frac{2\omega}{3}\right) - 2|\zeta_{\mathrm{IR}}| \cos 2\omega = 0. \tag{5.16}$$

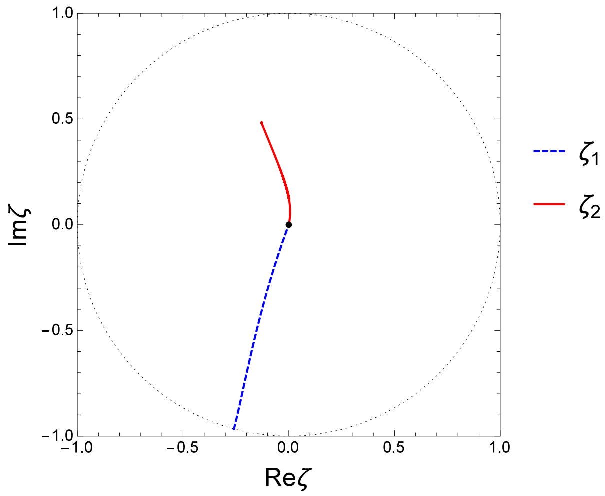

Figure 10. A typical flow in $\omega = \pi/8$ theory obtained by choosing $\sigma_{1\mathrm{IR}} = 0.58351\pi$ and $\zeta_{2\mathrm{IR}} = \frac{\sqrt{2}-\sqrt{6}}{8} + \mathrm{i}\frac{\sqrt{2+\sqrt{3}}}{4}$.

An example of such a flow in $\omega = \pi/8$ theory is plotted in figure 10, for which the IR behavior is given by

$$\begin{aligned} \delta r_1 &= 3.14822\,\rho^2, & \delta\sigma_1 &= -2.87451\,\rho^4 \\ \delta\mathrm{Re}\zeta_2 &= 3.90762\,\rho^{1.75672}, & \delta\mathrm{Im}\zeta_2 &= -9.20492\,\rho^{1.75672}. \end{aligned} \tag{5.17}$$

In this case, it should be noted that the convenient variables to study the perturbation of ζ_2 are ($\delta\mathrm{Re}\zeta_2$, $\delta\mathrm{Im}\zeta_2$) rather than (δr_2, $\delta\sigma_2$). When $\rho \to 0$, the metric appears to be of the same singular form as (4.55). The UV expansion of this solution is characterised by

$$\{\alpha_1, \beta_1, \tilde{\alpha}_1, \tilde{\beta}_1\} = \{-0.985129, -1.38687, 0.179916, 0.365543\}. \tag{5.18}$$

To achieve a proper understanding of the nature of the IR singularities of the geometry, it seems to be indispensable to embed the lower-dimensional solution into the UV-complete string or M-theory. Some tentative lower-dimensional criteria for characterising a physically-allowable IR singularity without reference to string theory were proposed in [43]. However, as pointed out by [51], there exist solutions that satisfy these lower-dimensional criteria but which nonetheless lift to higher-dimensional solutions with unphysical singularities. Since the higher-dimensional origin of the ω-deformed theories is currently unknown, we must necessarily postpone attempting to give a complete interpretation of the ω-deformed supersymmetric domain-wall solutions which generically have IR singularities.

