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A cornucopia of AdS_5 vacua

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Abstract: We report on a systematic search for AdS_5 vacua corresponding to critical points of the potential in the five-dimensional $\mathcal{N} = 8$ SO(6) gauged supergravity. By employing Google's TensorFlow Machine Learning library, we find the total of 32 critical points including 5 previously known ones. All 27 new critical points are non-supersymmetric. We compute the mass spectra of scalar fluctuatons for all points and find that the non-supersymmetric AdS_5 vacua are perturbatively unstable. Many of the new critical points can be found analytically within consistent truncations of the $\mathcal{N} = 8$ supergravity with respect to discrete subgroups of the $\mathrm{S}(\mathrm{O}(6) \times \mathrm{GL}(2, \mathbb{R}))$ symmetry of the potential. In particular, we discuss in detail a $\mathbb{Z}_2^3$-invariant truncation with 10 scalar fields and 15 critical points. We also compute explicitly the scalar potential in a $\mathbb{Z}_2^2$-invariant extension of that truncation to 18 scalar fields and reproduce 17 of the 32 critical points from the numerical search. Finally, we show that the full potential as a function of 42 scalar fields can be studied analytically using the so-called solvable parametrization. In particular, we find that all critical points lie in a $\mathbb{Z}_2$-invariant subspace spanned by 22 scalar fields.

Keywords: AdS-CFT Correspondence, Gauge-gravity correspondence, Supergravity Models, Superstring Vacua

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JHEP07(2020)240

Contents

1 Introduction

The AdS/CFT correspondence is deeply rooted in string theory and its low-energy supergravity limits. Therefore, it is important to understand fully the landscape of consistent AdS backgrounds in string theory. A fruitful strategy has been to identify a consistent Kaluza-Klein (KK) truncation of a ten- or eleven-dimensional supergravity to lower d dimensions and to study critical points of the scalar potential in the resulting gauged supergravity. Each critical point with a negative value of the potential leads to an AdS_d solution and thus candidate AdS background of string theory.

JHEP07(2020)240

Our goal in this paper is to use a mixture of old analytic and modern numerical methods to search systematically for critical points of the scalar potential in $\mathcal{N} = 8$ SO(6) gauged supergravity in five dimensions [1–3]. This is interesting for several reasons. First, there is now a complete, constructive proof that this five-dimensional supergravity is a consistent KK truncation of type IIB string theory on S^5 [4–8]. In particular, this means that all AdS_5 vacua corresponding to critical points of the supergravity potential can be uplifted to AdS solutions of string theory. Secondly, the problem should be amenable to similar computational techniques based on Machine Learning that were successfully applied in [9] to find hundreds of new critical points of the scalar potential in the de Wit-Nicolai SO(8) gauged supergravity in four dimensions [10]. Finally, by extrapolating the results in [11], it is natural to expect that a large fraction of the critical points might be accessible analytically, or semi-analytically, within a suitable truncation with respect to a discrete subgroup of the full symmetry group of the theory.

Through holography, the SO(6) gauged supergravity has been an indispensable tool for studying the $\mathcal{N} = 4$ SYM theory and its deformations. Indeed, AdS_5 vacua are dual to conformal fixed points obtained by deforming $\mathcal{N} = 4$ SYM and domain wall solutions between these critical points are dual to RG flows between the CFTs [12–14]. From this perspective, one would also like to determine the stability of those vacua. If an AdS_5 solution is supersymmetric, it is necessarily stable [15] and the dual CFT is unitary. However, if there are scalar fluctuations with negative masses violating the Breitenlohner-Freedman (BF) bound [16], the dual operators have complex dimensions and the dual CFT is not unitary. In fact, it has been argued in [17] using the Weak Gravity Conjecture [18] that all non-supersymmetric vacua in string theory should be unstable. The violation of the BF bound for a given AdS solution is then the simplest sign of that instability.

It is perhaps surprising that not much progress has been made in classifying AdS_5 vacua of the SO(6) gauged supergravity since the initial discovery in 1998 of five critical points listed in table 1[1] in an SU(2)-invariant sector of the theory [4]. One reason might be that the Leigh-Strassler analysis [20] of $\mathcal{N} = 1$ deformations of $\mathcal{N} = 4$ SYM suggests that there should be no other supersymmetric critical points beyond the $\mathcal{N} = 8$ point, `T0750000`, and the $\mathcal{N} = 2$ point, `T0839947`, already found in [4]. The other three points in table 1 are non-supersymmetric and perturbatively unstable as discussed further in appendix D. It is then reasonable to expect that any missing point is non-supersymmetric and thus perturbatively unstable as well. Note, however, that the latter need not be necessarily true given that there is a perturbatively stable yet non-supersymmetric SO(3) $\times$ SO(3)-invariant AdS_4 solution in four dimensions [21, 22],[2] and there are multiple examples of perturbatively stable AdS_3 vacua in three-dimensional supergravities [25, 26].

Given the large number of known critical points of the scalar potentials in maximal gauged supergravities in three [25, 26] and four dimensions [9, 11, 19, 21, 22, 27–29], it is

[1]Following the convention for labelling critical points in four-dimensional supergravity [19], we propose to denote the points in five dimensions according to the value of the first seven digits in their cosmological constant by $\mathtt{Tn_1n_2n_3n_4n_5n_6n_7}$.

[2]However, it has been shown recently that this solution is unstable in string theory due to brane-jet instability [23] and higher KK-modes violating the BF bound [24].

Point	Symmetry	$\mathcal{P}_*$	SUSY	BF Stability
T0750000	SO(6)	$-\frac{3}{4}$	$\mathcal{N}=8$	S
T0780031	SO(5)	$-\frac{3^{5/3}}{8}$	—	U
T0839947	$\mathrm{SU}(2)\times\mathrm{U}(1)$	$-\frac{2^{4/3}}{3}$	$\mathcal{N}=2$	S
T0843750	SU(3)	$-\frac{27}{32}$	—	U
T0870297	$\mathrm{SU}(2)\times\mathrm{U}(1)^2$	$-\frac{3}{8}\left(\frac{25}{2}\right)^{1/3}$	—	U

Table 1. The SU(2)-invariant extrema [4].

to be expected that there are comparably many AdS_5 vacua of the five-dimensional SO(6) gauged supergravity beyond the ones in table 1. It is the lower symmetry (less than SU(2)) of these vacua that makes looking for them a challenging problem.

Recall that the potential of the $\mathcal{N}=8$ $d=5$ supergravity is a function on the 42-dimensional scalar manifold, which is a coset of the maximally noncompact group $\mathrm{E}_{6(6)}$ modded by its compact subgroup, USp(8). In the conventions of [2],[3] the potential can be written as

$$\mathcal{P}=-\frac{1}{32}\,g^2\,\left[\,2(W_{ab})^2-(W_{abcd})^2\,\right]\,, \tag{1.1}$$

which looks deceptively simple until fully unpacked. Indeed, the W_{ab} and W_{abcd} tensors are quadratic in the components of the scalar 27-bein, $\mathcal{V}=(\mathcal{V}^{IJab},\mathcal{V}_{I\alpha}{}^{ab})$, which, modulo a linear transformation, is a group element of $\mathrm{E}_{6(6)}$ obtained by exponentiating non-compact elements, $\Phi=\sum\phi_A T_A$, of the Lie algebra $\mathfrak{e}_{6(6)}$, where T_A are some fixed generators and ϕ_A are the 42 scalars fields. It follows from the construction of the W-tensors that the potential is manifestly invariant under the SO(6) gauge symmetry acting on the $I,J=1,\ldots,6$ indices as well as the axion-dilaton $\mathrm{SL}(2,\mathbb{R})$ that acts on the $\alpha=7,8$ index of the 27-bein. This reduces the number of independent degrees of freedom in (1.1) to $42-15-3=24$. In fact, by including discrete symmetries one can show that the actual symmetry is $\mathrm{S}(\mathrm{O}(6)\times\mathrm{GL}(2,\mathbb{R}))$ [30, 31], which we will exploit in section 2. When viewed as a function on $\mathrm{E}_{6(6)}$, the potential (1.1) is also invariant under local USp(8) transformations acting on the $a,b=1,\ldots,8$ indices of the 27-bein, but that symmetry is already fixed by the USp(8) gauge choice in Φ.

The problem now is to compute the potential, $\mathcal{P}(\phi_A)$, as an explicit function of the scalar fields and then find its critical points. Analytically, this cannot be done in full generality. A time-tested method, first used by Warner [21, 27] in four dimensions, is to truncate the potential of interest to a smaller number of fields that are invariant under some subgroup, G, of the full symmetry group of the theory. The critical points of the truncated potential are then automatically critical points of the full potential. For a judicious choice of the subgroup, G, one may end up with an analytically tractable problem leading to a potential with new critical points. As we discuss briefly in section 2, this method has not been too successful thus far in five dimensions beyond the original analysis in [4]. The

[3] See also appendix A.

JHEP07(2020)240

scalar potentials in various truncations considered over the years in the literature either did not include new critical points or were deemed too complicated to attempt extremization.

Another way to make progress is to attack the problem numerically. This has been initiated about ten years ago by one of the authors and resulted in around 40 new AdS_4 vacua [19, 22, 26, 28, 32] in the de Wit-Nicolai SO(8) gauged supergravity for the total of 50 critical points known in 2013.[4] Recently, a more powerful numerical code using Machine Learning (ML) and Google's TensorFlow libraries [33] was developed in [9] and led to the total of 194 points that include 2 additional ones found in the follow up analytic work [11]. It is rather straightforward to port the ML code included with [9] from four to five dimensions and, in fact, considerably simplify it using the new publicly available TensorFlow2 libraries[5] as well as by exploiting symmetries of the potential.

By performing a systematic, numerical search using the new ML code, we find the total of 32 AdS_5 vacua in $\mathcal{N}=8$ $d=5$ SO(6) gauged supergravity. Those include the 5 classic ones in table 1. We also compute the gravitini and scalar spectra at each point, which are needed to determine unbroken supersymmetries and the BF (in)stability. We find that all 27 new points are non-supersymmetric, which is compatible with the expectation that the dual $\mathcal{N}=4$ SYM theory does not admit relevant deformations, apart from the one in [20], which lead to interacting supersymmetric CFTs. All new points have BF unstable scalar modes, which is perhaps disappointing, but not unexpected. Hence our results further support the instability conjecture for non-supersymmetric AdS vacua in string theory [17].

It turns out that many of the new AdS_5 vacua can also be found using more analytic methods. We generalize here an observation in [11] about the existence of a special truncation in four dimensions in which the scalar manifold is a product of mutually commuting Poincaré disks. That truncation arises from the subalgebra $\mathfrak{su}(1,1)^7\subset\mathfrak{e}_{7(7)}$ and can be obtained by imposing a discrete $\mathbb{Z}_2^3$ symmetry on the scalar fields. This truncation is quite remarkable in that it is very easy to analyze analytically and yet its potential captures 25% of the 194 known critical points.

A natural question is whether there exists a similarly marvelous truncation for the $\mathcal{N}=8$ supergravity in five dimensions. We find that indeed it does and corresponds to the embedding $\mathfrak{o}(1,1)^2\oplus\mathfrak{su}(1,1)\subset\mathfrak{e}_{6(6)}$ for which the scalar coset is a product of 6 simple commuting factors,

$$\mathcal{M}_{(10)}\equiv \mathrm{O}(1,1)^2\times\left(\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}\right)^4\,, \tag{1.2}$$

that is 2 half-lines and 4 Poincaré disks. In fact, there exist two different consistent truncations for which the full scalar potential has been already worked out in the literature. Both use a $\mathbb{Z}_2^3$ symmetry and have the same looking coset, but preserve different amount of the $\mathrm{SO}(6)\times\mathrm{SL}(2,\mathbb{R})$ symmetry. The first one, found 20 years ago in [34], has an $\mathrm{U}(1)^4$ unbroken symmetry so that the truncated potential can be reduced to 6 scalar fields. The second truncation, found quite recently in [31], breaks all continuous symmetries and the potential is a function of all 10 fields. For that reason we will refer to them as the 6-scalar

[4]Those include the 7 original points found in the "classic period" [21, 27] and one further point in [29].
[5]Cf. https://blog.tensorflow.org/2019/09/tensorflow-20-is-now-available.html.

and the 10-scalar model, respectively. As we show in section 2, all critical points in the 6-scalar model lie within the 10-scalar model. We find that the latter has 15 AdS_4 vacua, with many of the new critical points computable exactly and a few remaining ones easily accessible to standard numerical routines for example in Mathematica.

The 6-scalar and the 10-scalar models arise by imposing different $\mathbb{Z}_2$ symmetry on the same intermediate truncation of the $\mathcal{N}=8$ supergravity with respect to $\mathbb{Z}_2^2 \subset \mathrm{SO}(6)$. That truncation has the scalar coset,

$$\mathcal{M}_{(18)} \equiv \mathrm{O}(1,1)^2 \times \frac{\mathrm{SO}(4,4)}{\mathrm{S}(\mathrm{O}(4)\times\mathrm{O}(4))}\,, \tag{1.3}$$

and preserves $\mathrm{U}(1)^4$ continuous symmetry. This means that the potential in this model depends on $18-4=14$ scalar fields. By carefully choosing the parametrization of both factors, we are able to compute the potential in a closed analytic form and determine, once more using a simple numerical routine, that it has in total 17 critical points. This shows that more than 50% of all critical points are accessible analytically either exactly or by using a high level numerical routines to solve a system of explicit equations for the extrema of the potential.

An even stronger validation of the numerical results of the TensorFlow search can be obtained by employing the so-called solvable parametrization [35, 36] of the full scalar coset. Indeed, we find that in this theory, as opposed to the ones in three or four dimensions, it is possible to compute the potential as an explicit function of all 42 scalar fields and then perform a search for critical points using Mathematica routines. This parametrizaton also allows us to consider systematically the $\mathbb{Z}_2$-invariant truncations that encompass the analytic results above. In particular, it turns out that all 32 critical points can be found within a 22-dimensional subspace of the coset manifold that is invariant under the special $\mathbb{Z}_2$ symmetry used to arrive at the 10-scalar model.

We begin in section 2 with a detailed discussion of the 10-scalar model and compute analytically whenever possible its 15 critical points. In section 3 we describe the numerical search performed with TensorFlow and elucidate relevant differences between the computational strategies in the $d=5$ search here and the $d=4$ search in [9]. In section 4, we reproduce all critical points using the solvable parametrzation and summarize various $\mathbb{Z}_2$-invariant truncations. We conclude with some open questions in section 5. A lot of technical details can be found in the appendices. Throughout the paper we use the same conventions as in [2]. However, to avoid any ambiguities, in appendix A we summarize the details of our parametrization of the scalar coset of the SO(6) supergravity. Appendix B has a detailed discussion of the consistent truncation to 18 scalars in (1.3). We present a careful derivation of the full potential in this 14-scalars model and find its critical points. The results of the full numerical search can be found in appendix C. We give a list of all 31 critical points together with their locations (partially canonicalized) and the mass spectra. Finally, in appendix D, we collect some old results for the scalar mass spectra around the critical points in table 1, most of which where known to many but never published.

