Published for SISSA by Springer

Received: *February 17, 2022*
Revised: *April 21, 2022*
Accepted: *June 2, 2022*
Published: *July 1, 2022*

Aiming for unification of $L_\mu - L_\tau$ and the standard model gauge group

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Abstract: In this letter we show a kind of $L_\mu - L_\tau$ gauge symmetry can be unified into a simple group E_7 with the standard model gauge symmetry in the context of coset space unification. We also discuss some implication from one of this kind of unification.

Keywords: Grand Unification, Theories of Flavour, New Gauge Interactions, Supersymmetry

ArXiv ePrint: 2106.01520

https://doi.org/10.1007/JHEP07(2022)011

Contents

1 Introduction

The standard model (SM) is a very successful theory and well established. Indeed almost all terrestrial experiments are explained by SM. However, there are several questions: how to explain the lepton flavor violation appearing in neutrino oscillation [1], the existence of dark matter, the baryon asymmetry, the discrepancy of muon anomalous magnetic moment between the theoretical [2] and experimental value [3], etc. To explain (some of) them many trials have been made by extending the SM. As one of the direction in these years many physicists extend the gauge symmetry, say, $L_\mu - L_\tau$ [4–7]. It gives plausible explanation [8–10] for muon anomalous magnetic moment and, in addition, may give a solution to Hubble inconsistency [11, 12] and IceCube Gap too [13–16].

Besides those phenomenological questions, there are fundamental and/or conceptual questions in SM: why are there three gauge groups SU(3), SU(2), U(1)? Why does nature has matters, in other words, for example why do quarks behave as a triplet of SU(3)? Why are there three copies of materials? Why does exist three generations? The former can be explained partly by the unification of the gauge group, the Grand Unified Theory (GUT).

Then it is natural to ask whether $L_\mu - L_\tau$ can be unified with the standard model gauge group, i.e. grander unified theory. Naively it looks difficult since (i) only leptons have $L_\mu - L_\tau$ charge and (ii) there is a generation dependence. In unified theories quarks are unified into same multiplet with leptons and hence not only leptons but also quarks should have $L_\mu - L_\tau$ charge. Therefore we have to give up a simple (only leptophilic) $L_\mu - L_\tau$ and we also assign its charge to quarks. On the contrary generation dependence means that $L_\mu - L_\tau$ is gauged family symmetry. Implementing them appropriately, in this letter we will see a kind of $L_\mu - L_\tau$ and SM gauge group are unified into a simple group E_7 within the context of coset space unification [17], a supersymmetric extension of nonlinear sigma model.

In section 2 we give a short review of coset space unification. Then In section 3 we show candidate assignments of $L_\mu - L_\tau$ and their interpretation. Finally we give a summary and discussion in section 4.

SU(5)	$\mathrm{U}(1)_1$	$\mathrm{U}(1)_2$	$\mathrm{U}(1)_3$
$\mathbf{10}_1$	0	0	4
$\mathbf{10}_2$	0	3	-1
$\mathbf{10}_3$	2	-1	-1
$\mathbf{5}^*_1$	0	3	3
$\mathbf{5}^*_2$	2	-1	3
$\mathbf{5}^*_3$	2	2	-2
$\mathbf{1}_1$	0	3	-5
$\mathbf{1}_2$	2	-1	-5
$\mathbf{1}_3$	2	-4	0

Table 1. U(1) charges of the NG multiplets. The $\mathrm{U}(1)_1$, $\mathrm{U}(1)_2$ and $\mathrm{U}(1)_3$ are the unbroken U(1)'s of coset-subspaces $\mathrm{E}_7/\mathrm{E}_6{\times}\mathrm{U}(1)$, $\mathrm{E}_6/\mathrm{SO}(10){\times}\mathrm{U}(1)$ and $\mathrm{SO}(10)/\mathrm{SU}(5){\times}\mathrm{U}(1)$, respectively.

