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The NSVZ β-function and the Schwinger-Dyson equations for $\mathcal{N}=1$ SQED with N_f flavors, regularized by higher derivatives

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Abstract: The effective diagram technique based on the Schwinger-Dyson equations is constructed for $\mathcal{N}=1$ SQED with N_f flavors, regularized by higher derivatives. Using these effective diagrams, it is possible to derive the exact NSVZ relation between the β-function and the anomalous dimension of the matter superfields exactly in all loops, if the renormalization group functions are defined in terms of the bare coupling constant. In particular, we verify that all integrals which give the β-function defined in terms of the bare coupling constant are integrals of double total derivatives and prove some identities relating Green functions.

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1 Introduction

The existence of ultraviolet divergences is a long standing problem of quantum field theory. An important step towards solving this problem is a discovery of supersymmetry [1, 2]. It is well known that the behavior of supersymmetric theories in the ultraviolet region is better due to non-renormalization theorems. In particular, the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory is finite [3–6]. Divergences in $\mathcal{N} = 2$ supersymmetric Yang-Mills theories exist only in the one-loop approximation [7]. Even in $\mathcal{N} = 1$ supersymmetric theories the superpotential is not renormalized [8]. However, the β-function in $\mathcal{N} = 1$ supersymmetric Yang-Mills theories receives quantum corrections in all orders. Nevertheless, this β-function is related with the anomalous dimension of the matter superfields. This relation is called the exact Novikov, Shifman, Vainshtein, and Zakharov (NSVZ) β-function [9–11]. In the original papers this β-function was obtained using arguments based on the structure of instanton contributions (for review, see [12]) or on the supermultiplet structure of anomalies. In particular, in the lowest orders the relation between the β-function and anomalies was investigated in [10, 13–15] and exactly in all orders in [10, 16, 17]. This was done using the Adler-Bardeen theorem [18] for the axial anomaly, a relation between the anomaly of energy-momentum tensor trace and a β-function, and a supermultiplet structure of anomalies. Another derivation of the exact NSVZ β-function based on anomalies was made in [19]. This β-function was also obtained in [20] using the non-renormalization theorem for the topological term. In [21] the rescaling anomaly is used for explanation of the higher order corrections to the NSVZ β-function.

In this paper we consider the $\mathcal{N} = 1$ supersymmetric electrodynamics (SQED) with N_f flavors, for which the NSVZ β-function is written as [22, 23]

$$\beta(\alpha) = \frac{\alpha^2 N_f}{\pi}\Big(1 - \gamma(\alpha)\Big). \tag{1.1}$$

The NSVZ β-function can be compared with the results of explicit calculations in the lowest orders of the perturbation theory. In the one- and two-loop approximations a β-function for supersymmetric theories was first calculated in [24] and [25], respectively, using the dimensional regularization [26–29]. However, because the dimensional regularization breaks supersymmetry [30], most calculations in supersymmetric theories were made with the dimensional reduction proposed in [31] (see [32] for a recent review). Using this regularization and the $\overline{\text{DR}}$-scheme, which is a modification of the $\overline{\text{MS}}$-scheme [33], the β-function of the $\mathcal{N} = 1$ supersymmetric Yang-Mills theory with matter was calculated in the three- [34, 35] and four-loop [36] approximations. The results coincide with the NSVZ β-function in one- and two-loop approximations (where the β-function is scheme-independent). In the higher

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loops the NSVZ β-function can be obtained after a special redefinition of the coupling constant [35, 37]. (Using such a redefinition the result for the four-loop β-function was correctly predicted in [38] before the explicit calculation made in [36].)

However, the regularization by the dimensional reduction is not self-consistent [39]. The inconsistencies can be removed only if one breaks the manifest supersymmetry [40, 41]. Therefore, supersymmetry can be broken by higher orders quantum corrections [40]. This was verified explicitly: in the $\mathcal{N} = 2$ supersymmetric Yang-Mills theory without matter superfields obtaining the three-loop β-function by different methods (using various Green functions) gives different results [40, 42]. (The calculation made in [42] showed that this does not take place for the $\mathcal{N} = 1$ supersymmetric Yang-Mills theory at three-loop level, as it was argued in [40].) In the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory the dimensional reduction does not break supersymmetry even in the four-loop approximation [43]. Nevertheless, with the dimensional reduction one can expect breaking of supersymmetry by quantum corrections in higher loops (see table 1 in [40]).

Although the dimensional reduction is the most popular regularization for calculations in supersymmetric theories, other methods are also used. For example, using a method based on the operator product expansion two-loop β-functions of scalar, spinor, and $\mathcal{N} = 1$ supersymmetric electrodynamics were calculated in [44]. With the differential renormalization [45] a two-loop β-function of the $\mathcal{N} = 1$ supersymmetric Yang-Mills theory was found in [46]. Another regularization used for calculations in supersymmetric theories is the higher covariant derivative regularization proposed in [47, 48]. This regularization was subsequently generalized to the supersymmetric case in [49, 50]. It can be also applied in $\mathcal{N} = 2$ supersymmetric theories [51, 52]. The higher covariant derivative regularization leads to loop integrals which have complicated structure. That is why this regularization is not frequently used for explicit calculations. However, it is quite possible. For example, a one-loop β-function of the (non-supersymmetric) Yang-Mills theory was calculated in [53]. After essential corrections introduced in the subsequent papers [54, 55] the well-known one-loop result [56, 57] was reobtained (although the original calculation made in [53] gave a different result). One can prove that at the one-loop level the higher covariant derivative regularization always produces the same result for a β-function as the dimensional regularization [58].

Quantum corrections obtained with the higher covariant derivative regularization in supersymmetric theories appear to have an interesting feature: the β-function defined in terms of the bare coupling constant is given by integrals of total derivatives with respect to a loop momentum [59–62] and even by integrals of double total derivatives [63–66]. Thus, it is possible to calculate one of the loop integrals analytically and reduce a number of the integrations over loop momentums. At least, in the Abelian case this allows to prove that the β-function and the anomalous dimension of the matter superfields defined in terms of the bare coupling constant satisfy the NSVZ relation [67–69]. (For a fixed regularization) these renormalization group functions are scheme independent (see, e.g., [69]), so that the NSVZ β-function is obtained for an arbitrary renormalization prescription. However, if the renormalization group functions are defined by the standard way in terms of the renormalized coupling constant, they depend on the subtraction scheme [70]. In this case the NSVZ β-function is obtained in a special subtraction scheme. If the theory is regularized by higher derivatives, such a scheme can be obtained in all orders by imposing simple boundary conditions on the renormalization constants [68, 69]. So far there is no similar prescription in the case of using the dimensional reduction, and the NSVZ scheme should be constructed in each order of the perturbation theory after calculating the renormalization group functions.

Thus, using the higher covariant derivative regularization one can naturally construct the scheme in which the β-function coincides with the exact NSVZ β-function at least in the Abelian case. Certainly, it is desirable to generalize the results to the non-Abelian case. However, in the non-Abelian case the calculations with the higher covariant derivative regularization were performed only in the one- and two-loop approximations, where the β-function is scheme-independent. Nevertheless, in both cases the β-function appears to be given by integrals of double total derivatives and coincide with the NSVZ expression. This allows to suggest that the structure of quantum corrections in the non-Abelian case is similar to the case of $\mathcal{N}=1$ SQED. However, the method used in [67] (which was proposed in [71]) is not convenient for generalizing the results to the non-Abelian case. Possibly, using this method one can prove the factorization of integrands into the double total derivatives, but obtaining the exact β-function by this method seems to be a very complicated problem. Even in $\mathcal{N}=1$ SQED for this purpose it is necessary to compare coefficients of different Feynman diagrams [67]. From the other side, the NSVZ expression naturally appears in case of using another method proposed in [72]. It is based on substituting solutions of the Ward (or Slavnov-Taylor) identities into the Schwinger-Dyson equations. The Schwinger-Dyson equations can be used for making calculations in a certain approximation as in [73], where the four-loop anomalous dimension of quenched QED was obtained by this method. However, they can also allow to find results which are exact in all orders. In particular, in Abelian supersymmetric theories by using the Schwinger-Dyson equation it is possible to present the two-point Green function of the gauge superfield as a sum of two effective diagrams. One of them is related with the two-point Green functions of the matter superfields and gives the exact NSVZ β-function. The second effective diagram cannot be expressed in terms of these two-point Green functions. However, calculations made in [74, 75] show that this effective diagram (in the limit of the vanishing external momentum) is always given by integrals of total derivatives and vanishes, as it was suggested in [72]. This feature was not so far explained within a method based on using the Schwinger-Dyson equations that is the main obstacle for deriving the exact NSVZ β-function by this method. Thus, it is desirable to understand, why the second effective diagram vanishes, especially because it seems that the considered technique can be generalized to the the non-Abelian case. (Vanishing of this diagram can be interpreted as a special identity relating some Green functions.)

In this paper we complete derivation of the NSVZ β-function started in [72]. In particular, we directly prove that the second effective diagram vanishes, and the β-function is given by integrals of double total derivatives. The method used in this paper seems to be applicable in the non-Abelian case. That is why throughout the paper we try to use the notation, which can be also used for non-Abelian supersymmetric Yang-Mills theories.

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The paper is organized as follows: in section 2 we describe $\mathcal{N}=1$ SQED with N_f flavors, regularized by higher derivatives, and introduce the notation. In section 3 we write the Schwinger-Dyson equations for the considered theory and present the β-function (and its derivative with respect to a specially introduced parameter g) as a sum of effective diagrams. In section 4 we prove that the β-function is given by integrals of total derivatives and is equal to the exact NSVZ β-function. Also in this section we present a direct proof of a special identity for Green functions, which was proposed in [72]. In section 5 we prove that the β-function is given by integrals of double total derivatives and present the derivation of the exact NSVZ β-function from this fact. A large number of technical details are collected in appendixes.

2 $\mathcal{N}=1$ SQED with N_f flavors, regularized by higher derivatives

In this paper we derive the NSVZ β-function for $\mathcal{N}=1$ SQED with N_f flavors in all orders using the technique based on the Schwinger-Dyson equations. It is convenient to write the action for this theory in terms of superfields, because in this case supersymmetry is a manifest symmetry [76, 77]. In this notation in the massless limit the action is given by

$$S=\frac{1}{4e_0^2}\mathrm{Re}\int d^4x\,d^2\theta\,W^aW_a+\frac{1}{4}\sum_{\alpha=1}^{N_f}\int d^4x\,d^4\theta\,\Big(\phi_\alpha^*e^{2V}\phi_\alpha+\widetilde{\phi}_\alpha^*e^{-2V}\widetilde{\phi}_\alpha\Big). \tag{2.1}$$

In order to regularize this theory, we modify its action by adding a term with higher derivatives [47, 48]:

$$S_{\mathrm{reg}}=\frac{1}{4e_0^2}\mathrm{Re}\int d^4x\,d^2\theta\,W^aR(\partial^2/\Lambda^2)W_a+\frac{1}{4}\sum_{\alpha=1}^{N_f}\int d^4x\,d^4\theta\,\Big(\phi_\alpha^*e^{2V}\phi_\alpha+\widetilde{\phi}_\alpha^*e^{-2V}\widetilde{\phi}_\alpha\Big). \tag{2.2}$$

The higher derivatives are included into the function R, which satisfies the conditions $R(0)=1$ and $R(\infty)=\infty$. For example, it is possible to choose $R=1+\partial^{2n}/\Lambda^{2n}$. The term with higher derivatives increases a degree of the momentum in the propagator of the gauge superfield. As a consequence, most loop integrals become convergent in the ultraviolet region. An accurate analysis shows that after introducing the higher derivative term divergences remain only in the one-loop approximation [78]. In order to cancel these remaining one-loop divergences, one should insert the Pauli-Villars determinants $\det(V,M_I)$ into the generating functional [79]:

$$Z=\int DV\,D\phi\,D\widetilde{\phi}\,\prod_{I=1}^{n}(\det(V,M_I))^{c_IN_f}\exp\Big(iS_{\mathrm{reg}}[V,\phi,\widetilde{\phi}]+iS_{\mathrm{gf}}[V]+iS_{\mathrm{source}}\Big). \tag{2.3}$$

For fixing a gauge

$$S_{\mathrm{gf}}=-\frac{1}{64e_0^2}\int d^4x\,d^4\theta\,\Big(VR(\partial^2/\Lambda^2)D^2\bar{D}^2V+VR(\partial^2/\Lambda^2)\bar{D}^2D^2V\Big) \tag{2.4}$$

is added to the classical action, while ghosts can be omitted in the Abelian case. The masses of the Pauli-Villars fields should be proportional to the parameter Λ:

$$M_I=a_I\Lambda, \tag{2.5}$$

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where a_I are some real constants which do not depend on the bare coupling constant. The coefficients c_I should satisfy the conditions

$$\sum_{I=1}^{n} c_I = 1; \qquad \sum_{I=1}^{n} c_I M_I^2 = 0, \tag{2.6}$$

which ensure cancelation of the remaining one-loop divergences. For simplicity, in this paper we use the following choice of this coefficients:

$$c_I = (-1)^{P_I+1}, \tag{2.7}$$

where P_I is an integer. In this case for even P_I we can present the Pauli-Villars determinants as functional integrals over the commuting (chiral) Pauli-Villars superfields. For odd P_I the Pauli-Villars superfields are anticommuting. Therefore, P_I is a Grassmannian parity of the Pauli-Villars superfields, and

$$\prod_{I=1}^{n}\Big(\det(V,M_I)\Big)^{c_I N_f} = \int \prod_{\alpha=1}^{N_f}\prod_{I=1}^{n} D\phi_{\alpha I} D\widetilde{\phi}_{\alpha I}\, \exp(iS_{\rm PV}), \tag{2.8}$$

where the action for the Pauli-Villars superfields is

$$S_{\rm PV} = \sum_{I=1}^{n}\sum_{\alpha=1}^{N_f}\Big\{\frac{1}{4}\int d^8x\,\Big(\phi_{\alpha I}^* e^{2V}\phi_{\alpha I} + \widetilde{\phi}_{\alpha I}^* e^{-2V}\widetilde{\phi}_{\alpha I}\Big)$$
$$+\Big(\frac{1}{2}\int d^4x\, d^2\theta\, M_I \phi_{\alpha I}\widetilde{\phi}_{\alpha I} + \frac{1}{2}\int d^4x\, d^2\bar\theta\, M_I \phi_{\alpha I}^*\widetilde{\phi}_{\alpha I}^*\Big)\Big\} \tag{2.9}$$

with

$$\int d^8x \equiv \int d^4x\, d^4\theta. \tag{2.10}$$

In order to simplify subsequent equations and make the calculations similar to a non-Abelian case, we also introduce the notation

$$\phi_i \equiv (\phi_{\alpha I},\, \widetilde{\phi}_{\alpha I}); \qquad \phi^{*i} \equiv (\phi_{\alpha I}^*,\, \widetilde{\phi}_{\alpha I}^*), \quad i = 1, \ldots 2(n+1)N_f, \tag{2.11}$$

where the usual fields ϕ_α and $\widetilde{\phi}_\alpha$ by definition correspond to $I=0$. The sum of mass terms can be written as

$$S_m \equiv \frac{1}{2}\sum_{I=0}^{n}\sum_{\alpha=1}^{N_f}\Big(\int d^4x\, d^2\theta\, M_I\phi_{\alpha I}\widetilde{\phi}_{\alpha I} + \mbox{c.c.}\Big)$$
$$\equiv \frac{1}{4}\int d^4x\, d^2\theta\, M^{ij}\phi_i\phi_j + \frac{1}{4}\int d^4x\, d^2\bar\theta\, M^*_{ij}\phi^{*i}\phi^{*j}, \tag{2.12}$$

where $M_0 = 0$, because the usual fields, which corresponds to $I=0$, are considered in the massless limit. Due to the gauge invariance the mass matrix satisfies the equation

$$(T)_m{}^i M^{mj} + (T)_m{}^j M^{im} = (-1)^{P_j}(MT)^{j\,i} + (MT)^{ij} = 0, \tag{2.13}$$

where

$$(T)_i{}^j \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \delta_{\alpha\beta} \cdot \delta_{IJ} \equiv (-1)^{P_i} (T)^j{}_i \tag{2.14}$$

is a generator of the U(1) group in the considered representation and P_i is a Grassmanian parity of the superfield ϕ_i.

It is convenient to introduce sources both for the usual superfields V, ϕ, and $\widetilde{\phi}$ and for the Pauli-Villars superfields:

$$S_{\text{source}} \equiv \int d^8x\, V J + \Big(\int d^4x\, d^2\theta\, \phi_i j^i + \int d^4x\, d^2\bar{\theta}\, \phi^{*i} j_i^* \Big). \tag{2.15}$$

Eq. (2.3) is a standard definition of the generating functional for the considered theory, regularized by higher derivatives. However, it is convenient to use the background field method and introduce some auxiliary sources. In the Abelian case for this purpose we make the substitution $V \to V + \boldsymbol{V}$, where $\boldsymbol{V}$ is the background field. Also in the kinetic terms of the matter superfields we introduce the auxiliary real parameter g according to the prescription

$$e^{2V} \to 1 + g(e^{2V} - 1); \qquad e^{-2V} \to 1 + g(e^{-2V} - 1). \tag{2.16}$$

Then the usual kinetic terms are obtained for $g = 1$. It is important that this substitution is made only for the quantum gauge field, which is an integration variable in the generating functional. Moreover, we introduce the auxiliary sources ϕ_{0i} and $\widetilde{\phi}_{0i}$[1] for each pair of the matter superfields (including the Pauli-Villars fields) according to the prescription

$$\begin{aligned} S_{\text{matter}} \to \frac{1}{4} \sum_{I=0}^{n} \sum_{\alpha=1}^{N_f} \int d^8x \Big[&(\phi_{\alpha I}^* + \phi_{0\alpha I}^*) e^{2\boldsymbol{V}} \Big(1 + g(e^{2V} - 1)\Big) (\phi_{\alpha I} + \phi_{0\alpha I}) \\ &+ (\widetilde{\phi}_{\alpha I}^* + \widetilde{\phi}_{0\alpha I}^*) e^{-2\boldsymbol{V}} \Big(1 + g(e^{-2V} - 1)\Big) (\widetilde{\phi}_{\alpha I} + \widetilde{\phi}_{0\alpha I}) \Big] + S_m. \end{aligned} \tag{2.17}$$

From eq. (2.17) we see that, by definition, the parameter g is present only in vertices containing internal lines of the gauge superfield. It is important that introducing the parameter g we break the quantum gauge invariance. As a consequence, it is impossible to use Ward identities for Green functions containing external lines of the quantum gauge field. However, the background gauge invariance

$$\boldsymbol{V} \to \boldsymbol{V} - \frac{1}{2}(A + A^*); \qquad V \to V; \qquad \phi \to e^A \phi; \qquad \widetilde{\phi} \to e^{-A} \widetilde{\phi}, \tag{2.18}$$

where A is an arbitrary chiral superfield, is unbroken.

Thus, the generating functional is given by the following expression:

$$Z \equiv e^{iW} = \int D\mu \exp\Big(iS_{\text{total}} + iS_{\text{gf}} + iS_{\text{source}} \Big), \tag{2.19}$$

[1] It is important that we do not impose the chirality condition on the fields ϕ_{0i} and $\widetilde{\phi}_{0i}$.

where $D\mu$ denotes the integration measure and

$$S_{\text{total}} = \frac{1}{4e_0^2}\text{Re}\int d^4x\, d^2\theta\, W^a R(\partial^2/\Lambda^2)W_a + \frac{1}{4e_0^2}\text{Re}\int d^4x\, d^2\theta\, \boldsymbol{W}^a\boldsymbol{W}_a$$
$$+\frac{1}{4}\sum_{I=0}^{n}\sum_{\alpha=1}^{N_f}\int d^8x\,\Big[(\phi_\alpha^* + \phi_{0\alpha}^*)e^{2\boldsymbol{V}}\Big(1+g(e^{2V}-1)\Big)(\phi_\alpha+\phi_{0\alpha}) + (\widetilde{\phi}_\alpha^* + \widetilde{\phi}_{0\alpha}^*)e^{-2\boldsymbol{V}}$$
$$\times\Big(1+g(e^{-2V}-1)\Big)(\widetilde{\phi}_\alpha+\widetilde{\phi}_{0\alpha})\Big]_I + \sum_{I=0}^{n}\sum_{\alpha=1}^{N_f}\Big(\frac{1}{2}\int d^4x\, d^2\theta\, M\phi_\alpha\widetilde{\phi}_\alpha + \text{c.c.}\Big)_I, \tag{2.20}$$

where $\boldsymbol{W}_a = \bar{D}^2 D_a\boldsymbol{V}/4$ is the field strength for the background gauge superfield $\boldsymbol{V}$. (It is easy to see that terms linear in the quantum field V can be omitted. Also, it is not necessary to introduce the regulator in the part of the action which depends only on the background field.) The effective action is defined by the standard way as

$$\Gamma[V,\boldsymbol{V},\phi_i] = W - S_{\text{source}}, \tag{2.21}$$

where the sources should be expressed in terms of fields through solving the equations

$$\phi_i = (-1)^{P_i}\frac{\delta W}{\delta j^i};\qquad \phi^{*i} = (-1)^{P_i}\frac{\delta W}{\delta j_i^*};\qquad V = \frac{\delta W}{\delta J}. \tag{2.22}$$

Differentiating the effective action we obtain

$$j^i = -\frac{\delta\Gamma}{\delta\phi_i};\qquad j_i^* = -\frac{\delta\Gamma}{\delta\phi^{*i}};\qquad J = -\frac{\delta\Gamma}{\delta V}. \tag{2.23}$$

Below we will see that it is not convenient to consider (ϕ_i, V) as independent variables. A more convenient choice is (ϕ_i, J), where J is a source for the quantum gauge superfield V. That is why below instead of the effective action we will mostly use the Routhian

$$\gamma[J,\boldsymbol{V},\phi_i] = W - \Big(\int d^4x\, d^2\theta\,\phi_i j^i + \text{c.c.}\Big), \tag{2.24}$$

where it is necessary to express only the sources j^i in terms of J and ϕ_i.

Due to the background gauge invariance (2.18) the two-point function of the background gauge superfield is transversal:

$$\Gamma^{(2)}_{\boldsymbol{V}} = -\frac{1}{16\pi}\int\frac{d^4p}{(2\pi)^4}\, d^4\theta\,\boldsymbol{V}(\theta,-p)\,\partial^2\Pi_{1/2}\boldsymbol{V}(\theta,p)\, d^{-1}(\alpha_0,\Lambda/p), \tag{2.25}$$

where $\alpha_0 = e_0^2/4\pi$ is a bare coupling constant and the supersymmetric transversal projector is given by

$$\partial^2\Pi_{1/2} = -\frac{1}{8}D^a\bar{D}^2D_a. \tag{2.26}$$

In this paper we will calculate the β-function defined in terms of the bare coupling constant

$$\beta\Big(\alpha_0(\alpha,\Lambda/\mu)\Big) = \frac{d\alpha_0(\alpha,\Lambda/p)}{d\ln\Lambda}\Big|_{\alpha=\text{const}}, \tag{2.27}$$

where $\alpha = \alpha(\alpha_0, \Lambda/p)$ is a renormalized coupling constant. It is determined by the requirement that the function $d(\alpha_0(\alpha, \Lambda/\mu), \Lambda/p)$ is finite in the limit $\Lambda \to \infty$. The anomalous dimension can be defined similarly:

$$\gamma\Big(\alpha_0(\alpha, \Lambda/\mu)\Big) \equiv -\frac{d\ln Z(\alpha, \Lambda/\mu)}{d\ln\Lambda}\Big|_{\alpha=\text{const}}, \tag{2.28}$$

where Z is a renormalization constant for the matter superfield, which is constructed by requiring finiteness of the function ZG in the limit $\Lambda \to \infty$. It is easy to see that the β-function (2.27) and the anomalous dimension (2.28) do not depend on a choice of the renormalized coupling constant α and the renormalization constant Z (see, e.g., [68]). The renormalization group functions (2.27) and (2.28) differ from the standard ones defined in terms of the renormalized coupling constant

$$\begin{aligned}\widetilde\beta\Big(\alpha(\alpha_0, \Lambda/\mu)\Big) &\equiv \frac{d\alpha(\alpha_0, \Lambda/\mu)}{d\ln\mu}\Big|_{\alpha_0=\text{const}};\\ \widetilde\gamma\Big(\alpha(\alpha_0, \Lambda/\mu)\Big) &\equiv \frac{d\ln Z(\alpha(\alpha_0, \Lambda/\mu), \Lambda/\mu)}{d\ln\mu}\Big|_{\alpha_0=\text{const}},\end{aligned} \tag{2.29}$$

which are scheme-dependent. However [68, 69], the functions (2.27) and (2.28) can be obtained from the renormalization group functions (2.29) by imposing the boundary conditions

$$Z_3(\alpha, x_0) = 1; \qquad Z(\alpha, x_0) = 1 \tag{2.30}$$

on the renormalization constants, where x_0 is an arbitrary fixed value of $\ln\Lambda/\mu$.[2]

In order to find the β-function (2.27) we calculate the expression

$$\frac{d}{d\ln\Lambda}\Big(d^{-1}(\alpha_0, \Lambda/p) - \alpha_0^{-1}\Big)\Big|_{p=0} = -\frac{d\alpha_0^{-1}}{d\ln\Lambda} = \frac{\beta(\alpha_0)}{\alpha_0^2}, \tag{2.31}$$

where Λ and α are considered as independent variables. This expression is well defined if the right hand side is expressed in terms of the bare coupling constant α_0. The left hand side of the expression (2.31) can be obtained from the two-point Green function of the background gauge superfield after the substitution

$$\boldsymbol{V}(x,\theta) \to \bar\theta^{\dot a}\bar\theta_{\dot a}\theta^b\theta_b \equiv \theta^4. \tag{2.32}$$

Strictly speaking, the part of the effective action corresponding to the two-point function of the gauge superfield is infinite after this substitution, because it is proportional to

$$\int d^4x \to \infty. \tag{2.33}$$

However, this procedure can be rigorously formulated by inserting a regulator $I(x)$

$$\boldsymbol{V}(x,\theta) \to \bar\theta^{\dot a}\bar\theta_{\dot a}\theta^b\theta_b \cdot I(x) \equiv \theta^4 \cdot I(x) \approx \theta^4, \tag{2.34}$$

[2]These boundary conditions are imposed only in a single point. They should not be confused with the condition $Z_3 = 1$ following from the conformal symmetry (see, e.g., [80]), which is valid for arbitrary values of $\ln\Lambda/\mu$.

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which is approximately equal to 1 at finite x^μ and tends to 0 at the large scale $R \to \infty$. Then in the leading order in R the considered part of the effective action is proportional to

$$\mathcal{V}_4 \equiv \int d^4x\, I^2 \sim R^4 \to \infty. \tag{2.35}$$

All terms containing the derivatives of the regulator I are suppressed as $1/R\Lambda \to 0$ and can be omitted. That is why below we do not explicitly write the regulator I, but assume that $\mathcal{V}_4$ is finite and tends to infinity. Actually this corresponds to taking the limit of the vanishing external momentum $p \sim R^{-1} \sim (\mathcal{V}_4)^{-1/4} \to 0$:

$$\frac{1}{2\pi}\mathcal{V}_4 \cdot \frac{d}{d\ln\Lambda}\Big(d^{-1}(\alpha_0, \Lambda/p) - \alpha_0^{-1}\Big)\Big|_{p=0} = \frac{1}{2\pi}\mathcal{V}_4 \cdot \frac{\beta(\alpha_0)}{\alpha_0^2}\Big|_{p=0} = \frac{d(\Delta\Gamma^{(2)}_{\boldsymbol{V}})}{d\ln\Lambda}\Big|_{\boldsymbol{V}(x,\theta)=\theta^4}, \tag{2.36}$$

where

$$\Delta\Gamma \equiv \Gamma - \frac{1}{4e_0^2}\mathrm{Re}\int d^4x\, d^2\theta\, \boldsymbol{W}^a \boldsymbol{W}_a. \tag{2.37}$$

The expressions for the two-point Green functions of the matter superfields can be found using arguments based on the chirality. Taking into account that the two-point functions of the matter superfields constructed from Γ and γ evidently coincide, they can be written as

$$A_{xy} \equiv \begin{pmatrix} \dfrac{\delta^2\gamma}{\delta(\phi_i)_x \delta(\phi^{*j})_y} & \dfrac{\delta^2\gamma}{\delta(\phi_i)_x \delta(\phi_j)_y} \\ \\ \dfrac{\delta^2\gamma}{\delta(\phi^{*i})_x \delta(\phi^{*j})_y} & \dfrac{\delta^2\gamma}{\delta(\phi^{*i})_x \delta(\phi_j)_y} \end{pmatrix} = \begin{pmatrix} G_j{}^i \frac{\bar D_x^2 D_x^2}{16}\delta^8_{xy} & -\frac{1}{4}(MJ)^{j\,i}\bar D_x^2 \delta^8_{xy} \\ \\ -\frac{1}{4}(MJ)^*_{j\,i} D_x^2 \delta^8_{xy} & G^j{}_i \frac{D_x^2 \bar D_x^2}{16}\delta^8_{xy} \end{pmatrix}, \tag{2.38}$$

where the fields are set to 0, $G_i{}^j$ and $(MJ)^{ij}$ are functions of ∂^2, and in our notation

$$\begin{aligned} G_i{}^j &\equiv (-1)^{P_i} G^j{}_i = \delta_{\alpha\beta}\cdot \delta_{IJ}\cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} G_I(\partial^2); \\ (MJ)^{ij} &= (MJ)^*_{ij} = \delta_{\alpha\beta}\cdot \delta_{IJ}\cdot \begin{pmatrix} 0 & M_I \\ (-1)^{P_I} M_I & 0 \end{pmatrix} J_I(\partial^2). \end{aligned} \tag{2.39}$$

(2×2 matrixes correspond to the fields ϕ and $\widetilde{\phi}$. The function J is real as a consequence of the CP-invariance.) By definition the matrix A^{-1} constructed from the inverse Green functions satisfies the condition

$$\int d^8y\, (A^{-1})_{xy} \begin{pmatrix} 0 & \bar D^2/8\partial^2 \\ D^2/8\partial^2 & 0 \end{pmatrix}_y A_{yz} = \begin{pmatrix} 0 & -\bar D^2/2 \\ -D^2/2 & 0 \end{pmatrix} \delta^8_{xz}. \tag{2.40}$$

An explicit expression for the matrix A^{-1} can be easily found:

$$(A^{-1})_{xy} \equiv \begin{pmatrix} \left(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*j})_y}\right)^{-1} & \left(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_j)_y}\right)^{-1} \\ \left(\frac{\delta^2\gamma}{\delta(\phi^{*i})_x\delta(\phi^{*j})_y}\right)^{-1} & \left(\frac{\delta^2\gamma}{\delta(\phi^{*i})_x\delta(\phi_j)_y}\right)^{-1} \end{pmatrix}$$
$$= -\frac{1}{\partial^2 G^2 + |MJ|^2}\begin{pmatrix} G^j{}_i\frac{\bar{D}_x^2 D_x^2}{4}\delta^8_{xy} & (MJ)^*_{ij}\bar{D}_x^2\delta^8_{xy} \\ (MJ)^{ij}D_x^2\delta^8_{xy} & G_j{}^i\frac{D_x^2\bar{D}_x^2}{4}\delta^8_{xy} \end{pmatrix}, \tag{2.41}$$

where the operator $\partial^2 G^2 + |MJ|^2$ is defined by the following prescription:

$$(\partial^2 G^2 + |MJ|^2)_i{}^k \equiv \partial^2 G_i{}^j G_j{}^k + (MJ)^*_{j\,i}(MJ)^{j\,k} = \delta_{\alpha\beta}\cdot\delta_{IJ}\cdot\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}(\partial^2 G_I^2 + M_I^2 J_I^2). \tag{2.42}$$

3 Schwinger-Dyson equations

Making the change of variables $\phi_i \to \phi_i + A_i$, where A_i are arbitrary chiral superfields, in the generating functional (2.19), we obtain the equation

$$-\frac{1}{2}\bar{D}^2\frac{\delta\Gamma}{\delta\phi_{0i}} - \frac{\delta\Gamma}{\delta\phi_i} + \frac{1}{2}M^{ij}\phi_j = 0. \tag{3.1}$$

This equation can be considered as a Schwinger-Dyson equation for the matter superfields.

Similarly, the Schwinger-Dyson equation for the two-point Green function of the gauge superfield can be written as [72]

$$\frac{\delta(\Delta\Gamma)}{\delta\boldsymbol{V}_x} = \frac{1}{2}\sum_{I=0}^{n}\sum_{\alpha=1}^{N_f}\Big\langle(\phi^*_\alpha + \phi^*_{0\alpha})e^{2\boldsymbol{V}}\Big(1 + g(e^{2V}-1)\Big)(\phi_\alpha + \phi_{0\alpha})$$
$$-(\widetilde{\phi}^*_\alpha + \widetilde{\phi}^*_{0\alpha})e^{-2\boldsymbol{V}}\Big(1 + g(e^{-2V}-1)\Big)(\widetilde{\phi}_\alpha + \widetilde{\phi}_{0\alpha})\Big\rangle_I, \tag{3.2}$$

where

$$\langle A\rangle \equiv \frac{1}{Z}\int D\mu\, A[V,\phi_i]\exp\Big(iS_{\text{total}} + iS_{\text{gf}} + iS_{\text{Source}}\Big) \tag{3.3}$$

and the sources should be expressed in terms of the fields using eq. (2.22). Because in this paper we use the background field method, eq. (3.2) can be simply obtained by differentiation of the effective action with respect to the background field $\boldsymbol{V}$. It is easy to see that the Schwinger-Dyson equation (3.2) can be equivalently rewritten in terms of derivatives with respect to the sources

$$\frac{\delta(\Delta\Gamma)}{\delta\boldsymbol{V}_x} = 2(T)^j{}_i\Big(\frac{1}{i}\frac{\delta}{\delta(j^j)_x}\frac{\delta\Gamma}{\delta(\phi_{0i})_x} + \frac{\delta\Gamma}{\delta(\phi_{0i})_x}(\phi_j + \phi_{0j})_x\Big)$$
$$= 2(T)^j{}_i\Big(\frac{1}{i}\frac{\delta}{\delta(j^*_i)_x}\frac{\delta\Gamma}{\delta(\phi_0^{*j})_x} + \frac{\delta\Gamma}{\delta(\phi_0^{*j})_x}(\phi^{*i} + \phi_0^{i*})_x\Big), \tag{3.4}$$

$$\Delta\Gamma^{(2)}_{\boldsymbol{V}} = \frac{1}{2}\int d^8x\, d^8y\, \boldsymbol{V}_x \boldsymbol{V}_y \frac{\delta^2(\Delta\Gamma)}{\delta\boldsymbol{V}_x \delta\boldsymbol{V}_y} =$$

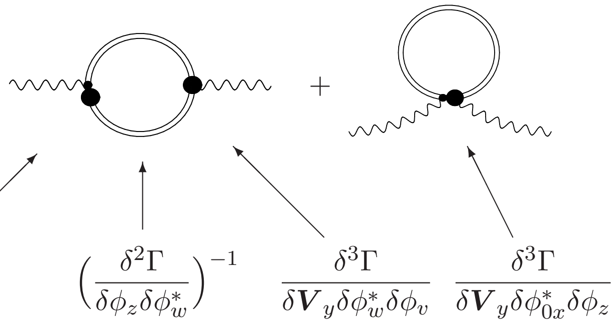

Figure 1. The Schwinger-Dyson equation for the two-point function of the gauge superfield. Below we present Feynman rules (for simplicity, in the massless case). In the massive case the effective diagrams are the same.

where the derivatives with respect to the sources are constructed according to the prescription

$$\frac{\delta}{\delta(j^i)_x} = -\int d^8y \Bigg[\Big(\frac{\delta^2\Gamma}{\delta(\phi_i)_x\delta(\phi^{*j})_y}\Big)^{-1}\frac{\bar{D}^2_y}{8(\partial^2)_y}\frac{\delta}{\delta(\phi^{*j})_y}$$
$$+\Big(\frac{\delta^2\Gamma}{\delta(\phi_i)_x\delta(\phi_j)_y}\Big)^{-1}\frac{D^2_y}{8(\partial^2)_y}\frac{\delta}{\delta(\phi_j)_y} + \Big(\frac{\delta^2\Gamma}{\delta(\phi_i)_x\delta V_y}\Big)^{-1}\frac{\delta}{\delta V_y}\Bigg]. \tag{3.5}$$

In order to verify this equation it is necessary to apply it to $(j^k)_z$ taking into account eqs. (2.23) and (2.40). Differentiating eq. (3.4) with respect to $\boldsymbol{V}_y$ and setting all fields (including ϕ_0) to 0, we obtain the Schwinger-Dyson equation for the two-point Green function of the gauge superfield. Details of this calculation are presented in appendix A. Following [72] it is convenient to formulate the result in the graphical form. It is presented in figure 1. In this figure external lines correspond to the gauge superfield $\boldsymbol{V}$, and, for simplicity, expressions for vertices and propagators are written for the massless case. In the analytical form the Schwinger-Dyson equation presented in figure 1 is written as

$$\frac{1}{2}\int d^8x\, d^8y\, \boldsymbol{V}_x\boldsymbol{V}_y\frac{\delta^2(\Delta\Gamma)}{\delta\boldsymbol{V}_y\delta\boldsymbol{V}_x} = -i(T)^j{}_i\int d^8x\, d^8y\, d^8z\, \boldsymbol{V}_x\boldsymbol{V}_y\Bigg(\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_x}\frac{\delta^2\Gamma}{\delta\boldsymbol{V}_y\delta(\phi_{0k})_z}$$
$$\times\Big[\delta^8_{xz}\delta^i_k + \frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^k)_z}\frac{\delta\gamma}{\delta(\phi_{0i})_x}\Big] + \frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_x}\frac{\delta^2\Gamma}{\delta\boldsymbol{V}_y\delta(\phi_0^{*k})_z}\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^*_k)_z}\frac{\delta\gamma}{\delta(\phi_{0i})_x}\Bigg), \tag{3.6}$$

where all fields are set to 0, and we use the notation

$$\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_x} = -\int d^8w\Bigg(\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x\delta(\phi_m)_w}\Big)^{-1}_{\phi,\phi_0,V=0}\frac{\delta}{\delta(\phi_{0m})_w} + \Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x\delta(\phi^{*m})_w}\Big)^{-1}_{\phi,\phi_0,V=0}\frac{\delta}{\delta(\phi_0^{*m})_w}\Bigg). \tag{3.7}$$

Note that all fields here are set to 0 in contrast to eq. (3.5). Due to this condition the derivatives $\boldsymbol{\delta}/\boldsymbol{\delta}j$ (anti)commute. However, below we will not usually write explicitly the condition $\phi, \phi_0, V = 0$ as in eq. (3.7).