Note added in version 1. The results in this paper were reported in seminars and at a conference [37, 38]. While we were preparing this manuscript, we became aware of the recent work [39], which finds 32 critical points of the scalar potential in $\mathcal{N}=8$ $d=5$ gauged supergravity using TensorFlow. The authors of [39] also calculate the gravitini spectra and find no new supersymmetric points. Apart from solution #26 in [39], which is missing from our list, we find a complete match between their values of the potential at critical points and the ones in our search. This provides yet another validation of the two numerical searches.

Note added. Since the posting of the original version of this paper, we found all 32 critical points using our TensorFlow code that will soon be open sourced in the Google Research M-theory repository.[6] These results are also independently confirmed by the new search using a combination of analytic and numerical routines in Mathematica based on the solvable parametrization of the potential as summarized in the newly added section 4.

2 The 10-scalar model

As explained in the Introduction, one way to deal with the complexity of the five-dimensional SO(6) gauged supergravity is to look for consistent truncations of the theory by imposing invariance under a subgroup, G, of the full symmetry group. In most examples $G\subset \mathrm{SO}(6)$, or $\mathrm{SO}(6)\times \mathrm{SL}(2,\mathbb{R})$, but the most interesting truncation discussed in this section is when $G\subset \mathrm{S}(\mathrm{O}(6)\times \mathrm{GL}(2,\mathbb{R}))$. Several such consistent truncations have been studied in the literature in various contexts, see for example [6, 12, 13, 30, 40–44], but no new critical points have been found after a systematic search within an SU(2) invariant truncation in [4]. Other SU(2)-invariant truncations listed in table A.1 in [14] merely reproduce a subset of critical points in [4]. The same is true for an U(1)-invariant truncation in [44]. There are, however, two truncations with respect to discrete symmetries obtained in [34] and [31], respectively, in which "holomorphic" superpotentials, and thus the full scalar potentials, are known explicitly. It appears that those potentials have never been fully analyzed. Therefore we begin our search by carefully examining these two models.

2.1 The consistent truncation

Motivated by this we start our discussion by studying the critical points of the consistent truncation in [31] which is invariant under a $\mathbb{Z}_2^3$ subgroup of $\mathrm{S}(\mathrm{O}(6)\times\mathrm{GL}(2,\mathbb{R}))$ and contains 10 out of the 42 scalar fields of the maximal theory. The procedure to obtain this truncation is outlined in detail in [31] and so we will be brief. Consider the O(6) matrices

$$\begin{aligned} P_1 &= \mathrm{diag}(-1,-1,1,1,1,1)\,,\\ P_2 &= \mathrm{diag}(1,1,-1,-1,1,1)\,,\\ P_3 &= \mathrm{diag}(1,-1,1,-1,1,-1)\,, \end{aligned} \tag{2.1}$$

and the following GL(2) matrices

$$Q=\mathrm{diag}(-1,-1)\,,\qquad Q'=\mathrm{diag}(-1,1)\,. \tag{2.2}$$

[6] https://github.com/google-research/google-research/tree/master/m_theory.

The truncation consists of the five-dimensional metric in addition to all fields that are invariant under the action of P_1Q, P_2Q, P_3Q'. Even though the third matrix P_3Q' is not inside $\mathrm{SL}(6,\mathbb{R})\times\mathrm{SL}(2,\mathbb{R})$, it is still a valid discrete symmetry to impose as explained in [30, 31]. In this paper we focus on AdS_5 vacua of the theory and so are only concerned with scalar fields that are invariant. Form fields must be set to zero. Imposing these symmetries leaves ten scalars parametrizing the scalar manifold

$$\mathcal{M}_{(10)} = \mathrm{O}(1,1)\times\mathrm{O}(1,1)\times\left(\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}\right)^4 . \tag{2.3}$$

These consist of five $\mathbf{20'}$ scalars, four $\mathbf{10}\oplus\overline{\mathbf{10}}$ scalars, and the dilaton. Two of the $\mathbf{20'}$ scalars are singled out as they parametrize the two $\mathrm{O}(1,1)$-factors in (2.3). An explicit parametrization of the coset (2.3) is given by specifying the generators of $\mathfrak{e}_{6(6)}$ that are invariant with respect to our choice of discrete symmetries. As explained in appendix A we use an $\mathrm{SL}(6,\mathbb{R})\times\mathrm{SL}(2,\mathbb{R})$ basis to define our generators. In particular the two $\mathrm{O}(1,1)$-factors correspond to the generators $\mathfrak{g}_\alpha$ and $\mathfrak{g}_\beta$ defined by

$$\begin{aligned}\mathfrak{g}_\alpha &= \widehat{\Lambda}^1{}_1+\widehat{\Lambda}^2{}_2-\widehat{\Lambda}^3{}_3-\widehat{\Lambda}^4{}_4\,,\\ \mathfrak{g}_\beta &= \widehat{\Lambda}^1{}_1+\widehat{\Lambda}^2{}_2+\widehat{\Lambda}^3{}_3+\widehat{\Lambda}^4{}_4-2\widehat{\Lambda}^5{}_5-2\widehat{\Lambda}^6{}_6\,,\end{aligned} \tag{2.4}$$

using the notation in appendix A. The remaining scalars are best parametrized in terms of one of the non-compact generators of $\mathfrak{su}(1,1)$ together with the compact one. The remaining non-compact generator can be obtained as the commutator of the other two. Using the notation in appendix A the four compact generators $\mathfrak{r}$ and the non-compact generators $\mathfrak{t}$ are specified by

$$\begin{aligned}
\mathfrak{t}_1 &= \frac{1}{\sqrt{2}}(\widehat{\Sigma}_{1357}-\widehat{\Sigma}_{2468})\,, & \mathfrak{r}_1 &= \frac{1}{\sqrt{2}}(\widehat{\Sigma}_{1357}+\widehat{\Sigma}_{2468})\,,\\
\mathfrak{t}_2 &= \frac{1}{\sqrt{2}}(\widehat{\Sigma}_{2367}-\widehat{\Sigma}_{1458})\,, & \mathfrak{r}_2 &= -\frac{1}{\sqrt{2}}(\widehat{\Sigma}_{2367}+\widehat{\Sigma}_{1458})\,,\\
\mathfrak{t}_3 &= \frac{1}{\sqrt{2}}(\widehat{\Sigma}_{2457}-\widehat{\Sigma}_{1368})\,, & \mathfrak{r}_3 &= -\frac{1}{\sqrt{2}}(\widehat{\Sigma}_{2457}+\widehat{\Sigma}_{1368})\,,\\
\mathfrak{t}_4 &= \frac{1}{\sqrt{2}}(\widehat{\Sigma}_{1467}-\widehat{\Sigma}_{2358})\,, & \mathfrak{r}_4 &= \frac{1}{\sqrt{2}}(\widehat{\Sigma}_{1467}+\widehat{\Sigma}_{2358})\,.
\end{aligned} \tag{2.5}$$

The full $\mathrm{E}_{6(6)}$ group element is constructed as follows

$$\mathcal{V} = \exp(\alpha\mathfrak{g}_\alpha)\cdot\exp(\beta\mathfrak{g}_\beta)\cdot\prod_{i=1}^{4}\exp(-\omega_i\mathfrak{r}_i)\cdot\exp(\rho_i\mathfrak{t}_i)\cdot\exp(\omega_i\mathfrak{r}_i)\,. \tag{2.6}$$

Notice that the commutator of the $\mathrm{SU}(1,1)$ generators $\mathfrak{t}_i$ and $\mathfrak{r}_i$ gives a linear combination of $\mathbf{20'}$ generators $\widehat{\Lambda}^I{}_J$ in addition to the dilaton generator $\mathfrak{g}_{\text{dilaton}} = \widehat{\Lambda}^7{}_7-\widehat{\Lambda}^8{}_8$. It thus follows that one of the ten scalars is the dilaton. Since the scalar potential of the full $\mathrm{SO}(6)$ gauged theory does not depend on the dilaton, the same will be true in the truncated 10-scalar model. The way we have parametrized the manifold, $\mathcal{M}_{(10)}$, in (2.6), the dilaton is mixed with all the other SU(1,1) scalars and isolating it is difficult. The action of $SL(2,\mathbb{R})$ on $\mathcal{V}$ is given by the transformation

$$\mathcal{V} \mapsto \mathcal{V} \cdot \exp(t\, \mathfrak{g}_{\text{dilaton}}) \,, \tag{2.7}$$

which leaves the potential invariant. Even though in principle it should be possible to translate what this action implies for the scalars ρ_i and ω_i, in practice the transformation is a complicated simultaneous action on all eight fields.

The scalar potential of this truncation can be compactly written as

$$\mathcal{P} = \frac{1}{32} e^{\mathcal{K}} \left(\frac{1}{6} |\partial_\alpha \mathcal{W}|^2 + \frac{1}{2} |\partial_\beta \mathcal{W}|^2 + \mathcal{K}^{i\bar{\jmath}} D_i \mathcal{W} D_{\bar{\jmath}} \overline{\mathcal{W}} - \frac{8}{3} |\mathcal{W}|^2 \right) , \tag{2.8}$$

where the Kähler covariant derivative is $D_i \mathcal{F} \equiv \partial_i \mathcal{F} + \mathcal{F} \partial_i \mathcal{K}$, the Kähler potential is

$$\mathcal{K} = - \sum_{i=1}^{4} \log(1 - |z_i|^2) \,, \tag{2.9}$$

and determines the kinetic terms through the Kähler metric $\mathcal{K}_{i\bar{\jmath}} \equiv \frac{\partial \mathcal{K}}{\partial z^i \partial \bar{z}^{\bar{\jmath}}}$ and its inverse $\mathcal{K}^{i\bar{\jmath}}$. The superpotential is [31]

$$\begin{aligned} \mathcal{W} = {} & e^{-4\alpha}(1 + z_1 z_2 - z_1 z_3 - z_1 z_4 - z_2 z_3 - z_2 z_4 + z_3 z_4 + z_1 z_2 z_3 z_4) \\ & + e^{2\alpha+2\beta}(1 + z_1 z_2 + z_1 z_3 + z_1 z_4 + z_2 z_3 + z_2 z_4 + z_3 z_4 + z_1 z_2 z_3 z_4) \\ & + e^{2\alpha-2\beta}(1 - z_1 z_2 + z_1 z_3 - z_1 z_4 - z_2 z_3 + z_2 z_4 - z_3 z_4 + z_1 z_2 z_3 z_4) \,. \end{aligned} \tag{2.10}$$

The complex scalars z_i are related to the ρ_i and ω_i in (2.6) as follows:

$$z_j = i \tanh \frac{\rho_j}{2}\, e^{-i\omega_j} \,. \tag{2.11}$$

The 10-scalar model exhibits a number of discrete symmetries, some of which were identified in [31]. For example $z_i \mapsto \pm \bar{z}_i$ and $z_i \mapsto -z_i$. Here we would like to point out a rather large group of symmetries that leaves the superpotential invariant. It can be specified by

$$\begin{aligned} e_1 : \quad & \alpha \mapsto -\frac{\alpha+\beta}{2} \,, \quad \beta \mapsto \frac{-3\alpha+\beta}{2} \,, \quad z_1 \mapsto -z_2 \mapsto z_1 \,. \\ e_2 : \quad & \alpha \mapsto -\frac{\alpha+\beta}{2} \,, \quad \beta \mapsto \frac{3\alpha-\beta}{2} \,, \quad z_2 \mapsto z_3 \mapsto -z_4 \mapsto z_2 \,. \\ e_3 : \quad & \alpha \mapsto \frac{-\alpha+\beta}{2} \,, \quad \beta \mapsto \frac{3\alpha+\beta}{2} \,, \quad z_1 \mapsto -z_2 \mapsto z_4 \mapsto -z_3 \mapsto z_1 \,. \end{aligned} \tag{2.12}$$

These satisfy $e_1^2 = e_2^3 = e_3^4 = e_1 e_2 e_3 = 1$ and therefore generate the group S_4. As we show in the next section this model has 15 critical points including all five of [4]. Furthermore, by computing the masses of the 10 scalar fields we have checked that all non-supersymmetric critical points are perturbatively unstable within the 10-scalar model.

We note that a simpler six-scalar model can be obtained by setting the real parts of all z_i scalars to zero. The potential then reduces to that of [34], see also [41]. In [34] a 10-scalar truncation was considered which is different from the one we have been discussing

JHEP07(2020)240

here. This latter truncation has a potential which only depends on six fields. Explicitly, the potential for this 6-scalar model can be obtained from the one in (2.8) by setting:

$$\begin{aligned} z_1 &= i \tanh \frac{1}{2}(\varphi_1 - \varphi_2 - \varphi_3 + \varphi_4)\,, \\ z_2 &= i \tanh \frac{1}{2}(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)\,, \\ z_3 &= i \tanh \frac{1}{2}(\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4)\,, \\ z_4 &= i \tanh \frac{1}{2}(\varphi_1 - \varphi_2 + \varphi_3 - \varphi_4)\,, \end{aligned} \tag{2.13}$$

The potential can also be written in terms of a superpotential,

$$\mathcal{P} = \frac{1}{8}\left(\frac{1}{6}(\partial_\alpha W)^2 + \frac{1}{2}(\partial_\beta W)^2 + (\partial_i W)^2 - \frac{8}{3}W^2\right)\,, \tag{2.14}$$

where ∂_i denotes a derivative with respect to φ_i and [34]

$$\begin{aligned} W = &\frac{1}{4}\mathrm{e}^{-4\alpha}(+\cosh 2\varphi_1 - \cosh 2\varphi_2 - \cosh 2\varphi_3 - \cosh 2\varphi_4) \\ &+ \frac{1}{4}\mathrm{e}^{2\alpha+2\beta}(-\cosh 2\varphi_1 - \cosh 2\varphi_2 + \cosh 2\varphi_3 - \cosh 2\varphi_4) \\ &+ \frac{1}{4}\mathrm{e}^{2\alpha-2\beta}(-\cosh 2\varphi_1 + \cosh 2\varphi_2 - \cosh 2\varphi_3 - \cosh 2\varphi_4)\,. \end{aligned} \tag{2.15}$$

Four of the points in table 1 are critical points of the 6-scalar potential in (2.14). Only the SO(5) invariant point, `T0780031`, lies outside it. There are in total eight critical points in the 6-scalar model.

2.2 Critical points

Here we provide a list of the 15 critical points of the 10-scalar model potential in (2.8). Some of these can be obtained analytically. For the others we have used the `FindRoot[ · ]` routine in Mathematica.

T0750000 [1–3].

$$z_a = 0\,, \quad \alpha = \beta = 0\,. \tag{2.16}$$

$$\mathcal{P} = -\frac{3}{4} = -0.750000\,. \tag{2.17}$$

Symmetry: SO(6) , $\mathcal{N} = 8$.
Comment: critical point of the 6-scalar model.

T0780031 [2].

$$z_2 = z_3 = 0\,, \quad z_1 = z_4 = 2 - \sqrt{3}\,, \quad \beta = 3\alpha = \frac{\log 3}{8}\,. \tag{2.18}$$

$$\mathcal{P} = -\frac{3 \times 3^{2/3}}{8}\,. \tag{2.19}$$

Symmetry: SO(5) , $\mathcal{N} = 0$.

