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# A non supersymmetric ${ m SO}(10)$ grand unified model for all the physics below $M_{ m GUT}$

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ABSTRACT: We present a renormalizable non supersymmetric Grand Unified SO(10) model which, at the price of a large fine tuning, is compatible with all compelling phenomenological requirements below the unification scale and thus realizes a minimal extension of the SM, unified in SO(10) and describing all known physics below  $M_{\rm GUT}$ . These requirements include coupling unification at a large enough scale to be compatible with the bounds on proton decay; a Yukawa sector in agreement with all the data on quark and lepton masses and mixings and with leptogenesis as the origin of the baryon asymmetry of the Universe; an axion arising from the Higgs sector of the model, suitable to solve the strong CP problem and to account for the observed amount of Dark Matter. The above constraints imposed by the data are very stringent and single out a particular breaking chain with the Pati-Salam group at an intermediate scale  $M_I \sim 10^{11}$  GeV.

KEYWORDS: GUT, Beyond Standard Model, Gauge Symmetry

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#### 1 Introduction

The first LHC runs at 7–8 TeV have led to the discovery of a candidate Higgs boson and to the non observation of new particles or exotic phenomena: no signals of new physics have been found neither by direct production of new particles nor in electroweak precision tests nor in flavour physics. The Standard Model (SM) has so far been confirmed by experiment beyond all expectations. This is surprising since the hierarchy problem [1–5] and, to some extent, the elegant WIMP (Weakly Interacting Massive Particle) solution of Dark Matter strongly suggested the presence of new physics near the Fermi scale. But as well known the hierarchy problem is one of "naturalness": the SM theory is renormalizable, finite, well defined and predictive once the dependence on the cut off is absorbed in a redefinition of masses and couplings. Thus the theory can indeed work in practice and the hierarchy problem only arises at the conceptual level if one looks at the cut off as a parameterization of our ignorance on the new physics that will modify the theory at large energy scales.

The hierarchy problem is not the only naturalness problem in fundamental physics: the observed value of the cosmological constant  $\Lambda$  also poses a tremendous, unsolved naturalness problem [6]. Yet the value of  $\Lambda$  is close to the Weinberg upper bound for galaxy formation [7]. According to the anthropic point of view, possibly our Universe is just one of infinitely many bubbles (Multiverse) continuously created from the vacuum by quantum fluctuations. Different physics takes place in different Universes according to the multitude of string theory solutions ( $\sim 10^{500}$ ) [8, 9]. Perhaps we live in a very unlikely Universe but the only one that allows our existence [10–12]. In the context of the SM one can argue against this view since plenty of models have been formulated that easily reduce the fine tuning from  $10^{14}$  to  $10^2$ : so why make our Universe so terribly unlikely? If to the SM we

add, say, supersymmetry, does the Universe become less fit for our existence? By comparison the case of the cosmological constant is a lot different because the context is not as fully specified as for the SM

However, as the criterium of naturalness has so far failed, we are lacking at present a reliable argument on where precisely the new physics threshold should be located. Because of the serious arguments against applying the anthropic philosophy to the SM, many still remain confident that some new physics will appear not too far from the weak scale; still, given the failure so far to detect new physics, there has been a revival of models that ignore the fine tuning problem while trying to accommodate the known facts. For example, several fine tuned Supersymmetric (SUSY) extensions of the SM have been studied like split SUSY [13–18] or large scale SUSY [19, 20]. There have also been reappraisals of non SUSY Grand Unified Theories (GUT) where again one completely disregards fine tuning: in some recent papers [21–26] several aspects of the problem have been discussed. Here we would like to establish whether a relatively simple non SUSY SO(10) GUT extension of the SM exists which is able to reproduce all the really compelling data that demand new physics beyond the SM at scales at and below  $M_{GUT}$ . We consider here a renormalizable model but in principle one could also study the same problem in SO(10) versions where each large Higgs representation is replaced by products of smaller field multiplets. In our resulting model the SM spectrum is completed by the just discovered light Higgs and no other new physics is present in the LHC range. At the GUT scale of  $M_{\rm GUT} \geq 10^{16} \, {\rm GeV}$  the unifying for shorthand), valid at an intermediate scale, typically  $M_I \sim 10^{10} - 10^{12} \, \mathrm{GeV}$ . Note that, in general, unification in SU(5) would not work because a group of rank larger than 4 is required in order to allow for a two step (at least) breaking needed, in the absence of SUSY, to restore coupling unification and to avoid a too fast proton decay (an alternative within SU(5) is to assume some ad hoc intermediate threshold with a set of new particles that modify the evolution towards unification [27]). The proposed non-SUSY SO(10) model, with a single intermediate breaking scale  $M_I$  between  $M_{GUT}$  and the electroweak scale, is compatible with the following requirements:

- 1- unification of couplings at a large enough scale  $M_{\rm GUT}$  compatible with the existing bounds on the proton life-time;
- 2- a Yukawa sector in agreement with all data on flavour physics, fermion masses and mixings, also including neutrinos, as well as with leptogenesis as the origin of the baryon asymmetry of the Universe;
- 3- an axion, which arises from the Higgs sector of the model, suitable to solve the strong CP problem and account for the observed amount of Dark Matter.

There is no item in the list concerning the onset of inflation in that we assume that the inflaton is a gravitationally coupled singlet (or even a larger sector of particles) without SM gauge interactions. It turns out that imposing all these requirements is very constraining, so that most of the possible breaking chains of SO(10) must be discarded and the PS symmetry at the intermediate scale emerges as the optimal solution. We show that all these different

phenomena can be satisfied in this fully specified, although schematic, GUT model, with a single intermediate scale at  $M_I \sim 10^{11} \, {\rm GeV}$ . In fact, within this breaking chain, the see-saw and leptogenesis mechanisms can both be made compatible with  $M_I \sim 10^{11} \, {\rm GeV}$ , which is consistent with the theoretical lower limit on the lightest heavy right-handed neutrino for sufficient leptogenesis [28] given by  $M_{\nu_1} \gtrsim 10^9 \, \mathrm{GeV}$ . The same intermediate scale  $M_I$ is also suitable for the axion to reproduce the correct Dark Matter abundance. Given the indicative character of our study we limit ourselves to Leading Order (LO) evolution, a relatively economic choice of SO(10) Higgs multiplets, a crude threshold matching and a sketchy approach to leptogenesis and to axions. We are aware that such a GUT model is terribly fine tuned, because of its explicit hierarchy problem that is also manifest in the necessary huge splittings within the Higgs multiplets (a sort of generalized doublet-triplet splitting problem). We consider this model as an extreme reference case where a minimum of new physics is introduced to realize SO(10) Grand Unification and to accommodate all unavoidable experimental requirements. Note that, given the experimental values of  $m_H$ ,  $m_t$  and  $\alpha_s(m_Z)$ , the vacuum instability occurring in the SM near the intermediate scale  $M_{\nu_1}$  [29] is maintained in this model because below  $M_{\nu_1}$  the coupling evolution is the same as in the SM. However the evolution is somewhat distorted above the intermediate scale.