6 Conclusions

This paper has addressed the problem of finding new solutions in the ω-deformed SO(8) gauged $\mathcal{N} = 8$ supergravities. For the the solutions to have holographic dual interpretations, our focus has been on new AdS stationary points and supersymmetric domain-wall solutions asymptotic to the $\mathcal{N} = 8$ AdS vacuum in the UV. The investigation of the new solutions in the complete $\mathcal{N} = 8$ supergravity is extremely complicated, since the theory contains 70 scalars. In order to render the problem tractable, we considered a consistently-truncated sector of the SO(8) gauged $\mathcal{N} = 8$ supergravity, by keeping the fields invariant under the subgroup $\mathrm{SO}(3)_D \times \mathrm{SO}(3)_R$ of SO(8). By construction, the truncated theory preserves $\mathcal{N} = 1$ supersymmetry. Besides the metric, the bosonic sector of the truncated theory consists of four scalar fields, parameterising an $\frac{\mathrm{SL}(2,R)}{\mathrm{SO}(2)} \times \frac{\mathrm{SL}(2,R)}{\mathrm{SO}(2)}$ coset. The scalar potential depends on the ω-parameter explicitly, and can be reformulated using an ω-dependent superpotential in the standard way. We gave an extensive discussion of the stationary points of the scalar potential. In addition to the previously-known G_2 or SO(7)-invariant points, there are two $\mathrm{SO}(3)_D \times \mathrm{SO}(3)_R$-invariant critical points captured by the $\mathrm{SO}(3)_D \times \mathrm{SO}(3)_R$-invariant sector. One of them preserves $\mathcal{N} = 3$ supersymmetry in the full $\mathcal{N} = 8$ theory [23], while the other one, which had not been found previously, is non-supersymmetric but nonetheless stable. The cosmological constants of these two critical points depend on the value of the ω parameter. In each case the value of the cosmological constant diverges in the $\omega \to 0$ limit, indicating the absence of these two new stationary points in the original de Wit-Nicolai theory.

We then looked for supersymmetric domain-wall solutions, which satisfy a set of first-order equations required by the $\mathcal{N} = 1$ supersymmetry. These equations describe a gradient flow in the scalar coset manifold, with the superpotential being the "potential" whose gradient drives the flow. A general feature of the domain-wall solutions is that the scalar fields flow to infinite values at an IR cutoff where the metric becomes singular.

In the original de Wit-Nicolai theory, supersymmetry of the flow solution does not force the pseudoscalar fields to be turned on. In fact, when the pseudoscalars are turned off, explicit supersymmetric domain-wall solutions exist, and they preserve enhanced 16 supercharges. These solutions can lift to solutions in M-theory describing the Coulomb-branch flows on M2-branes spreading out into six possible distributions in the transverse space [19]. We showed explicitly that the VEVs of the dimension-1 primary operators driving the Coulomb-branch flow are encoded in the leading UV expansion coefficients of the scalar fields of the 4-dimensional theory. The physically-allowed IR-singular solutions are the ones sourced by M2-branes of positive tension.

When the pseudoscalars are turned on, the complexity of the flow equations prevents us from obtaining the solutions analytically. We were only able to obtain numerical solutions, by integrating the flow equations from the IR to the UV. The solutions we found correspond to RG flows driven by both the VEVs and also by supersymmetric mass deformations. The supersymmetric mass terms correspond to the non-vanishing UV expansion coefficients of the pseudoscalars. The competition between the VEVs and the mass terms leads to a variety of possible IR singularities in the geometry. Lifting to M-theory, the physical solutions approach the Coulomb-branch flow of dielectric M5-branes wrapping on S^3 in the deep IR.

In the ω-deformed theories, within the same truncated scalar sector, supersymmetry of the domain-wall solutions requires that the pseudoscalars be active. The singular IR behaviors of the solutions are similar to those arising in the $\omega = 0$ case. However, a proper understanding of the nature of these IR singularities demands an embedding into the UV-complete string or M-theory, which is beyond our current knowledge. We hope to return to this problem in future.

Although in this work we have been focusing on supersymmetric domain-wall solutions, it is also worthwhile to study other types of solutions, both supersymmetric and non-supersymmetric. In particular, there may be new scalar hairy AdS black-hole solutions within the $\mathrm{SO}(3)_D \times \mathrm{SO}(3)_R$-invariant sector, which would be the holographic duals of conformal field theories at non-zero temperature. The diverse AdS scalar hairy black holes may bring some new understanding of the phase structures of the dual field theory. In fact, the $\mathrm{SO}(3)_D \times \mathrm{SO}(3)_R$-invariant sector can include an $\mathcal{N} = 1$ vector multiplet. This would provide a new arena within which to look for new U(1)-charged AdS dyonic black holes in SO(8) gauged $\mathcal{N} = 8$ supergravity, in addition to those that were found within the truncation to the STU supergravity model [52–55], and the $\mathrm{SO}(3)\times\mathrm{SO}(3)$-invariant sector that was studied in [21, 22]. The inclusion of both electric and magnetic charges will lead to a richer structure in the phase diagrams for AdS black holes.