T0839947 [4].

$$z_1 = i(\sqrt{3}-2)\,, \quad z_2 = z_3 = -z_4 = -z_1\,, \qquad 3\alpha = \beta = \frac{1}{4}\log 2\,. \tag{2.20}$$

$$\mathcal{P} = -\frac{2\times 2^{1/3}}{3} \approx -0.839947\,. \tag{2.21}$$

Symmetry: SU(2) × U(1) , $\mathcal{N}=2$.
Comment: critical point of the 6-scalar model.

T0843750 [2].

$$z_1 = -i\sqrt{5-2\sqrt{6}}\,, \quad z_2 = z_4 = -z_3 = -z_1\,, \qquad \alpha = \beta = 0\,. \tag{2.22}$$

$$\mathcal{P} = -\frac{27}{32} = -0.843750\,. \tag{2.23}$$

Symmetry: SU(3) , $\mathcal{N}=0$.
Comment: critical point of the 6-scalar model.

T0870297 [4].

$$z_1 = z_2 = 0\,, \quad z_3 = -z_4 = i\,\frac{2\sqrt{10}+\sqrt{15}-5}{2\sqrt{10}+\sqrt{15}+5} \qquad \beta = -3\alpha = -\frac{1}{8}\log 10\,. \tag{2.24}$$

$$\mathcal{P} = -\frac{3\times 5^{2/3}}{8\times 2^{1/3}}\,. \tag{2.25}$$

Symmetry: SU(2) × U(1) , $\mathcal{N}=0$.
Comment: critical point of the 6-scalar model. In [4], see table 1, the symmetry of this point is listed as SU(2) × U(1)^2. The second U(1) factor is the compact generator of SL(2, ℝ) which lies outside of the SO(6) gauge algebra.

T0878939. This point has $\mathcal{N}=0$ and is a critical point of the 6-scalar model found by setting

$$z_1 = z_2 = 0\,, \quad z_3 = -z_4 = i\frac{\sqrt{-X^2-(1-Y)^4Y^4+2XY^2(1+6Y+Y^2)}}{X+(1-Y)^2Y^2+2\sqrt{X}Y(1+Y)}\,, \tag{2.26}$$

with

$$\alpha = \frac{1}{24}\log X\,, \quad \beta = \frac{1}{2}\log Y - \frac{1}{8}\log X\,. \tag{2.27}$$

The potential is then

$$\mathcal{P} = -\frac{X^2+Y^4(1-Y^2)^2-2XY^2(3+4Y+3Y^2)}{16X^{1/3}Y^2(X+Y^2(1-Y)^2)}\,. \tag{2.28}$$

This potential has two critical points, `T0878939` and `T1001482` which are correlated as follows. First one takes one of the four real roots of the equation

$$8-44Y+33Y^2+74Y^3+33Y^4-44Y^5+8Y^6 = 0\,. \tag{2.29}$$

Note that the equation is self-reciprocal or palindromic and therefore the solutions come in inverse pairs which lead to the same cosmological constant. Use a given solution for Y to find X as a solution of the equation

$$5X^2 + (1-Y)^4 Y^4 + 2XY^2(1-10Y+Y^2) = 0\,. \tag{2.30}$$

where the solution for X must be correlated with the solution of Y. That is the choice of sign in the above second order equation for X is correlated with which of the two different solution we start with for Y. For `T0878939` we then find the approximate values

$$X = 0.006865\,, \quad Y = 0.283702\,, \quad \mathcal{P} = -0.878939\,. \tag{2.31}$$

The value of $\mathcal{P}$ can be obtained as a root of the polynomial

$$729 + 5723136\,\mathcal{P}^3 + 14123008\,\mathcal{P}^6 + 8388608\,\mathcal{P}^9\,. \tag{2.32}$$

Note that the `T0870298` is another critical point of (2.28) with $X=10$, $Y=1$.

T0887636. This point has $\mathcal{N}=0$ and is a critical point of the 6-scalar model located at

$$\begin{aligned} z_2 &= -z_3 = i\,\frac{1-\sqrt{Y+\sqrt{Y^2-1}}}{1+\sqrt{Y+\sqrt{Y^2-1}}}\,, \qquad\qquad \beta = 3\alpha = \frac{1}{8}\log\left(\frac{20+4\sqrt{34}}{3}\right)\,,\\ z_1 &= i\,\frac{X+\sqrt{X^2-1}-\sqrt{Y+\sqrt{Y^2-1}}}{X+\sqrt{X^2-1}+\sqrt{Y+\sqrt{Y^2-1}}}\,,\\ z_4 &= i\,\frac{(X+\sqrt{X^2-1})\sqrt{Y+\sqrt{Y^2-1}}-1}{(X+\sqrt{X^2-1})\sqrt{Y+\sqrt{Y^2-1}}+1}\,, \end{aligned} \tag{2.33}$$

where the potential reduces to

$$\mathcal{P} = \frac{(1+Y)^{1/3}\Big(2(1+Y)(Y^2-3) - X^2(7+3Y)\Big)}{16\times 2^{2/3}X^{4/3}}\,, \tag{2.34}$$

and

$$X^2 = \frac{1}{243}(88+40\sqrt{34})\,, \quad Y = \frac{1}{9}(-1+2\sqrt{34})\,. \tag{2.35}$$

The value of the potential is

$$\mathcal{P} = -\frac{(196079+33524\sqrt{34})^{1/3}}{3^{7/3}\times 2^{8/3}} \approx -0.887636\,, \tag{2.36}$$

which is the smaller of the two real roots of the polynomial

$$107811 + 100392448\mathcal{P}^3 + 143327232\mathcal{P}^6\,. \tag{2.37}$$

T0892913.

$$z_3 = i\frac{1}{\sqrt{5}}\,, \quad z_1 = z_2 = z_4 = 0\,, \quad 3\alpha = \beta = \frac{1}{2}\log 2\ . \tag{2.38}$$

$$\mathcal{P} = -\frac{9}{8\times 2^{1/3}}\,. \tag{2.39}$$

Symmetry: $\mathcal{N}=0$.
Comment: critical point of the 6-scalar model.

JHEP07(2020)240

T0964525.

$$\begin{aligned} &\alpha = -0.0262713\,, && \beta = 0.254756\,, && z_1 = 0.224701 - i\,0.487424\,, \\ &z_2 = 0.0709794\,, && z_3 = -0.256605\,, && z_4 = 0.0116728 - i\,0.507927\,. \end{aligned} \tag{2.40}$$

$$\mathcal{P} \approx -0.9645259\,. \tag{2.41}$$

Symmetry: $\mathcal{N} = 0$.

T0982778.

$$z_1 = -z_4 = \frac{\sqrt{19-4\sqrt{22}}}{\sqrt{3}}\,, \quad z_2 = z_3 = 0.280116 + i\,0.485175\,, \quad \beta = 3\alpha = \frac{1}{8}\log(3/2)\,. \tag{2.42}$$

$$\mathcal{P} = -\frac{3\times 3^{2/3}}{4\times 2^{2/3}}\,. \tag{2.43}$$

Symmetry: $\mathcal{N} = 0$.
Comment: the values of z_1 and z_2 can be obtained as roots of the polynomials $3 - 38X^2 + 3X^4$ and $4 + 14Y^2 + 45Y^4 + 14Y^6 + 4Y^8$, respectively.

T1001482. This point has $\mathcal{N} = 0$ and is a critical point of the 6-scalar model obtained in the same way as `T0878939` with the following approximate values for the roots of the polynomials in (2.29) and (2.30)

$$X = 0.097733\,, \quad Y = 0.337328\,, \quad \mathcal{P} = -1.001482\,. \tag{2.44}$$

Note that $\mathcal{P}$ is a root of the polynomial in (2.37).

T1125000. This point has $\mathcal{N} = 0$ and is located at

$$\begin{aligned} &z_1 = -z_3 = -\sqrt{1+2Y-Y^2} + iY\,, && z_2 = -\bar{z}_4 = \frac{1-X-z_1(1+X)}{1+X-z_1(1-X)}\,, \\ &\beta = \frac{1}{2}\log X\,, && \alpha = 0\,, \end{aligned} \tag{2.45}$$

which gives the potential

$$\mathcal{P} = -\frac{9}{8}\,. \tag{2.46}$$

The value of X is a root of the polynomial $1 - 5X^2 + X^4$. The value of Y is unfixed. This is due to the fact that the five-dimensional dilaton is a flat direction in the potential. Therefore we can fix Y to any convenient value by an $\mathrm{SL}(2,\mathbb{R})$ symmetry transformation. The only constraint when fixing Y is that one has to ensure that all four scalars z_a lie inside the unit disk.

T1304606.

$$\begin{aligned} &\alpha = 0.0713344\,, && \beta = 0.214003\,, \\ &z_1 = 0.340985 - i\,0.385628\,, && z_2 = 0.109181 + i\,0.698203\,, \\ &z_3 = 0.0805304 - i\,0.315369\,, && z_4 = -0.481872 - i\,0.341603\,. \end{aligned} \tag{2.47}$$

$$\mathcal{P} \approx -1.304606\,. \tag{2.48}$$

Symmetry: $\mathcal{N} = 0$.

T1417411.

$$z_1 = -z_4 = i\,\frac{\sqrt{(9-4\sqrt{2})(1+i\,4\sqrt{3})}}{7}\,,\quad z_2 = z_3 = i(\sqrt{2}-1)\,,\quad \beta = 3\alpha = \frac{1}{4}\log 2\,. \tag{2.49}$$

$$\mathcal{P} = -\frac{9}{4\times 2^{2/3}}\,. \tag{2.50}$$

Symmetry: $\mathcal{N} = 0$.

T1501862.

$$\begin{aligned}
\alpha &= 0.0766018\,, & \beta &= 0.0519887\,,\\
z_1 &= -0.214941 + i\,0.285334\,, & z_2 &= -0.0554356 + i\,0.297182\,,\\
z_3 &= 0.483533 + i\,0.610042\,, & z_4 &= 0.293764 - i\,0.686\,.
\end{aligned} \tag{2.51}$$

$$\mathcal{P} \approx -1.501862\,. \tag{2.52}$$

Symmetry: $\mathcal{N} = 0$.

3 Critical points with TensorFlow

Following the basic strategy explained in [9], the numerical search for critical points was performed with TensorFlow. In this section, we want to elucidate relevant differences between the computational strategies used for $d = 4$ in this earlier publication and $d = 5$ supergravity in this article. These mainly come from two sources, differences in physics, and also advances in the software ecosystem.

3.1 TensorFlow and other options

The commonly used conventions for de Wit-Nicolai maximal gauged $d = 4$ supergravity use complex $E_{7(7)}$ generator matrices. When employing numerical minimization with backpropagation as an effective strategy to search for vacuum solutions of the equations of motion, the stationarity condition is a smooth $\mathbb{R}^{70} \to \mathbb{R}$ function. Using Machine Learning terminology, one would regard this as the 'Loss Function'. If we want to keep the code in close alignment with the formulae from the published literature, we hence need a framework for reverse-mode automatic differentiation (AD) that supports Einstein summation, taking (ideally also higher) derivatives of matrix exponentiation, complex matrix exponentiation, and, importantly, taking gradients of $\mathbb{R}^n \to \mathbb{R}$ computations even if intermediate steps involve complex quantities and holomorphic functions.

It is especially this last point that is slightly subtle and apparently not widely appreciated in the Machine Learning world, which makes TensorFlow at the time of this writing (to the best of the authors' knowledge) the only AD framework with which the $d = 4$ calculation could be done using the established conventions. We want to briefly explain why.

For loss functions that involve complex intermediate quantities, it is not sufficient for a computational framework to simply support complex derivatives: it must be able to in particular correctly handle the case that a real-valued result is the magnitude-square of

JHEP07(2020)240

a complex intermediate result, schematically: $y = f_j(z_k(x_m)) \cdot \overline{f_j(z_k(x_m))}$, with the f_j being holomorphic functions of the intermediate complex quantities z_k that in turn are functions of the real input parameters x_k. When backpropagating such an expression, the AD framework repeatedly answers the question by how much the final result would change, relative to ε, if one interrupted the calculation right after the currently-in-focus intermediate quantity q_n was obtained and changed it $q_n \to q_n + \varepsilon$. This answer, i.e. the sensitivity of the end result on q_n, is found by referring, in every step, to the already-known sensitivities for later intermediate quantities q_{n+k}. Starting with the sensitivity of the end result on the end result, which is 1, we proceed through the entire computation a second time, in reverse, to ultimately obtain the sensitivities of the end result on the input parameters, i.e. the gradient. For a product of the above schematic form, the sensitivity of the end result y on the intermediate quantity $z_k(x_m)$ is $\overline{f_j(z_k(x_m))} \cdot \partial_{z_k} f_j(z_k(x_m))$, and the sensitivity of y on the intermediate quantity $\overline{z_k(x_m)}$ is the complex-conjugate of this value. Clearly, a reverse mode automatic differentiation framework that only knows about holomorphic derivatives and not this subtlety involving complex conjugation will not be able to produce the expected gradients. TensorFlow uses a modified definition of a 'complex gradient' that is *not* the holomorphic derivative, but also involves complex conjugation in precisely the way that is needed to make this case work.[7]

While the 56-dimensional fundamental representation of $E_{7(-133)}$ is pseudoreal (i.e. does not permit all-real generator matrices), this is not the case for $E_{7(7)}$, closely paralleling the familiar situation for SU(2) and $SL(2,\mathbb{R})$. It is indeed possible to translate de Wit-Nicolai supergravity from the 'SU(8)-aligned' basis that makes fermion couplings look simple to a '$SL(8,\mathbb{R})$-aligned' basis with all-real $E_{7(7)}$ generator matrices, and this alternative description has been used e.g. in [45] to great effect. In maximal gauged five-dimensional supergravity, the commonly used conventions employ a real basis for the corresponding $E_{6(6)}$ generator matrices of size 27×27, and so there would be the option to also base the computation on some other reverse-mode AD numerical framework, such as perhaps the — in comparison to TensorFlow — much more lightweight 'JAX' library [46].

For this work, we nevertheless decided to stay with TensorFlow, partly out of the desire to develop further software tools for supergravity research that are generally applicable also in situations where complex derivatives occur.

3.2 The $d = 5$ calculation

As in maximal four-dimensional supergravity, critical points of the equations of motion are saddle points, except for the maximum at the origin with unbroken SO(6) symmetry. For this work, we did not use a stationarity condition that is expressed in terms of the gradient of the potential with respect to an infinitesimal frame change that multiplies the Vielbein matrix from one side, as in (2.8) and (2.9) of [9]. Rather, we took as stationarity condition the length-squared of the gradient of the potential, and let TensorFlow work out the gradient of this (scalar) stationarity condition. The theory of Automated Differentiation tells us that the computational effort for obtaining the gradient of a scalar function is no

[7]For technical details, cf. https://github.com/tensorflow/tensorflow/issues/3348.

JHEP07(2020)240

more than six times the effort to compute the function (ignoring the effect of caches), and so computing the gradient of the stationarity-condition here is no more than $6^2\times$ the effort of evaluating the potential, which is quite affordable with only 42 parameters.