Based on the earlier results in [21–26, 30–35], we are led to the following breaking chain:

$$SO(10) \xrightarrow{M_{GUT}-210_H} 4_C 2_L 2_R \xrightarrow{M_I-126_H, 45_H} 3_C 2_L 1_Y \xrightarrow{M_Z-10_H} 3_C 1_Y, \qquad (1.1)$$

with the Pati-Salam (PS) group  $4_C 2_L 2_R$  being the intermediate gauge symmetry group. For the breaking of SO(10) to the PS group we adopt a  $210_H$  of Higgs, which gives suitable values for  $M_{\rm GUT}$  and  $M_I$ . The representation 54 also leads to the PS intermediate group but it leaves the left-right symmetry unbroken and, moreover, the mass scales  $M_{\rm GUT}$  and  $M_I$  turn out to be not appropriate. As discussed in section 3 different intermediate groups like  $3_C 2_L 2_R 1$  or  $4_C 2_L 1_R$  fail to end up with a running compatible with all the constraints (we confirm and extend to our case the results of ref. [35]). The breaking down to the SM is achieved by using a  $\overline{126}_H$  (and a less relevant  $45_H$ ) and the final step to the  $3_C 1_Y$ is obtained by means of a  $10_H$ . In order to have a suitable axion Dark Matter candidate we also introduce the  $45_H$  representation, with a mass scale close to  $M_I$  and a specified transformation property under a Peccei-Quinn symmetry. Both  $\overline{126}_H$  and  $10_H$  are also necessary to generate fermion masses (for simplicity, we do not introduce a  $120_H$  that, in principle, could also contribute to fermion masses). We perform a full 3-generation study of fermion masses and mixings. The previous works that are closest to the present approach are those in refs. [21, 25, 26, 34, 36]. We differ from ref. [21] in the symmetry breaking chain (those authors choose a 54 for the breaking at  $M_{\rm GUT}$ ) and for the additional representation  $45_H$  involved in the PQ symmetry. Also they do not make a full 3-generation fit and do not discuss leptogenesis. In ref. [25] there is a detailed fit of fermion masses but no discussion of leptogenesis, axions and the SO(10) breaking chain. We differ from ref. [26] in that the authors assume a particular form of lepton-quark symmetry, do not discuss the PQ symmetry and do not perform a detailed fit of all fermion masses. Finally, in ref. [34] and ref. [36] the same Yukawa sector as ours is considered but neither the implications for leptogenesis nor for axions are discussed.

$10_H$	$(1,2,2) \oplus (6,1,1)$
16	$(4,2,1)\oplus(\overline{4},1,2)$
$45_H$	$(1,1,3) \oplus (1,3,1) \oplus (6,2,2) \oplus (15,1,1)$
$\overline{126}_H$	$(6,1,1) \oplus (10,1,3) \oplus (\overline{10},3,1) \oplus (15,2,2)$
$210_H$	$(1,1,1) \oplus (15,1,3) \oplus (15,1,1) \oplus (15,3,1) \oplus \ (\overline{10},2,2) \ \oplus (10,2,2) \oplus (6,2,2)$

Table 1. Higgs multiplets under the Pati-Salam group.

The Dark Matter problem is one of the strongest evidences for new physics. In this model it should be solved by axions [37–43]. It must be said that axions have the problem that their mass should be adjusted to reproduce the observed amount of Dark Matter. In this respect the WIMP solution, like the neutralinos in SUSY models, is more attractive because the observed amount of Dark Matter is more guaranteed in this case. Neutrino masses and mixing originate in this GUT extended SM from lepton number violation, Majorana masses and the see-saw mechanism while baryogenesis occurs through leptogenesis [44–46]. One should one day observe proton decay and neutrino-less beta decay. None of the alleged indications for new physics at colliders should survive (in particular even the claimed muon (g-2) discrepancy [47–49] should be attributed, if not to an experimental problem, to an underestimate of the theoretical errors or, otherwise, to some specific addition to the model [50]). This model is in line with the non observation of  $\mu \to e \gamma$  at MEG [51], of the electric dipole moment of the neutron [52] etc. It is a very important challenge to experiment to falsify this scenario by establishing a firm evidence of new physics at the LHC or at another "low energy" experiment.

The plan of this article is as follows: in section 2 we elucidate the role of the various Higgs representations useful for our purposes; in section 3 we discuss the evolution of the gauge couplings from  $M_Z$  up to the GUT scale, with the PS intermediate gauge group, and argue on why other possible breaking patterns are less suitable to accommodate the requirements listed in the above-mentioned points 1–3. Section 4 is devoted to the study of the fit of the fermion masses, mixing and leptogenesis whereas in section 5 we present the implication of our results on the mass and Dark Matter contribution of cold axions. In section 6 we draw our conclusions.

#### 2 Higgs representations

In the following, we describe in more detail the SO(10) representations needed to realize the program outlined above. It is useful to classify the various submultiplets in terms of their PS quantum numbers; they are reported in table 1.