Besides the ω-deformed SO(8) gauging, $\mathcal{N} = 8$ supergravity allows other possible dyonic gaugings, which also admit supersymmetric AdS vacua [23]:

- $\mathrm{SO}(1,7)$ and $[\mathrm{SO}(1,1) \times \mathrm{SO}(6)] \ltimes T^{12}$ supergravities with $\mathcal{N} = 4$ supersymmetry.
- $\mathrm{SO}(1,7)$ and $\mathrm{ISO}(1,7)$ gauged supergravities with $\mathcal{N} = 3$ supersymmetry.

Interestingly, the dyonic $\mathrm{ISO}(1,7)$ gauged maximal supergravity can arise from a consistent reduction of massive type IIA supergravity [56], allowing a stringy interpretation of the physical consequences of the dyonic gauging, from which many details of the CFTs dual to the supersymmetric AdS vacua can be deduced. Therefore, it should be worthwhile to study the supersymmetric domain-wall solutions, and other type of solutions, in $\mathcal{N} = 8$ supergravities with different dyonic gaugings.

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A Branching rules and invariant 4-forms

The embedding of the $\mathrm{SO}(3)_D \times \mathrm{SO}(3)_R$ that we are considering in this paper into SO(8) can be described via the chain of embeddings

$$\begin{aligned}&\mathrm{SO}(8) \to \mathrm{SO}(3)\times\mathrm{SO}(5) \to \mathrm{SO}(3)\times\mathrm{SO}(4) \to \mathrm{SO}(3)\times\mathrm{SO}(3)_L\times\mathrm{SO}(3)_R \to \mathrm{SO}(3)_D\times\mathrm{SO}(3)_R\,,\\ &35_s \to (3,10)+(1,5) \to (3,6)+\ldots(1,1)+\ldots \to (3,3,1)+\ldots(1,1,1)\ldots \to (1,1)+\ldots(1,1)\ldots\end{aligned}$$

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The branching of 35_c goes the same way. Starting with $\mathrm{SO}(3)\times\mathrm{SO}(5)$, we have

$$\begin{aligned}
&dx^1\wedge dx^2\wedge dx^3\wedge dx^{\hat a}\in(1,5) && \text{(A.1)}\\
&dx^i\wedge dx^j\wedge dx^{\hat a}\wedge dx^{\hat b}\in(3,10)\text{with } i=1,2,3\in\mathrm{SO}(3)\quad\text{and}\quad \hat a\,\hat b=4\ldots8\in\mathrm{SO}(5).
\end{aligned}$$

Reducing to $\mathrm{SO}(3)\times\mathrm{SO}(4)$ gives

$$\begin{aligned}
&dx^1\wedge dx^2\wedge dx^3\wedge dx^8\in(1,1) && \text{(A.2)}\\
&dx^i\wedge dx^j\wedge dx^a\wedge dx^b\in(3,6)\text{with } i=1,2,3\in\mathrm{SO}(3)\quad\text{and}\quad a\,b=4\ldots7\in\mathrm{SO}(4).
\end{aligned}$$

The decomposition under $\mathrm{SO}(3)\times\mathrm{SO}(3)_L\times\mathrm{SO}(3)_R$ gives

$$\begin{aligned}
&dx^1\wedge dx^2\wedge dx^3\wedge dx^8 &&\in(1,1,1)\\
&\epsilon^i_{jk}\eta^{\hat i}_{ab}dx^i\wedge dx^j\wedge dx^a\wedge dx^b &&\in(3,3,1)\text{ with }\hat\imath=1,2,3\in\mathrm{SO}(3)_L,
\end{aligned}\tag{A.3}$$

where $\eta^{\hat i}_{ab}$ are the 't Hooft matrices.