For the de Wit-Nicolai theory, $\mathfrak{spin}(8)$ symmetry can be employed to rotate a solution in such a way that one of the two symmetric traceless matrices $M_{\alpha\beta}, M_{\dot\alpha\dot\beta}$ that describe the location of a critical point (cf. (D.3) in [9]) gets diagonalized. For five-dimensional maximal supergravity, we first performed a scan in the full 42-dimensional parameter space, starting from 100 000 seeded pseudorandom starting locations, and then checked that we could indeed re-identify all solutions found in this way by performing another (similarly large) scan using a reduced coordinate-parametrization that set the non-diagonal entries of the $\Lambda^I{}_J$ and also the two $\mathrm{SL}(2,\mathbb{R})/\mathrm{SO}(2)$ axion-dilaton parameters to zero. As the volume of the SO(6) orbit of a solution is a function of the distance from the origin, one would naturally expect these two different scanning methods to produce any given solution with very different probability, and so using only the latter, reduced, parametrization, might have increased the risk of overlooking solutions. Also, the conjecture that one can indeed always set the axion-dilaton parameters to zero seems to be currently unproven.

As for the $d = 4$ calculation, we employed residual unbroken SO(6) symmetry that is associated with degenerate entries on the diagonal of $\Lambda^I{}_J$ to further reduce the number of non-zero $\Sigma_{ijk;\alpha}$-coefficients, but there is no guarantee in our tables that the number of parameters found in each case is indeed minimal.

Given that TensorFlow currently is limited to performing calculations with at most IEEE 754 64-bit float precision, and also the inherent problems of solving nonlinear equation systems via minimization to good accuracy, we found it effective to further increase the accuracy of a solution-candidate as obtained from minimization via a modified multidimensional Newton method. Here, one has to be careful due to the presence of flat ("Goldstone mode") directions in the potential and hence also stationarity condition.

3.3 Modern TensorFlow

In this work, TensorFlow mostly serves as a "fast numerical gradients" library for high-dimensional numerical minimization. While it is useful to adopt Machine Learning terminology for easier communication with other (mostly Machine Learning) users of TensorFlow, this is not strictly necessary. Due to the public release of TensorFlow2 in September 2019,[8] which moves away from the explicit meta-programming paradigm, much of the scaffolding that was used on the example Colab notebook[9] published alongside [9] can be eliminated. In particular, the need for continuation-passing techniques (such as provided by: `call_with_critical_point_scanner()`) in order to evaluate a function "in session context" is now gone.

There broadly are two major approaches to reverse mode Automatic Differentiation (AD), program-transformation based AD and tape-based AD. TensorFlow1 was based on program transformation, where the 'program' is a description of a calculation in terms of a

[8] Cf. https://blog.tensorflow.org/2019/09/tensorflow-20-is-now-available.html.
[9] https://aihub.cloud.google.com/u/0/p/products%2F74df893f-1ede-49c5-9f83-cfb290c05386/v/2.

JHEP07(2020)240

(tensor-)arithmetic graph that can be evaluated on general purpose CPUs or alternatively also hardware that is more specialized towards parallel numerics, i.e. GPUs or Google's Tensor Processing Units[10] (TPUs). The Python programming language is here used as a 'Meta-Language' to manipulate 'graph' objects that represent computations.

TensorFlow2 tries to hide much of this meta-programming complexity by making the graph invisible to the user and mostly following the 'tape-based' paradigm. Here, the idea is that the sequence of computational steps in a calculation for which we want to have a fast and accurate gradient are recorded on a 'tape'. Once the calculation is done, the tape is 'played in reverse', in each step updating sensitivities of the final result on intermediate quantities, in their natural latest-to-earliest order. Pragmatically, this means that a TensorFlow2 '`Tensor`' object can be seen as an envelope around a NumPy array that can be tracked on a tape, but otherwise is passed around and manipulated mostly like an array of numbers. This in particular means that with TensorFlow2, interfacing with optimizers such as `scipy.optimize.fmin_bfgs()` no longer requires a TensorFlow-provided wrapper such as `ScipyOptimizerInterface()`, or initiating numerical evaluation through an explicitly managed 'session', but instead can be done by simply wrapping up numpy-arrays in TensorFlow tensors for gradient computations, roughly along these lines:

```
def tf_minimize(tf_func, x0):
  """Minimizes a TensorFlow tf.Tensor -> tf.Tensor function."""
  def f_opt(xs):
    return tf_func(tf.constant(xs, dtype=tf.float64)).numpy()
  def fprime_opt(xs):
    t_xs = tf.constant(xs, dtype=tf.float64)
    tape = tf.GradientTape()
    with tape:
      tape.watch(t_xs)
      t_val = tf_func(t_xs)
    return tape.gradient(t_val, t_xs).numpy()
  opt = scipy.optimize.fmin_bfgs(
    f_opt, numpy.array(x0), fprime=fprime_opt, disp=0)
  return f_opt(opt), opt
```

4 Critical points from a solvable parametrization

A solvable parametrization of the scalar cosets in supergravity theories [35, 36] arises from the Iwasawa decomposition [47] of noncompact semisimple Lie groups, $G = KDN$, where K is the maximal compact subroup, D is a maximal "noncompact torus" and N is a noncompact, nilpotent subgroup. The scalar vielbein is then globally given by the group elements

$$\mathcal{V} = \exp\left(\sum_{i=1}^{\ell} \varphi_i\, \mathfrak{h}_i\right) \exp\left(\sum_{\alpha\in\Delta_+} x_\alpha\, \mathfrak{e}_\alpha\right), \tag{4.1}$$

[10]Cf. https://tinyurl.com/y6gmwfes.

where $\mathfrak{h}_i$ are generators of a noncompact Cartan subalgebra and $\mathfrak{e}_\alpha$ are the corresponding positive root generators. A clear advantage of this parametrization is that the first exponential of commuting generators is easy to compute, while the second one collapses to a polynomial.

In this section we summarize the results obtained by applying the solvable parametrization to the full scalar coset $\mathrm{E}_{6(6)}/\mathrm{USp}(8)$ of the $\mathcal{N}=8$ $d=5$ supergravity.[11] It turns out that the current computational capabilities of Mathematica run on a laptop suffice to obtain a closed form analytic expression for the full potential as a function of all 42 scalar fields and then search numerically for its critical points.

4.1 Solvable parametrization

The simplest choice for the noncompact Cartan subalgebra is to take the diagonal generators in $\mathfrak{sl}(6,\mathbb{R})\times\mathfrak{sl}(2,\mathbb{R})\subset\mathfrak{e}_{6(6)}$,

$$\begin{aligned}
\mathfrak{h}_1 &= \frac{1}{\sqrt{2}}(\widehat{\Lambda}^1{}_1-\widehat{\Lambda}^2{}_2)\,,\\
\mathfrak{h}_2 &= \frac{1}{\sqrt{6}}(\widehat{\Lambda}^1{}_1+\widehat{\Lambda}^2{}_2-2\widehat{\Lambda}^3{}_3)\,,\\
\mathfrak{h}_3 &= \frac{1}{2\sqrt{3}}(\widehat{\Lambda}^1{}_1+\widehat{\Lambda}^2{}_2+\widehat{\Lambda}^3{}_3-3\widehat{\Lambda}^4{}_4)\,,\\
\mathfrak{h}_4 &= \frac{1}{2\sqrt{5}}(\widehat{\Lambda}^1{}_1+\widehat{\Lambda}^2{}_2+\widehat{\Lambda}^3{}_3+\widehat{\Lambda}^4{}_4-4\widehat{\Lambda}^5{}_5)\,,\\
\mathfrak{h}_5 &= \frac{1}{\sqrt{30}}(\widehat{\Lambda}^1{}_1+\widehat{\Lambda}^2{}_2+\widehat{\Lambda}^3{}_3+\widehat{\Lambda}^4{}_4+\widehat{\Lambda}^5{}_5-5\widehat{\Lambda}^6{}_6)\,,\\
\mathfrak{h}_6 &= \frac{1}{\sqrt{2}}(\widehat{\Lambda}^7{}_7-\widehat{\Lambda}^8{}_8)\,,
\end{aligned}\tag{4.2}$$

which are normalized such that $\mathrm{Tr}\,\mathfrak{h}_i\mathfrak{h}_j=6\,\delta_{ij}$. A natural set of the corresponding positive and negative root generators, $\mathfrak{e}_\alpha$ and $\mathfrak{f}_\alpha$, respectively, is given in table 2. We parametrize the positive roots, $\alpha\in\Delta_+$, in terms of their coordinates in the simple root basis,

$$[n_1n_2n_3n_4n_5n_6]\qquad\longleftrightarrow\qquad\alpha=\sum_{i=1}^{6}n_i\alpha_i\,,\tag{4.3}$$

where the simple roots, α_i, are given explicitly by

$$\begin{aligned}
\alpha_1 &= \left(\sqrt{2},0,0,0,0,0\right)\,, & \alpha_2 &= \left(-\frac{1}{\sqrt{2}},\sqrt{\frac{3}{2}},0,0,0,0\right)\,,\\
\alpha_3 &= \left(0,-\sqrt{\frac{2}{3}},\frac{2}{\sqrt{3}},0,0,0\right)\,, & \alpha_4 &= \left(0,0,-\frac{\sqrt{3}}{2},\frac{\sqrt{5}}{2},0,0\right)\,,\\
\alpha_5 &= \left(0,0,0,-\frac{2}{\sqrt{5}},\sqrt{\frac{6}{5}},0\right)\,, & \alpha_6 &= \left(0,0,-\frac{\sqrt{3}}{2},-\frac{3}{2\sqrt{5}},-\sqrt{\frac{3}{10}},\frac{1}{\sqrt{2}}\right)\,.
\end{aligned}\tag{4.4}$$

[11] In the context of this theory, the solvable parametrization was first employed in [48] to compute the full scalar potential in an SO(3)-invariant truncation with a coset $\mathrm{G}_{2(2)}/\mathrm{SO}(4)$.

JHEP07(2020)240

The generators $\Xi^{\pm}_{IJK\alpha}$ are defined by

$$\Xi^{+}_{IJK\alpha} \equiv \frac{1}{\sqrt{2}}\,\widehat{\Sigma}^{IJK\alpha}\,, \qquad \Xi^{-}_{IJK\alpha} \equiv \frac{1}{\sqrt{2}}\,\widehat{\Sigma}_{IJK\alpha}\,. \tag{4.5}$$

The normalization is chosen such that

$$\operatorname{Tr}\mathfrak{e}_\alpha\mathfrak{e}_\beta = \operatorname{Tr}\mathfrak{f}_\alpha\mathfrak{f}_\beta = 0\,, \qquad \operatorname{Tr}\mathfrak{e}_\alpha\mathfrak{f}_\beta = 6\,\delta_{\alpha\beta}\,. \tag{4.6}$$

One should note that the root generators, $\mathfrak{e}_\alpha$ and $\mathfrak{f}_\alpha$ are combinations of both compact and noncompact generators of $\mathfrak{e}_{6(6)}$. Specifically, the compact generators are spanned by $(\mathfrak{e}_\alpha - \mathfrak{f}_\alpha)$, while the noncompact ones by $(\mathfrak{e}_\alpha + \mathfrak{f}_\alpha)$ and the Cartan generators, $\mathfrak{h}_i$. As a result, the relation between the scalar fields in the symmetric gauge used in the previous sections and the solvable parametrization here is highly nonlinear.[12] In particular, the action of the SO(6) gauge group, which is very simple in the symmetric gauge, becomes completely obscured in the solvable parametrization.

4.2 The scalar potential and critical points

A direct evaluation of the exponentials for the scalar 27-bein in (4.1) shows that the second factor is a polynomial of degree 18. After symbolic substitution for the nonvanishing matrix elements of the scalar vielbein, it turns out possible to generate a close form expression for the full scalar potential by following the usual steps in [2]. The resulting analytic expression in terms of the 42 scalar fields, φ_i, $i = 1,\ldots,6$, and x_α, $\alpha \in \Delta_+$, is given as supplementary material with this submission (file "Solvable_Potential.txt").[13] One can check numerically that, as expected, the potential does not depend on the $\mathrm{SL}(2,\mathbb{R})$ scalar corresponding to the maximal root [123212].

After evaluating symbolically the gradient of the potential with respect to all scalar fields, we have performed an exhaustive numerical search for the critical points using the `FindRoot[ · ]` routine in Mathematica starting at random points on the scalar coset. Since the solvable parametrization of the coset does not lead to any coordinate singularities, unlike the polar parametrization used in a similar numerical search in appendix B, all zeros of the gradient correspond to actual critical points of the potential. The resulting list of critical points found in this search is the same as the one found using TensorFlow in section 3 that are given in appendix C. This provides a completely independent consistency check between the two searches within the numerical accuracy of the Mathematica routines.

4.3 $\mathbb{Z}_2$ truncations

Given an analytic expression for the full potential, it is now straightforward to explore various truncations to smaller sectors. In particular, truncations with respect to the $\mathbb{Z}_2$ discrete symmetries considered in section 2 and appendix B amount to setting various subsets of the scalar fields to zero. This results in simpler potentials, whose critical points can be determined using the same routine as for the full potential above.

[12] At a given point on the scalar coset, the relation between the two sets of fields is easily determined, at least numerically, from the USp(8)-invariant product of the scalar vielbein and its transpose.

[13] The scalar fields in the file are φ[i] and x[$\mathtt{n}_1,\ldots,\mathtt{n}_6$], where $[n_1 \ldots n_6]$ denotes the root as in (4.3).

