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Backreacted D0/D4 background

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Abstract: We construct a supergravity background corresponding to a backreacted D0/D4-brane system. The background is holographically dual to the Venecianno limit of the Berkoos-Douglas matrix model. It is known that the localized D0/D4 system is unstable when the D0-branes are within the D4-branes. To circumvent this difficulty we separate the D4s from the D0s, which are placed at the origin, and restore the symmetry of the combined system by distributing the D4-branes on a spherical shell around the D0-branes. The backreacted solution is first obtained perturbatively in N_f/N_c and displayed analytically to 1st order. A non-perturbative numerical solution is then presented.

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1 Introduction

The general two D-brane intersection was discussed in [1] and it was pointed out that the method of Cherkis and Hashimoto [2] does not yield a solution in terms of elementary functions for the D0/D4 intersection. The problem was not further pursued there and the more serious difficulty of the instability when the branes are not separated was therefore not encountered. By separating the branes and distributing the D4's on a spherical shell we are able to provide both an analytic perturbative solution and a numerical non-perturbative solution.

Much progress has been made in improving the understanding of Gauge/Gravity duality especially in low dimensional settings. The existence of explicit dual geometries to known quantum theories has enabled non-perturbative studies of the corresponding field theories make quantitative comparisons with the predicted results from their gravitational duals. This work is still in its primitive stage and much remains to be done. In particular there is as yet no known backreacted dual geometry which is accessible to numerical lattice field theory techniques. Our present study is a first step in this direction.

Of the low dimensional gauge/gravity pairs the most studied is the BFSS model and its massive deformation the BMN matrix model. The gravitational dual of the BFSS model is the solution of IIA supergravity with a stack of coincident D0-branes or equivalently its lift to 11-dimensional supergravity. This geometry can be conveniently probed using D4-branes [3] which can be displaced from the origin where the D0-branes are located and in the presence of a black hole the shape of the condensate is determined by the background geometry. Checks of the condensate provide strong probes of the background geometry. In particular the comparison of the condensate as computed from a Born-Infeld probe action [3] on the geometry with a non-perturbative lattice study of the gauge theory provides a strong test of the gravity predictions and probes the dual geometry in layers associated with the location of the D4-branes. A further check is provided by comparison of the mass susceptibility of the condensate [4]. It is noteworthy that all these checks give excellent agreement between the matrix model and its gravitational dual.

To go beyond the probe limit one needs to solve for the backreacted geometry taking the non-perturbative effects of the D4-branes into account. It is not difficult to set up the necessary supergravity equations, however the simplest situation,[1] where the D0-branes all sit at one point within the overlapping D4-branes, suffers an instability [5] reflected by divergent gravity solutions. In the dual field theory this corrspond to the limit of vanishing mass of the fundamental fields.

To overcome this difficulty and preserve spherical symmetry we distribute the D4-branes on a spherical shell around the D0-branes.[2] More specifically we displace the D4s from the D0-branes, hence introducing a mass for the fundamental fields and further distribute the displacement of the D4s on a spherical shell around the D0s. Then, just as in electrostatics, the solution interior to the shell is that in the absence of D4-branes while in the exterior one has the spherically symmetric solution of the combined system with continuity of the geometry required on the shell. We are then in a position to solve the resulting D0/D4 system. We begin by studying it perturbatively in N_f/N_c. The resulting 1st order perturbative backreaction is sufficient to guide the full non-perturbative solution.

Unfortunately, we have not yet found the solution in the presence of a black hole which would be dual to the Berkooz-Douglas model in a thermal bath but we are optimistic that a numerical solution can be constructed in this case also.

The principal results of the paper are:

- We study the case of D4-branes separated from the D0-branes and distributed (smeared) over an S^4 orthogonal to the D4s and surrounding the D0s and find the resulting backreacted geometry.

[1] Certain aspects of the abelian D0-D4 bound state have been studied in ref. [6].

[2] For more backreacted solutions in the context of the gauge/gravity correspondence we refer the reader to [9]–[18] for localized and [19]–[37] for smeared solutions.

- We provide an explicit perturbative solution to the leading backreaction of the D4-branes on the D0-geometry in a perturbative expansion in $\frac{N_f}{N_c}$.

- We find a numerical solution for general $\frac{N_f}{N_c}\frac{\lambda}{2m_q^3}$ where λ is the 't Hooft coupling and m_q is the bare mass of the fundamental flavours.

The paper is layed out as follows: in section 2 we briefly review the matrix model, then in section 3 we set up and exhibit the dual geometry with the relevant partitial differential equation, (3.18), necessary to find the non-perturbative geometry. In section 3.4 we exhibit the perturbative solution for a generic distribution of D4-branes that preserve the spherical symmetry of the overlapping D0/D4 intersection. In section 4 we solve for the explicit solution to first order with the D4s distributed on a spherical shell as in figure 1 and in section 4.3 we exhibit the non-perturbative solution for this shell distribution. The bulk of the paper closes with a discussion. The paper closes with some technical appendices, in particular in appendix C we present the equivalently smeared D2/D6 system.

2 The dual matrix model

In this paper we address the dual geometry to the Berkooz Douglas matrix model.

When fundamental flavours are added to the maximally supersymmetric matrix model (the BFSS model) the resulting is know as the Berkooz-Douglas matrix model [7, 8]. The resulting Lagrangian is then:

$$\begin{aligned}\mathcal{L} = \frac{1}{g^2}\mathbf{Tr}\left(\frac{1}{2}D_0X^aD_0X^a + \frac{i}{2}\lambda^{\dagger\rho}D_0\lambda_\rho + \frac{1}{2}D_0\bar{X}^{\rho\dot{\rho}}D_0X_{\rho\dot{\rho}} + \frac{i}{2}\theta^{\dagger\dot{\rho}}D_0\theta_{\dot{\rho}}\right)\\ + \frac{1}{g^2}\mathbf{tr}\left(D_0\bar{\Phi}^\rho D_0\Phi_\rho + i\chi^\dagger D_0\chi\right) + \mathcal{L}_{\text{int}}\,, \qquad (2.1)\end{aligned}$$

where:

$$\begin{aligned}\mathcal{L}_{\text{int}} = \frac{1}{g^2}\mathbf{Tr}\left(\frac{1}{4}\left[X^a,X^b\right]\left[X^a,X^b\right] + \frac{1}{2}\left[X^a,\bar{X}^{\rho\dot{\rho}}\right]\left[X^a,X_{\rho\dot{\rho}}\right] - \frac{1}{4}\left[\bar{X}^{\alpha\dot{\alpha}},X_{\beta\dot{\alpha}}\right]\left[\bar{X}^{\beta\dot{\beta}},X_{\alpha\dot{\beta}}\right]\right)\\ + \frac{1}{g^2}\mathbf{tr}\left(\bar{\Phi}^\alpha\left[\bar{X}^{\beta\dot{\alpha}},X_{\alpha\dot{\alpha}}\right]\Phi_\beta + \frac{1}{2}\bar{\Phi}^\alpha\Phi_\beta\bar{\Phi}^\beta\Phi_\alpha - \bar{\Phi}^\alpha\Phi_\alpha\bar{\Phi}^\beta\Phi_\beta\right)\\ + \frac{1}{g^2}\mathbf{Tr}\left(\frac{1}{2}\bar{\lambda}^\rho\gamma^a\left[X^a,\lambda_\rho\right] + \frac{1}{2}\bar{\theta}^{\dot{\alpha}}\gamma^a\left[X^a,\theta_{\dot{\alpha}}\right] - \sqrt{2}\,i\,\varepsilon_{\alpha\beta}\,\bar{\theta}^{\dot{\alpha}}\left[X_{\beta\dot{\alpha}},\lambda_\alpha\right]\right)\\ + \frac{1}{g^2}\mathbf{tr}\left(\sqrt{2}\,i\,\varepsilon_{\alpha\beta}\,\bar{\chi}\lambda_\alpha\Phi_\beta - \sqrt{2}\,i\,\varepsilon_{\alpha\beta}\,\bar{\Phi}^\alpha\bar{\lambda}_\beta\chi\right)\\ - \frac{1}{g^2}\sum_{i=1}^{N_f}\left(\left(\bar{\Phi}^\rho\right)^i\left(X^a - m_i^a\mathbb{1}\right)\left(X^a - m_i^a\mathbb{1}\right)\left(\Phi_\rho\right)_i + \bar{\chi}^i\gamma^a\left(X^a - m_i^a\mathbb{1}\right)\chi_i\right)\,. \qquad (2.2)\end{aligned}$$

The indices $a = 1,\ldots,5$ correspond to the directions transverse to the D4-brane, while m_i^a are the components of the bare masses of the flavours and correspond to the positions of the D4-branes. The **Tr** denotes trace over the SU(N) colour gauge indices, while **tr** denotes a trace over the flavours.

The adjoint fermions λ^ρ and $\theta^{\dot\alpha}$ (the BFSS fermions) are four eight-component Weyl fermions of six dimensions correspondingly of positive and negative chirality and satisfying the reality conditions (simplectic majorana):

$$\lambda_\alpha = \varepsilon_{\alpha\beta}\,\lambda^{c\,\beta}; \qquad \theta_{\dot\alpha} = -\varepsilon_{\dot\alpha\dot\beta}\,\theta^{c\,\dot\beta}\,, \tag{2.3}$$

where:

$$\psi^c \equiv C_6^{-1}\bar\psi^T\,. \tag{2.4}$$

In the final line quadratic in the fields the masses of the different N_f-fundamental multiplets are to be distributed on an S^4, so that $m_i^a = m_q n_i^a$ and the n_i^a are N_f vectors distributed spherically symmetrically[3] on S^4. In the large N_f limit the sum will become an integral and it is this case that is of interest in this paper.

3 Uplift of the D0/D4 intersection

It is well known that D0-branes solutions of type IIA supergravity can be obtained by dimensional reduction of solutions to eleven dimensional supergravity with momentum along the $\mathcal{M}$-theory circle. On the other hand the D4-branes solutions of type IIA supergravity are obtained after dimensional reduction of an $\mathcal{M}$5-brane solution of eleven dimensional supergravity. This is why the eleven dimensional uplift of the backreacted D0/D4-brane intersection can be obtained by considering an $\mathcal{M}$5-brane geometry with momentum along the $\mathcal{M}$-theory circle. This construction is a magnetic dual analogue of the construction used by Cherkis and Hashimoto [2] to obtain the uplift of the backreacted D2/D6-brane intersection.