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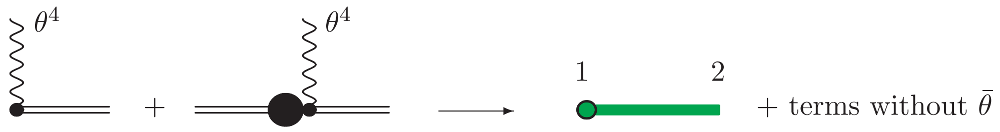


Figure 2. The sum of two effective lines.

The Schwinger-Dyson equation (3.6) can be simplified after the substitution (2.32). (As we already mentioned above, this substitution automatically gives $p = 0$.) For this purpose it is convenient to use the identity

$$\begin{aligned}
&(T)^j{}_i \int d^8x\,(\theta^4)_x\bigg\{\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_x}\frac{\delta}{\delta(\phi_{0i})_x}+\int d^8y\,\bigg[\Big(\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^k)_y}\frac{\delta\gamma}{\delta(\phi_{0i})_x}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_x}\frac{\delta}{\delta(\phi_{0k})_y}+\Big(\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^*_k)_y}\frac{\delta\gamma}{\delta(\phi_{0i})_x}\Big)\\
&\times\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_x}\frac{\delta}{\delta(\phi_0^{*k})_y}\bigg]\bigg\}=\int d^8x\,\bigg\{(T)^j{}_i\Big(\Big[\theta^a\theta_a\bar\theta^{\dot b}\frac{\bar D_{\dot b}D^2}{4\partial^2}+2i\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\frac{\partial_\mu}{\partial^2}-\frac{D^2}{4\partial^2}\theta^a\theta_a\Big]\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}\Big)\frac{\delta}{\delta\phi_{0i}}\\
&-i(MT)^{ij}\,\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\Big(\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^i}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}\bigg\},
\end{aligned}\tag{3.8}$$

which is proved in appendix B. It is convenient to define the operator which contains all terms of the first degree in $\bar\theta$ in eq. (3.8):

$$\begin{aligned}
\text{GreenLine}[1,2]\equiv\int d^8x\,\Big((T)^j{}_i\Big(\theta^a\theta_a\bar\theta^{\dot b}\frac{\bar D_{\dot b}D^2}{4\partial^2}+2i\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\frac{\partial_\mu}{\partial^2}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^j}\cdot\frac{\delta}{\delta_1\phi_{0i}}\\
-i(MT)^{ij}\,\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\Big(\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^i}\Big)\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^j}\Big).
\end{aligned}\tag{3.9}$$

We will graphically denote this operator by a green effective line with the ends 1 and 2. In figure 2 the identity (3.8) is presented in the graphical form. The indexes 1 and 2 in the right hand side of eq. (3.9) point the vertices to which the corresponding derivatives act. (Sometimes we will omit these indexes if they coincide.) Actually this expression can be considered as a modification of the effective propagator (multiplied by two derivatives with respect to the ϕ_0). Also we will also use other effective lines. Our notation is presented in figure 3. Note that in the case of using color lines we do not sometimes explicitly draw the external lines. Instead of them we draw a small circle, to which we attach corresponding θ-s.

Using the identity (3.8) it is possible to rewrite the Schwinger-Dyson equation (3.6) in a different form. The result is presented in figure 4. In the analytical form it can be written as

$$\frac{1}{2}\int d^8x\,d^8y\,(\theta^4)_x(\theta^4)_y\frac{d}{d\ln\Lambda}\frac{\delta^2(\Delta\Gamma)}{\delta\boldsymbol{V}_x\delta\boldsymbol{V}_y}=-i\frac{d}{d\ln\Lambda}\int d^8y\,(\theta^4)_y\,\text{GreenLine}\cdot\frac{\delta\Gamma}{\delta\boldsymbol{V}_y}+\delta,\tag{3.10}$$

where

$$\delta\equiv i\int d^8x\,d^8y\,(\theta^4)_y(T)^j{}_i\Big(\frac{D^2}{4\partial^2}\theta^a\theta_a\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}\Big)_x\frac{\delta^2\Gamma}{\delta(\phi_{0i})_x\delta\boldsymbol{V}_y}.\tag{3.11}$$

The green effective line can be presented as a sum of the blue and yellow lines, see figure 5:

$$\text{GreenLine}[1,2]=2\cdot\text{BlueLine}_{\dot b}[\theta^a\theta_a\bar\theta^{\dot b};1,2]+\text{YellowLine}_\mu[\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b;1,2],\tag{3.12}$$

$$\text{UsualLine}[1,2] = \int d^8x \left(\frac{\delta}{\delta_2\phi_{0i}}\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^i} + \frac{\delta}{\delta_2\phi_0^{*i}}\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j_i^*}\right)$$

$$\text{GreenLine}[1,2] = \int d^8x \Big((T)^j{}_i\Big(\theta^a\theta_a\bar\theta^{\dot b}\frac{\bar D_{\dot b}D^2}{4\partial^2} + 2i\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\frac{\partial_\mu}{\partial^2}\Big)$$
$$\times\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^j}\cdot\frac{\delta}{\delta_1\phi_{0i}} - i(MT)^{ij}\,\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\Big(\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^i}\Big)\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^j}\Big)$$

$$\text{GreenWithCross}[1,2] = -\int d^8x\,\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\Big(i(MT)^{ij}\Big(\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^i}\Big)\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^j}\Big)$$

$$\text{BrownLine}_\mu[1,2] = \int d^8x\,\Big((T)^j{}_i\Big(-\frac{2i\partial_\mu}{\partial^2} - (\gamma_\mu)^{a\dot b}\theta_a\frac{\bar D_{\dot b}D^2}{4\partial^2}\Big)$$
$$\times\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^j}\cdot\frac{\delta}{\delta_1\phi_{0i}} + i(MT)^{ij}\Big(\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^i}\Big)\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^j}\Big)$$

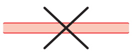
$$\text{BrownWithCross}_\mu[1,2] = i(MT)^{ij}\int d^8x\,\Big(\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^i}\Big)\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^j}$$

$$\text{BlueLine}^{\dot b}[\alpha;1,2] = \int d^8x\,\alpha\,(T)^j{}_i\Big(\frac{\bar D^{\dot b}D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^j}\Big)\cdot\frac{\delta}{\delta_1\phi_{0i}}$$

$$\text{RedLine}^{\dot b}[1,2] = \int d^8x\,\Big((T)^j{}_i\Big(\theta^a\theta_a\frac{\bar D^{\dot b}D^2}{\partial^2} - i(\gamma^\mu)^{a\dot b}\theta_a\frac{\bar D^2 D^2\partial_\mu}{2\partial^4}\Big)$$
$$\times\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^j}\cdot\frac{\delta}{\delta_1\phi_{0i}} - i(MT)^{ij}(\gamma^\mu)^{a\dot b}\theta_a\Big(\frac{D^2\partial_\mu}{4\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^i}\Big)\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^j}\Big)$$

$$\text{RedWithCross}^{\dot b}[1,2] = -i(MT)^{ij}\int d^8x\,(\gamma^\mu)^{a\dot b}\theta_a\Big(\frac{D^2\partial_\mu}{4\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^i}\Big)\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^j}$$

$$\text{PinkLine}^{\dot b}[1,2] = \int d^8x\,\Big((T)^j{}_i\Big(\theta^a\theta_a\frac{\bar D^{\dot b}D^2}{2\partial^2} - i(\gamma^\mu)^{a\dot b}\theta_a\frac{\bar D^2 D^2\partial_\mu}{8\partial^4}\Big)$$
$$\times\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^j}\cdot\frac{\delta}{\delta_1\phi_{0i}} - i(MT)^{ij}(\gamma^\mu)^{a\dot b}\theta_a\Big(\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^i}\Big)\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^j}\Big)$$



$$\text{PinkWithCross}^{\dot b}[1,2] = -i(MT)^{ij}\int d^8x\,(\gamma^\mu)^{a\dot b}\theta_a\Big(\frac{D^2\partial_\mu}{8\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^i}\Big)\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^j}$$

$$\text{YellowLine}_\mu[\alpha;1,2] = \int d^8x\,\alpha\,\Big(2i(T)^j{}_i\Big(\frac{\partial_\mu}{\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^j}\Big)\cdot\frac{\delta}{\delta_1\phi_{0i}}$$
$$-i(MT)^{ij}\Big(\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^i}\Big)\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^j}\Big)$$

$$\text{LineWithDot}[\alpha;1,2] \equiv \frac{i}{4}\int d^8x\,\alpha\,\Big(\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j_i^*}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^i} + \frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j_i^*}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^i} + M^{ij}\Big(\frac{D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^i}\Big)$$
$$\times\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^j} + M^*_{ij}\Big(\frac{\bar D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j_i^*}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j_j^*} + M^{ij}\Big(\frac{D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^i}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^j} + M^*_{ij}\Big(\frac{\bar D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j_i^*}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j_j^*}\Big)$$

Figure 3. Definitions of some effective lines which are used in this paper. Expressions for the lines with a cross are obtained after extracting terms proportional to masses of the Pauli-Villars fields.

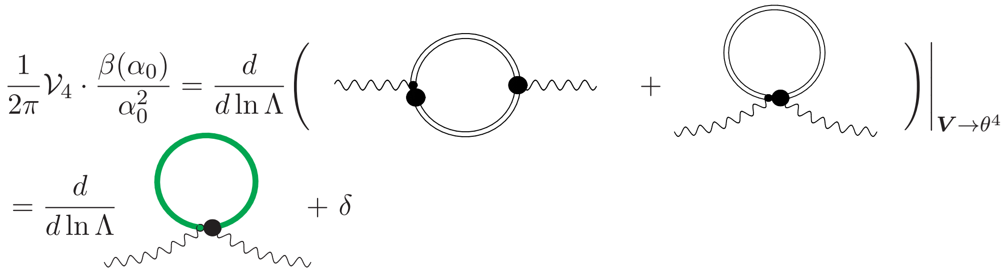


Figure 4. Obtaining the β-function (defined in terms of the bare coupling constant) from the Schwinger-Dyson equation. The additional term δ is given by eq. (3.11).

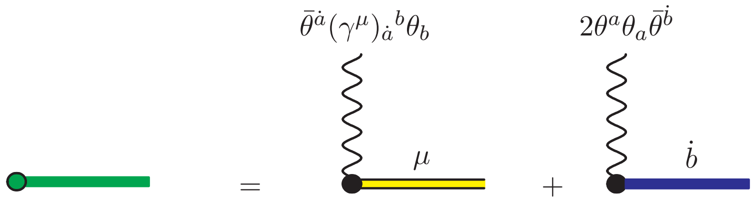


Figure 5. The green effective line can be presented as a sum of the blue and yellow effective lines.

where

$$\begin{aligned}
\text{BlueLine}^{\dot b}[\alpha;1,2] &\equiv \int d^8x\,\alpha\,(T)^j{}_i\Big(\frac{\bar D^{\dot b}D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^j}\Big)\cdot\frac{\delta}{\delta_1\phi_{0i}}; \qquad (3.13)\\
\text{YellowLine}_\mu[\alpha;1,2] &\equiv \int d^8x\,\alpha\,\Big(2i(T)^j{}_i\Big(\frac{\partial_\mu}{\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^j}\Big)\cdot\frac{\delta}{\delta_1\phi_{0i}} - i(MT)^{ij}\Big(\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^i}\Big)\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^j}\Big).
\end{aligned}$$

Using eq. (3.12) and the identity

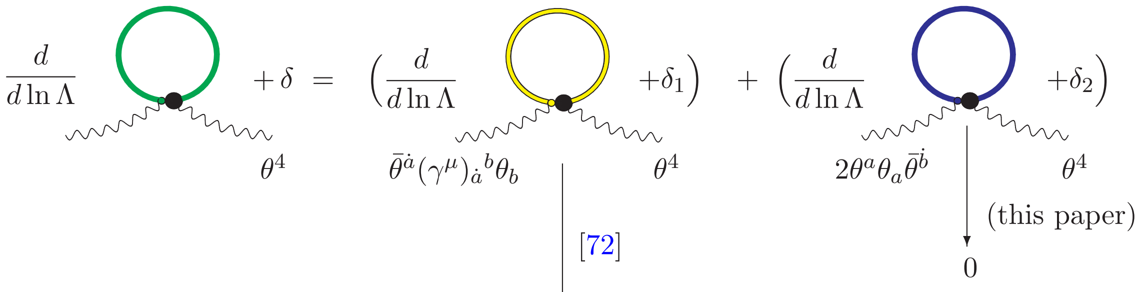
$$\frac{D^2}{4\partial^2}\theta^a\theta_a = \theta^a\theta_a\frac{D^2}{4\partial^2} + \Big(\theta^a\frac{D_a}{\partial^2} - \frac{1}{\partial^2}\Big) \qquad (3.14)$$

it is possible to split the effective diagram presented in figure 4 into two parts. This is shown in figure 6. In this figure we use the notation

$$\begin{aligned}
\delta_1 &\equiv i\int d^8x\,d^8y\,(\theta^4)_y(T)^j{}_i\Big(\frac{\theta^a D_a - 1}{\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^j}\Big)_x\frac{\delta^2\Gamma}{\delta(\phi_{0i})_x\delta\boldsymbol{V}_y};\\
\delta_2 &\equiv i\int d^8x\,d^8y\,(\theta^4)_y(T)^j{}_i\Big(\theta^a\theta_a\frac{D^2}{4\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^j}\Big)_x\frac{\delta^2\Gamma}{\delta(\phi_{0i})_x\delta\boldsymbol{V}_y} = \delta - \delta_1. \qquad (3.15)
\end{aligned}$$

The result for the effective diagram with the yellow effective line (including δ_1) can be expressed in terms of the anomalous dimension of the matter superfield [72].[3] For this purpose it is necessary to substitute the solution of the Ward identity for the effective vertex. The result is also presented in figure 6. It is given by an integral of a total derivative with respect to a loop momentum. Such a structure allows to reduce a number

[3] In [72] only the case $N_f = 1$ is considered.



$$N_f\mathcal{V}_4\frac{d}{d\ln\Lambda}\int\frac{d^4q}{(2\pi)^4}\frac{4}{q^2}\frac{d}{dq^2}\bigg(2\ln G-\sum_{I=1}^{n}c_I\Big(\ln(q^2G^2+M^2J^2)+\frac{M^2J}{q^2G^2+M^2J^2}\Big)_I\bigg)$$

Figure 6. The result of substituting the solution of Ward identities into the Schwinger-Dyson equation (for the diagram with the yellow effective line).

of momentum integrations and relate the β-function with the anomalous dimension of the matter superfield. Really, calculating the integral in the four-dimensional spherical coordinates we obtain

$$N_f\mathcal{V}_4\frac{d}{d\ln\Lambda}\int\frac{d^4q}{(2\pi)^4}\frac{4}{q^2}\frac{d}{dq^2}\bigg(2\ln G-\sum_{I=1}^{n}c_I\Big(\ln(q^2G^2+M^2J^2)+\frac{M^2J}{q^2G^2+M^2J^2}\Big)_I\bigg)$$
$$=\frac{1}{2\pi}\mathcal{V}_4\cdot\frac{N_f}{\pi}\Big(1-\gamma(\alpha_0)\Big), \tag{3.16}$$

where we take into account that for a function $f(q^2)$ which rapidly decreases at the infinity

$$\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2}\frac{df}{dq^2}=\frac{1}{16\pi^2}\int\limits_0^\infty dq^2\frac{df}{dq^2}=\frac{1}{16\pi^2}\Big(f(\infty)-f(0)\Big)=-\frac{1}{16\pi^2}f(0). \tag{3.17}$$

(The functions considered here rapidly decrease at the infinity due to the higher derivative regularization.) As a consequence, the contribution of the diagram with the yellow effective line gives the exact NSVZ β-function

$$\beta(\alpha_0)=\frac{\alpha_0^2N_f}{\pi}\Big(1-\gamma(\alpha_0)\Big). \tag{3.18}$$

However, using this method it is impossible to calculate the diagram with the blue effective line in figure 6. Some explicit calculations in the lowest (three- and four-) loops [75] show that this diagram plus δ_2 is also given by an integral of a total derivative and vanishes. In the graphical form this is presented in figures 6 and 7. This equality can be considered as a nontrivial relation between Green functions [72]. It was proved indirectly in [67] using a method proposed in [71]. In particular, it is possible to prove that the integrand corresponding to this diagram is a total derivative. In the analytical form the equality presented in figure 7 can be written as

$$-i\frac{d}{d\ln\Lambda}\int d^8x\,d^8y\,(\theta^4)_y(T)^j{}_i\Big(\theta^a\theta_a\bar\theta^{\dot b}\frac{\bar D_{\dot b}D^2}{4\partial^2}-\theta^a\theta_a\frac{D^2}{4\partial^2}\Big)_x\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_x}\frac{\delta}{\delta(\phi_{0i})_x}\frac{\delta\Gamma}{\delta\boldsymbol{V}_y}=0. \tag{3.19}$$

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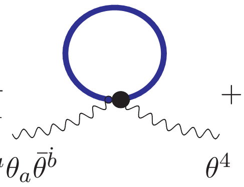
$$2 \cdot \frac{d}{d\ln\Lambda}$$



$$+\ \delta_2 \ =\ 0.$$

Figure 7. This identity was suggested in [72]. In this paper this equality is proved.

In this paper we prove this identity directly. Moreover, we prove that the β-function is given by integrals of double total derivatives. In order to do this, it is necessary to use two ideas.

1. First, it is necessary to rewrite the effective vertices in the diagrams presented in figure 4 or figure 7 using the Schwinger-Dyson equation one more time. This procedure was first proposed in [81]. Let us, for example, start with eq. (3.10) and substitute the expression for $\delta\Gamma/\delta\boldsymbol{V}_y$ from the Schwinger-Dyson equation (3.4). It is convenient to write the result in terms of the Routhian γ, because in this case the number of effective diagrams is less. Details of this calculation are presented in appendix C. The result can be written in the following form:

$$\frac{1}{2}\int d^8x\, d^8y\,(\theta^4)_x(\theta^4)_y \frac{d}{d\ln\Lambda}\frac{\delta^2\Delta\Gamma}{\delta\boldsymbol{V}_x\delta\boldsymbol{V}_y} = -2\frac{d}{d\ln\Lambda}(\text{GreenLine})^2\cdot\gamma + \Delta, \tag{3.20}$$

where

$$\Delta \equiv -i\frac{d}{d\ln\Lambda}\int d^8x\, d^8y\,(\theta^4)_x\Big(\frac{D^2}{4\partial^4}\Big)_y\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_j)_y}\Big)^{-1}\times$$
$$\times\Big\{C(R)_k{}^i M^{j\,k}\delta^8_{xy} - (MT)^{im}(MT)^{lj}\Big(\frac{D^2}{32\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_m)_x\delta(\phi_l)_y}\Big)^{-1}\Big\} \tag{3.21}$$

with

$$C(R)_i{}^m = (T)_i{}^k(T)_k{}^m = \begin{pmatrix} 1 & 0\\ 0 & 1\end{pmatrix}\cdot\delta_{\alpha\beta}\cdot\delta_{IJ}. \tag{3.22}$$

(Δ can be graphically interpreted as a one-loop effective diagram. However, in order to avoid too large number of effective diagrams we write this term explicitly.) Note that this expression does not contain infrared divergences due to the differentiation with respect to $\ln\Lambda$, which should be made before the momentum integration.

A simple (qualitative) graphical interpretation of results obtained in appendix C is presented in figure 8. (A white line can be substituted by any other effective line.) In particular, if this rule is applied to the diagram in the left hand side of the equation presented in figure 6, then a β-function will be determined by the two-loop effective diagram presented in figure 9. As earlier, it is convenient to split this effective diagram into two parts using eq. (3.12). The result is graphically presented in figure 9, where (see appendix C.2)

$$\Delta_2 = \delta_2;\qquad \Delta_1 = \Delta - \Delta_2. \tag{3.23}$$

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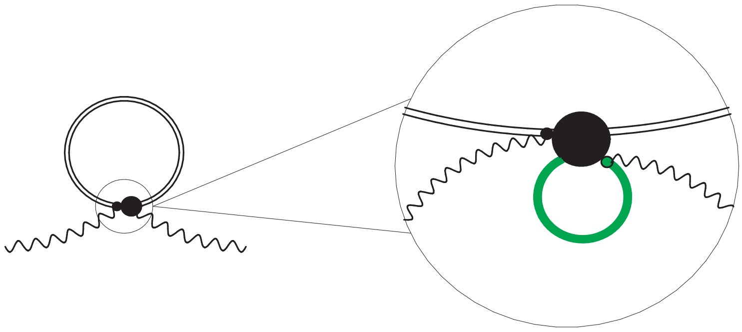


Figure 8. Applying the Schwinger-Dyson equation to the effective vertex we can see "the inner structure" of the effective diagram. (The large circles can correspond to any effective line.)

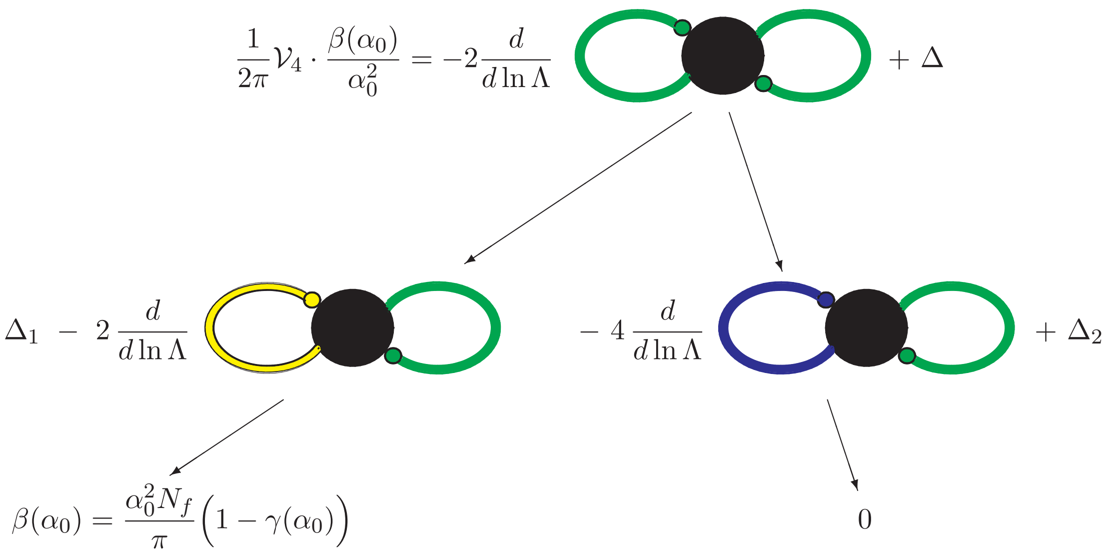


Figure 9. Applying the Schwinger-Dyson equation one more time, it is possible to obtain that a β-function is determined by the two-loop effective diagrams.

In appendix C.2 the expression Δ_2 is also written in terms of the functions G and MJ. Δ_1 can be easily found using eq. (3.21). After simple transformations we obtain

$$\begin{aligned}\Delta_1 &= -iC(R)_k{}^i\frac{d}{d\ln\Lambda}\int d^8x\,d^8y\,(\theta^4)_y\Big(\frac{D^2}{4\partial^4}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_j)_y}\Big)^{-1}\\ &\quad\times\Big\{M^{jk}\delta^8_{xy}-M^{jn}M^{mk}\Big(\frac{D^2}{32\partial^4}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_m)_x\delta(\phi_n)_y}\Big)^{-1}-\frac{\delta^2\gamma}{\delta(\phi_k)_x\delta(\phi_{0j})_y}\Big\}; \qquad (3.24)\\ \Delta_2 &= -2iC(R)_i{}^j\frac{d}{d\ln\Lambda}\int d^8x\,d^8y\,(\theta^4)_y\frac{1}{\partial^2}\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x(\phi_k)_y}\Big)^{-1}\frac{\delta^2\gamma}{\delta(\phi_{0i})_x\delta(\phi_{0k})_y}. \qquad (3.25)\end{aligned}$$

2. Now let us proceed to the second idea. An attempt to present all two-loop effective diagrams in figure 9 as integrals of total derivatives encounters considerable problems. The reason can be understood from the results of [67]. The matter is that the total derivative in this case nontrivially depends on the number of vertices in a diagram.

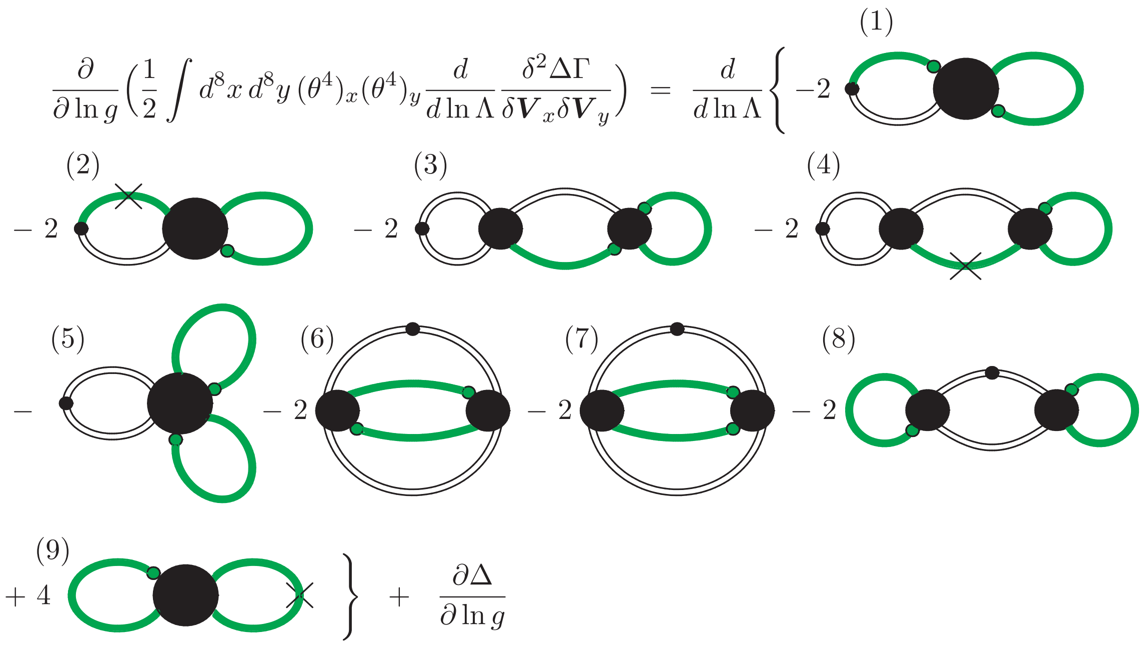


Figure 10. These diagrams are obtained after differentiating the effective diagram presented in figure 9 with respect to the parameter g. Below we will see that this trick allows to write the result as an integral of a double total derivative. The term $\partial\Delta/\partial\ln g$ is given by eq. (D.20).

Therefore, it seems impossible to write the total derivatives in the form of effective diagrams. However, the solution can be found. For this purpose we introduce the parameter g according to the prescription (2.16).

Let us differentiate the upper diagram in figure 9 with respect to the parameter g using the identity

$$\begin{aligned}
\frac{\partial\gamma}{\partial\ln g} = \frac{1}{2}\int d^8x\,\Big\{ & -\frac{1}{2}(\phi^{*i}+\phi_0^{*i})(\phi_i+\phi_{0i}) + (\phi_i+\phi_{0i})_x\frac{\delta\gamma}{\delta(\phi_{0i})_x} + (\phi^{*i}+\phi_0^{*i})_x\frac{\delta\gamma}{\delta(\phi_0^{*i})_x}\\
& -\frac{i}{2}\Big(\frac{\delta^2\gamma}{\delta\phi_i\delta\phi^{*i}}\Big)^{-1} - iM^{ji}\Big(\frac{D^2}{16\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_j)_y}\Big)^{-1}_{y=x}\\
& -iM^*_{ji}\Big(\frac{\bar D^2}{16\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi^{*i})_x\delta(\phi^{*j})_y}\Big)^{-1}_{y=x}\Big\},
\end{aligned} \tag{3.26}$$

which is proved in appendix D.1. The technique constructed in this appendix allows to calculate the derivative of eq. (3.20) with respect to the parameter g. The result is presented in figure 10. The term $\partial\Delta/\partial\ln g$ is calculated in appendix D.4 and is given by eq. (D.20). The diagrams presented in figure 10 are obtained by differentiating the upper diagram in figure 9. Technical details of this calculation are presented in appendixes D.2 and D.3. Here we briefly discuss the result. Diagrams (1)–(4) and (9) in this figure come from the derivative of the green effective line. Actually, it is necessary to differentiate the inverse Green functions inside $\boldsymbol{\delta}/\boldsymbol{\delta}j$. All terms corresponding to the two-loop effective diagrams are included into diagrams (1), (2), and (9). Similarly, terms corresponding to three-loop effective diagrams are included into diagrams (3) and (4).

A green effective line with a cross (in diagrams (4) and (9)) corresponds to the operator

$$\text{GreenWithCross}[1,2] = -i(MT)^{ij}\int d^8x\,\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\Big(\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^i}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^j}, \tag{3.27}$$

which is obtained from the operator GreenLine by keeping only terms proportional to the Pauli-Villars masses. Integrating by parts and using eq. (2.13) it is easy to see that this operator is symmetric with respect to the permutation of the points 1 and 2:

$$\text{GreenWithCross}[1,2] = \text{GreenWithCross}[2,1]. \tag{3.28}$$

Due to this symmetry of arguments we do not mark out one of the ends by a circle. The lines with crosses appear, because terms with the masses are quadratic in $\boldsymbol{\delta}/\boldsymbol{\delta} j$, while the other terms are linear in $\boldsymbol{\delta}/\boldsymbol{\delta} j$.

A line with a dot corresponds to the operator

$$\begin{aligned}\text{LineWithDot}[\alpha;1,2] \equiv \frac{i}{4}\int d^8x\,(\alpha)_x\Big(&\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j_i^*}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^i} + \frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j_i^*}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^i} + M^{ij}\Big(\frac{D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^i}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^j}\\
&+M^*_{ij}\Big(\frac{D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j_i^*}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j_j^*} + M^{ij}\Big(\frac{D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^i}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^j} + M^*_{ij}\Big(\frac{D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j_i^*}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j_j^*}\Big)\end{aligned} \tag{3.29}$$

with $\alpha = 1$. Diagram (1) contains the operator

$$\begin{aligned}\text{GreenWhiteLine}[1,2] \equiv& -\frac{1}{2}\cdot\text{GreenLine}[1,3]\cdot\text{UsualLine}[2,3]\\
&\times\int d^8x\Big(\phi_0^{*i}\phi_{0i}+M^{ij}\phi_{0i}\frac{D^2}{8\partial^2}\phi_{0j}+M^*_{ij}\phi_0^{*i}\frac{\bar D^2}{8\partial^2}\phi_0^{*j}\Big)_{[3]},\end{aligned} \tag{3.30}$$

where

$$\text{UsualLine}[2,3] = \int d^8x\,\Big(\frac{\delta}{\delta_3\phi_{0i}}\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^i} + \frac{\delta}{\delta_3\phi_0^{*i}}\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j_i^*}\Big). \tag{3.31}$$

In eq. (3.30) the subscript [3] means that only the derivatives $\delta/\delta_3\phi_0$ nontrivially act to the argument of the bracket. Similarly, diagram (2) in figure 10 contains the operator

$$\begin{aligned}\text{GreenWithCrossWhite}[1,2] \equiv& -\frac{1}{2}\cdot\text{GreenWithCross}[1,3]\cdot\text{UsualLine}[2,3]\\
&\times\int d^8x\,\Big(\phi_0^{*i}\phi_{0i}+M^{ij}\phi_{0i}\frac{D^2}{8\partial^2}\phi_{0j}+M^*_{ij}\phi_0^{*i}\frac{\bar D^2}{8\partial^2}\phi_0^{*j}\Big)_{[3]}.\end{aligned} \tag{3.32}$$

Diagrams (5)–(8) correspond to differentiation of the four-point function (the large black circle in figure 9). Technical details of the corresponding calculation can be found in appendix D.2.

Below we try to avoid writing large analytical expressions corresponding to effective Feynman diagrams. Instead of this, we write numerical coefficients for all diagrams, so that the analytical expression can be unambiguously constructed using the definitions of the effective lines. As an example, here we present an analytical expression for the sum of diagrams presented in figure 10:

$$
\begin{aligned}
&\frac{\partial}{\partial\ln g}\Big(\frac{1}{2}\int d^8x\, d^8y\, (\theta^4)_x(\theta^4)_y \frac{d}{d\ln\Lambda}\frac{\delta^2\Delta\Gamma}{\delta \boldsymbol{V}_x\delta\boldsymbol{V}_y}\Big)\\
&= \frac{d}{d\ln\Lambda}\Big(-2\cdot \text{GreenWhiteLine}\cdot\text{GreenLine}\cdot\gamma\\
&-2\cdot\text{GreenWithCrossWhite}\cdot\text{GreenLine}\cdot\gamma\\
&-2\cdot\text{GreenLine}[2,1]\,\text{UsualLine}[1,2]\,\text{LineWithDot}[1;1,1]\,\text{GreenLine}[2,2]\Big(\gamma[1]\,\gamma[2]\Big)\\
&-2\cdot\text{GreenWithCross}[2,1]\,\text{UsualLine}[1,2]\,\text{LineWithDot}[1;1,1]\,\text{GreenLine}[2,2]\Big(\gamma[1]\,\gamma[2]\Big)\\
&-\text{LineWithDot}[1]\cdot(\text{GreenLine})^2\cdot\gamma\\
&-2\cdot\text{LineWithDot}[1;1,2]\,\text{UsualLine}[1,2]\,\text{GreenLine}[1,2]\,\text{GreenLine}[2,1]\,\Big(\gamma[1]\,\gamma[2]\Big)\\
&-2\cdot\text{LineWithDot}[1;1,2]\,\text{UsualLine}[1,2]\,(\text{GreenLine}[2,1])^2\,\Big(\gamma[1]\,\gamma[2]\Big)\\
&-2\cdot\text{LineWithDot}[1;1,2]\,\text{UsualLine}[1,2]\,\text{GreenLine}[1,1]\,\text{GreenLine}[2,2]\Big(\gamma[1]\,\gamma[2]\Big)\\
&+4\cdot\text{GreenLine}\cdot\text{GreenWithCross}\cdot\gamma\Big)+\frac{\partial\Delta}{\partial\ln g}.
\end{aligned}\tag{3.33}
$$

Expressions for all differential operators corresponding to the various lines can be found in figure 3. Let us remind that if ends of an effective line coincide, we sometimes omit numbers which numerate them. In section 5 we prove that the sum of diagrams presented in figure 10 in the momentum representation is given by integrals of double total derivatives.

The method considered in this paper also allows to prove the identity presented in figure 7 directly. This is made in section 4.2. In particular, we prove that the contribution of this effective diagram to the β-function is given by a vanishing integral of a total derivative. In order to do this, it is convenient to differentiate the diagram with the blue effective line presented in figure 9 (or, equivalently, in figure 7) with respect to the parameter g. In the graphical form the result is given by a sum of diagrams presented in figure 11. The corresponding analytical expression can be unambiguously constructed using the expressions for the effective lines presented in figure 3.