JHEP07(2020)240

$\alpha \in \Delta_+$	$\mathfrak{e}_\alpha$	$\mathfrak{f}_\alpha$	S_1	S_2	S_3
[100000]	$\widehat{\Lambda}^1{}_2$	$\widehat{\Lambda}^2{}_1$	$*$	$*$	
[010000]	$\widehat{\Lambda}^2{}_3$	$\widehat{\Lambda}^3{}_2$			
[001000]	$\widehat{\Lambda}^3{}_4$	$\widehat{\Lambda}^4{}_3$	$*$	$*$	
[000100]	$\widehat{\Lambda}^4{}_5$	$\widehat{\Lambda}^5{}_4$	$*$		
[000010]	$\widehat{\Lambda}^5{}_6$	$\widehat{\Lambda}^6{}_5$	$*$	$*$	
[000001]	Ξ^+_{4567}	Ξ^-_{4567}		$*$	
[110000]	$\widehat{\Lambda}^1{}_3$	$\widehat{\Lambda}^3{}_1$			$*$
[011000]	$\widehat{\Lambda}^2{}_4$	$\widehat{\Lambda}^4{}_2$			$*$
[001100]	$\widehat{\Lambda}^3{}_5$	$\widehat{\Lambda}^5{}_3$	$*$		$*$
[000110]	$\widehat{\Lambda}^4{}_6$	$\widehat{\Lambda}^6{}_4$	$*$		$*$
[001001]	Ξ^+_{3567}	Ξ^-_{3567}		$*$	$*$
[111000]	$\widehat{\Lambda}^1{}_4$	$\widehat{\Lambda}^4{}_1$			
[011100]	$\widehat{\Lambda}^2{}_5$	$\widehat{\Lambda}^5{}_2$		$*$	
[001110]	$\widehat{\Lambda}^3{}_6$	$\widehat{\Lambda}^6{}_3$	$*$		
[011001]	Ξ^+_{2567}	Ξ^-_{2567}	$*$		
[001101]	Ξ^+_{3467}	Ξ^-_{3467}			
[111100]	$\widehat{\Lambda}^1{}_5$	$\widehat{\Lambda}^5{}_1$		$*$	$*$
[011110]	$\widehat{\Lambda}^2{}_6$	$\widehat{\Lambda}^6{}_2$		$*$	$*$
[111001]	Ξ^+_{1567}	Ξ^-_{1567}	$*$		$*$
[011101]	Ξ^+_{2467}	Ξ^-_{2467}	$*$	$*$	$*$
[001111]	Ξ^+_{3457}	Ξ^-_{3457}			$*$
[111110]	$\widehat{\Lambda}^1{}_6$	$\widehat{\Lambda}^6{}_1$		$*$	
[111101]	Ξ^+_{1467}	Ξ^-_{1467}	$*$	$*$	
[012101]	Ξ^+_{2367}	Ξ^-_{2367}	$*$	$*$	
[011111]	Ξ^+_{2457}	Ξ^-_{2457}	$*$	$*$	
[112101]	Ξ^+_{1367}	Ξ^-_{1367}	$*$	$*$	$*$
[111111]	Ξ^+_{1457}	Ξ^-_{1457}	$*$	$*$	$*$
[012111]	Ξ^+_{2357}	Ξ^-_{2357}	$*$	$*$	$*$
[122101]	Ξ^+_{1267}	Ξ^-_{1267}			
[112111]	Ξ^+_{1357}	Ξ^-_{1357}	$*$	$*$	
[012211]	Ξ^+_{2347}	Ξ^-_{2347}	$*$		
[122111]	Ξ^+_{1257}	Ξ^-_{1257}			$*$
[112211]	Ξ^+_{1347}	Ξ^-_{1347}	$*$		$*$
[122211]	Ξ^+_{1247}	Ξ^-_{1247}		$*$	
[123211]	Ξ^+_{1237}	Ξ^-_{1237}		$*$	$*$
[123212]	$\widehat{\Lambda}^7{}_8$	$\widehat{\Lambda}^8{}_7$	$*$	$*$	

Table 2. Root generators of $E_{6(6)}$.

JHEP07(2020)240

Point	$S_{1,2}$	S_3	S_1S_2	$S_{1,2}S_3$	$S_1S_2S_3$
T0750000	*	*	*	*	*
T0780031	*	*	*	*	*
T0839947	*	*	*	*	*
T0843750	*	*	*	*	*
T0870297	*	*	*	*	*
T0878939	*	*	*	*	*
T0887636	*	*	*	*	*
T0892913	*	*	*	*	*
T0963952		*			
T0964097	*	*		*	
T0964525	*	*	*	*	*
T0982778	*	*	*	*	*
T1001482	*	*	*	*	*
T1054687	*	*	*	*	
T1073529	*	*		*	
T1125000	*	*	*	*	*
T1297247		*			
T1302912	*	*		*	
T1304606	*	*	*	*	*
T1319179	*	*		*	
T1382251		*			
T1391035	*	*			
T1416746	*	*			
T1417411	*	*	*	*	*
T1460654		*			
T1460729	*	*		*	
T1497042	*	*		*	
T1499666	*	*			
T1501862	*	*	*	*	*
T1510900	*	*		*	
T1547778	*	*	*	*	
T1738407	*	*		*	
Total	28	32	17	25	15

Table 3. Critical points from discrete truncations.

The three $\mathbb{Z}_2$ symmetries we want to discuss here are generated by

$$S_1 \equiv P_1 Q\,, \qquad S_2 \equiv P_2 Q\,, \qquad S_3 \equiv P_3 Q'\,, \tag{4.7}$$

where P_1, P_2, P_3 and Q, Q' are given in (2.1) and (2.2), respectively. Clearly, the Cartan generators, $\mathfrak{h}_i$, commute with these symmetries. The root generators that are even (invariant) under a given symmetry are labelled by the star in table 2. The remaining generators are odd and the corresponding scalar fields are set to zero in the truncations.

The results of our searches for critical points in various $\mathbb{Z}_2$, $\mathbb{Z}_2^2$ and $\mathbb{Z}_2^3$-invariant sectors are summarized in table 3. Since S_1 and S_2 are conjugate under the adjoint action of SO(6), the two $\mathbb{Z}_2$-invariant truncations yield the same set of points, albeit with different sets of scalar fields. The truncation to the sector invariant under S_1 and S_2 reproduces the points found in appendix B, where we use a completly different parametrization of the coset. The combined truncation with respect to S_1, S_2 and S_3 yields the 10-scalar model and we reproduce the results in section 2.

What is surprising and new here is that *all* critical points are found within a truncation with respect to the special $\mathbb{Z}_2$ symmetry generated by $S_3 \in \mathrm{S}(\mathrm{O}(6)\times \mathrm{GL}(2,\mathbb{R}))$. The 22 scalars in this truncation parametrize the coset

$$\mathcal{M}_{(22)} \equiv \frac{\mathrm{SL}(6,\mathbb{R})}{\mathrm{SO}(6)} \times \frac{\mathrm{SL}(2,\mathbb{R})}{\mathrm{SO}(2)}\,, \tag{4.8}$$

with, however, different $\mathrm{SL}(6,\mathbb{R})$ and $\mathrm{SL}(2,\mathbb{R})$ than those generated by $\widehat{\Lambda}^I{}_J$ and $\widehat{\Lambda}^\alpha{}_\beta$ in (A.4). From the last column in table 2 we see that the relevant $\mathfrak{sl}(6,\mathbb{R})\oplus\mathfrak{sl}(2,\mathbb{R})$ subalgebra of $\mathfrak{e}_{6(6)}$ is spanned by the "even" root generators with $n_1+\ldots+n_6 \in 2\,\mathbb{Z}$. It would be interesting to understand the a priori reason for the "critical efficiency" of this truncation.

5 Conclusions

In this paper we presented a numerical exploration of the AdS_5 vacua corresponding to critical points of the scalar potential of the SO(6) maximal gauged supergravity. Out of the 31 critical points, we find that there are only 2 that are supersymmetric and perturbatively stable. Usually one would dismiss the 29 unstable AdS_5 solutions as physically irrelevant. Nevertheless, the existence of these critical points may point towards some interesting dynamics in the supersymmetry broken phases of the planar $\mathcal{N}=4$ SYM theory. As discussed in [49], AdS vacua with scalars below the BF bound signal the loss of conformality in the dual QFT. Perhaps some of the perturbatively unstable vacua admit an interpretation along these lines and can be viewed as holographic duals to complex CFTs [50, 51]. Alternatively these unstable vacua may serve as lampposts for other type of approximately conformal QFT dynamics similar to the ones studied in [52]. To understand this question better one can study holographic RG flows represented by domain wall solutions connecting our new vacua. This can be done most explicitly for the 10-scalar and 6-scalar consistent truncations. For example, if there are supersymmetric RG flows that closely approach some of the unstable AdS_5 vacua this may suggest an approximately conformal supersymmetric phase of $\mathcal{N} = 4$ SYM. It should also be noted that the 10- and 14-scalar consistent truncations have wider applications in the context of holography. As emphasized in [31, 53] they can be used to study the holographic dual description of the $\mathcal{N} = 1^*$ mass deformation of $\mathcal{N} = 4$ SYM on $\mathbb{R}^4$ and S^4 for general values of the complex mass parameters.

All of the AdS_5 vacua we constructed can be uplifted to solutions of type IIB supergravity using the explicit formulae in [8]. This will result in ten-dimensional AdS_5 solutions with non-trivial fluxes on S^5. Given that the new critical points are perturbatively unstable, they can be used as a test ground for exploring the general mechanisms responsible for instabilities in non-supersymmetric flux compactifications. In addition, using the ten-dimensional uplift may allow for the possibility of stabilizing some of the AdS_5 vacua by projecting out the unstable modes using an appropriate orbifold action in type IIB string theory [54].

Finally, we note that there are other gaugings that lead to maximal supergravity theories in five dimensions, see [2, 55]. It will be interesting to apply similar numerical and analytical tools to study the critical points of these theories.

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A Conventions

Throughout this paper we use the same conventions as in [2], which the reader should consult for details. Here we summarize an explicit parametrization of the scalar manifold

$$\mathcal{M}_{(42)} \equiv \frac{\mathrm{E}_{6(6)}}{\mathrm{USp}(8)}\,, \tag{A.1}$$

of the $\mathcal{N} = 8$ $d = 5$ supergravity as needed for the truncations in section 2 and appendix B, an explicit construction of the potential in section 3, and specifying the location of its critical points in appendix C.

The most straightforward description of the $\mathfrak{e}_{6(6)}$ generators in the so-called $\mathrm{SL}(6,\mathbb{R})\times\mathrm{SL}(2,\mathbb{R})$ basis is through their action on 27-dimensional vectors with components $(z_{IJ}, z^{I\alpha})$, $z_{IJ} = -z_{JI}$,[14]

$$\begin{aligned}\delta z_{IJ} &= -\Lambda^K{}_I z_{KJ} - \Lambda^K{}_J z_{IK} + \Sigma_{IJK\beta} z^{K\beta}\,,\\ \delta z^{I\alpha} &= \Lambda^I{}_K z^{K\alpha} + \Lambda^\alpha{}_\beta z^{I\beta} + \Sigma^{KLI\alpha} z_{KL}\,,\end{aligned} \tag{A.2}$$

[14]For the corresponding 27×27 matrix, see (A.36) in [2].

JHEP07(2020)240

where $(\Lambda^I{}_J)$ and $(\Lambda^\alpha{}_\beta)$ are real matrices in $\mathfrak{sl}(6,\mathbb{R})$ and $\mathfrak{sl}(2,\mathbb{R})$, respectively, and $\Sigma_{IJK\alpha} = \Sigma_{[IJK]\alpha}$ is real with

$$\Sigma^{IJK\alpha} = \frac{1}{6}\epsilon^{IJKLMN}\epsilon^{\alpha\beta}\Sigma_{LMN\beta}\,. \tag{A.3}$$

Note that the transformation (A.2) can be extended to arbitrary $(\Lambda^I{}_J) \in \mathfrak{gl}(6,\mathbb{R})$ and $(\Lambda^\alpha{}_\beta) \in \mathfrak{gl}(2,\mathbb{R})$. This can be used to introduce a convenient basis of generators $(\widehat{\Lambda}^I{}_J, \widehat{\Lambda}^\alpha{}_\beta, \widehat{\Sigma}_{IJK\alpha})$ in $\mathfrak{e}_{6(6)} \oplus \mathbb{R}^2$ defined by the following nonvanishing parameters in (A.2) for each generator:[15]

$$\begin{aligned}
\widehat{\Lambda}^I{}_J:&\qquad \Lambda^I{}_J = 1\,, && I,J = 1,\ldots,6\,,\\
\widehat{\Lambda}^\alpha{}_\beta:&\qquad \Lambda^\alpha{}_\beta = 1\,, && \alpha,\beta = 7,8\,,\\
\widehat{\Sigma}_{IJK\alpha}:&\quad \Sigma_{IJK\alpha} = \Sigma_{KIJ\alpha} = \ldots = -\Sigma_{KJI\alpha} = 1\,, && I<J<K\,.
\end{aligned} \tag{A.4}$$

The coset, $E_{6(6)}/\mathrm{USp}(8)$, has a trivial topology of $\mathbb{R}^{42}$ and, via the exponential map, is isomorphic to the corresponding quotient of the Lie algebras, $\mathfrak{e}_{6(6)}/\mathfrak{usp}(8)$. The usual choice of the coset representatives is then given by the noncompact generators for which

$$\Lambda^I{}_J = \Lambda^J{}_I\,, \qquad \Lambda^\alpha{}_\beta = \Lambda^\beta{}_\alpha, \qquad \Sigma_{IJK\alpha} = \Sigma^{IJK\alpha}\,. \tag{A.5}$$

An ordered set of the $20+2+20$ independent parameters in (A.5) provides then global coordinates on the scalar manifold, $\mathcal{M}_{(42)}$.

B 14-scalar model

In this appendix we present a truncation of the potential to a 14-scalar model that arises as an intermediate step in the construction of the 6-scalar model in [34] and/or the 10-scalar model [31] discussed in section 2. The main result is an explicit, albeit rather complicated, form of the scalar potential in this sector. It yields a subset of 17 extrema of the full potential.

B.1 $\mathbb{Z}_2 \times \mathbb{Z}_2$-invariant truncations

There are two equivalent methods to obtain the 14-scalar model we are interested in. The first one is to truncate with respect to a $\mathbb{Z}_2 \times \mathbb{Z}_2 \subset \mathrm{SO}(6)$ symmetry generated by [34]

$$g_1 = \mathrm{diag}(-1,-1,-1,-1,1,1) \qquad \text{and} \qquad g_2 = \mathrm{diag}(1,1,-1,-1,-1,-1)\,. \tag{B.1}$$

The second method is to use $\mathbb{Z}_2 \times \mathbb{Z}_2 \subset \mathrm{S}(\mathrm{O}(6)\times \mathrm{GL}(2,\mathbb{R}))$ generated by P_1Q and P_2Q [31], where

$$P_1 = \mathrm{diag}(-1,-1,1,1,1,1)\,,\quad P_2 = \mathrm{diag}(1,1,-1,-1,1,1)\,,\quad Q = \mathrm{diag}(-1,-1)\,. \tag{B.2}$$

are the same as in (2.1) and (2.2). The truncations with respect to the $\mathbb{Z}_2 \times \mathbb{Z}_2$ in (B.1) or (B.2), respectively, yield the same set of invariant generators of $\mathfrak{o}(1,1)^2 \times \mathfrak{so}(4,4) \subset \mathfrak{e}_{6(6)}$, with the resulting scalar coset

$$\mathcal{M}_{\mathrm{O}(1,1)^2} \times \mathcal{M}_{\mathrm{SO}(4,4)} \equiv \mathrm{O}(1,1)^2 \times \frac{\mathrm{SO}(4,4)}{\mathrm{SO}(4)\times\mathrm{SO}(4)}\,. \tag{B.3}$$

To compute the potential, we need a workable parametrization of the second factor.

[15]Note that unlike [2] we use the range $\alpha,\beta = 7,8$ for the $\mathrm{SL}(2,\mathbb{R})$ indices.