Starting from the most general invariant ansatz for the eleven dimensional metric consistent with the above assumptions and imposing the requirement to preserve 1/4 of the original supersymmetry of the background one can reduce the anzatz to a form depending on a single harmonic function. Indeed, the most general SO(5) $\times$ SO(4) anzatz is:

$$\begin{aligned} ds_{11}^2 = &-K_1(u,v)\,dt^2 + K_3(u,v)\,(dx_{11} + A_0(u,v)\,dt)^2 + K_2(u,v)\left(du^2 + u^2 d\Omega_3^2\right) + \\ &+ K_4(u,v)\left(dv^2 + v^2 d\Omega_4^2\right)\,, \end{aligned} \tag{3.1}$$

$$\mathcal{F}_{(4)} = F'(v)\;v^4\,\sin^3\psi\,\sin\tilde\alpha\,\cos\tilde\alpha\,d\psi\wedge d\tilde\alpha\wedge d\tilde\beta\wedge d\tilde\gamma\,, \tag{3.2}$$

$$d\Omega_3^2 = d\alpha^2 + \sin^2\alpha\,d\beta^2 + \cos^2\alpha\,d\gamma^2\,, \tag{3.3}$$

$$d\Omega_4^2 = d\psi^2 + \sin^2\psi\,d\tilde\Omega_3^2\,, \qquad d\tilde\Omega_3^2 = d\tilde\alpha^2 + \sin^2\tilde\alpha\,d\tilde\beta^2 + \cos^2\tilde\alpha\,d\tilde\gamma^2\,. \tag{3.4}$$

Requiring that the $\mathcal{M}$5-brane charge is fixed to Q_5 determines the function $F(v)$. Indeed,

$$\int \mathcal{F}_{(4)} = \frac{8}{3}\pi^2\,v^4\,F'(v) = -Q_5\,. \tag{3.5}$$

results in:

$$F(v) = 1 + \frac{Q_5}{8\pi^2 v^3} \equiv 1 + \frac{v_5^3}{v^3}\,, \tag{3.6}$$

[3]This can be done by sprinkling using a Poisson distribution.

where without loss of generality we set $F(\infty) = 1$. It is straightforward to show (see appendix A for details) that the solution preserving supersymmetry is given by:

$$K_1 = \left(1 + \frac{v_5^3}{v^3}\right)^{-1/3} H(u,v)^{-1} \tag{3.7}$$

$$K_2 = \left(1 + \frac{v_5^3}{v^3}\right)^{-1/3} \tag{3.8}$$

$$K_3 = \left(1 + \frac{v_5^3}{v^3}\right)^{-1/3} H(u,v) \tag{3.9}$$

$$K_4 = \left(1 + \frac{v_5^3}{v^3}\right)^{2/3} \tag{3.10}$$

$$A_0(u,v) = H(u,v)^{-1} - 1 \tag{3.11}$$

where in equation (3.11) we have fixed a constant of integration demanding that if $H \to 1$ at infinity then $A_0 \to 0$. The resulting metric can be written in the format:

$$ds_{11}^2 = \left(1 + \frac{v_5^3}{v^3}\right)^{-1/3} \left(-H\left(u,v\right)^{-1} dt^2 + H\left(u,v\right)\left(dx_{11} + \left(H\left(u,v\right)^{-1} - 1\right) dt\right)^2 + du^2 + u^2\, d\Omega_3^2\right) + \left(1 + \frac{v_5^3}{v^3}\right)^{2/3} \left(dv^2 + v^2\, d\Omega_4^2\right) . \tag{3.12}$$

We observe that supersymmetry does not restrict the shape of the function $H(u,v)$. The equation of motion for H can be obtained either by using the Einstein equations or by requiring that the angular momentum along x_{11} is conserved.[4]

3.1 The function H(u,v)

In this subsection we obtain the equation of motion for $H(u,v)$ by demanding that the current associated with the angular momentum along x_{11} is conserved. The metric (3.12) has the Killing vector $\xi = \partial/\partial_{x_{11}}$ which we can use to define the angular momentum along x_{11} as:

$$J_{x_{11}} \propto \int_{\partial\Sigma} \star\left(\nabla_\mu\, \xi_\nu\, dx^\mu \wedge dx^\nu\right) = \int_\Sigma d \star\left(\nabla_\mu\, \xi_\nu\, dx^\mu \wedge dx^\nu\right) , \tag{3.13}$$

where Σ is a constant time slice of the geometry and $\partial\Sigma$ is its boundary. Demanding that the definition of $J_{x_{11}}$ is independent on the choice of the slice requires that the variation of $J_{x_{11}}$ with respect to deformations of the surface would vanish:

$$\delta J_{x_{11}} = \int_{\delta\Sigma} d \star\left(\nabla_\mu\, \xi_\nu\, dx^\mu \wedge dx^\nu\right) = 0\,. \tag{3.14}$$

Therefore, we obtain:

$$d \star\left(\nabla_\mu\, \xi_\nu\, dx^\mu \wedge dx^\nu\right) = \left[\left(1 + \frac{v_5^3}{v^3}\right)^{-1} \Box_5(v) + \Box_4(u)\right] H(u,v)\, \omega_{(10)} = 0\,, \tag{3.15}$$

[4] After reduction to 10D this corresponds to the conserved D0-brane Ramond-Ramond charge.

where $\omega_{(10)} = du \wedge dv \wedge \omega_{(8)}$ is the volume form of Σ. And the differential operators are given by:

$$\Box_5(v) = \frac{1}{v^4}\partial_v\left(v^4\partial_v\right) \tag{3.16}$$

$$\Box_4(u) = \frac{1}{u^3}\partial_u\left(u^3\partial_u\right) . \tag{3.17}$$

The equation of motion can be written as:

$$\partial_v^2 H(u,v) + \frac{4}{v}\partial_v H(u,v) + \left(1+\frac{v_5^3}{v^3}\right)\left(\partial_u^2 H(u,v) + \frac{3}{u}\partial_u H(u,v)\right) = 0\,. \tag{3.18}$$

One can show that equation (3.18) can be obtained from the Einstein equations. It can also be obtained by requiring Ramond-Ramond charge conservation in the dimensionally reduced ten dimensional metric. Note also that the metric (3.12) and the harmonic equation (3.18) can be obtained in a very elegant way using the Garfinkle-Vachaspati method [10, 11] using considerations very similar to those performed in ref. [12], we refer the reader to appendix B for details of the derivation.

3.2 Dimensional reduction

Using the standard ansatz:

$$ds_{11}^2 = e^{-\frac{2}{3}\Phi} g_{\mu\nu}dx^\mu dx^\nu + e^{\frac{4}{3}\Phi}(dx_{11} + A_\mu dx^\mu)^2\,. \tag{3.19}$$

It is straightforward to obtain the type IIA metric:

$$\begin{aligned} ds_{10}^2 = &-H(u,v)^{-1/2}\left(1+\frac{v_4^3}{v^3}\right)^{-1/2} dt^2 + H(u,v)^{1/2}\left[\left(1+\frac{v_4^3}{v^3}\right)^{-1/2}\left(du^2+u^2\,d\Omega_3^2\right) + \right.\\ &\left.\left(1+\frac{v_4^3}{v^3}\right)^{1/2}\left(dv^2+v^2\,d\Omega_4^2\right)\right] \end{aligned} \tag{3.20}$$

$$e^\Phi = \left(1+\frac{v_4^3}{v^3}\right)^{-1/4} H(u,v)^{3/4} \tag{3.21}$$

$$C_1 = \left(H(u,v)^{-1} - 1\right) dt \tag{3.22}$$

$$F_4 = -3\, v_4^3\, \omega_{S^4}\,, \tag{3.23}$$

where we have renamed v_5 to v_4 and ω_{S^4} is the volume form of the unit S^4. The parameter v_4^3 is proportional to the number of D4-branes, N_f:

$$v_4^3 = N_f\, \pi\, g_s\, \alpha'^{3/2}\,. \tag{3.24}$$

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3.3 Decoupling limit

Let us consider the $v \to \infty$ limit of equation (3.18) (which is equivalent to the $v_4 \to 0$ limit). In this limit the differential operator reduces to the Laplacian in 9D, and has an SO(9) symmetric solution:

$$H_0(u,v) = 1 + \frac{r_0^7}{(u^2+v^2)^{7/2}}\,, \tag{3.25}$$

which is the harmonic function of the D0-brane in the absence of D4-branes. This is not surprising since at large v (far from the D4-branes) the effect of the D4-branes dies out, the SO(9) symmetry is restored and the form (3.25) follows from Ramond-Ramond charge conservation. The parameter r_0^7 is proportional to the number of D0-branes, N_c:

$$r_0^7 = N_c\, 60\, \pi^3\, g_s\, \alpha'^{7/2}\,. \tag{3.26}$$

Given that the u dependent part in equation (3.18) is the same as in flat space we consider a Fourier transform along u:

$$H(u,v) = 1 + \frac{r_0^7}{(2\pi)^4}\int d^4p\, e^{i\,\vec{p}.\vec{u}}\, h(p,v) = 1 + \frac{r_0^7}{4\,\pi^2}\int_0^\infty dp\, p^2\, \frac{J_1(p\,u)}{u}\, h(p,v)\,. \tag{3.27}$$

where the Fourier transformed function h satisfies:

$$\partial_v^2 h(p,v) + \frac{4}{v}\partial_v h(p,v) - p^2\left(1+\frac{v_4^3}{v^3}\right)h(p,v) = 0\,, \tag{3.28}$$

To make contact with the dual field theory we consider the near horizon (decoupling) limit. To this end we define new variables:

$$U = u/\alpha'\,, \qquad V = v/\alpha', \qquad P = p\,\alpha' \tag{3.29}$$

Now taking the limit $\alpha' \to 0$ while keeping the new radial coordinates U and V fixed will zoom in the region near the core of the D0-branes. Leaving only the leading contribution to the metric we obtain:

$$1+\frac{v_4^3}{v^3} = 1 + \frac{N_f \pi g_s \alpha'^{-3/2}}{V^3} = 1 + \frac{N_f 4\pi^3 g_{YM}^2}{V^3} = 1 + \frac{N_f}{N_c}\frac{4\pi^3\lambda}{V^3}\,, \tag{3.30}$$

where the relation $g_{YM}^2 = (2\pi)^{p-2}\, g_s\, \alpha'^{\frac{p-3}{2}}$ for $p=0$ have been used. As one can see the warp factor of the D4-branes survives the decoupling limit reflecting the fact that only the fundamental fields sourced by the D4-branes contribute to the dynamics. We also have

$$\frac{r_0^7}{4\,\pi^2} = \frac{N_c\, 60\, \pi^3\, g_s\, \alpha'^{7/2}}{4\,\pi^2} = 60\pi^3 N_c\, g_{YM}^2\, (\alpha')^5 = 60\pi^3\lambda\,(\alpha')^5\,, \tag{3.31}$$

where $\lambda = N_c\, g_{YM}^2$ is the t'Hooft coupling. Furthermore, in the limit $v_4 \to 0$ the function $h(p,v)$ is given by the Fourier transform of equation (3.25):

$$h_0(p,v) = \frac{4\pi^2}{15} e^{-p\,v}\frac{1+pv}{v^3} \tag{3.32}$$

which suggests that $h(p,v)$ scales as $1/v^3$ under the transformation (3.29) or rather:

$$h(p,v) = (\alpha')^{-3}\tilde{h}(P,V)\,. \tag{3.33}$$

Therefore we have:

$$H(u,v) = 1 + \frac{\tilde{H}(U,V)}{(\alpha')^2} = (\alpha')^{-2}\tilde{H}(U,V) + O\left(\alpha'^0\right)\,, \tag{3.34}$$

where

$$\tilde{H}(U,V) = 60\,\pi^3\,\lambda \int_0^\infty dP\,P^2\,\frac{J_1(P\,U)}{U}\,\tilde{h}(P,V)\,. \tag{3.35}$$

The decoupled metric is then given by:

$$ds_{10}^2/\alpha' = -\tilde{H}^{-1/2}\left(1+\frac{N_f}{N_c}\frac{4\pi^3\lambda}{V^3}\right)^{-1/2} dt^2 + \tilde{H}^{1/2}\left[\left(1+\frac{N_f}{N_c}\frac{4\pi^3\lambda}{V^3}\right)^{-1/2}\left(dU^2+U^2\,d\Omega_3^2\right)+\right.$$
$$\left.\left(1+\frac{N_f}{N_c}\frac{4\pi^3\lambda}{V^3}\right)^{1/2}\left(dV^2+V^2\,d\Omega_4^2\right)\right] \tag{3.36}$$

Note that the functions $h(p,v)$ and $\tilde{h}(P,V)$ satisfy practically the same equation. This is why we continue to consider both the decoupled solution and the flat solution simultaneously.