In the Yukawa sector, the (1,2,2) of the  $10_H$  representation can be decomposed into  $(1,2,2) = (1,2,+\frac{1}{2}) \oplus (1,2,-\frac{1}{2}) \equiv H_u \oplus H_d$  under the  $3_C 2_L 1_Y$  group; if  $10_H = 10_H^*$  then  $H_u^* = H_d$  as in the SM. It has been shown, for instance in [21], that in the limit of  $V_{cb} = 0$  (with  $V_{ij}$  the CKM matrix) the ratio  $m_t/m_b$  should be close to 1, thus contradicting the experimental fact that, even at the GUT scale,  $m_t/m_b \gg 1$ . On the other hand, although

the  $10_H$  is a real representation from the SO(10) point of view, its components can be chosen either real or complex. In the latter case,  $10_H \neq 10_H^*$  and then  $H_u^* \neq H_d$ . An extra symmetry, in our case the Peccei-Quinn U(1)<sub>PQ</sub>, is present in the lagrangian to forbid the Yukawa couplings related to  $10_H^*$  (see below for a detailed discussion). This solves the problem and helps in keeping the parameter space at an acceptable level. In the following for vacuum expectation values (vevs) we will use a short-hand notation like:

$$k_{u,d} = \langle (1,2,2)_{u,d} \rangle_{10} \,.$$
 (2.1)

These vev's, of the order of the EW scale, generate fermion masses and break  $3_C 2_L 1_Y$  down to  $3_C 1_Q$ . The  $\overline{126}_H$  contains two important vevs:

$$v_R = \langle (10, 1, 3) \rangle_{126}, \qquad v_{u,d} = \langle (15, 2, 2)_{u,d} \rangle_{126}.$$
 (2.2)

The first one,  $v_R$ , of order  $M_I$ , is needed to generate the right-handed neutrino mass matrix and to break the PS group down to the SM; while  $v_{u,d}$  contribute to the fermion mass matrices and thus must be of the order of the EW scale. We assume that only a single light Higgs doublet remains with components in different PS representations. For this to work, an  $\mathcal{O}(1)$  mixing between the (1,2,2) of the  $10_H$  and the (15,2,2) of the  $\overline{126}_H$  is needed in the effective electroweak doublet; such a mixing occurs below the PS mass scale, so the (15,2,2) fields must have masses around  $M_I$ .

The role of the  $45_H$  representation is better understood in connection with the  $U(1)_{PQ}$  Peccei-Quinn symmetry [37–41] and the axion solution of the Dark Matter problem [42, 43]. With a complex  $10_H$  we can transform the fields participating to the fermion mass matrices as follows:

$$16_H \to e^{i\alpha} 16_H , 10_H \to e^{-2i\alpha} 10_H , \overline{126}_H \to e^{-2i\alpha} \overline{126}_H .$$
 (2.3)

This solves the problem of the Yukawa couplings of the  $10_H^*$  discussed above and leads to the axion as Dark Matter candidate. No other multiplets in the model have a non vanishing PQ charge, except for the  $45_H$  (see below). It is expected that the  $\mathrm{U}(1)_{PQ}$  be broken by a nonzero  $\langle \overline{126}_H \rangle$  at the scale of  $\mathrm{SU}(2)_R$  breaking, otherwise  $10_H$  would drive the  $\mathrm{U}(1)$  breaking to give  $M_{PQ} \approx M_W$ , which is ruled out by experiments. Note that the (10,1,3) component of  $\overline{126}_H$  responsible for the PS breaking contains a component (1,1,3,-1) under the group  $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ . However a single  $\overline{126}_H$  is not enough since a linear combination of the  $\mathrm{U}(1)_{PQ}$ ,  $T_{3R}$  and B-L remains unbroken. This problem is solved by introducing another representation that can be either a 16 [32] or a  $45_H$  [33] or another 126, as suggested in [21]. We have studied the evolution of couplings in the three cases and found that the  $45_H$  representation leads to the most suitable value of  $M_I$ . The  $45_H$  contains the multiplet (1,1,3) under PS with vanishing B-L number. Assigning to such a multiplet a PQ charge  $\alpha' \neq \alpha$  and a vev at the same scale  $M_I$  we can break the previous degeneracy and have a viable axion candidate.

It is clear that all components of the  $\overline{126}_H$ ,  $10_H$  and  $45_H$  not involved in the breaking chain and in the Yukawa sector must live at a much higher scale (the GUT scale) in order not to be in conflict with the assumed breaking pattern and with the bounds on proton

	$210_H$	$\overline{126}_{H}$	$45_H$	$10_H$
$M_{ m GUT}$	all components	$(6,1,1), (\overline{10},3,1)$	(1,3,1), (6,2,2), (15,1,1)	(6,1,1)
$M_I$	_	(10,1,3),(15,2,2)	(1, 1, 3)	_
EW	_	_	_	(1, 2, 2)

Table 2. Mass scales of the Higgs components.

decay. For this large separation of scales we have no better motivation than to invoke the extended survival hypothesis (ESH) [30, 32, 53] which is the assumption that at any scale, the only scalar multiplets present are those that develop VEVs at smaller scales. According to this assumption, we summarize in table 2 the mass scales of the Higgs components.

#### 3 Coupling evolution

In this section we show that the Higgs representations discussed in the previous section are enough to guarantee a sufficiently large GUT scale  $M_{\rm GUT}$  and an intermediate scale  $M_I$  compatible with the see-saw mechanism for neutrino masses, an acceptable amount of leptogenesis and a viable axion as a Dark Matter candidate. As already specified in the Introduction, we restrict ourselves to 1-loop accuracy; the evolution equations between two generic scales  $M_{1,2}$  are then given by the standard formulae:

$$\alpha_i^{-1}(M_2) = \alpha_i^{-1}(M_1) - \frac{a_i}{2\pi} \log \frac{M_2}{M_1},$$
(3.1)

where the coefficients  $a_i$  depend on the representations of fermions and scalars lighter than  $M_2$ . In the remaining of this section, we adopt the following short-hand notation for the fields relevant for the coupling evolution:

$$(1,2,2) \equiv \Phi$$
,  $(10,1,3) \equiv \Delta_R$ ,  $(15,2,2) \equiv \Sigma$ ,  $(1,1,3) \equiv \sigma$ .  $(3.2)$ 

Among the Higgs fields only  $\Phi$  is involved in the evolution of the  $2_L$  and  $1_Y$  couplings between the EW mass and the intermediate mass scale  $M_I$ ; from  $M_I$  to  $M_{\rm GUT}$  we have to take into account the contributions of  $\Phi$ ,  $\Sigma$ ,  $\Delta_R$  and  $\sigma$ : the fields  $\Phi$  and  $\Delta_R$  contribute to  $\alpha_{4C}$ ,  $\Phi$  and  $\Sigma$  to the SU(2)<sub>L</sub> group (with coupling  $\alpha'_{2L}$  and  $\beta$ -function coefficient  $\alpha'_{2L}$ ) and all of them to  $\alpha_{2R}$ .