Finally, the reduction to $\mathrm{SO}(3)_D\times\mathrm{SO}(3)_R$, where $\mathrm{SO}(3)_D$ is the diagonal in $\mathrm{SO}(3)\times\mathrm{SO}(3)_L$, gives

$$\begin{aligned}
&dx^1\wedge dx^2\wedge dx^3\wedge dx^8 &&\in(1,1)\\
&\epsilon^i_{jk}\eta^i_{ab}dx^i\wedge dx^j\wedge dx^a\wedge dx^b &&\in(1,1)\text{ with } i=1,2,3\in\mathrm{SO}(3)_D
\end{aligned}\tag{A.4}$$

Note that i is now taken to be an $\mathrm{SO}(3)_D$ index, and has been contracted. The 8-dimensional Hodge dual forms are also invariant, corresponding to scalars that will be retained. They can be written as

$$\begin{aligned}
&\eta^i_{ab}\eta^i_{cd}dx^a\wedge dx^b\wedge dx^c\wedge dx^d=dx^4\wedge dx^5\wedge dx^6\wedge dx^7\\
&\eta^i_{ab}dx^i\wedge dx^a\wedge dx^b\wedge dx^8
\end{aligned}\tag{A.5}$$

We take the 't Hooft matrices to be given by

$$\eta^1=\begin{pmatrix}0&-1&0&0\\1&0&0&0\\0&0&0&1\\0&0&-1&0\end{pmatrix},\quad \eta^2=\begin{pmatrix}0&0&-1&0\\0&0&0&-1\\1&0&0&0\\0&1&0&0\end{pmatrix},\quad \eta^3=\begin{pmatrix}0&0&0&-1\\0&0&1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix}.\tag{A.6}$$

These transform as the **3** of $so(3)_L$, whose generators are chosen to be

$$B_1=-\frac{1}{2}(R^{01}-R^{23}),\quad B_2=-\frac{1}{2}(R^{02}-R^{31}),\quad B_3=-\frac{1}{2}(R^{03}-R^{12}).\tag{A.7}$$

The 't Hooft matrices are invariant under $so(3)_R$, whose generators are chosen to be

$$A_1=\frac{1}{2}(R^{01}+R^{23}),\quad A_2=\frac{1}{2}(R^{02}+R^{31}),\quad A_3=\frac{1}{2}(R^{03}+R^{12}),\tag{A.8}$$

where the R^{ij} are the $so(4)$ generators, with $(R^{ij})_{ij}=-(R^{ij})_{ji}=1$, and all other elements equal to zero.

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B Conventions

The 8D gamma matrices admits a real representation:

$$\begin{aligned}
&\Gamma^1 = \sigma_2 \otimes \sigma_2 \otimes \sigma_1 \otimes \sigma_0, && \Gamma^2 = \sigma_2 \otimes \sigma_3 \otimes \sigma_0 \otimes \sigma_2,\\
&\Gamma^3 = \sigma_2 \otimes \sigma_0 \otimes \sigma_2 \otimes \sigma_3, && \Gamma^4 = \sigma_2 \otimes \sigma_0 \otimes \sigma_2 \otimes \sigma_1,\\
&\Gamma^5 = \sigma_2 \otimes \sigma_1 \otimes \sigma_0 \otimes \sigma_2, && \Gamma^6 = \sigma_2 \otimes \sigma_2 \otimes \sigma_3 \otimes \sigma_0,\\
&\Gamma^7 = -\sigma_2 \otimes \sigma_2 \otimes \sigma_2 \otimes \sigma_2, && \Gamma^8 = \sigma_1 \otimes \sigma_0 \otimes \sigma_0 \otimes \sigma_0,\\
&\Gamma^9 = \Gamma^1 \dots \Gamma^8 = -\sigma_3 \otimes \sigma_0 \otimes \sigma_0 \otimes \sigma_0, &&
\end{aligned} \tag{B.1}$$

where σ_1, σ_2 and σ_3 are the standard Pauli matrices, and σ_0 denotes the 2×2 identity matrix. The gamma matrices are all block off-diagonal:

$$\Gamma^i = \begin{pmatrix} 0 & \hat{\Gamma}^i_{I\alpha} \\ (\hat{\Gamma}^i)^{\mathrm{T}}_{\alpha I} & 0 \end{pmatrix}. \tag{B.2}$$

The 8×8 matrices $\hat{\Gamma}^i_{I\alpha}$ are the triality tensors, which map between the three 8-dimensional representations of SO(8). Note that since $\hat{\Gamma}^8_{I\alpha} = \delta_{I\alpha}$, the I and α indices are equivalent under SO(7).