3.4 Perturbative solution far from the D4-brane

The fact that the solution is tractable in the regime $v \gg v_4$ instructs us to consider the expansion:

Next we expand:

$$h(p,v) = \frac{1}{v^3}\sum_{n=0}^\infty (p\,v_4)^{3n}\,h_n(p,v) \tag{3.37}$$

and substitute in equation (3.28) to obtain:

$$\partial_v^2 h_n(p,v) - \frac{2}{v}\partial_v h_n(p,v) - p^2 h_n(p,v) = \frac{h_{n-1}(p,v)}{p\,v^3}\,, \tag{3.38}$$

where the convention $h_{-1}(p,v) \equiv 0$ was used. The homogeneous part of equation (3.38) has the general solution:

$$\tilde{h}_n(p,v) = \frac{4\pi^2}{15}\,A_n(p)\,e^{-p\,v}(1+p\,v) + \frac{4\pi^2}{15}\,B_n(p)\,e^{p\,v}\,(1-p\,v)\,. \tag{3.39}$$

It is easy to check that at $n=0$ we have $h_0 = \tilde{h}_0$ with $A_0(p)=1$ and $B_0(p)=0$. Next we construct a Green's function satisfying:

$$\partial_v^2 G_p(v,v') - \frac{2}{v}\partial_v G_p(v,v') - p^2 G_p(v,v') = \delta(v-v') \tag{3.40}$$

and vanishing as $v\to\infty$. We obtain:

$$G_p(v,v') = \frac{\Theta(v'-v)e^{-p(v'-v)}(1-p\,v)(1+p\,v') + \Theta(v-v')e^{-p(v-v')}(1+p\,v)(1-p\,v')}{2p^3 v'^2}\,. \tag{3.41}$$

The general solution of equation (3.38) can now be written as:

$$h_n(p,v) = \int^{\infty} dv'\, G_p(v,v') \frac{h_{n-1}(p,v')}{p\,v'^3} + \frac{4\pi^2}{15} A_n(p)\, e^{-p\,v}(1+p\,v)\,. \tag{3.42}$$

Note that we have intentionally omitted the lower boundary of the integral in equation (3.42) since its dependence can always be absorbed in a redefinition of the constant $A_n(p)$. In more details we have:

$$\begin{aligned}\int^{\infty} dv'\, G_p(v,v') f(v') \equiv e^{-p\,v}(1+p\,v) \int^{v} dv' \frac{e^{p\,v'}(1-p\,v')}{2p^3 v'^2} f(v') + \\ + e^{p\,v}(1-p\,v) \int_{v}^{\infty} dv' \frac{e^{-p\,v'}(1+p\,v')}{2p^3 v'^2} f(v')\,,\end{aligned} \tag{3.43}$$

where in the first integral we have taken the primitive function evaluated at v. The freedom to add a constant to the primitive function reflects the freedom to redefine the constant $A_n(p)$ in equation (3.42). Next we define:

$$\tilde{G}_p^{k+1}(v,v') = \int^{\infty} dv_1 \frac{G_p(v,v_1)}{p\,v_1^3} \int^{\infty} dv_2 \frac{G_p(v_1,v_2)}{p\,v_2^3} \cdots \int^{\infty} dv_k \frac{G_p(v_{k-1},v_k)}{p\,v_k^3} \frac{G_p(v_k,v')}{p\,v'^3} \tag{3.44}$$

with the convention $\tilde{G}_p^1(v,v') = G_p(v,v')/(p\,v'^3)$ and $\tilde{G}_p^0(v,v') = \delta(v-v')$. Now using recursively (3.42) for the general solution of (3.38) regular at large v, we can write:

$$h_n(p,v) = \frac{4\pi^2}{15} \sum_{k=0}^{n} A_k(p) \int^{\infty} dv'\, \tilde{G}_p^{n-k}(v,v')\, e^{-p\,v'}(1+p\,v')\,, \tag{3.45}$$

where $A_0(p) = 1$ is fixed by D0-brane charge conservation and the rest of the constants remain undetermined. Note that the undetermined constants $A_k(p)$ are constants only in v and the undetermined behaviour in p can describe a very large family of possible solutions once the Fourier transform is performed. Formally, we can write down the solution valid for $v > v_4$:

$$H(u,v) = 1 + \frac{r_0^7}{15} \int_0^{\infty} dp\, p^2 \frac{J_1(p\,u)}{u} \sum_{n=0}^{\infty} \frac{(p\,v_4)^{3n}}{v^3} \sum_{k=0}^{n} A_k(p) \int^{\infty} dv'\, \tilde{G}_p^{n-k}(v,v')\, e^{-p\,v'}(1+p\,v')\,, \tag{3.46}$$

which (using that we know the solution at $n=0$) can be written as:

$$\begin{aligned}H(u,v) = 1 + \frac{r_0^7}{(u^2+v^2)^{7/2}} + \\ + \frac{r_0^7}{15} \int_0^{\infty} dp\, p^2 \frac{J_1(p\,u)}{u} \sum_{n=1}^{\infty} \frac{(p\,v_4)^{3n}}{v^3} \sum_{k=0}^{n} A_k(p) \int^{\infty} dv'\, \tilde{G}_p^{n-k}(v,v')\, e^{-p\,v'}(1+p\,v')\,.\end{aligned} \tag{3.47}$$

We can also substitute in equation (3.37) to write down the solution for the Fourier transformed $h(p,v)$:

$$h\left(p,v\right) = \frac{4\pi^2}{15\,v^3}\Bigg[e^{-p\,v}\left(1+p\,v\right) + \sum_{n=1}^{\infty}\left(p\,v_4\right)^{3n}\sum_{k=0}^{n}A_k\left(p\right)\int^{\infty}dv'\,\tilde{G}_p^{n-k}\left(v,v'\right)\,e^{-p\,v'}\left(1+p\,v'\right)\Bigg] \tag{3.48}$$

In practice, we quickly loose analytic tractability of the perturbative solution (3.47) when attempting to evaluate the higher order contributions. In the next section we will "regulate" the geometry by introducing a hollow shell of partially smeared D4-branes. This will allows us to determined the constants of integration $A_k(p)$ and construct a perturbative solution.

4 Massive shell

To obtain a non-perturbative solution one needs to impose appropriate boundary conditions both near the core of the geometry and at infinity. While the boundary condition at infinity is physically clear (asymptotic flatness) it is not as clear in the massless case near the core of the geometry. In fact, the authors of [5] have argued that the massless solution is unstable and the D0-branes would desolve inside the D4-branes if the two stacks of D-branes are not separated.

To circumvent this difficulty we separate the D4-branes from the D0-branes and distribute them spherically symmetrically (i.e. smear them) along the S^4 directions of the $\mathbb{R}^5$ transverse to the D4-branes (see figure 1). The result is a shell of radius v_0, where in the decoupling limit v_0 is related to the bare mass of the fundamental flavours m_q in the holographic theory via:

$$m_q = \frac{v_0}{2\pi\alpha'}\,. \tag{4.1}$$

Smearing is equivalent to considering flavours with masses uniformly distributed[5] on a unit S^4. Equation (3.5) now becomes:

$$\int\limits_{v>v_0}\mathcal{F}_{(4)} = \frac{8}{3}\pi^2\,v^4\,F'(v)\Big|_{v>v_0} = -Q_5\,, \tag{4.2}$$

$$\int\limits_{v<v_0}\mathcal{F}_{(4)} = \frac{8}{3}\pi^2\,v^4\,F'(v)|_{v<v_0} = 0\,. \tag{4.3}$$

Given that the function $F(v)$ determines the components of the metric (see appendix A and equations (3.7)–(3.12) we require that $F(v)$ is continuous across the shell. Therefore, equation (3.6) is modified to:

$$F(v) = \begin{cases} 1 + v_4^3/v^3 \text{ for } v \geq v_0 \\ 1 + v_4^3/v_0^3 \text{ for } v < v_0 \end{cases} \tag{4.4}$$

[5]Note that the masses of the fundamental flavours are five dimensional vectors see [3].

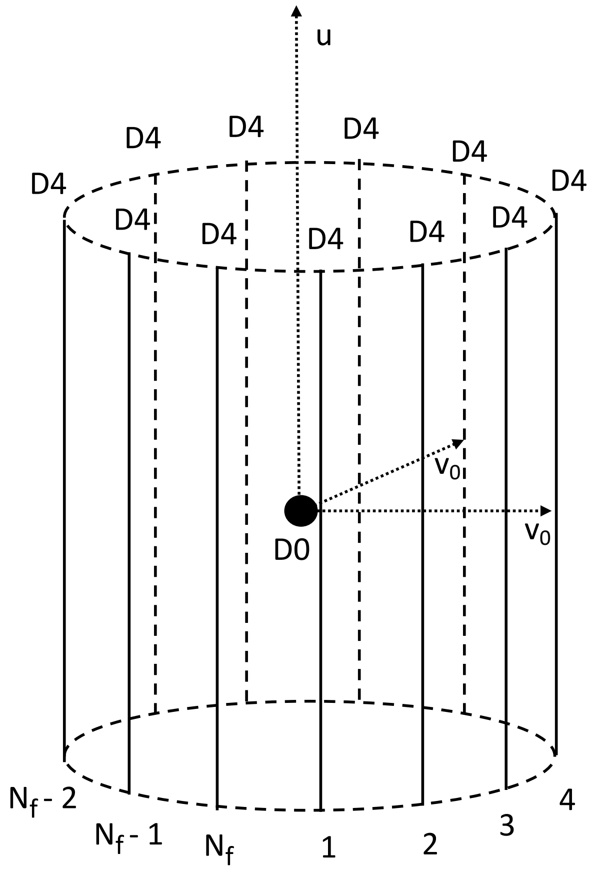

Figure 1. The D0-branes at the origin are surrounded by uniform density of D4-branes separated in the $\mathbb{R}^5$ transverse to the D4-branes and a distance $v = v_0$ from the D0-branes.

Note that the solution outside of the shell (for $v > v_0$) is the same as if all the D4-branes were concentrated at the origin (similarly to the Birkhoff's theorem in General Relativity and the shell theorem in Newtonian gravity), since we preserved the symmetry and charges of the massless case ($v_0 = 0$). Note also that the Rammond-Rammong field has jump in its first derivative at the shell. In fact these leads to cusp in the matric leading to a delta function in the Einstein tensor. We refer the reader to appendix D for a discussion on the origin of the delta function source.