B.2 Polar parametrization of the coset

In the vector representation of SO(4, 4), the compact S(O(4) × O(4)) subgroup is given by block matrices

$$O = \begin{pmatrix} O_1 & 0 \\ 0 & O_2 \end{pmatrix}, \qquad O_1, O_2 \in \mathrm{O}(4)\,, \quad O_1 O_2 \in \mathrm{SO}(4)\,. \tag{B.4}$$

The non-compact generators are of the form

$$X = \begin{pmatrix} 0 & M \\ M^T & 0 \end{pmatrix}, \tag{B.5}$$

and the 4 × 4 matrices, M, provide global coordinates on the coset. Now, note that

$$OXO^T = \begin{pmatrix} 0 & O_1 M O_2^T \\ O_2 M^T O_1^T & 0 \end{pmatrix}, \tag{B.6}$$

and use the fact that any generic real matrix can be diagonalized by two orthogonal matrices, that is

$$M = O_1 \Lambda O_2^T\,. \tag{B.7}$$

The diagonal matrix, Λ, consists of 4 commuting, noncompact generators. Furthermore, any 4 such generators are conjugate under the action of the compact subgroup. The idea now is to parametrize the $\mathcal{M}_{\mathrm{SO}(4,4)}$ coset in terms of Euler angles for O_1 and O_2 and the four parameters in Λ.

To this end, we first decompose the compact generators of $\mathfrak{so}(4,4) \subset \mathfrak{e}_{6(6)}$ into generators of 4 mutually commuting $\mathfrak{su}(2)$'s, which are labelled by $\alpha, \beta, \gamma, \delta$.[16] Inside the $\mathfrak{e}_{6(6)}$, one can choose those generators as follows:

$$\begin{aligned}
\mathfrak{r}_1^{(\alpha)} &= \frac{1}{\sqrt{2}}\Big(-\widehat{\Sigma}_{1357} + \widehat{\Sigma}_{1368} + \widehat{\Sigma}_{1458} + \widehat{\Sigma}_{1467} + \widehat{\Sigma}_{2358} + \widehat{\Sigma}_{2367} + \widehat{\Sigma}_{2457} - \widehat{\Sigma}_{2468}\Big)\,,\\
\mathfrak{r}_2^{(\alpha)} &= \frac{1}{\sqrt{2}}\Big(-\widehat{\Sigma}_{1358} - \widehat{\Sigma}_{1367} - \widehat{\Sigma}_{1457} + \widehat{\Sigma}_{1468} - \widehat{\Sigma}_{2357} + \widehat{\Sigma}_{2368} + \widehat{\Sigma}_{2458} + \widehat{\Sigma}_{2467}\Big)\,,\\
\mathfrak{r}_3^{(\alpha)} &= \widehat{A}^1{}_2 + \widehat{A}^3{}_4 + \widehat{A}^5{}_6 + \widehat{A}^7{}_8\,,
\end{aligned} \tag{B.8}$$

$$\begin{aligned}
\mathfrak{r}_1^{(\beta)} &= \frac{1}{\sqrt{2}}\Big(-\widehat{\Sigma}_{1357} + \widehat{\Sigma}_{1368} - \widehat{\Sigma}_{1458} - \widehat{\Sigma}_{1467} - \widehat{\Sigma}_{2358} - \widehat{\Sigma}_{2367} + \widehat{\Sigma}_{2457} - \widehat{\Sigma}_{2468}\Big)\,,\\
\mathfrak{r}_2^{(\beta)} &= \frac{1}{\sqrt{2}}\Big(\widehat{\Sigma}_{1358} + \widehat{\Sigma}_{1367} - \widehat{\Sigma}_{1457} + \widehat{\Sigma}_{1468} - \widehat{\Sigma}_{2357} + \widehat{\Sigma}_{2368} - \widehat{\Sigma}_{2458} - \widehat{\Sigma}_{2467}\Big)\,,\\
\mathfrak{r}_3^{(\beta)} &= \widehat{A}^1{}_2 + \widehat{A}^3{}_4 - \widehat{A}^5{}_6 - \widehat{A}^7{}_8\,,
\end{aligned} \tag{B.9}$$

$$\begin{aligned}
\mathfrak{r}_1^{(\gamma)} &= \frac{1}{\sqrt{2}}\Big(-\widehat{\Sigma}_{1357} - \widehat{\Sigma}_{1368} + \widehat{\Sigma}_{1458} - \widehat{\Sigma}_{1467} - \widehat{\Sigma}_{2358} + \widehat{\Sigma}_{2367} - \widehat{\Sigma}_{2457} - \widehat{\Sigma}_{2468}\Big)\,,\\
\mathfrak{r}_2^{(\gamma)} &= \frac{1}{\sqrt{2}}\Big(\widehat{\Sigma}_{1358} - \widehat{\Sigma}_{1367} + \widehat{\Sigma}_{1457} + \widehat{\Sigma}_{1468} - \widehat{\Sigma}_{2357} - \widehat{\Sigma}_{2368} + \widehat{\Sigma}_{2458} - \widehat{\Sigma}_{2467}\Big)\,,\\
\mathfrak{r}_3^{(\gamma)} &= \widehat{A}^1{}_2 - \widehat{A}^3{}_4 + \widehat{A}^5{}_6 - \widehat{A}^7{}_8
\end{aligned} \tag{B.10}$$

[16] Note that α and β have different meaning in the main text than in this appendix.

JHEP07(2020)240

$$\mathfrak{r}_1^{(\delta)} = \frac{1}{\sqrt{2}}\left(-\widehat{\Sigma}_{1357} - \widehat{\Sigma}_{1368} - \widehat{\Sigma}_{1458} + \widehat{\Sigma}_{1467} + \widehat{\Sigma}_{2358} - \widehat{\Sigma}_{2367} - \widehat{\Sigma}_{2457} - \widehat{\Sigma}_{2468}\right),$$
$$\mathfrak{r}_2^{(\delta)} = \frac{1}{\sqrt{2}}\left(\widehat{\Sigma}_{1358} - \widehat{\Sigma}_{1367} - \widehat{\Sigma}_{1457} - \widehat{\Sigma}_{1468} + \widehat{\Sigma}_{2357} + \widehat{\Sigma}_{2368} + \widehat{\Sigma}_{2458} - \widehat{\Sigma}_{2467}\right), \tag{B.11}$$
$$\mathfrak{r}_3^{(\delta)} = -\widehat{A}^1{}_2 + \widehat{A}^3{}_4 + \widehat{A}^5{}_6 - \widehat{A}^7{}_8,$$

where $\widehat{A}^I{}_J = \widehat{\Lambda}^I{}_J - \widehat{\Lambda}^J{}_I$. They satisfy,

$$[\mathfrak{r}_i, \mathfrak{r}_j] + 4\epsilon_{ijk}\mathfrak{r}_k = 0, \qquad \operatorname{Tr}\mathfrak{r}_i\mathfrak{r}_j = -48\,\delta_{ij}, \tag{B.12}$$

within each $\mathfrak{su}(2)$ subalgebra.

The group elements of these four commuting SU(2)'s inside $E_{6(6)}$ are now parametrized by the Euler angles $\alpha_1, \ldots, \delta_3$ defined by

$$g(\varphi_1, \varphi_2, \varphi_3) = \exp(\varphi_1\mathfrak{r}_3^{(\varphi)}) \cdot \exp(\varphi_2\mathfrak{r}_1^{(\varphi)}) \cdot \exp(\varphi_3\mathfrak{r}_3^{(\varphi)}), \qquad \varphi = \alpha,\ \beta,\ \gamma,\ \delta. \tag{B.13}$$

By simultanously diagonalizing the four Casimir operators, one can bring the group element of the compact subgroup into a block diagonal form corresponding to the branching

$$\begin{aligned}\mathbf{27} \longrightarrow\ & 3\times(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})+ \\ & +(\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{1})+(\mathbf{2},\mathbf{1},\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1})+(\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{2})+(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{2})+(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{2}),\end{aligned} \tag{B.14}$$

where the 4×4 blocks of each SU(2) are of the form

$$\begin{pmatrix} s_1 & s_2 & \sqrt{2}\,s_3 & \sqrt{2}\,s_4 \\ -s_2 & s_1 & -\sqrt{2}\,s_4 & \sqrt{2}\,s_3 \\ -s_3/\sqrt{2} & s_4/\sqrt{2} & s_1 & -s_2 \\ -s_4/\sqrt{2} & \sqrt{3}/\sqrt{2} & s_2 & s_1 \end{pmatrix}, \tag{B.15}$$

modulo a permutation of signs between some terms that make the two SU(2)'s in each $(\mathbf{2},\mathbf{2})$ block commute. The s_i's above are

$$\begin{aligned} s_1 &= \cos 2\varphi_2 \cos 2(\varphi_1+\varphi_3), \quad & s_2 &= \cos 2\varphi_2 \sin 2(\varphi_1+\varphi_3), \\ s_3 &= \sin 2\varphi_2 \cos 2(\varphi_1-\varphi_3), \quad & s_4 &= \sin 2\varphi_2 \sin 2(\varphi_1-\varphi_3), \end{aligned} \tag{B.16}$$

for each of the angles $\varphi = \alpha, \beta, \gamma, \delta$. Note that

$$s_1^2 + s_2^2 + s_3^2 + s_4^2 = 1, \tag{B.17}$$

so that each block is simply a unit quaternion. Then the 24×24 block of the $E_{6(6)}$ matrix corresponding to the second line in (B.14) is a diagonal matrix parametrized by 4 commuting quaternions, $q_\alpha, \ldots, q_\delta$.

Next we choose 4 commuting noncompact generators, cf. (2.5),

$$\begin{aligned} \mathfrak{g}_1 &= \frac{1}{\sqrt{2}}\left(\widehat{\Sigma}_{1357} - \widehat{\Sigma}_{2468}\right), \quad & \mathfrak{g}_2 &= \frac{1}{\sqrt{2}}\left(\widehat{\Sigma}_{1467} - \widehat{\Sigma}_{2358}\right), \\ \mathfrak{g}_3 &= \frac{1}{\sqrt{2}}\left(\widehat{\Sigma}_{2367} - \widehat{\Sigma}_{1458}\right), \quad & \mathfrak{g}_4 &= \frac{1}{\sqrt{2}}\left(\widehat{\Sigma}_{2457} - \widehat{\Sigma}_{1368}\right). \end{aligned} \tag{B.18}$$

Then the scalar 27-bein

$$\mathcal{V}_{\mathrm{SO}(4,4)}(\alpha,\beta,\gamma,\delta;\rho) = g(\alpha)\dots g(\delta)\exp\left(\sum_i \rho_i \mathfrak{g}_i\right) g(\delta)^{-1}\dots g(\alpha)^{-1} \in \mathrm{E}_{6(6)}\,, \tag{B.19}$$

parametrizes the coset $\mathcal{M}_{\mathrm{SO}(4,4)}$. The matrix $\mathcal{V}_{\mathrm{SO}(4,4)}$ is somewhat sparse with 195 out of $27^2 = 729$ nonzero entries. In the following, it will be useful to work with the corresponding matrix obtained by replacing the nonvanishing entries in $\mathcal{V}_{\mathrm{SO}(4,4)}$ with symbolic entries, say m_{ij}.

Adding the $\mathcal{M}_{\mathrm{O}(1,1)}$ factor does not change much. We choose generators, cf. (2.4),

$$\begin{aligned} \tilde{\mathfrak{g}}_1 &= \frac{3}{2}\left(\widehat{\Lambda}^1{}_1 + \widehat{\Lambda}^2{}_2 - \widehat{\Lambda}^3{}_3 - \widehat{\Lambda}^4{}_4\right)\,, \\ \tilde{\mathfrak{g}}_2 &= \frac{5}{3}\left(\widehat{\Lambda}^1{}_1 + \widehat{\Lambda}^2{}_2 + \widehat{\Lambda}^3{}_3 + \widehat{\Lambda}^4{}_4 - 2\widehat{\Lambda}^5{}_5 - 2\widehat{\Lambda}^6{}_6\right)\,, \end{aligned} \tag{B.20}$$

with the corresponding group element

$$\mathcal{V}_{\mathrm{O}(1,1)^2}(\xi_1,\xi_2) = \exp(\xi_1\,\tilde{\mathfrak{g}}_1 + \xi_2\,\tilde{\mathfrak{g}}_2)\,. \tag{B.21}$$

This matrix is diagonal and simply "decorates" the m_{ij}'s in (B.19) by exponential factors. Finally, the full scalar 27-bein is

$$\mathcal{V}(\xi;\alpha,\beta,\gamma,\delta;\rho) = \mathcal{V}_{\mathrm{O}(1,1)^2}(\xi)\cdot\mathcal{V}_{\mathrm{SO}(4,4)}(\alpha,\beta,\gamma,\delta;\rho)\,. \tag{B.22}$$

B.3 Computation of the potential

Using symbolic representation of $\mathcal{V}$, the potential is a sum of 2784 terms quartic in m_{ij}'s, which fall into 6 different groups depending on the $\mathrm{O}(1,1)^2$ prefactors,

$$e^{-3\xi_1-\frac{10}{3}\xi_2}\,,\qquad e^{3\xi_1-\frac{10}{3}\xi_2}\,,\qquad e^{-\frac{40}{3}\xi_2}\,,\qquad e^{\frac{20}{3}\xi_2}\,,\qquad e^{-6\xi_1+\frac{20}{3}\xi_2}\,,\qquad e^{6\xi_1+\frac{20}{3}\xi_2}\,. \tag{B.23}$$

After substituting for m_{ij}'s, we find the prefactors in (B.23) are multiplied by 48, 48, 18, 48, 18, 18 different quartic products of $\cosh\rho_{ij}$ and $\sinh\rho_{ij}$, $\rho_{ij} = \rho_i - \rho_j$, respectively, for the total of 198 terms. In turn, each of those terms is multiplied by a homogenous polynomial of order 16 in 16 different s_i's (B.15) for the 12 Euler angles. A typical number of terms in those trigonometric polynomials is on the order of 40,000. That number is drastically reduced upon repeated use of (B.17), usually to less than a 100. Finally, the substitution of explicit s_i's in terms of the angles further collapses each group to a relatively small number of terms.

In the last stage, all dependence of the potential on the four angles α_1, β_1, γ_1 and δ_1 disappears and one is left with the potential that depends on 8 Euler angles and 6 noncompact fields. This is a nice consistency check for this long calculation. The $\mathbb{Z}_2\times\mathbb{Z}_2$ truncation preserves $\mathrm{U}(1)^4 \subset \mathrm{SO}(6)\times\mathrm{SL}(2,\mathbb{R})$, generated by $\mathfrak{r}_3^{(\varphi)}$'s, which is a symmetry of the potential. Hence the latter should be a function of $18-4=14$ independent scalar fields, as indeed it is.

Even a simplified expression for the potential is too long to be written down in a reasonable amount of space here. Instead, it is made available as a Mathematica input file, see section B.5.

JHEP07(2020)240

T0750000	T0780031	T0839947	T0843750	T0870297	T0878939
T0887636	T0892913	T0964525	T0982778	T1001482	T1054687
T1125000	T1304606	T1417411	T1501862	T1547778	

Table 4. The critical points in the $\mathbb{Z}_2 \times \mathbb{Z}_2$-invariant sector.

Point	ξ_1	ξ_2	ρ_1	ρ_2	ρ_3	ρ_4	$\alpha_{2/3}$	$\beta_{2/3}$	$\gamma_{2/3}$	$\delta_{2/3}$
T1054687	0	0	0.59672	−0.00571	0.60944	0.59486	0.05047	0.38645	0.52874	0.28433
							0.46797	1.17929	1.64474	0.85757
T1547778	0.14931	0.04479	0.34047	1.36783	0.56949	0.58893	0.58938	1.5804	1.05899	1.38975
							0.23497	0.41012	1.56041	1.36012

Table 5. Positions of T1054687 and T1547778 in the polar coordinates.

