Published for SISSA by Springer

Received: April 14, 2015
Revised: June 16, 2015
Accepted: July 15, 2015
Published: August 4, 2015

Generalized supersymmetric cosmological term in N=1 supergravity

P.K. Concha,[a,b,c] **E.K. Rodríguez**[a,b,c] **and P. Salgado**[a]

[a] *Departamento de Física, Universidad de Concepción, Casilla 160-C, Concepción, Chile*
[b] *Dipartimento di Scienza Applicata e Tecnologia (DISAT), Politecnico di Torino, Corso Duca degli Abruzzi 24, I-10129 Torino, Italia*
[c] *Istituto Nazionale di Fisica Nucleare (INFN) Sezione di Torino, Via Pietro Giuria 1, 10125 Torino, Italia*

E-mail: patrickconcha@udec.cl, everodriguez@udec.cl, pasalgad@udec.cl

Abstract: An alternative way of introducing the supersymmetric cosmological term in a supergravity theory is presented. We show that the AdS-Lorentz superalgebra allows to construct a geometrical formulation of supergravity containing a generalized supersymmetric cosmological constant. The $N = 1$, $D = 4$ supergravity action is built only from the curvatures of the AdS-Lorentz superalgebra and corresponds to a MacDowell-Mansouri like action. The extension to a generalized AdS-Lorentz superalgebra is also analyzed.

Keywords: Supergravity Models, Gauge Symmetry

ArXiv ePrint: 1504.01898

doi:10.1007/JHEP08(2015)009

Contents

1 Introduction

A good candidate to describe the dark energy corresponds to the cosmological constant [1, 2]. It is well known that a cosmological term can be introduced in a $D = 4$ gravity theory using the Anti de Sitter (AdS) algebra. In particular the supersymmetric extension of gravity including a cosmological term can be obtained in a geometric formulation. In this framework, supergravity is built from the curvatures of the $\mathfrak{osp}\,(4|1)$ superalgebra and the action is known as the MacDowell-Mansouri action [3].

Recently it was presented in ref. [4] an alternative way of introducing the generalized cosmological constant term using the Maxwell algebra. It is usually accepted that the symmetries of Minkowski spacetime are described by the Poincaré algebra. In refs. [5, 6] this spacetime was generalized extending its symmetries from the Poincaré to the Maxwell symmetries whose generators satisfy the following commutation relations

$$[P_a, P_b] = \Lambda Z_{ab}, \qquad Z_{ab} = -Z_{ba}\ , \tag{1.1}$$

$$[J_{ab}, Z_{cd}] = \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc}, \tag{1.2}$$

$$[Z_{ab}, Z_{cd}] = 0, \qquad [Z_{ab}, P_c] = 0. \tag{1.3}$$

Here Z_{ab} correspond to tensorial Abelian charges and the constant Λ can be related to the cosmological constant when $[\Lambda] = M^2$. If we put $\Lambda = e$, where e is the electromagnetic coupling constant, we have the possible description of spacetime in presence of a constant electromagnetic background field.

The deformations of the Maxwell symmetries lead to the $\mathfrak{so}(D-1,2) \oplus \mathfrak{so}(D-1,1)$ or $\mathfrak{so}(D,1) \oplus \mathfrak{so}(D-1,1)$ algebra [7, 8]. In this case the Z_{ab} generators are non-abelian. If spacetime symmetries are considered local symmetries then it is possible to construct Chern-Simons gravity actions where dark energy could be interpreted as part of the metric of spacetime.

Subsequently it was shown in ref. [9] that the generalized cosmological constant term can also be included in a Born-Infeld like action built from the curvatures of the AdS-Lorentz[1] ($AdS - \mathcal{L}_4$) algebra. Alternatively the AdS-Lorentz algebra can be obtained as an abelian semigroup expansion (S-expansion) of the AdS algebra using $S_{\mathcal{M}}^{(2)}$ as the relevant semigroup [10].

The S-expansion procedure is based on combining the multiplication law of a semigroup S with the structure constants of a Lie algebra $\mathfrak{g}$ [11]. The new Lie algebra obtained using this method is called the S-expanded algebra $\mathfrak{G} = S \times \mathfrak{g}$. Diverse (super)gravity theories have been extensively studied using the S-expansion approach. In particular, interesting results have been obtained in refs. [12–21]. An alternative expansion method can be found in ref. [22].

In this paper we analyze the consequence of considering the supersymmetric extension of the AdS-Lorentz algebra in the construction of a supergravity theory. This superalgebra has the following anticommutation relation,

$$\{Q_\alpha, Q_\beta\} = -\frac{1}{2}\left[\left(\gamma^{ab}C\right)_{\alpha\beta} Z_{ab} - 2\left(\gamma^a C\right)_{\alpha\beta} P_a\right], \tag{1.4}$$

where Q_α represents a 4-component Majorana spinor charge. Unlike the Maxwell superalgebra the new generators Z_{ab} are not abelian and behave as a Lorentz generator,

$$[Z_{ab}, Z_{cd}] = \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc}. \tag{1.5}$$

The presence of the Z_{ab} generators implies the introduction of a new bosonic "matter" field k^{ab} which modifies the definition of the different curvatures. In particular, we are interested in studying the geometrical consequences of including the generators $Z_{ab} = [P_a, P_b]$ in supergravity. Although the same non-commutativity is present in the Maxwell symmetries, it was shown in ref. [20] that the supergravity action à la MacDowell-Mansouri based on the Maxwell superalgebra does not reproduce the cosmological constant term in the action. Then, the AdS-Lorentz superalgebra seems to be a better candidate in order to introduce the cosmological term in supergravity, in presence of the bosonic generators Z_{ab}.

On the other hand, as shown in ref. [23, 24] the four-dimensional renormalized action for AdS gravity, which corresponds to the bosonic MacDowell-Mansouri action, is equivalent on-shell to the square of the Weyl tensor describing conformal gravity. Then, the supergravity action à la MacDowell-Mansouri suggests a superconformal structure which represents an additional motivation in our construction.

It is the purpose of this work to construct a supergravity action which contains a generalized supersymmetric cosmological constant from the AdS-Lorentz superalgebra. To this aim, we apply the S-expansion method to the $\mathfrak{osp}(4|1)$ superalgebra and we build a MacDowell-Mansouri like action with the expanded 2-form curvatures. The result presented here corresponds to an alternative way of introducing the supersymmetric cosmological term and can be seen as the supersymmetric extension of refs. [4, 9]. We extend our result introducing the generalized minimal AdS-Lorentz superalgebra and we build a more general $D = 4$, $N = 1$ supergravity action involving a supersymmetric cosmological term.

[1] Also known as $\mathfrak{so}(D-1,1) \oplus \mathfrak{so}(D-1,2)$ algebra.

JHEP08(2015)009

This work is organized as follows: in section 2 we review the construction of the AdS-Lorentz superalgebra using the S-expansion procedure. Sections 3 and 4 contain our main results. In section 3, we present the $D = 4$, $N = 1$ supergravity action including a generalized supersymmetric cosmological constant. We show that this action corresponds to a MacDowell-Mansouri like action built from the curvatures of the AdS-Lorentz superalgebra. In section 4 we extend our results to the generalized minimal AdS-Lorentz superalgebra. Section 5 concludes the work with some comments about possible development and usefulness of our results.

2 *AdS*−Lorentz superalgebra and the abelian semigroup expansion procedure

The abelian semigroup expansion procedure (S-expansion) is a powerful tool in order to derive new Lie (super)algebras [11]. Furthermore, the S-expansion method has the advantage to provide with an invariant tensor for the S-expanded algebra $\mathfrak{G} = S \times \mathfrak{g}$ in terms of an invariant tensor for the original algebra $\mathfrak{g}$.

Following refs. [11, 18], it is possible to obtain the AdS−Lorentz superalgebra as an S-expansion of the $\mathfrak{osp}\,(4|1)$ superalgebra using $S_{\mathcal{M}}^{(2)}$ as the abelian semigroup.

Before applying the S-expansion method it is necessary to consider a decomposition of the original algebra $\mathfrak{g} = \mathfrak{osp}\,(4|1)$ in subspaces V_p,

$$\begin{aligned} \mathfrak{g} = \mathfrak{osp}\,(4|1) &= \mathfrak{so}\,(3,1) \oplus \frac{\mathfrak{osp}\,(4|1)}{\mathfrak{sp}\,(4)} \oplus \frac{\mathfrak{sp}\,(4)}{\mathfrak{so}\,(3,1)} \\ &= V_0 \oplus V_1 \oplus V_2, \end{aligned} \tag{2.1}$$

where V_0 is generated by the Lorentz generator $\tilde{J}_{ab}$, V_1 corresponds to the fermionic subspace generated by a 4-component Majorana spinor charge $\tilde{Q}_\alpha$ and V_2 corresponds to the AdS boost generated by $\tilde{P}_a$. The $\mathfrak{osp}\,(4|1)$ generators satisfy the following (anti)commutation relations

$$\left[\tilde{J}_{ab}, \tilde{J}_{cd}\right] = \eta_{bc}\tilde{J}_{ad} - \eta_{ac}\tilde{J}_{bd} - \eta_{bd}\tilde{J}_{ac} + \eta_{ad}\tilde{J}_{bc}, \tag{2.2}$$

$$\left[\tilde{J}_{ab}, \tilde{P}_c\right] = \eta_{bc}\tilde{P}_a - \eta_{ac}\tilde{P}_b, \tag{2.3}$$

$$\left[\tilde{P}_a, \tilde{P}_b\right] = \tilde{J}_{ab}, \tag{2.4}$$

$$\left[\tilde{J}_{ab}, \tilde{Q}_\alpha\right] = -\frac{1}{2}\left(\gamma_{ab}\tilde{Q}\right)_\alpha, \qquad \left[\tilde{P}_a, \tilde{Q}_\alpha\right] = -\frac{1}{2}\left(\gamma_a\tilde{Q}\right)_\alpha, \tag{2.5}$$

$$\left\{\tilde{Q}_\alpha, \tilde{Q}_\beta\right\} = -\frac{1}{2}\left[\left(\gamma^{ab}C\right)_{\alpha\beta}\tilde{J}_{ab} - 2\left(\gamma^a C\right)_{\alpha\beta}\tilde{P}_a\right]. \tag{2.6}$$

Here, γ_a are Dirac matrices and C stands for the charge conjugation matrix.