4.1 Solution inside the shell

To write down the solution for the background inside the shell we define:

$$\begin{aligned} \hat{t} &= \left(1 + \frac{v_4^3}{v_0^3}\right)^{-1/4} t\,, \\ \hat{u} &= \left(1 + \frac{v_4^3}{v_0^3}\right)^{-1/4} u\,, \\ \hat{v} &= \left(1 + \frac{v_4^3}{v_0^3}\right)^{1/4} v\,. \end{aligned} \tag{4.5}$$

The solution inside the shell ($v < v_0$) is then given by:

$$ds_{10}^2 = -H(\hat{u},\hat{v})^{-1/2} d\hat{t}^{\,2} + H(\hat{u},\hat{v})^{1/2} \left[d\hat{u}^2 + \hat{u}^2\, d\Omega_3^2 + d\hat{v}^2 + \hat{v}^2\, d\Omega_4^2 \right], \tag{4.6}$$

$$e^{\Phi} = \left(1 + \frac{v_4^3}{v_0^3}\right)^{-1/4} H(\hat{u},\hat{v})^{3/4}, \tag{4.7}$$

$$C_1 = \left(H(\hat{u},\hat{v})^{-1} - 1\right) \left(1 + \frac{v_4^3}{v_0^3}\right)^{1/4} d\hat{t}\,. \tag{4.8}$$

The analogue of equation (3.18) is then:

$$\partial_v^2 H(u,v) + \frac{4}{v}\partial_v H(u,v) + \left(1 + \frac{v_4^3}{v_0^3}\right)\left(\partial_u^2 H(u,v) + \frac{3}{u}\partial_u H(u,v)\right) = 0\,, \tag{4.9}$$

which under the change of variables (4.5) becomes:

$$\partial_{\hat{v}}^2 H(\hat{u},\hat{v}) + \frac{4}{\hat{v}}\partial_{\hat{v}} H(\hat{u},\hat{v}) + \left(\partial_{\hat{u}}^2 H(\hat{u},\hat{v}) + \frac{3}{\hat{u}}\partial_{\hat{u}} H(\hat{u},\hat{v})\right) = 0\,, \tag{4.10}$$

with the maximally symmetric solution:

$$H\left(\hat{u},\hat{v}\right) = 1 + \frac{\left(1 + v_4^3/v_0^3\right)^{-1/4} r_0^7}{\left(\hat{u}^2 + \hat{v}^2\right)^{7/2}}\,. \tag{4.11}$$

4.2 First order solution

Our strategy is to solve (petrubatively) the Fourier transformed equation of motion (3.28) by using the Fourier transformed solution inside the shell and imposing continuity of the solution at the shell.[6] To obtain a closed form solution we consider a perturbative expansion in small v_4/v_0. In the near horizon limit the ratio v_4/v_0 can be related to the physical parameters of the dual gauge theory via:

$$\frac{v_4^3}{v_0^3} = \frac{N_f}{N_c}\frac{\lambda}{2m_q^3} \tag{4.12}$$

and requirement $v_4 \ll v_0$ can be written as $m_q^3 \gg \frac{N_f}{2N_c}\lambda$, that is the bare mass of the fundamental flavours is much larger than the energy scale set by the t'Hooft coupling. Note that since $v > v_0$ outside of the shell, this implies that $v \gg v_4$ and we can apply the formalism fom section 3.4. Restricting to first order in equation (3.48) and performing the necessary integration we get:

$$h\left(p,v\right) = \frac{4\pi^2}{15} e^{-p\,v}\left[\frac{\left(1+p\,v\right)}{v^3} + \frac{v_4^3}{v^3}\left(\frac{p^2}{4v} + A_1\left(p\right)\left(1+p\,v\right)\right) + O\left(p^6 v_4^6\right)\right] \tag{4.13}$$

[6] To verify the validity of this approach in appendix C we have revisited the backreacted D2/D6 system, we have constructed an analytic solution with a massive shell of smeared D6-branes. Remarkably, but not unexpectedly, in the limit of vanishing radius of the shell (which is the limit of vanishing fundamental mass) we recover the original solution from ref. [2], which was obtained exploiting properties of the Taub-NUT geometry.

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To determine the constant of integration $A_1(p)$ we expand we first restore the original variables in equation (4.11) to obtain:

$$H(u,v) = 1 + \frac{\gamma^3\, r_0^7}{(u^2+\gamma^2\, v^2)^{7/2}}\,, \tag{4.14}$$

$$\text{where} \quad \gamma^2 = 1 + \frac{v_4^3}{v_0^3}\,, \tag{4.15}$$

which is valid inside the shell ($v \leq v_0$). The corresponding expression expressed as a Fourier transformed is then given by:

$$1 + \frac{\gamma^3\, r_0^7}{(u^2+\gamma^2\, v^2)^{7/2}} = 1 + \frac{r_0^7}{4\,\pi^2}\int_0^\infty dp\, p^2\, \frac{J_1(p\, u)}{u}\, \frac{4\,\pi^2}{15}\, e^{-\gamma\, p\, v}\frac{1+\gamma\, p\, v}{v^3}\,. \tag{4.16}$$

Next, we expand the solution inside the cavity (4.14) to obtain:

$$H(u,v) = 1 + \frac{r_0^7}{(u^2+v^2)^{\frac{7}{2}}}\left(1 + \frac{v_4^3}{v_0^3}\frac{3u^2-4v^2}{2(u^2+v^2)} + O\left(\frac{v_4^6}{v_0^6}\right)\right)\,. \tag{4.17}$$

From the expansion of the Fourier transformed expression inside the cavity (4.16) at $v = v_0$ we get:

$$h(p, v_0) = \frac{4\pi^2}{15}e^{-p\, v_0}\left(\frac{1+p\, v_0}{v_0^3} - \frac{v_4^3}{v_0^3}\frac{p^2}{2v_0} + O\left(\frac{v_4^6}{v_0^6}\right)\right), \tag{4.18}$$

consistent with the Fourier transform of the expansion (4.17). Comparing inside and outside the shell, i.e. equations (4.13) and (4.18) at $v = v_0$, for the constant of integration $A_1(p)$ we obtain:

$$A_1(p) = -\frac{3}{4\, v_0}\frac{p^2}{1+p\, v_0} \tag{4.19}$$

and hence the Fourier transformed function $h(p,v)$ is given by:

$$h(p,v) = \frac{4\pi^2}{15}e^{-p\, v}\left[\frac{(1+p\, v)}{v^3} + \frac{v_4^3}{v^3}\left(\frac{p^2}{4v} - \frac{3p^2}{4v_0}\frac{1+p\, v}{1+p\, v_0}\right) + O\left(p^6 v_4^6\right)\right] \tag{4.20}$$

For the function $H(u,v)$ outside the shell ($v > v_0$) we obtain:

$$\begin{aligned} H(u,v) = 1 &+ \frac{r_0^7}{(u^2+v^2)^{\frac{7}{2}}}\left\{1 + \frac{v_4^3}{v^3}\left(\frac{3u^2-4v^2}{u^2+v^2}\frac{3v-v_0}{4v_0}\right)\right\} \\ &- r_0^7\frac{v_4^3}{v^3}\frac{(v-v_0)}{20u\, v_0}\int_0^\infty e^{-p\, v}\frac{p^5\, J_1(p\, u)}{1+p\, v_0}dp + O\left(\frac{v_4^6}{v_0^6}\right) \end{aligned} \tag{4.21}$$

We can combine equations (4.17) and (4.21) into the following expression:

$$H(u,v) = 1 + \frac{r_0^7}{(u^2+v^2)^{\frac{7}{2}}}\left\{1 + \frac{v_4^3}{v_0^3}H_1\left(\frac{u}{v_0}, \frac{v}{v_0}\right)\right\} + O\left(\frac{v_4^6}{v_0^6}\right) \tag{4.22}$$

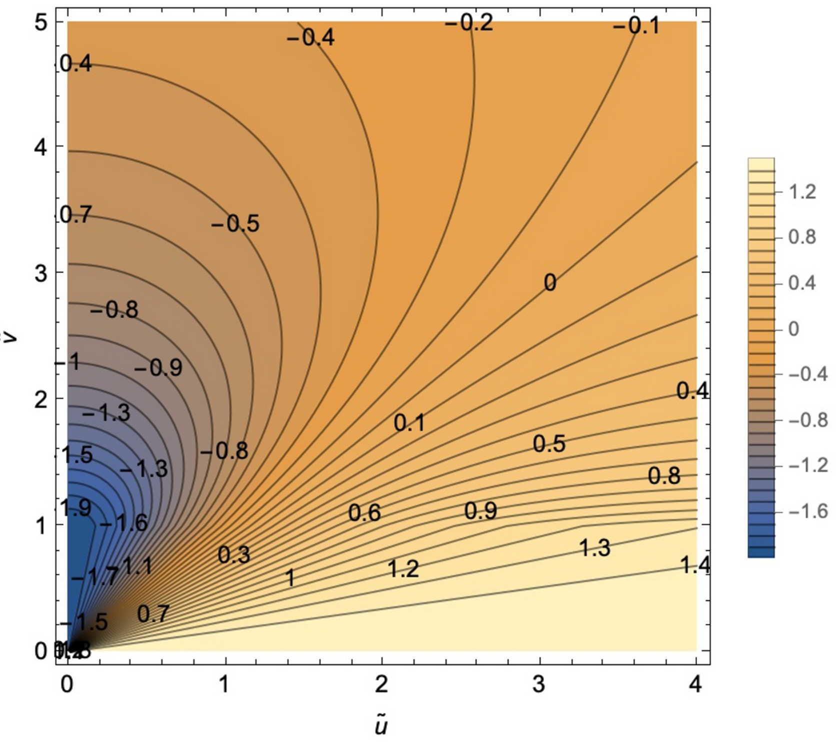

Figure 2. A contour plot of the function H_1 from equation (4.23).

where the function H_1 is given by:

$$H_1(\tilde{u},\tilde{v}) = \begin{cases} \frac{3\tilde{u}^2-4\tilde{v}^2}{\tilde{u}^2+\tilde{v}^2}\frac{3\tilde{v}-1}{4\tilde{v}^3} - \frac{(\tilde{v}-1)(\tilde{u}^2+\tilde{v}^2)^{\frac{7}{2}}}{20\tilde{u}\,\tilde{v}^3}\int\limits_0^\infty e^{-\tilde{p}\,\tilde{v}}\frac{\tilde{p}^5\,J_1(\tilde{p}\,\tilde{u})}{1+\tilde{p}}d\tilde{p} & \text{for } \tilde{v} > 1 \\ \frac{3\tilde{u}^2-4\tilde{v}^2}{2(\tilde{u}^2+\tilde{v}^2)} & \text{for } \tilde{v} \leq 1 \end{cases} \tag{4.23}$$

with $\tilde{u} = u/v_0$, $\tilde{v} = v/v_0$ and $\tilde{p} = v_0\,p$. A plot of the function H_1 is presented in figure 2. As one can see, it is continuous with a range $[-2, 1.5]$. Therefore, if one keeps the perturbative parameter small ($v_4 \ll v_0$) the perturbative expansion is well defined and valid everywhere (except at the origin where H diverges).