We have six evolution equations (three below and three above  $M_I$ ) and six unknown to fix: the three couplings at  $M_I$ , the unified coupling  $\alpha(M_{\rm GUT})$  and the scales  $M_I$  and  $M_{\rm GUT}$ . The following input values and matching conditions are imposed [54, 55]:

$$\alpha_{3C}(M_Z) = 0.1176 \pm 0.002 \qquad \alpha_{2L}(M_Z) = 0.033812 \pm 0.000021$$

$$\alpha_{1_Y}(M_Z) = 0.016946 \pm 0.000006 \qquad (3.3)$$

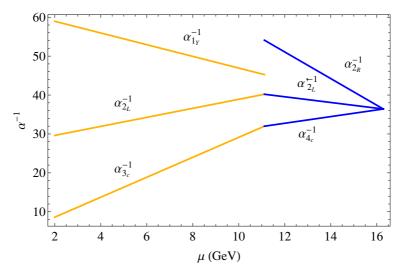
$$\alpha_{3C}(M_I) = \alpha_{4C}(M_I) \qquad \alpha_{2L}(M_I) = \alpha'_{2L}(M_I)$$

$$\alpha_{1_Y}^{-1}(M_I) = \frac{3}{5}\alpha_{2R}^{-1}(M_I) + \frac{2}{5}\alpha_{4C}^{-1}(M_I)$$

$$\alpha_{4C}(M_{GUT}) = \alpha_{2L}(M_{GUT}) \qquad \alpha_{4C}(M_{GUT}) = \alpha'_{2L}(M_{GUT}).$$

$a_3$	$a_{2_L}$	$a_{1_Y}$	$a_4$	$a'_{2_L}$	$a_{2_R}$
-7	$-\frac{19}{6}$	$\frac{41}{10}$	$-\frac{7}{3}$	2	$\frac{28}{3}$

**Table 3**.  $\beta$ -function coefficients.



**Figure 1**. Evolution of the coupling constants from  $M_Z$  to  $M_{\rm GUT}$ .

The  $\beta$ -function coefficients  $a_i$  for the various gauge groups (computed following [56] and [57]) are listed in table 3.

The numerical RGE solutions are shown in figure 1; we obtain:

$$M_I = (1.3 \pm 0.2) \cdot 10^{11} \,\text{GeV} \qquad M_{\text{GUT}} = (1.9 \pm 0.6) \cdot 10^{16} \,\text{GeV}$$
 (3.4)

with  $\alpha_U^{-1} \sim 36.4$  (or  $\alpha_U \sim 0.027$ ). The stated errors only take into account the propagated uncertainties from the SM coupling constants and the Z boson mass. Two-loop effects in the matching conditions and possible threshold effects are not included. The jump of  $a_{1_Y}$  at the intermediate scale  $M_I$  is due to the fact that the generator of  $1_Y$  is a sum of terms from the two different factors  $SU(2)_R$  and  $SU(4)_C$  in the PS gauge group, with matching condition as given in eq. (3.3).

From the order of magnitude estimate of the proton lifetime, given by:

$$\tau \sim \frac{M_{\text{GUT}}^4}{\alpha_U^2 m_p^5},\tag{3.5}$$

and by using the values of  $M_{\rm GUT}$  and  $\alpha_U$  obtained from our coupling evolution, we get:

$$\tau^{\text{model}} \sim 5 \cdot 10^{36} \,\mathrm{y} \,,$$
(3.6)

to be compared to the present experimental limit [58]:

$$\tau^{\text{exp}} \equiv \tau(p \to e^+ \pi^0) \gtrsim 10^{34} \,\text{y} \,.$$
 (3.7)

We see that the model is confortably consistent with the experimental limit. Notice that the proton decay can also be mediated by colored scalar triplets contained in the  $\overline{126}_H$  representation with masses  $M_T$  around the intermediate scale  $M_I$ ; these contributions can be estimated as (to be compared with eq. (3.5)):

$$\Gamma \sim \frac{g_T^2 \, m_p^5}{M_T^4} \tag{3.8}$$

where  $g_T$  is a product of coupling of the Higgses to the fermions in the amplitudes for proton decay and  $M_T$  is the mass of the Higgses. Since the color singlet component of the 15-dimensional Higgs multiplet is a component of the physical Higgs particle then also the colour triplet Higgs boson couples to fermions proportionally to the fermion masses, modulo some Clebsch-Gordan factors. Then one can expect that the largest contribution comes from a vertex with u and s quarks (times a Cabibbo suppression factor), thus causing the proton to decay into a K meson and a lepton. We estimate:

$$\Gamma \sim \frac{m_u^2 \, m_d \, \sin^2 \theta_C \, m_s \, m_p^5}{v_{15}^4 \, M_T^4} \,,$$
 (3.9)

where  $v_{15}$  is either  $v_u$  or  $v_d$ , defined in our eq. (2.3), that we assume of the order of the electroweak scale. To give a lower bound on the triplet Higgs mass, we compare the previous estimate with the experimental limit on  $p \to e^+ K^0$  or  $p \to \mu^+ K^0$  or  $p \to \nu K^+$ , all with limits around  $\tau \gtrsim 2 \times 10^{33}$  y [58], and we derive

$$M_T \gtrsim 10^{10-11} \,\text{GeV} \,.$$
 (3.10)

This bound can be easily satisfied in our model considering that the scalar triplets are expected to have masses around  $M_I \sim 10^{11} \,\text{GeV}$ . A comparable result also holds for the colored Higgs contributions to the  $p \to e^+ \pi^0$  channel. For this channel, both the prediction and the experimental bound on the life-time are larger.

We have studied alternative breaking chains to verify whether they could be compatible with the requirements outlined in the Introduction. If instead of using a  $210_H$  to break SO(10) down to the PS groups we had adopted a  $54_H$ , the intermediate gauge group would have been  $PS \times P$ , where P is a parity symmetry that enforces the left and right couplings to be equal from  $M_I$  to  $M_{GUT}$ . In this case, the Higgs boson sector must be modified to become more left-right symmetric and the values of the mass scales and couplings would be changed. Such a breaking chain has been already discussed in [35]: from their analysis and the present bounds on the proton life-time this chain is ruled out. We have repeated the analysis, adding the (15,2,2) multiplet of the  $126_H$  representation and a  $45_H$  (or  $16_H$ ) at the intermediate scale (all not included in [35]) to get a viable axion candidate. We confirm that, with present data on the proton decay, the  $PS \times P$  predicts a too short life-time,  $\tau \sim 10^{32}$  y. Other interesting possibilities for the intermediate gauge groups are those involving  $3_C 2_L 2_R 1_X$ , where X = (B - L)/2. In this case, adding the contributions of (1,2,2,0) of  $126_H$  and that of (1,1,3,0) of the  $45_H$  (or that of the (1,1,2,-1/2) of the  $16_H$ ), we always get a too small intermediate scale,  $M_I \sim 10^9$  GeV, incompatible with the

see-saw mechanism and difficult to reconcile with the axion explanation of Dark Matter. If the symmetry group also includes the P parity, we get  $M_I \sim (0.4-1) \ 10^{11} \,\text{GeV}$  (depending on whether we use the  $16_H$  or  $45_H$ , respectively) and a value of the proton life-time roughly one-to-two orders of magnitude smaller than its present experimental bound. We conclude that the SO(10) breaking chains  $3_C 2_L 2_R 1_X$ , with or without P-parity, are not suitable to fulfill all the phenomenological constraints considered in this paper.