2 Coset space unification

We first review the structure of coset space unification. Three family fermions including right-handed neutrinos naturally are accommodated in the coset-space family unification [18] in supersymmetric (SUSY) GUTs. Coset-spaces based on E_7 are known as unique choices to contain three families of quarks and leptons [19]. Among them $\mathrm{E}_7/\mathrm{SU}(5){\times}\mathrm{U}(1)^3$ is the most interesting, since it contains also three families of right-handed neutrinos as Nambu-Goldstone (NG) multiplets [20]. This model contains three families of $\mathbf{10}_i+\mathbf{5}^*_i+\mathbf{1}_i$ ($i=1,2,3$) as NG multiplets. Though in addition, there is an extra $\mathbf{5}$, we ignore it in this letter. Here, the SU(5) is the usual GUT gauge group. Their quantum numbers under the unbroken subgroup are given in table 1. Incidentally, though there is an extra $\mathbf{5}$, we will ignore it in this letter hereafter as it is irrelevant. These U(1)'s are interpreted as those from the breaking chain

$$
\begin{aligned}
\mathrm{E}_7 &\longrightarrow \mathrm{E}_6\times\mathrm{U}(1)_1 \longrightarrow \mathrm{SO}(10)\times\mathrm{U}(1)_1\times\mathrm{U}(1)_2 \\
&\longrightarrow \mathrm{SU}(5)\times\mathrm{U}(1)_1\times\mathrm{U}(1)_2\times\mathrm{U}(1)_3\,.
\end{aligned}
\tag{2.1}
$$

The subscripts of fields in table 1 (and also following tables) are not generation indices but the embedding (or definition) of fields into E_7 adjoint representation. For example at the first breaking, $\mathbf{1}_3$, $\mathbf{1}_2$, $\mathbf{5}^*_3$, $\mathbf{5}^*_2$, and $\mathbf{10}_3$ appear in the spectrum. Similarly $\mathbf{1}_1$ $\mathbf{5}^*_1$ $\mathbf{10}_2$ does at the second breaking. At the last $\mathbf{10}_1$ arises.

3 Rearrangement of U(1) charge for $\mathbf{L}_\mu-\mathbf{L}_\tau$

As U(1)'s are commutable and hence we can take linear combination, that is,

$$
Q_i = a_{ij}q_j, \tag{3.1}
$$

where q_j is given in the table 1 and Q_i's are the new U(1) charges. The existence of $\mathrm{L}_\mu-\mathrm{L}_\tau$ indicates that one of rearranged U(1), say new $\mathrm{U}(1)_3$, charge Q_3 for a pair of $\mathbf{10}$ and $\mathbf{5}$

SU(5)	$\mathrm{U}(1)_1$	$\mathrm{U}(1)_2$	$\mathrm{U}(1)_{3=\mu-\tau}$
$\mathbf{10}_1$	0	3	1
$\mathbf{10}_2$	0	3	−1
$\mathbf{10}_3$	2	−2	0
$\mathbf{5}^*_1$	0	6	0
$\mathbf{5}^*_2$	2	1	1
$\mathbf{5}^*_3$	2	1	−1
$\mathbf{1}_1$	0	0	−2
$\mathbf{1}_2$	2	−5	−1
$\mathbf{1}_3$	2	−5	1

Table 2. U(1) charges of the NG multiplets in breaking (3.2). The $\mathrm{U}(1)_1$, $\mathrm{U}(1)_2$ and $\mathrm{U}(1)_3$ are the unbroken U(1)'s of coset-subspaces $\mathrm{E}_7/\mathrm{E}_6\times\mathrm{U}(1)$, $\mathrm{E}_6/\mathrm{SU}(5)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ and SU(2)/U(1), respectively.

must be $\pm1, 0$. Indeed there are three kinds of such recombination, given by

$$a_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{5}{4} & -\frac{3}{4} \\ 0 & -\frac{1}{4} & \frac{1}{4} \end{pmatrix} \tag{3.2}$$

$$= \begin{pmatrix} \frac{2}{3} & \frac{1}{12} & \frac{1}{4} \\ 0 & \frac{5}{4} & -\frac{3}{4} \\ -\frac{1}{3} & \frac{1}{12} & \frac{1}{4} \end{pmatrix} \tag{3.3}$$

$$= \begin{pmatrix} \frac{2}{3} & \frac{1}{3} & 1 \\ -\frac{5}{3} & -\frac{5}{6} & -\frac{1}{2} \\ -\frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}. \tag{3.4}$$

Each solution gives independent breaking chain.

