Erratum: Transverse momentum in double parton scattering: factorisation, evolution and matching

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1 Colour structure of DPDs

The analysis of the colour structure of two-gluon DPDs is incomplete in the original manuscript and needs to be extended. Consider a quantity with four adjoint indices that transforms like an overall colour singlet. One can first couple the indices pairwise to one of the combinations $R = 1, A, S, 10, \bar{10}, 27$. An overall singlet is then obtained if the colour representations two index pairs are in conjugate colour representations. This includes the cases where one pair is in the antisymmetric octet and the other in the symmetric octet. These cases were omitted in [1], see equations (6.a), (6.b) and (7) in that work. This mistake was repeated in equation (2.121) of [2].

In appendix A of [3] it was shown that the two-gluon distributions corresponding to $R = 10$ and $\bar{10}$ in the previous construction are equal. The proof given there makes use of the colour decompositions given in [1, 2] and breaks down once the missing combinations with one symmetric and one antisymmetric octet are included. We hence must extend the colour decompositions in the present work by adding the mixed octet combinations and restoring separate quantities for $R = 10$ and $\bar{10}$.

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In the list of projection operators P_R in (4.3), we hence replace $P_D = P_{10} + P_{\overline{10}}$ with the separate projectors

$$\begin{aligned} P_{10}^{aa'bb'} &= \frac{1}{4}(\delta^{ab}\delta^{a'b'} - \delta^{ab'}\delta^{a'b}) - \frac{1}{2}P_A^{aa'bb'} - \frac{i}{4}(d^{abc}f^{a'b'c} + f^{abc}d^{a'b'c}), \\ P_{\overline{10}}^{aa'bb'} &= \frac{1}{4}(\delta^{ab}\delta^{a'b'} - \delta^{ab'}\delta^{a'b}) - \frac{1}{2}P_A^{aa'bb'} + \frac{i}{4}(d^{abc}f^{a'b'c} + f^{abc}d^{a'b'c}) \end{aligned} \quad (1.1)$$

for the decuplet and antidecuplet. These projectors agree with $P_{aa'bb'}^{10}$ and $P_{aa'bb'}^{\overline{10}}$ in equation (1.19) of [4]. They also agree with the corresponding projectors in equation (12) of [1], provided that we identify $\langle a'b'|P_R|ab\rangle = P_R^{ab a'b'}$ in analogy to equation (2) of that paper. Note that the form of these projectors is correct for $SU(N)$ and not restricted to $N = 3$ [4]. The symmetry relation (4.7) is not valid for decuplet projectors and must be corrected to read

$$P_R^{r\ s} = P_{\overline{R}}^{s\ r}, \quad (1.2)$$

where $\overline{R}$ is the conjugate representation of R . It is understood that $\overline{R} = 10$ for $R = \overline{10}$ and that $\overline{R} = R$ for the singlet, all octets, and for $R = 27$. The projector relation (4.8) remains valid.

For the mixed gluon octet channels, we introduce the tensors

$$P_{AS}^{aa'bb'} = \frac{1}{\sqrt{N^2 - 4}}f^{aa'c}d^{bb'c}, \quad P_{SA}^{aa'bb'} = \frac{1}{\sqrt{N^2 - 4}}d^{aa'c}f^{bb'c}, \quad (1.3)$$

which satisfy the relations

$$P_{AS}^{a\ b} = P_{SA}^{b\ a}, \quad P_{AS}^{a\ a} = P_{SA}^{a\ a} = 0. \quad (1.4)$$

They are not projectors, since they do not satisfy (4.8).

We also note that (4.8) is incorrect for the octet tensors with mixed fundamental and adjoint indices: the contraction $P_A^{aa'ii'}P_S^{i'ibb'}$ is not zero but equal to $P_{AS}^{aa'bb'}$. The octet tensors in (4.4) are therefore not projectors either. Fortunately, the incorrect instances of (4.8) were not used in our calculations.

To have a uniform notation for calculations, we write the projectors as

$$P_R^{r\ s} = P_{\overline{R}}^{s\ r}, \quad (1.5)$$

where the double indices r and s are both in the fundamental or both in the adjoint representation. We furthermore write $P_{R_1 R_2}^{i\ a} = P_{R_2 R_1}^{a\ i}$ for the tensors in (4.4), with $R_1 = 1, 8$ for the fundamental indices and $R_2 = 1, A, S$ for the adjoint ones. We then have the simple rules

$$P_{R_1 R_2}^{r\ s} = P_{R_2 R_1}^{s\ r}, \quad P_{R_1 R_2}^{r\ s} P_{R_3 R_4}^{s\ t} = \delta_{R_2 \overline{R_3}} P_{R_1 R_4}^{r\ t}. \quad (1.6)$$

The corrected form of (4.9) hence reads

$$P_{R_1 R_2}^{r\ s} P_{R_3 R_4}^{r\ s} = \delta_{R_2 \overline{R_4}} P_{R_1 R_3}^{r\ r} = \delta_{R_1 \overline{R_3}} \delta_{R_2 \overline{R_4}} m(R_1), \quad (1.7)$$

where the definition (4.10) of the multiplicity $m(R)$ remains valid as it stands.¹

¹For general N , one obtains multiplicities $m(8) = N^2 - 1$ for the octet and $m(10) = (N^2 - 1)(N^2 - 4)/4$ for the decuplet representations.

The tensor $P_{R_1 R_2}$ couples two double-index objects in the representations R_1 and R_2 to an overall singlet. Hence, the two representations must have the same multiplicity. Formally one can impose this by defining $P_{RR'} = 0$ for $m(R) \neq m(R')$. One should then also define $P_{1010} = P_{\overline{10}\overline{10}} = 0$.

An object with four colour indices that transforms as an overall colour singlet can now be represented in terms of the tensors $P_{RR'}$. The corrected form of the decomposition (4.11) reads

$$M^{rs} = \sum_{RR'} \frac{1}{m(R)} P_{RR'}^{rs} \left(P_{\overline{R}\overline{R}'}^{tu} M^{tu} \right). \quad (1.8)$$

A simple example illustrating the necessity to include $RR' = AS$ in the sum over representation pairs is $M^{aa'bb'} = f^{aa'c} d^{bb'c}$. The relation (4.12) correctly reads

$$M_1^{rs} M_2^{rs} = \sum_{RR'} \frac{1}{m(R)} \left(P_{RR'}^{rs} M_1^{rs} \right) \left(P_{\overline{R}\overline{R}'}^{tu} M_2^{tu} \right). \quad (1.9)$$

Note that, for each contracted double index, the corresponding representations in M_1 and M_2 are conjugate to each other as a consequence of (1.7).