### 4 Fermion masses, mixings and leptogenesis

Fermion masses arise from the following Yukawa interactions (for economy of parameters we are omitting the possible contribution of a  $120_H$ ):

$$L_Y = 16(h\,10_H + f\,\overline{126}_H)16\,. \tag{4.1}$$

where the coupling matrices h and f are complex symmetric matrices. Recalling the vev definitions in eqs. (2.1), (2.2) the fermion mass matrices of the model have the following form:

$$M_u = h k_u + f v_u,$$
  $M_d = h k_d + f v_d$   
 $M_{\nu}^D = h k_u - 3 f v_u,$   $M_l = h k_d - 3 f v_d,$   $M_{\nu}^M = f v_R.$  (4.2)

We can rewrite the previous expressions in a more compact form, suitable for a fit to masses and mixing angles:

$$M_u = r_v(H + s F),$$
  $M_d = H + F$  (4.3)  
 $M_\nu^D = r_v(H - 3sF)$   $M_l = H - 3F$   $M_\nu^M = r_R^{-1} F,$ 

where  $H = h k_d$ ,  $F = f v_d$ ,  $r_v = k_u/k_d$ ,  $s = v_u/r_v v_d$  and  $r_R = v_d/v_R$ . Since the charged lepton masses are known with high accuracy, we prefer to express  $M_u$  and the neutrino matrices in terms of  $M_d$  and  $M_l$  (the latter with eigenvalues at  $M_{\rm GUT}$  given by [59]  $m_e$ =0.46965 MeV,  $m_{\mu}$ =99.1466 MeV and  $m_{\tau}$ =1.68558 GeV) as follows [25]:

$$M_{u} = r_{v} \left( \frac{3+s}{4} M_{d} + \frac{1-s}{4} M_{l} \right),$$

$$M_{\nu}^{D} = r_{v} \left( \frac{3(1-s)}{4} M_{d} + \frac{1+3s}{4} M_{l} \right),$$

$$M_{\nu}^{M} = \frac{r_{R}^{-1}}{4} (M_{d} - M_{l}).$$
(4.4)

With respect to the existing literature, in our fit we also add the additional requirement of a quantitatively successful leptogenesis [60, 61]:

$$\eta_B^{CMB} = (5.7 \pm 0.6) \times 10^{-10}$$
 (90% CL - deuterium only). (4.5)

The obvious way to compute the baryon-to-photon number ratio would be to implement the Boltzmann equations in the fit and then evaluate  $\eta_B$  following, for example, the prescription

given in refs. [62, 63]. However, this procedure is too long and complicated for the precision we are aiming to in this work. Instead, we can use approximate expressions, depending on the heavy neutrino spectrum of the theory. For example, since leptogenesis is expected to occur at a temperature of the order of  $M_{\nu_1}$ , if  $10^9 < M_{\nu_1} < 10^{12}$  GeV only the  $\tau$  Yukawa coupling is in equilibrium and the muon and electron asymmetries are indistinguishable. In this case one can adopt a two-flavor approach with only  $N_1$  contributing to the asymmetry if the right-handed spectrum satisfies the condition  $(M_{2,3} - M_1)/M_1 \gg 1$  or adding the  $N_2$  contribution if  $(M_{\nu_2} - M_{\nu_1})/M_{\nu_1} \sim \mathcal{O}(1)$ . Since the heavy spectrum and the Dirac mass matrix are not known a priori, that is before making the fit to the observables at the GUT scale, we adopt the following general algorithm:

- 1- we assume to work with a given number of flavours and active right-handed neutrinos;
- 2- in the fit we implement simplified solutions of the Boltzmann equations (see, for instance, [62]);
- 3- after the fit, we check a posteriori that the adopted assumptions in step (1) are correct;
- 4- in the case of a positive answer, we use the heavy spectrum and the Dirac mass matrix obtained from the fit to solve numerically the Boltzmann equations and get a more precise determination of  $\eta_B$ .

Obviously, if the assumptions in (1) are not correct, we have to modify our approximate formulae and run the chain again. It has been emphasized in [26, 64] that in SO(10) with see-saw and renormalizable Yukawa couplings, there is a strict quark-lepton relation not very suitable for implementing the mechanism of baryogenesis through leptogenesis. In fact, the hierarchy of the eigenvalues of the Dirac mass matrix  $M_{\nu}^{D}$ , implied by the quark-lepton connection, implies a strong hierarchy among the heavy right-handed neutrinos via the see-saw formula and this, in general, produces a too small lightest right-handed mass to generate a successful leptogenesis. For a possible way out we assume the easiest possibility, that is that  $N_1$  and  $N_2$  are sufficiently close in mass to contribute both to  $\eta_B$ ; we further assume to work in the two-flavour regime, since we expect the lightest right-handed neutrinos with mass of the order of  $M_I \sim 10^{11} \,\text{GeV}$ , then in a range where only the  $\tau$  Yukawa coupling is in equilibrium.

To estimate the number of independent parameters we proceed as follows. Eq. (4.2) contains 24 real parameters in h and f and 5 (in principle) complex Yukawas, for a total of 34 parameters. Working in the basis where the charged leptons are diagonal allows to remove 3 angles and 6 phases contained in the unitary matrix W such that  $M_d^{\text{diag}} = W M_d W^T$ , so we are left with 25 parameters. From eq. (4.4) we see that the 7 quantities h, f,  $k_u$ ,  $k_d$ ,  $v_u$ ,  $v_d$ ,  $v_R$  only appear in the 3 combinations  $r_v$ , s and  $r_R$ , so we have 6 and not 10 parameters, which means 21 independent parameters. Moreover, since  $r_v$  and  $r_R$  appear as overall factors in eq. (4.4), they can be taken as real, so the total number of parameters is 19, of which 7 are phases: 12 in the down mass matrix, 3 in the charged lepton mass matrix, 2 contained in the complex s parameter and one each in  $r_v$  and  $r_R$ .