The subspace structure may be written as

$$\begin{aligned} &[V_0, V_0] \subset V_0, \quad [V_1, V_1] \subset V_0 \oplus V_2, \\ &[V_0, V_1] \subset V_1, \quad [V_1, V_2] \subset V_1, \\ &[V_0, V_2] \subset V_2, \quad [V_2, V_2] \subset V_0. \end{aligned} \tag{2.7}$$

JHEP08(2015)009

Following the definitions of ref. [11], let $S_{\mathcal{M}}^{(2)} = \{\lambda_0, \lambda_1, \lambda_2\}$ be an abelian semigroup whose elements satisfy the multiplication law,

$$\lambda_\alpha \lambda_\beta = \begin{cases} \lambda_{\alpha+\beta}, & \text{if } \alpha+\beta \leq 2 \\ \lambda_{\alpha+\beta-2}, & \text{if } \alpha+\beta > 2 \end{cases} \tag{2.8}$$

Let us consider the subset decomposition $S_{\mathcal{M}}^{(2)} = S_0 \cup S_1 \cup S_2$, with

$$S_0 = \{\lambda_0, \lambda_2\}\,, \tag{2.9}$$
$$S_1 = \{\lambda_1\}\,, \tag{2.10}$$
$$S_2 = \{\lambda_2\}\,. \tag{2.11}$$

One sees that this decomposition is said to be resonant since it satisfies the same structure as the subspaces V_p [compare with eqs. (2.7)]

$$\begin{array}{ll} S_0 \cdot S_0 \subset S_0, & S_1 \cdot S_1 \subset S_0 \cap S_2, \\ S_0 \cdot S_1 \subset S_1, & S_1 \cdot S_2 \subset S_1, \\ S_0 \cdot S_2 \subset S_2, & S_2 \cdot S_2 \subset S_0. \end{array} \tag{2.12}$$

Following theorem IV.2 of ref. [11], we can say that the superalgebra

$$\mathfrak{G}_R = W_0 \oplus W_1 \oplus W_2, \tag{2.13}$$

is a resonant subalgebra of $S_{\mathcal{M}}^{(2)} \times \mathfrak{g}$, where

$$W_0 = (S_0 \times V_0) = \{\lambda_0, \lambda_2\} \times \left\{\tilde{J}_{ab}\right\} = \left\{\lambda_0 \tilde{J}_{ab}, \lambda_2 \tilde{J}_{ab}\right\}, \tag{2.14}$$
$$W_1 = (S_1 \times V_1) = \{\lambda_1\} \times \left\{\tilde{Q}_\alpha\right\} = \left\{\lambda_1 \tilde{Q}_\alpha\right\}, \tag{2.15}$$
$$W_2 = (S_2 \times V_2) = \{\lambda_2\} \times \left\{\tilde{P}_a\right\} = \left\{\lambda_2 \tilde{P}_a\right\}. \tag{2.16}$$

Thus the new superalgebra is generated by $\{J_{ab}, P_a, Z_{ab}, Q_\alpha\}$, where these new generators can be written as

$$\begin{aligned} J_{ab} &= \lambda_0 \tilde{J}_{ab}, \\ Z_{ab} &= \lambda_2 \tilde{J}_{ab}, \\ P_a &= \lambda_2 \tilde{P}_a, \\ Q_\alpha &= \lambda_1 \tilde{Q}_\alpha. \end{aligned}$$

The expanded generators satisfy the (anti)commutation relations

$$[J_{ab}, J_{cd}] = \eta_{bc} J_{ad} - \eta_{ac} J_{bd} - \eta_{bd} J_{ac} + \eta_{ad} J_{bc}, \tag{2.17}$$
$$[J_{ab}, Z_{cd}] = \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc}, \tag{2.18}$$
$$[Z_{ab}, Z_{cd}] = \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc}, \tag{2.19}$$
$$[J_{ab}, P_c] = \eta_{bc} P_a - \eta_{ac} P_b, \qquad [P_a, P_b] = Z_{ab}, \tag{2.20}$$

$$[Z_{ab}, P_c] = \eta_{bc} P_a - \eta_{ac} P_b, \tag{2.21}$$

$$[J_{ab}, Q_\alpha] = -\frac{1}{2} (\gamma_{ab} Q)_\alpha, \qquad [P_a, Q_\alpha] = -\frac{1}{2} (\gamma_a Q)_\alpha, \tag{2.22}$$

$$[Z_{ab}, Q_\alpha] = -\frac{1}{2} (\gamma_{ab} Q)_\alpha, \tag{2.23}$$

$$\{Q_\alpha, Q_\beta\} = -\frac{1}{2} \left[\left(\gamma^{ab} C \right)_{\alpha\beta} Z_{ab} - 2 (\gamma^a C)_{\alpha\beta} P_a \right], \tag{2.24}$$

where we have used the multiplication law of the semigroup (2.8) and the commutation relations of the original superalgebra. The new superalgebra obtained after a resonant $S_{\mathcal{M}}^{(2)}$-expansion of $\mathfrak{osp}(4|1)$ corresponds to the $AdS-$Lorentz superalgebra $sAdS - \mathcal{L}_4$ in four dimensions. The details of its construction can be found in ref. [18]. An extensive study of the relation between Lie algebras and the semigroup expansion method can be found in ref. [25].

One can see that the $AdS-$Lorentz superalgebra contains the $AdS - \mathcal{L}_4$ algebra $= \{J_{ab}, P_a, Z_{ab}\}$[2] as a subalgebra. The $AdS - \mathcal{L}_4$ algebra and its generalization have been extensively studied in ref. [9]. In particular it was shown that this algebra allows to include a generalized cosmological constant in a Born-Infeld gravity action.

On the other hand it is well known that an Inönü-Wigner contraction of the $AdS-$Lorentz superalgebra leads to the Maxwell superalgebra. In fact, the rescaling

$$Z_{ab} \to \mu^2 Z_{ab}, \quad P_a \to \mu P_a \quad \text{and} \quad Q_\alpha \to \mu Q_\alpha \tag{2.25}$$

provide the Maxwell superalgebra in the limit $\mu \to \infty$.

3 Generalized supersymmetric cosmological term from $AdS-$Lorentz superalgebra

In ref. [3] it was introduced a geometric formulation of $N = 1$, $D = 4$ supergravity using the $\mathfrak{osp}(4|1)$ gauge fields. The resulting action is known as the MacDowell-Mansouri action whose geometrical interpretation can be found in ref. [28]. In a very similar way to ref. [20] in which a MacDowell-Mansouri like action was built for the minimal Maxwell superalgebra, we will construct an action for the AdS-Lorentz superalgebra using the useful properties of the S-expansion procedure.

We have shown in the previous section that the $D = 4$ $AdS-$Lorentz superalgebra can be found as an S-expansion of the $\mathfrak{osp}(4|1)$ superalgebra. Following the definitions of ref. [11], let $S_{\mathcal{M}}^{(2)} = \{\lambda_0, \lambda_1, \lambda_2\}$ be an abelian semigroup whose elements satisfy the multiplication law (2.8). After the extraction of a resonant subalgebra one finds the $AdS-$Lorentz superalgebra whose generators $\{J_{ab}, P_a, Z_{ab}, Q_\alpha\}$ satisfy the commutations relations (2.17)–(2.24).

In order to write down an action for $AdS-$Lorentz superalgebra we start from the one-form connection

$$A = A^A T_A = \frac{1}{2}\omega^{ab} J_{ab} + \frac{1}{l} e^a P_a + \frac{1}{2} k^{ab} Z_{ab} + \frac{1}{\sqrt{l}} \psi^\alpha Q_\alpha, \tag{3.1}$$

[2] Also known as Poincaré semi-simple extended algebra.

where the one-form gauge fields are given in terms of the components of the $\mathfrak{osp}\,(4|1)$ connection,

$$
\begin{aligned}
\omega^{ab} &= \lambda_0 \tilde{\omega}^{ab}, \\
e^{a} &= \lambda_2 \tilde{e}^{a}, \\
k^{ab} &= \lambda_2 \tilde{\omega}^{ab}, \\
\psi^{\alpha} &= \lambda_1 \tilde{\psi}^{\alpha}.
\end{aligned}
$$

The associated two-form curvature $F = dA + A \wedge A$ is given by

$$
F = F^{A} T_{A} = \frac{1}{2} R^{ab} J_{ab} + \frac{1}{l} R^{a} P_{a} + \frac{1}{2} F^{ab} Z_{ab} + \frac{1}{\sqrt{l}} \Psi^{\alpha} Q_{\alpha}, \tag{3.2}
$$

where

$$
\begin{aligned}
R^{ab} &= d\omega^{ab} + \omega^{a}_{\ c} \omega^{cb}, \\
R^{a} &= de^{a} + \omega^{a}_{\ b} e^{b} + k^{a}_{\ b} e^{b} - \frac{1}{2} \bar{\psi} \gamma^{a} \psi, \\
F^{ab} &= dk^{ab} + \omega^{a}_{\ c} k^{cb} - \omega^{b}_{\ c} k^{ca} + k^{a}_{\ c} k^{cb} + \frac{1}{l^{2}} e^{a} e^{b} + \frac{1}{2l} \bar{\psi} \gamma^{ab} \psi, \\
\Psi &= d\psi + \frac{1}{4} \omega_{ab} \gamma^{ab} \psi + \frac{1}{2l} e^{a} \gamma_{a} \psi + \frac{1}{4} k_{ab} \gamma^{ab} \psi.
\end{aligned}
$$

The one-forms ω^{ab}, e^{a}, ψ and k^{ab} are the spin connection, the vielbein, the gravitino field and a bosonic "matter" field, respectively. Here ψ corresponds to a Majorana spinor which satisfies $\bar{\psi} = \psi C$, where C is the charge conjugation matrix. Naturally when $F = 0$ the Maurer-Cartan equations for the $AdS-$Lorentz superalgebra are satisfied.

In order to interpret the gauge field as the vielbein, it is necessary to introduce a length scale l. In fact, if we choose the Lie algebra generators T_A to be dimensionless then the 1-form connection fields $A = A^{A}_{\mu} T_{A} dx^{\mu}$ must also be dimensionless. Nevertheless, the vielbein $e^{a} = e^{a}_{\ \mu} dx^{\mu}$ must have dimensions of length if it is related to the spacetime metric $g_{\mu\nu}$ through $g_{\mu\nu} = e^{a}_{\ \mu} e^{b}_{\ \nu} \eta_{ab}$. Thus the "true" gauge field must be of the form e^{a}/l. In the same way we must consider that $\psi/\sqrt{l}$ is the "true" gauge field of supersymmetry since the gravitino $\psi = \psi_{\mu} dx^{\mu}$ has dimensions of $(\text{length})^{1/2}$.