In fact, one can see that the range of values of the function H_1 is seeded at the origin where the function H_1 is multi-valued. Indeed, consider the correction (4.23) inside the shell ($\tilde{v} \leq 1$) evaluated on the ray $\tilde{u} = \kappa\,\tilde{v}$. We obtain:

$$H_1(\kappa\tilde{v},\tilde{v}) = \frac{3(\kappa\tilde{v})^2 - 4\tilde{v}^2}{2((\kappa\tilde{u})^2 + \tilde{v}^2)} = \frac{3\kappa^2 - 4}{2(\kappa^2+1)} \tag{4.24}$$

Clearly the possible values of the parameter κ are in the range $[0, \infty)$. In the limit $\kappa \to 0$ we get $H_1 \to -2$, and in the limit $\kappa \to \infty$ we get $H_1 \to \frac{3}{2}$, which is the observed range of the correction function $H_1 \in [-2, 1.5]$. We can now understand the contour lines in figure 2 as representing a $\sim 1/(u^2+v^2)^{7/2}$ fall of the full function $H(u, v)$ along the contour with the coefficient of proportionality seeded at the origin.

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4.3 Non-perturbative numerical solution

In this section we follow the same strategy, namely to use the closed form solution inside the shell to specify the boundary conditions at the shell. However, instead of solving perturbatively the equation of motion (3.28) we solve it numerically.

The non-petrubative solution inside the shell and its Fourier transform are given in equations (4.14) and (4.16), respectively. The Fourier transform of $H(u,v)$ outside the shell is given by equation (3.27) and satisfies equation (3.28). Our strategy is to solve numerically equation (3.28) for $h(p,v)$ by imposing continuity at $v = v_0$ and regularity at infinity. One can show that for large v the Fourier transform function $h(p,v)$ has the asymptotic form:

$$h(p,v) = A(p)\, e^{-p\,v} \left(\frac{1}{v^3} + \frac{p}{v^2} + O\left(\frac{1}{v^4}\right)\right) + B(p)\, e^{p\,v} \left(\frac{1}{v^3} - \frac{p}{v^2} + O\left(\frac{1}{v^4}\right)\right) . \tag{4.25}$$

It is clear that regularity of the solution requires $B(p) = 0$. On the other hand continuity at $v = v_0$ requires:

$$h(p,v_0) = \frac{4\,\pi^2}{15}\, e^{-\gamma\,p\,v_0} \frac{1+\gamma\,p\,v_0}{v_0^3} . \tag{4.26}$$

The boundary condition (4.26) at $v = v_0$ and the regularity condition at $v = \infty$ ($B(p) = 0$ in (4.25) are sufficient to obtain a unique numerical solution to equation (3.28). For more details on the numerical techniques that we used we refer the reader to section E in the appendix. Here we simply present the numerical profile of the function $H(u,v)$. It is again convenient to introduce the dimensionless variables $\tilde{u} = u/v_0$, and $\tilde{v} = v/v_0$, and define $\tilde{v}_4 = v_4/v_0$. Then we can write the correction function H_c as in equation (4.22):

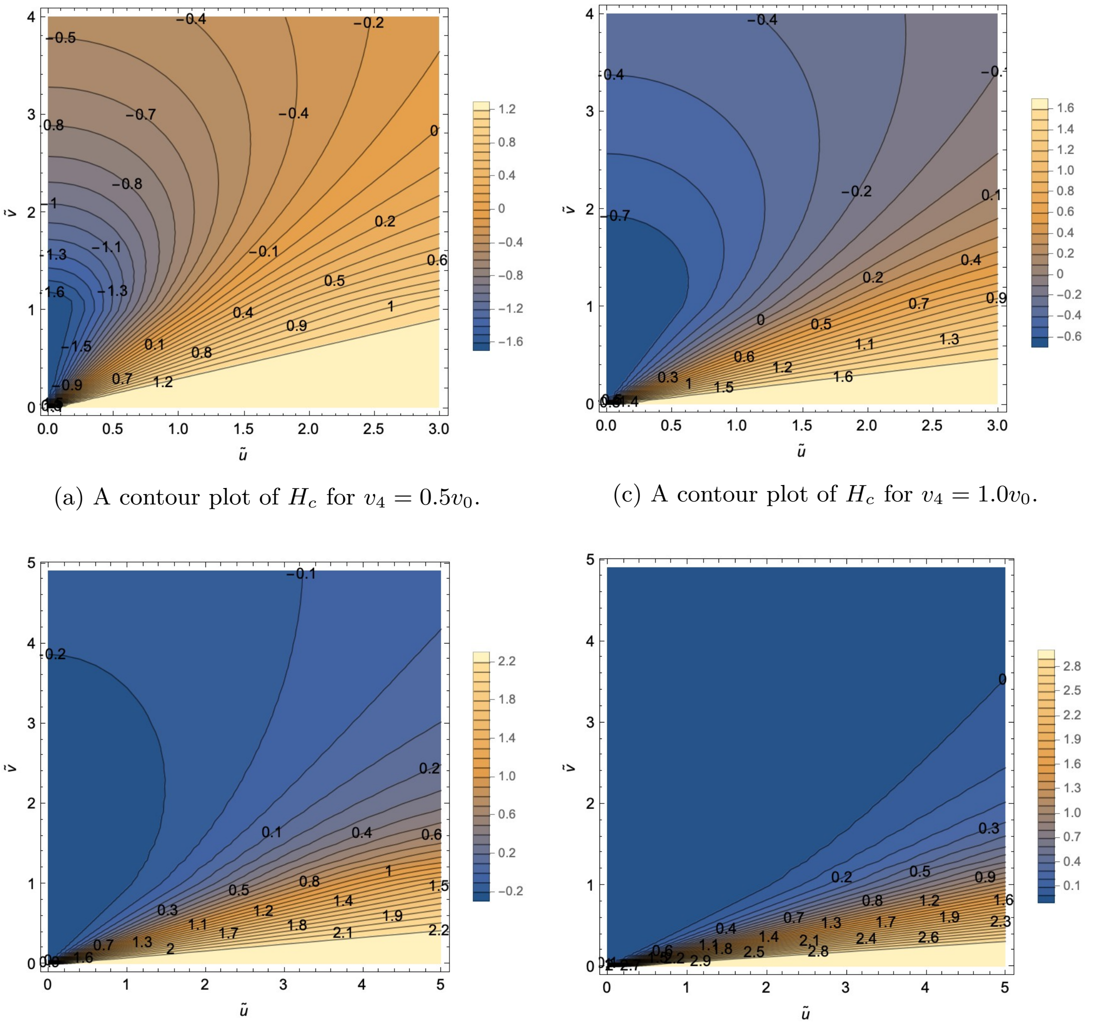
$$H(u,v) = 1 + \frac{r_0^7}{(u^2+v^2)^{\frac{7}{2}}} \left[1 + \frac{v_4^3}{v_0^3} H_c\left(\frac{u}{v_0}, \frac{v}{v_0}, \frac{v_4}{v_0}\right)\right] . \tag{4.27}$$

Note that unlike the first order correction function H_1, in (4.22), the correction function H_c is non-perturbative and thus dependent on v_4/v_0. In figure 3 we have provided contour plots of the correction function H_c for different values of the parameter v_4/v_0 whose physical meaning is given by equation (4.12). As one can see for $v_4 < v_0$ the contour plot is very similar to the one for the first order correction presented in figure 2. The analogies with the perturbative studies go even further. One can again see that the range of the correction function is seeded at the shell and is determined by the solution inside the shell. Indeed, let us evaluate the correction function $H_c(u,v)$ inside the shell along a ray $\tilde{u} = \kappa\,v$ starting at the origin. Using equation (4.14) For the we obtain:

$$H_c\left(\kappa\tilde{v},\tilde{v},\tilde{v}_4\right) = \frac{v_0^3}{v_4^3}\left(H\left(\kappa\tilde{v},\tilde{v}\right)\left((\kappa\tilde{v})^2+\tilde{v}^2\right)^{7/2}-1\right) = \frac{\left(\kappa^2+1\right)^{7/2} v_0^9\left(v_0^3+v_4^3\right)^{3/2}}{v_4^3\left(\left(\kappa^2+1\right)v_0^3+v_4^3\right)^{7/2}} - \frac{v_0^3}{v_4^3} , \tag{4.28}$$

which is a constant. The parameter κ is again in the range $[0,\infty)$, which cover all of the internal region of the shell. Furthermore one can check that the right-hand side of equation (4.28) is monotonically increasing function of κ. Therefore, to we obtain the minimum value of the correction function inside the shell, we take the limit $\kappa \to 0$ to obtain:

$$-\frac{v_0^3\left(2v_0^3+v_4^3\right)}{\left(v_0^3+v_4^3\right)^{2}} = -2 + \frac{3v_4^3}{v_0^3} + O\left(\frac{v_6}{v_0^6}\right) , \tag{4.29}$$



(a) A contour plot of H_c for $v_4 = 0.5v_0$.

(c) A contour plot of H_c for $v_4 = 1.0v_0$.

(b) A contour plot of H_c for $v_4 = 1.5v_0$.

(d) A contour plot of H_c for $v_4 = 2.0v_0$.

Figure 3. Contour plots of the correction function H_c from equation (4.27) for different values of the parameter v_4/v_0.

which to leading order agrees with the lower limit for the first order correction H_1. In the same way if take the limit $\kappa \to \infty$ we obtain the maximum value of the correction function H_c inside the shell:

$$\frac{(v_0^3 + v_4^3)^{3/2} - v_0^{9/2}}{v_0^{3/2} v_4^3} = \frac{3}{2} + \frac{3v_4^3}{8v_0^3} + O\left(\frac{v_6}{v_0^6}\right) , \tag{4.30}$$

which again to leading order agrees with the maximum value of the first order correction H_1. Furthermore examining again the contour plots in figure 3, we verify that the range ot the correction function H_c outside of the shell is seeded at the shell and one can understand the contour lines as curves with a $\sim 1/(u^2+v^2)^{7/2}$ fall off with a constant of proportionality seeded at the origin, where the correction function H_c is multi-valued while the full function H diverges.

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5 Discussion

We have provided a solution to a D0/D4 system where the D4-branes are displaced from the D0-branes with the displacement lying on a spherical shell around the D0s. The leading perturbative solution in N_f/N_c is given by (4.22) and (4.23) and presented in graphical form in figure 2. Just as in electrostatics, the solution interior to the shell is the same as that in the absence of the D4s, however the interior expression is modified from

$$H(u,v) = 1 + \frac{r_0^7}{r^7} \qquad \text{to} \qquad H(u,v) = 1 + \frac{\gamma^3 r_0^7}{r^7} \tag{5.1}$$

where $\gamma^2 = 1 + \frac{N_f}{N_c}\frac{\lambda}{2m_q^3}$ and $r^2 = u^2 + \gamma^2 v^2$ is the interior radial coordinate. The dependence on N_f/N_c may appear strange, however, it arises since the parameter r_0 is measured at infinity by following a direction radially outwards in u at fixed v. The non-perturbative (in N_f/Nc) solution, as can be seen from figure 3, is quite similar to the leading perturbative solution. The principal effect of increasing v_4/v_0 is that the geometry outside of the shell approaches that of the near horizon limit of the D4-branes. Indeed, let us consider the dilaton e^Φ from equation (3.21). If we use the analytic solution inside the shell (4.14) evaluated at $v = v_0$, it is easy to show that in the limit $v_4 \to \infty$, while keeping v_0 fixed, we have:

$$e^\Phi = \left(1 + \frac{v_4^3}{v_0^3}\right)^{-1/4} \left(1 + \frac{\left(1 + \frac{v_4^3}{v_0^3}\right)^{3/2} r_0^7}{\left(u^2 + \left(1 + \frac{v_4^3}{v_0^3}\right) v_0^2\right)^{7/2}}\right)^{3/4} = \left(\frac{v_0}{v_4}\right)^{3/4} + O\left[\left(\frac{v_0}{v_4}\right)^{15/4}\right], \tag{5.2}$$

which is the behaviour of the dilaton in the near horizon limit of the D4-brane background.