$m_u  ({ m MeV})$	$0.495 \pm 0.185$	$ V_{us} $	$0.2254 \pm 0.0006$
$m_d  ({ m MeV})$	$1.155 \pm 0.495$	$ V_{cb} $	$0.04194 \pm 0.0006$
$m_s  ({ m MeV})$	$22.0 \pm 7.0$	$ V_{ub} $	$0.00369 \pm 0.00013$
$m_c  ({ m GeV})$	$0.235 \pm 0.035$	J	$3.16 \pm 0.1) \times 10^{-5}$
$m_b  ({ m GeV})$	$1.00 \pm 0.04$	$\sin^2 \theta_{12}^l$	$0.308 \pm 0.017$
$m_t  ({ m GeV})$	$74.15 \pm 3.85$	$\sin^2 \theta_{23}^l$	$0.3875 \pm 0.0225$
r	$0.031 \pm 0.001$	$\sin^2 \theta_{13}^l$	$0.0241 \pm 0.0025$

**Table 4.** Input values at the scale  $M_{\rm GUT} = 2 \times 10^{16} \, {\rm GeV}$ .

We do not fit neither the lepton masses nor  $r_R$  so, in total, we have to estimate 15 real parameters (12 in the down mass matrix, 2 contained in the complex s parameter and the real  $r_v$ ) to fit the 15 observables summarized in table 4 and in eq. (4.5). The relations in eq. (4.4) hold at the GUT scale, so we have to take into account the Standard Model running of the quark masses and mixing and charged leptons masses from low energies up to  $M_{\rm GUT}$ ; for a precise calculation, the contribution of the Higgs states with masses around  $M_I$  (see table 2) cannot be ignored for the running from  $M_I$  to  $M_{GUT}$ ; however, their effect amounts to a minor correction because the corresponding beta function coefficients are not large and the running involves a logarithmic dependence on the ratio  $M_{\rm GUT}/M_I$ which is quite smaller than  $M_I/M_Z$ . The values of the fermion masses at  $M_{GUT}$  reported in table 4 are taken from [59]; for the neutrino parameters  $(\theta_{ij}^l)$  and r) we used the recent results in [65] and for the CKM parameters the ones quoted in [66]. Although the last two sets of observables are taken at the lowest mass scale in the running, we do not expect in this case sizable corrections in the evolution, as it can be appreciated comparing our table 4 with table VI in [25]. Notice that for the data with asymmetric  $1\sigma$  error, we take the value in the center of the interval as the best fit and consider a symmetric  $1\sigma$  interval. This simplifying choice has a negligible impact on the results of our fit.

To check whether the model is able to reproduce the experimental values of fermion masses, mixing and  $\eta_B$ , we perform a  $\chi^2$  analysis using:

$$\chi^2 = \sum_i \left(\frac{P_i - O_i}{\sigma_i}\right)^2 \,, \tag{4.6}$$

where the  $P_i$  denote the theoretical values of the observables and the  $O_i$  are the experimental values extrapolated to  $M_{\text{GUT}}$  (with  $\sigma_i$  being the corresponding  $1\sigma$  uncertainties). We get a reasonably good minimum of the  $\chi^2$ , namely, for 15 data points:

$$\chi^2_{\rm min} = 17.4\,;$$
 (4.7)

the best fit solutions (and the related pulls) of the observables are given in table 5. We observe that all the experimental data are reproduced within  $3\sigma$ , the largest contributions coming from the b quark mass and to a lesser extent also from the c and s masses

obs.	fit	pull	obs.	fit	pull
$m_u(\text{MeV})$	0.49	0.03	$ V_{us} $	0.225	0.038
$m_d({ m MeV})$	0.78	0.75	$ V_{cb} $	0.042	-0.208
$m_s({ m MeV})$	32.5	-1.50	$ V_{ub} $	0.0038	-0.659
$m_c({ m GeV})$	0.287	-1.49	J	$3.1\times10^{-5}$	0.589
$m_b({ m GeV})$	1.11	-2.77	$\sin^2 \theta_{12}^l$	0.318	0.611
$m_t({ m GeV})$	71.4	0.70	$\sin^2 \theta_{23}^l$	0.353	-1.548
r	0.031	0.10	$\sin^2 \theta_{13}^l$	0.0222	-0.758
$\eta_B$	$5.699 \times 10^{-10}$	-0.001			

**Table 5**. Best fit solutions for the fermion observables at the scale  $M_{\rm GUT} = 2 \cdot 10^{16} \, {\rm GeV}$ .

and the atmospheric neutrino mixing angle  $\theta_{23}$ . One could argue that an extra source of error on the quark masses can well arise from the distortions on the mass evolution induced by the deformed evolution with respect to the SM that occur above  $M_I$ . On the other hand, the tendency of the atmospheric angle to drift toward small values (compared to the input in table 4), is mainly due to the stringent requirement of a successful leptogenesis.<sup>1</sup> In fact, as a check, we have redone the fit only including the 14 observables in table 4. We have obtained a very good  $\chi^2$  minimum,  $\chi^2_{\min}/dof \sim 0.95$ , very close to the results shown in [25]. Equipped with the neutrino mass matrices as obtained from the best fit parameter values (which indeed made the heavy neutrino masses very hierarchical,  $M_{\nu_1}: M_{\nu_2}: M_{\nu_3} \sim 1:10:100$ ), we computed the resulting baryon-to-photon number ratio, obtaining  $\eta_B \sim -10^{11}$ , which is wrong in sign and magnitude.