The first recombination leads new U(1) charges given in table 2. These U(1)'s are interpreted as residual one from the breaking chain

$$\begin{aligned} \mathrm{E}_7 &\longrightarrow \mathrm{E}_6\times\mathrm{U}(1)_1 \longrightarrow \mathrm{SU}(5)\times\mathrm{SU}(2)\times\mathrm{U}(1)_1\times\mathrm{U}(1)_2 \\ &\longrightarrow \mathrm{SU}(5)\times\mathrm{U}(1)_1\times\mathrm{U}(1)_2\times\mathrm{U}(1)_{3=\mu-\tau}\,. \end{aligned} \tag{3.5}$$

In this chain both ($\mathbf{5}^*_3$, $\mathbf{5}^*_2$) and ($\mathbf{10}_2$, $\mathbf{10}_1$) appear as SU(2) doublet at the second breaking.

The second one arises from the breaking chain

$$\begin{aligned} \mathrm{E}_7 &\longrightarrow \mathrm{SO}(10)\times\mathrm{SU}(2)\times\mathrm{U}(1)_1 \\ &\longrightarrow \mathrm{SU}(5)\times\mathrm{SU}(2)\times\mathrm{U}(1)_1\times\mathrm{U}(1)_2 \\ &\longrightarrow \mathrm{SU}(5)\times\mathrm{U}(1)_1\times\mathrm{U}(1)_2\times\mathrm{U}(1)_{3=\mu-\tau}\,. \end{aligned} \tag{3.6}$$

Their U(1) charges are shown in table 3. In this chain both SO(10) $\mathbf{16}$=($\mathbf{10}_1$+$\mathbf{5}^*_1$+$\mathbf{1}_1$) and ($\mathbf{10}_3$+$\mathbf{5}^*_3$+$\mathbf{1}_3$) form an SU(2) doublet at the first breaking. Note that U(1) charges for $\mathbf{1}_1$

SU(5)	$\mathrm{U}(1)_1$	$\mathrm{U}(1)_2$	$\mathrm{U}(1)_{3=\mu-\tau}$
$\mathbf{10}_1$	1	-1	1
$\mathbf{10}_2$	0	4	0
$\mathbf{10}_3$	1	-1	-1
$\mathbf{5}_1^*$	1	3	1
$\mathbf{5}_2^*$	2	-2	0
$\mathbf{5}_3^*$	1	3	-1
$\mathbf{1}_1$	1	-5	1
$\mathbf{1}_2$	0	0	-2
$\mathbf{1}_3$	1	-5	-1

Table 3. U(1) charges of the NG multiplets in breaking (3.3). The $\mathrm{U}(1)_1$, $\mathrm{U}(1)_2$ and $\mathrm{U}(1)_3$ are the unbroken U(1)'s of coset-subspaces $\mathrm{E}_7/\mathrm{SO}(10)\times\mathrm{SU}(2)\times\mathrm{U}(1)$, $\mathrm{SO}(10)/\mathrm{SU}(5)\times\mathrm{U}(1)$ and $\mathrm{SU}(2)/\mathrm{U}(1)$, respectively.

SU(5)	$\mathrm{U}(1)_1$	$\mathrm{U}(1)_2$	$\mathrm{U}(1)_{3=\mu-\tau}$
$\mathbf{10}_1$	4	2	0
$\mathbf{10}_2$	0	-3	1
$\mathbf{10}_3$	0	-3	-1
$\mathbf{5}_1^*$	4	-1	1
$\mathbf{5}_2^*$	4	-1	-1
$\mathbf{5}_3^*$	0	-6	0
$\mathbf{1}_1$	-4	-5	-1
$\mathbf{1}_2$	-4	-5	1
$\mathbf{1}_3$	0	0	-2

Table 4. U(1) charges of the NG multiplets in breaking (3.4). The $\mathrm{U}(1)_1$, $\mathrm{U}(1)_2$ and $\mathrm{U}(1)_3$ are the unbroken U(1)'s of coset-subspaces $\mathrm{E}_7/\mathrm{SU}(6)\times\mathrm{SU}(2)\times\mathrm{U}(1)$, $\mathrm{SU}(6)/\mathrm{SU}(5)\times\mathrm{SU}(1)$ and $\mathrm{SU}(2)/\mathrm{U}(1)$, respectively.

are reversed from the naive change by (3.3). It is due to the fact that we always have a freedom of choice to extracting a representation $\mathbf{r}$ or $\mathbf{r}^*$ as NG boson. As we need GUT representation while we have no choice to select say, $\mathbf{10}^*$, we can switch $\mathbf{1}$ to $\mathbf{1}^*$. It may lead drastic change of phenomenology though we will not touch this point in this letter.