From the Bianchi identity $\nabla F = 0$, with $\nabla = d + [A, \cdot]$, it is possible to write down the Lorentz covariant exterior derivatives of the curvatures as

$$
DR^{ab} = 0, \tag{3.3}
$$

$$
DR^{a} = R^{a}_{\ b} e^{b} + R^{c} k_{c}^{\ a} + \bar{\psi} \gamma^{a} \Psi, \tag{3.4}
$$

$$
\begin{aligned}
DF^{ab} = {} & R^{a}_{\ c} k^{cb} - R^{b}_{\ c} k^{ca} + F^{a}_{\ c} k^{cb} - F^{b}_{\ c} k^{ca} + \frac{1}{l^{2}} \left(R^{a} e^{b} - e^{a} R^{b} \right) \\
& + \frac{1}{l} \bar{\Psi} \gamma^{ab} \psi,
\end{aligned} \tag{3.5}
$$

$$
\begin{aligned}
D\Psi = {} & \frac{1}{4} R_{ab} \gamma^{ab} \psi + \frac{1}{4} F_{ab} \gamma^{ab} \psi - \frac{1}{4} k_{ab} \gamma^{ab} \Psi + \frac{1}{2l} R^{a} \gamma_{a} \psi \\
& - \frac{1}{2l} e^{a} \gamma_{a} \Psi.
\end{aligned} \tag{3.6}
$$

JHEP08(2015)009

The general form of the MacDowell-Mansouri action built with the $\mathfrak{osp}\,(4|1)$ two-form curvature is given by

$$S = 2\int \langle F \wedge F \rangle = 2\int F^A \wedge F^B \langle T_A T_B \rangle \,, \tag{3.7}$$

with the following choice of the invariant tensor

$$\langle T_A T_B \rangle = \begin{cases} \langle J_{ab} J_{cd} \rangle = \epsilon_{abcd} \\ \langle Q_\alpha Q_\beta \rangle = 2\,(\gamma_5)_{\alpha\beta} \,. \end{cases} \tag{3.8}$$

It is important to note that if $\langle T_A T_B \rangle$ is an invariant tensor for the $\mathfrak{osp}\,(4|1)$ superalgebra then the action corresponds to a topological invariant. The action can be seen as the supersymmetric generalization of the $D = 4$ Born-Infeld action in which the action is built from the AdS two-form curvature using $\langle T_A T_B \rangle$ as an invariant tensor for the Lorentz group.

In order to build a MacDowell-Mansouri like action for the $AdS-$Lorentz superalgebra we will consider the S-expansion of $\langle T_A T_B \rangle$ and the 2-form curvature given by (3.2).

Thus, the action for the $AdS-$Lorentz superalgebra can be written as

$$S = 2\int F^A \wedge F^B \langle T_A T_B \rangle_{sAdS-\mathcal{L}_4} \,, \tag{3.9}$$

where $\langle T_A T_B \rangle_{sAdS-\mathcal{L}_4}$ can be derived from the original components of the invariant tensor (3.8). Using Theorem VII.1 of ref. [11], it is possible to show that the non-vanishing components of $\langle T_A T_B \rangle_{sAdS-\mathcal{L}_4}$ are given by

$$\langle J_{ab} J_{cd} \rangle_{sAdS-\mathcal{L}_4} = \alpha_0 \langle J_{ab} J_{cd} \rangle \,, \tag{3.10}$$

$$\langle J_{ab} Z_{cd} \rangle_{sAdS-\mathcal{L}_4} = \alpha_2 \langle J_{ab} J_{cd} \rangle \,, \tag{3.11}$$

$$\langle Z_{ab} Z_{cd} \rangle_{sAdS-\mathcal{L}_4} = \alpha_2 \langle J_{ab} J_{cd} \rangle \,, \tag{3.12}$$

$$\langle Q_\alpha Q_\beta \rangle_{sAdS-\mathcal{L}_4} = \alpha_2 \langle Q_\alpha Q_\beta \rangle \,, \tag{3.13}$$

where α_0 and α_2 are dimensionless arbitrary independent constants. This choice of the invariant tensor breaks the AdS-Lorentz supergroup to their Lorentz like subgroup.

Then considering the non-vanishing components of the invariant tensor (3.10)–(3.13) and the 2-form curvature (3.2), it is possible to write down an action as

$$S = 2\int \left(\frac{1}{4}\alpha_0 \epsilon_{abcd} R^{ab} R^{cd} + \frac{1}{2}\alpha_2 \epsilon_{abcd} R^{ab} F^{cd} + \frac{1}{4}\alpha_2 \epsilon_{abcd} F^{ab} F^{cd} + \frac{2}{l}\alpha_2 \bar{\Psi}\gamma_5 \Psi \right). \tag{3.14}$$

Explicitly, the action takes the form

$$\begin{aligned} S = \int \frac{\alpha_0}{2}\epsilon_{abcd} R^{ab} R^{cd} + \alpha_2 \epsilon_{abcd} \Big(& R^{ab} Dk^{cd} + R^{ab} k^c{}_e k^{ed} + \frac{1}{l^2} R^{ab} e^c e^d \\ & + \frac{1}{2l} R^{ab} \bar{\psi}\gamma^{cd}\psi + \frac{1}{2} Dk^{ab} Dk^{cd} + Dk^{ab} k^c{}_e k^{ed} + \frac{1}{l^2} Dk^{ab} e^c e^d \\ & + \frac{1}{2l} Dk^{ab} \bar{\psi}\gamma^{cd}\psi + \frac{1}{2} k^a{}_f k^{fb} k^c{}_g k^{gd} + \frac{1}{l^2} k^a{}_f k^{fb} e^c e^d + \frac{1}{2l} k^a{}_f k^{fb} \bar{\psi}\gamma^{cd}\psi \\ & + \frac{1}{2l^3} e^a e^b \bar{\psi}\gamma^{cd}\psi + \frac{1}{2l^4} e^a e^b e^c e^d \Big) + \alpha_2 \left(\frac{4}{l} D\bar{\psi}\gamma_5 D\psi + \frac{4}{l^2}\bar{\psi} e^a \gamma_a \gamma_5 D\psi \right. \end{aligned}$$

$$+\frac{2}{l}D\bar{\psi}\gamma_5 k_{ab}\gamma^{ab}\psi+\frac{1}{l^3}\bar{\psi}e^a\gamma_a\gamma_5 e^b\gamma_b\psi+\frac{1}{l^2}\bar{\psi}e^a\gamma_a\gamma_5 k^{bc}\gamma_{bc}\psi$$
$$+\frac{1}{4l}\bar{\psi}k_{ab}\gamma^{ab}\gamma_5 k_{cd}\gamma^{cd}\psi\Big). \tag{3.15}$$

The action can be written in a more compact way using the gamma matrix identity

$$\gamma_{ab}\gamma_5=-\frac{1}{2}\epsilon_{abcd}\gamma^{cd}, \tag{3.16}$$

and the gravitino Bianchi identity

$$DD\psi=\frac{1}{4}R^{ab}\gamma_{ab}\psi. \tag{3.17}$$

In fact one can see that

$$\frac{1}{2}\epsilon_{abcd}R^{ab}\bar{\psi}\gamma^{cd}\psi+4D\bar{\psi}\gamma_5 D\psi=d\left(4D\bar{\psi}\gamma_5\psi\right),$$
$$\frac{1}{2}\epsilon_{abcd}Dk^{ab}\bar{\psi}\gamma^{cd}\psi+2D\bar{\psi}\gamma_5 k^{ab}\gamma_{ab}\psi=d\left(\bar{\psi}k^{ab}\gamma_{ab}\gamma_5\psi\right).$$

Furthermore it is possible to show that

$$\bar{\psi}e^a\gamma_a\gamma_5 e^b\gamma_b\psi=\frac{1}{2}e^a e^b\bar{\psi}\gamma^{cd}\psi\epsilon_{abcd},$$
$$\frac{1}{4}\bar{\psi}k_{ab}\gamma^{ab}\gamma_5 k_{cd}\gamma^{cd}\psi=-\frac{1}{2}k^a{}_f k^{fb}\bar{\psi}\gamma^{cd}\psi\epsilon_{abcd},$$
$$\bar{\psi}e^a\gamma_a\gamma_5 k^{bc}\gamma_{bc}\psi=\epsilon_{abcd}k^{ab}e^c\bar{\psi}\gamma^d\psi,$$

where we have used the following identities

$$\gamma_a\gamma_b=\gamma_{ab}+\eta_{ab},$$
$$\gamma^{ab}\gamma^{cd}=\epsilon^{abcd}\gamma_5-4\delta^{[a}_{[c}\gamma^{b]}_{d]}-2\delta^{ab}_{cd},$$
$$\gamma^c\gamma^{ab}=-2\gamma^{[a}\delta^{b]}_c-\epsilon^{abcd}\gamma_5\gamma_d,$$

and the fact that $\gamma_5\gamma_a$ is an antisymmetric matrix. Thus the MacDowell-Mansouri like action for the $AdS-$Lorentz superalgebra takes the form

$$S=\int\frac{\alpha_0}{2}\epsilon_{abcd}R^{ab}R^{cd}+\frac{\alpha_2}{l^2}\left(\epsilon_{abcd}R^{ab}e^c e^d+4\bar{\psi}e^a\gamma_a\gamma_5 D\psi\right)$$
$$+\alpha_2\epsilon_{abcd}\left(R^{ab}Dk^{cd}+R^{ab}k^c{}_e k^{ed}+\frac{1}{2}Dk^{ab}Dk^{cd}+Dk^{ab}k^c{}_e k^{ed}+\frac{1}{2}k^a{}_f k^{fb}k^c{}_g k^{gd}\right)$$
$$+\alpha_2\epsilon_{abcd}\left(\frac{1}{l^2}Dk^{ab}e^c e^d+\frac{1}{l^2}k^a{}_f k^{fb}e^c e^d+\frac{1}{l^3}e^a e^b\bar{\psi}\gamma^{cd}\psi\right.$$
$$\left.+\frac{1}{l^2}k^{ab}e^c\bar{\psi}\gamma^d\psi+\frac{1}{2l^4}e^a e^b e^c e^d\right)+\alpha_2 d\left(4D\bar{\psi}\gamma_5\psi+\bar{\psi}k^{ab}\gamma_{ab}\gamma_5\psi\right). \tag{3.18}$$

This action has been intentionally separated in five pieces where the first term is proportional to α_0 and corresponds to the Gauss Bonnet term. The second term contains the

Einstein-Hilbert term plus the Rarita-Schwinger (RS) Lagrangian describing pure supergravity. The third piece corresponds to a Gauss Bonnet like term containing the new super $AdS-$Lorentz fields. This piece does not contribute to the dynamics and can be written as a boundary term. The fourth term corresponds to a generalized supersymmetric cosmological term which contains the usual supersymmetric cosmological constant plus three additional terms depending on k^{ab}. The last piece is a boundary term.

One can see that the MacDowell-Mansouri like action built using the useful definitions of the S-expansion procedure describes a supergravity theory with a generalized supersymmetric cosmological term.

From (3.18) we can see that the bosonic part of the action corresponds to the one found for $AdS-$Lorentz algebra in ref. [9]. Besides, the action contains the generalized cosmological term introduced in ref. [4] for the Maxwell algebra.