Various derivations and results in the paper must be adjusted to reflect the change from one to two representation labels and the conjugation of representations (which is trivial for all representations apart from the decuplet and antidecuplet). We present the necessary changes in the order in which they appear in the manuscript.

The corrected form for the colour decomposition (4.13) of a DPD is

$$F_{a_1 a_2}^{r_1 r_2} = \sum_{R_1 R_2} \frac{1}{\mathcal{N}_{a_1} \mathcal{N}_{a_2}} \frac{1}{\varepsilon(R_1) \varepsilon(R_2)} \frac{1}{\sqrt{m(R_1)}} {}^{R_1 R_2} F_{a_1 a_2} P_{R_1 R_2}^{r_1 r_2} \quad (1.10)$$

with

$${}^{R_1 R_2} F_{a_1 a_2} = \mathcal{N}_{a_1} \mathcal{N}_{a_2} \varepsilon(R_1) \varepsilon(R_2) \frac{1}{\sqrt{m(R_1)}} P_{\overline{R_1} \overline{R_2}}^{\underline{s}_1 \underline{s}_2} F_{a_1 a_2}^{\underline{s}_1 \underline{s}_2}, \quad (1.11)$$

where instead of $\varepsilon_a(R)$ from (4.15) we now use

$$\varepsilon(A) = i, \quad \varepsilon(R) = 1 \quad \text{for } R \neq A. \quad (1.12)$$

With our new notation, the definition (4.18) of colour projected twist-two operators reads

$${}^R O_a^r = \mathcal{N}_a \varepsilon(R) P_{\overline{R}\overline{R}}^{rs} O_a^s, \quad (1.13)$$

and (4.19) must be modified to

$$2\pi\delta(p^+ - p'^+) 2p^+ {}^{R_1 R_2} F_{\text{us}, a_1 a_2} = \frac{1}{\sqrt{m(R_1)}} P_{\overline{R_1} \overline{R_2}}^{rs} \langle p' | {}^{R_1} O_{a_1}^r {}^{R_2} O_{a_2}^s | p \rangle. \quad (1.14)$$

For $R_2 = \overline{R_1}$, one can eliminate the projector on the r.h.s. by using that $P_{\overline{R}\overline{R}}^{rs} {}^R O_a^s = {}^R O_a^r$.

We modify the colour projection (4.20) of the soft matrix as

$${}^{R_1 R_2, R'_1 R'_2} S_{a_1 a_2} = \frac{\varepsilon(R_1) \varepsilon(R_2)}{\varepsilon(R'_1) \varepsilon(R'_2)} \frac{1}{\sqrt{m(R_1) m(R'_1)}} P_{\overline{R_1} \overline{R'_1}}^{r_1 r'_1} S_{a_1 a_2}^{r_1 r'_1, \underline{s}_1 \underline{s}'_1} P_{\overline{R'_2} \overline{R_2}}^{\underline{s}_2 \underline{s}'_2}. \quad (1.15)$$

The same holds for the colour space matrices S^{-1} , s , s^{-1} , and K . Together with (1.9), this leads to a modification of the multiplication rule for matrices in representation space, which we define in general by

$${}^{R_1 R_2, R'_1 R'_2} (M_1 \cdot M_2) = \sum_{R'_1 R'_2} {}^{R_1 R_2, \bar{R}'_1 \bar{R}'_2} M_1 {}^{R'_1 R'_2, R''_1 R''_2} M_2, \quad (1.16)$$

where the projections of M_1 and M_2 on representations are defined as in (1.15). The correct form of (4.21) then reads

$${}^{R_1 R_2, R'_1 R'_2} (S_{a_1 a_2}^{-1} \cdot S_{a_1 a_2}) = {}^{R_1 R_2, R'_1 R'_2} (s_{a_1 a_2}^{-1} \cdot s_{a_1 a_2}) = \delta_{R_1 \bar{R}'_1} \delta_{R_2 \bar{R}'_2}, \quad (1.17)$$

and corresponding changes are to be made in (4.22) and (4.23). Again as a consequence of (1.9), the expression (4.24) for the cross section is modified to

$$X = \frac{H_{a_1 b_1}}{\mathcal{N}_{a_1} \mathcal{N}_{b_1}} \frac{H_{a_2 b_2}}{\mathcal{N}_{a_2} \mathcal{N}_{b_2}} \sum_{R_1 R_2} \eta(R_1) \eta(R_2) {}^{\bar{R}_1 \bar{R}_2} F_{b_1 b_2} {}^{R_1 R_2} F_{a_1 a_2} \quad (1.18)$$

with

$$\eta(R) = \varepsilon^2(R), \quad (1.19)$$

so that $\eta(A) = -1$ and $\eta(R) = 1$ for $R \neq A$.

We now discuss the necessary adjustments to the symmetry properties derived in section 4.3. Using that $P_{R_1 R_2}^{aa' bb'} = [P_{\bar{R}_1 \bar{R}_2}^{aa' bb'}]^*$, one finds that (4.27) and (4.28) must be corrected to

$${}^{R_1 R_2, \bar{R}'_1 \bar{R}'_2} S_{a_1 a_2} = \left({}^{R'_1 R'_2, \bar{R}_1 \bar{R}_2} S_{a_1 a_2} \right)^*, \quad (1.20)$$

$$\begin{aligned} {}^{R_1 R_2, \bar{R}'_1 \bar{R}'_2} S_{a_1 a_2} &= \frac{\eta(R'_1) \eta(R'_2)}{\eta(R_1) \eta(R_2)} \left({}^{\bar{R}_1 \bar{R}_2, R'_1 R'_2} S_{\bar{a}_1 \bar{a}_2} \right)^* \\ &= \frac{\eta(R'_1) \eta(R'_2)}{\eta(R_1) \eta(R_2)} {}^{\bar{R}'_1 \bar{R}'_2, R_1 R_2} S_{\bar{a}_1 \bar{a}_2}. \end{aligned} \quad (1.21)$$

An analogous correction is to be made for (4.29). The result that the soft matrix in representation space is real valued remains true, so that the complex conjugation in (1.20) and (1.21) can be omitted. Doing this in the first line of (1.21), one finds that

$${}^{10 \bar{10}, \bar{10} 10} S_{gg} = {}^{\bar{10} 10, 10 \bar{10}} S_{gg}, \quad {}^{10 \bar{10}, 10 \bar{10}} S_{gg} = {}^{\bar{10} 10, \bar{10} 10} S_{gg}, \quad (1.22)$$

which is expected because charge conjugation interchanges the representations 10 and $\bar{10}$. Furthermore, one finds that ${}^{R_1 R_2, R'_1 R'_2} S_{gg} = 0$ if there is an odd number of representations A and no representation is equal to 10 or $\bar{10}$. This reflects the fact that the antisymmetric and the symmetric gluon octets transform under charge conjugation with a relative minus sign. Corresponding results hold for the Collins-Soper Kernel K_{gg} .