The final one corresponds to the breaking chain

$$\begin{aligned}\mathrm{E}_7 &\longrightarrow (\mathrm{SU}(6)\times\mathrm{SU}(2)\times\mathrm{U}(1)_1)\\ &\longrightarrow \mathrm{SU}(5)\times\mathrm{SU}(2)\times\mathrm{U}(1)_1\times\mathrm{U}(1)_2\\ &\longrightarrow \mathrm{SU}(5)\times\mathrm{U}(1)_1\times\mathrm{U}(1)_2\times\mathrm{U}(1)_{3=\mu-\tau}\,. \end{aligned} \tag{3.7}$$

Their U(1) charges are shown in table 4. In this chain both $(\mathbf{5}_1^*, \mathbf{5}_2^*)$ and $(\mathbf{10}_2, \mathbf{10}_3)$ appear as SU(2) doublet at the second breaking.

Again U(1) charges for $\mathbf{1}_1$ and $\mathbf{1}_2$ are reversed from the naive change by (3.4). In addition in this breaking chain we should not have the first stage, that is, we should interpret that E_7 breaks directly to $\mathrm{SU}(5)\times\mathrm{SU}(2)\times\mathrm{U}(1)$. Otherwise we could not realize

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three **10**'s. These lead drastic change of phenomenology too though we will not touch this point in this letter.

Thus there are essentially these three chains. It is understood by following two steps. The first one is that there are four maximal subgroups of E_7 including SU(5), which are $E_6\times$U(1), SO(12)$\times$SU(2), SU(8), and SU(6)$\times$SU(3). The second is among them we can directly check that it is impossible to get three **10** and three $\mathbf{5}^*$ via SU(8) by direct calculation. We note also that these three chains are independent. It is understood by the fact that U(1) charges for "right-handed neutrinos" are different. Therefore in each chain, in principle, we will have a quite different phenomenology for, at least, neutrino physics.

Incidentally, new $\mathrm{U}(1)_1$ and $\mathrm{U}(1)_2$ for the first breaking chain while q_2 and q_3 are exchangeable in the second and the third chain. This exchange may lead to different phenomenology.

Matter assignment — examples. To discuss phenomenology it is necessary to specify an assignment of fermions. To do so we need to know the breaking parameter [17]. While the first breaking chain looks similar to the previous model the others look quite different because the origin of the right-handed neutrino(s) is different. Therefore, we show examples from the breaking chain (3.5). In this U(1) assignment, μ-flavored doublet belongs to $\mathbf{5}_2^*$ and τ-flavored one does $\mathbf{5}_3^*$. Correspondingly μ-flavored singlet belongs to $\mathbf{10}_2$ and τ-flavored one does belongs to $\mathbf{10}_1$. The remaining e-flavord leptons are contained in $\mathbf{5}_1^*$ and $\mathbf{10}_3$. The embedding of other fermions is arbitrary. Though it is determined by the mass spectrum of fermions. To do so we need breaking parameters but it is totally beyond the scope of this letter. Instead of specifying breaking parameters, we show two possible examples of the emmbeddings as examples.

The first one is

$$\mathbf{10}_1 = (t^c, \{t_L, b_L\}, \tau^c) \quad \mathbf{5}_1^* = (d^c, \{\nu_{Le}, e_L\}), \tag{3.8}$$

$$\mathbf{10}_2 = (c^c, \{c_L, s_L\}, \mu^c) \quad \mathbf{5}_2^* = (s^c, \{\nu_{L\mu}, \mu_L\}), \tag{3.9}$$

$$\mathbf{10}_3 = (u^c, \{u_L, d_L\}, e^c) \quad \mathbf{5}_3^* = (b^c, \{\nu_{L\tau}, \tau_L\}). \tag{3.10}$$

This keeps the naive structure of generation though from the breaking pattern it may be difficult to assign the 1st (3rd) generation into $\mathbf{10}_{3(1)}$.