Note that our fit has no degrees of freedom (15 observables vs 15 parameters): due to the high non linearity of the problem there is no perfect matching but the average squared deviation  $\chi^2/15$  is of order 1. We are also in the position to make a list of predictions including the light neutrino masses, the heavy right-handed masses, the values of the three CP-violating phases (the Dirac  $\delta$  and the two Majorana phases  $\varphi_{1,2}$ , extracted according to the convention used in [68]) and the value of the effective mass in the neutrinoless double beta decay rate,  $m_{ee}$ . They are summarized in the following table 6. The light neutrino spectrum corresponds to the normal hierarchy case. The rate of neutrinoless double beta decay is too small to be detected by experiments planned for the near future. The masses of the heavy right handed neutrinos are in the range  $10^{11}$ - $10^{12}$  GeV. The light neutrino masses  $m_{\nu_i}$  are reproduced from the see-saw formula  $m_{\nu} \sim y^2 v^2/M_R$  with  $v \sim m_{\rm top}$  by Yukawa couplings of the order of  $10^{-1}$ - $10^{-2}$  which come out from fitting the fermion masses; the sum of neutrino masses  $\Sigma m_{\nu_i} \sim 0.065 \, {\rm eV}$  is compatible with the constraints from Cosmology [69],  $\Sigma m_{\nu_i} \lesssim (0.1-1) \, {\rm eV}$  at  $2\sigma$ .

Some additional comments on  $\eta_B$  are worthwhile: the right-handed neutrino masses in

<sup>&</sup>lt;sup>1</sup>Notice that it is a general property of type-I see-saw in renormalizable SO(10) theories to favor small atmospheric mixing, at least in the 2-family case [67].

light $\nu$ masses (eV)	heavy $\nu$ masses $(10^{11}\mathrm{GeV})$	phases (°)	$m_{ee} \; (eV)$
.0046	1.00	$\delta = 88.6$	$5 \times 10^{-4}$
.0098	1.09	$\phi_1 = -33.2$	
.0504	21.4	$\phi_2 = 15.7$	

**Table 6.** Predicted values of the light neutrino masses, of the heavy right-handed masses, of the leptonic CP-violating phases and of  $m_{ee}$ .

table 6 confirm that our decision to work in the two-flavour regime is the most appropriate for the study of leptogenesis in this model; in addition, given that  $(M_{\nu_2} - M_{\nu_1})/M_{\nu_1} \sim 10\%$ , it was correct to take into account the contribution of  $N_2$  also (we verified that the contribution of  $N_3$  is irrelevant). The baryon asymmetry can now be computed solving numerically the kinetic equations, obtaining:

$$\eta_B \sim 5 \times 10^{-10} \,,$$
(4.8)

in agreement with the experimental value.

Additional decay channels for the Majorana neutrinos should be taken into account, involving the RH gauge bosons and also the colour singlet scalars in the (10,1,3) representation, all with masses around  $M_I$  (notice that the (1,1,3) contained in the  $45_H$  does not couple to fermions). In a (1,1,3) we have a triplet of  $SU(2)_R$  with 3 complex fields or 6 real fields. Three of these fields become the longitudinal modes of  $W_R$  and  $Z_R$  and three remain in the spectrum. In the following we only explicitly refer to the effects on leptogenesis of the RH gauge bosons: their longitudinal modes are clearly included while for the 3 physical scalar modes the discussion would go along similar lines. Note that our aim here is simply to establish that there are no obvious no go theorems that prevent the present framework to agree with the data while a precise quantitave account would need the specification not only of the detailed spectrum of our model but also of the cosmological parameters assumed in the analysis. To show that the inclusion of the contributions from, let us say,  $W_R$  does not spoil our results on  $\eta_B$ , we can work in the approximation where the Majorana  $N_1$  gives the largest contribution to  $\eta_B$ ; in this case, the total decay width of the Majorana neutrino is:

$$\Gamma_{N_1} = \frac{(M_{\nu_D}^{\dagger} M_{\nu_D})_{11}}{4 \pi v_u^2} M_{\nu_1} (1 + X), \qquad (4.9)$$

where  $v_u \sim 100 \,\text{GeV}$  and 1+X is the dilution factor. It turns out that the case  $M_{\nu_1} > M_{W_R}$  is untenable, since the decays of Majorana neutrinos into two-body final states of the type  $N \to l_R + W_R$  are too fast, thus causing  $X \sim \mathcal{O}(10^4 - 10^5)$  [70–72]. We then have to rely in a scenario where  $M_{\nu_1} < M_{W_R}$ ; in this case,  $N_1$  mainly decays into three-body final states, with a width given by:

$$\Gamma_3 = \frac{3 g_R^4}{2^{10} \pi^3} \frac{M_{\nu_1}^5}{M_{W_R}^4}; \tag{4.10}$$

consequently, the value of X is [70–72]:

$$X = \frac{3 g_{\rm A}^4 v_u^2}{2^8 \pi^2 (M_{\nu_D}^{\dagger} M_{\nu_D})_{11} a_w^2}, \tag{4.11}$$

where  $g_R \sim 0.24$  is the SU(2)<sub>R</sub> coupling at the PS scale (see figure 1),  $v_u \sim 100\,\mathrm{GeV}$  and  $a_w = \left(\frac{M_{W_R}}{M_{\nu_1}}\right)^2 > 1$ . The entries of  $M_{\nu_D}$  are known from our fit procedure,  $(M_{\nu_D})_{11} \sim \mathcal{O}(1)\,\mathrm{GeV}$  in the basis where the Majorana mass matrix is diagonal and with real entries; our estimate gives  $X < 10^{-2}/a_w^2$ , so in principle there is a broad range of values for the ratio  $a_w$  that corresponds to a negligible correction to  $\Gamma_{N_1}$  computed in our model. However, we also have to check that the three-body decay width  $\Gamma_3$  satisfies the out-of-equilibrium condition  $\Gamma_3 < H$ , where  $H = 1.66\,g_\star^{1/2}\,M_{\nu_1}^2/m_{pl}$  is the expansion rate of the Universe, with  $g_\star = 106.75$  and  $m_{pl}$  is the Planck mass. In our case, this condition translates into the lower limit:

$$M_{\nu_1} > \frac{3 g_A^4 m_{pl}}{1.66 \cdot g_{\star}^{1/2} 2^{10} \pi^3 a_w^2} \sim 2 \cdot 10^{11} / a_w^2 \text{ GeV} .$$
 (4.12)

Then, simply choosing  $a_w^2 \gtrsim 2$ , the value of  $M_{\nu_1}$  found in our fit is compatible with the out-of-equilibrium condition and  $M_{W_R}$  is still of order of the intermediate scale  $M_I$ . A more precise determination of this bound relies on the solution of the Boltzmann equations extended to include the right-handed gauge sector, which is beyond the scope of the paper.