Using that $\varepsilon(R_1) \varepsilon(R_2) P_{\bar{R}_1 \bar{R}_2}^{aa' bb'} = [\varepsilon(R_1) \varepsilon(R_2) P_{R_1 R_2}^{a'a b'b}]^*$, one finds that ${}^{R_1 R_2} F_{a_1 a_2}(x_i, \mathbf{y})$ is real valued in all colour channels except for the decuplets, for which one has ${}^{10 \bar{10}} F_{gg}(x_i, \mathbf{y}) = [{}^{\bar{10} 10} F_{gg}(x_i, \mathbf{y})]^*$.

The soft factor for collinear DPDs. The relations (4.39) and (4.40) become

$$\begin{aligned} W_{ab} W_{b'a'}^\dagger P_{RR'}^{bb'cc'} &= P_{RR'}^{aa'bb'} W_{bc} W_{c'b'}^\dagger, \\ W_{ab} W_{b'a'}^\dagger P_{RR'}^{bb'k'k} &= P_{RR'}^{aa'j'j} W_{jk} W_{k'j'}^\dagger, \quad W_{ij} W_{j'i'}^\dagger P_{RR'}^{jj'aa'} = P_{RR'}^{ii'bb'} W_{ab} W_{b'a'}^\dagger, \end{aligned} \quad (1.23)$$

so that (4.41) generalises to

$$(S_{a_1 a_2})^{\dots \frac{r}{s} \dots} P_{RR'}^{\frac{s}{t}} = P_{RR'}^{\frac{r}{s}} (S_{a_1 a_2})^{\dots \frac{s}{t} \dots}. \quad (1.24)$$

Note that these relations do *not* involve conjugation of representation labels. As a corollary, one obtains

$$P_{RR'}^{\frac{r_1 s}{t_1}} S_{a_1 a_2}^{\frac{s}{t_2}, t_1 t_2} = S_{a_1 a_2}^{\frac{r_1 t_2, s}{t_1}} P_{RR'}^{\frac{s}{t_1}} \quad (1.25)$$

and its analogue for the second parton. Using these results and the rules in (1.6), one can generalise equations (4.41) to (4.48) to all valid combinations of representations. We find that for the collinear soft factor the first two representations must be conjugate to the second two,

$$R_1 R_2, R'_1 R'_2 S_{a_1 a_2} = \delta_{R_1 \bar{R}'_1} \delta_{R_2 \bar{R}'_2} R_1 R_2, \bar{R}_1 \bar{R}_2 S_{a_1 a_2}, \quad (1.26)$$

and that for singlet and octet channels, the soft factors are equal for the quark and gluon representations. We thus get unity in the singlet sector,

$$^1S = ^{11,11}S_{qq} = ^{11,11}S_{gg} = ^{11,11}S_{gg} = ^{11,11}S_{gg} = 1, \quad (1.27)$$

and three non-trivial factors

$$\begin{aligned} ^8S &= ^{88,88}S_{qq} = ^{8A,8A}S_{qq} = ^{8S,8S}S_{qq} = ^{A8,A8}S_{qq} = ^{S8,S8}S_{qq} \\ &= ^{AA,AA}S_{gg} = ^{SS,SS}S_{gg} = ^{AS,AS}S_{gg} = ^{SA,SA}S_{gg}, \\ ^{10}S &= ^{10\bar{10},\bar{10}10}S_{gg} = ^{\bar{10}10,10\bar{10}}S_{gg}, \\ ^{27}S &= ^{2727,2727}S_{gg}, \end{aligned} \quad (1.28)$$

where for the decuplet sector we used (1.22). Analogous results hold for the Collins-Soper Kernel J for collinear DPDs. On the left-hand side of (1.28), we have labelled the soft factors by one representative of the different labels on the right-hand sides. Taking a different representative, one has $^8S = ^AS = ^SS$ and $^{10}S = ^{\bar{10}}S$.

The extended soft factor can be expressed in terms of the one for colour singlet production, and in generalisation of (4.43), we have

$$\begin{aligned} P_{\bar{R}_1 \bar{R}_2}^{\frac{r_1 t_2}{s_1}} P_{\bar{R}'_1 \bar{R}'_2}^{\frac{u_1 t_2}{s_2}} P_{\bar{R}_3 \bar{R}_3'}^{\frac{s_1 t_1}{t_3}} P_{\bar{R}_4 \bar{R}_4'}^{\frac{s_2 t_2}{t_4}} (S_{a_1 a_2})_{\frac{s_1 s_2, t_1 t_2}{t_3 t_4}}^{\frac{r_1 t_2, u_1 t_2}{t_3 t_4}} \\ = \delta_{R_1 \bar{R}_3} \delta_{R_2 \bar{R}_4} \delta_{R'_1 \bar{R}'_3} \delta_{R'_2 \bar{R}'_4} m(R_1)^{R_1} S, \end{aligned} \quad (1.29)$$

where the combination of tensors on the left and Kronecker deltas on the right implies that all representations must have the same multiplicity. This can be used to correct the

argument in section 4.5 for the production of final states with net colour. In the definition (4.51), the order of colour indices in H_{ab} must be interchanged (see figure 3), so that

$$X = H_{a_1 b_1}^{t_1 s_1} H_{a_2 b_2}^{t_2 s_2} [F_{\text{us}, b_1 b_2}^T(Y_R) S^{-1}(Y_R - Y_L)]^{r_1 r_2} \times [S(Y_R - Y_L)]_{s_1 s_2, t_1 t_2}^{r_1 r_2, u_1 u_2} [S^{-1}(Y_R - Y_L) F_{\text{us}, a_1 a_2}(Y_L)]^{u_1 u_2}. \quad (1.30)$$

Correcting (4.52) to include all relevant colour channels, we obtain as a replacement of (4.53) the final expression

$$X = \sum_{R_1 R_2 R'_1 R'_2} \bar{R}_1 R'_1 H_{a_1 b_1} \bar{R}_2 R'_2 H_{a_2 b_2} \bar{R}'_1 \bar{R}'_2 F_{b_1 b_2}^{R_1 R_2} F_{a_1 a_2} \quad (1.31)$$

with hard-scattering factors

$$^{RR'} H_{ab} = \frac{1}{\mathcal{N}_a \mathcal{N}_b} \frac{1}{\varepsilon(R) \varepsilon(R')} \frac{1}{m(R)} P_{\bar{R} \bar{R}'}^{st} H_{ab}^{st}. \quad (1.32)$$

For colour singlet final states, one has $^{RR'} H_{ab} = \delta_{\bar{R} \bar{R}'} \eta(R) H_{ab} / (\mathcal{N}_a \mathcal{N}_b)$ with H_{ab} defined in (4.55). Then (1.31) reduces to (1.18).