4 Total derivatives and the NSVZ boldmath β-function

4.1 The effective diagram with the yellow line

In order to prove that the β-function (defined in terms of the bare coupling constant) is determined by integrals of total derivatives, it is convenient to use the coordinate representation. In the coordinate representation an integral of a total derivative can be written as

$$\text{Tr}[x^\mu, \text{something}] = 0, \tag{4.1}$$

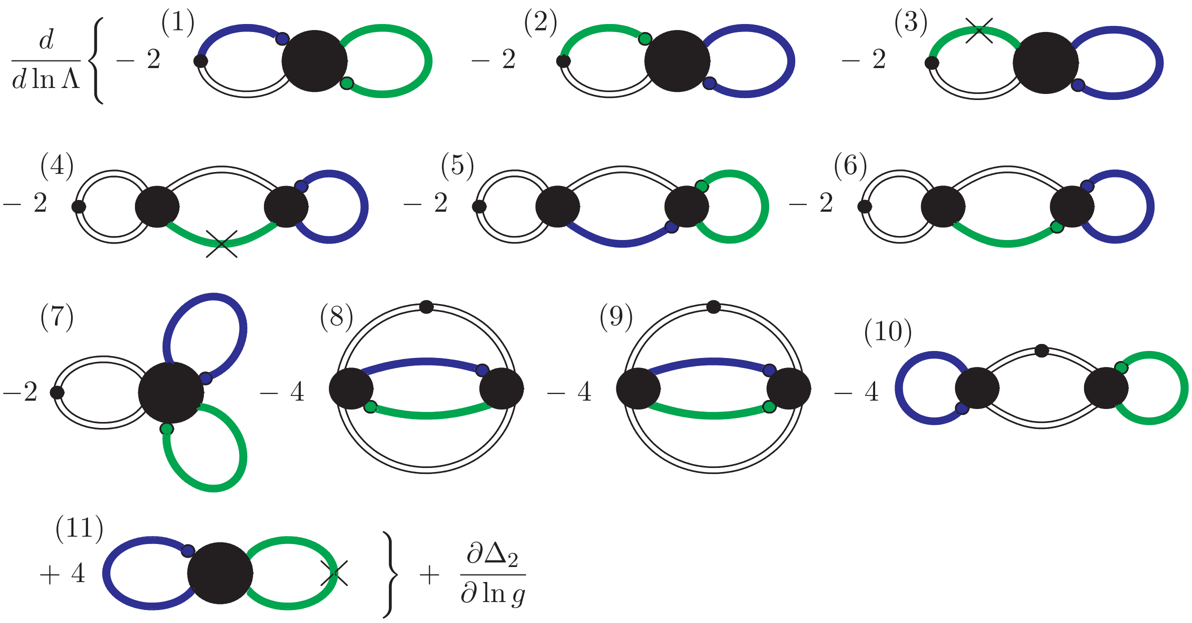


Figure 11. Diagrams obtained after differentiating the effective diagram with a blue effective line presented in figure 9 with respect to the parameter g. Here the blue effective line corresponds to the operator $\text{Blueline}_{\dot b}[\theta^a\theta_a\bar\theta^{\dot b}]$ and the line with a dot corresponds to the operator LineWithDot[1].

where

$$\operatorname{Tr} M \equiv \operatorname{tr}\int d^8x\, M_{xx}; \qquad [\alpha, A_{xy}] \equiv (\alpha)_x A_{xy} - A_{xy}(\alpha)_y, \tag{4.2}$$

and tr denotes the usual matrix trace. We will try to present (the sum of) expressions for the effective diagrams as such traces of commutators. First, as a simple example, we consider a diagram with the yellow effective line presented in figure 9 and verify that the sum of the this diagram and Δ_1 is given by an integral of a total derivative. This sum (for $N_f = 1$) has been already calculated in [72] by substituting solutions of the Ward identities for the effective vertices. In this paper we reobtain the result by a different method, which is also used for calculation of the other diagram (which has not been calculated in [72].)

The expression for the considered diagram is written as

$$-2\cdot \text{YellowLine}_\mu[\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b]\cdot \text{GreenLine}\cdot\gamma = \text{YellowLine}_\mu[\theta^4]\cdot \text{BrownLine}^\mu\cdot\gamma, \tag{4.3}$$

where we used the identity proved in appendix F.1 and the notation

$$\text{BrownLine}_\mu[1,2] \equiv \int d^8x\,\bigg\{(T)^j{}_i\Big(-\frac{2i\partial_\mu}{\partial^2} - (\gamma^\mu)^{a\dot b}\theta_a\frac{\bar D_{\dot b}D^2}{4\partial^2}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^j}\frac{\delta}{\delta_1\phi_{0i}}$$
$$+i(MT)^{ij}\Big(\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^i}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^j}\bigg\}. \tag{4.4}$$

This operator is very useful, because by the help of this notation commutators of various Green functions with

$$(y_\mu)^* \equiv x_\mu - i\bar\theta^{\dot a}(\gamma_\mu)_{\dot a}{}^b\theta_b \tag{4.5}$$

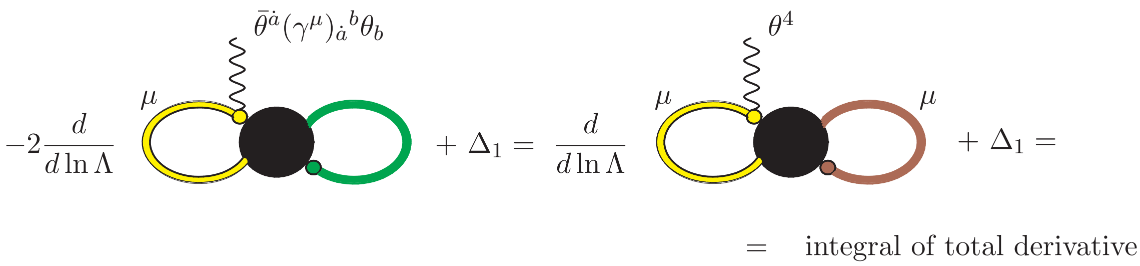


Figure 12. Using two-loop effective diagrams it is possible to present the effective diagram with the yellow line (including δ_1) in figure 6 (or a corresponding diagram in figure 9) as an integral of a total derivative.

can be written in a very compact form. The details of the corresponding calculations are given in appendix E.1. Here we present only the results. First, we introduce the following notation:

$$\Big[(T)y_\mu^*, \frac{\delta^2\gamma}{\delta(\phi_{0i})_y\delta(\phi_0^{*j})_z}\Big] \equiv -(T)_k{}^i(y_\mu^*)_y\frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_0^{*j})_z} + (T)_j{}^k(y_\mu^*)_z\frac{\delta^2\gamma}{\delta(\phi_{0i})_y\delta(\phi_0^{*k})_z};$$
$$\Big[(T)y_\mu^*, \frac{\delta^2\gamma}{\delta(\phi_{0i})_y\delta(\phi_{0j})_z}\Big] \equiv -(T)_k{}^i(y_\mu^*)_y\frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_{0j})_z} - (T)_k{}^j(y_\mu^*)_z\frac{\delta^2\gamma}{\delta(\phi_{0i})_y\delta(\phi_{0k})_z}, \qquad (4.6)$$

etc. Commutators with other Green functions (with an arbitrary number of indexes) can be constructed similarly. (Each index gives a term in the sum; for upper indexes the sign is "$-$", and for lower indexes the sign is "$+$".) Then the result of appendix E.1 can be written as

$$\Big[(T)y_\mu^*, \frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_0^{*l})_z}\Big] = \text{BrownLine}_\mu\cdot\frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_0^{*l})_z};$$
$$\Big[(T)y_\mu^*, \frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_{0l})_z}\Big] = \text{BrownLine}_\mu\cdot\frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_{0l})_z} \qquad (4.7)$$

etc., where all fields should be set to 0. These identities allow to rewrite the considered contribution as an integral of a total derivative in the momentum representation. A simple graphical version of the result is presented in figure 12. Below we prove the last equality in this figure. For this purpose we substitute the explicit expression for the operator $\text{YellowLine}_\mu[\theta^4]$ into eq. (4.3). Then the diagram with the brown effective line is written in the form

$$\frac{d}{d\ln\Lambda}\text{BrownLine}^\mu\cdot\int d^8x\,\theta^4\Big\{2i(T)^j{}_i\Big(\frac{\partial_\mu}{\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}\Big)\frac{\delta\gamma}{\delta\phi_{0i}} - i(MT)^{ij}\Big(\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^i}\Big)\frac{\boldsymbol{\delta}\gamma}{\boldsymbol{\delta}j^j}\Big\}. \qquad (4.8)$$

Using the identities (4.7) we can present the expression (4.8) in the form

$$
\begin{aligned}
&\frac{d}{d\ln\Lambda}\int d^8x\, d^8y\, (\theta^4)_x\Big\{-2i(T)^k{}_i\Big(\frac{\partial_\mu}{\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_x\delta(\phi^{*j})_y}\Big)^{-1}\Big[(T)y^*_\mu,\frac{\delta^2\gamma}{\delta(\phi_0^{*j})_y\delta(\phi_{0i})_x}\Big]\\
&-2i(T)^k{}_i\Big(\frac{\partial_\mu}{\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_x\delta(\phi_j)_y}\Big)^{-1}\Big[(T)y^*_\mu,\frac{\delta^2\gamma}{\delta(\phi_{0j})_y\delta(\phi_{0i})_x}\Big]-i\int d^8z\,(MT)^{ij}\\
&\times\bigg(\Big(\frac{\partial_\mu D^2}{16\partial^4}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*m})_y}\Big)^{-1}\Big[(T)y^*_\mu,\frac{\delta^2\gamma}{\delta(\phi_0^{*m})_y\delta(\phi_{0n})_z}\Big]\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x\delta(\phi_n)_z}\Big)^{-1}\\
&+\Big(\frac{\partial_\mu D^2}{16\partial^4}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_m)_y}\Big)^{-1}\Big[(T)y^*_\mu,\frac{\delta^2\gamma}{\delta(\phi_{0m})_y\delta(\phi_{0n})_z}\Big]\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x\delta(\phi_n)_z}\Big)^{-1}\\
&+\Big(\frac{\partial_\mu D^2}{16\partial^4}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*m})_y}\Big)^{-1}\Big[(T)y^*_\mu,\frac{\delta^2\gamma}{\delta(\phi_0^{*m})_y\delta(\phi_0^{*n})_z}\Big]\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x\delta(\phi^{*n})_z}\Big)^{-1}\\
&+\Big(\frac{\partial_\mu D^2}{16\partial^4}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_m)_y}\Big)^{-1}\Big[(T)y^*_\mu,\frac{\delta^2\gamma}{\delta(\phi_{0m})_y\delta(\phi_0^{*n})_z}\Big]\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x\delta(\phi^{*n})_z}\Big)^{-1}\bigg)\Big\}. \qquad (4.9)
\end{aligned}
$$

In order to write eq. (4.9) as an integral of a total derivative in the momentum space we (anti)commute the generators with the Green functions. Using the results of appendix E.3 we obtain

$$
\begin{aligned}
&\frac{d}{d\ln\Lambda}\int d^8x\,(\theta^4)_x C(R)_i{}^k\bigg\{\int d^8y\,\bigg(-2i\Big(\frac{\partial_\mu}{\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi^{*j})_y\delta(\phi_k)_x}\Big)^{-1}\Big[y^*_\mu,\frac{\delta^2\gamma}{\delta(\phi_0^{*j})_y\delta(\phi_{0i})_x}\Big]\\
&-2i\Big(\frac{\partial_\mu}{\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_y\delta(\phi_k)_x}\Big)^{-1}\Big[y^*_\mu,\frac{\delta^2\gamma}{\delta(\phi_{0j})_y\delta(\phi_{0i})_x}\Big]+iM^{jn}M^{mi}\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_x\delta(\phi_j)_y}\Big)^{-1}\\
&\times\Big(\frac{\partial_\mu D^2}{16\partial^4}\Big)_x\Big(\frac{\partial_\mu D^2}{8\partial^4}\Big)_y\Big(\frac{\delta^2\gamma}{\delta(\phi_m)_x\delta(\phi_n)_y}\Big)^{-1}\bigg)+iM^{ij}\Big(\frac{\partial_\mu D^2}{16\partial^4}\Big)_x\Big[y^*_\mu,\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x\delta(\phi_k)_y}\Big)^{-1}\Big]_{y=x}\\
&-iM^{ij}\Big(\frac{\partial^\mu}{\partial^2}\Big)_x\Big(\frac{\partial_\mu D^2}{8\partial^4}\Big)_y\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x\delta(\phi_k)_y}\Big)^{-1}_{y=x}\bigg\}. \qquad (4.10)
\end{aligned}
$$

Then we add Δ_1 to eq. (4.10) and write the result in the momentum representation substituting explicit expressions for the (inverse) Green functions. Details of this calculation are presented in appendix G. Taking into account eqs. (2.39) and (2.42) the result for the considered contribution (written in the momentum representation after the Wick rotation in the Euclidian space) can be written as

$$
\begin{aligned}
\mathcal{V}_4 N_f\frac{d}{d\ln\Lambda}\int\frac{d^4q}{(2\pi)^4}\frac{2q^\mu}{q^4}\frac{\partial}{\partial q^\mu}\Big\{&2\ln G\\
&+\sum_{I=1}^{n}(-1)^{P_I}\Big(\ln(q^2G^2+M^2J^2)+\frac{M^2J}{q^2G^2+M^2J^2}\Big)_I\Big\}, \qquad (4.11)
\end{aligned}
$$

where we separate the main contribution of the fields ϕ_α and $\widetilde{\phi}_\alpha$ (corresponding to $I=0$) and contributions of the Pauli-Villars fields (corresponding to $I\geq 1$). This expression agrees with the result obtained in [72] by a different method for $N_f = 1$. (In [72] the considered contribution was calculated by substituting expressions for vertices obtained by solving the Ward identities.)

Thus, the sum of the considered effective diagram and Δ_1 is given by the integral of a total derivative, which can be easily calculated using the identity

$$\int \frac{d^4q}{(2\pi)^4}\frac{q^\mu}{q^4}\frac{\partial}{\partial q^\mu}f(q) = \frac{1}{(2\pi)^4}\oint\limits_{S^3_\varepsilon} dS_\mu \frac{q^\mu}{q^4}f(q) = -\frac{1}{8\pi^2}f(0) = -2\pi^2\int\frac{d^4q}{(2\pi)^4}\delta^4(q)f(q), \tag{4.12}$$

where $f(q^2)$ is a function which rapidly decreases at the infinity, and S^3_ε is a 3-sphere in the momentum space surrounding the point $q = 0$ with the radius $\varepsilon \to 0$.

Assuming that the other contributions vanish (we prove this statement in the next section) we obtain the NSVZ relation for the renormalization group functions defined in terms of the bare coupling constant. Really, terms containing the Pauli-Villars masses are convergent and finite beyond the one-loop approximation, because these masses are proportional to the parameter Λ. Therefore,

$$\frac{1}{2\pi}\mathcal{V}_4\cdot\frac{\beta(\alpha_0)}{\alpha_0^2} = \frac{1}{2\pi^2}\mathcal{V}_4 N_f\Big(\sum_{I=1}^n c_I - \frac{d\ln G}{d\ln\Lambda}\Big|_{q=0}\Big) = \frac{1}{2\pi^2}\mathcal{V}_4 N_f\Big(1-\gamma(\alpha_0)\Big). \tag{4.13}$$

Thus, for $\mathcal{N}=1$ SQED with N_f flavors we obtain the NSVZ β-function

$$\beta(\alpha_0) = \frac{\alpha_0^2 N_f}{\pi}\Big(1-\gamma(\alpha_0)\Big). \tag{4.14}$$

4.2 The effective diagram with the blue line

In order to prove that the β-function defined in terms of the bare coupling constant is given by integrals of total derivatives it is also necessary to present the expression for the last diagram (with the blue effective line) in figure 6 plus δ_2 as a trace of a commutator. Calculations in the lowest orders allow to suggest that this contribution is always given by integral of a total derivative and vanishes [72]. An indirect proof of this fact is actually given in [67] by a different method. In this section we present a direct proof. In order to do this, it is necessary to differentiate the generating functional with respect to the auxiliary parameter g, introduced in eq. (2.16). Next, we prove the identity presented in figure 13 in a graphical form. In this figure an arc with an arrow denotes a trace of a commutator with y^*_μ. Certainly, the corresponding analytical expression can be easily constructed:

$$\begin{aligned}&2\frac{d}{d\ln\Lambda}\frac{\partial}{\partial\ln g}\int d^8x\,(\theta^4)_x \text{BlueLine}_{\dot{b}}[\theta^a\theta_a\bar\theta^{\dot{b}}]\cdot\frac{\delta\Gamma}{\delta\boldsymbol{V}_x} + \delta_2 = \\ &(\gamma^\mu)^{a\dot{b}}\frac{d}{d\ln\Lambda}\text{Tr}\Big[(T)y^*_\mu,\ \text{LineWithDot}[\theta^4]\cdot\text{BlueLine}_{\dot{b}}[\theta_a]\cdot\gamma\Big] = 0.\end{aligned} \tag{4.15}$$

For the simplest cases the operation $[(T)\alpha,\ldots]$ was defined in the previous section. The natural generalization of this definition can be formulated as follows: if a tensor B has a lower index i corresponding to a superfield in the point x, then $[(T)\alpha, B]$[4] includes

$$(T)_i{}^j\alpha_x(B_j)_x. \tag{4.16}$$

[4] For the case $P_\alpha = P_B = 1$ we use the notation $\{(T)\alpha, B\}$.

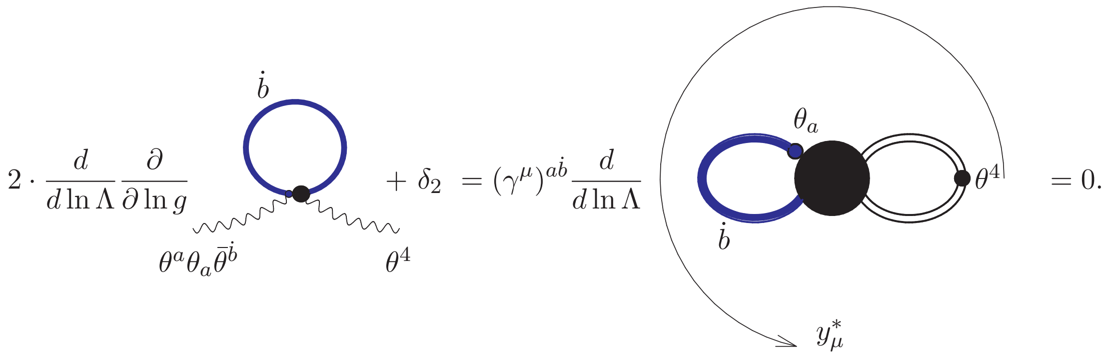


Figure 13. This identity is needed for proving that the β-function (defined in terms of the bare coupling constant) is given by integrals of total derivatives and satisfies the NSVZ relation.

Similarly, for an upper index i this expression includes

$$-\,(T)_j{}^i\alpha_x(B^j)_x. \tag{4.17}$$

For example, applying this rule we can easily obtain eq. (4.6). However, it is necessary to define the operation $[(T)y^*_\mu,\ldots]$ (and other similar operations) more accurately for diagrams containing closed loops of the matter superfields. First, let us explain, how to construct the expression in the right hand side of eq. (4.15). It can be schematically written as

$$\text{Diagram}^\mu \equiv \int d\mu\, \text{Line}^\mu\cdot\text{Vertex}\cdot\text{Line}, \tag{4.18}$$

where $d\mu$ denotes the integration measure, and we omitted indexes for simplicity. The whole expression in the right hand side (for simplicity, without the derivative $d/d\ln\Lambda$) can be written as

$$\text{Tr}\Big[(T)y^*_\mu,\text{Diagram}^\mu\Big] \equiv \int d\mu\,\Big[(T)y^*_\mu,\text{Line}^\mu\Big]\cdot\text{Vertex}\cdot\text{Line} + \int d\mu\,\text{Line}^\mu \cdot\Big[(T)y^*_\mu,\text{Vertex}\Big]\cdot\text{Line} + \int d\mu\,\text{Line}^\mu\cdot\text{Vertex}\cdot\Big[(T)y^*_\mu,\text{Line}\Big] = 0. \tag{4.19}$$

(It is easy to see that all terms in this sum cancel each other.) The operation $[(T)y^*_\mu,\ldots]$ in eq. (4.15) is constructed formally according to eq. (4.19). In the momentum representation the expression (4.19) is given by an integral of a total derivative, because

$$\begin{aligned}\int d^8x\,\Big[(T)y^*_\mu, X_{(i,x)}{}^{(i,x)}\Big] &= \int d^8x\,d^8y\,\delta^8_{xy}\Big((y^*_\mu)_x T_i{}^j X_{(j,x)}{}^{(i,y)} - (y^*_\mu)_y T_j{}^i X_{(i,x)}{}^{(j,y)}\Big)\\ &= \text{Tr}\Big[y^*_\mu,\, T_j{}^i X_i{}^j\Big] = -\int\frac{d^4q}{(2\pi)^4}\frac{\partial}{\partial q^\mu}\int d^4\theta\, T_j{}^i X_j{}^i(q,\theta).\end{aligned} \tag{4.20}$$

(The last equation is written in the Euclidian space after the Wick rotation.) Therefore, the equation presented in figure 13 implies that the derivative of the effective diagram in the left hand side with respect to $\ln g$ is given by an integral of a total derivative. Moreover, the result is 0, because the integrand does not contain singularities. Then we integrate the

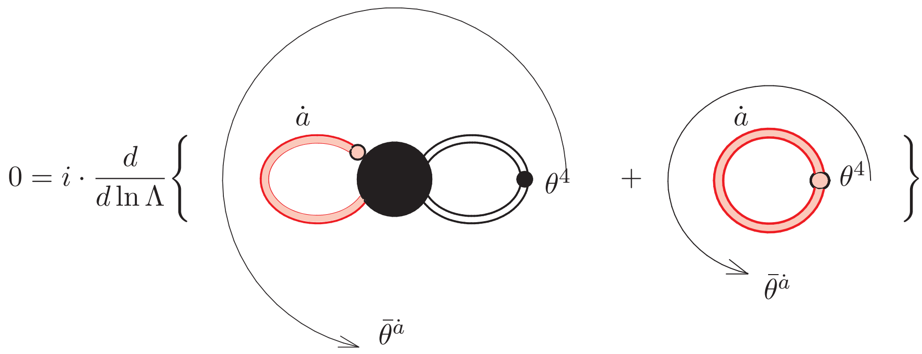


Figure 14. We add these terms to the diagram in the right hand side of figure 13.

considered equality over $\ln g$ from $g = 0$ to $g = 1$. The theory corresponding to $g = 0$ does not contain quantum gauge field, and quantum corrections to the Green function of the gauge superfield are given only by one-loop diagrams. It is easy to see that in the one-loop approximation the effective diagram in the left hand side of figure 13 vanishes. Therefore, (because the original theory corresponds to $g = 1$) this effective diagram is also given by an integral of a total derivative and is equal to 0 for $g = 1$.

Thus, taking into account the results of the previous section, the identity presented in figure 13 allows to prove that the β-function defined in terms of the bare coupling constant is given by integrals of total derivatives and satisfies the NSVZ relation. Let us proceed to proving this identity. The expressions

$$\Big[(T)y_\mu^*, \text{Vertex}\Big]; \qquad \Big[(T)y_\mu^*, \text{Line}\Big]; \qquad \Big[(T)y_\mu^*, \text{Line}^\mu\Big] \tag{4.21}$$

(and other similar expressions) can be calculated using the Schwinger-Dyson equations. The details of the corresponding calculations are presented in appendix E. It is convenient to add the (vanishing) diagrams presented in figure 14, where

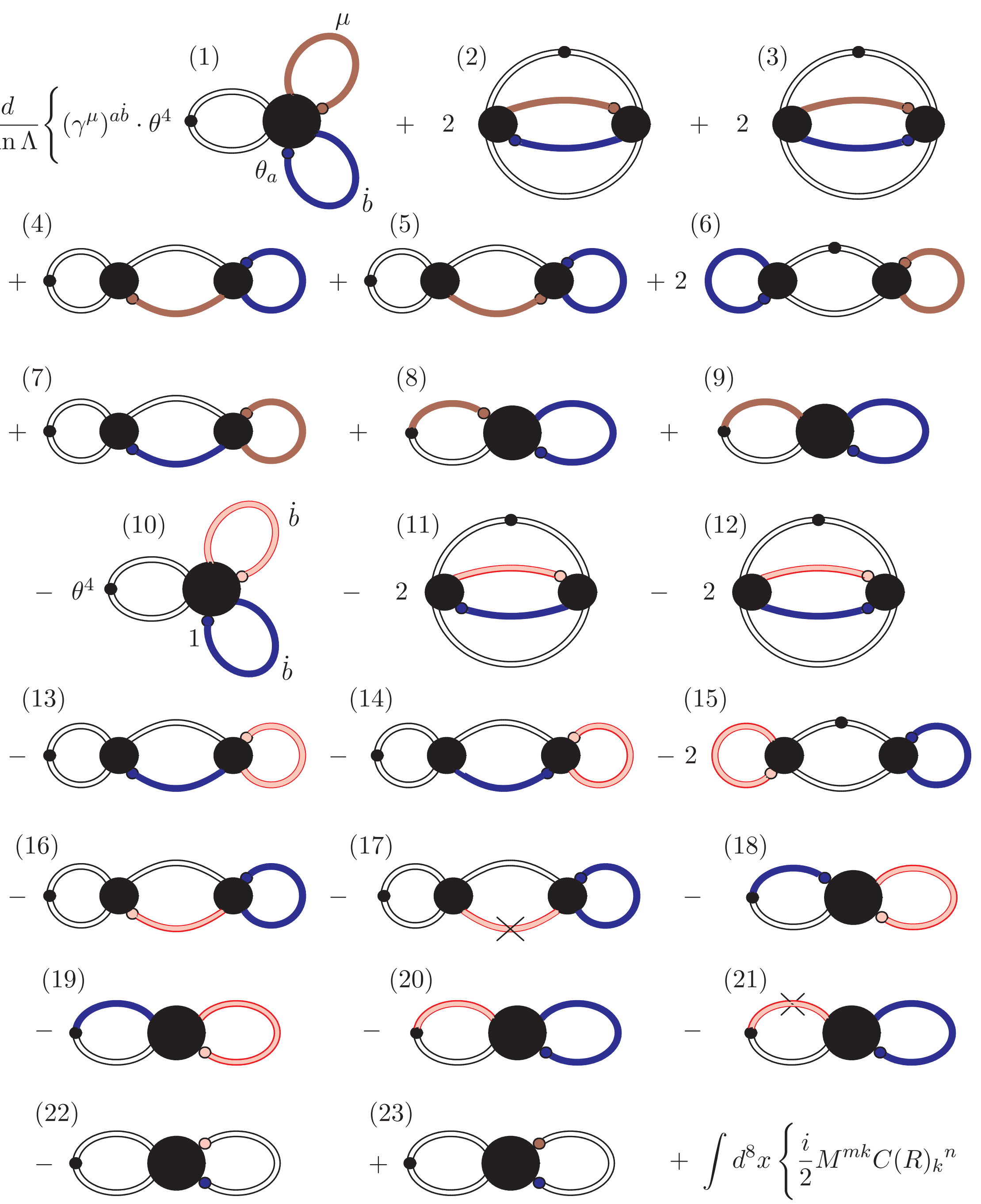
$$\begin{aligned}\text{PinkLine}^{\dot{b}}[1,2] = \int d^8x \Big\{ &(T)^j{}_i \Big(\theta^a \theta_a \frac{\bar{D}^{\dot{b}} D^2}{2\partial^2} - i(\gamma^\mu)^{a\dot{b}}\theta_a \frac{\bar{D}^2 D^2 \partial_\mu}{8\partial^4}\Big) \frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^j} \cdot \frac{\delta}{\delta_1 \phi_{0i}} \\ &- i(MT)^{ij} (\gamma^\mu)^{\dot{b}a} \theta_a \Big(\frac{D^2 \partial_\mu}{16\partial^4} \frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^i}\Big) \cdot \frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^j} \Big\},\end{aligned} \tag{4.22}$$

to the diagram presented in the right hand side of figure 13. It is easy to see that the diagrams presented in figure 14 encode the commutator

$$\begin{aligned}&-\frac{1}{2} \cdot \frac{d}{d\ln\Lambda} \text{Tr}\,(\theta^4)_x \Big\{ (T)\bar{\theta}^{\dot{a}},\ \text{PinkLine}_{\dot{a}} \cdot \Big(\frac{\boldsymbol{\delta}^2 \gamma}{\boldsymbol{\delta} j_i^* \boldsymbol{\delta} j^i} + i\phi_0^{*i}\phi_{0i} + M^{ik} \Big(\frac{D^2}{8\partial^2} \frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^i}\Big) \frac{\boldsymbol{\delta}\gamma}{\boldsymbol{\delta} j^k} \\ &+ iM^{ik} \Big(\frac{D^2}{8\partial^2}\phi_{0i}\Big)\phi_{0k} + M^*_{ik} \Big(\frac{\bar{D}^2}{8\partial^2} \frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j_i^*}\Big) \frac{\boldsymbol{\delta}\gamma}{\boldsymbol{\delta} j_k^*} + iM^*_{ik} \Big(\frac{\bar{D}^2}{8\partial^2}\phi_0^{*i}\Big)\phi_0^{*k} \Big)_x \Big\}.\end{aligned} \tag{4.23}$$

In the graphical form the result for the sum of all commutators is presented in figure 15. The expressions for the effective lines are collected in figure 3. Let us briefly explain, how the diagrams presented in figure 15 are constructed.

$$\frac{d}{d\ln\Lambda}\Bigg\{(\gamma^\mu)^{a\dot b}\cdot\theta^4$$



$$+\int d^8x\,\Bigg\{\frac{i}{2}M^{mk}C(R)_k{}^n$$

$$\times\Bigg(\mathrm{LineWithDot}[\theta^4]\cdot\left(\frac{D^2}{8\partial^4}\right)_x\frac{\boldsymbol{\delta}^2\gamma}{\boldsymbol{\delta}(j^m)_x\boldsymbol{\delta}(j^n)_y}\Bigg|_{y=x}+\int d^8y\left(\frac{\partial_\mu}{\partial^2}\right)_x\left(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_m)_y}\right)^{-1}$$

$$\times\mathrm{LineWithDot}[\theta^4]\cdot\left(\frac{D^2\partial_\mu}{4\partial^4}\right)_y\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^n)_y}\frac{\delta\gamma}{\delta(\phi_{0i})_x}\Bigg)-\frac{i}{2}(\theta^4)_x(\gamma^\mu)^{\dot a b}\mathrm{BlueLine}_{\dot a}[\theta_b]\Bigg((MT)^{ik}$$

$$\times\left(\frac{D^2\partial_\mu}{4\partial^4}\right)_x\frac{\boldsymbol{\delta}^2\gamma}{\boldsymbol{\delta}(j^i)_x\boldsymbol{\delta}(j^k)_y}+(TM^*)_{ik}\left(\frac{\bar D^2\partial_\mu}{4\partial^4}\right)_x\frac{\boldsymbol{\delta}^2\gamma}{\boldsymbol{\delta}(j_i^*)_x\boldsymbol{\delta}(j_k^*)_y}\Bigg)_{y=x}-(\gamma^\mu)^{\dot a b}(\bar\theta^{\dot c}\bar\theta_{\dot c}\theta_b)_x(TM^*)_{ik}$$

$$\times\left(\frac{i\bar D^2\partial_\mu}{16\partial^4}\right)_x\mathrm{PinkLine}_{\dot a}\frac{\boldsymbol{\delta}^2\gamma}{\boldsymbol{\delta}(j_i^*)_x\boldsymbol{\delta}(j_k^*)_y}\Bigg|_{y=x}\Bigg\}\Bigg\}.$$

Figure 15. These diagrams are obtained after calculating the commutators in the right hand side of figures 13 and 14. In all these diagrams the line with a dot corresponds to the operator LineWithDot$[\theta^4]$.

1. A commutator $[(T)y^*_\mu, \text{Vertex}]$ (where Vertex denotes the four-point Green function in figure 13) is calculated according to the rules derived in appendix E.1. From this commutator we obtain diagrams (1)–(5) in figure 15. A similar commutator $[(T)\bar\theta^{\dot b}, \text{Vertex}]$ (which appears in the first diagram in figure 14) gives diagrams (10)–(14).

2. It is also necessary to calculate commutators with the inverse Green functions which are contained inside the derivatives $\boldsymbol{\delta}/\boldsymbol{\delta}j$. The calculation of these commutators is described in appendix E.3. Commuting $(T)y^*_\mu$ with the operator LineWithDot gives diagrams (6), (8), and (9). In diagrams (8) and (9) the left effective line is given by the operators

$$\text{BrownWhiteLine}_\mu[1,2] \equiv -\frac{1}{2}\cdot\text{BrownLine}_\mu[1,3]\cdot\text{UsualLine}[2,3] \times\int d^8x\,\theta^4\Big(\phi_0^{*i}\phi_{0i}+M^{ij}\phi_{0i}\frac{D^2}{8\partial^2}\phi_{0j}+M^*_{ij}\phi_0^{*i}\frac{\bar D^2}{8\partial^2}\phi_0^{*j}\Big)_{[3]}. \tag{4.24}$$

$$\text{WhiteBrownLine}_\mu[1,2] \equiv -\frac{1}{2}\cdot\text{BrownLine}_\mu[3,1]\cdot\text{UsualLine}[2,3] \times\int d^8x\,\theta^4\Big(\phi_0^{*i}\phi_{0i}+M^{ij}\phi_{0i}\frac{D^2}{8\partial^2}\phi_{0j}+M^*_{ij}\phi_0^{*i}\frac{\bar D^2}{8\partial^2}\phi_0^{*j}\Big)_{[3]}, \tag{4.25}$$

respectively. As earlier, the subscript [3] means that only the derivatives $\delta/\delta_3\phi_0$ nontrivially act on the expression in the brackets.

Similarly, commuting $(T)\bar\theta^{\dot b}$ with the operator LineWithDot in the first diagram in figure 14 gives diagrams (15), (18), and (19). In diagrams (18) and (19) we use the operators (with $\alpha=\theta^4$ and $\beta=1$)

$$\text{BlueWhiteLine}^{\dot b}[\alpha,\beta;1,2] \equiv -\frac{1}{2}\cdot\text{BlueLine}^{\dot b}[\beta;1,3]\cdot\text{UsualLine}[2,3] \times\int d^8x\,\alpha\Big(\phi_0^{*i}\phi_{0i}+M^{ij}\phi_{0i}\frac{D^2}{8\partial^2}\phi_{0j}+M^*_{ij}\phi_0^{*i}\frac{\bar D^2}{8\partial^2}\phi_0^{*j}\Big)_{[3]}; \tag{4.26}$$

$$\text{WhiteBlueLine}^{\dot b}[\alpha,\beta;1,2] \equiv -\frac{1}{2}\cdot\text{BlueLine}^{\dot b}[\beta;3,1]\cdot\text{UsualLine}[2,3] \times\int d^8x\,\alpha\Big(\phi_0^{*i}\phi_{0i}+M^{ij}\phi_{0i}\frac{D^2}{8\partial^2}\phi_{0j}+M^*_{ij}\phi_0^{*i}\frac{\bar D^2}{8\partial^2}\phi_0^{*j}\Big)_{[3]}, \tag{4.27}$$

respectively. For example, the operator WhiteBlueLine can be explicitly written as

$$\text{WhiteBlueLine}^{\dot b}[\alpha,\beta;1,2] = -\frac{1}{2}(-1)^{P_\alpha(1+P_\beta)}\int d^8x\,\alpha\Big(\beta\, T^j{}_i\Big(\frac{\bar D^{\dot b}D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^j}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^*_i} -\beta\,(MT)^{ij}\frac{D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^i}\frac{\bar D^{\dot b}D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^j}-(MT)^{ij}\frac{D^2}{8\partial^2}\,\beta\,\frac{\bar D^{\dot b}D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^i}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^j}\Big). \tag{4.28}$$

(The other similar operators have much more complicated form.)

3. Commuting $(T)y_\mu^*$ with the inverse Green functions inside the operator BlueLine in a diagram presented in figure 13 we obtain diagrams (7) and (23). The effective line with two color disks in diagram (23) encodes the operator

$$-i(T)^j{}_i(T)^l{}_k\int d^8x\,d^8y\left(\theta_a\frac{\bar D_{\dot b}D^2}{8\partial^2}\right)_x\left(-\frac{2i\partial_\mu}{\partial^2}-(\gamma_\mu)^{c\dot d}\theta_c\frac{\bar D_{\dot d}D^2}{4\partial^2}\right)_y$$
$$\left(\frac{\delta^2\gamma}{\delta(\phi_l)_y\delta(\phi_j)_x}\right)^{-1}\frac{\delta}{\delta(\phi_{0k})_y}\frac{\delta}{\delta(\phi_{0i})_x}. \tag{4.29}$$

Moreover, we also obtain the commutator

$$\left[y_\mu^*,(\gamma^\mu)^{a\dot b}\theta_a\frac{\bar D_{\dot b}D^2}{8\partial^2}\right](T)_j{}^k(T)^j{}_i\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^k}\cdot\frac{\delta}{\delta_1\phi_{0i}}=(T)_j{}^k(T)^j{}_i\left\{\bar\theta_{\dot a},i\theta^c\theta_c\frac{\bar D^{\dot a}D^2}{2\partial^2}+(\gamma^\mu)^{\dot a b}\theta_b\right.$$
$$\left.\times\frac{\bar D^2D^2\partial_\mu}{8\partial^4}\right\}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^k}\cdot\frac{\delta}{\delta_1\phi_{0i}}+(T)_j{}^k(MT)^{ij}\left\{\bar\theta_{\dot a},(\gamma^\mu)^{\dot a b}\theta_b\frac{D^2\partial_\mu}{16\partial^4}\right\}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^i}\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^k}. \tag{4.30}$$

Note that the last term evidently vanishes. It is included for the convenience, because due to its presence this commutator cancels the corresponding contribution from $\{(T)\bar\theta^{\dot a},\ \mathrm{PinkLine}_{\dot a}\}$. Moreover, commuting $(T)\bar\theta^{\dot a}$ with the inverse Green functions inside the operator $\mathrm{PinkLine}_{\dot a}$ in the first diagram in figure 14, we obtain diagrams (16), (17) and (22). The effective line with two color disks in diagram (22) corresponds to

$$-i(T)^j{}_i(T)^l{}_k\int d^8x\,d^8y\left(\theta^c\theta_c\frac{\bar D_{\dot b}D^2}{2\partial^2}-i(\gamma^\mu)_{\dot b}{}^c\theta_c\frac{\bar D^2D^2\partial_\mu}{8\partial^4}\right)_x$$
$$\left(\frac{\bar D^{\dot b}D^2}{8\partial^2}\right)_y\left(\frac{\delta^2\gamma}{\delta(\phi_l)_y\delta(\phi_j)_x}\right)^{-1}\frac{\delta}{\delta(\phi_{0k})_y}\frac{\delta}{\delta(\phi_{0i})_x}. \tag{4.31}$$

Similar commutators in the second diagram in figure 14 give diagrams (20) and (21).