#### 5 Axions as dark matter particles

Here we attempt to give an estimate of the axion mass which is crucial for axion interpretation of Dark Matter. On the other hand, the axion mechanism gives a solution to the strong CP problem without need to impose an additional constraint in the fitting procedure. The mass can be computed using [43]:

$$m_a = \frac{z^{\frac{1}{2}}}{1+z} \frac{f_\pi m_\pi}{F_a} \,, \tag{5.1}$$

where  $z = m_u/m_d$  and  $F_a$  is the axion decay constant. For our numerical estimate, we fix  $f_{\pi} = 92 \,\text{MeV}$  and  $m_{\pi} = 138 \,\text{MeV}$  and take  $F_a = M_I = 1.3 \times 10^{11}$ ; with the parameter z in the interval 0.35 - 0.60, we get  $m_a \sim (4.3 - 4.7) \times 10^{-5} \,\text{eV}$ , compatible with astrophysical bounds [73]. In the case where inflation occurs after the PQ phase transition, the cosmological energy density of cold axions can be estimated from [73]:

$$\Omega_a h^2 \approx 0.7 \left( \frac{F_a}{10^{12} \,\text{GeV}} \right)^{\frac{7}{6}} \left( \frac{\alpha}{\pi} \right)^2$$
(5.2)

where h is the present-day Hubble expansion parameter and  $\alpha$  is the "initial misalignment angle", varying in the interval  $(-\pi,\pi)$ . In order to estimate whether our axions are able to fill the experimental bound [74]:

$$\Omega_a h^2 = 0.1199 \pm 0.0027 \tag{5.3}$$

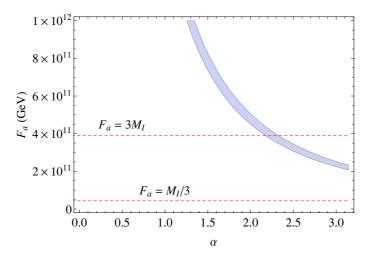


Figure 2. The region in the  $(\alpha, F_a)$  parameter space where the energy density of cold axions saturates eq. (5.3) at  $3\sigma$  level. The horizontal lines correspond to  $F_a = 1/3 M_I$  and  $F_a = 3M_I$ .

we use eq. (5.2) with  $F_a$  and  $\alpha$  as free parameters and plot the region where the bound (5.3) is satisfied at  $3\sigma$  level. This is shown in figure 2; since  $F_a$  should be of the order of  $M_I$ , where the PQ symmetry is broken, we consider the range  $F_a = (1/3 M_I, 3M_I)$ , enclosed in the horizontal lines. As we can see, values of  $F_a$  close to  $M_I$  are perfectly viable to make the axion of our model the relevant component of the cold Dark Matter, at least for values of  $\alpha \gtrsim 2$ .

In the case where inflation occurs after the PQ phase transition (the situation we discussed above), the axion is subject to quantum fluctuations, which leaves a distinctive imprint on the cosmic microwave background (CMB) spectrum. When the temperature is comparable to  $\Lambda_{QCD}$ , such fluctuations induce non-vanishing density perturbations and then a contribution to the total power spectrum of the CMB. The recent Planck results [75] are in tension with the axion contribution to the CMB computed for the values of  $\alpha$  and  $F_a$  of our model. However, there are several ways to avoid this constraint, such as, for example, invoking an axion decay constant during inflation larger than the present one [76], or theories where the scalar fields are coupled to gravity in a non-minimal way [77], or even scenarios in which the QCD coupling becomes large at an intermediate or high energy scale in the very early Universe [78].

#### 6 Conclusions

The SM has passed all sorts of tests at Colliders and the general expectation of discovering new physics at the LHC has been so far frustrated. Yet the SM cannot explain a number of phenomena like Dark Matter, Baryogenesis and neutrino Majorana masses not to mention Dark Energy, inflation and quantum gravity. Due to these problems the horizon of particle physics must necessarily be extended up to  $M_{\rm GUT}$  and even to  $M_{\rm Planck}$ . A suitable arena for enlarging the SM to very large energy scales is provided by GUT's. In particular SO(10) GUT's are very attractive with the spectacular success of the 16 dimensional representation

that reproduces the quantum numbers of all the fermions in one generation, including the right handed neutrinos. In this work we have studied a non SUSY SO(10) GUT that could extend the validity of the SM up to  $M_{\rm GUT}$  describing fermion masses and mixings, Majorana masses, neutrinos with see-saw, Baryogenesis and Dark Matter explained by axions. The possibility of accommodating all compelling phenomena that demand new physics below  $M_{\rm GUT}$  in a non SUSY SO(10) model is highly non trivial. In fact, it singles out a particular breaking chain with a Pati-Salam symmetry at an intermediate mass scale  $M_I \sim 10^{11} \, {\rm GeV}$ . We have shown that a reasonable fit to the data can be obtained in this framework; of course, the price to pay is a very large level of fine tuning. As our goal was to establish an existence proof rather than a completely realistic model, we believe that the rough approximations used for the running and the matching can be considered as adequate for our purposes. In general the idea of an SO(10) GUT is very appealing but all its practical realizations are clumsy, more so in the non SUSY case because of the hierarchy problem and the need of an intermediate symmetry breaking scale in order to obtain a precise coupling unification and a large enough unification scale to be compatible with the existing bounds on proton decay. We discussed here a renormalizable model. Alternatively one could allow for non renormalizable couplings in order to work with smaller Higgs representations. We see no obstacle in principle to produce a successful model also in this case, but we leave it for further study. In conclusion we have considered worthwhile to study the possibility of an all-comprehensive version of non SUSY SO(10) GUT and we have built and compared in detail with experiment an example of such a theory.

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### A Best fit parameter values

Here we list the best fit values of the 15 parameters used in our fit procedure. The 12 elements in  $M_d$  are as follows:

$$M_d (\text{GeV}) = \begin{pmatrix} (-.0034, .0004) & (-7.7 \times 10^{-6}, -.0098) & (-.0112, -.0712) \\ (-7.7 \times 10^{-6}, -.0098) & (.0108, .0010) & (.2162, .0060) \\ (-.0112, -.0712) & (.2162, .0060) & (1.062, -.0584) \end{pmatrix},$$

The complex parameter s and the real parameter  $r_v$  are

$$s = (.37, -.079)$$
  $r_v = 60.03$ . (A.1)

The value of  $r_R$  can be fixed, for example, from the solar mass difference  $\Delta m_{12}^2 = 7.54 \times 10^{-5} \, \text{eV}^2$ ; it turns out that  $r_R^{-1} = 1.21 \times 10^{13}$  [65].

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