B.4 The critical points

We have found 17 critical points of the scalar potential in this $\mathbb{Z}_2 \times \mathbb{Z}_2$-invariant sector using the `FindRoot[ · ]` routine in Mathematica starting at random locations on the scalar manifold. Those points are listed in table 4. As expected, they include all critical points found in the 10-scalar model in section 2, with only two additional ones, T1054687 and T1547778, whose positions in the polar parametrization used here are given in table 5.[17]

A major inconvenience when working with the polar-type coordinates, as compared to the ones used in sections 2 and 3, is the presence of coordinate singularities in the parametrization of the scalar coset. As a result the search routine yields a large fraction of "fake critical points". Those are then eliminated by an explicit check of criticality, that is by evaluating the potential to the first order in ϵ on the scalar vielbein

$$\left(1 + \epsilon \sum_A \psi_A T_A\right) \mathcal{V}_* \,, \tag{B.24}$$

where $\mathcal{V}_*$ is the presumed critical point. The sum in (B.24) runs over all 78 generators of $\mathfrak{e}_{6(6)}$ and to eliminate a point it is sufficient to verify that (B.24) does not vanish for some random values of the parameters ψ_A.

The numerical search for critical points in this sector appears to be quite efficient, so we believe that there should be no missing critical points from our search. It is then quite remarkable that the 17 points found here constitute more than 50% of all critical points found by the TensorFlow search in section 3.

B.5 Supplementary material

A text file with a Mathematica input for the full potential in the $\mathbb{Z}_2 \times \mathbb{Z}_2$-invariant sector in this section is available for download as supplementary material with this submission (file

[17]Note that

$$-\frac{3^{5/3}\cdot 5}{2^{13/3}} = -1.547778397919356258066223415191758573521977177024293751706188 7\ldots\,,$$

which agrees with the value of the potential for T1547778 to the numerical accuracy we tested it.

"CriticalPoints17.txt"). The potential depends on 14 scalar fields, which are denoted by the same symbols as in the text above. The locations of the critical points can be found in a Mathematica input file which is available for download as supplementary material with this submission (file "PotentialSO44.txt").

C Critical points and mass spectra

In this appendix, we list numerical data obtained from TensorFlow on the locations, gravitino and scalar mass spectra, cosmological constant, as well as residual gauge symmetry and supersymmetry. Gravitino masses are normalized relative to the AdS radius such that for every unbroken supersymmetry, there is a $m^2/m_0^2[\psi] = 1$ gravitino, and the BF bound is $m^2/m_0^2[\phi] \geq -4$. The (symmetric-traceless) $\Lambda^I{}_J$ parameters have been diagonalized. The diagonal $\Lambda^I{}_I$ entries listed sum to zero as expected, hiding a linear constraint on the numerical data.

This list of solutions, produced by starting numerical optimization from 10^5 random points, is likely to be mostly complete. Notably, an independent second deep scan that used a modified 'loss function' to guide the search towards supersymmetric solutions did not find any supersymmetric critical points beyond the two already known ones.

In the list of solutions we give the location on the scalar manifold in terms of the generators in appendix A. The $\mathfrak{e}_{6(6)}$ element is constructed as a linear combination of the generators $\widehat{\Lambda}^\alpha{}_\beta$, $\widehat{\Lambda}^I{}_J$, and $\widehat{\Sigma}_{IJK\alpha}$. The coefficient of $\widehat{\Lambda}^\alpha{}_\beta$ is set to zero for all solutions as explained in section 3. The coefficients of $\widehat{\Lambda}^I{}_J$ are denoted by $\Lambda^I{}_J$, and the coefficients multiplying $\widehat{\Sigma}_{IJK7}$ are $\pm\sqrt{2}\Sigma_{\pm(IJK;1+\cdots;2)}$. Only nonzero coefficients of the $\widehat{\Sigma}_{IJK\alpha}$-generators are displayed. This accounts for all non-compact generators. The group element is obtained by exponentiating the linear combination just described.

$$\textbf{T0750000}: V/g^2 = -0.75000000,\ \mathcal{N} = 8,\ \mathfrak{so}(6) \to \mathfrak{su}(4) \quad \text{(C.1)}$$

$m^2/m_0^2[\psi] : 1.000_{\times 8}$

$m^2/m_0^2[\phi] : -4.000_{\times 20}, -3.000_{\times 20}, 0.000_{\times 2}$

$\Lambda^1{}_1 = \Lambda^2{}_2 = \Lambda^3{}_3 = \Lambda^4{}_4 = \Lambda^5{}_5 = \Lambda^6{}_6 = 0$

$$\textbf{T0780031}: V/g^2 = -0.78003143,\ \mathfrak{so}(6) \to \mathfrak{so}(5) \quad \text{(C.2)}$$

$m^2/m_0^2[\psi] : 1.185_{\times 8}$

$m^2/m_0^2[\phi] : -5.333_{\times 14}, -2.000_{\times 20}, 0.000_{\times 7}, 8.000$

$\Lambda^1{}_1 = \Lambda^2{}_2 = \Lambda^3{}_3 = \Lambda^4{}_4 = \Lambda^5{}_5 \approx -0.09155, \Lambda^6{}_6 \approx 0.45776$

$$\textbf{T0839947}: V/g^2 = -0.83994737,\ \mathcal{N} = 2,\ \mathfrak{so}(6) \to \mathfrak{su}(2) + \mathfrak{u}(1) \quad \text{(C.3)}$$

$m^2/m_0^2[\psi]: 1.000_{\times 2}, 1.361_{\times 4}, 1.778_{\times 2}$

$m^2/m_0^2[\phi]: -4.000_{\times 3}, -3.750_{\times 12}, -3.437_{\times 4}, -3.000_{\times 2}, -2.438_{\times 4}, -1.292, 0.000_{\times 13},$
$3.000_{\times 2}, 9.292$

$\Lambda^1{}_1 = \Lambda^2{}_2 = \Lambda^3{}_3 = \Lambda^4{}_4 \approx -0.11552, \Lambda^5{}_5 = \Lambda^6{}_6 \approx 0.23105$

$\Sigma_{+125;1+346;2} \approx -0.27465, \Sigma_{+126;2+345;1} = \Sigma_{+135;2+246;1} = \Sigma_{+136;1+245;2} \approx 0.27465$

T0843750 : $V/g^2 = -0.84375000,\ \mathfrak{so}(6) \to \mathfrak{su}(3)$ (C.4)

$m^2/m_0^2[\psi]: 1.210_{\times 6}, 2.000_{\times 2}$

$m^2/m_0^2[\phi]: -4.444_{\times 12}, -1.778_{\times 12}, 0.000_{\times 17}, 8.000$

$\Lambda^1{}_1 = \Lambda^2{}_2 = \Lambda^3{}_3 = \Lambda^4{}_4 = \Lambda^5{}_5 = \Lambda^6{}_6 \approx 0.00000$

$\Sigma_{+123;1+456;2} \approx 0.29343, \Sigma_{+123;2-456;1} \approx 0.02565, \Sigma_{+124;1-356;2} \approx 0.00727,$
$\Sigma_{+124;2+356;1} \approx 0.02437, \Sigma_{+125;1+346;2} \approx 0.01975, \Sigma_{+125;2-346;1} \approx -0.30327,$
$\Sigma_{+126;1-345;2} \approx -0.08422, \Sigma_{+126;2+345;1} \approx 0.02034, \Sigma_{+134;1+256;2} \approx 0.01948,$
$\Sigma_{+134;2-256;1} \approx -0.31733, \Sigma_{+135;1-246;2} \approx 0.00577, \Sigma_{+135;2+246;1} \approx 0.00693,$
$\Sigma_{+136;1+245;2} \approx 0.12256, \Sigma_{+136;2-245;1} \approx -0.01130, \Sigma_{+145;1+236;2} \approx 0.32741,$
$\Sigma_{+145;2-236;1} \approx 0.01353, \Sigma_{+146;1-235;2} \approx -0.01950, \Sigma_{+146;2+235;1} \approx -0.08083,$
$\Sigma_{+156;1+234;2} \approx -0.01220, \Sigma_{+156;2-234;1} \approx -0.12477$

T0870297 : $V/g^2 = -0.87029791,\ \mathfrak{so}(6) \to \mathfrak{su}(2) + \mathfrak{u}(1)$ (C.5)

$m^2/m_0^2[\psi]: 1.440_{\times 4}, 1.600_{\times 4}$

$m^2/m_0^2[\phi]: -5.440_{\times 6}, -4.000_{\times 4}, -2.560_{\times 8}, -2.400_{\times 6}, 0.000_{\times 13}, 3.360_{\times 2}, 9.600, 10.400_{\times 2}$

$\Lambda^1{}_1 = \Lambda^2{}_2 = \Lambda^3{}_3 = \Lambda^4{}_4 \approx -0.19188, \Lambda^5{}_5 = \Lambda^6{}_6 \approx 0.38376$

$\Sigma_{+135;2+246;1} = \Sigma_{+136;1+245;2} \approx -0.35635$

T0878939 : $V/g^2 = -0.87893974,\ \mathfrak{so}(6) \to \mathfrak{u}(1)$ (C.6)

$m^2/m_0^2[\psi]: 1.358_{\times 4}, 1.802_{\times 4}$

$m^2/m_0^2[\phi]: -5.827, -5.221_{\times 4}, -4.990, -4.937_{\times 2}, -4.780_{\times 2}, -4.475, -2.522_{\times 2}, -2.288_{\times 4},$
$-1.532_{\times 4}, 0.000_{\times 16}, 0.820, 4.437, 9.803, 12.622_{\times 2}$

$\Lambda^1{}_1 = \Lambda^2{}_2 \approx -0.21480, \Lambda^3{}_3 = \Lambda^4{}_4 \approx -0.20031, \Lambda^5{}_5 = \Lambda^6{}_6 \approx 0.41511$

JHEP07(2020)240

$\Sigma_{+136;2-245;1} = \Sigma_{+146;1-235;2} \approx 0.34860$

$$\textbf{T0887636} : V/g^2 = -0.88763615,\ \mathfrak{so}(6) \to \mathfrak{u}(1) \tag{C.7}$$

$m^2/m_0^2[\psi] : 1.483_{\times 2}, 1.604_{\times 4}, 1.838_{\times 2}$

$m^2/m_0^2[\phi] : -5.907_{\times 2}, -5.489_{\times 4}, -5.208, -4.731, -4.581_{\times 2}, -4.054_{\times 2}, -2.293_{\times 4},$
$-1.182_{\times 2}, -0.253_{\times 2}, 0.000_{\times 16}, 2.070_{\times 2}, 3.344, 9.651, 14.863_{\times 2}$

$\Lambda^1{}_1 = \Lambda^2{}_2 = \Lambda^3{}_3 = \Lambda^4{}_4 \approx -0.22251, \Lambda^5{}_5 = \Lambda^6{}_6 \approx 0.44502$

$\Sigma_{+125;1+346;2} \approx 0.16666, \Sigma_{+126;2+345;1} \approx 0.03828, \Sigma_{+135;2+246;1} \approx -0.14968,$
$\Sigma_{+136;1+245;2} \approx 0.14968, \Sigma_{+145;1+236;2} \approx -0.15351, \Sigma_{+146;2+235;1} \approx 0.37172$

$$\textbf{T0892913} : V/g^2 = -0.89291309,\ \mathfrak{so}(6) \to \mathfrak{u}(1) + \mathfrak{u}(1) \tag{C.8}$$

$m^2/m_0^2[\psi] : 1.667_{\times 8}$

$m^2/m_0^2[\phi] : -6.000_{\times 4}, -5.572, -5.000_{\times 4}, -4.520, -3.500_{\times 4}, -0.833_{\times 4}, 0.000_{\times 16}, 1.667_{\times 4},$
$2.667, 9.572, 16.000, 16.520$

$\Lambda^1{}_1 = \Lambda^2{}_2 = \Lambda^3{}_3 = \Lambda^4{}_4 \approx -0.23105, \Lambda^5{}_5 = \Lambda^6{}_6 \approx 0.46210$

$\Sigma_{+125;1+346;2} \approx -0.00001, \Sigma_{+125;2-346;1} \approx 0.00001, \Sigma_{+126;1-345;2} \approx -0.00198,$
$\Sigma_{+126;2+345;1} \approx -0.00217, \Sigma_{+135;1-246;2} \approx 0.35483, \Sigma_{+135;2+246;1} \approx -0.32504$

$$\textbf{T0963952} : V/g^2 = -0.96395224,\ \mathfrak{so}(6) \to \emptyset \tag{C.9}$$

$m^2/m_0^2[\psi] : 1.624_{\times 2}, 1.753_{\times 2}, 1.924_{\times 2}, 2.063_{\times 2}$

$m^2/m_0^2[\phi] : -5.874, -5.026_{\times 2}, -4.449_{\times 2}, -4.076_{\times 2}, -4.017_{\times 2}, -4.010_{\times 2}, -3.485_{\times 2}, -2.623,$
$-0.499, -0.156_{\times 2}, 0.000_{\times 17}, 6.042_{\times 2}, 7.106_{\times 2}, 9.730, 11.012_{\times 2}, 12.757$

$\Lambda^1{}_1 \approx -0.32137, \Lambda^2{}_2 = \Lambda^3{}_3 \approx -0.24369, \Lambda^4{}_4 = \Lambda^5{}_5 \approx 0.24012, \Lambda^6{}_6 \approx 0.32852$

$\Sigma_{+126;2+345;1} = \Sigma_{+136;1+245;2} \approx 0.17139, \Sigma_{+146;1-235;2} = \Sigma_{+156;2-234;1} \approx 0.52343$

$$\textbf{T0964097} : V/g^2 = -0.96409727,\ \mathfrak{so}(6) \to \emptyset \tag{C.10}$$

$m^2/m_0^2[\psi] : 1.701_{\times 4}, 1.985_{\times 4}$

JHEP07(2020)240

$$m^2/m_0^2[\phi]: -5.922, -5.120, -5.100, -4.711, -4.163, -4.146, -4.097, -4.056, -4.040, -3.996, -3.943, -3.455, -3.450, -2.553, -0.456, 0.000_{\times 18}, 0.137, 5.735, 5.821, 7.182, 7.379, 9.682, 10.417, 11.561, 13.019$$

$\Lambda^1{}_1 \approx -0.32305, \Lambda^2{}_2 \approx -0.24370, \Lambda^3{}_3 \approx -0.23916, \Lambda^4{}_4 \approx 0.22698, \Lambda^5{}_5 \approx 0.24757, \Lambda^6{}_6 \approx 0.33137$

$\Sigma_{+136;2-245;1} \approx 0.21467, \Sigma_{+146;2+235;1} \approx -0.51776, \Sigma_{+156;1+234;2} \approx 0.54357$

$$\textbf{T0964525}: V/g^2 = -0.96452592,\ \mathfrak{so}(6) \to \mathfrak{u}(1) \tag{C.11}$$