4. Some terms (for example, the effective one-loop diagrams) do not have simple graphical interpretation. We write their sum explicitly in figure 15.

In order to prove identity (4.15), it is necessary to verify that the sum of the diagrams presented in figure 15 coincides with the sum of the diagrams presented in figure 11. (Certainly, the explicitly written terms should be also taken into account). This is made in appendix H, using the identity

$$2\cdot\mathrm{BlueLine}_{\dot b}[\theta^a\theta_a\bar\theta^{\dot b};1,2]\cdot\mathrm{GreenLine}[3,4]+2\cdot\mathrm{GreenLine}[1,2]\cdot\mathrm{BlueLine}_{\dot b}[\theta^a\theta_a\bar\theta^{\dot b};3,4]+O(\theta^3)$$
$$=(\theta^4)_z\Big(\mathrm{BlueLine}_{\dot b}[1;1,2]\cdot\mathrm{PinkLine}^{\dot b}[3,4]+\mathrm{PinkLine}_{\dot b}[1,2]\cdot\mathrm{BlueLine}^{\dot b}[1;3,4] \tag{4.32}$$
$$-(\gamma^\mu)^{a\dot b}\mathrm{BlueLine}_{\dot b}[\theta_a;1,2]\cdot\mathrm{BrownLine}_\mu[3,4]-(\gamma^\mu)^{a\dot b}\mathrm{BrownLine}_\mu[1,2]\cdot\mathrm{BlueLine}_{\dot b}[\theta_a;3,4]\Big).$$

This identity is proved in appendix F.3. Its graphical version is presented in figure 16. According to this figure the sum of four diagrams with the same topology containing the lines given in the left hand side of this figure is equal to the sum of two diagrams containing the lines in the right hand side of this figure. An example of applying this identity is presented in figure 17.

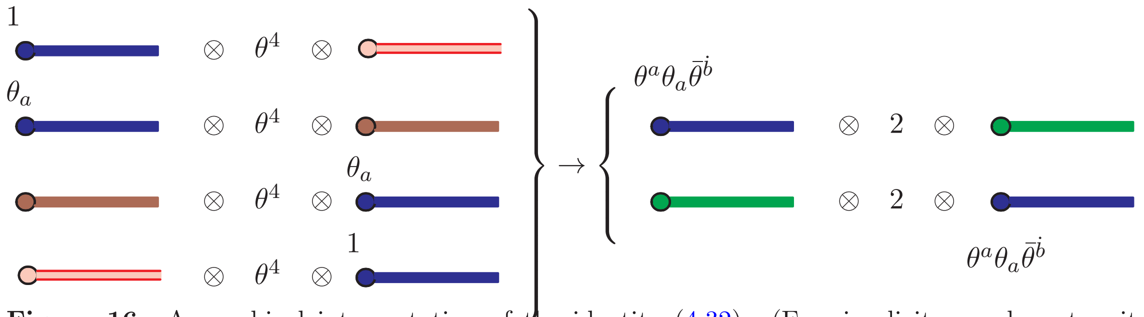


Figure 16. A graphical interpretation of the identity (4.32). (For simplicity we do not write indexes corresponding to various lines.)

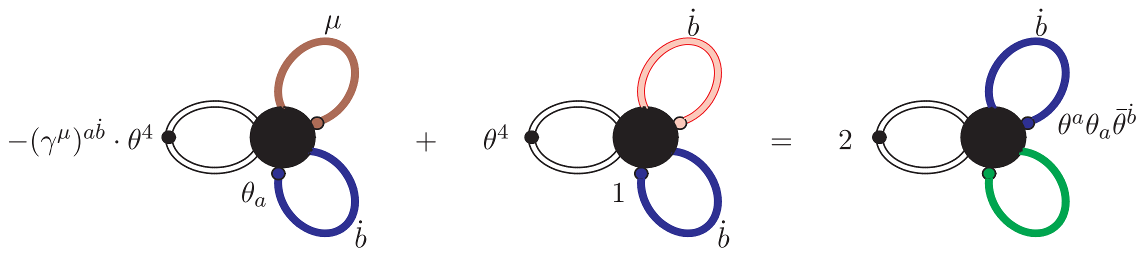


Figure 17. An example of applying the identity (4.32). This example illustrates how to prove an identity presented in figure 13. (The diagrams in this figure are symmetric with respect to permutations of the effective lines.)

Thus, we have obtained that the β-function (2.27) is determined by the integrals of total derivatives and coincides with the NSVZ β-function. Also we have proved the identity (3.19) directly.

5 Double total derivatives

5.1 Factorization of integrands into double total derivatives

In the previous section we have proved that a β-function of $\mathcal{N}=1$ SQED, regularized by higher derivatives, is given by integrals of total derivatives. This allows to calculate one of the loop integrals and obtain the exact NSVZ relation for the renormalization group functions defined in terms of the bare coupling constant. However, according to [63–67] the β-function is given by integrals of double total derivatives. In this section we prove this statement for $\mathcal{N}=1$ SQED with N_f flavors, regularized by higher derivatives, in all orders. For this purpose we differentiate the two-point Green function of the background gauge superfield with respect to $\ln g$. The required statement follows from the identity

$$
\begin{aligned}
&\frac{d}{d\ln\Lambda}\frac{\partial}{\partial\ln g}\Big(\frac{1}{2}\int d^8x\, d^8y\,(\theta^4)_x(\theta^4)_y\frac{\delta^2\Delta\Gamma}{\delta\boldsymbol{V}_x\delta\boldsymbol{V}_y}\Big)\\
&=\frac{i}{4}C(R)_i{}^j\frac{d}{d\ln\Lambda}\mathrm{Tr}\,(\theta^4)_x\Big[y_\mu^*,\Big[y_\mu^*,\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x\delta(\phi^{*i})_y}\Big)^{-1}+M^{ik}\Big(\frac{D^2}{8\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_x\delta(\phi_j)_y}\Big)^{-1}\\
&+M^*_{jk}\Big(\frac{\bar D^2}{8\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi^{*k})_x\delta(\phi^{*i})_y}\Big)^{-1}\Big]\Big]_{y=x}-\text{singularities}\;=\;-\text{singularities},
\end{aligned}\tag{5.1}
$$

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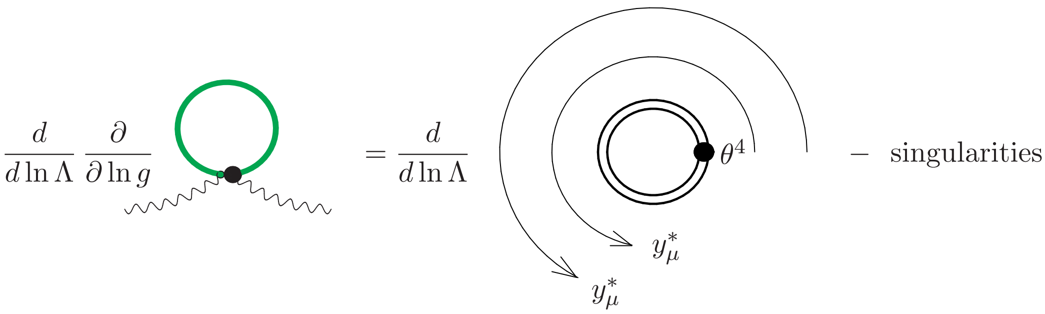


Figure 18. A graphical presentation of the double total derivatives in eq. (5.1).

where

$$[y^*_\mu, A_{xy}] \equiv (y^*_\mu)_x A_{xy} - A_{xy}(y^*_\mu)_y \tag{5.2}$$

and "$-$ singularites" means that singular contributions containing δ-functions (see below) should be subtracted from this expression. (The expression in the right hand side should be accurately defined for diagrams which include closed loops of the matter superfields. We will discuss this definition below.) Note that the last equality in eq. (5.1) evidently follows from eq. (4.1). In the graphical form the identity (5.1) is presented in figure 18. According to eq. (5.1) the β-function of $\mathcal{N} = 1$ SQED with N_f flavors, regularized by higher derivatives, in the momentum representation is given by not only by integrals of total derivatives, but by integrals of *double* total derivatives. (It is necessary to take into account that θ^4 and y^*_μ commute.)

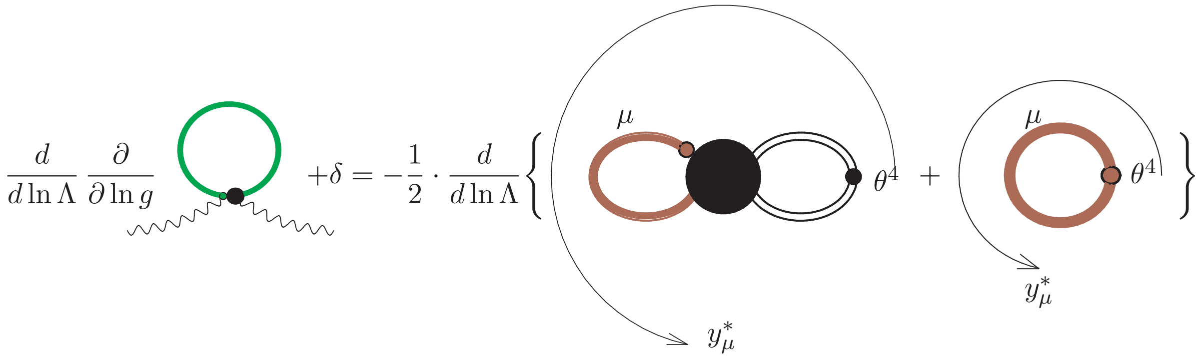
Using the operation $[(T)y^*_\mu, \ldots]$, defined by eq. (4.6), the equality (5.1) can be rewritten in the form

$$\frac{d}{d\ln\Lambda}\frac{\partial}{\partial\ln g}\Big(\frac{1}{2}\int d^8x\, d^8y\, (\theta^4)_x(\theta^4)_y \frac{\delta^2\Delta\Gamma}{\delta\boldsymbol{V}_x\delta\boldsymbol{V}_y}\Big)$$
$$= \frac{i}{4}\cdot\frac{d}{d\ln\Lambda}\mathrm{Tr}\,(\theta^4)_x\Big[(T)y^*_\mu, \Big[(T)y^*_\mu, \Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*i})_y}\Big)^{-1} + M^{ik}\Big(\frac{D^2}{8\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_x\delta(\phi_i)_y}\Big)^{-1}$$
$$+ M^*_{ik}\Big(\frac{\bar D^2}{8\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi^{*k})_x\delta(\phi^{*i})_y}\Big)^{-1}\Big]\Big]_{y=x} - \text{singularities}. \tag{5.3}$$

Then the commutators in the right hand side of this equation can be calculated using identities obtained in appendix E. First, it is necessary to calculate the inner commutator. The right hand side of eq. (5.3) can be equivalently rewritten in the form

$$-\frac{i}{4}\cdot\frac{d}{d\ln\Lambda}\,\mathrm{Tr}\,(\theta^4)_x\Big[(T)y^*_\mu,\ \mathrm{BrownLine}^\mu\cdot\Big(\frac{\boldsymbol{\delta}^2\gamma}{\boldsymbol{\delta}j^*_i\boldsymbol{\delta}j^i} + i\phi_0^{*i}\phi_{0i} + M^{ik}\Big(\frac{D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^i}\Big)\frac{\boldsymbol{\delta}\gamma}{\boldsymbol{\delta}j^k}$$
$$+ iM^{ik}\Big(\frac{D^2}{8\partial^2}\phi_{0i}\Big)\phi_{0k} + M^*_{ik}\Big(\frac{\bar D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^*_i}\Big)\frac{\boldsymbol{\delta}\gamma}{\boldsymbol{\delta}j^*_k} + iM^*_{ik}\Big(\frac{\bar D^2}{8\partial^2}\phi_0^{*i}\Big)\phi_0^{*k}\Big) - (MT)^{ik}\Big(\frac{D^2\partial^\mu}{4\partial^4}\Big)_x$$
$$\times\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_x\delta(\phi_i)_y}\Big)^{-1}_{y=x} - (TM^*)_{ik}\Big(\frac{\bar D^2\partial^\mu}{4\partial^4}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi^{*k})_x\delta(\phi^{*i})_y}\Big)^{-1}_{y=x}\Big] - \text{singularities}. \tag{5.4}$$

(This equation and eq. (4.19) accurately define the expression in the right hand side for diagrams with closed loops of the matter superfields.) The terms containing the operator



$$+\frac{i}{4}\cdot\frac{d}{d\ln\Lambda}\mathrm{Tr}\,(\theta^4)_x\Big[(T)y^*_\mu,\,(MT)^{ik}\Big(\frac{D^2\partial^\mu}{4\partial^4}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_x\delta(\phi_i)_y}\Big)^{-1}_{y=x}$$

$$+(TM^*)_{ik}\Big(\frac{\bar{D}^2\partial^\mu}{4\partial^4}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi^{*k})_x\delta(\phi^{*i})_y}\Big)^{-1}_{y=x}\Big] - \text{singularities.}$$

Figure 19. A graphical presentation of total derivatives which are obtained after calculating the inner commutator in eq. (5.1). This is a graphical form of eq. (5.4).

$\mathrm{BrownLine}^\mu$ in this equality can be easily presented in a graphical form. The result is shown in figure 19. In order to avoid too large number of effective lines some terms are written explicitly.

Then it is necessary to calculate the second commutator. This can be done similarly to the calculation made in the previous section. As earlier, it is convenient to add some terms to the expression in the right hand side of figure 19. They are presented in figure 20. These effective diagrams correspond to the analytical expression

$$-\frac{1}{4}\cdot\frac{d}{d\ln\Lambda}\mathrm{Tr}\,(\theta^4)_x\Big[(T)\theta^{\dot{a}},\,\mathrm{RedLine}_{\dot{a}}\cdot\Big(\frac{\boldsymbol{\delta}^2\gamma}{\boldsymbol{\delta}j^*_i\boldsymbol{\delta}j^i}+i\,\phi_0^{*i}\phi_{0i}+M^{ik}\Big(\frac{D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^i}\Big)\frac{\boldsymbol{\delta}\gamma}{\boldsymbol{\delta}j^k}$$
$$+i\,M^{ik}\Big(\frac{D^2}{8\partial^2}\phi_{0i}\Big)\phi_{0k}+M^*_{ik}\Big(\frac{\bar{D}^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^*_i}\Big)\frac{\boldsymbol{\delta}\gamma}{\boldsymbol{\delta}j^*_k}+i\,M^*_{ik}\Big(\frac{\bar{D}^2}{8\partial^2}\phi_0^{*i}\Big)\phi_0^{*k}\Big)\Big]=0, \qquad (5.5)$$

where the operator

$$\mathrm{RedLine}^{\dot{a}}[1,2] = \int d^8x\,\Big((T)^j{}_i\Big(\theta^b\theta_b\frac{\bar{D}^{\dot{a}}D^2}{\partial^2}-i(\gamma^\mu)^{\dot{a}b}\theta_b\frac{\bar{D}^2D^2\partial_\mu}{2\partial^4}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2j^j}\cdot\frac{\delta}{\delta_1\phi_{0i}}$$
$$-i(MT)^{ij}(\gamma^\mu)^{\dot{a}b}\theta_b\Big(\frac{D^2\partial_\mu}{4\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2j^i}\Big)\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1j^j}\Big)_x \qquad (5.6)$$

is denoted by the red line.

The commutators are calculated using equations derived in appendix E. In a graphical form the result is presented in figure 21. (The singular contributions will be calculated in the next section.) Let us describe this calculation in details.

1. Commuting $(T)y^*_\mu$ with the four point function according to the prescription presented in appendix E.1 gives diagrams (1)–(4) and 1/2 of diagram (5) in figure 21.

2. Commuting $(T)\bar{\theta}^{\dot{a}}$ with the four-point vertices in the first effective diagram in figure 20 we obtain diagrams (13)–(17) in figure 21. The details of this calculation are presented in appendix E.2.

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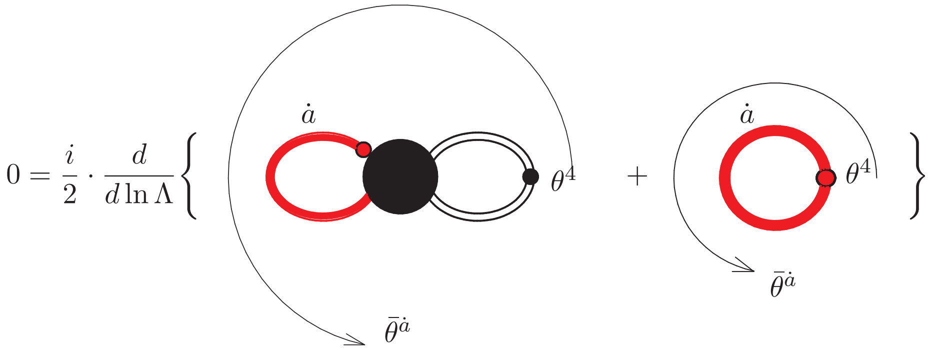


Figure 20. It is convenient to add these terms to the diagrams in the right hand side of figure 19.

3. The other 1/2 of diagram (5) and diagrams (6) and (11) in figure 21 are obtained if $(T)y^*_\mu$ is commuted with the inverse Green functions coming from the derivatives $\boldsymbol{\delta}/\boldsymbol{\delta}j$ which are contained in the brown effective line (in the first effective diagram in figure 19). In diagram (11) the effective line with two brown disks denotes the operator

$$-i(T)^j{}_i(T)^l{}_k\int d^8x\,d^8y\,\Big(-\frac{2i\partial^\mu}{\partial^2}+(\gamma^\mu)^{a\dot{b}}\theta_a\frac{\bar{D}^{\dot{b}}D^2}{4\partial^2}\Big)_x\Big(-\frac{2i\partial_\mu}{\partial^2}+(\gamma_\mu)^{c\dot{d}}\theta_c\frac{\bar{D}^{\dot{d}}D^2}{4\partial^2}\Big)_y$$
$$\times\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_y\delta(\phi_j)_x}\Big)^{-1}\frac{\delta}{\delta(\phi_{0k})_y}\frac{\delta}{\delta(\phi_{0i})_x}. \tag{5.7}$$

Similarly, diagrams (12), (18), and (19) in figure 21 are obtained from the first effective diagram in figure 20, if $(T)\bar{\theta}^{\dot{a}}$ is commuted with the inverse Green functions contained in the red effective line. In diagram (12) the effective line with two color disks denotes the operator

$$-i(T)^j{}_i(T)^l{}_k\int d^8x\,d^8y\,\Big(\frac{\bar{D}_{\dot{b}}D^2}{8\partial^2}\Big)_x\Big(\theta^a\theta_a\frac{\bar{D}^{\dot{b}}D^2}{2\partial^2}-i(\gamma^\mu)^{a\dot{b}}\theta_a\frac{\bar{D}^2D^2\partial_\mu}{4\partial^2}\Big)_y$$
$$\times\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_y\delta(\phi_j)_x}\Big)^{-1}\frac{\delta}{\delta(\phi_{0k})_y}\frac{\delta}{\delta(\phi_{0i})_x}. \tag{5.8}$$

In addition to these diagrams the considered commutators give terms containing the operator LineWithDot$[\theta^4]$ which are explicitly written in figure 21.

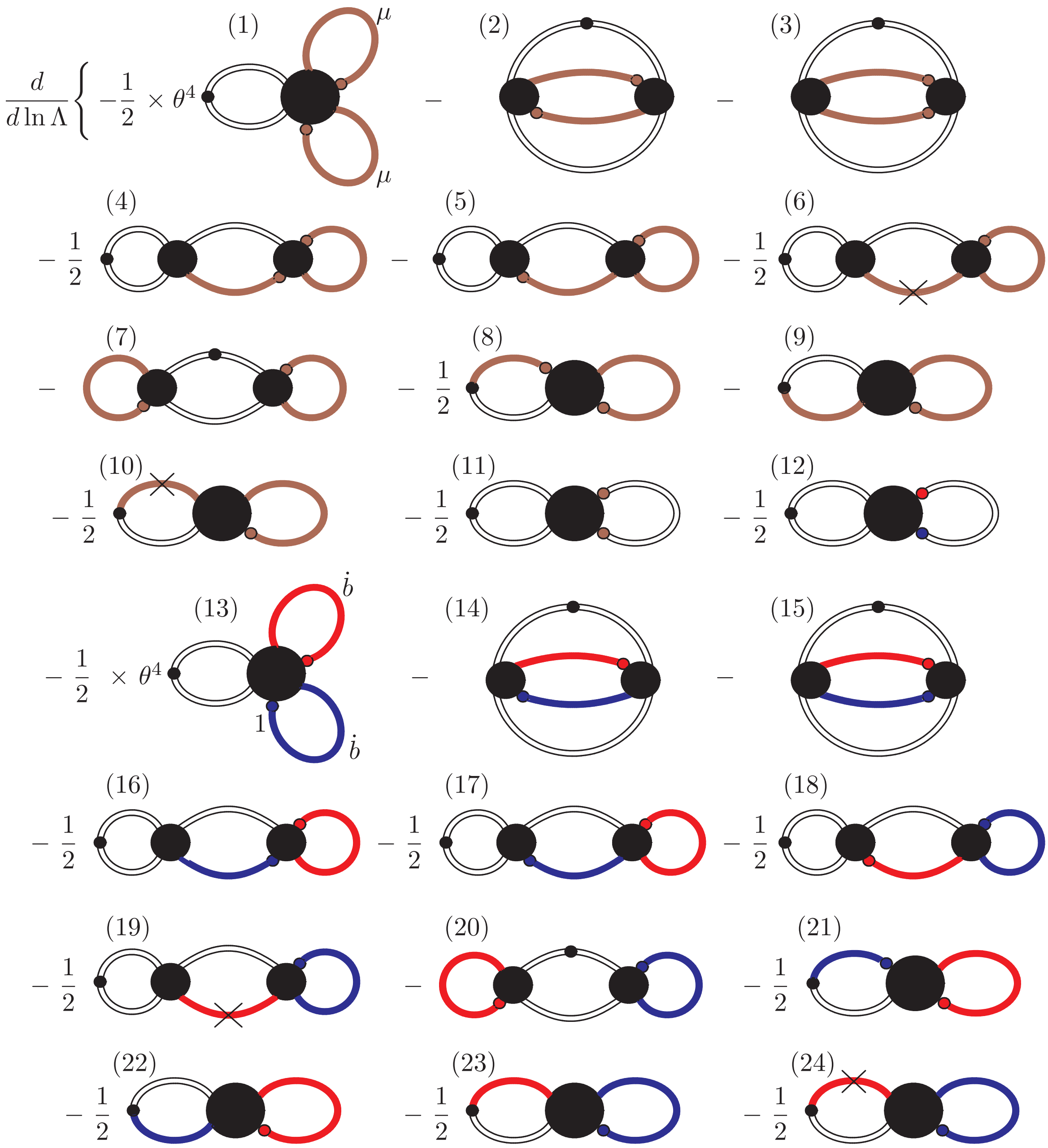
4. It is also necessary to commute $(T)y^*_\mu$ with the operators

$$i\frac{\bar{D}^2D^2\partial^\mu}{8\partial^4}-(\gamma^\mu)^{a\dot{b}}\theta_a\frac{\bar{D}_{\dot{b}}D^2}{4\partial^2}\qquad\text{and}\qquad\frac{D^2\partial_\mu}{16\partial^4}, \tag{5.9}$$

which are contained in the brown effective line. Taking into account that

$$[x^\mu,\frac{\partial_\mu}{\partial^4}]=[-i\frac{\partial}{\partial p_\mu},-\frac{ip_\mu}{p^4}]=-2\pi^2\delta^4(p_E)=-2\pi^2i\delta^4(p)=-2\pi^2i\delta^4(\partial), \tag{5.10}$$

we obtain



$$
\begin{aligned}
&+\text{One-Loop}+\frac{i}{2}\int d^8x\,d^8y\,\bigg(C(R)_k{}^n M^{jk}\delta^8_{xy}\left(\frac{D^2}{4\partial^4}\right)_x+(MT)^{mn}(MT)^{ij}\left(\frac{D^2\partial^\mu}{8\partial^4}\right)_y\\
&\times\left(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_m)_y}\right)^{-1}\left(\frac{D^2\partial_\mu}{8\partial^4}\right)_x\bigg)\cdot\text{LineWithDot}[\theta^4]\cdot\frac{\boldsymbol{\delta}^2\gamma}{\boldsymbol{\delta}(j^j)_x\boldsymbol{\delta}(j^n)_y}+\frac{i}{2}\int d^8x\,(\theta^4)_x\text{BrownLine}^\mu\\
&\times\bigg((MT)^{ik}\left(\frac{D^2\partial_\mu}{4\partial^4}\right)_x\frac{\boldsymbol{\delta}^2\gamma}{\boldsymbol{\delta}(j^i)_x\boldsymbol{\delta}(j^k)_y}\bigg|_{y=x}+(TM^*)_{ik}\left(\frac{\bar D^2\partial_\mu}{4\partial^4}\right)_x\frac{\boldsymbol{\delta}^2\gamma}{\boldsymbol{\delta}(j_i^*)_x\boldsymbol{\delta}(j_k^*)_y}\bigg|_{y=x}\bigg)-\int d^8x\,(\gamma^\mu)^{\dot a b}\\
&\times(\bar\theta^{\dot c}\bar\theta_{\dot c}\theta_b)_x(TM^*)_{ik}\left(\frac{i\bar D^2\partial_\mu}{32\partial^4}\right)_x\text{RedLine}_{\dot a}\cdot\frac{\boldsymbol{\delta}^2\gamma}{\boldsymbol{\delta}(j_i^*)_x\boldsymbol{\delta}(j_k^*)_y}\bigg|_{y=x}\bigg\}.
\end{aligned}
$$

Figure 21. These diagrams are obtained by calculating two commutators in eq. (5.1). Constructing expressions corresponding to these diagrams we assume that the first spinor index is lower and the second one is upper. The expression One-Loop is given by eq. (5.14).

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$$\left[y^*_\mu,\, i\frac{\bar D^2 D^2\partial^\mu}{8\partial^4} - (\gamma^\mu)^{a\dot b}\theta_a\frac{\bar D_{\dot b}D^2}{4\partial^2}\right] = \left\{\bar\theta_{\dot a}, -i\theta^c\theta_c\frac{\bar D^{\dot a}D^2}{\partial^2} - (\gamma^\mu)^{\dot a b}\theta_b\frac{\bar D^2 D^2\partial_\mu}{2\partial^4}\right\} + \frac{\pi^2}{4}\delta^4(\partial)\bar D^2 D^2;$$
$$\left[y^*_\mu, \frac{iD^2\partial_\mu}{16\partial^4}\right] = -\left\{\bar\theta_{\dot a}, (\gamma^\mu)^{\dot a b}\theta_b\frac{D^2\partial_\mu}{4\partial^4}\right\} + \frac{\pi^2}{8}\delta^4(\partial)D^2. \qquad (5.11)$$

As a consequence, the terms which do not contain δ-functions cancel the corresponding terms coming from the diagrams presented in figure 20.

5. Commuting $(T)y^*_\mu$ with the operator LineWithDot$[\theta^4]$ we obtain diagrams (7), (8), and the other 1/2 of diagram (9) in figure 21. These diagrams are constructed using the effective lines defined by eqs. (4.24) and (4.25). Also the considered commutators and the commutators written in figure 19 explicitly give the terms containing the operator BrownLine$^\mu$ in figure 21.

6. Diagrams (20), (21), and (22) are obtained if $(T)\bar\theta^{\dot a}$ is commuted with the effective line containing a dot. Expressions for diagrams (21) and (22) are constructed using the notation (4.26) and (4.27), respectively. Also we obtain a term containing the operator RedLine$^{\dot a}$ explicitly written in figure 21.

7. The second effective diagram in figure 19 gives 1/2 of diagram (9) and diagram (10). Similarly, diagrams (23) and (24) are obtained from the second effective diagram in figure 20. In these diagrams we use the notation

$$\begin{aligned}\text{RedWhiteLine}_{\dot a}[\theta^4;1,2] \equiv & -\frac{1}{2}\cdot\text{RedLine}_{\dot a}[1,3]\cdot\text{UsualLine}[2,3]\\ &\times\int d^8x\,(\theta^4)_x\Big(\phi_0^{*i}\phi_{0i} + M^{ij}\phi_{0i}\frac{D^2}{8\partial^2}\phi_{0j} + M^*_{ij}\phi_0^{*i}\frac{\bar D^2}{8\partial^2}\phi_0^{*j}\Big)_{[3]};\end{aligned} \qquad (5.12)$$

$$\begin{aligned}\text{WhiteRedLine}_{\dot a}[\theta^4;1,2] \equiv & -\frac{1}{2}\cdot\text{RedLine}_{\dot a}[3,1]\cdot\text{UsualLine}[2,3]\\ &\times\int d^8x\,(\theta^4)_x\Big(\phi_0^{*i}\phi_{0i} + M^{ij}\phi_{0i}\frac{D^2}{8\partial^2}\phi_{0j} + M^*_{ij}\phi_0^{*i}\frac{\bar D^2}{8\partial^2}\phi_0^{*j}\Big)_{[3]}.\end{aligned} \qquad (5.13)$$

8. Also the considered diagrams and terms written explicitly in figure 19 give some contributions which can be graphically presented as one-loop effective diagrams. In figure 21 they are denoted by One-Loop. This expression has the following form:

$$\begin{aligned}\text{One-Loop} = & -i\frac{d}{d\ln\Lambda}\int d^8x\, d^8y\,(\theta^4)_x\bigg\{\bigg(C(R)_j{}^i M^{mj}\delta^8_{xy}\Big(\frac{D^2}{4\partial^4}\Big)_x - (MT)^{nm}(MT)^{ij}\Big(\frac{D^2\partial^\mu}{8\partial^4}\Big)_y\\ &\times\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x\delta(\phi_n)_y}\Big)^{-1}\Big(\frac{D^2\partial_\mu}{8\partial^4}\Big)_x\bigg)\bigg(-\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_m)_y}\Big)^{-1} + \int d^8z\,\bigg[\frac{1}{4}\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_y\delta(\phi_k)_z}\Big)^{-1}\\ &\times\Big(\frac{\delta^2\gamma}{\delta(\phi^{*k})_z\delta(\phi_m)_x}\Big)^{-1} + \frac{1}{4}\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_y\delta(\phi^{*k})_z}\Big)^{-1}\Big(\frac{\delta^2\gamma}{\delta(\phi_m)_x\delta(\phi_k)_z}\Big)^{-1} + M^{kl}\Big(\frac{\delta^2\gamma}{\delta(\phi_m)_y\delta(\phi_k)_z}\Big)^{-1}\\ &\times\Big(\frac{D^2}{16\partial^2}\Big)_z\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_z\delta(\phi_i)_x}\Big)^{-1} + M^*_{kl}\Big(\frac{\delta^2\gamma}{\delta(\phi_m)_y\delta(\phi^{*k})_z}\Big)^{-1}\Big(\frac{\bar D^2}{16\partial^2}\Big)_z\Big(\frac{\delta^2\gamma}{\delta(\phi^{*l})_z\delta(\phi_i)_x}\Big)^{-1}\bigg]\bigg)\bigg\}.\end{aligned} \qquad (5.14)$$

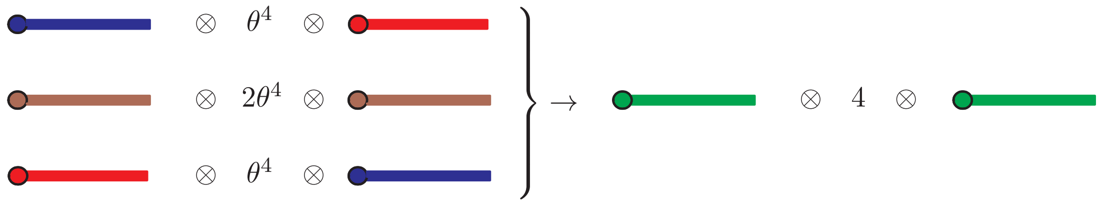

Figure 22. A graphical interpretation of the identity (5.15).

The sum of diagrams presented in figure 21 should be compared with the sum of diagrams presented in figure 10. (Certainly, the explicitly written terms should be also taken into account.) For this purpose it is necessary to use the identity

$$
\begin{aligned}
&(\theta^4)_z\Big(\text{BlueLine}_{\dot{b}}[1;1,2]\cdot\text{RedLine}^{\dot{b}}[3,4]+\text{RedLine}_{\dot{b}}[1,2]\cdot\text{BlueLine}^{\dot{b}}[1;3,4] \\
&+2\cdot\text{BrownLine}^{\mu}[1,2]\cdot\text{BrownLine}_{\mu}[3,4]\Big)=4\cdot\text{GreenLine}[1,2]\cdot\text{GreenLine}[3,4]+O(\theta^3),
\end{aligned}
\tag{5.15}
$$

derived in appendix F.4. This identity is graphically presented in figure 22. As earlier, this figure should be understood as follows: we find a sum of three diagrams with the same topology which contain effective lines presented in the left hand side of figure 22 and θ^4 in an auxiliary (but fixed) position. Then this sum can be replaced by a single diagram with the same topology containing two green effective lines. Using this identity we see that

1. The sum of diagrams (1) and (13) in figure 21 gives diagram (5) in figure 10.

2. The sum of diagrams (2) and (14) in figure 21 gives diagram (6) in figure 10.

3. The sum of diagrams (3) and (15) in figure 21 gives diagram (7) in figure 10.

4. The sum of diagrams (7) and (20) in figure 21 gives diagram (8) in figure 10.

5. The sum of the expression One-Loop and the terms containing the operator LineWithDot explicitly written in figure 21 is equal to $\partial\Delta/\partial\ln g$, which is calculated in appendix D.4. (It is evident that θ^4 in the terms with the operator LineWithDot can be shifted to an arbitrary point of the diagram.)