One can note that if we omit the boundary terms in (3.18), the action can be written as

$$\begin{aligned} S = \int \frac{\alpha_2}{l^2} \left(\epsilon_{abcd} R^{ab} e^c e^d + 4\bar{\psi} e^a \gamma_a \gamma_5 D\psi \right) + \alpha_2 \epsilon_{abcd} \left(\frac{1}{l^2} Dk^{ab} e^c e^d + \frac{1}{l^2} k^a{}_f k^{fb} e^c e^d \right. \\ \left. + \frac{1}{l^3} e^a e^b \bar{\psi} \gamma^{cd} \psi + \frac{1}{l^2} k^{ab} e^c \bar{\psi} \gamma^d \psi + \frac{1}{2l^4} e^a e^b e^c e^d \right), \end{aligned} \tag{3.19}$$

or equivalently

$$\begin{aligned} S = \int \frac{\alpha_2}{l^2} \left(\epsilon_{abcd} R^{ab} e^c e^d + 4\bar{\psi} e^a \gamma_a \gamma_5 D\psi \right) \\ + \alpha_2 \epsilon_{abcd} \left(\frac{2}{l^2} k^{ab} \hat{T}^c e^d + \frac{1}{l^2} k^a{}_f k^{fb} e^c e^d + \frac{1}{l^3} e^a e^b \bar{\psi} \gamma^{cd} \psi + \frac{1}{2l^4} e^a e^b e^c e^d \right), \end{aligned} \tag{3.20}$$

where we have used

$$\epsilon_{abcd} Dk^{ab} e^c e^d = 2\epsilon_{abcd} k^{ab} T^c e^d + d \left(\frac{1}{l^2} \epsilon_{abcd} k^{ab} e^c e^d \right),$$

$$\hat{T}^a = De^a - \frac{1}{2} \bar{\psi} \gamma^a \psi = T^a - \frac{1}{2} \bar{\psi} \gamma^a \psi.$$

Interestingly if we consider $k^{ab} = 0$ in our action we obtain the usual MacDowell-Mansouri action for the $Osp\,(4|1)$ supergroup.

In order to obtain the field equations let us compute the variation of the Lagrangian with respect to the different super $AdS-$Lorentz fields. The variation of the Lagrangian with respect to the spin connection ω^{ab}, modulo boundary terms, is given by

$$\begin{aligned} \delta_\omega \mathcal{L} &= \frac{\alpha_2}{l^2} \epsilon_{abcd} \left(2\delta\omega^{ab} De^c e^d + 2\delta\omega^a{}_f k^{fb} e^c e^d \right) + \frac{\alpha_2}{l^2} \bar{\psi} e^a \gamma_a \gamma_5 \delta\omega^{cd} \gamma_{cd} \psi \\ &= \frac{2\alpha_2}{l^2} \epsilon_{abcd} \delta\omega^{ab} \left(T^c + k^c{}_f e^f - \frac{1}{2} \bar{\psi} \gamma^c \psi \right) e^d \\ &= \frac{2\alpha_2}{l^2} \epsilon_{abcd} \delta\omega^{ab} R^c e^d. \end{aligned} \tag{3.21}$$

Here we see that $\delta_\omega \mathcal{L} = 0$ leads to the following field equation for the $AdS-$Lorentz supertorsion

$$\epsilon_{abcd} R^a e^d = 0. \tag{3.22}$$

JHEP08(2015)009

On the other hand, the variation of the Lagrangian with respect to the vielbein e^a is given by

$$
\begin{aligned}
\delta_e\mathcal{L} &= \frac{\alpha_2}{l^2}\epsilon_{abcd}\left(2R^{ab}e^c + 2Dk^{ab}e^c + 2k^a{}_f k^{fb}e^c + \frac{2}{l}\bar{\psi}\gamma^{ab}\psi e^c + \frac{2}{l^2}e^a e^b e^c\right)\delta e^d \\
&\quad + \frac{\alpha_2}{l^2}\left(4\bar{\psi}\gamma_d\gamma_5 D\psi + \bar{\psi}\gamma_d\gamma_5 k^{ab}\gamma_{ab}\psi\right)\delta e^d. \\
&= \frac{2\alpha_2}{l^2}\epsilon_{abcd}\left(R^{ab}e^c + F^{ab}e^c\right)\delta e^d + \frac{\alpha_2}{l^2}\left(4\bar{\psi}\gamma_d\gamma_5\Psi\right)\delta e^d,
\end{aligned}
\tag{3.23}
$$

where we have used the $AdS-$Lorentz 2-form curvatures (3.2) and the fact that

$$
\begin{aligned}
\epsilon_{abcd}\bar{\psi}\gamma^{ab}\psi e^c &= 2\bar{\psi}\gamma_d\gamma_5 e^c\gamma_c\psi, \\
\epsilon_{abcd}k^{ab}e^c\bar{\psi}\gamma^d\psi &= \bar{\psi}e^a\gamma_a\gamma_5 k^{bc}\gamma_{bc}\psi.
\end{aligned}
$$

Then the field equation is obtained imposing $\delta_e\mathcal{L} = 0$

$$
2\epsilon_{abcd}\left(R^{ab} + F^{ab}\right)e^c + 4\bar{\psi}\gamma_d\gamma_5\Psi = 0. \tag{3.24}
$$

One can see that the rescaling

$$
k^{ab} \to \mu^2 k^{ab}, \quad e^a \to \mu e^a \quad \text{and} \quad \psi \to \sqrt{\mu}\psi
$$

and dividing (3.23) by μ^2 provide us with the usual field equation for supergravity in the limit $\mu \to 0$,

$$
\epsilon_{abcd}R^{ab}e^c + 4\bar{\psi}\gamma_d\gamma_5 D\psi = 0, \tag{3.25}
$$

where D corresponds to the Lorentz covariant exterior derivative.

The variation of the Lagrangian with respect to the new $AdS-$Lorentz field k^{ab}, modulo boundary terms, gives

$$
\begin{aligned}
\delta_k\mathcal{L} &= \frac{\alpha_2}{l^2}\epsilon_{abcd}\left(2\delta k^{ab}De^c e^d + 2\delta k^a{}_f k^{fb}e^c e^d + \frac{1}{l^2}\delta k^{ab}\bar{\psi}\gamma^d\psi e^c\right) \\
&= \frac{2\alpha_2}{l^2}\epsilon_{abcd}\delta k^{ab}\left(T^c + k^c{}_f e^f - \frac{1}{2}\bar{\psi}\gamma^c\psi\right)e^d \\
&= \frac{2\alpha_2}{l^2}\epsilon_{abcd}\delta k^{ab}R^c e^d,
\end{aligned}
\tag{3.26}
$$

where we have used the gamma matrix identities

$$
\begin{aligned}
\gamma_{ab}\gamma_5 &= -\frac{1}{2}\epsilon_{abcd}\gamma^{cd}, \\
\gamma^c\gamma^{ab} &= -2\gamma^{[a}\delta_c^{b]} - \epsilon^{abcd}\gamma_5\gamma_d.
\end{aligned}
$$

Here we see that $\delta_k\mathcal{L} = 0$ leads to the same field equation than $\delta_\omega\mathcal{L} = 0$

$$
\epsilon_{abcd}R^a e^d = 0. \tag{3.27}
$$

Let us consider the variation of the Lagrangian with respect to the gravitino field ψ, modulo boundary terms,

$$
\delta_\psi\mathcal{L} = \frac{\alpha_2}{l^2}\left(4\delta\bar{\psi}e^a\gamma_a\gamma_5 D\psi + 4D\bar{\psi}e^a\gamma_a\gamma_5\delta\psi - 4\bar{\psi}De^a\gamma_a\gamma_5\delta\psi\right.
$$

$$
\begin{aligned}
&+2\delta\bar{\psi}e^{a}\gamma_{a}\gamma_{5}k^{bc}\gamma_{bc}\psi+4\delta\bar{\psi}\gamma_{a}\gamma_{5}k_{b}^{a}e^{b}\psi\Big)+\frac{\alpha_{2}}{l^{2}}\epsilon_{abcd}\left(\frac{2}{l}e^{a}e^{b}\delta\bar{\psi}\gamma^{cd}\psi\right)\\
&=\frac{\alpha_{2}}{l^{2}}\Big(4\delta\bar{\psi}e^{a}\gamma_{a}\gamma_{5}D\psi+4\delta\bar{\psi}\gamma_{a}\gamma_{5}D\psi e^{a}+4\delta\bar{\psi}De^{a}\gamma_{a}\gamma_{5}\psi\\
&\quad+2\delta\bar{\psi}e^{a}\gamma_{a}\gamma_{5}k^{bc}\gamma_{bc}\psi+4\delta\bar{\psi}\gamma_{a}\gamma_{5}k_{b}^{a}e^{b}\psi\Big)+\frac{4\alpha_{2}}{l^{3}}\delta\bar{\psi}e^{a}\gamma_{a}\gamma_{5}e^{b}\gamma_{b}\psi\\
&=\frac{\alpha_{2}}{l^{2}}\delta\bar{\psi}\left(8e^{a}\gamma_{a}\gamma_{5}\Psi+4\gamma_{a}\gamma_{5}\psi De^{a}+4\gamma_{a}\gamma_{5}k_{b}^{a}e^{b}\psi\right). \qquad (3.28)
\end{aligned}
$$

Then, using the definition of the supertorsion

$$R^{a}=De^{a}+k^{a}{}_{b}e^{b}-\frac{1}{2}\bar{\psi}\gamma^{a}\psi,$$

and the Fierz identity

$$\gamma_{a}\psi\bar{\psi}\gamma^{a}\psi=0,$$

we find the following field equation,

$$8e^{a}\gamma_{a}\gamma_{5}\Psi+4\gamma_{a}\gamma_{5}\psi R^{a}=0. \qquad (3.29)$$

We can see that the introduction of a generalized supersymmetric cosmological constant leads to field equations very similar to those of $\mathfrak{osp}\,(4|1)$ supergravity. The differences appear in the definition of the two-form curvatures due to the presence of the new matter field k^{ab}.

Let us note that, from eqs. (3.22) and (3.27), the equation of motion coming from the variation of the Lagrangian with respect to the bosonic field k^{ab} reduces to that of the spin connection ω^{ab}.

$$\epsilon_{abcd}R^{a}e^{d}=\epsilon_{abcd}\left(T^{c}+k^{c}{}_{f}e^{f}-\frac{1}{2}\bar{\psi}\gamma^{c}\psi\right)e^{d}=0. \qquad (3.30)$$

Interestingly, we can define a new bosonic field as

$$\varpi^{ab}=\omega^{ab}+k^{ab}, \qquad (3.31)$$

and its respective covariant derivative,

$$\mathcal{D}=d+\varpi. \qquad (3.32)$$

Then, the equation of motion can be written as

$$\epsilon_{abcd}\left(\mathcal{D}e^{c}-\frac{1}{2}\bar{\psi}\gamma^{c}\psi\right)e^{d}=0. \qquad (3.33)$$

This allows to express the bosonic field ϖ^{ab} in terms of the vielbein e^{a} and gravitino fields ψ^{α}. This may be solved considering the following decomposition,

$$\varpi^{ab}=\mathring{\varpi}^{ab}+\tilde{\varpi}^{ab}, \qquad (3.34)$$

where $\mathring{\varpi}^{ab}$ corresponds to the solution of $\mathcal{D}e^{c}=0$ and it is given by

$$\mathring{\varpi}_{\mu}^{ab}=\left(e_{\lambda}^{c}\partial_{[\mu}e_{\nu]}^{d}\eta_{cd}+e_{\nu}^{c}\partial_{[\lambda}e_{\mu]}^{d}\eta_{cd}-e_{\mu}^{c}\partial_{[\nu}e_{\lambda]}^{d}\eta_{cd}\right)e^{\lambda|a}e^{\nu|b}. \qquad (3.35)$$

Now we have that

$$\mathcal{D}e^a = de^a + \mathring{\varpi}^{ab} e_b + \tilde{\varpi}^{ab} e_b = \frac{1}{2}\bar{\psi}\gamma^a\psi, \tag{3.36}$$

implies

$$\tilde{\varpi}^{ab}_{[\mu} e_{\nu]b} = \frac{1}{2}\bar{\psi}_\mu \gamma^a \psi_\nu. \tag{3.37}$$

Then we may solve $\tilde{\varpi}^{ab}$ in terms of the two other fields,

$$\tilde{\varpi}^{ab}_\mu = \frac{1}{4} e^{a|\lambda} e^{b|\nu} \left(\bar{\psi}_\mu\gamma_\lambda\psi_\nu + \bar{\psi}_\lambda\gamma_\nu\psi_\mu - \bar{\psi}_\nu\gamma_\mu\psi_\lambda - \bar{\psi}_\mu\gamma_\nu\psi_\lambda - \bar{\psi}_\nu\gamma_\lambda\psi_\mu + \bar{\psi}_\lambda\gamma_\mu\psi_\nu\right). \tag{3.38}$$

Thus, the bosonic field ϖ^{ab} is completely determined in terms of e^a_μ and ψ^α_μ and does not carry additional physical degrees of freedom. In particular, when the supertorsion $R^a = \mathcal{D}e^c - \frac{1}{2}\bar{\psi}\gamma^c\psi$ is set equal to zero, the number of bosonic degrees of freedom is two as the Einstein-Hilbert gravity theory and corresponds to the remaining components of the vielbein.