With this assignment, the coupling of fermions with $\mathrm{L}_\mu - \mathrm{L}_\tau$ gauge boson Z' is given by

$$\begin{aligned}\mathcal{L}_{Z'} = g_{Z'}\{&(\bar{\mu}\gamma^\rho\mu + \bar{\nu}_{L\mu}\gamma^\rho\nu_{L\mu}) - (\bar{\tau}\gamma^\rho\tau + \bar{\nu}_{L\tau}\gamma^\rho\nu_{L\tau}) \\ &+(\bar{c}\gamma^\rho\gamma_5 c - \bar{s}\gamma^\rho s) - (\bar{t}\gamma^\rho\gamma_5 t - \bar{b}\gamma^\rho b)\}Z'_\rho.\end{aligned} \tag{3.11}$$

Leptons have a vector coupling with $\mathrm{L}_\mu - \mathrm{L}_\tau$ gauge boson appropriately. On the contrary quarks have an axial vector coupling for c and t while a vector coupling for s and b. At this moment there is no strong constraint from experiments.

The second one is

$$\mathbf{10}_1 = (u^c, \{u_L, d_L\}, \tau^c) \quad \mathbf{5}_1^* = (b^c, \{\nu_{Le}, e_L\}), \tag{3.12}$$

$$\mathbf{10}_2 = (c^c, \{c_L, s_L\}, \mu^c) \quad \mathbf{5}_2^* = (d^c, \{\nu_{L\mu}, \mu_L\}), \tag{3.13}$$

$$\mathbf{10}_3 = (t^c, \{t_L, b_L\}, e^c) \quad \mathbf{5}_3^* = (s^c, \{\nu_{L\tau}, \tau_L\}). \tag{3.14}$$

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This emmbeddings is achieved by assuming that the prediction for quark mixing given in [17] holds even after the recombination. By this the embedding for quark doublets is also determined. We still have the freedom for the embedding of right-handed quarks. Here we examine one of the possibilities that look most interesting.

With this assignment, the coupling of fermions with $\mathrm{L}_\mu - \mathrm{L}_\tau$ gauge boson Z' is given by

$$\begin{aligned}\mathcal{L}_{Z'} = g_{Z'}\{&(\bar{\mu}\gamma^\rho\mu + \bar{\nu}_{L\mu}\gamma^\rho\nu_{L\mu}) - (\bar{\tau}\gamma^\rho\tau + \bar{\nu}_{L\tau}\gamma^\rho\nu_{L\tau}) \\ &-(\bar{u}\gamma^\rho\gamma_5 u + \bar{d}\gamma^\rho\gamma_5 d) + (\bar{c}\gamma^\rho\gamma_5 c + \bar{s}\gamma^\rho\gamma_5 s)\}Z'_\rho.\end{aligned} \tag{3.15}$$

Again leptons have a vector coupling with $\mathrm{L}_\mu - \mathrm{L}_\tau$ gauge boson appropreately. On the contrary quarks have an axial vector coupling with it. Therefore in the non-relativistic limit there is no connection between quarks and leptons mediated by the $\mathrm{L}_\mu - \mathrm{L}_\tau$ gauge boson. For example there is no constraint from atomic physics.

There may be an effect on meson decay. However, as there is no direct coupling of Z' to electrons, very tiny effects are expected and hence we would expect that this model is also free from constraints on mesons.

Another implication is on proton decay. Though for dimension-5 operators we have no indication, for dimension-6 operator that is that mediated by gauge bosons we have an interesting “prediction”. Proton decay is induced by $\overline{\mathbf{10}}_1\mathbf{10}_1\bar{\mathbf{5}}^*_2\mathbf{5}^*_2$. It leads

$$p \to \mu^+\pi^0. \tag{3.16}$$

Instead of e^+ we will observe μ^+. If we find this then it is an prominent signature of the scenario.

4 Summary and discussion

In this pape, we show a unification of SM gauge and $\mathrm{L}_\mu - \mathrm{L}_\tau$ gauge symmetry into the simple group E_7 in the context of coset space unification. There are three types of unification that will lead to different phenomenology, at least for neutrino. To check it we need to specify breaking chains and breaking parameters as in [17].

Even though details are a matter of breaking parameters, we show two examples of matter assignment for the first breaking chain to show that this framework contains a plenty of models. Indeed derived low energy Lagrangians are quite different from each other and possible predictions are distinctive.

In addition to those breaking parameters for matter assignment, we have to specify the breaking mechanism to seek the final theory. Mechanism is strongly related with not only what kind of matter appear in the spectrum but also breaking parameters which determine low energy Lagrangian, say yukawa terms. There are several breaking methods: spontaneous symmetry breaking, Coset Space Dimensional Reduction [21, 22], Difference of boundary condition between bosons and fermions [23, 24], Non-linear realization [18, 25–28], Hosotani mechanism [29], etc.