Our study is the begining of a larger one. The next step is to include the presence of a black hole in the dual geometry. This would be dual to the finite temperature Berkooz-Douglas model and would provide predictions for observables such as the internal energy and condensate of the matrix model as a function of temperature. Further generalisations would involve the mass deformed BD-model [38], i.e. the BMN model with fundamental flavours. This however would be especially challenging since the dual geometry to the BMN model is itself quite complicated.

Acknowledgments

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A Supersymmetry analysis

We impose the requirement that the gravitino variation vanish. Which is equivalent to showing the existence of a Killing spinor ε satisfying:

$$\delta\psi_\mu = \nabla_\mu \varepsilon + \frac{1}{12}\left(\Gamma_\mu \frac{1}{4!} F_{\lambda_1\lambda_2\lambda_3\lambda_4}\Gamma^{\lambda_1\lambda_2\lambda_3\lambda_4} - \frac{1}{2} F_{\mu\lambda_1\lambda_2\lambda_3}\Gamma^{\lambda_1\lambda_2\lambda_3}\right)\varepsilon\,. \tag{A.1}$$

One can easily check that:

$$\frac{1}{4!} F_{\lambda_1 \lambda_2 \lambda_3 \lambda_4} \Gamma^{\lambda_1 \lambda_2 \lambda_3 \lambda_4} = \frac{F'(v)}{K_4(u,v)^2} \bar{\Gamma}^{78910} , \tag{A.2}$$

$$\frac{1}{2} F_{\mu \lambda_1 \lambda_2 \lambda_3} \Gamma^{\lambda_1 \lambda_2 \lambda_3} = \begin{cases} 3 \frac{F'(v)}{K_4(u,v)^2} \Gamma_\mu \bar{\Gamma}^{78910} & \mu \in S^4 \\ 0 & \mu \notin S^4 \end{cases} , \tag{A.3}$$

where $\bar{\Gamma}^a$ are the flat gamma matrices. Therefore, the gravitino equation has the following form:

$$\delta\psi_\mu = \nabla_\mu \varepsilon - \frac{1}{6} \frac{F'(v)}{K_4(u,v)^2} \Gamma_\mu \bar{\Gamma}^{78910} \varepsilon \quad \text{for} \quad \mu \in S^4 , \tag{A.4}$$

$$\delta\psi_\mu = \nabla_\mu \varepsilon + \frac{1}{12} \frac{F'(v)}{K_4(u,v)^2} \Gamma_\mu \bar{\Gamma}^{78910} \varepsilon \quad \text{for} \quad \mu \notin S^4 . \tag{A.5}$$

Before we proceed to solve these equations we fix our conventions for the projections on the Killing spinor ε and 11D Clifford algebra: we consider:

$$\bar{\Gamma}^{012345678910} = \delta_1 , \tag{A.6}$$

$$\bar{\Gamma}^{012345} \varepsilon = \delta_2 \, \varepsilon , \tag{A.7}$$

$$\bar{\Gamma}_{01} \varepsilon = \delta_3 \, \varepsilon , \tag{A.8}$$

where:

$$\delta_1^2 = \delta_2^2 = \delta_3^2 = 1 . \tag{A.9}$$

The first equality (A.6) reflects the freedom that one has when using the 10D chirality matrix to construct the 11D Clifford algebra. The first projection (A.7) reflects the fact that the $\mathcal{M}$5-brane breaks the 11D Poincare invariance down to $\mathcal{P}_4 \times \mathrm{SO}(5)$, while the second projection reflects the fact that momentum along the x_{11} direction (labelled by '1' in the index notations) breaks the 11D Poincare invariance down to SO(9) invariance (rotational summetry in the transverse directions), which is also the symmetry of the D0-brane background in 10D. We see that the projections (A.7), (A.8) leave intact 1/4 of the original supersymmetry of the background which is expected for the D0/D4 brane intersection. We proceed by writing down the components of the gravitino equations.

Component along t. We obtain:

$$\begin{aligned} 0 = E_0^t \, \partial_t \, \varepsilon + \frac{\partial_u K_1 \bar{\Gamma}_{02} + \partial_v K_1 \bar{\Gamma}_{06}}{4 K_1 \, K_2^{1/2}} \varepsilon + \frac{K_3^{1/2} \left(\partial_u \, A_0 \, \bar{\Gamma}_{12} + \partial_v \, A_0 \, \bar{\Gamma}_{16} \right)}{4 K_1^{1/2} \, K_2^{1/2}} \varepsilon \\ + \frac{1}{12} \frac{F'(v)}{K_4^2} \bar{\Gamma}_0 \bar{\Gamma}^{78910} \varepsilon , \end{aligned} \tag{A.10}$$

where we have omitted the arguments of the functions K_i and A_0. Equations (A.6)–(A.8) imply that:

$$\bar{\Gamma}^{78910}\,\varepsilon = \delta_1\,\delta_2\,\bar{\Gamma}_6\,\varepsilon \tag{A.11}$$

$$\bar{\Gamma}_{1\,2}\,\varepsilon = -\delta_3\,\bar{\Gamma}_{0\,2}\,\varepsilon \tag{A.12}$$

$$\bar{\Gamma}_{1\,6}\,\varepsilon = -\delta_3\,\bar{\Gamma}_{0\,6}\,\varepsilon\,. \tag{A.13}$$

Substituting in equation (A.10) and equating to zero the coefficients in front of the independent projections we obtain:

$$0 = \partial_t\,\varepsilon \tag{A.14}$$

$$0 = \frac{\partial_u K_1}{K_1} - \delta_3\left(\frac{K_3}{K_1}\right)^{1/2}\partial_u\,A_0 \tag{A.15}$$

$$0 = \frac{\partial_v K_1}{K_1} - \delta_3\left(\frac{K_3}{K_1}\right)^{1/2}\partial_v\,A_0 + \delta_1\,\delta_2\,\frac{1}{3}\,\frac{F'(v)}{K_4^{3/2}}\,. \tag{A.16}$$

Component along x_{11}. We obtain:

$$0 = E_0^t\,\partial_t\,\varepsilon + E_1^{x_{11}}\,\partial_{x_{11}}\,\varepsilon - \frac{K_3^{1/2}\left(\partial_u A_0\,\bar{\Gamma}_{0\,2} + \partial_v A_0\,\bar{\Gamma}_{0\,6}\right)}{4K_1^{1/2}K_2^{1/2}}\,\varepsilon + \frac{\partial_u K_3\,\bar{\Gamma}_{1\,2} + \partial_v K_3\,\bar{\Gamma}_{1\,6}}{4K_3\,K_4^{1/2}}\,\varepsilon$$
$$+\frac{1}{12}\,\frac{F'(v)}{K_4^2}\,\bar{\Gamma}_1\,\bar{\Gamma}^{78910}\,\varepsilon\,. \tag{A.17}$$

Using equations (A.11)–(A.13) as well equations (A.14)–(A.16) we obtain:

$$0 = \partial_{x_{11}}\,\varepsilon \tag{A.18}$$

$$0 = \delta_3\,\frac{\partial_u K_3}{K_3} + \left(\frac{K_3}{K_1}\right)^{1/2}\partial_u A_0 \tag{A.19}$$

$$0 = \delta_3\,\frac{\partial_v K_3}{K_3} + \left(\frac{K_3}{K_1}\right)^{1/2}\partial_v A_0 + \delta_1\,\delta_2\,\delta_3\,\frac{1}{3}\,\frac{F'(v)}{K_4^{3/2}}\,. \tag{A.20}$$

Combining equations (A.15)–(A.16) and (A.19)–(A.20) we obtain:

$$0 = \frac{\partial_u K_1}{K_1} + \frac{\partial_u K_3}{K_3} \tag{A.21}$$

$$F'(v) = -\frac{3}{2}\delta_1\,\delta_2\,K_4^{3/2}\left(\frac{\partial_v K_1}{K_1} + \frac{\partial_v K_3}{K_3}\right)\,. \tag{A.22}$$

Component along u. We obtain:

$$0 = E_2^u\,\partial_u\,\varepsilon - \frac{K_3^{1/2}\,\partial_u A_0}{K_1^{1/2}\,K_2^{1/2}}\,\bar{\Gamma}_{0\,1}\,\varepsilon + \frac{\partial_v K_2}{K_2\,K_4^{1/2}}\,\bar{\Gamma}_{2\,6}\,\varepsilon + \frac{1}{3}\,\frac{F'(v)}{K_4^2}\,\bar{\Gamma}_2\,\bar{\Gamma}^{78910}\,\varepsilon\,. \tag{A.23}$$

Using (A.8) and (A.11) and setting to zero the independent components, we obtain:

$$E_2^u\,\partial_u\,\varepsilon = \delta_3\,\frac{K_3^{1/2}\,\partial_u A_0}{K_1^{1/2}\,K_2^{1/2}}\,\varepsilon\,, \tag{A.24}$$

$$F'(v) = -3\,\delta_1\,\delta_2\,K_4^{3/2}\,\frac{\partial_v\,K_2}{K_2}\,. \tag{A.25}$$

Component along v. We obtain:

$$0 = E_6^v\,\partial_v\varepsilon - \frac{K_3^{1/2}\,\partial_v A_0}{4K_1^{1/2}K_4^{1/2}}\,\bar\Gamma_{0\,1}\,\varepsilon - \frac{\partial_u K_4}{4K_1^{1/2}\,K_4}\,\bar\Gamma_{2\,6}\,\varepsilon + \frac{1}{12}\,\frac{F'(v)}{K_4^2}\,\bar\Gamma_6\,\bar\Gamma^{7\,8\,9\,10}\,\varepsilon\,. \tag{A.26}$$

Using equations (A.11)–(A.13) as well equations (A.14)–(A.16) and setting to zero the independent components we obtain:

$$\partial_u K_4 = 0\,, \tag{A.27}$$

$$E_6^v\,\partial_v\varepsilon = \delta_3\,\frac{K_3^{1/2}\,\partial_v A_0}{4K_1^{1/2}K_4^{1/2}}\,\varepsilon - \delta_1\,\delta_2\,\frac{1}{12}\,\frac{F'(v)}{K_4^2}\,\varepsilon\,. \tag{A.28}$$