On the other hand, although the Lagrangian is built from the AdS-Lorentz superalgebra it is not invariant under gauge transformations. In fact, the Lagrangian does not correspond to a Yang-Mills Lagrangian, nor a topological invariant.

As we can see the variation of the action (3.18) under gauge supersymmetry can be obtained using $\delta R = [\epsilon, R]$,

$$\delta_{\text{susy}} S = -\frac{4\alpha_2}{l^2}\int R^a \bar{\Psi}\gamma_a\gamma_5\epsilon. \tag{3.39}$$

Thus in order to have gauge supersymmetry invariance it is necessary to impose the supertorsion constraint

$$R^a = 0. \tag{3.40}$$

However this leads to express the spin connection ω^{ab} in terms of the others fields $\{e^a, k^{ab}, \psi\}$.

Nevertheless, it is possible to have supersymmetry invariance in the first formalism adding an extra piece to the gauge transformation $\delta\omega^{ab}$ such that the variation of the action can be written as

$$\delta S = -\frac{4\alpha_2}{l^2}\int R^a \left[\bar{\Psi}\gamma_a\gamma_5\epsilon - \frac{1}{2}\epsilon_{abcd} e^b \delta_{\text{extra}}\omega^{cd}\right], \tag{3.41}$$

where the supersymmetry invariance is fullfilled when

$$\delta_{\text{extra}}\omega^{ab} = 2\epsilon^{abcd}\left(\bar{\Psi}_{ec}\gamma_d\gamma_5\epsilon + \bar{\Psi}_{de}\gamma_c\gamma_5\epsilon - \bar{\Psi}_{cd}\gamma_e\gamma_5\epsilon\right) e^e, \tag{3.42}$$

with $\bar{\Psi} = \bar{\Psi}_{ab} e^a e^b$.

Thus, the action (3.18) in the first order formalism is invariant under the following supersymmetry transformations

$$\delta\omega^{ab} = 2\epsilon^{abcd}\left(\bar{\Psi}_{ec}\gamma_d\gamma_5\epsilon + \bar{\Psi}_{de}\gamma_c\gamma_5\epsilon - \bar{\Psi}_{cd}\gamma_e\gamma_5\epsilon\right) e^e, \tag{3.43}$$

$$\delta k^{ab} = -\frac{1}{l}\bar{\epsilon}\gamma^{ab}\psi, \tag{3.44}$$

$$\delta e^a = \bar{\epsilon}\gamma^a\psi, \tag{3.45}$$

$$\delta\psi = d\epsilon + \frac{1}{4}\omega^{ab}\gamma_{ab}\epsilon + \frac{1}{4}k^{ab}\gamma_{ab}\epsilon + \frac{1}{2l}e^a\gamma_a\epsilon. \tag{3.46}$$

Let us note that supersymmetry is not a gauge symmetry of the action, since it is broken to a Lorentz like symmetry. In particular, the supersymmetry transformations leaving the action invariant do not close off-shell. While the super AdS-Lorentz gauge variation close off-shell by construction.

4 The generalized minimal $AdS-$Lorentz superalgebra

In this section, we show that a particular choice of an abelian semigroup S leads to a new Lie superalgebra. For this purpose we will consider the $\mathfrak{osp}\,(4|1)$ superalgebra as a starting point.

Let us consider a decomposition of the original superalgebra $\mathfrak{g} = \mathfrak{osp}\,(4|1)$ as

$$\begin{aligned} \mathfrak{g} = \mathfrak{osp}\,(4|1) &= \mathfrak{so}\,(3,1) \oplus \frac{\mathfrak{osp}\,(4|1)}{\mathfrak{sp}\,(4)} \oplus \frac{\mathfrak{sp}\,(4)}{\mathfrak{so}\,(3,1)} \\ &= V_0 \oplus V_1 \oplus V_2, \end{aligned} \tag{4.1}$$

where V_0, V_1 and V_2 satisfy (2.7) and correspond to the Lorentz subspace, the fermionic subspace and the AdS-boost, respectively.

Let $S_{\mathcal{M}}^{(4)} = \{\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ be the abelian semigroup whose elements satisfy the following multiplication law

$$\lambda_\alpha\lambda_\beta = \begin{cases} \lambda_{\alpha+\beta}, & \text{if } \alpha+\beta \leq 4 \\ \lambda_{\alpha+\beta-4}, & \text{if } \alpha+\beta > 4 \end{cases} \tag{4.2}$$

Let us consider the decomposition $S = S_0 \cup S_1 \cup S_2$, with

$$S_0 = \{\lambda_0, \lambda_2, \lambda_4\}\,, \tag{4.3}$$

$$S_1 = \{\lambda_1, \lambda_3\}\,, \tag{4.4}$$

$$S_2 = \{\lambda_2, \lambda_4\}\,. \tag{4.5}$$

One can see that this decomposition satisfies the same structure as the subspaces V_p, then we say that the decomposition is resonant [compare with eqs. (2.7)]

$$\begin{aligned} &S_0 \cdot S_0 \subset S_0, \quad S_1 \cdot S_1 \subset S_0 \cap S_2, \\ &S_0 \cdot S_1 \subset S_1, \quad S_1 \cdot S_2 \subset S_1, \\ &S_0 \cdot S_2 \subset S_2, \quad S_2 \cdot S_2 \subset S_0. \end{aligned} \tag{4.6}$$

Following theorem IV.2 of ref. [11], we say that the superalgebra

$$\mathfrak{G}_R = W_0 \oplus W_1 \oplus W_2$$

is a resonant subalgebra of $S_{\mathcal{M}}^{(4)} \times \mathfrak{g}$, where

$$W_0 = (S_0 \times V_0) = \{\lambda_0, \lambda_2, \lambda_4\} \times \left\{\tilde{J}_{ab}\right\} = \left\{\lambda_0\tilde{J}_{ab}, \lambda_2\tilde{J}_{ab}, \lambda_4\tilde{J}_{ab}\right\}, \tag{4.7}$$

$$W_1 = (S_1 \times V_1) = \{\lambda_1, \lambda_3\} \times \left\{\tilde{Q}_\alpha\right\} = \left\{\lambda_1 \tilde{Q}_\alpha, \lambda_3 \tilde{Q}_\alpha\right\}, \tag{4.8}$$

$$W_2 = (S_2 \times V_2) = \{\lambda_2, \lambda_4\} \times \left\{\tilde{P}_a\right\} = \left\{\lambda_2 \tilde{P}_a, \lambda_4 \tilde{P}_a\right\}. \tag{4.9}$$

Then the new superalgebra is generated by $\left\{J_{ab}, P_a, \tilde{Z}_a, \tilde{Z}_{ab}, Z_{ab}, Q_\alpha, \Sigma_\alpha\right\}$ with

$$\begin{aligned}
&J_{ab} = \lambda_0 \tilde{J}_{ab}, \quad P_a = \lambda_2 \tilde{P}_a, \\
&\tilde{Z}_{ab} = \lambda_2 \tilde{J}_{ab}, \quad \tilde{Z}_a = \lambda_4 \tilde{P}_a, \\
&Z_{ab} = \lambda_4 \tilde{J}_{ab}, \quad Q_\alpha = \lambda_1 \tilde{Q}_\alpha, \\
&\Sigma_\alpha = \lambda_3 \tilde{Q}_\alpha.
\end{aligned} \tag{4.10}$$

where $\tilde{J}_{ab}, \tilde{P}_a$ and $\tilde{Q}_\alpha$ are the $\mathfrak{osp}(4|1)$ generators. The new generators satisfy the (anti)commutation relations

$$[J_{ab}, J_{cd}] = \eta_{bc} J_{ad} - \eta_{ac} J_{bd} - \eta_{bd} J_{ac} + \eta_{ad} J_{bc}, \tag{4.11}$$

$$[Z_{ab}, Z_{cd}] = \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc}, \tag{4.12}$$

$$[J_{ab}, Z_{cd}] = \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc}, \tag{4.13}$$

$$\left[J_{ab}, \tilde{Z}_{cd}\right] = \eta_{bc} \tilde{Z}_{ad} - \eta_{ac} \tilde{Z}_{bd} - \eta_{bd} \tilde{Z}_{ac} + \eta_{ad} \tilde{Z}_{bc}, \tag{4.14}$$

$$\left[\tilde{Z}_{ab}, \tilde{Z}_{cd}\right] = \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc}, \tag{4.15}$$

$$\left[\tilde{Z}_{ab}, Z_{cd}\right] = \eta_{bc} \tilde{Z}_{ad} - \eta_{ac} \tilde{Z}_{bd} - \eta_{bd} \tilde{Z}_{ac} + \eta_{ad} \tilde{Z}_{bc}, \tag{4.16}$$

$$[J_{ab}, P_c] = \eta_{bc} P_a - \eta_{ac} P_b, \qquad [Z_{ab}, P_c] = \eta_{bc} P_a - \eta_{ac} P_b, \tag{4.17}$$

$$\left[\tilde{Z}_{ab}, P_c\right] = \eta_{bc} \tilde{Z}_a - \eta_{ac} \tilde{Z}_b, \qquad \left[J_{ab}, \tilde{Z}_c\right] = \eta_{bc} \tilde{Z}_a - \eta_{ac} \tilde{Z}_b, \tag{4.18}$$

$$\left[\tilde{Z}_{ab}, \tilde{Z}_c\right] = \eta_{bc} P_a - \eta_{ac} P_b, \qquad \left[Z_{ab}, \tilde{Z}_c\right] = \eta_{bc} \tilde{Z}_a - \eta_{ac} \tilde{Z}_b, \tag{4.19}$$