6. The sum of diagrams (5), (17), and (18) in figure 21 is equal to diagram (a) in figure 23. Although this diagram is absent in figure 10, it is equal to diagram (4) in this figure (see the first string in figure 23). In order to see this, we note that the left part of this diagram is proportional to

$$
\begin{aligned}
&\Big(\frac{\partial}{\partial\ln g}-1\Big)\frac{\delta^2\gamma}{\delta(\phi_{0i})_x\delta(\phi^{*j})_y}-\frac{D_y^2}{8}\delta^8_{xy}\sim D_x^2\delta^8_{xy} \\
\text{or}\qquad &\Big(\frac{\partial}{\partial\ln g}-1\Big)\frac{\delta^2\gamma}{\delta(\phi_{0i})_x\delta(\phi_j)_y}\sim D_x^2\bar{D}_x^2\delta^8_{xy}.
\end{aligned}
\tag{5.16}
$$

In both cases there is the projector D_x^2 acting on the remaining part of the green

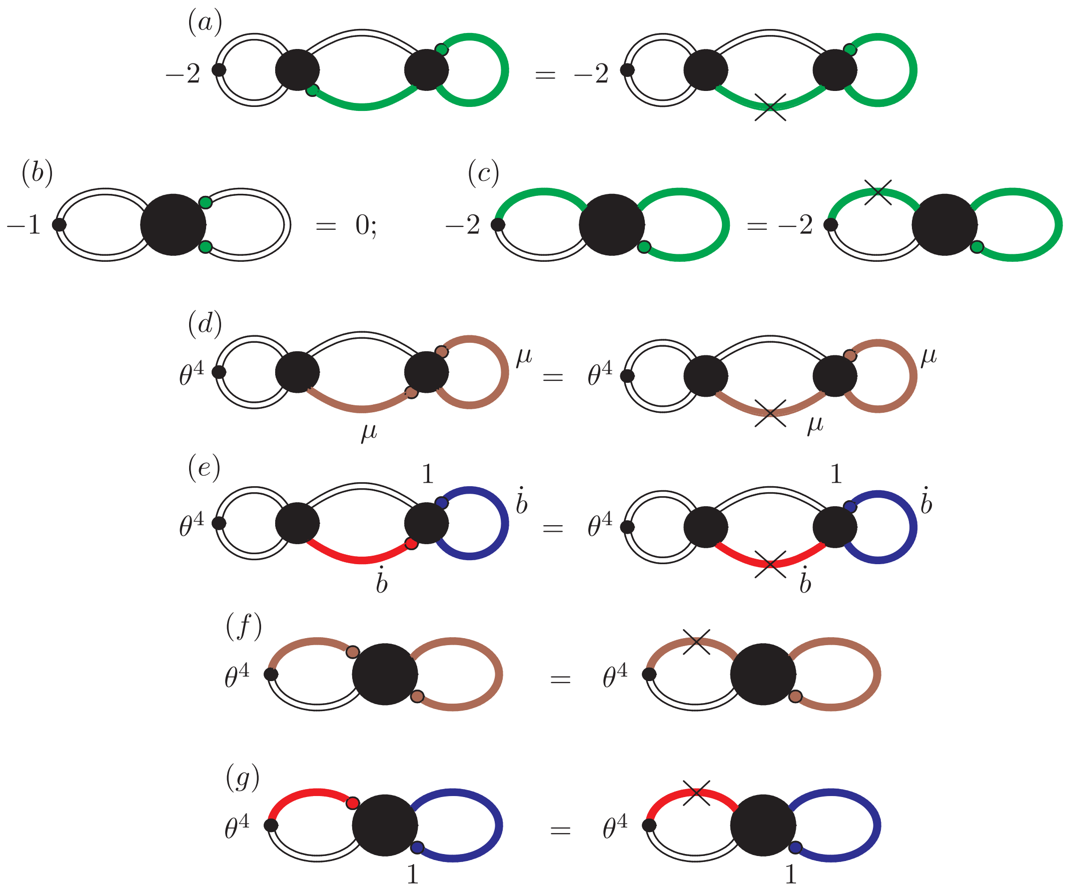

Figure 23. Some useful identities for the effective diagrams.

effective line:

$$D^2\Big(\theta^a\theta_a\bar\theta^{\dot b}\frac{\bar D_{\dot b}D^2}{4\partial^2}-i\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\frac{\bar D^2D^2\partial_\mu}{8\partial^4}\Big)=0. \tag{5.17}$$

Therefore, the part of the green line containing $\delta/\delta\phi_0$ vanishes. The remaining part of the green line is denoted by the green line with a cross. Thus, we prove the identity presented in the first string of figure 23. Using this identity we see that the sum of the considered diagrams gives diagram (4) in figure 10.

7. The sum of diagrams (11) and (12) is equal to diagram (b) in figure 23, where the line with two green disks corresponds to the operator

$$-i(T)^j{}_i(T)^l{}_k\int d^8x\,d^8y\,\Big(-2i\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\frac{\partial^\mu}{\partial^2}+\theta^a\theta_a\bar\theta^{\dot b}\frac{\bar D_{\dot b}D^2}{4\partial^2}\Big)_x$$
$$\times\Big(-2i\bar\theta^{\dot c}(\gamma^\nu)_{\dot c}{}^d\theta_d\frac{\partial_\nu}{\partial^2}+\theta^c\theta_c\bar\theta^{\dot d}\frac{\bar D_{\dot d}D^2}{4\partial^2}\Big)_y\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_y\delta(\phi_j)_x}\Big)^{-1}\frac{\delta}{\delta(\phi_{0k})_y}\frac{\delta}{\delta(\phi_{0i})_x}. \tag{5.18}$$

Using the above arguments it is easy to prove that this diagram vanishes. Really, its left side is proportional to

$$\Big(\frac{\partial}{\partial\ln g}-1\Big)\frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_{0i})_x}. \tag{5.19}$$

This Green function can contain parts proportional to δ^8_{xy}, $D^2\delta^8_{xy}$, $\bar{D}^2\delta^8_{xy}$, $(D^2\bar{D}^2)_x\delta^8_{xy}$, and $(\bar{D}^2 D^2)_x\delta^8_{xy}$. However, it is easy to see that all these structures give 0 if they act on the product

$$\Big(\theta^a\theta_a\bar{\theta}^{\dot{b}}\frac{\bar{D}_{\dot{b}}D^2}{4\partial^2} - i\bar{\theta}^{\dot{a}}(\gamma^\mu)_{\dot{a}}{}^b\theta_b\frac{\bar{D}^2 D^2\partial_\mu}{8\partial^4}\Big)_x\Big(\theta^c\theta_c\bar{\theta}^{\dot{d}}\frac{\bar{D}_{\dot{d}}D^2}{4\partial^2} - i\bar{\theta}^{\dot{c}}(\gamma^\nu)_{\dot{c}}{}^d\theta_d\frac{\bar{D}^2 D^2\partial_\nu}{8\partial^4}\Big)_y. \tag{5.20}$$

(It is necessary to take into account the integral over $d^8x\,d^8y$ and note that terms which do not contain θ^4 vanish.) Therefore, the sum of diagrams (11) and (12) vanishes.

8. The sum of diagrams (9), (22), and (23) gives diagram (c) in figure 23. Using the identity (5.17) we obtain that the considered sum of diagrams is equal to diagram (2) in figure 10. This equality is presented in figure 23.

9. Let us consider a sum of diagrams (4), (6), (16), and (19) in figure 21. First, it is necessary to note that diagrams (4) and (6) are equal (see the third string in figure 23). In order to see this, we consider terms which do not contain the masses in the left brown line. In these terms the right vertex with the right brown line has the following structure

$$\begin{aligned} &(D^2)_y\Big[(T)y^*_\mu,\ \frac{\delta^2\gamma}{\delta(\phi_{0i})_x\delta(\phi_0^{*j})_y}\Big] \sim D^2_x\delta^8_{xy}\\ \text{or}\qquad &(\bar{D}^2)_y\Big[(T)y^*_\mu,\ \frac{\delta^2\gamma}{\delta(\phi_{0i})_x\delta(\phi_{0j})_y}\Big] \sim D^2_x\bar{D}^2_x\delta^8_{xy}. \end{aligned} \tag{5.21}$$

In order to verify the last equality we note that from dimensional arguments and the Feynman rules

$$\frac{\delta^2\gamma}{\delta(\phi_{0i})_x\delta(\phi_{0j})_y} = f_1(\partial^2)D^2\delta^8_{xy} + f_2(\partial^2)\bar{D}^2\delta^8_{xy}. \tag{5.22}$$

Therefore, this part of the diagram contains the projector D^2_x acting on the left brown line. Taking into account that

$$D^2\Big(i\frac{\bar{D}^2 D^2\partial_\mu}{8\partial^4} - (\gamma_\mu)^{a\dot{b}}\theta_a\frac{\bar{D}_{\dot{b}}D^2}{4\partial^2}\Big) = 0, \tag{5.23}$$

we see that the part of the left brown line containing $\delta/\delta\phi_0$ vanishes. The remaining part of the brown line is equal to the brown line with a cross.

Moreover, diagram (19) is equal to diagram (e) in figure 23 multiplied by $-1/2$. In order to prove this identity we again consider terms which do not contain masses in the red effective line of diagram (e). Then the right vertex with the blue effective line is proportional to

$$\begin{aligned} &(D^2)_y\Big[(T)\bar{\theta}^{\dot{b}},\ \frac{\delta^2\gamma}{\delta(\phi_{0i})_x\delta(\phi_0^{*j})_y}\Big] \sim \Big[\bar{\theta}^{\dot{b}},\ D^2_x\delta^8_{xy}\Big] = 0\\ \text{or}\qquad &(\bar{D}^2)_y\Big[(T)\bar{\theta}^{\dot{b}},\ \frac{\delta^2\gamma}{\delta(\phi_{0i})_x\delta(\phi_{0j})_y}\Big] = 0. \end{aligned} \tag{5.24}$$

(The last equality can be also verified using eq. (5.22).) Therefore, the part of the red line containing $\delta/\delta\phi_0$ vanishes. The remaining part of the red line is equal to the red line with a cross.

Taking into account identities presented in the third and forth lines of figure 23, we obtain the sum of effective diagrams to which we can apply the identity (F.26). The result is given by diagram (3) in figure 10.

10. The sum of diagrams (8), (10), (21), and (24) is investigated similarly to the previous group of diagrams. For this purpose it is necessary to take into account identities presented in the last two strings of figure 23, which can be proved exactly as in the previous case. As usually, the red effective line in diagram (g) corresponds to the operator $\text{RedWhiteLine}_{\dot a}[\theta^4]$, and the red line with a cross in diagram (24) corresponds to the operator

$$\text{RedWhiteWithCross}_{\dot a}[\theta^4;1,2] \equiv -\frac{1}{2}\cdot \text{RedWithCross}_{\dot a}[1,3]\cdot \text{UsualLine}[2,3] \times \int d^8x\,(\theta^4)_x\Big(\phi_0^{*i}\phi_{0i} + M^{ij}\phi_{0i}\frac{D^2}{8\partial^2}\phi_{0j} + M^*_{ij}\phi_0^{*i}\frac{\bar D^2}{8\partial^2}\phi_0^{*j}\Big)_{[3]}. \tag{5.25}$$

Using these identities we see that the sum of the considered diagrams is equal to diagram (1) in figure 10.

11. It is easy to see that terms proportional to (TM^*) explicitly written in figure 21 cancel each other. For this purpose it is necessary to use the algebraic identity

$$\theta^c\theta_c A\theta_a = -(-1)^{P(A)}\theta_a A\theta^c\theta_c + O(\theta) \tag{5.26}$$

and its consequence, eq. (F.3), which is proved in appendix F.1.

12. The term proportional to (MT) and containing the operator BrownLine_μ explicitly written in figure 21 can be presented in the form

$$4\cdot \text{GreenLine}\cdot \text{GreenWithCross}\cdot\gamma \tag{5.27}$$

using the identity (5.26). This expression coincides with diagram (9) in figure 10.

Collecting the results we see that the sum of diagrams presented in figure 21 is equal to the sum of diagrams presented in figure 10. This completes the proof of the identity (5.1).

5.2 Derivation of the NSVZ β-function

Diagrams presented in figure 21 (the sum of which is equal to the sum of diagrams presented in figure 10) are obtained after calculating commutators with y^*_μ. However,

$$\text{Tr}[y^*_\mu, A] = 0. \tag{5.28}$$

As a consequence, the sum of diagrams presented in figure 10 is equal to the sum of terms containing the δ-singularities in eq. (5.1) with an opposite sign. Now, let us calculate these singular contributions starting from eq. (5.4), which can be written in the form

$$\begin{aligned}
&-\frac{i}{4}\frac{d}{d\ln\Lambda}\mathrm{Tr}\,(\theta^4)_x\Big[(T)y^*_\mu,\ \mathrm{BrownLine}^\mu\cdot\Big(\frac{\boldsymbol{\delta}^2\gamma}{\boldsymbol{\delta}j^*_i\boldsymbol{\delta}j^i}+M^{ik}\Big(\frac{D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^i}\Big)\frac{\boldsymbol{\delta}\gamma}{\boldsymbol{\delta}j^k}+M^*_{ik}\Big(\frac{\bar D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^*_i}\Big)\frac{\boldsymbol{\delta}\gamma}{\boldsymbol{\delta}j^*_k}\\
&+i\,\phi_0^{*i}\phi_{0i}\Big)+(MT)^{ki}\Big\{\Big(\frac{\bar D^2D^2\partial^\mu}{8\partial^4}\Big)_y\Big(\frac{D^2}{8\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_y\delta(\phi_k)_x}\Big)^{-1}_{y=x}+\int d^8y\,\Big\{M^{mn}\Big(\frac{D^2\partial^\mu}{8\partial^4}\Big)_y\\
&\times\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_y\delta(\phi_m)_x}\Big)^{-1}\Big(\frac{D^2}{8\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_y\delta(\phi_n)_x}\Big)^{-1}+M^*_{mn}\Big(\frac{D^2\partial^\mu}{8\partial^4}\Big)_y\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_y\delta(\phi^{*m})_x}\Big)^{-1}\Big(\frac{\bar D^2}{8\partial^2}\Big)_x\\
&\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_y\delta(\phi^{*n})_x}\Big)^{-1}\Big\}\Big\}-(MT)^{ik}\Big(\frac{D^2\partial^\mu}{4\partial^4}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_x\delta(\phi_i)_y}\Big)^{-1}_{y=x}-(TM)^*_{ik}\Big(\frac{\bar D^2\partial^\mu}{4\partial^4}\Big)_x\\
&\times\Big(\frac{\delta^2\gamma}{\delta(\phi^{*k})_x\delta(\phi^{*i})_y}\Big)^{-1}_{y=x}\Big]-\text{singularities}.
\end{aligned}\qquad(5.29)$$

The singular part of this expression is calculated using eq. (5.10). The result with the opposite sign (which is equal to the sum of the considered diagrams) is given by

$$\begin{aligned}
&\frac{\pi^2}{8}C(R)_i{}^j\frac{d}{d\ln\Lambda}\int d^8x\,\delta^4(\partial_\alpha)_x\Big\{\Big(\frac{\delta}{\delta(\phi_{0i})_z}\Big(\bar D^2D^2\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_x}\Big)-\frac{1}{2}M^{ki}\Big(D^2\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^k)_x}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_z}\Big)\\
&\times\Big(\mathrm{LineWithDot}[\theta^4]\cdot\gamma-\frac{1}{2}\int d^8y\,(\theta^4\phi_0^{*k}\phi_{0k})_y\Big)-(\theta^4)_xM^*_{kj}(\bar D^2)_x\Big(\frac{\delta^2\gamma}{\delta(\phi^{*i})_x\delta(\phi^{*k})_z}\Big)^{-1}\\
&-2(\theta^4)_xM^{ki}(D^2)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x\delta(\phi_k)_z}\Big)^{-1}+M^{ki}(D^2)_x\int d^8y\,(\theta^4)_y\Big(M^{mn}\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x\delta(\phi_m)_y}\Big)^{-1}\\
&\times\Big(\frac{D^2}{16\partial^2}\Big)_y\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_z\delta(\phi_n)_y}\Big)^{-1}+M^*_{mn}\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x\delta(\phi^{*m})_y}\Big)^{-1}\Big(\frac{\bar D^2}{16\partial^2}\Big)_y\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_z\delta(\phi^{*n})_y}\Big)^{-1}\Big)\Big\}_{z=x}.
\end{aligned}\qquad(5.30)$$

This expression contains explicit dependence only on θ^4. Commutation of θ^4 produces terms of the third and lower degrees of θ. All these terms vanish after integrating over the anticommuting variables. Therefore, it is possible to shift θ^4 to an arbitrary point of the diagram. Let us shift θ^4 to the point x:

$$\mathrm{LineWithDot}[\theta^4]\to(\theta^4)_x\cdot\mathrm{LineWithDot}[1].\qquad(5.31)$$

Next, we use the equalities

$$\begin{aligned}
&\frac{\delta^2}{\delta(\phi_{0i})_x\delta(\phi_{0j})_y}\Big(\mathrm{LineWithDot}[1]\cdot\gamma-\frac{1}{2}\int d^8x\,\phi_0^{*k}\phi_{0k}\Big)=2\Big(\frac{\partial}{\partial\ln g}-1\Big)\frac{\delta^2\gamma}{\delta(\phi_{0i})_x\delta(\phi_{0j})_y};\\
&\frac{\delta^2}{\delta(\phi_{0i})_x\delta(\phi_0^{*j})_y}\Big(\mathrm{LineWithDot}[1]\cdot\gamma-\frac{1}{2}\int d^8x\,\phi_0^{*k}\phi_{0k}\Big)=2\Big(\frac{\partial}{\partial\ln g}-1\Big)\frac{\delta^2\gamma}{\delta(\phi_{0i})_x\delta(\phi_0^{*j})_y},
\end{aligned}\qquad(5.32)$$

which are proved in appendix D. It is easy to see that all terms which do not contain the derivatives with respect to $\ln g$ vanish. Really, according to eq. (B.2)

$$(-2)\Big(\bar D^2D^2\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_x}\Big)\frac{\delta\gamma}{\delta(\phi_{0i})_y}=2(\bar D^2D^2)_x\delta^8_{xy}\delta^i_j+2M^{ik}(D^2)_y\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x\delta(\phi_k)_y}\Big)^{-1}.\qquad(5.33)$$

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(The first term in this expression vanishes after differentiation with respect to $\ln\Lambda$.) Similarly,

$$M^{ki}\Big(D^2\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^k)_x}\Big)\frac{\boldsymbol{\delta}\gamma}{\boldsymbol{\delta}(j^j)_x} = M^{ki}\bigg\{(D^2)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_y\delta(\phi_k)_x}\Big)^{-1}_{y=x} - \int d^8y\,\bigg(M^{nm}(D^2)_x$$
$$\times\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_x\delta(\phi_m)_y}\Big)^{-1}\Big(\frac{D^2}{16\partial^2}\Big)_y\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x\delta(\phi_n)_y}\Big)^{-1} + M^*_{nm}(D^2)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_x\delta(\phi^{*m})_y}\Big)^{-1}$$
$$\times\Big(\frac{\bar{D}^2}{16\partial^2}\Big)_y\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x\delta(\phi^{*n})_y}\Big)^{-1}\bigg)\bigg\}. \tag{5.34}$$

Collecting all terms, the considered singular contribution can be presented as

$$\frac{\pi^2}{4}C(R)_i{}^j\frac{d}{d\ln\Lambda}\int d^8x\,\theta^4\delta^4(\partial_\alpha)_x\bigg(\frac{\delta}{\delta(\phi_{0i})_y}\bar{D}^2D^2\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_x} - \frac{1}{2}M^{ki}(D^2)_x\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^k)_x}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_y}\bigg)\frac{\partial\gamma}{\partial\ln g}. \tag{5.35}$$

Using the equation for the derivative of the inverse matrix it is possible to rewrite this expression in a simpler form

$$\frac{\pi^2}{4}C(R)_i{}^j\frac{d}{d\ln\Lambda}\int d^8x\,\theta^4\delta^4(\partial_\alpha)_x\bigg(\frac{\delta}{\delta(\phi_{0i})_y}\bar{D}^2D^2\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_x}\frac{\partial\gamma}{\partial\ln g}$$
$$+\frac{1}{2}M^{ki}(D^2)_x\frac{\partial}{\partial\ln g}\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_y\delta(\phi_k)_x}\Big)^{-1}\bigg)_{y=x} \tag{5.36}$$

Then we express the derivatives with respect to sources in terms of the derivatives with respect to fields and substitute explicit expressions for the Green functions, for example,

$$\frac{\delta^2\gamma}{\delta(\phi_i)_y\delta(\phi_0^{*j})_x} = -\frac{1}{8}G_j{}^i\bar{D}_x^2\delta^8_{xy};\qquad \frac{\delta^2\gamma}{\delta(\phi_i)_y\delta(\phi_{0j})_x} = -\frac{1}{32\partial^2}\Big((MJ)^{ji} - M^{ji}\Big)D_x^2\bar{D}_x^2\delta^8_{xy}. \tag{5.37}$$

After calculating the integrals over $d^4\theta$ in the Euclidean space the result can be presented in the following form:

$$\frac{\partial}{\partial\ln g}\frac{d}{d\ln\Lambda}\Big(\frac{1}{2}\int d^8x\,d^8y\,(\theta^4)_x(\theta^4)_y\frac{\delta^2\Delta\Gamma}{\delta\boldsymbol{V}_x\delta\boldsymbol{V}_y}\Big) = -\frac{1}{4\pi^2}\mathcal{V}_4N_f\frac{\partial}{\partial\ln g}$$
$$\times\frac{d}{d\ln\Lambda}\bigg(2\ln G + \sum_{I=1}^{n}(-1)^{P_I}\Big(\ln(q^2G^2+M^2J^2) + \frac{M^2J}{(q^2G^2+M^2J^2)}\Big)_I\bigg)_{q=0}. \tag{5.38}$$

This result is very similar to eq. (4.11), which was found earlier by a different method. The only difference is the presence of the derivative with respect to $\ln g$. As earlier, we obtain

$$\frac{\partial}{\partial\ln g}\frac{d}{d\ln\Lambda}\Big(\frac{1}{2}\int d^8x\,d^8y\,(\theta^4)_x(\theta^4)_y\frac{\delta^2\Delta\Gamma}{\delta\boldsymbol{V}_x\delta\boldsymbol{V}_y}\Big) = -\frac{1}{2\pi^2}\mathcal{V}_4N_f\frac{\partial}{\partial\ln g}\frac{d}{d\ln\Lambda}\ln G. \tag{5.39}$$

Let us integrate this equation over $\ln g$ from $g = 0$ to $g = 1$. The considered theory coincides with $\mathcal{N} = 1$ SQED with N_f flavors for $g = 1$. Therefore, at the upper limit

$$\frac{d}{d\ln\Lambda}\Big(\frac{1}{2}\int d^8x\, d^8y\, (\theta^4)_x(\theta^4)_y \frac{\delta^2\Delta\Gamma}{\delta\boldsymbol{V}_x\delta\boldsymbol{V}_y}\Big)_{g=1} = \frac{1}{2\pi}\mathcal{V}_4\cdot\frac{\beta(\alpha_0)}{\alpha_0^2};$$
$$\frac{d}{d\ln\Lambda}\ln G\Big|_{g=1} = \gamma(\alpha_0). \tag{5.40}$$

For $g = 0$ the considered theory does not contain the *quantum* gauge field. Therefore, for $g = 0$ only one-loop diagrams contribute to the two-point Green function of the gauge superfield, and

$$\frac{d}{d\ln\Lambda}\Big(\frac{1}{2}\int d^8x\, d^8y\, (\theta^4)_x(\theta^4)_y \frac{\delta^2\Delta\Gamma}{\delta\boldsymbol{V}_x\delta\boldsymbol{V}_y}\Big)_{g=0} = \frac{1}{2\pi}\mathcal{V}_4\cdot\frac{\beta_{1-\text{loop}}(\alpha_0)}{\alpha_0^2};$$
$$\frac{d}{d\ln\Lambda}\ln G\Big|_{g=0} = 0, \tag{5.41}$$

where

$$\beta_{1-\text{loop}} = \frac{\alpha_0^2}{\pi}N_f. \tag{5.42}$$

Thus, after the integration we obtain the NSVZ relation

$$\beta(\alpha_0) = \frac{\alpha_0^2 N_f}{\pi}(1-\gamma(\alpha_0)) \tag{5.43}$$

for the renormalization group functions defined in terms of the bare coupling constant.

6 Conclusion

In this paper we present a derivation of the NSVZ relation for $\mathcal{N} = 1$ SQED with N_f flavors, regularized by higher derivatives, using a method based on the effective diagram technique and the Schwinger-Dyson equations. We prove that with this regularization the exact NSVZ β-function relates the renormalization group functions defined in terms of the bare coupling constant. (If the renormalization group functions are defined in terms of the renormalized coupling constant, the NSVZ scheme can be easily constructed by imposing the simple boundary conditions (2.30) on the renormalization constants [68, 69].) The technique based on the Schwinger-Dyson equations seems to be more convenient for generalization of the results to the non-Abelian case than another method discussed in [67].

The method considered in this paper allows to easily calculate a contribution to the β-function proportional to the anomalous dimension of the matter superfields. For this purpose expressions for the effective vertices are found by solving the Ward (or Slavnov-Taylor) identities [72]. However, in order to prove that the other contributions vanish for the considered theory, it is necessary to essentially modify the method. First, a β-function should be written in terms of two-loop effective diagrams. Moreover, it is necessary to introduce an auxiliary parameter g and perform a differentiation with respect to $\ln g$. The derivative of the two-point function of the gauge superfield with respect to $\ln g$ can be presented as a sum of three-loop effective diagrams. After these modifications it is possible to find the remaining contribution to the β-function defined in terms of the bare coupling constant. In this paper we obtain that this contribution vanishes. Moreover, we prove that the β-function is given by integrals of double total derivatives in agreement with the results of [63, 67]. Such a structure allows to calculate one of the loop integrals and obtain the NSVZ β-function in all orders. The origin of the exact NSVZ β-function can be easily explained, because taking one of loop integrals we relate the β-function in a certain order with the anomalous dimension in the previous order.

The results obtained in this paper can be verified by explicit calculations in the lowest loops. The three-loop calculation will be described in the forthcoming paper.

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A The Schwinger-Dyson equation for the two-point Green function of the gauge superfield

We are interested in the expression

$$\frac{1}{2}\int d^8x\, d^8y\, \boldsymbol{V}_x \boldsymbol{V}_y \frac{\delta^2 \Delta\Gamma}{\delta \boldsymbol{V}_x \delta \boldsymbol{V}_y}, \tag{A.1}$$

where the effective action Γ is given by eq. (2.21) and all fields are set to 0. In order to calculate this expression we differentiate the Schwinger-Dyson equation (3.4) with respect to $\boldsymbol{V}_y$ and set all fields to 0. Then the result is multiplied by $\boldsymbol{V}_x\boldsymbol{V}_y/2$. After integrating over $d^8x\, d^8y$ we rewrite eq. (A.1) as

$$\frac{1}{i}(T)^j{}_i \int d^8x\, d^8y\, \boldsymbol{V}_x \boldsymbol{V}_y \frac{\delta}{\delta \boldsymbol{V}_y}\Big(\frac{\delta}{\delta(j^j)_x}\frac{\delta\Gamma}{\delta(\phi_{0i})_x}\Big). \tag{A.2}$$

In order to present this expression as a sum of effective diagrams, it is convenient to commute $\delta/\delta\boldsymbol{V}$ and $\delta/\delta j$. Differentiating inverse Green functions inside the derivatives $\delta/\delta j^j$ and simplifying the result using eq. (3.1), it is easy to see that

$$\Big[\frac{\delta}{\delta \boldsymbol{V}_y}, \frac{\delta}{\delta(j^j)_x}\Big] = \int d^8z \Bigg(\frac{\delta}{\delta(j^j)_x}\frac{\delta^2\Gamma}{\delta \boldsymbol{V}_y \delta(\phi_{0k})_z}\cdot\frac{\delta}{\delta(j^k)_z}$$
$$+\frac{\delta}{\delta(j^j)_x}\frac{\delta^2\Gamma}{\delta \boldsymbol{V}_y \delta(\phi_0^{*k})_z}\cdot\frac{\delta}{\delta(j_k^*)_z} + \frac{\delta}{\delta(j^j)_x}\frac{\delta^2\Gamma}{\delta \boldsymbol{V}_y \delta V_z}\cdot\frac{\delta}{\delta J_z}\Bigg), \tag{A.3}$$

where the derivatives with respect to sources should be expressed in terms of the derivatives with respect to fields. Really, for example,

$$\int d^8z\, \frac{\delta^2\Gamma}{\delta \boldsymbol{V}_y \delta(\phi_{0k})_z}\cdot\frac{\delta}{\delta(j^k)_z} = \int d^8z\, \frac{\delta}{\delta \boldsymbol{V}_y}\Big(-\frac{1}{2}\bar{D}_z^2\Big)\frac{\delta\Gamma}{\delta(\phi_{0k})_z}\cdot\frac{D^2}{8\partial^2}\frac{\delta}{\delta(j^k)_z}$$
$$= \int d^8z\, \frac{\delta^2\Gamma}{\delta \boldsymbol{V}_y \delta(\phi_k)_z}\cdot\frac{D^2}{8\partial^2}\frac{\delta}{\delta(j^k)_z} \tag{A.4}$$

due to the chirality of the derivative with respect to the source j. All vertices containing odd degrees of the matter superfields vanish after setting the fields to 0. Two-point Green functions of the matter superfields constructed from the functionals Γ and γ evidently coincide (again, after setting the fields to 0). Using these facts eq. (A.2) can be written in the form

$$\frac{1}{2}\int d^8x\, d^8y\, \boldsymbol{V}_x \boldsymbol{V}_y \frac{\delta^2\Delta\Gamma}{\delta\boldsymbol{V}_y\delta\boldsymbol{V}_x} = -i(T)^j{}_i \int d^8x\, d^8y\, d^8z\, \boldsymbol{V}_x \boldsymbol{V}_y \Bigg(\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_x}\frac{\delta^2\Gamma}{\delta\boldsymbol{V}_y\delta(\phi_{0k})_z}$$
$$\times\Big[\delta^8_{xz}\delta^i_k + \frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^k)_z}\frac{\delta\gamma}{\delta(\phi_{0i})_x}\Big] + \frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_x}\frac{\delta^2\Gamma}{\delta\boldsymbol{V}_y\delta(\phi_0^{*k})_z}\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^*_k)_z}\frac{\delta\gamma}{\delta(\phi_{0i})_x}\Bigg), \tag{A.5}$$

where the derivative $\boldsymbol{\delta}/\boldsymbol{\delta}j^i$ is defined by eq. (3.7). It differs from the derivative $\delta/\delta j^i$, because all fields in the inverse two-point Green functions are set to 0. The graphical interpretation of this result is presented in figure 1.

B The identity for the effective lines

In order to simplify the calculations we use the substitution $\boldsymbol{V}\to\theta^4$. Then we find a sum of effective lines presented in the left hand side of figure 2. In the analytical form this sum corresponds to the expression

$$(T)^j{}_i\int d^8x\,(\theta^4)_x\Bigg\{\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_x}\frac{\delta}{\delta(\phi_{0i})_x} + \int d^8y\,\Bigg[\Big(\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^k)_y}\frac{\delta\gamma}{\delta(\phi_{0i})_x}\Big)\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_x}\frac{\delta}{\delta(\phi_{0k})_y}$$
$$+\Big(\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^*_k)_y}\frac{\delta\gamma}{\delta(\phi_{0i})_x}\Big)\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_x}\frac{\delta}{\delta(\phi_0^{*k})_y}\Bigg]\Bigg\}, \tag{B.1}$$

where the fields in the two-point functions are set to 0. Note that in our notation effective lines include derivatives which act on the vertices attached to the line. This allows to considerably simplify expressions for the multiloop effective diagrams.

Using eq. (3.1) and arguments based on chirality it is easy to verify that

$$\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^k)_y}\frac{\delta\gamma}{\delta(\phi_{0i})_x} = \Big(\frac{\bar D^2 D^2}{16\partial^2}\Big)_y\delta^8_{xy}\delta^i_k + M^{im}\Big(\frac{D^2}{16\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_y\delta(\phi_m)_x}\Big)^{-1};$$
$$\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^*_k)_y}\frac{\delta\gamma}{\delta(\phi_{0i})_x} = M^{im}\Big(\frac{D^2}{16\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi^{*k})_y\delta(\phi_m)_x}\Big)^{-1}. \tag{B.2}$$

(The fields are set to 0.) In order to prove these equations, it is necessary to apply to them the operator $-\bar D_x^2/2$. Substituting the Green functions (B.2) into eq. (B.1) after some simple transformations we rewrite the considered expression as

$$\int d^8x\,\Bigg\{(T)^j{}_i\Bigg[\Big(1+\frac{\bar D^2 D^2}{16\partial^2}\Big)\theta^4\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}\Bigg]\frac{\delta}{\delta\phi_{0i}} + \theta^4(MT)^{ij}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^i}\Big(\frac{D^2}{16\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}\Big)\Bigg\}. \tag{B.3}$$

The first term in this expression can be transformed using the identity

$$\Big(1+\frac{\bar D^2 D^2}{16\partial^2}\Big)\theta^4\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j} = \Big(\theta^a\theta_a\bar\theta^{\dot b}\frac{\bar D_{\dot b}D^2}{4\partial^2} + 2i\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\frac{\partial_\mu}{\partial^2} - \frac{D^2}{4\partial^2}\theta^a\theta_a\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}. \tag{B.4}$$

The degree of θ in the last term of eq. (B.3) can be also decreased by the help of eq. (2.13):

$$\int d^8x\,\theta^4(MT)^{ij}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^i}\left(\frac{D^2}{16\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^j}\right)=-\int d^8x\,\theta^4(MT)^{ij}\left(\frac{\bar D^2D^2}{16\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^i}\right)\left(\frac{D^2}{16\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^j}\right)$$
$$=\int d^8x\,\theta^a\theta_a\bar\theta^{\dot b}(MT)^{ij}\left(\frac{\bar D_{\dot b}D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^i}\right)\left(\frac{D^2}{16\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^j}\right). \tag{B.5}$$

Then integrating by parts gives

$$(MT)^{ij}\int d^8x\left(\left(\theta_a\bar\theta^{\dot b}\frac{D^a\bar D_{\dot b}D^2}{32\partial^4}-\bar\theta^{\dot b}\frac{\bar D_{\dot b}D^2}{32\partial^4}\right)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^i}\right)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^j}=(MT)^{ij}\int d^8x\left(\left(-i\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\right.\right.$$
$$\left.\left.\times\frac{D^2\partial_\mu}{16\partial^4}-\frac{D^2}{16\partial^4}\right)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^i}\right)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^j}=-i(MT)^{ij}\int d^8x\,\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\Big(\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^i}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^j}. \tag{B.6}$$

Therefore, the expression (B.1) can be rewritten in the following form:

$$\int d^8x\,\Big\{(T)^j{}_i\Big(\theta^a\theta_a\bar\theta^{\dot b}\frac{\bar D_{\dot b}D^2}{4\partial^2}+2i\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\frac{\partial_\mu}{\partial^2}-\frac{D^2}{4\partial^2}\,\theta^a\theta_a\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^j}\frac{\delta}{\delta\phi_{0i}}$$
$$-i(MT)^{ij}\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\Big(\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^i}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^j}\Big\}. \tag{B.7}$$

C The Schwinger-Dyson equation in terms of two-loop effective diagrams

C.1 β-function in terms of two-loop effective diagrams

In order to rewrite the Schwinger-Dyson equation as a sum of two-loop effective diagrams, let us start with eq. (3.10) and substitute $\delta\Gamma/\delta\boldsymbol{V}_y$ from eq. (3.4) [81]:

$$\frac{1}{2\pi}\mathcal{V}_4\cdot\frac{\beta(\alpha_0)}{\alpha_0^2}=-i\frac{d}{d\ln\Lambda}\int d^8x\,d^8y\,\Big[(T)^j{}_i\Big(\theta^a\theta_a\bar\theta^{\dot b}\frac{\bar D_{\dot b}D^2}{4\partial^2}+2i\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\frac{\partial_\mu}{\partial^2}-\frac{D^2}{4\partial^2}\,\theta^a\theta_a\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^j}$$
$$\times\frac{\delta}{\delta\phi_{0i}}-i(MT)^{ij}\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\Big(\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^i}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^j}\Big]_x(\theta^4)_y(T)^l{}_k\Big[\frac{2}{i}\frac{\delta}{\delta j^l}\frac{\delta\Gamma}{\delta\phi_{0k}}+2\frac{\delta\Gamma}{\delta\phi_{0k}}(\phi_l+\phi_{0l})\Big]_y. \tag{C.1}$$

(Certainly, all fields in this expression should be set to 0 after calculation of the derivatives.) First, let us find a contribution of the second term in the last square brackets (or, equivalently, terms containing $\phi_l+\phi_{0l}$). We will denote this contribution by Δ. After calculating the derivatives nontrivial terms can be written as

$$\Delta=2i\frac{d}{d\ln\Lambda}\int d^8x\,d^8y\,(\theta^4)_y(T)_k{}^l\Big\{(T)^j{}_l\Big(\frac{D^2}{4\partial^2}\Big)_x(\theta^a\theta_a)_x\Big(\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_x}\frac{\delta\gamma}{\delta(\phi_{0k})_y}\Big)\delta^8_{xy}$$
$$+(T)^j{}_i\frac{\delta^2\gamma}{\delta(\phi_{0i})_x\delta(\phi_{0k})_y}\Big(\theta^a\theta_a\bar\theta^{\dot b}\frac{\bar D_{\dot b}D^2}{4\partial^2}+2i\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\frac{\partial_\mu}{\partial^2}-\frac{D^2}{4\partial^2}\,\theta^a\theta_a\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_y\delta(\phi_j)_x}\Big)^{-1}$$
$$-i(MT)^{ij}\Big(\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_x}\frac{\delta\gamma}{\delta(\phi_{0k})_y}\Big)\Big(\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\frac{D^2\partial_\mu}{16\partial^4}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_y\delta(\phi_i)_x}\Big)^{-1}\Big\}. \tag{C.2}$$

Here instead of the effective action Γ we use the Routhian γ, because this considerably simplifies the calculations. In order to rewrite all equations in terms of γ, it is necessary to take into account that

$$\frac{\delta\Gamma}{\delta\phi_{0i}}[V,\phi,\phi_0] = \frac{\delta\gamma}{\delta\phi_{0i}}[J,\phi,\phi_0]. \tag{C.3}$$

Note that some terms in eq. (C.2) vanish, because they are proportional to θ in a more than the forth degree. For example, all terms in the second string of this equation vanish due to this reason. Really, taking into account that

$$\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_y\delta(\phi_j)_x}\Big)^{-1} \sim \bar D^2\delta_{xy}, \tag{C.4}$$

we can commute θ_b with this Green function. Then the result will be proportional to $(\theta^4)_y(\theta_b)_y = 0$. In order to simplify the remaining terms we use eq. (B.2). Taking into account that some terms vanish after differentiation with respect to $\ln\Lambda$ we obtain

$$\Delta = \frac{d}{d\ln\Lambda}\int d^8x\, d^8y\, (\theta^4)_y\Big\{iC(R)_k{}^j M^{km}\Big(\frac{D^2}{2\partial^2}\Big)_x(\theta^a\theta_a)_x\Big(\frac{D^2}{16\partial^2}\Big)_y\Big(\frac{\delta^2\gamma}{\delta(\phi_m)_y\delta(\phi_j)_x}\Big)^{-1}\delta^8_{xy}$$
$$+\Big[\frac{\bar D^2 D^2}{8\partial^2}\delta^8_{xy}(T)_j{}^l + (MT)^{ml}\Big(\frac{D^2}{8\partial^2}\Big)_y\Big(\frac{\delta^2\gamma}{\delta(\phi_m)_y\delta(\phi_j)_x}\Big)^{-1}\Big](MT)^{ij}(\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b)_x\Big(\frac{D^2\partial_\mu}{16\partial^4}\Big)_x$$
$$\times\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_y\delta(\phi_i)_x}\Big)^{-1}\Big\}. \tag{C.5}$$

Shifting $(\theta^a\theta_a)_x$ and $(\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b)_x$ to the point y this expression can be rewritten as

$$\Delta \equiv -i\frac{d}{d\ln\Lambda}\int d^8x\, d^8y\, (\theta^4)_y\Big(\frac{D^2}{4\partial^4}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_j)_y}\Big)^{-1}$$
$$\times\Big\{C(R)_k{}^i M^{jk}\delta^8_{xy} - (MT)^{im}(MT)^{lj}\Big(\frac{D^2}{32\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_m)_x\delta(\phi_l)_y}\Big)^{-1}\Big\}. \tag{C.6}$$

It is important that this expression does not contain infrared singularities due to the differentiation with respect to $\ln\Lambda$, which should be made before calculating the momentum integral.