Evolution of DPDs. The colour structure of DPDF and DTMD evolution equations and their solutions must be adjusted for the number of representation labels and the difference between a representation and its conjugate.

The Collins-Soper kernel for DTMDs must have four representation labels, as does the soft factor from which it is derived. The Collins-Soper equation (5.11) then becomes

$$\begin{aligned} & \frac{\partial}{\partial \log \zeta} {}^{R_1 R_2} F_{a_1 a_2}(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta) \\ &= \frac{1}{2} \sum_{R'_1 R'_2} {}^{R_1 R_2, \bar{R}'_1 \bar{R}'_2} K_{a_1 a_2}(\mathbf{z}_i, \mathbf{y}; \mu_i) {}^{R'_1 R'_2} F_{a_1 a_2}(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta) \end{aligned} \quad (1.33)$$

with a kernel satisfying

$$\frac{\partial}{\partial \log \mu_1} {}^{R_1 R_2, R'_1 R'_2} K_{a_1 a_2}(\mathbf{z}_i, \mathbf{y}; \mu_i) = -\gamma_{K, a_1}(\mu_1) \delta_{R_1 \bar{R}'_1} \delta_{R_2 \bar{R}'_2} \quad (1.34)$$

and a corresponding equation for μ_2 . The matrix exponential in (5.14) is now defined as the exponential series with the matrix multiplication in (1.16). Using the analogue of (1.21) for $K_{a_1 a_2}$, the relation (5.18) becomes

$$\begin{aligned} & \eta(R_1) \eta(R_2) {}^{\bar{R}_1 \bar{R}_2, R'_1 R'_2} \exp [M_{b_1 b_2} L] {}^{\bar{R}'_1 \bar{R}'_2} F_{b_1 b_2} \\ &= \eta(R'_1) \eta(R'_2) {}^{\bar{R}'_1 \bar{R}'_2} F_{b_1 b_2} {}^{R'_1 R'_2, \bar{R}_1 \bar{R}_2} \exp [M_{a_1 a_2} L], \end{aligned} \quad (1.35)$$

where we have abbreviated the logarithm by L . The matrix $M_{a_1 a_2}$ is defined by the updated version of (5.16),

$$\begin{aligned} & {}^{R_1 R_2, R'_1 R'_2} K_{a_1 a_2}(\mathbf{z}_i, \mathbf{y}; \mu_i) \\ &= \delta_{R_1 \bar{R}'_1} \delta_{R_2 \bar{R}'_2} [{}^{11} K_{a_1}(\mathbf{z}_1; \mu_1) + {}^{11} K_{a_2}(\mathbf{z}_2; \mu_2)] + {}^{R_1 R_2, R'_1 R'_2} M_{a_1 a_2}(\mathbf{z}_i, \mathbf{y}), \end{aligned} \quad (1.36)$$

where $^{11}K_a$ is the Collins-Soper kernel for a single-parton TMD. The corrected form of (5.19) will be given below.

The kernel for the rapidity evolution of DPDFs derives from the collinear soft factor and hence is labelled by a single representation. We thus have

$$\frac{\partial}{\partial \log \zeta} {}^{R_1 R_2} F(x_i, \mathbf{y}; \mu_i, \zeta) = \frac{1}{2} {}^{R_1} J(\mathbf{y}; \mu_i) {}^{R_1 R_2} F(x_i, \mathbf{y}; \mu_i, \zeta) \quad (1.37)$$

as update of (5.22), where instead of $^{R_1} J$ we could also write $^{R_2} J$. By contrast, the DGLAP kernels for DPDFs carry two colour indices, and the evolution equation for the first parton scale has the structure

$$\frac{\partial}{\partial \log \mu_1} {}^{R_1 R_2} F_{a_1 a_2} = 2 \sum_{b_1, R'_1} {}^{R_1} \bar{R}'_1 P_{a_1 b_1} \otimes_{x_1} {}^{R'_1 R_2} F_{b_1 a_2}. \quad (1.38)$$

The Collins-Soper equation (5.25) for the evolution kernel should be modified to

$$\frac{\partial}{\partial \log \zeta} {}^{RR'} P_{ab}(x; \mu, \zeta) = -\frac{1}{4} \delta_{R\bar{R}'} \delta_{ab} \delta(1-x) {}^R \gamma_J(\mu), \quad (1.39)$$

and corresponding changes are to be made in (5.26). As a consequence of charge conjugation invariance, one has

$$^{10\bar{10}} P_{gg} = {}^{\bar{10}10} P_{gg}, \quad {}^{AS} P_{gg} = {}^{SA} P_{gg} = 0, \quad (1.40)$$

see the discussion after equation (1.21).

Short-distance matching. We now specify how the definitions and main results for the short-distance matching of DTMDs in section 6 are to be adjusted. The necessary adjustments of intermediate steps and of corollaries are easy to deduce. Starting in section 6.2, we define colour projections

$${}^R C_{S,a}(\mathbf{z}) = P_{\bar{R}R}^{s\bar{t}} C_{S,a}^{s\bar{t}}(\mathbf{z})/m(R) \quad (1.41)$$

for the matching kernel of the TMD soft factor for small $\mathbf{z}$. The two representation labels of $P^{s\bar{t}}$ in (1.41) must be conjugate to each other, because $P_{AS}^{s\bar{t}} C_{S,g}^{s\bar{t}} = P_{SA}^{s\bar{t}} C_{S,g}^{s\bar{t}} = 0$ due to charge conjugation invariance, and because the double indices $\underline{s}$ and $\bar{t}$ are either both in the fundamental or both in the adjoint representation. From (6.2) we then obtain

$${}^{R_1 R_2, R'_1 R'_2} S_{a_1 a_2}(\mathbf{z}_i, \mathbf{y}) = \delta_{R_1 \bar{R}'_1} \delta_{R_2 \bar{R}'_2} {}^{R_1} C_{S,a_1}(\mathbf{z}_1) {}^{R_2} C_{S,a_2}(\mathbf{z}_2) {}^{R_1} S(\mathbf{y}) \quad (1.42)$$

instead of (6.4). The matching of the twist-two operators at small $\mathbf{z}$ reads

$$O_a^r(x, \mathbf{y}, \mathbf{z}) = \sum_b C_{us,ab}^{r\bar{s}}(x', \mathbf{z}) \otimes_x O_b^s(x', \mathbf{y}). \quad (1.43)$$