Some of those mechanisms requires gauge symmetry [21, 22, 28]. Note that the coset space unification is a kind of non-linear realization and hence only the global symmetry is relevant. To construct a full gauge theory we have to ensure that the global symmetry can be gauged. Indeed these breaking mechanisms rely on the fact that the fundamental symmetry, which is G of coset G/H, is gauged. By combining with these mechanism we will find a way to gauge SU(5).

Finally we make a comment on probably more serious problem. It is that non-linear supersymmetric theory here is in generalanomalous [30]. Indeed in this paper extra 5plet is omitted in the tables as mentioned at the first paragraph of section 2. However any nonlinear model should be an effectie theory which is derived from a non-anomalous theory. It means that there must be a way to evade anomaly even within non-linear theory. One of the ways is to add $\bar{5}$ to the theory [19]. It is allowed thoeretically [31]. By this an anomaly-free is indeed constructed.

All of the details are beyond the scope of this work, thus these will be made in the future.

Acknowledgments

The author thanks MIZOGUCHI, Shun'ya for helpful implication during the workshop "Particle and phenomenology workshop 2020" held in the Nambu Yoichiro Institute of Theoretical and Experimental Physics (NITEP) at Osaka City University. The author is also grateful to TAKANISHI, Yasutaka for useful comment. This work was supported by JSPS KAKENHI Grants No. JP18H01210 and MEXT KAKENHI Grant No. JP18H05543.

References

[1] Super-Kamiokande collaboration, *Evidence for oscillation of atmospheric neutrinos*, *Phys. Rev. Lett.* **81** (1998) 1562 [hep-ex/9807003] [inSPIRE].

[2] T. Aoyama et al., *The anomalous magnetic moment of the muon in the Standard Model*, *Phys. Rept.* **887** (2020) 1 [arXiv:2006.04822] [inSPIRE].

[3] Muon $g-2$ collaboration, *Measurement of the Positive Muon Anomalous Magnetic Moment to* 0.46 *ppm*, *Phys. Rev. Lett.* **126** (2021) 141801 [arXiv:2104.03281] [inSPIRE].

[4] R. Foot, *New Physics From Electric Charge Quantization?*, *Mod. Phys. Lett. A* **6** (1991) 527 [inSPIRE].

[5] X.-G. He, G.C. Joshi, H. Lew and R.R. Volkas, *New-Z' phenomenology*, *Phys. Rev. D* **43** (1991) 22 [inSPIRE].

[6] R. Foot, X.-G. He, H. Lew and R.R. Volkas, *Model for a light Z' boson*, *Phys. Rev. D* **50** (1994) 4571 [hep-ph/9401250] [inSPIRE].

[7] X.-G. He, G.C. Joshi, H. Lew and R.R. Volkas, *Simplest Z' model*, *Phys. Rev. D* **44** (1991) 2118 [inSPIRE].

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[8] S.N. Gninenko and N.V. Krasnikov, *The Muon anomalous magnetic moment and a new light gauge boson*, Phys. Lett. B **513** (2001) 119 [hep-ph/0102222] [INSPIRE].

[9] S. Baek, N.G. Deshpande, X.-G. He and P. Ko, *Muon anomalous $g-2$ and gauged $L_\mu - L_\tau$ models*, Phys. Rev. D **64** (2001) 055006 [hep-ph/0104141] [INSPIRE].

[10] E. Ma, D.P. Roy and S. Roy, *Gauged $L_\mu - L_\tau$ with large muon anomalous magnetic moment and the bimaximal mixing of neutrinos*, Phys. Lett. B **525** (2002) 101 [hep-ph/0110146] [INSPIRE].

[11] M. Escudero, D. Hooper, G. Krnjaic and M. Pierre, *Cosmology with A Very Light $L_\mu - L_\tau$ Gauge Boson*, JHEP **03** (2019) 071 [arXiv:1901.02010] [INSPIRE].

[12] T. Araki et al., *Resolving the Hubble tension in a $\mathrm{U}(1)_{L_\mu - L_\tau}$ model with the Majoron*, Prog. Theor. Exp. Phys. **2021** (2021) 103B05 [arXiv:2103.07167] [INSPIRE].