Let us now consider the remaining terms in eq. (C.1) (which are obtained from the first term in the last square brackets). In order to rewrite them in a more convenient form, we commute the derivatives $\delta/\delta j$ and $\delta/\delta\phi_0$ (including $\delta/\delta\phi_0$ which is contained in $\boldsymbol{\delta}/\boldsymbol{\delta} j^i$) taking into account that

$$\Big[\frac{\delta}{\delta(\phi_{0k})_z},\frac{\delta}{\delta(j^n)_y}\Big\} \equiv \frac{\delta}{\delta(\phi_{0k})_z}\frac{\delta}{\delta(j^n)_y} - (-1)^{P_kP_n}\frac{\delta}{\delta(j^n)_y}\frac{\delta}{\delta(\phi_{0k})_z}$$
$$= (-1)^{P_kP_n}\int d^8w\,\Big(\frac{\delta}{\delta(j^n)_y}\frac{\delta^2\gamma}{\delta(\phi_{0k})_z\delta(\phi_{0l})_w}\cdot\frac{\delta}{\delta(j^l)_w} + \frac{\delta}{\delta(j^n)_y}\frac{\delta^2\gamma}{\delta(\phi_{0k})_z\delta(\phi_0^{*l})_w}\cdot\frac{\delta}{\delta(j^*_l)_w}\Big). \tag{C.7}$$

Note that so far we did not set the fields to 0. Because the Routhian γ is used instead of the effective action Γ, the expression in the right hand side does not contain derivatives with respect to the gauge superfield V. (This is the main reason, why in the subsequent equations we use the functional γ.) As a consequence, setting all fields equal to 0 we obtain

$$\Big(\hat{A}\frac{\delta}{\delta j^j}\Big)_y\frac{\delta}{\delta(\phi_{0i})_y}(T)^l{}_k\int d^8x\,(\theta^4)_x\frac{\delta}{\delta(j^l)_x}\frac{\delta\Gamma}{\delta(\phi_{0k})_x}$$
$$= (T)^l{}_k\int d^8x\,d^8z\,(\theta^4)_x\bigg(\Big[\delta^8_{xz}\delta^k_m+\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^m)_z}\frac{\delta\gamma}{\delta(\phi_{0k})_x}\Big]\Big(\hat{A}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}\Big)_y\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^l)_x}\frac{\delta^2\gamma}{\delta(\phi_{0m})_z\delta(\phi_{0i})_y}$$
$$+\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^*_m)_z}\frac{\delta\gamma}{\delta(\phi_{0k})_x}\cdot\Big(\hat{A}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}\Big)_y\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^l)_x}\frac{\delta^2\gamma}{\delta(\phi_0^{*m})_z\delta(\phi_{0i})_y}\bigg), \tag{C.8}$$

where $\hat{A}$ is an operator acting on the coordinates y which does not contain $\bar{\theta}^{\dot{a}}\bar{\theta}_{\dot{a}}$. Its explicit form in the considered case can be found from eq. (C.1). Then it is possible to substitute explicit expressions for the two-point Green functions and repeat all transformations made in appendix B. It is easy to see that the result can be written as

$$\Big(\hat{A}\frac{\delta}{\delta j^j}\Big)_y\frac{\delta}{\delta(\phi_{0i})_y}(T)^l{}_k\int d^8x\,(\theta^4)_x\frac{\delta}{\delta(j^l)_x}\frac{\delta\Gamma}{\delta(\phi_{0k})_x}=\text{GreenLine}\cdot\Big(\hat{A}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}\Big)_y\frac{\delta\gamma}{\delta(\phi_{0i})_y}, \tag{C.9}$$

where we take into account that only terms proportional to $\bar{\theta}^2$ nontrivially contribute to the result. (In particular, a term without $\bar{\theta}$ in eq. (B.7) gives a vanishing contribution.) Therefore, finally we obtain

$$\frac{1}{2\pi}\mathcal{V}_4\cdot\frac{\beta(\alpha_0)}{\alpha_0^2}=-2\frac{d}{d\ln\Lambda}(\text{GreenLine})^2\gamma+\Delta, \tag{C.10}$$

where Δ is given by eq. (C.6). The first term in this expression can be graphically presented as a two-loop effective diagram, while the second one corresponds to a one-loop effective diagram.

C.2 The identity (3.19) in terms of two-loop effective diagrams

In order to prove the identity (3.19), it is necessary to present its left hand side as a sum of two-loop effective diagrams. For this purpose we use the Schwinger-Dyson equation (3.4). The first term in eq. (3.19) can be presented as a two-loop effective diagram

$$-4\cdot\text{BlueLine}_{\dot{b}}[\theta^a\theta_a\bar{\theta}^{\dot{b}}]\cdot\text{GreenLine}\cdot\gamma \tag{C.11}$$

similarly to the calculation made in the previous section. In the graphical form this diagram (containing one blue effective line and one green effective line) is shown in figure 9. All terms which could be interpreted as one-loop effective diagrams in this case vanish.

However, the second term in eq. (3.19) can be written as a one-loop effective diagram. Let us remind that a contribution of this term is denoted by δ_2. In order to write all two-loop contributions in the same form we also use the notation $\Delta_2=\delta_2$. Using the Schwinger-Dyson equation (3.4) δ_2 can be written as

$$i(T)^j{}_i(T)^l{}_k\frac{d}{d\ln\Lambda}\int d^8x\,d^8y\,(\theta^4)_y\Big(\theta^a\theta_a\frac{D^2}{2\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}\Big)_x\frac{\delta}{\delta(\phi_{0i})_x}\Big[\frac{1}{i}\frac{\delta}{\delta j^l}\frac{\delta\Gamma}{\delta\phi_{0k}}+\frac{\delta\Gamma}{\delta\phi_{0k}}(\phi_l+\phi_{0l})\Big]_y. \tag{C.12}$$

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The first term in the square brackets vanishes, because it is proportional to the first degree of $\bar{\theta}$. This follows from eq. (C.9). Calculating the derivatives with respect to ϕ_0 it is easy to see that the only nontrivial term is

$$- i(T)_i{}^j(T)_k{}^l \frac{d}{d\ln\Lambda}\int d^8x\, d^8y\, (\theta^4)_y(\theta^a\theta_a)_x\Big(\frac{D^2}{2\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x(\phi_l)_y}\Big)^{-1}\frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_{0i})_x}. \tag{C.13}$$

Commuting θ-s with the covariant derivatives and using the identity

$$(T)_k{}^l\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x(\phi_l)_y}\Big)^{-1} = -(T)_j{}^l\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_x(\phi_k)_y}\Big)^{-1} \tag{C.14}$$

we obtain

$$\Delta_2 = -2iC(R)_i{}^j\frac{d}{d\ln\Lambda}\int d^8x\, d^8y\, (\theta^4)_y\frac{1}{\partial^2}\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x(\phi_k)_y}\Big)^{-1}\frac{\delta^2\gamma}{\delta(\phi_{0i})_x\delta(\phi_{0k})_y}. \tag{C.15}$$

This expression can be written in terms of the functions G and J. For this purpose we substitute the explicit expression for the inverse Green function from eq. (2.41). Then, it is necessary to use eq. (3.1), which allows to express the remaining Green function in terms of J:

$$\begin{aligned}(D^2)_y\frac{\delta^2\gamma}{\delta(\phi_0^{*j})_y\delta(\phi_{0i})_x} &= \frac{1}{4}G^i{}_j D^2\delta^8_{xy};\\ (\bar{D}^2)_x\frac{\delta^2\gamma}{\delta(\phi_{0j})_y\delta(\phi_{0i})_x} &= \Big((MJ)^{ij} - M^{ij}\Big)\Big(\frac{\bar{D}^2D^2}{16\partial^2}\Big)_x\delta^8_{xy}.\end{aligned} \tag{C.16}$$

Calculating the integrals over θ after the Wick rotation we obtain

$$\Delta_2 = -\mathcal{V}_4\cdot C(R)_i{}^j\frac{d}{d\ln\Lambda}\int\frac{d^4q}{(2\pi)^4}\Big(\frac{2}{q^4(q^2G^2+|MJ|^2)}(MJ)^*\Big)_{jk}\Big((MJ)^{ki} - M^{ki}\Big). \tag{C.17}$$

D Derivatives with respect to the parameter g

D.1 The derivative of the Routhian

Let us set the background gauge superfield to 0, $\boldsymbol{V} = 0$, and differentiate the Routhian γ with respect to the parameter g using the identity

$$\frac{\partial\gamma}{\partial\ln g} = \frac{\partial W}{\partial\ln g}. \tag{D.1}$$

The result can be written as

$$\frac{\partial\gamma}{\partial\ln g} = \Big\langle\frac{g}{4}\sum_{I=0}^{n}\sum_{\alpha=1}^{N_f}\int d^8x\,\Big\{(\phi^*+\phi_0^*)(e^{2V}-1)(\phi+\phi_0) + (\widetilde{\phi}^*+\widetilde{\phi}_0^*)(e^{-2V}-1)(\widetilde{\phi}+\widetilde{\phi}_0)\Big\}_{\alpha I}\Big\rangle. \tag{D.2}$$

In terms of the Routhian γ the right hand side of this equation can be presented in the form

$$\frac{\partial\gamma}{\partial\ln g} = \frac{1}{2}\int d^8x\,\Big\{\frac{1}{i}(-1)_i{}^j\frac{\delta}{\delta(j^j)_x}\frac{\delta\gamma}{\delta(\phi_{0i})_x} + (\phi_i+\phi_{0i})_x\frac{\delta\gamma}{\delta(\phi_{0i})_x} + \frac{1}{i}(-1)_i{}^j\frac{\delta}{\delta(j_i^*)_x}\frac{\delta\gamma}{\delta(\phi_0^{*j})_x}$$
$$+(\phi^{*i}+\phi_0^{*i})_x\frac{\delta\gamma}{\delta(\phi_0^{*i})_x} - \frac{i}{2}\Big(\frac{\delta^2\gamma}{\delta\phi^{*i}\delta\phi_i}\Big)^{-1} - \frac{1}{2}(\phi^{*i}+\phi_0^{*i})(\phi_i+\phi_{0i})\Big\}, \tag{D.3}$$

where

$$(-1)_i{}^j \equiv \delta_{ij}\cdot(-1)^{P_i}. \tag{D.4}$$

Let us consider the first term in this expression. Integrating by parts and using eq. (3.1) we obtain

$$\int d^8x\,(-1)_i{}^j\frac{\delta}{\delta j^j}\frac{\delta\gamma}{\delta\phi_{0i}} = -\int d^8x\,(-1)_i{}^j\Big(\frac{D^2}{16\partial^2}\frac{\delta}{\delta j^j}\Big)\bar D^2\frac{\delta\gamma}{\delta\phi_{0i}} = \int d^8x\,(-1)_i{}^j\Big(\frac{D^2}{8\partial^2}\frac{\delta}{\delta j^i}\Big)\Big(\frac{\delta\gamma}{\delta\phi_i}$$
$$-\frac{1}{2}M^{ij}\phi_j\Big) = \int d^8x\,\Big\{(-1)_i{}^i\Big(\frac{D^2\bar D^2}{16\partial^2}\Big)_x\delta^8_{xy} + M^{j\,i}\Big(\frac{D^2}{16\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_j)_y}\Big)^{-1}\Big\}_{x=y}, \tag{D.5}$$

where we take into account that $\delta\gamma/\delta\phi_i = -j^i$. Note that the first term in this expression vanishes, because

$$(-1)_i{}^i = \sum_{\alpha=1}^{N_f}\sum_{I=0}^{n}(-1)^{P_I} = N_f\Big(1-\sum_{I=1}^{n}c_I\Big) = 0 \tag{D.6}$$

due to the first equality in eq. (2.6). The third term in eq. (D.3) can be considered similarly. Thus, we obtain

$$\frac{\partial\gamma}{\partial\ln g} = \frac{1}{2}\int d^8x\,\Big\{-\frac{1}{2}(\phi^{*i}+\phi_0^{*i})(\phi_i+\phi_{0i})+(\phi_i+\phi_{0i})_x\frac{\delta\gamma}{\delta(\phi_{0i})_x}+(\phi^{*i}+\phi_0^{*i})_x\frac{\delta\gamma}{\delta(\phi_0^{*i})_x}$$
$$-\frac{i}{2}\Big(\frac{\delta^2\gamma}{\delta\phi_i\delta\phi^{*i}}\Big)^{-1} - iM^{j\,i}\Big(\frac{D^2}{16\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_j)_y}\Big)^{-1}_{y=x} - iM^*_{j\,i}\Big(\frac{\bar D^2}{16\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi^{*i})_x\delta(\phi^{*j})_y}\Big)^{-1}_{y=x}\Big\}. \tag{D.7}$$

D.2 Derivatives of effective vertices

Using the derivative of the Routhian γ with respect to $\ln g$ given by eq. (D.7) we can easily calculate the derivatives of various Green functions. For example,

$$\frac{\partial}{\partial\ln g}\frac{\delta^2\gamma}{\delta(\phi_{0i})_y\delta(\phi_{0j})_z} = \frac{\delta^2}{\delta(\phi_{0i})_y\delta(\phi_{0j})_z}\frac{\partial\gamma}{\partial\ln g}. \tag{D.8}$$

Substituting $\delta\gamma/\delta\ln g$ from eq. (D.7), differentiating, and then setting all fields to 0, we obtain

$$\frac{\partial}{\partial\ln g}\frac{\delta^2\gamma}{\delta(\phi_{0i})_y\delta(\phi_{0j})_z} = \frac{\delta^2\gamma}{\delta(\phi_{0i})_y\delta(\phi_{0j})_z} + \frac{i}{4}\int d^8x\,\Big(\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^k}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j_k^*} + M^{lk}\Big(\frac{D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^k}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^l}$$
$$+M^*_{lk}\Big(\frac{\bar D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j_k^*}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j_l^*}\Big)_x\cdot\frac{\delta^2\gamma}{\delta(\phi_{0i})_y\delta(\phi_{0j})_z}. \tag{D.9}$$

Similar expressions can be written for the derivatives of the other two-point functions. In order to rewrite the result in a more compact form, we use the notation (3.29). Then the derivatives of the two-point functions can be written as

$$\frac{\partial}{\partial\ln g}\Big(g^{-1}\frac{\delta^2\gamma}{\delta(\phi_{0i})_y\delta(\phi_{0j})_z}\Big) = \frac{1}{2g}\,\text{LineWithDot}[1]\cdot\frac{\delta^2\gamma}{\delta(\phi_{0i})_y\delta(\phi_{0j})_z};$$
$$\frac{\partial}{\partial\ln g}\Big(g^{-1}\frac{\delta^2\gamma}{\delta(\phi_{0i})_y\delta(\phi_0^{*j})_z}\Big) = -\frac{1}{4g}\delta^8_{yz}\delta^i_j + \frac{1}{2g}\,\text{LineWithDot}[1]\cdot\frac{\delta^2\gamma}{\delta(\phi_{0i})_y\delta(\phi_0^{*j})_z}. \qquad \text{(D.10)}$$

(The second equality is derived using the same method.) Derivatives of the other two-point functions can be written in a similar form.

In order to calculate the derivatives of the four-point functions we again use this method. It is convenient to introduce the notation

$$\text{UsualLine}[1,2] \equiv \int d^8x\,\Big(\frac{\delta}{\delta_2\phi_{0i}}\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^i} + \frac{\delta}{\delta_2\phi_0^{*i}}\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j_i^*}\Big)_x = \text{UsualLine}[2,1]. \qquad \text{(D.11)}$$

Then it is possible to write the derivatives of the four-point functions in a rather simple form. For example, taking into account that any Green function with an odd number of legs corresponding to the matter superfields vanishes, we obtain

$$\frac{\partial}{\partial\ln g}\Big(g^{-2}\frac{\delta^4\gamma}{\delta(\phi_{0i})_x\delta(\phi_{0j})_y\delta(\phi_{0k})_z\delta(\phi_{0l})_w}\Big) = \frac{1}{2g^2}\bigg(\text{LineWithDot}[1]\cdot \qquad \text{(D.12)}$$
$$\times\frac{\delta^4\gamma}{\delta(\phi_{0i})_x\delta(\phi_{0j})_y\delta(\phi_{0k})_z\delta(\phi_{0l})_w} + 2\cdot\text{LineWithDot}[1;1,2]\cdot\text{UsualLine}[1,2]$$
$$\times\Big(\frac{\delta^2\gamma[1]}{\delta(\phi_{0i})_x\delta(\phi_{0j})_y}\frac{\delta^2\gamma[2]}{\delta(\phi_{0k})_z\delta(\phi_{0l})_w} + \frac{\delta^2\gamma[1]}{\delta(\phi_{0i})_x\delta(\phi_{0k})_z}\cdot\frac{\delta^2\gamma[2]}{\delta(\phi_{0j})_y\delta(\phi_{0l})_w}\cdot(-1)^{P_jP_k}$$
$$+\frac{\delta^2\gamma[1]}{\delta(\phi_{0j})_y\delta(\phi_{0k})_z}\cdot\frac{\delta^2\gamma[2]}{\delta(\phi_{0i})_x\delta(\phi_{0l})_w}\cdot(-1)^{P_iP_j+P_iP_k}\Big)\bigg).$$

In this expression the derivatives $\boldsymbol{\delta}/\boldsymbol{\delta}_1 j$ act on $\gamma[1]$, and the derivatives $\boldsymbol{\delta}/\boldsymbol{\delta}_2 j$ act on $\gamma[2]$. The derivatives of the other four-point functions with respect to $\ln g$ can be written in a similar form.

D.3 Derivatives of effective lines

Expressions for the derivatives of effective vertices obtained in the previous section allow to find derivatives of effective diagrams. The effective diagrams include effective lines. Therefore, it is desirable to find derivatives of these effective lines, which, in particular, contain inverse Green functions inside the derivatives $\boldsymbol{\delta}/\boldsymbol{\delta}j$. The derivatives of the inverse Green functions can be easily calculated using eq. (D.10):

$$\frac{\partial}{\partial\ln g}\bigg(g\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_j)_y}\Big)^{-1}\bigg) = g\int d^8z\,\bigg[M^{lk}\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_k)_z}\Big)^{-1}\Big(\frac{D^2}{16\partial^2}\Big)_z\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_z\delta(\phi_j)_y}\Big)^{-1}$$
$$+M^*_{lk}\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*k})_z}\Big)^{-1}\Big(\frac{\bar D^2}{16\partial^2}\Big)_z\Big(\frac{\delta^2\gamma}{\delta(\phi^{*l})_z\delta(\phi_j)_y}\Big)^{-1} + \frac{1}{4}\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_k)_z}\Big)^{-1}\Big(\frac{\delta^2\gamma}{\delta(\phi^{*k})_z\delta(\phi_j)_y}\Big)^{-1}$$
$$+\frac{1}{4}\Big(\frac{\delta^2\gamma}{\delta(\phi^{*k})_z\delta(\phi_i)_x}\Big)^{-1}\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_z\delta(\phi_j)_y}\Big)^{-1}\bigg] - \frac{g}{2}\text{LineWithDot}[1]\cdot\frac{\boldsymbol{\delta}^2\gamma}{\boldsymbol{\delta}(j^j)_y\boldsymbol{\delta}(j^i)_x}. \qquad \text{(D.13)}$$

Similarly,

$$\frac{\partial}{\partial\ln g}\left(g\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*j})_y}\Big)^{-1}\right) = g\int d^8z\,\Bigg[M^{lk}\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_k)_z}\Big)^{-1}\Big(\frac{D^2}{16\partial^2}\Big)_z\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_z\delta(\phi^{*j})_y}\Big)^{-1}$$

$$+M^*_{lk}\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*k})_z}\Big)^{-1}\Big(\frac{\bar D^2}{16\partial^2}\Big)_z\Big(\frac{\delta^2\gamma}{\delta(\phi^{*l})_z\delta(\phi^{*j})_y}\Big)^{-1}+\frac{1}{4}\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_k)_z}\Big)^{-1}\Big(\frac{\delta^2\gamma}{\delta(\phi^{*k})_z\delta(\phi^{*j})_y}\Big)^{-1}$$

$$+\frac{1}{4}\Big(\frac{\delta^2\gamma}{\delta(\phi^{*k})_z\delta(\phi_i)_x}\Big)^{-1}\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_z\delta(\phi^{*j})_y}\Big)^{-1}\Bigg] - \frac{g}{2}\text{LineWithDot}[1]\cdot\frac{\boldsymbol{\delta}^2\gamma}{\boldsymbol{\delta}(j^*_j)_y\boldsymbol{\delta}(j^i)_x}. \qquad \text{(D.14)}$$

From these equations we obtain

$$\Big[\frac{\partial}{\partial\ln g},\, g\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^i)_x}\Big] = -\int d^8y\,\Bigg(\frac{\partial}{\partial\ln g}\Big[g\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_j)_y}\Big)^{-1}\Big]\frac{\delta}{\delta(\phi_{0j})_y}$$

$$+\frac{\partial}{\partial\ln g}\Big[g\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*j})_y}\Big)^{-1}\Big]\frac{\delta}{\delta(\phi_0^{*j})_y}\Bigg) = \frac{g}{4}\int d^8z\,\Bigg\{\Big(2\cdot\text{LineWithDot}[1]\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^i)_x}\frac{\delta\gamma}{\delta(\phi_{0k})_z}$$

$$+\Big(\frac{\delta^2\gamma}{\delta(\phi^{*k})_z\delta(\phi_i)_x}\Big)^{-1}+M^{kl}\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_l)_z}\Big)^{-1}\frac{D^2}{4\partial^2}\Big)\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^k)_z}+\Big(2\cdot\text{LineWithDot}[1]\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^i)_x}$$

$$\times\frac{\delta\gamma}{\delta(\phi_0^{*k})_z}+\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_k)_z}\Big)^{-1}+M^*_{kl}\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*l})_z}\Big)^{-1}\frac{\bar D^2}{4\partial^2}\Big)\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^*_k)_z}\Bigg\}. \qquad \text{(D.15)}$$

We can use this equation for differentiating the operator GreenLine (or other similar operators) with respect to $\ln g$. Let us consider, for the definiteness, the derivative of the green effective line. More exactly, let us differentiate

$$g\cdot\text{GreenLine}[1,2] = \int d^8x\,\Bigg\{g(T)^j{}_i\Big(\theta^a\theta_a\bar\theta^{\dot b}\frac{\bar D_{\dot b}D^2}{4\partial^2}+2i\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\frac{\partial_\mu}{\partial^2}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^j}\cdot\frac{\delta}{\delta_1\phi_{0i}}$$

$$-i\,g^{-1}g(MT)^{ij}\,\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\Big(\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^i}\Big)\cdot g\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1 j^j}\Bigg\}_x. \qquad \text{(D.16)}$$

It is convenient to write the result in the following form:

$$\Big[\frac{\partial}{\partial\ln g},\, g\cdot\text{GreenLine}[1,2]\Big] = \frac{g}{2}\Big\{\text{GreenWhiteLine}[1,2]+\text{GreenWithCrossWhite}[2,1]$$

$$-2\cdot\text{GreenWithCross}[1,2]+\text{LineWithDot}[1;3,3]\cdot\Big(\text{GreenLine}[1,3]\cdot\text{UsualLine}[2,3]$$

$$+\text{GreenWithCross}[2,3]\cdot\text{UsualLine}[1,3]\Big)\cdot\gamma[3]\Big\}. \qquad \text{(D.17)}$$

Let us briefly explain the derivation of this identity. In this expression all terms without the operator LineWithDot[1] are included into the effective lines

$$\text{GreenWhiteLine}[1,2] \equiv \frac{1}{2}\int d^8x\,d^8z\,\Bigg\{(T)^j{}_i\Big(\theta^a\theta_a\bar\theta^{\dot b}\frac{\bar D_{\dot b}D^2}{4\partial^2}+2i\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\frac{\partial_\mu}{\partial^2}\Big)_x$$

$$\times\Bigg(\Big(\Big(\frac{\delta^2\gamma}{\delta(\phi^{*k})_z\delta(\phi_j)_x}\Big)^{-1}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2(j^k)_z}+\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x\delta(\phi_k)_z}\Big)^{-1}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2(j^*_k)_z}+M^{lk}\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x\delta(\phi_k)_z}\Big)^{-1}$$

$$\times\frac{D^2}{4\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2(j^l)_z}+M^*_{lk}\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x\delta(\phi^{*k})_z}\Big)^{-1}\frac{\bar{D}^2}{4\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2(j^*_l)_z}\bigg)\frac{\delta}{\delta_1(\phi_{0i})_x}-i(MT)^{ij}\bar{\theta}^{\dot{a}}(\gamma^\mu)_{\dot{a}}{}^b\theta_b$$
$$\times\Big(\frac{D^2\partial_\mu}{16\partial^4}\Big)_x\bigg(\Big(\frac{\delta^2\gamma}{\delta(\phi^{*k})_z\delta(\phi_i)_x}\Big)^{-1}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2(j^k)_z}+\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_k)_z}\Big)^{-1}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2(j^*_k)_z}$$
$$+M^{lk}\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_k)_z}\Big)^{-1}\frac{D^2}{4\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2(j^l)_z}+M^*_{lk}\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*k})_z}\Big)^{-1}\frac{\bar{D}^2}{4\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2(j^*_l)_z}\bigg)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1(j^j)_x}\bigg\}\tag{D.18}$$

and

$$\text{GreenWithCrossWhite}[1,2]\equiv-\frac{i}{2}\int d^8x\,d^8z\,\bigg\{(MT)^{ij}\bar{\theta}^{\dot{a}}(\gamma^\mu)_{\dot{a}}{}^b\theta_b\Big(\frac{D^2\partial_\mu}{16\partial^4}\Big)_x$$
$$\times\bigg(\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*k})_z}\Big)^{-1}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2(j^k)_z}+\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_z\delta(\phi_i)_x}\Big)^{-1}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2(j^*_k)_z}+M^{kl}\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_k)_z}\Big)^{-1}$$
$$\times\frac{D^2}{4\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2(j^l)_z}+M^*_{kl}\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*k})_z}\Big)^{-1}\frac{\bar{D}^2}{4\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2(j^*_l)_z}\bigg)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1(j^j)_x}\bigg\}.\tag{D.19}$$

These expressions can be written in a more compact form, eqs. (3.30) and (3.32), respectively. The term with the operator GreenWithCross appears when we differentiate the factor g^{-1} in the mass term. The other terms containing the operator LineWithDot are obtained from the derivatives of $g\,\boldsymbol{\delta}/\boldsymbol{\delta}j$. However, it is necessary to take into account that the terms without masses are linear in this derivative, while the terms containing the masses are quadratic. That is why the result contains the sum of the operators GreenLine and GreenWithCross.

Derivatives of the other effective lines can be constructed similarly.

D.4 Derivative of Δ

The derivative of the additional contribution Δ, given by eq. (3.21), with respect to $\ln g$ can be calculated using eq. (D.13). The result is

$$\frac{\partial\Delta}{\partial\ln g}=-i\frac{d}{d\ln\Lambda}\int d^8x\,d^8y\,(\theta^4)_x\bigg\{\bigg(C(R)_k{}^iM^{jk}\delta^8_{xy}\Big(\frac{D^2}{4\partial^4}\Big)_x-(MT)^{im}(MT)^{nj}\Big(\frac{D^2\partial^\mu}{4\partial^4}\Big)_y$$
$$\times\Big(\frac{\delta^2\gamma}{\delta(\phi_m)_x\delta(\phi_n)_y}\Big)^{-1}\Big(\frac{D^2\partial_\mu}{16\partial^4}\Big)_x\bigg)\bigg(-\frac{1}{2}\text{LineWithDot}[1]\cdot\frac{\boldsymbol{\delta}^2\gamma}{\boldsymbol{\delta}(j^j)_y\boldsymbol{\delta}(j^i)_x}+\int d^8z$$
$$\times\bigg[\frac{1}{4}\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_k)_z}\Big)^{-1}\Big(\frac{\delta^2\gamma}{\delta(\phi^{*k})_z\delta(\phi_j)_y}\Big)^{-1}+\frac{1}{4}\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*k})_z}\Big)^{-1}\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_y\delta(\phi_k)_z}\Big)^{-1}$$
$$+M^{lk}\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_k)_z}\Big)^{-1}\Big(\frac{D^2}{16\partial^2}\Big)_z\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_z\delta(\phi_j)_y}\Big)^{-1}+M^*_{lk}\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*k})_z}\Big)^{-1}\Big(\frac{\bar{D}^2}{16\partial^2}\Big)_z$$
$$\times\Big(\frac{\delta^2\gamma}{\delta(\phi^{*l})_z\delta(\phi_j)_y}\Big)^{-1}\bigg]-\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_j)_y}\Big)^{-1}\bigg)\bigg\}.\tag{D.20}$$

The terms containing the operator LineWithDot[1] can be graphically presented as two-loop effective diagrams. The other terms correspond to one-loop effective diagrams.

E Calculation of commutators

E.1 Commutators with $(T)y_\mu^*$

In this section we calculate commutators of $(T)y_\mu^*$ with various Green functions. For this purpose we can use the Schwinger-Dyson equation (3.4), which is valid for any values of g. Let us multiply this equation by y_μ^* and integrate the result over d^8x:

$$\int d^8x\,(y_\mu^*)_x \frac{\delta(\Delta\Gamma)}{\delta \boldsymbol{V}_x} = \int d^8x\,(y_\mu^*)_x (T)^j{}_i \Big(\frac{2}{i}\frac{\delta}{\delta(j_i^*)_x}\frac{\delta\gamma}{\delta(\phi_0^{*j})_x} + 2\frac{\delta\gamma}{\delta(\phi_0^{*j})_x}(\phi^{*i}+\phi_0^{i*})_x\Big). \tag{E.1}$$

The first term in this expression vanishes. Really, using antichirality of the derivative with respect to j^* and integrating by parts this term can be rewritten as

$$\frac{2}{i}\int d^8x\,(y_\mu^*)_x (T)^j{}_i \Big(-\frac{D^2\bar{D}^2}{16\partial^2}\frac{\delta}{\delta j_i^*}\Big)_x \frac{\delta\gamma}{\delta(\phi_0^{*j})_x} = \frac{2}{i}\int d^8x\,(y_\mu^*)_x (T)^j{}_i \Big(-\frac{\bar{D}^2}{16\partial^2}\frac{\delta}{\delta j_i^*}\Big)_x D_x^2 \frac{\delta\gamma}{\delta(\phi_0^{*j})_x}. \tag{E.2}$$

Taking into account eq. (3.1) we obtain

$$\begin{aligned}&\frac{2}{i}\int d^8x\,(y_\mu^*)_x (T)^j{}_i \Big(\frac{\bar{D}^2}{8\partial^2}\frac{\delta}{\delta j_i^*}\Big)_x \Big(-j_j^* - \frac{1}{2}M^*_{jk}\phi^{*k}\Big)_x = \\ &i(TM^*)_{ik}\int d^8x\,(y_\mu^*)_x \Big(\frac{\bar{D}^2}{8\partial^2}\Big)_x \Big(\frac{D^2\bar{D}^2}{16\partial^2}\Big)_y \Big(\frac{\delta^2\gamma}{\delta(\phi^{*k})_y\delta(\phi^{*i})_x}\Big)^{-1}_{x=y} = 0.\end{aligned} \tag{E.3}$$

(The term with the derivative of j^* vanishes due to the evident identity $T^i{}_i = 0$. The last equality can be obtained by integrating D^2 by parts and taking into account eq. (2.13).) Note that the fields here are not yet set to 0. From the other side,

$$\int d^8x\,(y_\mu^*)_x \frac{\delta(\Delta\Gamma)}{\delta \boldsymbol{V}_x} = \int d^8x\,(y_\mu^*)_x (T)^j{}_i \Big(\frac{2}{i}\frac{\delta}{\delta(j^j)_x}\frac{\delta\gamma}{\delta(\phi_{0i})_x} + 2\frac{\delta\gamma}{\delta(\phi_{0i})_x}(\phi_j+\phi_{0j})_x\Big). \tag{E.4}$$

Comparing eq. (E.1) and eq. (E.4) we obtain

$$\begin{aligned}&\int d^8x\,(y_\mu^*)_x (T)_i{}^j \Big((\phi^{i*}+\phi_0^{i*})_x\frac{\delta\gamma}{\delta(\phi_0^{*j})_x} - (\phi_j+\phi_{0j})_x\frac{\delta\gamma}{\delta(\phi_{0i})_x}\Big)\\ &\quad = -i\int d^8x\,(y_\mu^*)_x (T)^j{}_i \Big(\frac{\delta}{\delta(j^j)_x}\frac{\delta\gamma}{\delta(\phi_{0i})_x}\Big).\end{aligned} \tag{E.5}$$

Differentiating eq. (E.5) with respect to various fields, it is possible to find

$$\begin{aligned}&\Big[(T)y_\mu^*, \frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_0^{*l})_z}\Big] \equiv -(y_\mu^*)_y(T)_i{}^k\frac{\delta^2\gamma}{\delta(\phi_{0i})_y\delta(\phi_0^{*l})_z} + (y_\mu^*)_z(T)_l{}^i\frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_0^{*i})_z};\\ &\Big[(T)y_\mu^*, \frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_{0l})_z}\Big] \equiv -(y_\mu^*)_y(T)_i{}^k\frac{\delta^2\gamma}{\delta(\phi_{0i})_y\delta(\phi_{0l})_z} - (y_\mu^*)_z(T)_i{}^l\frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_{0i})_z}\end{aligned} \tag{E.6}$$

and other similar expressions. (All fields in this commutators are set to 0.) For example, differentiating eq. (E.5) with respect to $(\phi_{0k})_y$ and $(\phi_{0l})_z$ and setting all fields to 0 we obtain

$$\Big[(T)y^*_\mu, \frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_{0l})_z}\Big] = -i\int d^8x\,(y^*_\mu)_x(T)^j{}_i\frac{\delta^2}{\delta(\phi_{0k})_y\delta(\phi_{0l})_z}\cdot\frac{\delta}{\delta(j^j)_x}\frac{\delta\gamma}{\delta(\phi_{0i})_x}. \tag{E.7}$$

Commuting the derivative with respect to the source j^j with the derivatives with respect to the fields ϕ_0 and taking into account that all Green functions with an odd number of ϕ-lines vanish, this expression can be presented in the form

$$\begin{aligned}
-i\int d^8x\,d^8w\,(y^*_\mu)_x(T)^j{}_i\Bigg[&\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_x}\frac{\delta^3\gamma}{\delta(\phi_{0k})_y\delta(\phi_{0l})_z\delta(\phi_{0m})_w}\cdot\Big(\delta^8_{xw}\delta^i_m+\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^m)_w}\frac{\delta\gamma}{\delta(\phi_{0i})_x}\Big)\\
&+\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_x}\frac{\delta^3\gamma}{\delta(\phi_{0k})_y\delta(\phi_{0l})_z\delta(\phi_0^{*m})_w}\cdot\Big(\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^*_m)_w}\frac{\delta\gamma}{(\phi_{0i})_x}\Big)\Bigg].
\end{aligned} \tag{E.8}$$

After substituting the two-point Green functions from eq. (B.2) the result is written as

$$\begin{aligned}
-i\int d^8x\,\Bigg[&(T)^j{}_i\Big(\Big(1+\frac{\bar D^2D^2}{16\partial^2}\Big)y^*_\mu\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}\Big)_x\frac{\delta^3\gamma}{\delta(\phi_{0l})_z\delta(\phi_{0k})_y\delta(\phi_{0i})_x}\\
&-(MT)^{ij}(y^*_\mu)_x\Big(\frac{D^2}{16\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^i}\Big)_x\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^j)_x}\frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_{0l})_z}\Bigg].
\end{aligned} \tag{E.9}$$

In order to simplify this expression we note that

$$\Big(1+\frac{\bar D^2D^2}{16\partial^2}\Big)y^*_\mu\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j} = \Big(\frac{2\partial_\mu}{\partial^2}-i(\gamma^\mu)^{\dot a b}\theta_b\frac{\bar D_{\dot a}D^2}{4\partial^2}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j} \tag{E.10}$$

and

$$\begin{aligned}
&\int d^8x\,(MT)^{ij}y^*_\mu\frac{D^2}{16\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^i}\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j} = -\int d^8x\,(MT)^{ij}y^*_\mu\frac{D^2}{16\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^i}\cdot\frac{\bar D^2D^2}{16\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j} = -\int d^8x\\
&\times(MT)^{ij}\bar D^{\dot a}y^*_\mu\frac{\bar D_{\dot a}D^2}{16\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^i}\cdot\frac{D^2}{16\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j} = 8\int d^8x\,(MT)^{ij}\theta^a\,(\gamma_\mu\gamma^\nu)_a{}^b\frac{D_b\partial_\nu}{16\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^i}\cdot\frac{D^2}{16\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}\\
&=\int d^8x\,(MT)^{ij}\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^i}\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}.
\end{aligned} \tag{E.11}$$

Using these identities we finally obtain

$$\Big[(T)y^*_\mu, \frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_{0l})_z}\Big] = \text{BrownLine}_\mu\cdot\frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_{0l})_z}, \tag{E.12}$$

where we have defined the operator

$$\begin{aligned}
\text{BrownLine}_\mu[1,2] \equiv \int d^8x\,\Big[&(T)_i{}^j\Big(\Big(-\frac{2i\partial_\mu}{\partial^2}-(\gamma^\mu)^{\dot a b}\theta_b\frac{\bar D_{\dot a}D^2}{4\partial^2}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^j}\Big)_x\frac{\delta}{\delta_1(\phi_{0i})_x}\\
&+i(MT)^{ij}\Big(\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^i}\Big)_x\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_1(j^j)_x}\Big].
\end{aligned} \tag{E.13}$$

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Similarly, we can differentiate eq. (E.5) with respect to $(\phi_{0k})_y$ and $(\phi_0^{*l})_z$ and set all fields equal to 0. Then, repeating the same operations as above, we find

$$\Big[(T)y_\mu^*, \frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_0^{*l})_z}\Big] = \text{BrownLine}_\mu\cdot\frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_0^{*l})_z}. \tag{E.14}$$

The commutators of $(T)y_\mu^*$ with four-point Green functions can be calculated by the same method. Differentiating eq. (E.5) with respect to $(\phi_{0i})_x$, $(\phi_{0j})_y$, $(\phi_{0k})_z$, and $(\phi_{0l})_w$ and setting all fields equal to 0, we obtain

$$\begin{aligned}&\Big[(T)y_\mu^*, \frac{\delta^4\gamma}{\delta(\phi_{0i})_x\delta(\phi_{0j})_y\delta(\phi_{0k})_z\delta(\phi_{0l})_w}\Big] = \text{BrownLine}_\mu\cdot\frac{\delta^4\gamma}{\delta(\phi_{0i})_x\delta(\phi_{j0})_y\delta(\phi_{k0})_z\delta(\phi_{l0})_w}\\
&+\text{BrownLine}_\mu[1,2]\cdot\text{UsualLine}[1,2]\cdot\bigg(\frac{\delta^2\gamma[1]}{\delta(\phi_{0i})_x\delta(\phi_{0j})_y}\cdot\frac{\delta^2\gamma[2]}{\delta(\phi_{0k})_z\delta(\phi_{0l})_w}+\frac{\delta^2\gamma[1]}{\delta(\phi_{0i})_x\delta(\phi_{0k})_z}\\
&\times\frac{\delta^2\gamma[2]}{\delta(\phi_{0j})_y\delta(\phi_{0l})_w}(-1)^{P_jP_k}+\frac{\delta^2\gamma[1]}{\delta(\phi_{0i})_x\delta(\phi_{0l})_w}\cdot\frac{\delta^2\gamma[2]}{\delta(\phi_{0j})_y\delta(\phi_{0k})_z}(-1)^{P_jP_l+P_kP_l}+\Big([1]\leftrightarrow[2]\Big)\bigg).\end{aligned} \tag{E.15}$$

Similar identities can be written for the derivatives with respect to ϕ_0^*. In this case in the left hand side for each ϕ_0^* it is necessary to replace $T_i{}^j$ by $-(T)_j{}^i$.