Components along S^3. We obtain:

$$E_i^{\eta_i}\,\partial_{\eta_i}\varepsilon + \frac{\tilde\omega_i^{a\,b}}{4\,u\,K_2^{1/2}}\bar\Gamma_{a\,b}\,\varepsilon - \frac{2K_2 + u\,\partial_u K_2}{4\,u\,K_2^{3/2}}\,\bar\Gamma_{2\,i}\,\varepsilon + \frac{\partial_v K_2}{4\,K_2\,K_4^{1/2}}\,\bar\Gamma_{i\,6}\,\varepsilon + \frac{1}{12}\,\frac{F'(v)}{K_4^2}\,\bar\Gamma_i\,\bar\Gamma^{7\,8\,9\,10}\,\varepsilon\,, \tag{A.29}$$

where $\tilde\omega_i^{a\,b}$ is the flat spin-connection on the unit S^3. Now using that:

$$E_i^{\eta_i}\partial_{\eta_i}\,\varepsilon = \frac{1}{u\,K_2^{1/2}}\,\tilde E_i^{\eta_i}\partial_{\eta_i}\,\varepsilon \tag{A.30}$$

we can write the first two terms in (A.29) as:

$$E_i^{\eta_i}\partial_{\eta_i}\varepsilon + \frac{\tilde\omega_i^{a\,b}}{4\,u\,K_2^{1/2}}\bar\Gamma_{a\,b}\,\varepsilon = \frac{1}{u\,K_2^{1/2}}\tilde\nabla_i\,\varepsilon\,, \tag{A.31}$$

where $\tilde\nabla_i = \tilde E_i^{\eta_i}\,\tilde\nabla_{\eta_i}$ is the covariant derivative along S^3 in flat coordinates. Note that even though S^3 is an odd sphere we can still construct a Killing spinor satisfying:

$$\tilde\nabla_i\varepsilon = \pm\frac{1}{2}\gamma\,\bar\Gamma_i\,\varepsilon\,, \tag{A.32}$$

where $\{\gamma\,,\,\bar\Gamma_i\} = 0$ and $\gamma^2 = 1$. We choose $\gamma = \bar\Gamma_2$ and a positive sign. Now using equation (A.11) and isolating the independent components we obtain:

$$\partial_u K_2 = 0\,, \tag{A.33}$$

$$F'(v) = -\delta_1\,\delta_2\,3\,K_4^{3/2}\,\frac{\partial_v K_2}{K_2}\,. \tag{A.34}$$

Components along S^4. We obtain:

$$E_m^{\xi_m}\partial_{\xi_m}\varepsilon + \frac{\tilde\omega_m^{a\,b}}{4\,v\,K_4^{1/2}}\,\bar\Gamma_{a\,b}\,\varepsilon - \frac{\partial_u K_4}{4\,K_2^{1/2}\,K_4}\,\bar\Gamma_{2\,m}\,\varepsilon - \frac{2K_4 + v\,\partial_v K_4}{4\,v\,K_4^{3/2}}\,\bar\Gamma_{6\,m}\,\varepsilon - \frac{1}{6}\,\frac{F'(v)}{K_4^2}\,\bar\Gamma_m\,\bar\Gamma^{7\,8\,9\,10}\,\varepsilon\,. \tag{A.35}$$

Now using that:

$$E_m^{\xi_m}\partial_{\xi_m}\,\varepsilon = \frac{1}{v\,K_4^{1/2}}\,\tilde{E}_m^{\xi_m}\partial_{\xi_m}\,\varepsilon\,, \tag{A.36}$$

we can write the first two terms in (A.35) as:

$$E_m^{\xi_m}\partial_{\xi_m}\varepsilon + \frac{\tilde{\omega}_m^{a\,b}}{4\,v\,K_4^{1/2}}\,\bar{\Gamma}_{a\,b}\,\varepsilon = \frac{\tilde{\nabla}_m\varepsilon}{v\,K_4^{1/2}}\,, \tag{A.37}$$

where $\tilde{\nabla}_m$ is the covariant derivative along the unit S^4 along the flat component m. Since S^4 is an even sphere we can write:

$$\tilde{\nabla}_m\varepsilon = \frac{1}{2}\bar{\Gamma}^{7\,8\,9\,10}\,\bar{\Gamma}_m\,\varepsilon = -\frac{1}{2}\bar{\Gamma}_m\,\bar{\Gamma}^{7\,8\,9\,10}\,\varepsilon = -\frac{1}{2}\,\bar{\Gamma}_{m\,6}\,\varepsilon\,, \tag{A.38}$$

where we used (A.11). Using all that and separating the independent components in equation (A.35) we obtain:

$$\partial_u K_4 = 0 \tag{A.39}$$

$$F'(v) = \delta_1\,\delta_2\,\frac{3}{2}\,K_4^{1/2}\,\partial_v K_4\,. \tag{A.40}$$

Finally, we put all equations together (already making some obvious simplifications):

$$0 = \frac{\partial_u K_1}{K_1} - \delta_3\left(\frac{K_3}{K_1}\right)^{1/2}\,\partial_u\,A_0 \tag{A.41}$$

$$0 = \frac{\partial_v K_1}{K_1} - \delta_3\left(\frac{K_3}{K_1}\right)^{1/2}\,\partial_v\,A_0 + \delta_1\,\delta_2\,\frac{1}{3}\frac{F'(v)}{K_4^{3/2}} \tag{A.42}$$

$$0 = \frac{\partial_u K_1}{K_1} + \frac{\partial_u K_3}{K_3} \tag{A.43}$$

$$F'(v) = -\frac{3}{2}\delta_1\,\delta_2\,K_4^{3/2}\left(\frac{\partial_v K_1}{K_1} + \frac{\partial_v K_3}{K_3}\right) \tag{A.44}$$

$$F'(v) = -3\,\delta_1\,\delta_2\,K_4^{3/2}\,\frac{\partial_v\,K_2}{K_2} \tag{A.45}$$

$$F'(v) = \delta_1\,\delta_2\,\frac{3}{2}\,K_4^{1/2}\,\partial_v K_4 \tag{A.46}$$

$$\partial_u K_2 = 0 \tag{A.47}$$

$$\partial_u K_4 = 0 \tag{A.48}$$

It is not difficult to check that with the choice $\delta_1 = \delta_2$ and $\delta_3 = 1$ we have the following solution to equations (A.41)–(A.48):

$$K_1 = \left(1 + \frac{v_5^3}{v^3}\right)^{-1/3} H(u,v)^{-1} \tag{A.49}$$

$$K_2 = \left(1 + \frac{v_5^3}{v^3}\right)^{-1/3} \tag{A.50}$$

$$K_3 = \left(1 + \frac{v_5^3}{v^3}\right)^{-1/3} H(u,v) \tag{A.51}$$

$$K_4 = \left(1 + \frac{v_5^3}{v^3}\right)^{2/3} \tag{A.52}$$

$$A_0(u,v) = H(u,v)^{-1} - 1 \tag{A.53}$$

$$F(v) = 1 + \frac{v_5^3}{v^3} \tag{A.54}$$

where in equation (A.53) we have fixed a constant of integration demanding that if $H \to 1$ at infinity then $A_0 \to 0$.

B Using the Garfinkle-Vachaspati method

The Garfinkle-Vachaspati (GV) method [10, 11] applies to space-times solutions extremizing the action:

$$S = \int d^d x \sqrt{-g} \left(R - \frac{1}{2} \sum_i \alpha_i(\phi) (\nabla \phi_i)^2 - \frac{1}{2} \sum_p \beta_p(\phi) F^2_{(p+1)} \right), \tag{B.1}$$

which clearly includes eleven dimensional supergravity. It is also required that the solutions to (B.1) admit hypersurface-orthogonal Killing field k^μ satisfying:

$$\begin{aligned} k_\mu\, k^\mu &= 0, \\ \nabla_{(\mu} k_{\nu)} &= 0, \\ \nabla_{[\mu} k_{\nu]} &= k_{[\mu} \nabla_{\nu]} S, \end{aligned} \tag{B.2}$$

where S is a scalar. The new deformed metric $G_{\mu\nu}$ is then constructed as:

$$G_{\mu\nu} = g_{\mu\nu} + e^S\, H\, k_\mu\, k_\nu\,, \tag{B.3}$$

where $g_{\mu\nu}$ is the old metric and the scalar H satisfies:

$$k^\mu \nabla_\mu H = 0 \quad \text{and} \quad \nabla^2 H = 0 \tag{B.4}$$

In ref. [12] the authors applied GV method to various membranes to study the possibility of asymptotically plane wave spacetimes which admit an event horizon. Our goal is more modest we will use the GV method as a shortcut to deform the $\mathcal{M}5$-brane solution into the uplift of the backreacted D0/D4-brane intersection. We start by the observation that the uplift of the D0-brane solution to eleven dimensions can be constructed as GV deformation of flat space-time. Indeed, it is easy to see that the metric:

$$ds_{11}^2 = -d\tilde{t}^2 + d\tilde{x}_{11}^2 + dr^2 + r^2 d\Omega_8^2 \,. \tag{B.5}$$

admits a hypersurface-orthogonal Killing field (with $S = 0$). To this end consider light-cone coordinates:

$$t = \left(\tilde{t} - \tilde{x}_{11}\right)/\sqrt{2} \tag{B.6}$$

$$x_{11} = \left(\tilde{t} + \tilde{x}_{11}\right)/\sqrt{2} \,. \tag{B.7}$$

The flat metric is now:

$$ds_{11}^2 = 2dt\, dx_{11} + dr^2 + r^2 d\Omega_8^2 \,. \tag{B.8}$$

and clearly the vector field $k = \partial/\partial x_{11}$ satisfies the requirements (B.2) with $S = 0$. Next we consider the deformation:

$$G_{\mu\nu} = g_{\mu\nu} + H\, \delta_\mu^{x_{11}} \delta_\nu^{x_{11}} \,, \tag{B.9}$$

to obtain:

$$\begin{aligned} ds_{11}^2 &= 2dt\, dx_{11} + H dx_{11}^2 + dr^2 + r^2 d\Omega_8^2 \\ &= -H^{-1} dt^2 + H\left(dx_{11} + H^{-1} dt\right)^2 + dr^2 + r^2 d\Omega_8^2 \,, \end{aligned} \tag{B.10}$$

which is the metric of the uplift of the D0-brane to eleven dimensions. Note that the choice $H \propto 1 + r_0^7/r^7$ satisfies equations (B.4).