[13] T. Araki, F. Kaneko, Y. Konishi, T. Ota, J. Sato and T. Shimomura, *Cosmic neutrino spectrum and the muon anomalous magnetic moment in the gauged $L_\mu - L_\tau$ model*, Phys. Rev. D **91** (2015) 037301 [arXiv:1409.4180] [INSPIRE].

[14] A. Kamada and H.-B. Yu, *Coherent Propagation of PeV Neutrinos and the Dip in the Neutrino Spectrum at IceCube*, Phys. Rev. D **92** (2015) 113004 [arXiv:1504.00711] [INSPIRE].

[15] T. Araki, F. Kaneko, T. Ota, J. Sato and T. Shimomura, *MeV scale leptonic force for cosmic neutrino spectrum and muon anomalous magnetic moment*, Phys. Rev. D **93** (2016) 013014 [arXiv:1508.07471] [INSPIRE].

[16] A. DiFranzo and D. Hooper, *Searching for MeV-Scale Gauge Bosons with IceCube*, Phys. Rev. D **92** (2015) 095007 [arXiv:1507.03015] [INSPIRE].

[17] J. Sato and T. Yanagida, *Large lepton mixing in a coset space family unification on $\mathrm{E}_7/\mathrm{SU}(5) \times \mathrm{U}(1)^3$*, Phys. Lett. B **430** (1998) 127 [hep-ph/9710516] [INSPIRE].

[18] W. Buchmüller, R.D. Peccei and T. Yanagida, *Quasi Nambu-Goldstone Fermions*, Nucl. Phys. B **227** (1983) 503 [INSPIRE].

[19] T. Kugo and T. Yanagida, *Unification of Families Based on a Coset Space $\mathrm{E}_7/\mathrm{SU}(5) \times \mathrm{SU}(3) \times \mathrm{U}(1)$*, Phys. Lett. B **134** (1984) 313 [INSPIRE].

[20] T. Yanagida and Y. Yasui, *Supersymmetric nonlinear sigma models based on exceptional groups*, Nucl. Phys. B **269** (1986) 575 [INSPIRE].

[21] N.S. Manton, *A New Six-Dimensional Approach to the Weinberg-Salam Model*, Nucl. Phys. B **158** (1979) 141 [INSPIRE].

[22] D. Kapetanakis and G. Zoupanos, *Coset space dimensional reduction of gauge theories*, Phys. Rept. **219** (1992) 4 [INSPIRE].

[23] J. Scherk and J.H. Schwarz, *Spontaneous Breaking of Supersymmetry Through Dimensional Reduction*, Phys. Lett. B **82** (1979) 60 [INSPIRE].

[24] J. Scherk and J.H. Schwarz, *How to Get Masses from Extra Dimensions*, Nucl. Phys. B **153** (1979) 61 [INSPIRE].

[25] M. Bando, T. Kuramoto, T. Maskawa and S. Uehara, *Structure of Nonlinear Realization in Supersymmetric Theories*, Phys. Lett. B **138** (1984) 94 [INSPIRE].

[26] M. Bando, T. Kuramoto, T. Maskawa and S. Uehara, *Nonlinear Realization in Supersymmetric Theories*, Prog. Theor. Phys. **72** (1984) 313 [INSPIRE].

JHEP07(2022)011

[27] M. Bando, T. Kuramoto, T. Maskawa and S. Uehara, *Nonlinear Realization in Supersymmetric Theories. Part 2*, Prog. Theor. Phys. **72** (1984) 1207 [INSPIRE].

[28] K. Higashijima and M. Nitta, *Supersymmetric nonlinear sigma models as gauge theories*, Prog. Theor. Phys. **103** (2000) 635 [hep-th/9911139] [INSPIRE].

[29] Y. Hosotani, *Dynamical Mass Generation by Compact Extra Dimensions*, Phys. Lett. B **126** (1983) 309 [INSPIRE].

[30] T. Moriya and Y. Yasui, *Anomalies in supersymmetric nonlinear σ models based on* E_l*-type groups*, J. Math. Phys. **27** (1986) 3040 [INSPIRE].

[31] S.R. Coleman, J. Wess and B. Zumino, *Structure of phenomenological Lagrangians. Part 1*, Phys. Rev. **177** (1969) 2239 [INSPIRE].

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