The $\mathrm{SO}(7)_+$ invariant tensor $C_{+\alpha\beta\rho\lambda}$ is proportional to $\sum_{i,j=1,7} \hat{\Gamma}^{ij\mathrm{T}}_{[\alpha\beta}\hat{\Gamma}^{ij\mathrm{T}}_{\rho\lambda]}$, whilst The $\mathrm{SO}(7)_-$ invariant tensor C_{-IJKL} is proportional to $\sum_{i,j=1,7} \hat{\Gamma}^{ij}_{[IJ}\hat{\Gamma}^{ij}_{KL]}$.

In the construction of the internal metric on the deformed 7-sphere when calculating the lifting from four to eleven dimensions, it is convenient to use a parameterisation of the eight Euclidean coordinates X^I describing the embedding of the unit S^7, with $X^I X^I = 1$, that is adapted to the symmetries of the system. We find that it is convenient to write

$$\begin{aligned}
&X^1 = \sin\chi\, \cos\xi\, \cos\tilde{\theta}\, \cos\varphi_1\,, && X^2 = \sin\chi\, \cos\xi\, \cos\tilde{\theta}\, \sin\varphi_1,\\
&X^3 = \sin\chi\, \cos\xi\, \sin\tilde{\theta}\, \cos\varphi_2\,, && X^4 = \sin\chi\, \cos\xi\, \sin\tilde{\theta}\, \sin\varphi_2,\\
&X^5 = \sin\chi\, \sin\xi\, \sin\theta\, \cos\varphi_3\,, && X^6 = \sin\chi\, \sin\xi\, \sin\theta\, \sin\varphi_3,\\
&X^7 = \sin\chi\, \sin\xi\, \cos\theta\,, && X^8 = \cos\chi.
\end{aligned} \tag{B.3}$$

The metric $d\Omega_7^2 = \delta_{AB} dX^A dX^B$ on the unit S^7 is now given by

$$d\Omega_7^2 = d\chi^2 + \sin^2\chi\, d\Omega_6^2, \tag{B.4}$$

with the metric $d\Omega_6^2$ on the unit S^6 given in terms of metrics $d\Omega_2^2$ and $d\Omega_3^2$ on a unit S^2 and a unit S^3 by

$$\begin{aligned}
&d\Omega_6^2 = d\xi^2 + \sin^2\xi\, d\Omega_2^2 + \cos^2\xi\, d\Omega_3^2,\\
&d\Omega_2^2 = d\theta^2 + \sin^2\theta\, d\varphi_3^2\,, \qquad d\Omega_3^2 = d\tilde{\theta}^2 + \cos^2\tilde{\theta}\, d\varphi_1^2 + \sin^2\tilde{\theta}\, d\varphi_2^2.
\end{aligned} \tag{B.5}$$

As a check of our uplift procedure, we can present explicitly the internal metric for the simpler G_2-invariant sector, which can be obtained from the $\mathrm{SO}(3)_D \times \mathrm{SO}(3)_R$ invariant sector by setting $\phi_1 = \phi_2$ and $\sigma_1 = -\sigma_2$. Using the uplift ansatz (4.58), we derive the internal metric for the G_2 invariant sector

$$ds_7^2 = \Delta^{-1}\left(a^{-3}d\chi^2 + \frac{\sin^2\chi}{a\left(a^2\cos^2\chi + b^2\sin\chi^2\right)}\,d\Omega_6^2\right), \tag{B.6}$$

where

$$\begin{aligned} a &= \cosh\phi_1 - \cos\sigma_1 \sinh\phi_1, \\ b &= \cosh\phi_1 + \cos\sigma_1 \sinh\phi_1, \\ \Delta &= a^{-1}\left(a^2\cos^2\chi + b^2\sin^2\chi\right)^{-2/3}. \end{aligned} \tag{B.7}$$

This result matches with the one given in [14].

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