In terms of the colour projected matching coefficients

$${}^{RR'} C_{us,ab} = \frac{\mathcal{N}_a}{\mathcal{N}_b} \frac{\varepsilon(R)}{\varepsilon(R')} \frac{1}{m(R)} P_{\bar{R}R'}^{r\bar{s}} C_{us,ab}^{r\bar{s}}, \quad (1.44)$$

one obtains the corrected form of (6.6) as

$${}^R O_a^r(x, \mathbf{y}, \mathbf{z}) = \sum_{b, R'} {}^{R\bar{R}'} C_{us,ab}(x', \mathbf{z}) \otimes_x P_{RR'}^{r\frac{s}{s}} {}^{R'} O_b^s(x', \mathbf{y}) \quad (1.45)$$

for the colour projected operators defined in (1.13). The possibility of quark-gluon mixing requires two colour labels R and R' for $C_{us,ab}$, in contrast to $C_{S,a}$. Adjusting the following steps in the manuscript and defining

$${}^{RR'} C_{ab}(x, \mathbf{z}; \mu, Y_C) = \lim_{Y_L \rightarrow -\infty} \frac{{}^{RR'} C_{us,ab}(x, \mathbf{z}; \mu, Y_L)}{\sqrt{{}^R C_{S,a}(\mathbf{z}; \mu, 2Y_C - 2Y_L)}}, \quad (1.46)$$

one finds that the correct colour structure for the matching of a DTMD onto a DPDF is

$${}^{R_1 R_2} F_{a_1 a_2}(\mathbf{z}_i, \mathbf{y}) = \sum_{b_1 b_2, R'_1 R'_2} {}^{R_1 \bar{R}'_1} C_{a_1 b_1}(\mathbf{z}_1) \otimes_{x_1} {}^{R_2 \bar{R}'_2} C_{a_2 b_2}(\mathbf{z}_2) \otimes_{x_2} {}^{R'_1 R'_2} F_{b_1 b_2}(\mathbf{y}), \quad (1.47)$$

where for brevity, we only give the transverse position arguments. The full version is given in (2.11) below. Due to charge conjugation invariance, one has

$${}^{10\bar{10}} C_{gg} = {}^{\bar{10}10} C_{gg}, \quad {}^{AS} C_{gg} = {}^{SA} C_{gg} = 0, \quad (1.48)$$

and analogous relations for $C_{us,gg}$.

The definitions of the kernel ${}^R K_a$ for the rapidity dependence of ${}^R C_{S,a}$ and of the associated anomalous dimension ${}^R \gamma_a$ remain as given in (6.14) and (6.18). Equation (6.15) hence becomes

$$\frac{\partial}{\partial \log \zeta} {}^{RR'} C_{ab}(x, \mathbf{z}; \mu, \zeta) = \frac{1}{2} {}^R K_a(\mathbf{z}; \mu) {}^{RR'} C_{ab}(x, \mathbf{z}; \mu, \zeta), \quad (1.49)$$

whereas the limiting expression (6.16) for small $\mathbf{z}_i$ should be replaced with

$$\begin{aligned} & {}^{R_1 R_2, R'_1 R'_2} K_{a_1 a_2}(\mathbf{z}_i, \mathbf{y}; \mu_i) \\ &= \delta_{R_1 \bar{R}'_1} \delta_{R_2 \bar{R}'_2} \left[{}^{R_1} K_{a_1}(\mathbf{z}_1; \mu_1) + {}^{R_2} K_{a_2}(\mathbf{z}_2; \mu_2) + {}^{R_1} J(\mathbf{y}; \mu_i) \right]. \end{aligned} \quad (1.50)$$

Charge conjugation invariance implies ${}^{10} C_{S,g} = {}^{\bar{10}} C_{S,g}$ and hence ${}^{10} K_g = {}^{\bar{10}} K_g$. The representation labels in (6.20) and (6.21) should be adapted as in (1.50) and (1.49), respectively. The corrected forms of (6.19), (6.22) and (6.24) will be given below. The functions g_F and g_K introduced in section 6.2.3 must have separate colour labels for the two partons, corresponding to the functions on the r.h.s. of their definitions in (6.31).

The result (6.36) for the expansion of the soft factor at small $\mathbf{z}_1$, $\mathbf{z}_2$ and $\mathbf{y}$ correctly reads

$${}^{R_1 R_2, R'_1 R'_2} S_{a_1 a_2}(\mathbf{z}_i, \mathbf{y}; \mu_i, Y) = {}^{R_1 R_2, R'_1 R'_2} C_{S, a_1 a_2}(\mathbf{z}_i, \mathbf{y}; \mu_i, Y). \quad (1.51)$$

The kernel $T_{a_0 \rightarrow a_1 a_2}$ for parton-level splitting in (6.38) and (6.42) should have separate labels R_1, R_2 for a_1, a_2 , as do the DPDs. The same holds for the twist-four function $G_{a_1 a_2}$ in (6.39) and (6.42).

In section 7.3.1, one should replace (7.22) with

$$\begin{aligned}
 2\pi\delta(p^+ - p'^+) 2p^+ {}^{RR'}\mathcal{M}_{ab}(x, z) &= \frac{1}{\mathcal{N}_b} \frac{1}{m(R)} P_{\overline{R}'R}^{r,s} \langle b, p', r' | {}^R O_a^s(x, \mathbf{y}, z) | b, p, r \rangle \\
 &= \varepsilon(R) \frac{\mathcal{N}_a}{\mathcal{N}_b} \frac{1}{m(R)} P_{\overline{R}'R}^{r,s} \langle b, p', r' | O_a^s(x, \mathbf{y}, z) | b, p, r \rangle, \\
 {}^R\mathcal{M}_{S,a}(z) &= \frac{1}{m(R)} P_{\overline{R}R}^{r,s} \langle 0 | O_{S,a}^{r,s}(\mathbf{y}, z) | 0 \rangle
 \end{aligned} \tag{1.52}$$

and the first equation in (7.23) with

$${}^{RR''}\mathcal{M}_{ac}(x, z) = \sum_{b,R'} {}^{R\overline{R}'}C_{us,ab}(x', z) \otimes_x {}^{R'R''}\mathcal{M}_{bc}(x'), \tag{1.53}$$

whilst the second equation in (7.23) remains valid as it stands. The adjustment of the remaining equations in that section is straightforward.