$$[P_a, P_b] = Z_{ab}, \qquad \left[\tilde{Z}_a, P_b\right] = \tilde{Z}_{ab}, \qquad \left[\tilde{Z}_a, \tilde{Z}_b\right] = Z_{ab}, \tag{4.20}$$

$$[J_{ab}, Q_\alpha] = -\frac{1}{2} (\gamma_{ab} Q)_\alpha, \qquad [P_a, Q_\alpha] = -\frac{1}{2} (\gamma_a \Sigma)_\alpha, \tag{4.21}$$

$$\left[\tilde{Z}_{ab}, Q_\alpha\right] = -\frac{1}{2} (\gamma_{ab} \Sigma)_\alpha, \qquad \left[\tilde{Z}_a, Q_\alpha\right] = -\frac{1}{2} (\gamma_a Q)_\alpha, \tag{4.22}$$

$$[Z_{ab}, Q_\alpha] = -\frac{1}{2} (\gamma_{ab} Q)_\alpha, \qquad [P_a, \Sigma_\alpha] = -\frac{1}{2} (\gamma_a Q)_\alpha, \tag{4.23}$$

$$[J_{ab}, \Sigma_\alpha] = -\frac{1}{2} (\gamma_{ab} \Sigma)_\alpha, \qquad \left[\tilde{Z}_a, \Sigma_\alpha\right] = -\frac{1}{2} (\gamma_a \Sigma)_\alpha, \tag{4.24}$$

$$\left[\tilde{Z}_{ab}, \Sigma_\alpha\right] = -\frac{1}{2} (\gamma_{ab} Q)_\alpha, \qquad [Z_{ab}, \Sigma_\alpha] = -\frac{1}{2} (\gamma_{ab} \Sigma)_\alpha, \tag{4.25}$$

$$\{Q_\alpha, Q_\beta\} = -\frac{1}{2} \left[\left(\gamma^{ab} C\right)_{\alpha\beta} \tilde{Z}_{ab} - 2 (\gamma^a C)_{\alpha\beta} P_a\right], \tag{4.26}$$

$$\{Q_\alpha, \Sigma_\beta\} = -\frac{1}{2} \left[\left(\gamma^{ab} C\right)_{\alpha\beta} Z_{ab} - 2 (\gamma^a C)_{\alpha\beta} \tilde{Z}_a\right], \tag{4.27}$$

$$\{\Sigma_\alpha, \Sigma_\beta\} = -\frac{1}{2} \left[\left(\gamma^{ab} C\right)_{\alpha\beta} \tilde{Z}_{ab} - 2 (\gamma^a C)_{\alpha\beta} P_a\right], \tag{4.28}$$

JHEP08(2015)009

where we have used the commutation relations of the original superalgebra and the multiplication law of the semigroup (4.2). The new superalgebra obtained after a resonant S-expansion of $\mathfrak{osp}\,(4|1)$ superalgebra corresponds to a generalized minimal AdS-Lorentz superalgebra in $D = 4$.

One can see that a new Majorana spinor charge Σ has been introduced as a direct consequence of the S-expansion procedure. The introduction of a second spinorial generator can be found in refs. [26, 27] in the supergravity and superstrings context, respectively.

Let us note that a generalized $AdS-$Lorentz algebra $= \left\{J_{ab}, P_a, \tilde{Z}_a, \tilde{Z}_{ab}, Z_{ab}\right\}$ forms a bosonic subalgebra of the new superalgebra and looks very similar to the $AdS - \mathcal{L}_6$ algebra introduced in ref. [9]. In fact one could identify $\tilde{Z}_{ab}$, Z_{ab} and $\tilde{Z}_a$ with $Z^{(1)}_{ab}$, $Z^{(2)}_{ab}$ and Z_a of $AdS - \mathcal{L}_6$ respectively. Nevertheless, the commutation relations (4.20) are subtly different of those of the $AdS - \mathcal{L}_6$ algebra. On the other hand, the usual $AdS - \mathcal{L}_4$ algebra $= \{J_{ab}, P_a, Z_{ab}\}$ is also a subalgebra.

It is interesting to observe that an Inönü-Wigner contraction of the new superalgebra leads to a generalization of the minimal Maxwell superalgebra introduced in ref. [29]. After the rescaling

$$\begin{gathered}\tilde{Z}_{ab} \to \mu^2 \tilde{Z}_{ab}, \quad Z_{ab} \to \mu^4 Z_{ab}, \quad P_a \to \mu^2 P_a, \\ \tilde{Z}_a \to \mu^4 \tilde{Z}_a, \quad Q_\alpha \to \mu Q_\alpha \quad \text{and} \quad \Sigma \to \mu^3 \Sigma,\end{gathered}$$

the limit $\mu \to \infty$ provides with a generalized minimal Maxwell superalgebra $s\mathcal{M}_4$ in $D = 4$. An extensive study of the minimal Maxwell superalgebra and its generalization has been done using expansion methods in refs. [30, 31]. On the other hand, it was shown in refs. [20, 32] that $D = 4$, $N = 1$ pure supergravity Lagrangian can be obtained as a quadratic expression in the curvatures associated with the minimal Maxwell superalgebra.

Analogously to the previous case, we can show that the generalized minimal AdS-Lorentz superalgebra found here can be used in order to build a more general supergravity action involving a generalized supersymmetric cosmological term.

As in the previous section, we start from the one-form gauge connection,

$$A = \frac{1}{2}\omega^{ab} J_{ab} + \frac{1}{l} e^a P_a + \frac{1}{2}\tilde{k}^{ab} \tilde{Z}_{ab} + \frac{1}{2} k^{ab} Z_{ab} + \frac{1}{l}\tilde{h}^a \tilde{Z}_a + \frac{1}{\sqrt{l}}\psi^\alpha Q_\alpha + \frac{1}{\sqrt{l}}\xi^\alpha \Sigma_\alpha, \tag{4.29}$$

where the one-form gauge fields are related to the $\mathfrak{osp}\,(4|1)$ gauge fields $\left(\tilde{\omega}^{ab}, \tilde{e}^a, \tilde{\psi}\right)$ as

$$\begin{aligned} \omega^{ab} &= \lambda_0 \tilde{\omega}^{ab}, & e^a &= \lambda_2 \tilde{e}^a, \\ \tilde{k}^{ab} &= \lambda_2 \tilde{\omega}^{ab}, & \psi^\alpha &= \lambda_1 \tilde{\psi}^\alpha, \\ k^{ab} &= \lambda_4 \tilde{\omega}^{ab}, & \xi^\alpha &= \lambda_3 \tilde{\psi}^\alpha, \\ \tilde{h}^a &= \lambda_4 e^a. \end{aligned}$$

Then the corresponding two-form curvature $F = dA + A \wedge A$ is given by

$$F = \frac{1}{2} R^{ab} J_{ab} + \frac{1}{l} R^a P_a + \frac{1}{2}\tilde{F}^{ab} \tilde{Z}_{ab} + \frac{1}{2} F^{ab} Z_{ab} + \frac{1}{l}\tilde{H}^a \tilde{Z}_a + \frac{1}{\sqrt{l}}\Psi^\alpha Q_\alpha + \frac{1}{\sqrt{l}}\Xi^\alpha \Sigma_\alpha, \tag{4.30}$$

where

$$R^{ab} = d\omega^{ab} + \omega^a_{\ c}\omega^{cb},$$

$$
\begin{aligned}
R^{a} &= de^{a} + \omega^{a}_{\ b}e^{b} + k^{a}_{\ b}e^{b} + \tilde{k}^{a}_{\ b}\tilde{h}^{b} - \frac{1}{2}\bar{\psi}\gamma^{a}\psi - \frac{1}{2}\bar{\xi}\gamma^{a}\xi, \\
\tilde{H}^{a} &= d\tilde{h}^{a} + \omega^{a}_{\ b}\tilde{h}^{b} + \tilde{k}^{a}_{\ b}e^{b} + k^{a}_{\ b}\tilde{h}^{b} - \bar{\psi}\gamma^{a}\xi, \\
\tilde{F}^{ab} &= d\tilde{k}^{ab} + \omega^{a}_{\ c}\tilde{k}^{cb} - \omega^{b}_{\ c}\tilde{k}^{ca} + k^{a}_{\ c}\tilde{k}^{cb} - k^{b}_{\ c}\tilde{k}^{ca} + \frac{2}{l^{2}}e^{a}\tilde{h}^{b} + \frac{1}{2l}\bar{\psi}\gamma^{ab}\psi + \frac{1}{2l}\bar{\xi}\gamma^{ab}\xi, \\
F^{ab} &= dk^{ab} + \omega^{a}_{\ c}k^{cb} - \omega^{b}_{\ c}k^{ca} + \tilde{k}^{a}_{\ c}\tilde{k}^{cb} + k^{a}_{\ c}k^{cb} + \frac{1}{l^{2}}e^{a}e^{b} + \frac{1}{l^{2}}\tilde{h}^{a}\tilde{h}^{b} + \frac{1}{l}\bar{\xi}\gamma^{ab}\psi, \\
\Psi &= d\psi + \frac{1}{4}\omega_{ab}\gamma^{ab}\psi + \frac{1}{4}k_{ab}\gamma^{ab}\psi + \frac{1}{4}\tilde{k}_{ab}\gamma^{ab}\xi + \frac{1}{2l}e_{a}\gamma^{a}\xi + \frac{1}{2}\tilde{h}_{a}\gamma^{a}\psi, \\
\Xi &= d\xi + \frac{1}{4}\omega_{ab}\gamma^{ab}\xi + \frac{1}{4}k_{ab}\gamma^{ab}\xi + \frac{1}{4}\tilde{k}_{ab}\gamma^{ab}\psi + \frac{1}{2l}e_{a}\gamma^{a}\psi + \frac{1}{2l}\tilde{h}_{a}\gamma^{a}\xi.
\end{aligned}
$$

Here the new Majorana field ξ is associated to the fermionic generator Σ, while the one-forms $\tilde{h}^{a}$, $\tilde{k}^{ab}$ and k^{ab} are the matter fields associated with the bosonic generators $\tilde{Z}_{a}$, $\tilde{Z}_{ab}$ and Z_{ab} respectively.