Encouraged by this observation we repeat the same procedure this time starting with the $\mathcal{M}$5-brane background written in light-cone coordinates:

$$ds_{11}^2 = \left(1 + \frac{v_5^3}{v^3}\right)^{-1/3} \left(2dt\, dx_{11} + du^2 + u^2\, d\Omega_3^2\right) + \left(1 + \frac{v_5^3}{v^3}\right)^{2/3} \left(dv^2 + v^2\, d\Omega_4^2\right) \tag{B.11}$$

One can check that the field $k = \partial/\partial x_{11}$ again satisfies equations (B.4) with $S = 0$ and we can apply the deformation (B.9). The result (after the translation $x_{11} \to x_{11} - t$) is the metric (3.12) which we duplicate below:

$$\begin{aligned} ds_{11}^2 = \left(1 + \frac{v_5^3}{v^3}\right)^{-1/3} &\left(-H(u,v)^{-1}\, dt^2 + H(u,v)\left(dx_{11} + (H(u,v)^{-1} - 1)\, dt\right)^2 + \right. \\ &\left. du^2 + u^2\, d\Omega_3^2\right) + \left(1 + \frac{v_5^3}{v^3}\right)^{2/3} \left(dv^2 + v^2\, d\Omega_4^2\right) \,. \end{aligned}$$

Note also that the first equation in (B.2) is satisfied with we consider $H = H(u,v)$, while the second equation is the harmonic equation (3.18):

$$\partial_v^2 H(u,v) + \frac{4}{v}\partial_v H(u,v) + \left(1 + \frac{v_5^3}{v^3}\right)\left(\partial_u^2 H(u,v) + \frac{3}{u}\partial_u H(u,v)\right) = 0 \,.$$

C The D2/D6 system revisited

In this section we revisit the D2/D6 system studied in ref. [2], where a fully localized supergravity solution of the system was constructed. The 10D metric obtained in ref. [2] is:

$$\begin{aligned} ds^2 = & H(y,r)^{-1/2}\left(1+\frac{2m}{r}\right)^{-1/2}\left(-dt^2+dx_1^2+dx_2^2\right) \\ & +H(y,r)^{1/2}\left(1+\frac{2m}{r}\right)^{-1/2}\left(dy^2+y^2d\Omega_3^2\right)+H(y,r)^{1/2}\left(1+\frac{2m}{r}\right)^{1/2}\left(dr^2+r^2d\Omega_2^2\right), \end{aligned} \tag{C.1}$$

where $H(y,r)$ is a solution of the harmonic equation:

$$\left(\frac{1}{1+\frac{2m}{r}}\right)\left(\frac{\partial^2}{\partial r^2}+\frac{2}{r}\frac{\partial}{\partial r}\right)H(y,r)+\nabla_y^2 H(y,r)=0\,. \tag{C.2}$$

The authors of ref. [2], considered the Fourier transform of H:

$$H(y,r)=1+Q_{M2}\int\frac{d^4p}{(2\pi)^4}e^{ipy}H_p(r)\,, \tag{C.3}$$

where $H_p(r)$ is a solution of the ordinary differential equation:

$$\left(\frac{1}{1+\frac{2m}{r}}\right)\left(\frac{\partial^2}{\partial r^2}+\frac{2}{r}\frac{\partial}{\partial r}\right)H_p(r)-p^2H_p(r)=0\,. \tag{C.4}$$

The solution of equation (C.4) regular at infinity is [2]:

$$H_p(r)=c_p e^{-pr}\mathcal{U}(1+pm,2,2pr)\,, \tag{C.5}$$

where $\mathcal{U}(a,b,z)$ is the confluent hypergeometric function. The normalization factor c_p was fixed in ref. [2] to:

$$c_p=\frac{\pi^2}{8}\frac{1}{m^2}(pm)^2\Gamma(pm)\,. \tag{C.6}$$

by requiring that in the limit $m\to\infty$ while keeping $z^2=8mr$ fixed, one has:

$$H_p(z)=\frac{\pi^2}{2z^2}\,pz\,K_1(pz)\,, \tag{C.7}$$

corresponding to:

$$H(y,z)=1+Q_{M2}\int\frac{d^4p}{(2\pi)^4}e^{ipy}H_p(z)=1+\frac{Q_{M2}}{(y^2+z^2)^3}\,. \tag{C.8}$$

To arrive at this result the authors of ref. [2] exploited the fact that in limit $m\to\infty$ (with fixed $z^2=8mr$) the 11D uplift of the geometry is that of a $M2$-membrane, which suggests the asymptotic form (C.8)). Clearly this is a property very specific to the D2/D6 intersection and the fact that the 11D uplift of the D6-branes is a magnetic monopole realised as a Taub-NUT geometry. We will show how to arrive at the same result following the method applied to the D0/D4 system.

To this end, we distribute the D6-branes on a shell of radius r_0 surrounding the D2-branes. inside the shell (for $r < r0$) the density of the D6-brane Ramond-Ramond charge vanishes and the corresponding warp factor is constant. Imposing continuity at the shell we arrive at the following form of the 10D metric for $r \leq r_0$:

$$\begin{aligned} ds^2 = {} & H(y,r)^{-1/2}\left(1+\frac{2m}{r_0}\right)^{-1/2}\left(-dt^2+dx_1^2+dx_2^2\right) \\ & +H(y,r)^{1/2}\left(1+\frac{2m}{r_0}\right)^{-1/2}\left(dy^2+y^2d\Omega_3^2\right)+H(y,r)^{1/2}\left(1+\frac{2m}{r_0}\right)^{1/2}\left(dr^2+r^2d\Omega_2^2\right). \end{aligned} \tag{C.9}$$

Now we define:

$$\tilde{t} = \left(1+\frac{2m}{r_0}\right)^{-1/4} t\,, \tag{C.10}$$

$$\tilde{x}_i = \left(1+\frac{2m}{r_0}\right)^{-1/4} x_i\,, \quad \text{for} \quad i=1,2\,; \tag{C.11}$$

$$\tilde{y} = \left(1+\frac{2m}{r_0}\right)^{-1/4} y\,, \tag{C.12}$$

$$\tilde{r} = \left(1+\frac{2m}{r_0}\right)^{1/4} r\,. \tag{C.13}$$

resulting in the metric:

$$ds^2 = H(\tilde{y},\tilde{r})^{-1/2}\left(-d\tilde{t}^2+d\tilde{x}_1^2+d\tilde{x}_2^2\right)+H(\tilde{y},\tilde{r})^{1/2}\left(d\tilde{y}^2+\tilde{y}^2d\Omega_3^2+d\tilde{r}^2+\tilde{r}^2d\Omega_2^2\right) \tag{C.14}$$

The most symmetric solution is then that of a D2-brane:

$$H(\tilde{y},\tilde{r}) = 1+\frac{Q_{D2}\left(1+\frac{2m}{r_0}\right)^{-3/4}}{(\tilde{y}^2+\tilde{r}^2)^{5/2}}\,, \tag{C.15}$$

where $Q_{D2} = (3/64m)Q_{M2}$. Going back to the original variables we have:

$$H(y,r) = 1+\frac{Q_{D2}\left(1+\frac{2m}{r_0}\right)^{1/2}}{\left(y^2+\left(1+\frac{2m}{r_0}\right)r^2\right)^{5/2}} = 1+Q_{M2}\int\frac{d^4p}{(2\pi)^4}e^{ipy}H_p^{(0)}(r,r_0)\,, \tag{C.16}$$

where:

$$H_p^{(0)}(r,r_0) = \frac{\pi^2}{16m}\frac{e^{-p\,r\left(1+\frac{2m}{r_0}\right)^{1/2}}}{r}\,. \tag{C.17}$$

Next we consider a solution of the form ([C.5]) outside of the shell ($r > r_0$):

$$H_p(r) = b_p e^{-p\,r}\mathcal{U}(1+pm,2,2pr)\,, \tag{C.18}$$

and impose continuity across the shell. Namely, we require that:

$$H_p(r_0) = H_p^{(0)}(r_0,r_0)\,, \tag{C.19}$$

which is:

$$b_p e^{-p r_0}\mathcal{U}(1+pm,2,2pr_0) = \frac{\pi^2}{16m}\frac{e^{-p\, r_0\left(1+\frac{2m}{r_0}\right)^{1/2}}}{r_0} \tag{C.20}$$

and can be used to determine the constant of integration b_p as a function of p. We obtain:

$$b_p = \frac{\pi^2}{16m}\frac{e^{-p\, r_0\left[\left(1+\frac{2m}{r_0}\right)^{1/2}-1\right]}}{r_0\,\mathcal{U}(1+pm,2,2pr_0)}\,. \tag{C.21}$$

Now taking the $r_0 \to 0$ limit:

$$\lim_{r_0\to 0} b_p = \lim_{r_0\to 0}\left(\frac{\pi^2}{16m}\frac{e^{-p\, r_0\left[\left(1+\frac{2m}{r_0}\right)^{1/2}-1\right]}}{r_0\,\mathcal{U}(1+pm,2,2pr_0)}\right) = \frac{\pi^2}{8}\frac{1}{m^2}(pm)^2\Gamma(pm)\,, \tag{C.22}$$

we recover the result (C.6).

D Supegravity action and delta function sources

The IIA Supergravity action coupled to D4-brane action is given by:

$$\begin{aligned} S_{IIA} = {}& \frac{1}{2\kappa_{10}^2}\int dx^{10}x\sqrt{-\tilde{g}}\left\{\tilde{\mathcal{R}} - \frac{1}{2}\partial_\mu\Phi\,\partial^\mu\Phi - \frac{e^{-\Phi}}{2}|H_{(3)}|^2 + \right. \\ & \left. -\frac{e^{\frac{3\Phi}{2}}}{2}|F_{(2)}|^2 - \frac{e^{\frac{\Phi}{2}}}{2}|F_{(4)}|^2\right\} - \frac{1}{4\kappa_{10}^2}\int B_{(2)}\wedge dC_{(3)}\wedge dC_{(3)} + \\ & N_f T_4\int d^5\xi\, e^{\Phi/4}\sqrt{\det[\hat{g}_{ab}+2\pi\alpha' e^{-\Phi/2}\mathcal{F}_{ab}]} + N_f T_4\int_{\mathcal{M}_5} C\wedge e^{\mathcal{F}}\,, \end{aligned} \tag{D.1}$$

where $\hat{g}_{ab}$ is the pull back of the metric on the worldvolume of the D4-branes and $\mathcal{F}_{ab} = B_{ab}/(2\pi\alpha') + F_{ab}$, where B_{ab} is the pullback of the Kalb-Rammond B-field and F_{ab} is the U(1) gauge field of the D4-branes. The last two terms in equation (D.1) are the DBI and WZ actions of the D4-brane. Choosing co-moving frame for the D4-brane $x^\mu = (\xi^a, y^m)$ the DBI action can be written in a ten dimensional form as:

$$S_{DBI} = \int d^{10}x\, e^{\Phi/4}\sqrt{\det\left[\hat{g}_{ab}+2\pi\alpha' e^{-\Phi/2}\mathcal{F}_{ab}\right]}\;\delta^{(5)}\left(x^m - X^m(\xi)\right)\,, \tag{D.2}$$

where the functions $X^m(\xi)$ for $m = 1,\ldots,5$ describe the embedding of the D4-brane. One can see that varying the action (D.1) with respect to the ten dimensional metric will result in a delta function source term in the Einstein equations.

E Numerical techniques

To solve numerically equation (3.28) we use shooting techniques available in the NDSolve method of Wolfram Mathematica. We also constructed the solution in python using the method 'odeint' from the scipy packages, which gave equivalent results. The results presented in the paper were obtained in Mathematica.

To employ a shooting technique we have to specify both the value and derivative of the function $h(p, v)$ and the shell $v = v_0$. While the value $h(p, v_0)$ is fixed by imposing continuity and using the analytic solution inside the shell, the first derivative is obtained using a searching procedure. The criteria to select the correct value of the first derivative is to obtain a regular solution at infinity. To this end one employs the perturbative solution at large v ($v \gg v_0$) and imposes the constraint that $B_n(p)$ in equation (3.39) vanishes.

The final step to obtain the numerical solution is to perform numerically the inverse Fourier transform integrating over the numerically generated function $h(p, v)$. A challenge here is integrating numurerically for large momenta p. To improve the accuracy of the behavior of the function $h(p, v)$ for large p ip to a cutoff λ_p is approximated parametrically and the integration in the open interval $[\Lambda_p, \infty)$ is performed analytically over the approximated function.

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