One-loop kernels. The kernel matrix (7.12) for Collins-Soper evolution of two-gluon DTMDs should be extended to include all colour channels. We refrain from giving the corresponding expressions here. In the limit $|z_1|, |z_2| \ll |\mathbf{y}|$, the elements of this matrix can be expressed in terms of the kernels ${}^R K_g$ and ${}^R J_g$ according to (1.50). We find that ${}^{10}K_g = 0$ and ${}^{10}J = {}^D J$ with ${}^D J$ given in (7.17).

The colour factors c_{ab} in (7.87) and $c_{a_0 \rightarrow a_1 a_2}$ in (7.97) now depend on two representation labels ($R_1 R_2$) referring to the parton pairs (ab) or ($a_1 a_2$). In addition to the results given in the manuscript, we find that both factors are zero for $R_1 R_2 = 10 \overline{10}$ and $\overline{10} 10$. This holds for general number of colours N . Both factors are also zero for $R_1 R_2 = AS$ and SA due to charge conjugation invariance.

2 Definition and rescaling of the rapidity parameter ζ

The definition (5.8) of the rapidity parameter in DPDs refers to the plus-momenta $x_1 p^+$ and $x_2 p^+$ of the two extracted partons, in generalisation of the definition (3.50) for single-parton TMDs. This leads to a problem in convolutions of DPDFs, which was overlooked and led to a number of mistakes in the original manuscript. The problem concerns the evolution equation (5.24) of DPDFs and the matching equation (6.11) of DTMDs onto DPDFs. The DPDFs on the right-hand-sides of these equations are to be taken at a fixed central rapidity Y_C . This is explicit in the steps leading to (6.11), and a corresponding derivation can be given for (5.24). This implies that in integrals over a momentum fraction of a DPD, the variable ζ does not remain constant but must change such that Y_C remains constant. Such a rescaling in all convolution integrals is possible, but we find it rather awkward. To avoid it, we express the rapidity dependence of DPDFs by the parameter

$$\zeta_p = 2(p^+)^2 e^{-2Y_C}, \tag{2.1}$$

which refers to the proton momentum. We use this parameter also for DTMDs, given their close connection with DPDFs. The handling of the rapidity dependence in DTMDs is then

different from the one for single-parton TMDs in the modern literature, notably in [5]. We note, however, that (2.1) corresponds to the definition of the ζ parameter in the original work of Collins and Soper [6]. The parameter ζ_p is then to be used instead of ζ from (5.8) as argument for DTMDs and DPDFs throughout the paper. In the following, we point out equations that change in a non-trivial way.

Defining the analogue of (2.1) for the left-moving proton as

$$\zeta_{\bar{p}} = 2(\bar{p}^-)^2 e^{2Y_C}, \quad (2.2)$$

the product of rapidity parameters is fixed to

$$\zeta_p \zeta_{\bar{p}} = s^2, \quad (2.3)$$

where $s = 2p^+ \bar{p}^-$ is the squared c.m. energy of the proton-proton collision (neglecting proton mass corrections). In terms of the invariant masses produced by the two hard scatters, one has $s = Q_1^2/(x_1 \bar{x}_1) = Q_2^2/(x_2 \bar{x}_2)$.

The definitions (2.1) and (2.2) refer to the Collins regulator for rapidity divergences. If one works in a different scheme, such as the δ regulator discussed in appendix B, the replacement of ζ by $\zeta_p = \zeta/(x_1 x_2)$ as argument of DPDs must be implemented accordingly.

DTMD evolution. Equations that involve DTMDs but no DPDFs are correct in the original manuscript and just need to be rewritten to reflect the change of argument in the distributions. In terms of the new rapidity parameter, the renormalisation group equation (5.7) for DTMDs reads

$$\frac{\partial}{\partial \log \mu_1} {}^{R_1 R_2} F_{a_1 a_2}(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta_p) = \gamma_{F, a_1}(\mu_1, x_1^2 \zeta_p) {}^{R_1 R_2} F_{a_1 a_2}(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta_p) \quad (2.4)$$

and in analogy for the derivative w.r.t. $\log \mu_2$. The rescaled rapidity parameters $x_1 \zeta/x_2$ and $x_2 \zeta/x_1$ in (5.13) should hence be replaced with $x_1^2 \zeta_p$ and $x_2^2 \zeta_p$, respectively.

Some attention is needed when choosing initial conditions for Collins-Soper evolution. In the matching relations for small $\mathbf{y}$ discussed in section 6.3, the rapidity parameter dependence of the DTMD arises from a short-distance matching kernel, which does not know about the proton momentum and hence can depend on $x_1 x_2 \zeta_p$ but not on ζ_p . Taking that kernel at a fixed scale ζ_0 so as to minimise corrections from higher orders thus gives the DTMD at $\zeta_p = \zeta_0/(x_1 x_2)$. Assuming that a corresponding initial condition is also useful at large $\mathbf{y}$, we therefore rewrite (5.17) in the form

$$\begin{aligned} & {}^{R_1 R_2} F_{a_1 a_2}(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta_p) \\ &= \exp \left\{ \int_{\mu_{01}}^{\mu_1} \frac{d\mu}{\mu} \left[\gamma_{a_1}(\mu) - \gamma_{K, a_1}(\mu) \log \frac{x_1 \sqrt{\zeta_p}}{\mu} \right] + {}^{11}K_{a_1}(\mathbf{z}_1; \mu_{01}) \log \frac{\sqrt{x_1 x_2 \zeta_p}}{\sqrt{\zeta_0}} \right. \\ & \quad \left. + \int_{\mu_{02}}^{\mu_2} \frac{d\mu}{\mu} \left[\gamma_{a_2}(\mu) - \gamma_{K, a_2}(\mu) \log \frac{x_2 \sqrt{\zeta_p}}{\mu} \right] + {}^{11}K_{a_2}(\mathbf{z}_2; \mu_{02}) \log \frac{\sqrt{x_1 x_2 \zeta_p}}{\sqrt{\zeta_0}} \right\} \\ & \times \sum_{R'_1 R'_2} {}^{R_1 R_2} \bar{R}'_1 \bar{R}'_2 \exp \left[M_{a_1 a_2}(\mathbf{z}_i, \mathbf{y}) \log \frac{\sqrt{x_1 x_2 \zeta_p}}{\sqrt{\zeta_0}} \right] \\ & \times {}^{R'_1 R'_2} F_{a_1 a_2}(x_i, \mathbf{z}_i, \mathbf{y}; \mu_{01}, \mu_{02}, \zeta_0/(x_1 x_2)). \end{aligned} \quad (2.5)$$