E.2 Commutators with $(T)\bar\theta$

Commutators with $(T)\bar\theta$ can be calculated by the same method as the commutators with $(T)y_\mu^*$. Exactly as in the previous section we obtain

$$\begin{aligned}\int d^8x\,(\bar\theta^{\dot a})_x(T)_i{}^j\Big((\phi^{i*}+\phi_0^{i*})_x\frac{\delta\gamma}{\delta(\phi_0^{*j})_x}-(\phi_j+\phi_{0j})_x\frac{\delta\gamma}{\delta(\phi_{0i})_x}\Big)&\\
=-i\int d^8x\,(\bar\theta^{\dot a})_x(T)^j{}_i\Big(\frac{\delta}{\delta(j^j)_x}\frac{\delta\gamma}{\delta(\phi_{0i})_x}\Big).&\end{aligned} \tag{E.16}$$

Differentiating this equation we obtain commutators with various Green functions. For example, the commutator with the two-point Green function can be written as

$$\begin{aligned}\Big[(T)\bar\theta^{\dot a}, \frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_{0l})_z}\Big] = -i\int d^8x\,\bigg[(T)_i{}^j\Big(\Big(1+\frac{\bar D^2D^2}{16\partial^2}\Big)\bar\theta^{\dot a}\frac{\boldsymbol\delta}{\boldsymbol\delta j^j}\Big)_x\frac{\delta^3\gamma}{\delta(\phi_{0k})_y\delta(\phi_{0l})_z\delta(\phi_{0i})_x}&\\
-(MT)^{ij}(\bar\theta^{\dot a})_x\Big(\frac{D^2}{16\partial^2}\frac{\boldsymbol\delta}{\boldsymbol\delta j^i}\Big)_x\frac{\boldsymbol\delta}{\boldsymbol\delta(j^j)_x}\frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_{0l})_z}\bigg].&\end{aligned} \tag{E.17}$$

This equation can be simplified using the relations

$$\begin{aligned}\Big(1+\frac{\bar D^2D^2}{16\partial^2}\Big)\bar\theta^{\dot a}\frac{\boldsymbol\delta}{\boldsymbol\delta j^j} &= \frac{\bar D^{\dot a}D^2}{8\partial^2}\frac{\boldsymbol\delta}{\boldsymbol\delta j^j};\\
\int d^8x\,(MT)^{ij}\,\bar\theta^{\dot a}\,\frac{D^2}{16\partial^2}\frac{\boldsymbol\delta}{\boldsymbol\delta j^i}\cdot\frac{\boldsymbol\delta}{\boldsymbol\delta j^j} &= \int d^8x\,(MT)^{ij}\,\bar\theta^{\dot a}\,\frac{\boldsymbol\delta}{\boldsymbol\delta j^i}\cdot\frac{D^2}{16\partial^2}\frac{\boldsymbol\delta}{\boldsymbol\delta j^j}=0.\end{aligned} \tag{E.18}$$

(The last equality follows from eq. (2.13).) Then we obtain

$$\Big[(T)\bar\theta^{\dot a},\ \frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_{0l})_z}\Big] = -i\cdot \text{BlueLine}^{\dot a}[1]\cdot\frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_{0l})_z}, \tag{E.19}$$

where we use the notation

$$\text{BlueLine}^{\dot a}[\alpha;1,2] = \int d^8x\,(\alpha)_x (T)^j{}_i\Big(\frac{\bar D^{\dot a}D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^j}\cdot\frac{\delta}{\delta_1\phi_{0i}}\Big)_x. \tag{E.20}$$

As always, the subscripts 1 and 2 denote end points of the line. If these points coincide, we sometimes for simplicity omit these indexes. Other commutators can be found similarly. For example,

$$\Big[(T)\bar\theta^{\dot a},\ \frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_0^{*l})_z}\Big] = -i\cdot \text{BlueLine}^{\dot a}[1]\cdot\frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_0^{*l})_z} \tag{E.21}$$

for the two-point function, or

$$\begin{aligned}&\Big[(T)\bar\theta^{\dot a},\ \frac{\delta^4\gamma}{\delta(\phi_{0i})_x\delta(\phi_{0j})_y\delta\phi_{0k})_z\delta(\phi_{0l})_w}\Big] = -i\cdot \text{BlueLine}^{\dot a}[1]\cdot\frac{\delta^4\gamma}{\delta(\phi_{0i})_x\delta(\phi_{j0})_y\delta(\phi_{k0})_z\delta(\phi_{l0})_w}\\
&-i\cdot\text{BlueLine}^{\dot a}[1;1,2]\cdot\text{UsualLine}[1,2]\cdot\Bigg(\frac{\delta^2\gamma[1]}{\delta(\phi_{0i})_x\delta(\phi_{0j})_y}\cdot\frac{\delta^2\gamma[2]}{\delta(\phi_{0k})_z\delta(\phi_{0l})_w}+\frac{\delta^2\gamma[1]}{\delta(\phi_{0i})_x\delta(\phi_{0k})_z}\\
&\times\frac{\delta^2\gamma[2]}{\delta(\phi_{0j})_y\delta(\phi_{0l})_w}(-1)^{P_jP_k}+\frac{\delta^2\gamma[1]}{\delta(\phi_{0i})_x\delta(\phi_{0l})_w}\cdot\frac{\delta^2\gamma[2]}{\delta(\phi_{0j})_y\delta(\phi_{0k})_z}(-1)^{P_jP_l+P_kP_l}+\Big([1]\leftrightarrow[2]\Big)\Bigg)
\end{aligned}\tag{E.22}$$

for the four-point function.

E.3 Commutators with propagators

In order to calculate commutators of $(T)y^*_\mu$ or $(T)\bar\theta$ with various Feynman diagrams according to the prescription (4.19), it is necessary to commute $(T)y^*_\mu$ or $(T)\bar\theta$ with inverse Green functions. Here we demonstrate, how this can be made. As a starting point we consider the identities (B.2), which can be presented in the following matrix form:

$$\begin{aligned}&\int d^8z\begin{pmatrix}\frac{\delta^2\gamma}{\delta(\phi_0^{*i})_x\delta(\phi_{0k})_z} & \frac{\delta^2\gamma}{\delta(\phi_0^{*i})_x\delta(\phi_0^{*k})_z}+M^*_{ki}\frac{\bar D^2}{16\partial^2}\delta^8_{xz}\\ \frac{\delta^2\gamma}{\delta(\phi_{0i})_x\delta(\phi_{0k})_z}+M^{ki}\frac{D^2}{16\partial^2}\delta^8_{xz} & \frac{\delta^2\gamma}{\delta(\phi_{0i})_x\delta(\phi_0^{*k})_z}\end{pmatrix}\\
&\times\begin{pmatrix}\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_z\delta(\phi^{*j})_y}\Big)^{-1} & \Big(\frac{\delta^2\gamma}{\delta(\phi_k)_z\delta(\phi_j)_y}\Big)^{-1}\\ \Big(\frac{\delta^2\gamma}{\delta(\phi^{*k})_z\delta(\phi^{*j})_y}\Big)^{-1} & \Big(\frac{\delta^2\gamma}{\delta(\phi^{*k})_z\delta(\phi_j)_y}\Big)^{-1}\end{pmatrix}=\begin{pmatrix}-\delta_i^j\Big(\frac{\bar D^2D^2}{16\partial^2}\Big)_x & 0\\ 0 & -\delta^i_j\Big(\frac{D^2\bar D^2}{16\partial^2}\Big)_x\end{pmatrix}\delta^8_{xy}.
\end{aligned}\tag{E.23}$$

Let us commute this equation with $(T)y^*_\mu$ or $(T)\bar\theta^a$, taking into account that the commutators in the right hand side do not vanish. We use the identities

$$\Big[y^*_\mu,\frac{\bar D^2 D^2}{16\partial^2}\Big]=\frac{\bar D^2 D^2\partial_\mu}{8\partial^4}+i(\gamma_\mu)^{a\dot b}\theta_a\frac{\bar D_{\dot b}D^2}{4\partial^2};\qquad \Big[\bar\theta^{\dot a},\frac{\bar D^2 D^2}{16\partial^2}\Big]=-\frac{\bar D^{\dot a}D^2}{8\partial^2};$$
$$D^2\Big[y^*_\mu,\frac{\bar D^2}{16\partial^2}\Big]D^2=-[y^*_\mu,D^2]=0;\qquad D^2\Big[\bar\theta^{\dot a},\frac{\bar D^2}{16\partial^2}\Big]D^2=0.\tag{E.24}$$

For example, let us consider the commutator

$$\begin{aligned}&\Big[(T)y^*_\mu,\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*j})_y}\Big)^{-1}\Big]\\
&\quad\equiv(y^*_\mu)_x(T)_i{}^m\Big(\frac{\delta^2\gamma}{\delta(\phi_m)_x\delta(\phi^{*j})_y}\Big)^{-1}-\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*m})_y}\Big)^{-1}(T)_m{}^j(y^*_\mu)_y.\end{aligned}\tag{E.25}$$

Commuting $(T)y^*_\mu$ with eq. (E.23) after some simple transformations we obtain

$$\begin{aligned}&\Big[(T)y^*_\mu,\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*j})_y}\Big)^{-1}\Big]=-\int d^8z\,d^8w\\
&\times\Big\{\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_k)_z}\Big)^{-1}\Big[(T)y^*_\mu,\frac{\delta^2\gamma}{\delta(\phi_{0k})_z\delta(\phi^{*l}_0)_w}\Big]\Big(\frac{\delta^2\gamma}{\delta(\phi^{*l})_w\delta(\phi^{*j})_y}\Big)^{-1}\\
&+\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_k)_z}\Big)^{-1}\Big[(T)y^*_\mu,\frac{\delta^2\gamma}{\delta(\phi_{0k})_z\delta(\phi_{0l})_w}\Big]\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_w\delta(\phi^{*j})_y}\Big)^{-1}\\
&+\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*k})_z}\Big)^{-1}\Big[(T)y^*_\mu,\frac{\delta^2\gamma}{\delta(\phi^{*k}_0)_z\delta(\phi^{*l}_0)_w}\Big]\Big(\frac{\delta^2\gamma}{\delta(\phi^{*l})_w\delta(\phi^{*j})_y}\Big)^{-1}\\
&+\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*k})_z}\Big)^{-1}\Big[(T)y^*_\mu,\frac{\delta^2\gamma}{\delta(\phi^{*k}_0)_z\delta(\phi_{0l})_w}\Big]\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_w\delta(\phi^{*j})_y}\Big)^{-1}\Big\}\\
&+(T)_i{}^m\Big(\frac{2\partial_\mu}{\partial^2}-i(\gamma_\mu)^{a\dot b}\theta_a\frac{\bar D_{\dot b}D^2}{4\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_m)_x\delta(\phi^{*j})_y}\Big)^{-1}\\
&+\int d^8z\,\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_k)_z}\Big)^{-1}(MT)^{lk}\Big(\frac{D^2\partial_\mu}{8\partial^4}\Big)_z\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_z\delta(\phi^{*j})_y}\Big)^{-1}.\end{aligned}\tag{E.26}$$

All commutators here can be calculated according to the prescription obtained in the previous section:

$$\begin{aligned}&\Big[(T)y^*_\mu,\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*j})_y}\Big)^{-1}\Big]=-\int d^8z\,d^8w\\
&\times\Big\{\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_k)_z}\Big)^{-1}\mbox{BrownLine}_\mu\cdot\frac{\delta^2\gamma}{\delta(\phi_{0k})_z\delta(\phi^{*l}_0)_w}\Big(\frac{\delta^2\gamma}{\delta(\phi^{*l})_w\delta(\phi^{*j})_y}\Big)^{-1}\\
&+\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_k)_z}\Big)^{-1}\mbox{BrownLine}_\mu\cdot\frac{\delta^2\gamma}{\delta(\phi_{0k})_z\delta(\phi_{0l})_w}\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_w\delta(\phi^{*j})_y}\Big)^{-1}\\
&+\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*k})_z}\Big)^{-1}\mbox{BrownLine}_\mu\cdot\frac{\delta^2\gamma}{\delta(\phi^{*k}_0)_z\delta(\phi^{*l}_0)_w}\Big(\frac{\delta^2\gamma}{\delta(\phi^{*l})_w\delta(\phi^{*j})_y}\Big)^{-1}\\
&+\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*k})_z}\Big)^{-1}\mbox{BrownLine}_\mu\cdot\frac{\delta^2\gamma}{\delta(\phi^{*k}_0)_z\delta(\phi_{0l})_w}\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_w\delta(\phi^{*j})_y}\Big)^{-1}\Big\}\end{aligned}$$

$$+(T)_i{}^m\Big(\frac{2\partial_\mu}{\partial^2}-i(\gamma_\mu)^{a\dot b}\theta_a\frac{\bar D_{\dot b}D^2}{4\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_m)_x\delta(\phi^{*j})_y}\Big)^{-1}$$
$$+\int d^8z\,\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_k)_z}\Big)^{-1}(MT)^{lk}\Big(\frac{D^2\partial_\mu}{8\partial^4}\Big)_z\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_z\delta(\phi^{*j})_y}\Big)^{-1}. \qquad (E.27)$$

* This expression can be written in a very compact form

$$\Big[(T)y^*_\mu,\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*j})_y}\Big)^{-1}\Big]=-\text{BrownLine}_\mu\cdot\Big(\frac{\boldsymbol{\delta}^2\gamma}{\boldsymbol{\delta}(j^*_j)_y\boldsymbol{\delta}(j^i)_x}+i(\phi_0^{*j})_y(\phi_{0i})_x\Big). \qquad (E.28)$$

(The fields here should be set to 0.) The other commutators can be considered similarly. Commutators with $(T)\bar\theta_{\dot a}$ are calculated by the exactly the same method. The result can be obtained by the substitution

$$\text{BrownLine}_\mu\to -i\cdot\text{BlueLine}_{\dot a}[1]. \qquad (E.29)$$

In particular, there are no terms containing the masses in the commutators with $(T)\bar\theta^{\dot a}$.

F Identities for effective lines

F.1 Proof of the identity (4.3)

In order to present the two-loop effective diagram with the yellow line presented in figure 9 (plus Δ_1) as an integral of a total derivative, we use the identity

$$2\cdot\text{YellowLine}_\mu[\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b]\cdot\text{GreenLine}\cdot\gamma=-\text{YellowLine}_\mu[\theta^4]\cdot\text{BrownLine}^\mu\cdot\gamma. \qquad (F.1)$$

In this section we prove this equality. Using the same method in the subsequent sections we prove more complicated identities relating the effective lines. First, we note that both sides of the considered equation are quadratic in $\bar\theta$. As a consequence, it is possible to shift $\bar\theta$ to an arbitrary point of a diagram, because the integral over d^8x does not vanish only if the integrand contains θ^4. Comparing definitions of the operators GreenLine and RedLine$_{\dot a}$ and taking into account the possibility of shifting $\bar\theta^{\dot a}$ we easily obtain (omitting unessential terms which do not contain $\bar\theta$)

$$4\cdot\text{GreenLine}[1,2]=(\bar\theta^{\dot c})_z\cdot\text{RedLine}_{\dot c}[1,2], \qquad (F.2)$$

where z is an arbitrary point of the considered supergraph. Therefore, taking into account that $\bar\theta^{\dot c}\bar\theta_{\dot a}=\delta^{\dot c}_{\dot a}\bar\theta^{\dot b}\bar\theta_{\dot b}/2$, we see that the equality (F.1) is equivalent to the identity

$$(\gamma^\mu)^{\dot ab}(\theta_b)_z\cdot\text{RedLine}_{\dot a}[1,2]=4(\theta^a\theta_a)_z\cdot\text{BrownLine}^\mu[1,2]+O(\theta). \qquad (F.3)$$

Let us compare terms quadratic in θ. In these terms we can make arbitrary shifts of θ, because terms proportional to the first degree of θ (denoted by $O(\theta)$) vanish after integrating over $d^4\theta$. Then the considered terms in the left hand side of eq. (F.3) can be written as

$$(\gamma_\mu)^{\dot ab}(\theta_b)_z\cdot\int d^8x(\gamma^\nu)_{\dot a}{}^c(\theta_c)_x\Big(8i(T)^j{}_i\frac{\partial_\nu}{\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}\frac{\delta}{\delta\phi_{0i}}-i(MT)^{ij}\frac{D^2\partial_\nu}{4\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^i}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}\Big)$$
$$=-4(\theta^b\theta_b)_z\cdot\int d^8x\Big(2i(T)^j{}_i\frac{\partial_\mu}{\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}\frac{\delta}{\delta\phi_{0i}}-i(MT)^{ij}\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^i}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}\Big)+O(\theta) \qquad (F.4)$$

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and coincide with the terms quadratic in θ in the right hand side of eq. (F.3). Terms proportional to the third degree of θ can be investigated similarly. In the left hand side such terms are given by

$$(\gamma^{\mu})^{\dot{a}b}(\theta_b)_z \cdot \int d^8x\,(T)^j{}_i\,(\theta^c\theta_c)_x \frac{\bar{D}_{\dot{a}}D^2}{\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^j}\frac{\delta}{\delta\phi_{0i}}. \tag{F.5}$$

In a Feynman graph the points z and x are connected by a sequence of vertices and propagators. This allows to write the left part of the above expression in the form ($P_A = 0$)

$$(\gamma^{\mu})^{\dot{a}b}\theta_b\,A\,\theta^c\theta_c = (\gamma^{\mu})^{\dot{a}b}[\theta_b,\,A]\,\theta^c\theta_c = (\gamma^{\mu})^{\dot{a}b}\theta^c\theta_c\,[\theta_b,\,A] + O(\theta) = -(\gamma^{\mu})^{\dot{a}b}\theta^c\theta_c\,A\theta_b + O(\theta), \tag{F.6}$$

where A is a differential operator, which does not explicitly depend on θ. (This operator encodes the sequence of vertices and propagators which connect the points z and x.) Similarly, for an arbitrary P_A

$$\theta_b\,A\,\theta^c\theta_c = -(-1)^{P_A}\theta^c\theta_c\,A\,\theta_b + O(\theta). \tag{F.7}$$

Terms $O(\theta)$ vanish after integration over $d^4\theta$. Omitting these terms we see that the last expression in eq. (F.6) corresponds to

$$-\,(\theta^c\theta_c)_z \cdot \int d^8x\,(T)^j{}_i\,(\gamma^{\mu})^{\dot{a}b}(\theta_b)_x \frac{\bar{D}_{\dot{a}}D^2}{\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta} j^j}\frac{\delta}{\delta\phi_{0i}} \tag{F.8}$$

and coincides with the terms cubic in θ in the right hand side of eq. (F.3). Thus, we have proved eq. (F.3) and eq. (F.1).

F.2 Auxiliary identities

In order to compare different groups of effective diagrams it is necessary to use some identities which relate various effective lines. All these identities follow from some simple commutators of θ^a with differential operators containing supersymmetric covariant derivatives and usual derivatives. In this subsection we prove some simple algebraic equalities which allow to relate various effective lines:

$$\theta_a AB\theta^b\theta_b + (-1)^{P_A+P_B}A\theta^b\theta_b B\theta_a - \theta_a A\theta^b\theta_b B = O(\theta); \tag{F.9}$$

$$\theta^b\theta_b AB\theta_a + (-1)^{P_A+P_B}\theta_a A\theta^b\theta_b B - A\theta^b\theta_b B\theta_a = O(\theta); \tag{F.10}$$

$$\theta^a\theta_a AB\theta^b\theta_b + 2(-1)^{P_A+P_B}\theta^a A\theta^b\theta_b B\theta_a - \theta^a\theta_a A\theta^b\theta_b B - A\theta^a\theta_a B\theta^b\theta_b = O(\theta), \tag{F.11}$$

where A and B are differential operators which do not explicitly depend on θ. Actually, these operators correspond to sequences of vertices and propagators in a Feynman graph connecting two fixed points.

In order to prove the first identity we rewrite its left hand side as

$$[\theta_a, A\}B\theta^b\theta_b + (-1)^{P_A}A[\theta_a, B\}\theta^b\theta_b + (-1)^{P_A+P_B}A\,\theta^b\theta_b[B,\theta_a\} - [\theta_a, A\}\theta^b\theta_b B. \tag{F.12}$$

Evidently, $[A, \theta_a\}$ and $[B, \theta_a\}$ do not explicitly depend on θ. Therefore, the whole expression is quadratic in θ, and shifts of $\theta^a\theta_a$ can change only the terms $O(\theta)$. As a consequence, the considered expression can be rewritten as

$$\theta^b\theta_b\Big([\theta_a, A\}B+(-1)^{P_A}A[\theta_a, B\}+(-1)^{P_A+P_B}A[B, \theta_a\}-[\theta_a, A\}B\Big)+O(\theta) = O(\theta). \tag{F.13}$$

The second and third identities can be proved similarly. For example, the left hand side of the third identity can be presented in the form

$$[\theta^a, [\theta_a, AB\}\}\theta^b\theta_b+2(-1)^{P_A+P_B}[\theta^a, A\}\theta^b\theta_b[B, \theta_a\}-[\theta^a, [\theta_a, A\}\}\theta^b\theta_b B-A[\theta^a, [\theta_a, B\}\}\theta^b\theta_b. \tag{F.14}$$

Again, shifts of $\theta^b\theta_b$ change only the terms $O(\theta)$ and this expression can be presented as

$$\theta^b\theta_b\Big([\theta^a, [\theta_a, A\}\}B + 2(-1)^{P_A}[\theta^a, A\}[\theta_a, B\} + A[\theta^a, [\theta_a, B\}\} - 2(-1)^{P_A}[\theta^a, A\}[\theta_a, B\}$$
$$-[\theta^a, [\theta_a, A\}\}B - A[\theta^a, [\theta_a, B\}\}\Big) + O(\theta) = O(\theta). \tag{F.15}$$

F.3 Proof of identity presented in figure 16

Using the identities (F.9)–(F.11) it is possible to prove the identity presented in figure 16, which can be written as

$$(\theta^4)_z\Big(\text{BlueLine}_{\dot b}[1;1,2]\cdot\text{PinkLine}^{\dot b}[3,4] + \text{PinkLine}_{\dot b}[1,2]\cdot\text{BlueLine}^{\dot b}[1;3,4]$$
$$-(\gamma^\mu)^{a\dot b}\text{BlueLine}_{\dot b}[\theta_a;1,2]\cdot\text{BrownLine}_\mu[3,4] - (\gamma^\mu)^{a\dot b}\text{BrownLine}_\mu[1,2]\cdot\text{BlueLine}_{\dot b}[\theta_a;3,4]\Big)$$
$$= 2\cdot\text{BlueLine}_{\dot b}[\theta^a\theta_a\bar\theta^{\dot b};1,2]\cdot\text{GreenLine}[3,4] + 2\cdot\text{GreenLine}[1,2]\cdot\text{BlueLine}_{\dot b}[\theta^a\theta_a\bar\theta^{\dot b};3,4]$$
$$+O(\theta^3), \tag{F.16}$$

where z is an arbitrary point of the considered supergraph. (Certainly, we assume that all effective lines are included into a connected Feynman graph.)

Both sides of the considered identity are quadratic in $\bar\theta$. Therefore, it is possible to shift $\bar\theta$ to an arbitrary point of the supergraph, because the terms $O(\bar\theta)$ vanish after the integration over $d^4\theta$. Using eq. (F.2) one can equivalently rewrite the identity (F.16) in the form

$$(\theta^a\theta_a)_z\Big(\text{BlueLine}_{\dot b}[1;1,2]\cdot\text{PinkLine}^{\dot b}[3,4] + \text{PinkLine}_{\dot b}[1,2]\cdot\text{BlueLine}^{\dot b}[1;3,4]$$
$$-(\gamma^\mu)^{a\dot b}\text{BlueLine}_{\dot b}[\theta_a;1,2]\cdot\text{BrownLine}_\mu[3,4]-(\gamma^\mu)^{a\dot b}\text{BrownLine}_\mu[1,2]\cdot\text{BlueLine}_{\dot b}[\theta_a;3,4]\Big)$$
$$=\frac{1}{4}\cdot\text{BlueLine}_{\dot b}[\theta^a\theta_a;1,2]\cdot\text{RedLine}^{\dot b}[3,4]+\frac{1}{4}\cdot\text{RedLine}_{\dot b}[1,2]\cdot\text{BlueLine}^{\dot b}[\theta^a\theta_a;3,4]+O(\theta). \tag{F.17}$$

This identity contains terms cubic in θ and terms quartic in θ, which will be considered separately. We start with the cubic terms. Let $A_{\dot b}$ denotes a sequence of lines and vertices connecting the points z and x which also includes terms coming from the operator

$$\int d^8x\,\Big((T)^j{}_i\Big(\frac{\bar D_{\dot b} D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_2 j^j}\Big)\frac{\delta}{\delta_1\phi_{0i}}\Big)_x. \tag{F.18}$$

Similarly, let B denotes a sequence of line and vertices connecting the points z and y. Note that in this operator we do not include terms coming from the operator

$$\int d^8y\,\Big((T)^j{}_i\,\frac{2i\partial_\mu}{\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_4 j^j}\frac{\delta}{\delta_3\phi_{0i}} - i(MT)^{ij}\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_4 j^i}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_3 j^j}\Big)_y \tag{F.19}$$

(Evidently, $P_A = 1$ and $P_B = 0$.) Then the contributions of the first and third terms in the left hand side of eq. (F.17) can be formally written in the form

$$A_{\dot b}\theta^a\theta_a B(\gamma^\mu)^{\dot b c}\theta_c + (\gamma^\mu)^{\dot b c}\theta_c A_{\dot b}\theta^a\theta_a B. \tag{F.20}$$

Using the identity (F.10) this expression can be rewritten as

$$(\gamma^\mu)^{\dot b c}\theta^a\theta_a A_{\dot b}B\theta_c + O(\theta) \tag{F.21}$$

and coincides with the contribution of the first term in the right hand side of eq. (F.17). (So far we discuss only terms cubic in θ.) Using the same method we prove that the second and fourth terms in the left hand side of eq. (F.17) give the second term in the right hand side. Therefore, the terms cubic in θ coincide. Let us now verify that terms quartic in θ are also the same in both sides of eq. (F.17). In this case the operators $A_{\dot b}$ and B are defined exactly as earlier. In particular, B denotes a sequence of line and vertices connecting the points z and y and does not include terms coming from the operator

$$\int d^8y\,\Big((T)^j{}_i\Big(\frac{\bar D_{\dot b}D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}_4 j^j}\Big)\frac{\delta}{\delta_3\phi_{0i}}\Big)_y. \tag{F.22}$$

(As earlier, in this case $P_A = 1$, $P_B = 0$.) Then terms quartic in θ can be formally presented in the form

$$4A_{\dot b}\,\theta^c\theta_c\,B\,\theta^d\theta_d + 4\theta^c\theta_c\,A_{\dot b}\,\theta^d\theta_d B + 4(\gamma^\mu)_{\dot d}{}^a\theta_a A^{\dot d}\,\theta^c\theta_c B(\gamma_\mu)_{\dot b}{}^e\theta_e. \tag{F.23}$$

It is easy to see that the last term in this equation can be equivalently rewritten as

$$8\,\theta^a A_{\dot b}\theta^c\theta_c B\theta_a. \tag{F.24}$$

This allows to apply eq. (F.11). As a result, we obtain that terms of the forth order in θ are given by the expression

$$4\,\theta^c\theta_c A_{\dot b}B\theta^d\theta_d + O(\theta), \tag{F.25}$$

which coincides with the corresponding terms in the right hand side of eq. (F.17). Taking into account that the cubic terms also coincide, we conclude that the identity (4.32) is proved.

F.4 Proof of identity presented in figure 22

In order to verify that a β-function is given by integrals of double total derivative we use the identity

$$\begin{aligned}&(\theta^4)_z\Big(\text{BlueLine}_{\dot b}[1;1,2]\cdot\text{RedLine}^{\dot b}[3,4] + \text{RedLine}_{\dot b}[1,2]\cdot\text{BlueLine}^{\dot b}[1;3,4]\\ &+2\cdot\text{BrownLine}^\mu[1,2]\cdot\text{BrownLine}_\mu[3,4]\Big) = 4\cdot\text{GreenLine}[1,2]\cdot\text{GreenLine}[3,4] + O(\theta^3),\end{aligned} \tag{F.26}$$

which is proved in this section. Exactly as in the previous section we note that both sides of this identity are quadratic in $\bar\theta$ and, therefore, it is possible to shift $\bar\theta$ to an arbitrary point of the supergraph. Using eq. (F.2) it is easy to see that eq. (F.26) can be equivalently written in the form

$$(\theta^a\theta_a)_z\Big(\text{BlueLine}_{\dot b}[1;1,2]\cdot\text{RedLine}^{\dot b}[3,4]+\text{RedLine}_{\dot b}[1,2]\cdot\text{BlueLine}^{\dot b}[1;3,4] \tag{F.27}$$
$$+2\cdot\text{BrownLine}^{\mu}[1,2]\cdot\text{BrownLine}_{\mu}[3,4]\Big)=-\frac{1}{8}\cdot\text{RedLine}^{b}[1,2]\cdot\text{RedLine}_{b}[3,4]+O(\theta).$$

This equation contains terms quadratic, cubic, and quartic in θ. The quartic terms in both sides are equal to the corresponding quartic terms in eq. (F.17), which are considered in the previous section, multiplied by 2. Similarly, terms cubic in θ are obtained by multiplying cubic terms in eq. (F.17) by 4. Therefore, it is necessary to consider only terms quadratic in θ. It is evident that in such terms θ-s can be shifted to an arbitrary point of the supergraph. Then the required equality of the quadratic terms follows from the algebraic identity

$$2\cdot\theta^a\theta_a\eta^{\mu\nu}=-\frac{1}{8}\cdot 4(\gamma^\mu)^{\dot b c}\theta_c\cdot 4(\gamma^\nu)_{\dot b}{}^{d}\theta_d. \tag{F.28}$$

(In the right hand side we shift both θ-s to the same point z.)

G Derivation of the expression (4.11)

The sum of Δ_1, which is given by eq. (3.24), and the expression (4.10) is

$$i\frac{d}{d\ln\Lambda}\int d^8x\,(\theta^4)_x C(R)_i{}^k\Big\{\int d^8y\Big(-\Big(\frac{2\partial_\mu}{\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi^{*j})_y\delta(\phi_k)_x}\Big)^{-1}\Big[y_\mu^*,\frac{\delta^2\gamma}{\delta(\phi_0^{*j})_y\delta(\phi_{0i})_x}\Big]$$
$$-\Big(\frac{2\partial_\mu}{\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_y\delta(\phi_k)_x}\Big)^{-1}\Big[y_\mu^*,\frac{\delta^2\gamma}{\delta(\phi_{0j})_y\delta(\phi_{0i})_x}\Big]+\Big(\frac{D^2}{4\partial^4}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_x\delta(\phi_j)_y}\Big)^{-1}\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_{0j})_y}\Big)$$
$$+M^{ij}\Big[y_\mu^*,\Big(\frac{\partial_\mu D^2}{16\partial^4}\Big)_y\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x\delta(\phi_k)_y}\Big)^{-1}\Big]_{y=x}-M^{ij}\Big(\frac{D^2}{8\partial^4}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x\delta(\phi_k)_y}\Big)^{-1}_{y=x}\Big\}. \tag{G.1}$$

Substituting the inverse Green functions from eq. (2.41) we obtain

$$iC(R)_i{}^k\frac{d}{d\ln\Lambda}\int d^8x\,(\theta^4)_x\Big\{\int d^8y\Big(\Big(\frac{\partial_\mu\bar D^2D^2}{2\partial^2(\partial^2G^2+|MJ|^2)}G\Big)_k{}^j\delta^8_{xy}\Big[y_\mu^*,\frac{\delta^2\gamma}{\delta(\phi_0^{*j})_y\delta(\phi_{0i})_x}\Big]$$
$$+\Big(\frac{2\partial_\mu\bar D^2}{\partial^2(\partial^2G^2+|MJ|^2)}(MJ)^*\Big)_{jk}\delta^8_{xy}\Big[y_\mu^*,\frac{\delta^2\gamma}{\delta(\phi_{0j})_y\delta(\phi_{0i})_x}\Big]-\Big(\frac{D^2\bar D^2}{4\partial^4(\partial^2G^2+|MJ|^2)}(MJ)^*\Big)_{kj}$$
$$\times\delta^8_{xy}\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_{0j})_y}\Big)+M^{ij}\Big[y_\mu^*,\Big(\frac{\partial_\mu D^2\bar D^2}{16\partial^4(\partial^2G^2+|MJ|^2)}(MJ)^*\Big)_{jk}\Big]_x\delta^8_{xy}\Big|_{y=x}$$
$$+M^{ij}\Big(\frac{1}{\partial^2G^2+|MJ|^2}(MJ)^*\Big)_{jk}\Big(\frac{D^2\bar D^2}{8\partial^4}\Big)_x\delta^8_{xy}\Big|_{y=x}\Big\}, \tag{G.2}$$

where all derivatives act on the point x. Taking into account that $[y_\mu^*,D^2]=0$ and $(\theta^4)_x[y_\mu^*,\bar D^2\delta^8_{xy}]=0$, it is possible to use eq. (C.16), which allows to express the remaining

Green functions in terms of G and J:

$$iC(R)_i{}^k \frac{d}{d\ln\Lambda}\int d^8x\,(\theta^4)_x\Big\{2\int d^8y\,\Big(\Big(\frac{\bar D^2 D^2\partial_\mu}{8\partial^2(\partial^2G^2+|MJ|^2)}G\Big)_k{}^j\,\delta^8_{xy}\Big[y^*_\mu, G^i{}_j\Big]_y\delta^8_{xy}$$
$$+\Big(\frac{\bar D^2 D^2\partial_\mu}{8\partial^2(\partial^2G^2+|MJ|^2)}(MJ)^*\Big)_{jk}\delta^8_{xy}\Big[y^*_\mu, \frac{(MJ)^{ij}}{\partial^2}-\frac{M^{ij}}{\partial^2}\Big]_y\delta^8_{xy}\Big)$$
$$-\Big(\frac{D^2\bar D^2}{8\partial^4(\partial^2G^2+|MJ|^2)}(MJ)^*\Big)_{jk}\Big((MJ)^{ij}-2M^{ij}\Big)\delta^8_{xy}\Big|_{y=x}$$
$$+M^{ij}\Big[y^*_\mu,\Big(\frac{\partial_\mu D^2\bar D^2}{16\partial^4(\partial^2G^2+|MJ|^2)}(MJ)^*\Big)_{kj}\Big]_x\delta^8_{xy}\Big|_{y=x}\Big\}. \tag{G.3}$$

Calculating the integrals over the anticommuting variables and using the identities

$$\bar D^2 D^2\delta^4(\theta_x-\theta_y)\Big|_{\theta_x=\theta_y}=4;\qquad \int d^4\theta\,\theta^4=4, \tag{G.4}$$

we obtain (in the Euclidian space after the Wick rotation)

$$\mathcal{V}_4\cdot C(R)_i{}^k\frac{d}{d\ln\Lambda}\int\frac{d^4q}{(2\pi)^4}\Big\{\frac{\partial G^i{}_j}{\partial q^\mu}\Big(\frac{2q_\mu}{q^2(q^2G^2+|MJ|^2)}G\Big)_k{}^j+\Big(\frac{\partial}{\partial q^\mu}\Big(\frac{(MJ)^{ij}}{q^2}\Big)$$
$$+\frac{q^\mu(MJ)^{ij}}{q^4}\Big)\Big(\frac{2q_\mu}{q^2(q^2G^2+|MJ|^2)}(MJ)^*\Big)_{jk}+\frac{q_\mu}{q^4}\frac{\partial}{\partial q^\mu}\Big(\frac{1}{(q^2G^2+|MJ|^2)}(MJ)^*\Big)_{jk}M^{ij}\Big\}. \tag{G.5}$$

Taking into account eqs. (2.39), (2.42), and (3.22) this expression can be presented as an integral of a total derivative:

$$\mathcal{V}_4\cdot N_f\sum_{I=0}^{n}(-1)^{P_I}\frac{d}{d\ln\Lambda}\int\frac{d^4q}{(2\pi)^4}\frac{2q^\mu}{q^4}\frac{\partial}{\partial q^\mu}\Big(\ln(q^2G^2+M^2J^2)+\frac{M^2J}{q^2G^2+M^2J^2}\Big)_I, \tag{G.6}$$

where P_I is a Grassmanian parity of the superfields ϕ_I and $\widetilde{\phi}_I$. This expression coincides with eq. (4.11).