Using the two-form curvature F it is possible to write the action for the generalized minimal $AdS-$Lorentz superalgebra as

$$
S = 2\int \langle F \wedge F\rangle = 2\int F^{A} \wedge F^{B} \langle T_{A}T_{B}\rangle_{S}\,, \tag{4.31}
$$

where $\langle T_{A}T_{B}\rangle_{S}$ corresponds to an S-expanded invariant tensor which is obtained from the original components of the invariant tensor (3.8). Using Theorem VII.1 of ref. [11], it is possible to show that the non-vanishing components of $\langle T_{A}T_{B}\rangle_{S}$ are given by

$$
\langle J_{ab}J_{cd}\rangle_{S} = \alpha_{0}\langle J_{ab}J_{cd}\rangle\,, \qquad \left\langle \tilde{Z}_{ab}\tilde{Z}_{cd}\right\rangle_{S} = \alpha_{4}\langle J_{ab}J_{cd}\rangle\,, \tag{4.32}
$$

$$
\left\langle J_{ab}\tilde{Z}_{cd}\right\rangle_{S} = \alpha_{2}\langle J_{ab}J_{cd}\rangle\,, \qquad \langle Z_{ab}Z_{cd}\rangle_{S} = \alpha_{4}\langle J_{ab}J_{cd}\rangle\,, \tag{4.33}
$$

$$
\left\langle \tilde{Z}_{ab}Z_{cd}\right\rangle_{S} = \alpha_{2}\langle J_{ab}J_{cd}\rangle\,, \qquad \langle J_{ab}Z_{cd}\rangle_{S} = \alpha_{4}\langle J_{ab}J_{cd}\rangle\,, \tag{4.34}
$$

$$
\langle Q_{\alpha}Q_{\beta}\rangle_{S} = \alpha_{2}\langle Q_{\alpha}Q_{\beta}\rangle\,, \qquad \langle \Sigma_{\alpha}\Sigma_{\beta}\rangle_{S} = \alpha_{2}\langle Q_{\alpha}Q_{\beta}\rangle\,, \tag{4.35}
$$

$$
\langle Q_{\alpha}\Sigma_{\beta}\rangle_{S} = \alpha_{4}\langle Q_{\alpha}Q_{\beta}\rangle\,, \tag{4.36}
$$

where α_{0}, α_{2} and α_{4} are dimensionless arbitrary independent constants and

$$
\begin{aligned}
\langle J_{ab}J_{cd}\rangle &= \epsilon_{abcd}, \\
\langle Q_{\alpha}Q_{\beta}\rangle &= 2\left(\gamma_{5}\right)_{\alpha\beta}.
\end{aligned}
$$

Then considering the two-form curvature (4.30) and the non-vanishing components of the invariant tensor (4.32)–(4.36) we found that the action can be written as a MacDowell-Mansouri like action,

$$
\begin{aligned}
S = 2\int \Big(&\frac{\alpha_{0}}{4}\epsilon_{abcd}R^{ab}R^{cd} + \frac{\alpha_{2}}{2}\epsilon_{abcd}R^{ab}\tilde{F}^{cd} + \frac{\alpha_{2}}{2}\epsilon_{abcd}\tilde{F}^{ab}F^{cd} + \frac{\alpha_{4}}{2}\epsilon_{abcd}R^{ab}F^{cd} \\
&\frac{\alpha_{4}}{4}\epsilon_{abcd}\tilde{F}^{ab}\tilde{F}^{cd} + \frac{\alpha_{4}}{2}\epsilon_{abcd}F^{ab}F^{cd} + \frac{2}{l}\alpha_{2}\bar{\Psi}\gamma_{5}\Psi + \frac{2}{l}\alpha_{2}\bar{\Xi}\gamma_{5}\Xi + \frac{4}{l}\alpha_{4}\bar{\Psi}\gamma_{5}\Xi\Big)\,.
\end{aligned} \tag{4.37}
$$

Since we are interested in obtaining the Einstein-Hilbert and the Rarita-Schwinger like Lagrangian with a generalized supersymmetric cosmological term, we shall consider only the piece proportional to α_4. Using the useful gamma matrix identities and the Bianchi identities ($dF + [A, F] = 0$) it is possible to write explicitly the α_4-term as

$$
\begin{aligned}
\mathcal{S} = \alpha_4 \int \epsilon_{abcd} & \left(R^{ab}\mathcal{K}^{cd} + \frac{1}{2}\tilde{\mathcal{K}}^{ab}\tilde{\mathcal{K}}^{cd} + \frac{1}{2}\mathcal{K}^{ab}\mathcal{K}^{cd} \right) \\
& + \frac{1}{l^2}\left(\epsilon_{abcd}R^{ab}e^c e^d + 4\bar{\psi}e^a\gamma_a\gamma_5 D\psi + 4\bar{\xi}e^a\gamma_a\gamma_5 D\xi \right) \\
& + \frac{1}{l^2}\left(\epsilon_{abcd}R^{ab}\tilde{h}^c\tilde{h}^d + 4\bar{\psi}\tilde{h}^a\gamma_a\gamma_5 D\xi + 4\bar{\xi}\tilde{h}^a\gamma_a\gamma_5 D\psi \right) \\
& + \frac{1}{l^2}\Bigg(2\epsilon_{abcd}\tilde{\mathcal{K}}^{ab}e^c\tilde{h}^d + \epsilon_{abcd}\mathcal{K}^{ab}e^c e^d + \epsilon_{abcd}\mathcal{K}^{ab}\tilde{h}^c\tilde{h}^d + \frac{1}{l^2}e^a e^b e^c e^d \\
& + \frac{6}{l^2}e^a e^b\tilde{h}^c\tilde{h}^d + \frac{1}{l^2}\tilde{h}^a\tilde{h}^b\tilde{h}^c\tilde{h}^d + \frac{2}{l}\bar{\psi}\gamma^{ab}\psi e^c\tilde{h}^d + \frac{2}{l}\bar{\psi}\gamma^{ab}\xi e^c e^d + \frac{2}{l}\bar{\psi}\gamma^{ab}\xi\tilde{h}^c\tilde{h}^d \\
& + \frac{2}{l}\bar{\xi}\gamma^{ab}\xi e^c\tilde{h}^d \Bigg) + \frac{1}{l^2}k^{ab}e^c\left\{ \bar{\psi}\gamma^d\psi + \bar{\xi}\gamma^d\xi \right\} + \frac{2}{l^2}\tilde{k}^{ab}e^c\bar{\psi}\gamma^d\xi + \frac{2}{l^2}k^{ab}\tilde{h}^a\bar{\psi}\gamma^d\xi \\
& + \frac{1}{l^2}\tilde{k}^{ab}\tilde{h}^c\left\{ \bar{\psi}\gamma^d\psi + \bar{\xi}\gamma^d\xi \right\} + d\left(\frac{8}{l}\bar{\xi}\gamma_5\nabla\psi \right),
\end{aligned}
\tag{4.38}
$$

where we have defined

$$
\begin{aligned}
\tilde{\mathcal{K}}^{ab} &= D\tilde{k}^{cb} + k^a{}_c\tilde{k}^{cb} + k^b{}_c\tilde{k}^{ac}, \\
\mathcal{K}^{ab} &= Dk^{ca} + \tilde{k}^a{}_c\tilde{k}^{cb} + k^a{}_c k^{cb}.
\end{aligned}
$$

Here we can see that the first piece corresponds to an Euler invariant term which can be seen as a Gauss-Bonnet like term and can be written as a boundary contribution. The second piece contains the Einstein-Hilbert term $\epsilon_{abcd}R^{ab}e^c e^d$ and the Rarita-Schwinger like Lagrangian. The novelty consists in the contribution of the new spinor field ξ which is related to the Majorana spinor charge Σ. The fourth term corresponds to a generalized supersymmetric cosmological term built from the new AdS-Lorentz fields. The last piece is a boundary term and does not contribute to the dynamics.

A significant difference with the previous case (see eq. (3.18)) is the presence of the matter field $\tilde{h}^a$ which is related to the new generator $\tilde{Z}_a$. In particular, if we consider $\tilde{h}^a = 0$ and omit boundary contributions, the term proportional to α_4 can be written as

$$
\begin{aligned}
\mathcal{S} = \alpha_4 \int \frac{1}{l^2} & \left(\epsilon_{abcd}R^{ab}e^c e^d + 4\bar{\psi}e^a\gamma_a\gamma_5\nabla\psi + 4\bar{\xi}e^a\gamma_a\gamma_5\nabla\xi \right) \\
& + \frac{1}{l^2}\left(\epsilon_{abcd}\mathcal{K}^{ab}e^c e^d + \frac{1}{l^2}e^a e^b e^c e^d + \frac{2}{l}\bar{\psi}\gamma^{ab}\xi e^c e^d \right),
\end{aligned}
\tag{4.39}
$$

with

$$
\begin{aligned}
\nabla\psi &= D\psi + \frac{1}{4}k_{ab}\gamma^{ab}\psi + \frac{1}{4}\tilde{k}_{ab}\gamma^{ab}\xi, \\
\nabla\xi &= D\xi + \frac{1}{4}k_{ab}\gamma^{ab}\xi + \frac{1}{4}\tilde{k}_{ab}\gamma^{ab}\psi.
\end{aligned}
$$

The action (4.39) corresponds to a four-dimensional supergravity action with a generalized supersymmetric cosmological term. Certainly, the choice of bigger semigroups would allow to construct larger AdS-Lorentz superalgebras introducing a more general cosmological term to supergravity. Nevertheless, this procedure would lead to more complicated actions which we do not consider here.

It is interesting to observe that an Inönü-Wigner contraction of the action (4.39) leads us to the $D = 4$ pure supergravity action. In fact after the rescaling

$$\omega_{ab} \to \omega_{ab}, \quad \tilde{k}_{ab} \to \mu^2 \tilde{k}_{ab}, \quad k_{ab} \to \mu^4 k_{ab},$$
$$e_a \to \mu^2 e_a, \quad \psi \to \mu\psi \quad \text{and} \quad \xi \to \mu^3 \xi,$$

and dividing the action by μ^4, the $D = 4$ pure supergravity action is retrieved by taking the limit $\mu \to \infty$,

$$\mathcal{S} = \alpha_4 \int \frac{1}{l^2} \left(\epsilon_{abcd} R^{ab} e^c e^d + 4\bar{\psi} e^a \gamma_a \gamma_5 D\psi \right). \tag{4.40}$$

It was shown in refs. [20, 32] that the $N = 1$, $D = 4$ pure supergravity can be derived from the minimal Maxwell superalgebra $s\mathcal{M}_4$. This result is not a surprise since the Inönü-Wigner contraction of the generalized minimal $AdS-$Lorentz superalgebra corresponds to the minimal Maxwell superalgebra $s\mathcal{M}_4$.

Let us note that the procedure considered here could be extended to bigger AdS-Lorentz superalgebras whose Inönü-Wigner contractions lead to the Maxwell superalgebras type defined in [31]. These Maxwell superalgebras correspond to the supersymmetric extension of the Maxwell algebras type introduced in refs. [19, 21].

5 Comments and possible developments

In the present work we have presented an alternative way of introducing the supersymmetric cosmological constant to supergravity. Based on the AdS-Lorentz superalgebra we have built the minimal $D = 4$ supergravity action which includes a generalized supersymmetric cosmological constant term. For this purpose we have applied the semigroup expansion method to the $\mathfrak{osp}\,(4|1)$ superalgebra allowing us to construct a MacDowell-Mansouri like action. The geometric formulation of the supergravity theory found here corresponds to a supersymmetric generalization of the results of refs. [4, 9].

Interestingly, we have shown that the AdS-Lorentz superalgebra allows to add new terms in the supergravity action à la MacDowell-Mansouri presented in [20], describing a generalized supersymmetric cosmological constant. In particular, unlike the Maxwell superalgebra, the bosonic fields k^{ab} associated to the generators Z_{ab} appear not only in the boundary terms, but also in the bulk Lagrangian.