Correspondingly, the expression (5.19) for the cross section level becomes

$$\begin{aligned}
 W_{a_1 a_2 b_1 b_2} = & \exp \left\{ \int_{\mu_{01}}^{\mu_1} \frac{d\mu}{\mu} \left[\gamma_{a_1}(\mu) - \gamma_{K,a_1}(\mu) \log \frac{Q_1^2}{\mu^2} \right] + {}^{11}K_{a_1}(\mathbf{z}_1; \mu_{01}) \log \frac{Q_1 Q_2}{\zeta_0} \right. \\
 & \left. + \int_{\mu_{02}}^{\mu_2} \frac{d\mu}{\mu} \left[\gamma_{a_2}(\mu) - \gamma_{K,a_2}(\mu) \log \frac{Q_2^2}{\mu^2} \right] + {}^{11}K_{a_2}(\mathbf{z}_2; \mu_{02}) \log \frac{Q_1 Q_2}{\zeta_0} \right\} \\
 & \times \Phi(\nu \mathbf{y}_+) \Phi(\nu \mathbf{y}_-) \sum_{R_1 R_2 R'_1 R'_2} \eta(R_1) \eta(R_2) {}^{\bar{R}_1 \bar{R}_2} F_{b_1 b_2}(\bar{x}_i, \mathbf{z}_i, \mathbf{y}; \mu_{01}, \mu_{02}, \zeta_0 / (\bar{x}_1 \bar{x}_2)) \\
 & \times {}^{R_1 R_2, \bar{R}'_1 \bar{R}'_2} \exp \left[M_{a_1 a_2}(\mathbf{z}_i, \mathbf{y}) \log \frac{Q_1 Q_2}{\zeta_0} \right] {}^{R'_1 R'_2} F_{a_1 a_2}(x_i, \mathbf{z}_i, \mathbf{y}; \mu_{01}, \mu_{02}, \zeta_0 / (x_1 x_2)) .
 \end{aligned} \tag{2.6}$$

Analogous variable substitutions should be made in equations (5.14), (6.20), (6.32), (6.44) and (6.47). Appropriate modifications should also be made for the rapidity parameters in the last sentence of section 5.3 and throughout section 6.5, including equations (6.52), (6.53), and (6.58).

DPDF evolution. The corrected DGLAP equation (5.24) for DPDFs is

$$\begin{aligned}
 & \frac{\partial}{\partial \log \mu_1} {}^{R_1 R_2} F_{a_1 a_2}(x_1, x_2, \mathbf{y}; \mu_i, \zeta_p) \\
 & = 2 \sum_{b_1, R'_1} {}^{R_1 \bar{R}'_1} P_{a_1 b_1}(x'_1; \mu_1, x_1^2 \zeta_p) \otimes_{x_1} {}^{R'_1 R_2} F_{b_1 a_2}(x'_1, x_2, \mathbf{y}; \mu_i, \zeta_p) ,
 \end{aligned} \tag{2.7}$$

where the rapidity argument of the evolution kernel P is rescaled in the same way as the anomalous dimensions in (2.4), thus referring to the plus-momentum of the parton associated with the renormalization scale μ_1 . An equation analogous to (2.7) holds of course for the derivative w.r.t. $\log \mu_2$.

With the corrected DGLAP equations for DPDFs, equations (5.27) to (5.29) must be modified as well. A correct way to separate the ζ dependence is

$$\begin{aligned}
 & {}^{R_1 R_2} F_{a_1 a_2}(x_i, \mathbf{y}; \mu_i, \zeta_p) \\
 & = \exp \left[{}^{R_1} J(\mathbf{y}; \mu_i) \log \frac{\sqrt{\zeta_p}}{\sqrt{\zeta_0}} \right] {}^{R_1 R_2} F_{a_1 a_2}(x_i, \mathbf{y}; \mu_i, \zeta_0) \\
 & = \exp \left[{}^{R_1} J(\mathbf{y}; \mu_i) \log \frac{\sqrt{\zeta_p}}{\sqrt{\zeta_0}} - \int_{\mu_0}^{\mu_1} \frac{d\mu}{\mu} {}^{R_1} \gamma_J(\mu) \log \frac{\sqrt{\zeta_0}}{\mu} - \int_{\mu_0}^{\mu_2} \frac{d\mu}{\mu} {}^{R_1} \gamma_J(\mu) \log \frac{\sqrt{\zeta_0}}{\mu} \right] \\
 & \quad \times {}^{R_1 R_2} \hat{F}_{a_1 a_2, \mu_0, \zeta_0}(x_i, \mathbf{y}; \mu_i) ,
 \end{aligned} \tag{2.8}$$

where $\hat{F}$ obeys the evolution equation

$$\begin{aligned}
 & \frac{\partial}{\partial \log \mu_1} {}^{R_1 R_2} \hat{F}_{a_1 a_2, \mu_0, \zeta_0}(x_1, x_2, \mathbf{y}; \mu_1, \mu_2) \\
 & = - {}^{R_1} \gamma_J(\mu_1) \log x_1 {}^{R_1 R_2} \hat{F}_{a_1 a_2, \mu_0, \zeta_0}(x_1, x_2, \mathbf{y}; \mu_1, \mu_2) \\
 & \quad + 2 \sum_{b_1, R'_1} {}^{R_1 \bar{R}'_1} P_{a_1 b_1}(x'_1; \mu_1, \mu_1^2) \otimes_{x_1} {}^{R'_1 R_2} \hat{F}_{b_1 a_2, \mu_0, \zeta_0}(x'_1, x_2, \mathbf{y}; \mu_1, \mu_2)
 \end{aligned} \tag{2.9}$$

and its analogue for μ_2 , with the initial condition

$$\begin{aligned} {}^{R_1 R_2} \widehat{F}_{a_1 a_2, \mu_0, \zeta_0}(x_i, \mathbf{y}; \mu_0, \mu_0) &= {}^{R_1 R_2} F_{a_1 a_2}(x_i, \mathbf{y}; \mu_0, \mu_0, \zeta_0) \\ &= \exp \left[{}^{R_1} J(\mathbf{y}; \mu_0, \mu_0) \log \sqrt{x_1 x_2} \right] {}^{R_1 R_2} F_{a_1 a_2}(x_i, \mathbf{y}; \mu_0, \mu_0, \zeta_0/(x_1 x_2)). \end{aligned} \quad (2.10)$$

In (2.8) it is essential that ζ_0 does *not* depend on x_1 or x_2 , so that the initial condition (2.10) is taken at constant $\zeta_p = \zeta_0$. In the second line of that equation, we have indicated that this can of course be related to an initial condition at $\zeta_p = \zeta_0/(x_1 x_2)$ by another step of Collins-Soper evolution.