H Relation between diagrams presented in figures 11 and 15

Now, let us compare sums of the diagrams presented in figures 11 and 15 (including the terms written explicitly) using the identity (4.32).

1. The sum of diagrams (1) and (10) in figure 15 is equal to diagram (7) in figure 11. This equality is illustrated in figure 17.

2. The sum of diagrams (2) and (11) in figure 15 gives diagram (8) in figure 11.

3. The sum of diagrams (3) and (12) in figure 15 gives diagram (9) in figure 11.

4. The sum of diagrams (6) and (15) in figure 15 gives diagram (10) in figure 11.

5. Let us consider the sum of diagrams (4), (7), (13), and (16) in figure 15. Using the identity (4.32) we see that this sum is equal to the sum of diagrams (a) and (g) in figure 24.

   Let us consider terms containing the derivative $\delta/\delta\phi_0$ inside the operator GreenLine in diagram (a). Then according to eq. (D.10) the left part of the diagram is proportional to

$$\begin{aligned} &\Big(\frac{\partial}{\partial \ln g}-1\Big)\frac{\delta^2\gamma}{\delta(\phi_{0i})_x\delta(\phi^{*j})_y}-\frac{D_y^2}{8}\delta^8_{xy}\sim D_x^2\delta^8_{xy}\\ \text{or}\qquad &\Big(\frac{\partial}{\partial \ln g}-1\Big)\frac{\delta^2\gamma}{\delta(\phi_{0i})_x\delta(\phi_j)_y}\sim D_x^2\bar D_x^2\delta^8_{xy}. \end{aligned} \tag{H.1}$$

   In both cases the projector D_x^2 acts on the green effective line. Taking into account the identity (5.17) we see that the part of the green line containing $\delta/\delta\phi_0$ vanishes. The remaining part of the green line is denoted by the green line with a cross. Thus, we prove the identity presented in the first string of figure 24 and obtain diagram (4) in figure 11.

6. Let us consider a sum of diagrams (5), (14), and (17) in figure 15. First, it is necessary to prove that diagram (17) is equal to diagram (c) in figure 24. For this purpose we consider terms which do not contain the masses in the operator PinkLine. Then it is easy to see that the right vertex with effective blue line is proportional to

$$\Big[(T)\bar\theta^{\dot b},\,\frac{\delta^2\gamma}{\delta(\phi_{0i})_x\delta(\phi^{*j})_y}\Big]=0\qquad\text{or}\qquad(\bar D^2)_y\Big[(T)\bar\theta^{\dot b},\,\frac{\delta^2\gamma}{\delta(\phi_{0i})_x\delta(\phi_{0j})_y}\Big]=0. \tag{H.2}$$

   Therefore, all such terms vanish. The remaining part of the operator PinkLine is denoted by the pink line with a cross. Thus, we verify the identity presented in the third string of figure 24. Using this result we can apply the identity (4.32) to the considered sum of diagrams. Then we obtain diagrams (5) and (6) in figure 11 and diagram (e) in figure 24.

7. Diagrams (8), (18), and (21) are considered similarly. Exactly as in the previous item we prove that diagram (21) is equal to diagram (d) in figure 24. Then applying the identity (4.32) we obtain diagrams (1) and (2) in figure 11 and diagram (f) in figure 24.

8. Applying the identity (4.32) to the sum of diagrams (9), (19), and (20) we obtain diagrams (b), (i), and (k) in figure 24. Using the equality (5.17) we obtain that diagram (b) is equal to diagram (3) in figure 11.

9. The sum of diagrams (22) and (23) gives diagram (h), which corresponds to

$$\begin{aligned} &2i(T)^j{}_i(T)^l{}_k\frac{d}{d\ln\Lambda}\int d^8x\,d^8y\,\Big(\theta^c\theta_c\bar\theta^{\dot d}\frac{\bar D_{\dot d}D^2}{4\partial^2}+2i\bar\theta^{\dot c}(\gamma^\mu)_{\dot c}{}^d\theta_d\frac{\partial_\mu}{\partial^2}\Big)_y\Big(\theta^a\theta_a\bar\theta^{\dot b}\frac{\bar D_{\dot b}D^2}{8\partial^2}\Big)_x\\ &\times\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_y\delta(\phi_j)_x}\Big)^{-1}\cdot\text{LineWithDot}[1]\cdot\frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_{0i})_x}. \end{aligned} \tag{H.3}$$

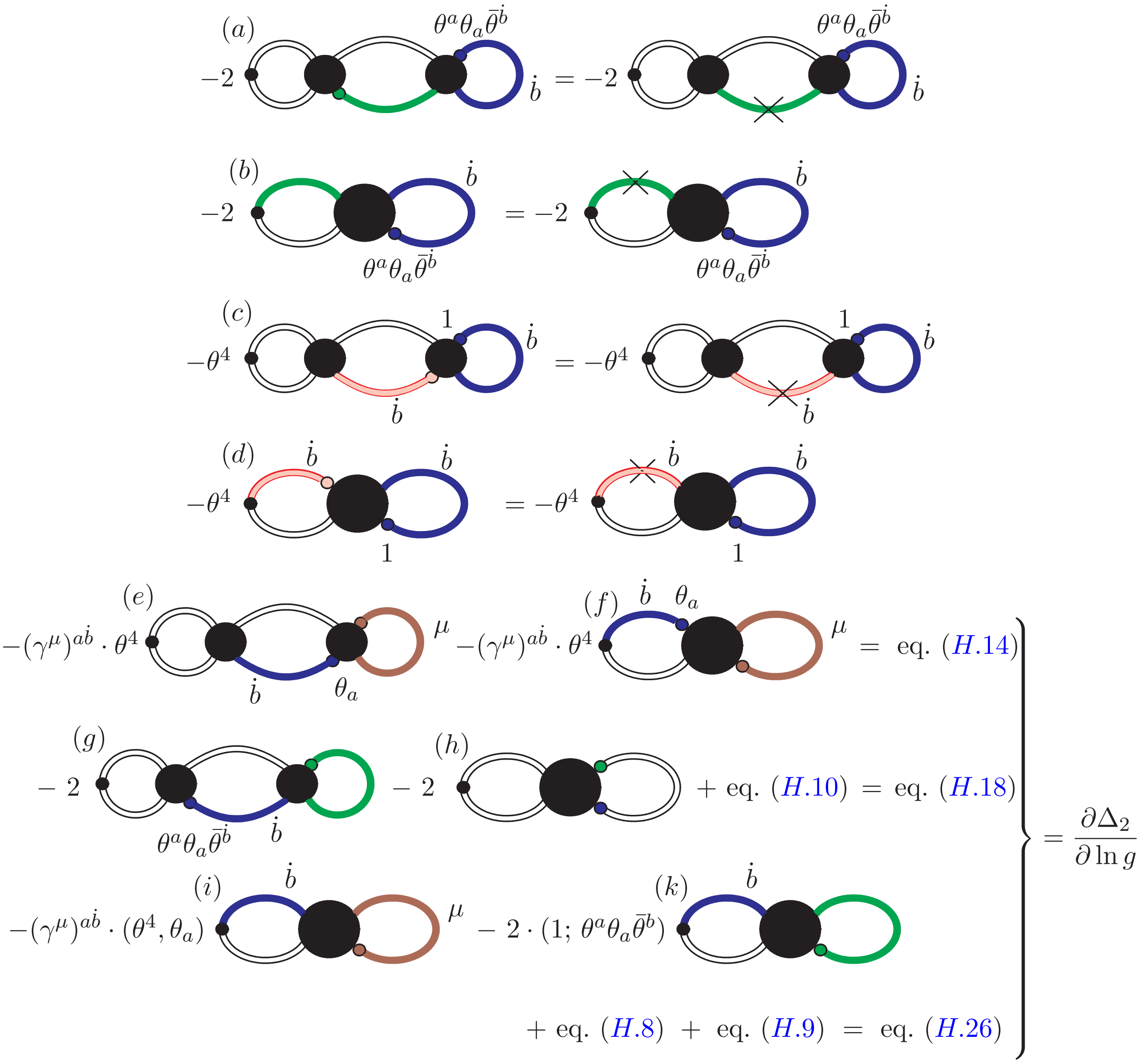

Figure 24. Some relations between effective diagrams needed for proving identity presented in figure 13. (For simplicity we omit the derivatives $d/d\ln\Lambda$ acting on these diagrams.)

Using eqs. (D.10) and commuting $(\theta^a\theta_a)_x$ with $(D^2)_x$ this expression can be presented in the form

$$-4g\,C(R)_i{}^j \frac{d}{d\ln\Lambda}\int d^8x\,d^8y\,\Big(\theta^c\theta_c\bar\theta^{\dot d}\frac{\bar D_{\dot d}D^2}{4\partial^2}+2i\bar\theta^{\dot c}(\gamma^\mu)_{\dot c}{}^d\theta_d\frac{\partial_\mu}{\partial^2}\Big)_y\Big(\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta^b\frac{\partial_\nu}{\partial^2}\Big)_x$$
$$\times\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_y\delta(\phi_j)_x}\Big)^{-1}\frac{\partial}{\partial\ln g}\Big(g^{-1}\frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_{0i})_x}\Big). \tag{H.4}$$

Taking into account the identity (F.7) after some simple transformations we write the result in the form

$$-2ig\,C(R)_i{}^j\frac{d}{d\ln\Lambda}\int d^8x\,d^8y\,(\theta^4)_x\Big(\frac{2\partial_\mu}{\partial^2}-i(\gamma_\mu)^{a\dot b}\theta_a\frac{\bar D_{\dot b}D^2}{4\partial^2}\Big)_y\Big(\frac{\partial_\mu}{\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_y\delta(\phi_j)_x}\Big)^{-1}$$
$$\times\frac{\partial}{\partial\ln g}\Big(g^{-1}\frac{\delta^2\gamma}{\delta(\phi_{0k})_y\delta(\phi_{0i})_x}\Big). \qquad \text{(H.5)}$$

10. Let us now consider the term

$$-i\frac{d}{d\ln\Lambda}\int d^8x\,(\theta^4)_x(\gamma^\mu)^{\dot a b}(MT)^{ik}\Big(\frac{D^2\partial_\mu}{8\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^i}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^k}\cdot\text{BlueLine}_{\dot a}[\theta_b]\cdot\gamma, \qquad \text{(H.6)}$$

which is written explicitly in figure 15. This expression is quadratic in $\bar\theta$. Therefore, it is possible to shift any $\bar\theta$ to an arbitrary point of the graph. Then using the identity (F.7) it is easy to see that the considered expression can be rewritten in the form

$$-i\frac{d}{d\ln\Lambda}\int d^8x\,(\bar\theta^{\dot c}(\gamma^\mu)_{\dot c}{}^d\theta_d)_x(MT)^{ik}\Big(\frac{D^2\partial_\mu}{4\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^i}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^k}\cdot\text{BlueLine}_{\dot b}[\theta^a\theta_a\bar\theta^{\dot b}]\cdot\gamma$$
$$=\frac{d}{d\ln\Lambda}\Big(4\cdot\text{GreenWithCross}\cdot\text{BlueLine}_b[\theta^a\theta_a\bar\theta^b]\cdot\gamma\Big). \qquad \text{(H.7)}$$

This result coincides with diagram (11) in figure 11.

11. The terms containing M^*, which are presented in figure 15 in the explicit form, are given by

$$-i(\gamma^\mu)^{\dot a b}(TM^*)_{ik}\frac{d}{d\ln\Lambda}\int d^8x\,\Big(2(\theta^4)_x\text{BlueLine}_{\dot a}[\theta_b]+(\bar\theta^{\dot c}\bar\theta_{\dot c}\theta_b)_x\text{PinkLine}_{\dot a}\Big)\Big(\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j_i^*}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j_k^*}\cdot\gamma$$
$$=-i(TM^*)_{ik}\frac{d}{d\ln\Lambda}\int d^8x\,(\theta^4)_x\text{BrownLine}^\mu\cdot\Big(\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j_i^*}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j_k^*}\cdot\gamma$$
$$=i\frac{d}{d\ln\Lambda}\int d^8x\,(\theta^4)_x(TM^*)_{ik}\Big(\frac{\bar D^2\partial^\mu}{16\partial^4}\Big)_x\Bigg(\Big[(T)y_\mu^*,\Big(\frac{\delta^2\gamma}{\delta(\phi^{*k})_y\delta(\phi^{*i})_x}\Big)^{-1}\Big]$$
$$-\int d^8z\,(MT)^{mn}\Big(\frac{D^2\partial^\mu}{8\partial^4}\Big)_z\Big(\frac{\delta^2\gamma}{\delta(\phi_m)_z\delta(\phi^{*k})_y}\Big)^{-1}\Big(\frac{\delta^2\gamma}{\delta(\phi_n)_z\delta(\phi^{*i})_x}\Big)^{-1}\Bigg)_{y=x}, \qquad \text{(H.8)}$$

where we use the results for $(T)y_\mu^*$ commutators with inverse Green functions obtained in appendix E.3 for deriving the last equality.

12. The first two terms explicitly written in figure 15 can be presented in the following form:

$$\int d^8x\,M^{mk}C(R)_k{}^n\Big(\frac{D^2}{8\partial^4}\Big)_x\text{LineWithDot}[\theta^4]\cdot\frac{\boldsymbol{\delta}^2\gamma}{\boldsymbol{\delta}(j^m)_x\boldsymbol{\delta}(j^n)_y}\Bigg|_{y=x}=-iC(R)_k{}^iM^{jk}$$
$$\int d^8x\,(\theta^4)_x\Bigg\{g^{-1}\Big(\frac{D^2}{8\partial^4}\Big)_x\frac{\partial}{\partial\ln g}\Bigg(g\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_j)_y}\Big)^{-1}\Bigg)_{y=x}-\int d^8z\,\Bigg(\Big(\frac{D^2}{16\partial^4}\Big)_x$$
$$\delta(\phi_m)_z\delta(\phi_i)_x\Big)^{-1}\Big(\frac{\delta^2\gamma}{\delta(\phi^{*m})_z\delta(\phi_j)_x}\Big)^{-1}+M^{mn}\Big(\frac{D^2}{16\partial^4}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_m)_z}\Big)^{-1}\Big(\frac{D^2}{8\partial^2}\Big)_z$$

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$$\delta(\phi_n)_z\delta(\phi_j)_x\Big)^{-1} + M^*_{mn}\Big(\frac{D^2}{16\partial^4}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi^{*m})_z}\Big)^{-1}\Big(\frac{\bar D^2}{8\partial^2}\Big)_z\Big(\frac{\delta^2\gamma}{\delta(\phi^{*n})_z\delta(\phi_j)_x}\Big)^{-1}\Big)\Big\}; \tag{H.9}$$

$$\int d^8x\, d^8y\,\Big\{M^{mk}C(R)_k{}^n\Big(\frac{\partial_\mu}{\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_m)_y}\Big)^{-1}\mathrm{LineWithDot}[\theta^4]\cdot\Big(\frac{D^2\partial_\mu}{4\partial^4}\Big)_y$$
$$\times\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}(j^n)_y}\frac{\delta\gamma}{\delta(\phi_{0i})_x}\Big\} = -ig\frac{d}{d\ln\Lambda}\int d^8x\, d^8y\, d^8z\,(\theta^4)_x\Big\{M^{mk}C(R)_k{}^n\Big(\frac{\partial_\mu}{\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_m)_y}\Big)^{-1}$$
$$\times\Big(\frac{D^2\partial_\mu}{4\partial^4}\Big)_y\Big[\Big(\frac{\delta^2\gamma}{\delta(\phi^{*p})_z\delta(\phi_n)_y}\Big)^{-1}\Big(\frac{\partial}{\partial\ln g}\Big(g^{-1}\frac{\delta^2\gamma}{\delta(\phi_{0i})_x\delta(\phi_0^{*p})_z}\Big) + \frac{1}{4g}\delta^8_{xz}\delta^i_p\Big)$$
$$+\Big(\frac{\delta^2\gamma}{\delta(\phi_p)_z\delta(\phi_n)_y}\Big)^{-1}\Big(g^{-1}\frac{\delta^2\gamma}{\delta(\phi_{0i})_x\delta(\phi_{0p})_z}\Big)\Big]\Big\}. \tag{H.10}$$

13. We have already obtained all diagrams presented in figure 11. However, there are also some additional contributions. Let us verify that the sum of them gives $\partial\Delta_2/\partial\ln g$.

    We will start with calculating the sum of diagrams (e) and (f) in figure 24. It is easy to see that the right part of these diagrams contains the operator D^2 acting on the right part of the blue effective line. Using the identity

$$D^2\Big((\gamma^\mu)^{a\dot b}\theta_a\frac{\bar D_{\dot b}D^2}{8\partial^2} + \frac{i\partial^\mu}{\partial^2}\Big) = 0 \tag{H.11}$$

    similar to item 6 it is possible to present the considered sum in the form:

$$\frac{1}{2}\cdot\frac{d}{d\ln\Lambda}\Big(\mathrm{LineWithDot}[\theta^4;1,1]\cdot\mathrm{YellowLine}^\mu[1,M=0;2,1]\cdot\mathrm{UsualLine}[1,2]\cdot$$
$$\times\mathrm{BrownLine}_\mu[2,2]\cdot\gamma[1]\cdot\gamma[2] + \mathrm{YellowWhiteLine}^\mu[\theta^4,M=0]\cdot\mathrm{BrownLine}_\mu\cdot\gamma\Big), \tag{H.12}$$

    where $M=0$ means that in the expression for the effective line it is necessary to set masses to 0. The operator $\mathrm{BrownLine}_\mu$ acting on derivatives of the Routhian γ gives commutators with $(T)y^*_\mu$, see eqs. (E.12) and (E.14). Taking into account eqs. (D.13) and (D.14) we see that the other operators give the derivative with respect to $\ln g$ and the considered sum of diagrams (e) and (f) can be rewritten as

$$2i\,(T)^l{}_i\frac{d}{d\ln\Lambda}\int d^8x\, d^8y\,(\theta^4)_x\,\frac{1}{g}\Big\{\frac{\partial}{\partial\ln g}\Big(g\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_x\delta(\phi_j)_y}\Big)^{-1}\Big)\Big(\frac{\partial_\mu}{\partial^2}\Big)_x\Big[(T)y^*_\mu,\frac{\delta^2\gamma}{\delta(\phi_{0j})_y\delta(\phi_{0i})_x}\Big]$$
$$+\frac{\partial}{\partial\ln g}\Big(g\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_x\delta(\phi^{*j})_y}\Big)^{-1}\Big)\Big(\frac{\partial_\mu}{\partial^2}\Big)_x\Big[y^*_\mu,\frac{\delta^2\gamma}{\delta(\phi_0^{*j})_y\delta(\phi_{0i})_x}\Big]\Big\}. \tag{H.13}$$

    In order to calculate this expression we substitute the inverse Green functions from eq. (2.41) and use eqs. (3.1) and (2.38). The result in the momentum representation is

$$\mathcal{V}_4\cdot 2C(R)_k{}^i\frac{d}{d\ln\Lambda}\int\frac{d^4q}{(2\pi)^4}\frac{q^\mu}{gq^2}\Big\{\frac{\partial}{\partial\ln g}\Big(\frac{g}{q^2G^2+|MJ|^2}G\Big)_i{}^j\frac{\partial G^k{}_j}{\partial q^\mu}$$
$$+\frac{\partial}{\partial\ln g}\Big(\frac{g}{q^2G^2+|MJ|^2}(MJ)^*\Big)_{ij}\frac{\partial}{\partial q^\mu}\Big(\frac{(MJ)^{jk}-M^{jk}}{q^2}\Big)\Big\}. \tag{H.14}$$

14. In order to simplify diagram (g) in figure 24 we use eq. (5.17). Then repeating the same arguments as for diagram (a) the derivative of this diagram with respect to $\ln\Lambda$ can be written as

$$\begin{aligned}
&\frac{d}{d\ln\Lambda}\,\text{LineWithDot}[1;1,1]\cdot\text{UsualLine}[1,2]\\
&\quad\times\text{YellowLine}_\mu[\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b, M=0;1,2]\cdot\text{GreenLine}[2,2]\cdot\gamma[1]\cdot\gamma[2]\\
&=-\frac{1}{8}\cdot\frac{d}{d\ln\Lambda}\,\text{LineWithDot}[1;1,1]\cdot\text{UsualLine}[1,2]\\
&\quad\times\text{YellowLine}_\mu[\bar\theta^{\dot c}\bar\theta_{\dot c}(\gamma^\mu)^{\dot a b}\theta_b, M=0;1,2]\cdot\text{RedLine}_{\dot a}[2,2]\cdot\gamma[1]\cdot\gamma[2]\\
&=-\frac{1}{2}\cdot\frac{d}{d\ln\Lambda}\,\text{LineWithDot}[1;1,1]\cdot\text{UsualLine}[1,2]\\
&\quad\times\text{YellowLine}_\mu[\theta^4, M=0;1,2]\cdot\text{BrownLine}^\mu[2,2]\cdot\gamma[1]\cdot\gamma[2].
\end{aligned}\tag{H.15}$$

This expression contains the operator BrownLine_μ acting on the two-point Green functions, which is related with $(T)y_\mu^*$ commutators according to the results obtained in appendix E.1. Similarly, the operator LineWithDot[1] acting on two-point functions is related with the derivatives of these functions with respect to $\ln g$ according to the results obtained in appendix D.2. Using these relations it is possible to rewrite diagram (g) as

$$\begin{aligned}
&2ig(T)^n{}_m\frac{d}{d\ln\Lambda}\int d^8x\,d^8y\,(\theta^4)_y\Bigg\{\frac{\partial}{\partial\ln g}\Bigg[\Big(g^{-1}\frac{\delta^2\gamma}{\delta(\phi_{0m})_y\delta(\phi_0^{*i})_x}\Big)+\frac{1}{4g}\delta^8_{xy}\delta_i^m\Bigg]\Big(\frac{\partial^\mu}{\partial^2}\Big)_y\Bigg(\Bigg[(T)y_\mu^*,\\
&\Big(\frac{\delta^2\gamma}{\delta(\phi^{*i})_x\delta(\phi_n)_y}\Big)^{-1}\Bigg]-(T)_n{}^j\Big(\frac{2\partial_\mu}{\partial^2}\Big)_y\Big(\frac{\delta^2\gamma}{\delta(\phi^{*i})_x\delta(\phi_j)_y}\Big)^{-1}-(MT)^{kl}\int d^8z\,\Big(\frac{D^2\partial_\mu}{8\partial^4}\Big)_z\\
&\times\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_z\delta(\phi^{*i})_x}\Big)^{-1}\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_z\delta(\phi_n)_y}\Big)^{-1}\Bigg)+\frac{\partial}{\partial\ln g}\Big(g^{-1}\frac{\delta^2\gamma}{\delta(\phi_{0m})_y\delta(\phi_{0i})_x}\Big)\Big(\frac{\partial_\mu}{\partial^2}\Big)_y\Bigg(\Bigg[(T)y_\mu^*,\\
&\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_n)_y}\Big)^{-1}\Bigg]-(T)_n{}^j\Big(\frac{2\partial_\mu}{\partial^2}\Big)_y\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_j)_y}\Big)^{-1}-(T)_i{}^j\Big(\frac{2\partial_\mu}{\partial^2}-i(\gamma_\mu)^{a\dot b}\theta_a\frac{\bar D_{\dot b}D^2}{4\partial^2}\Big)_x\\
&\times\Big(\frac{\delta^2\gamma}{\delta(\phi_j)_x\delta(\phi_n)_y}\Big)^{-1}-(MT)^{kl}\int d^8z\,\Big(\frac{D^2\partial_\mu}{8\partial^4}\Big)_z\Big(\frac{\delta^2\gamma}{\delta(\phi_k)_z\delta(\phi_i)_x}\Big)^{-1}\Big(\frac{\delta^2\gamma}{\delta(\phi_l)_z\delta(\phi_n)_y}\Big)^{-1}\Bigg)\Bigg\}.
\end{aligned}\tag{H.16}$$

Adding to this expression the contributions (H.5) and (H.10) we obtain a simpler expression

$$\begin{aligned}
&2ig(T)^n{}_m\frac{d}{d\ln\Lambda}\int d^8x\,d^8y\,(\theta^4)_y\Bigg\{\frac{\partial}{\partial\ln g}\Bigg[\Big(g^{-1}\frac{\delta^2\gamma}{\delta(\phi_{0m})_y\delta(\phi_0^{*i})_x}\Big)+\frac{1}{4g}\delta^8_{xy}\delta_i^m\Bigg]\Big(\frac{\partial^\mu}{\partial^4}\Big)_y\Big[(T)y_\mu^*,\,(\partial^2)_x\\
&\times\Big(\frac{\delta^2\gamma}{\delta(\phi^{*i})_x\delta(\phi_n)_y}\Big)^{-1}\Big]+\frac{\partial}{\partial\ln g}\Big(g^{-1}\frac{\delta^2\gamma}{\delta(\phi_{0m})_y\delta(\phi_{0i})_x}\Big)\Big(\frac{\partial_\mu}{\partial^4}\Big)_y\Big[(T)y_\mu^*,\,(\partial^2)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_i)_x\delta(\phi_n)_y}\Big)^{-1}\Big]\Bigg\}.
\end{aligned}\tag{H.17}$$

Substituting the inverse Green functions and using eq. (3.1) after the Wick rotation in the Euclidian space it can be rewritten in the form

$$\mathcal{V}_4\cdot 2C(R)_k{}^i\frac{d}{d\ln\Lambda}\int\frac{d^4q}{(2\pi)^4}\frac{q^\mu}{q^2}\Big\{-\frac{1}{q^2}\frac{\partial}{\partial q^\mu}\Big(\frac{gq^2}{q^2G^2+|MJ|^2}G\Big)^j{}_i\frac{\partial}{\partial\ln g}\Big(g^{-1}G_j{}^k-g^{-1}\delta_j^k\Big)$$
$$-\frac{\partial}{\partial q^\mu}\Big(\frac{gq^2}{q^2G^2+|MJ|^2}(MJ)^*\Big)_{ij}\frac{\partial}{\partial\ln g}\Big(\frac{(MJ)^{jk}-M^{jk}}{gq^4}\Big)\Big\}. \tag{H.18}$$

15. The sum of the expressions (H.14) and (H.18) is

$$\mathcal{V}_4\cdot 2N_f\sum_{I=0}^{n}(-1)^{P_I}\frac{d}{d\ln\Lambda}\int\frac{d^4q}{(2\pi)^4}\Big\{\frac{\partial}{\partial\ln g}\Big(\frac{M^2J(-2J+4)}{q^4(q^2G^2+M^2J^2)}\Big)$$
$$-\frac{q^\mu}{q^4}\frac{\partial}{\partial q^\mu}\Big(\frac{2q^2G+2M^2J}{q^2G^2+M^2J^2}\Big)\Big\}_I. \tag{H.19}$$

16. Diagram (k) contains

$$-2\cdot\text{WhiteBlueLine}_{\dot b}[1,\theta^a\theta_a\bar\theta^{\dot b}]$$
$$=\int d^8x\,\theta^a\theta_a\bar\theta^{\dot b}\Big\{(T)^k{}_i\frac{\bar D_{\dot b}D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^k}\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j_i^*}-(MT)^{ij}\frac{\bar D_{\dot b}D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^i}\frac{D^2}{4\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}\Big\}$$
$$=-i\int d^8x\,\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b\Big\{(T)^k{}_i\frac{\partial_\mu}{\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^k}\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j_i^*}-(MT)^{ij}\frac{D^2\partial_\mu}{4\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^i}\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}\Big\}. \tag{H.20}$$

Taking into account that the considered graph is quadratic in $\bar\theta$ and using eq. (F.7) we obtain

$$(\bar\theta^{\dot a}(\gamma^\mu)_{\dot a}{}^b\theta_b)_x\times\text{GreenLine}=-\frac{1}{8}(\gamma^\mu)^{a\dot b}(\bar\theta^{\dot c}\bar\theta_{\dot c}\theta_a)_x\times\text{RedLine}_{\dot b}+\ldots$$
$$=-\frac{1}{2}(\theta^4)_x\times\text{BrownLine}^\mu+\ldots, \tag{H.21}$$

where dots denote terms vanishing after integration over $d^4\theta$. Using this equation the derivative of the considered diagram with respect to $\ln\Lambda$ can be written in the form

$$\frac{i}{2}\cdot\frac{d}{d\ln\Lambda}\int d^8x\,(\theta^4)_x\Big((T)^k{}_i\frac{\partial_\mu}{\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^k}\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j_i^*}-(MT)^{ij}\frac{D^2\partial_\mu}{4\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^i}\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}\Big)\cdot\text{BrownLine}^\mu\cdot\gamma. \tag{H.22}$$

In this expression the operator BrownLine^μ acts on two-point Green functions. This allows to present the result in the form

$$\frac{i}{2}\cdot\frac{d}{d\ln\Lambda}\int d^8x\,(\theta^4)_x\Big\{-(T)^n{}_m\Big(\frac{\partial^\mu}{\partial^2}\Big)_x\Bigg(\Big[(T)y_\mu^*,\Big(\frac{\delta^2\gamma}{\delta(\phi^{*m})_y\delta(\phi_n)_x}\Big)^{-1}\Big]-(T)_n{}^p\Big(\frac{2\partial_\mu}{\partial^2}\Big)_x$$
$$\times\Big(\frac{\delta^2\gamma}{\delta(\phi^{*m})_y\delta(\phi_p)_x}\Big)^{-1}-(MT)^{pq}\int d^8z\,\Big(\frac{D^2\partial_\mu}{8\partial^4}\Big)_z\Big(\frac{\delta^2\gamma}{\delta(\phi_p)_z\delta(\phi^{*m})_y}\Big)^{-1}\Big(\frac{\delta^2\gamma}{\delta(\phi_q)_z\delta(\phi_n)_x}\Big)^{-1}\Bigg)$$
$$+(MT)^{nm}\Big(\frac{\partial_\mu}{\partial^2}\Big)_x\Big(\frac{D^2}{4\partial^2}\Big)_y\Bigg(\Big[(T)y_\mu^*,\Big(\frac{\delta^2\gamma}{\delta(\phi_m)_y\delta(\phi_n)_x}\Big)^{-1}\Big]-(T)_n{}^p\Big(\frac{2\partial_\mu}{\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_m)_y\delta(\phi_p)_x}\Big)^{-1}$$
$$-(MT)^{pq}\int d^8z\,\Big(\frac{D^2\partial_\mu}{8\partial^4}\Big)_z\Big(\frac{\delta^2\gamma}{\delta(\phi_p)_z\delta(\phi_m)_y}\Big)^{-1}\Big(\frac{\delta^2\gamma}{\delta(\phi_q)_z\delta(\phi_n)_x}\Big)^{-1}\Bigg)\Big\}_{y=x}. \tag{H.23}$$

17. Diagram (i) contains

$$-(\gamma^\mu)^{a\dot b}\text{WhiteBlueLine}_{\dot b}[\theta^4,\theta_a] = -\frac{1}{2}(\gamma^\mu)^{a\dot b}\int d^8x\,\theta^4(MT)^{ij}\Big(\frac{D^2}{8\partial^2}\,\theta_a\frac{\bar D_{\dot b}D^2}{8\partial^2}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^i}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}$$
$$= i\int d^8x\,\theta^4(MT)^{ij}\Big(\frac{D^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^i}\Big)\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^j}. \tag{H.24}$$

Therefore, the derivative of this diagram with respect to $\ln\Lambda$ can be written as

$$\frac{d}{d\ln\Lambda}\int d^8x\,(\theta^4)_x(MT)^{nm}\frac{iD^2\partial_\mu}{16\partial^4}\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^n}\cdot\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}j^m}\cdot\text{BrownLine}^\mu\cdot\gamma = \frac{d}{d\ln\Lambda}\int d^8x\,(\theta^4)_x$$
$$\times(MT)^{nm}\Big(\frac{iD^2\partial_\mu}{16\partial^4}\Big)_x\Bigg(-\Big[(T)y^*_\mu,\Big(\frac{\delta^2\gamma}{\delta(\phi_m)_y\delta(\phi_n)_x}\Big)^{-1}\Big]+(T)_n{}^p\Big(\frac{2\partial_\mu}{\partial^2}\Big)_x\Big(\frac{\delta^2\gamma}{\delta(\phi_m)_y\delta(\phi_p)_x}\Big)^{-1}$$
$$+(MT)^{pq}\int d^8z\,\Big(\frac{D^2\partial^\mu}{8\partial^4}\Big)_z\Big(\frac{\delta^2\gamma}{\delta(\phi_p)_z\delta(\phi_m)_y}\Big)^{-1}\Big(\frac{\delta^2\gamma}{\delta(\phi_q)_z\delta(\phi_n)_x}\Big)^{-1}\Bigg)_{y=x}. \tag{H.25}$$

18. It is easy to see that the sum of eq. (H.8), (H.9), (H.23), and (H.25) is given by

$$\mathcal{V}_4\cdot 2N_f\frac{d}{d\ln\Lambda}\sum_{I=0}^{n}(-1)^{P_I}\int\frac{d^4q}{(2\pi)^4}\Bigg\{-\frac{\partial}{\partial\ln g}\bigg(\frac{2M^2J}{q^4(q^2G^2+M^2J^2)}\bigg)$$
$$+\frac{q^\mu}{q^4}\frac{\partial}{\partial q^\mu}\bigg(\frac{2q^2G+2M^2J}{q^2G^2+|MJ|^2}\bigg)\Bigg\}_I. \tag{H.26}$$

19. The sum of eqs. (H.19) and (H.26) is equal to $\partial\Delta_2/\partial\ln g$ (Δ_2 is given by eq. (3.25) or eq. (C.17)):

$$-\mathcal{V}_4\cdot 2N_f\frac{d}{d\ln\Lambda}\sum_{I=0}^{n}(-1)^{P_I}\frac{\partial}{\partial\ln g}\int\frac{d^4q}{(2\pi)^4}\Big(\frac{2M^2J(J-1)}{q^4(q^2G^2+M^2J^2)}\Big)_I. \tag{H.27}$$

This completes the prove that the sum of diagrams presented in figure 11 is equal to the sum of diagrams presented in figure 15.

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