The presence of the bosonic fields k^{ab} in the boundary could be useful in the context of the duality between string theory realized on an asymptotically AdS space-time (times a compact manifold M) and the conformal field theory living on the boundary (AdS/CFT correspondence) [33–36]. Interestingly, as shown in ref. [23], the introduction of a topological boundary in a four-dimensional bosonic action is equivalent to the holographic renormalization procedure in the AdS/CFT context. At the supergravity level, it was shown in ref. [37] that the supersymmetry invariance of the supergravity action is recovered adding appropriate boundary terms and thus, reproducing the MacDowell-Mansouri action. Then, it is tempting to argue that the presence of the fields k^{ab} in the

JHEP08(2015)009

boundary would allow not only to recover the supersymmetry invariance in the geometric approach (rheonomic approach), but also to regularize the supergravity action in the holographic renormalization context.

We have also presented a more general supergravity action containing a cosmological constant (see eq. (4.38)). For this aim we have introduced the generalized minimal AdS-Lorentz superalgebra using a bigger semigroup. This new superalgebra requires the introduction of a second Majorana spinorial generator Σ leading to additional degress of freedom. In particular we have shown that the Inönü-Wigner contraction of the generalized AdS-Lorentz superalgebra leads to the minimal Maxwell superalgebra.

Our results provide one more example of the advantage of the semigroup expansion method in the geometrical formulation of a supergravity theory. The approach presented here could be useful in order to analyze a possible extension to higher dimensions. Nevertheless, it seems that in odd dimensions the Chern-Simons (CS) theory is the appropriate formalism in order to construct a supergravity action. For instance, for $D = 3$ interesting CS (super)gravity theories are obtained using the AdS-Lorentz (super)symmetries [10, 18, 38].

On the other hand, it would be interesting to study the extended supergravity in the geometrical formulation. A future work could be analyze the N-extended AdS-Lorentz superalgebra introduced in ref. [39] and the construction of N-extended supergravities. It seems that the semigroup expansion procedure used here could have an important role in the construction of matter-supergravity theories.

Acknowledgments

This work was supported in part by FONDECYT Grants N° 1130653. Two of the authors (P.K. C., E.K. R.) authors were supported by grants from the Comisión Nacional de Investigación Científica y Tecnológica (CONICYT) and from the Universidad de Concepción, Chile. P.K. C. and E.K. R. wish to thank L. Andrianopoli, R. D'Auria and M. Trigiante for their kind hospitality at Dipartimieto Scienza Applicata e Tecnologia (DISAT) of Politecnico di Torino and for enlightening discussions.

References

[1] J. Frieman, M. Turner and D. Huterer, *Dark energy and the accelerating universe*, *Ann. Rev. Astron. Astrophys.* **46** (2008) 385 [arXiv:0803.0982] [INSPIRE].

[2] T. Padmanabhan, *Dark energy and its implications for gravity*, *Adv. Sci. Lett.* **2** (2009) 174 [arXiv:0807.2356] [INSPIRE].

[3] S.W. MacDowell and F. Mansouri, *Unified geometric theory of gravity and supergravity*, *Phys. Rev. Lett.* **38** (1977) 739 [*Erratum ibid.* **38** (1977) 1376] [INSPIRE].

[4] J.A. de Azcarraga, K. Kamimura and J. Lukierski, *Generalized cosmological term from Maxwell symmetries*, *Phys. Rev.* **D 83** (2011) 124036 [arXiv:1012.4402] [INSPIRE].

[5] H. Bacry, P. Combe and J.L. Richard, *Group-theoretical analysis of elementary particles in an external electromagnetic field. 1. The relativistic particle in a constant and uniform field*, Nuovo Cim. **A 67** (1970) 267 [INSPIRE].

[6] R. Schrader, *The Maxwell group and the quantum theory of particles in classical homogeneous electromagnetic fields*, Fortsch. Phys. **20** (1972) 701 [INSPIRE].

[7] R. Durka, J. Kowalski-Glikman and M. Szczachor, *Gauged AdS-Maxwell algebra and gravity*, Mod. Phys. Lett. **A 26** (2011) 2689 [arXiv:1107.4728] [INSPIRE].

[8] D.V. Soroka and V.A. Soroka, *Semi-simple extension of the (super)Poincaré algebra*, Adv. High Energy Phys. **2009** (2009) 234147 [hep-th/0605251] [INSPIRE].

[9] P. Salgado and S. Salgado, $\mathfrak{so}(D-1,1)\otimes\mathfrak{so}(D-1,2)$ *algebras and gravity*, Phys. Lett. **B 728** (2014) 5 [INSPIRE].

[10] J. Diaz et al., *A generalized action for* $(2+1)$*-dimensional Chern-Simons gravity*, J. Phys. **A 45** (2012) 255207 [arXiv:1311.2215] [INSPIRE].

[11] F. Izaurieta, E. Rodriguez and P. Salgado, *Expanding Lie (super)algebras through Abelian semigroups*, J. Math. Phys. **47** (2006) 123512 [hep-th/0606215] [INSPIRE].

[12] F. Izaurieta, E. Rodriguez and P. Salgado, *Eleven-dimensional gauge theory for the M algebra as an Abelian semigroup expansion of* $\mathfrak{osp}(32|1)$, Eur. Phys. J. **C 54** (2008) 675 [hep-th/0606225] [INSPIRE].

[13] F. Izaurieta, E. Rodriguez, P. Minning, P. Salgado and A. Perez, *Standard general relativity from Chern-Simons gravity*, Phys. Lett. **B 678** (2009) 213 [arXiv:0905.2187] [INSPIRE].

[14] P.K. Concha, D.M. Peñafiel, E.K. Rodríguez and P. Salgado, *Even-dimensional general relativity from Born-Infeld gravity*, Phys. Lett. **B 725** (2013) 419 [arXiv:1309.0062] [INSPIRE].

[15] N. González, P. Salgado, G. Rubio and S. Salgado, *Einstein-Hilbert action with cosmological term from Chern-Simons gravity*, J. Geom. Phys. **86** (2014) 339 [INSPIRE].

[16] P. Salgado, R.J. Szabo and O. Valdivia, *Topological gravity and transgression holography*, Phys. Rev. **D 89** (2014) 084077 [arXiv:1401.3653] [INSPIRE].

[17] C. Inostroza, A. Salazar and P. Salgado, *Brans-Dicke gravity theory from topological gravity*, Phys. Lett. **B 734** (2014) 377 [INSPIRE].

[18] O. Fierro, F. Izaurieta, P. Salgado and O. Valdivia, $(2+1)$*-dimensional supergravity invariant under the AdS-Lorentz superalgebra*, arXiv:1401.3697 [INSPIRE].

[19] P.K. Concha, D.M. Penafiel, E.K. Rodriguez and P. Salgado, *Chern-Simons and Born-Infeld gravity theories and Maxwell algebras type*, Eur. Phys. J. **C 74** (2014) 2741 [arXiv:1402.0023] [INSPIRE].

[20] P.K. Concha and E.K. Rodríguez, $N=1$ *supergravity and Maxwell superalgebras*, JHEP **09** (2014) 090 [arXiv:1407.4635] [INSPIRE].

[21] P.K. Concha, D.M. Peñafiel, E.K. Rodríguez and P. Salgado, *Generalized Poincaré algebras and Lovelock-Cartan gravity theory*, Phys. Lett. **B 742** (2015) 310 [arXiv:1405.7078] [INSPIRE].

[22] J.A. de Azcarraga, J.M. Izquierdo, M. Picón and O. Varela, *Generating Lie and gauge free differential (super)algebras by expanding Maurer-Cartan forms and Chern-Simons supergravity*, Nucl. Phys. **B 662** (2003) 185 [hep-th/0212347] [INSPIRE].

[23] O. Mišković and R. Olea, *Topological regularization and self-duality in four-dimensional anti-de Sitter gravity*, *Phys. Rev.* **D 79** (2009) 124020 [arXiv:0902.2082] [INSPIRE].

[24] O. Mišković, R. Olea and M. Tsoukalas, *Renormalized AdS action and critical gravity*, *JHEP* **08** (2014) 108 [arXiv:1404.5993] [INSPIRE].

[25] L. Andrianopoli, N. Merino, F. Nadal and M. Trigiante, *General properties of the expansion methods of Lie algebras*, *J. Phys.* **A 46** (2013) 365204 [arXiv:1308.4832] [INSPIRE].

[26] R. D'Auria and P. Fré, *Geometric supergravity in $D = 11$ and its hidden supergroup*, *Nucl. Phys.* **B 201** (1982) 101 [*Erratum ibid.* **B 206** (1982) 496] [INSPIRE].

[27] M.B. Green, *Supertranslations, superstrings and Chern-Simons forms*, *Phys. Lett.* **B 223** (1989) 157 [INSPIRE].

[28] D.K. Wise, *The geometric role of symmetry breaking in gravity*, *J. Phys. Conf. Ser.* **360** (2012) 012017 [arXiv:1112.2390] [INSPIRE].

[29] S. Bonanos, J. Gomis, K. Kamimura and J. Lukierski, *Maxwell superalgebra and superparticle in constant gauge badkgrounds*, *Phys. Rev. Lett.* **104** (2010) 090401 [arXiv:0911.5072] [INSPIRE].

[30] J.A. de Azcarraga, J.M. Izquierdo, J. Lukierski and M. Woronowicz, *Generalizations of Maxwell (super)algebras by the expansion method*, *Nucl. Phys.* **B 869** (2013) 303 [arXiv:1210.1117] [INSPIRE].

[31] P.K. Concha and E.K. Rodríguez, *Maxwell superalgebras and Abelian semigroup expansion*, *Nucl. Phys.* **B 886** (2014) 1128 [arXiv:1405.1334] [INSPIRE].

[32] J.A. de Azcarraga and J.M. Izquierdo, *Minimal $D = 4$ supergravity from the super-Maxwell algebra*, *Nucl. Phys.* **B 885** (2014) 34 [arXiv:1403.4128] [INSPIRE].

[33] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Int. J. Theor. Phys.* **38** (1999) 1113 [hep-th/9711200] [INSPIRE].

[34] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from noncritical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109] [INSPIRE].

[35] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150] [INSPIRE].

[36] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [hep-th/9905111] [INSPIRE].

[37] L. Andrianopoli and R. D'Auria, *$N = 1$ and $N = 2$ pure supergravities on a manifold with boundary*, *JHEP* **08** (2014) 012 [arXiv:1405.2010] [INSPIRE].

[38] S. Hoseinzadeh and A. Rezaei-Aghdam, *$(2 + 1)$-dimensional gravity from Maxwell and semisimple extension of the Poincaré gauge symmetric models*, *Phys. Rev.* **D 90** (2014) 084008 [arXiv:1402.0320] [INSPIRE].

[39] D.V. Soroka and V.A. Soroka, *Semi-simple $o(N)$-extended super-Poincaré algebra*, arXiv:1004.3194 [INSPIRE].

JHEP08(2015)009