Reflecting these changes, the last sentence in section 7.3.5 should be modified as follows: “One can thus adapt numerical code for the one-loop evolution of colour singlet DPDs by rescaling the evolution kernels and adding the term with ${}^{R_1} \gamma_J$ from (2.9).”

Matching DTMDs to DPDEs. The corrected master formula (6.11) for the matching of a DTMD at large $\mathbf{y}$ reads

$$\begin{aligned} {}^{R_1 R_2} F_{a_1 a_2}(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta_p) &= \sum_{b_1 b_2, R'_1 R'_2} {}^{R_1} \bar{R}'_1 C_{a_1 b_1}(x'_1, \mathbf{z}_1; \mu_1, x_1^2 \zeta_p) \\ &\quad \otimes_{x_1} {}^{R_2} \bar{R}'_2 C_{a_2 b_2}(x'_2, \mathbf{z}_2; \mu_2, x_2^2 \zeta_p) \otimes_{x_2} {}^{R'_1 R'_2} F_{b_1 b_2}(x'_i, \mathbf{y}; \mu_i, \zeta_p). \end{aligned} \quad (2.11)$$

Corresponding substitutions for the rescaled rapidity parameters must be made in (6.21) and (G.1). Moreover, equation (6.19) is incorrect and should be replaced with

$$\begin{aligned} \frac{\partial}{\partial \log \mu} {}^{RR''} C_{ac}(x, \mathbf{z}; \mu, \zeta) &= \gamma_{F,a}(\mu, \zeta) {}^{RR''} C_{ac}(x, \mathbf{z}; \mu, \zeta) \\ &\quad - 2 \sum_{b, R'} \int_x^1 \frac{dx'}{x'} {}^{R\bar{R}'} C_{ab}(x', \mathbf{z}; \mu, \zeta) {}^{R'R''} P_{bc}\left(\frac{x}{x'}, \mu, \frac{\zeta}{x'^2}\right), \end{aligned} \quad (2.12)$$

which involves a special rescaling of the rapidity parameter in the evolution kernel P . The corrected form of (6.22) is

$$\begin{aligned} {}^{R_1 R_2} F_{a_1 a_2}(x_i, \mathbf{z}_i, \mathbf{y}; \mu_i, \zeta_p) &= \exp \left\{ \int_{\mu_{01}}^{\mu_1} \frac{d\mu}{\mu} \left[\gamma_{a_1}(\mu) - \gamma_{K,a_1}(\mu) \log \frac{x_1 \sqrt{\zeta_p}}{\mu} \right] + {}^{R_1} K_{a_1}(\mathbf{z}_1; \mu_{01}) \log \frac{x_1 \sqrt{\zeta_p}}{\mu_{01}} \right. \\ &\quad \left. + \int_{\mu_{02}}^{\mu_2} \frac{d\mu}{\mu} \left[\gamma_{a_2}(\mu) - \gamma_{K,a_2}(\mu) \log \frac{x_2 \sqrt{\zeta_p}}{\mu} \right] + {}^{R_2} K_{a_2}(\mathbf{z}_2; \mu_{02}) \log \frac{x_2 \sqrt{\zeta_p}}{\mu_{02}} \right\} \\ &\quad \times \sum_{b_1 b_2, R'_1 R'_2} {}^{R_1} \bar{R}'_1 C_{a_1 b_1}(x'_1, \mathbf{z}_1; \mu_{01}, \mu_{01}^2) \otimes_{x_1} {}^{R_2} \bar{R}'_2 C_{a_2 b_2}(x'_2, \mathbf{z}_2; \mu_{02}, \mu_{02}^2) \\ &\quad \otimes_{x_2} \exp \left[{}^{R_1} J(\mathbf{y}; \mu_{0i}) \log \frac{\sqrt{x'_1 x'_2 \zeta_p}}{\sqrt{\zeta_0}} \right] {}^{R'_1 R'_2} F_{b_1 b_2}(x'_i, \mathbf{y}; \mu_{0i}, \zeta_0/(x'_1 x'_2)). \end{aligned} \quad (2.13)$$

This is quite similar to (2.5), but now the initial condition for the rapidity parameter depends on the momentum fractions x'_1 and x'_2 of the DPD under the convolution integrals

with the matching kernels C . In (6.23) one should replace ζ with ζ_p and ζ_0 with $\zeta_0/(x'_1 x'_2)$ in the function arguments and in the explicit logarithm. In the cross-section level result (6.24), one should then replace the last line by

$$\begin{aligned} & \otimes_{x_2} [\Phi(\nu \mathbf{y})]^2 \exp \left[R_1 J(\mathbf{y}; \mu_{0i}) \log \frac{s \sqrt{x'_1 x'_2 \bar{x}'_1 \bar{x}'_2}}{\zeta_0} \right] \\ & \times {}^{R'_3 R'_4} F_{d_1 d_2}(\bar{x}'_i, \mathbf{y}; \mu_{0i}, \zeta_0/(\bar{x}'_1 \bar{x}'_2)) {}^{R'_1 R'_2} F_{c_1 c_2}(x'_i, \mathbf{y}; \mu_{0i}, \zeta_0/(x'_1 x'_2)), \end{aligned} \quad (2.14)$$

where as in (2.13) the constant ζ_0 is rescaled by the momentum fractions of the DPDs under the convolution integrals with the matching kernels.

Given the above modifications regarding $\hat{F}_{a_1 a_2, \mu_0, \zeta_0}$, we do not see a corrected form of (6.27) that would be particularly useful, so this equation and its discussion should be discarded.

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