$m^2/m_0^2[\psi]: 1.798_{\times 4}, 1.899_{\times 4}$

$$m^2/m_0^2[\phi]: -5.981, -5.350_{\times 2}, -4.441_{\times 2}, -4.119_{\times 4}, -3.992_{\times 2}, -3.453_{\times 2}, -2.393, 0.000_{\times 16}, 0.275_{\times 4}, 5.108_{\times 2}, 7.699_{\times 2}, 9.546, 11.108_{\times 2}, 13.615$$

$\Lambda^1{}_1 \approx -0.32814, \Lambda^2{}_2 = \Lambda^3{}_3 \approx -0.23391, \Lambda^4{}_4 = \Lambda^5{}_5 \approx 0.22848, \Lambda^6{}_6 \approx 0.33900$

$\Sigma_{+146;2+235;1} = \Sigma_{+156;1+234;2} \approx -0.55645$

$$\textbf{T0982778}: V/g^2 = -0.98277802,\ \mathfrak{so}(6) \to \mathfrak{u}(1) \tag{C.12}$$

$m^2/m_0^2[\psi]: 1.630_{\times 4}, 2.222_{\times 4}$

$$m^2/m_0^2[\phi]: -6.371, -5.431, -5.333_{\times 2}, -4.114_{\times 2}, -4.000_{\times 2}, -3.277, -3.033_{\times 2}, -2.846, -1.903, -1.127_{\times 2}, 0.000_{\times 17}, 2.194_{\times 2}, 6.611, 7.033_{\times 2}, 9.333, 9.799, 11.714_{\times 2}, 14.085$$

$\Lambda^1{}_1 = \Lambda^2{}_2 \approx -0.32349, \Lambda^3{}_3 \approx -0.19823, \Lambda^4{}_4 = \Lambda^5{}_5 \approx 0.25591, \Lambda^6{}_6 \approx 0.33339$

$\Sigma_{+135;2+246;1} = \Sigma_{+156;1+234;2} \approx 0.56869$

$$\textbf{T1001482}: V/g^2 = -1.00148265,\ \mathfrak{so}(6) \to \mathfrak{u}(1) \tag{C.13}$$

$m^2/m_0^2[\psi]: 1.744_{\times 4}, 2.188_{\times 4}$

$$m^2/m_0^2[\phi]: -6.436, -5.586_{\times 4}, -4.410_{\times 2}, -2.824, -2.719_{\times 2}, -2.306_{\times 4}, 0.000_{\times 16}, 3.171_{\times 2}, 5.354_{\times 2}, 7.693_{\times 4}, 8.788, 9.401, 11.392, 11.598$$

$\Lambda^1{}_1 = \Lambda^2{}_2 \approx -0.34956, \Lambda^3{}_3 = \Lambda^4{}_4 \approx 0.15576, \Lambda^5{}_5 = \Lambda^6{}_6 \approx 0.19379$

$\Sigma_{+136;2-245;1} = \Sigma_{+146;1-235;2} \approx -0.63157$

JHEP07(2020)240

$$\mathbf{T1054687}: V/g^2 = -1.05468750,\ \mathfrak{so}(6) \to \emptyset \tag{C.14}$$

$m^2/m_0^2[\psi] : 1.733_{\times 2}, 2.326_{\times 6}$

$m^2/m_0^2[\phi] : -6.512, -5.251_{\times 2}, -4.161_{\times 3}, -4.062_{\times 2}, -2.925_{\times 2}, -1.560_{\times 3}, 0.000_{\times 18}, 4.490_{\times 2}, 6.697_{\times 3}, 10.600, 15.735_{\times 3}, 16.638_{\times 2}$

$\Lambda^1{}_1 = \Lambda^2{}_2 = \Lambda^3{}_3 \approx -0.37177, \Lambda^4{}_4 = \Lambda^5{}_5 = \Lambda^6{}_6 \approx 0.37177$

$\Sigma_{+124;1-356;2} \approx -0.40776, \Sigma_{+124;2+356;1} \approx -0.09457, \Sigma_{+125;2-346;1} \approx -0.20112,$
$\Sigma_{+126;1-345;2} \approx -0.07976, \Sigma_{+126;2+345;1} \approx 0.00066, \Sigma_{+134;1+256;2} \approx 0.04717,$
$\Sigma_{+134;2-256;1} \approx 0.21114, \Sigma_{+135;1-246;2} \approx 0.34833, \Sigma_{+135;2+246;1} \approx -0.16791,$
$\Sigma_{+136;1+245;2} \approx -0.06930, \Sigma_{+136;2-245;1} \approx -0.14455, \Sigma_{+145;1+236;2} \approx -0.43585,$
$\Sigma_{+146;2+235;1} \approx -0.17902, \Sigma_{+156;1+234;2} \approx -0.00305$

$$\mathbf{T1073529}: V/g^2 = -1.07352975,\ \mathfrak{so}(6) \to \emptyset \tag{C.15}$$

$m^2/m_0^2[\psi] : 1.819_{\times 2}, 2.011_{\times 2}, 2.433_{\times 2}, 2.806_{\times 2}$

$m^2/m_0^2[\phi] : -6.536, -4.882_{\times 2}, -4.631_{\times 2}, -4.068_{\times 2}, -3.714_{\times 2}, -3.354, -2.637_{\times 2}, 0.000_{\times 17}, 1.273_{\times 2}, 5.470_{\times 2}, 8.041, 8.130_{\times 2}, 10.762, 14.391_{\times 2}, 19.457_{\times 2}, 21.617$

$\Lambda^1{}_1 = \Lambda^2{}_2 \approx -0.39487, \Lambda^3{}_3 \approx -0.35452, \Lambda^4{}_4 = \Lambda^5{}_5 \approx 0.34969, \Lambda^6{}_6 \approx 0.44490$

$\Sigma_{+124;2+356;1} \approx 0.47010, \Sigma_{+125;1+346;2} \approx -0.47010,$
$\Sigma_{+134;1+256;2} = \Sigma_{+146;2+235;1} = \Sigma_{+156;1+234;2} \approx -0.25201, \Sigma_{+135;2+246;1} \approx 0.25201$

$$\mathbf{T1125000}: V/g^2 = -1.12500000,\ \mathfrak{so}(6) \to \emptyset \tag{C.16}$$

$m^2/m_0^2[\psi] : 2.444_{\times 8}$

$m^2/m_0^2[\phi] : -7.325, -6.206, -4.769, -4.748_{\times 2}, -4.427, -3.550_{\times 2}, -3.280, -1.276, 0.000_{\times 18}, 4.806_{\times 2}, 4.883_{\times 2}, 5.333, 6.504, 7.861_{\times 2}, 10.829, 16.719, 18.634, 22.748_{\times 2}, 25.265$

$\Lambda^1{}_1 \approx -0.48597, \Lambda^2{}_2 = \Lambda^3{}_3 \approx -0.39170, \Lambda^4{}_4 = \Lambda^5{}_5 \approx 0.39170, \Lambda^6{}_6 \approx 0.48597$

$\Sigma_{+125;1+346;2} \approx 0.60475, \Sigma_{+134;2-256;1} \approx -0.60475$

$$\mathbf{T1297247}: V/g^2 = -1.29724786,\ \mathfrak{so}(6) \to \emptyset \tag{C.17}$$

$m^2/m_0^2[\psi] : 1.901_{\times 2}, 2.568_{\times 2}, 2.964_{\times 2}, 3.758_{\times 2}$

$m^2/m_0^2[\phi] : -5.445, -4.874, -4.060, -3.966, -3.942, -3.204, -3.186, -2.907, -1.014,$
$-0.934, 0.000_{\times 18}, 6.983, 7.652, 11.889, 12.146, 12.543, 13.041, 13.946, 14.490,$
$18.175, 18.187, 21.001, 21.574, 22.590, 22.900$

$\Lambda^1{}_1 = \Lambda^2{}_2 \approx -0.59541, \Lambda^3{}_3 = \Lambda^4{}_4 \approx 0.23362, \Lambda^5{}_5 = \Lambda^6{}_6 \approx 0.36178$

$\Sigma_{+123;1+456;2} \approx 0.48716, \Sigma_{+124;2+356;1} \approx -0.48716, \Sigma_{+125;1+346;2} = \Sigma_{+126;2+345;1} \approx 0.04462,$
$\Sigma_{+134;1+256;2} = \Sigma_{+156;1+234;2} \approx -0.37352, \Sigma_{+135;2+246;1} \approx 0.28567, \Sigma_{+136;1+245;2} \approx 0.33709,$
$\Sigma_{+145;1+236;2} \approx -0.47047, \Sigma_{+146;2+235;1} \approx 0.03521$

T1302912 : $V/g^2 = -1.30291232,\ \mathfrak{so}(6) \to \emptyset$ (C.18)

$m^2/m_0^2[\psi] : 1.868_{\times 2}, 2.939_{\times 2}, 2.966_{\times 2}, 3.715_{\times 2}$

$m^2/m_0^2[\phi] : -5.530, -5.163, -4.601, -4.088, -3.938, -3.876, -2.975, -1.241, -0.535,$
$0.000_{\times 18}, 1.055, 10.323, 11.248, 11.304, 11.795, 12.471, 12.633, 15.614, 15.958,$
$16.392, 16.582, 22.633, 22.985, 23.633, 24.258$

$\Lambda^1{}_1 \approx -0.62113, \Lambda^2{}_2 \approx -0.58860, \Lambda^3{}_3 \approx 0.24575, \Lambda^4{}_4 \approx 0.29746, \Lambda^5{}_5 \approx 0.32510,$
$\Lambda^6{}_6 \approx 0.34142$

$\Sigma_{+123;2-456;1} \approx 0.46285, \Sigma_{+124;1-356;2} \approx 0.38576, \Sigma_{+126;2+345;1} \approx -0.06801,$
$\Sigma_{+135;1-246;2} \approx -0.73256, \Sigma_{+145;2-236;1} \approx -0.31760, \Sigma_{+156;1+234;2} \approx -0.44500$

T1304606 : $V/g^2 = -1.30460644,\ \mathfrak{so}(6) \to \emptyset$ (C.19)

$m^2/m_0^2[\psi] : 1.863_{\times 2}, 3.045_{\times 4}, 3.623_{\times 2}$

$m^2/m_0^2[\phi] : -5.709, -5.072, -4.820, -4.308, -4.051, -3.851, -2.894, -0.409, 0.000_{\times 18},$
$0.705_{\times 2}, 10.707, 11.268, 12.495, 12.520_{\times 2}, 13.495, 14.491, 15.601_{\times 2}, 16.679,$
$22.729, 23.752_{\times 2}, 24.989$

$\Lambda^1{}_1 = \Lambda^2{}_2 \approx -0.60876, \Lambda^3{}_3 = \Lambda^4{}_4 \approx 0.28534, \Lambda^5{}_5 = \Lambda^6{}_6 \approx 0.32343$

$\Sigma_{+123;2-456;1} = \Sigma_{+124;1-356;2} \approx -0.41059, \Sigma_{+135;1-246;2} \approx 0.32857, \Sigma_{+145;2-236;1} \approx 0.87485$

T1319179 : $V/g^2 = -1.31917968,\ \mathfrak{so}(6) \to \emptyset$ (C.20)

$m^2/m_0^2[\psi] : 2.007_{\times 2}, 2.314_{\times 2}, 3.390_{\times 2}, 3.479_{\times 2}$

$m^2/m_0^2[\phi]: -6.083_{\times 2}, -4.130, -3.789_{\times 2}, -3.689, -3.654_{\times 2}, -0.971, 0.000_{\times 17}, 0.946_{\times 2}, 8.035, 9.713, 10.759_{\times 2}, 11.497, 12.191, 12.714, 17.658_{\times 2}, 17.814, 21.132_{\times 2}, 23.045, 23.374$

$\Lambda^1{}_1 = \Lambda^2{}_2 \approx -0.60188, \Lambda^3{}_3 = \Lambda^4{}_4 \approx 0.22066, \Lambda^5{}_5 = \Lambda^6{}_6 \approx 0.38122$

$\Sigma_{+123;2-456;1} = \Sigma_{+124;1-356;2} \approx 0.59938, \Sigma_{+135;1-246;2} \approx 0.31540, \Sigma_{+136;2-245;1} \approx -0.00424, \Sigma_{+145;2-236;1} \approx 0.24695, \Sigma_{+146;1-235;2} \approx 0.55810$

$$\mathbf{T1382251} : V/g^2 = -1.38225189,\ \mathfrak{so}(6) \to \emptyset \tag{C.21}$$

$m^2/m_0^2[\psi] : 2.157_{\times 2}, 2.626_{\times 2}, 3.824_{\times 2}, 4.425_{\times 2}$

$m^2/m_0^2[\phi] : -6.955, -5.615, -4.845, -3.442, -3.429, -3.048, -1.888, -0.873, 0.000_{\times 18}, 2.191, 4.190, 9.654, 9.868, 10.839, 11.089, 12.148, 14.269, 17.139, 17.449, 20.030, 20.351, 28.208, 28.346, 45.348, 45.357$

$\Lambda^1{}_1 = \Lambda^2{}_2 \approx -0.61986, \Lambda^3{}_3 = \Lambda^4{}_4 \approx 0.12499, \Lambda^5{}_5 = \Lambda^6{}_6 \approx 0.49487$

$\Sigma_{+123;1+456;2} = \Sigma_{+124;2+356;1} \approx -0.57846, \Sigma_{+125;2-346;1} = \Sigma_{+126;1-345;2} \approx 0.13899, \Sigma_{+134;1+256;2} = \Sigma_{+156;1+234;2} \approx -0.44722, \Sigma_{+135;1-246;2} \approx -0.37639, \Sigma_{+136;2-245;1} \approx 0.00529, \Sigma_{+145;2-236;1} \approx 0.14221, \Sigma_{+146;1-235;2} \approx -0.12738$

$$\mathbf{T1391035} : V/g^2 = -1.39103566,\ \mathfrak{so}(6) \to \emptyset \tag{C.22}$$

$m^2/m_0^2[\psi] : 2.576_{\times 4}, 3.726_{\times 4}$

$m^2/m_0^2[\phi] : -7.422, -5.873, -5.133, -3.439, -3.258, -3.039, -1.551, 0.000_{\times 18}, 0.990, 3.614, 5.111, 8.912, 9.520, 9.929, 11.031, 12.107, 15.840, 16.038, 16.254, 19.051, 20.981, 22.866, 23.627, 40.460, 40.498$

$\Lambda^1{}_1 \approx -0.65089, \Lambda^2{}_2 \approx -0.60760, \Lambda^3{}_3 \approx 0.08094, \Lambda^4{}_4 \approx 0.20776, \Lambda^5{}_5 \approx 0.48464, \Lambda^6{}_6 \approx 0.48515$

$\Sigma_{+123;1+456;2} \approx 0.61508, \Sigma_{+124;2+356;1} \approx -0.57701, \Sigma_{+126;1-345;2} \approx -0.20028, \Sigma_{+134;2-256;1} \approx -0.56777, \Sigma_{+136;1+245;2} \approx 0.17976, \Sigma_{+146;2+235;1} \approx 0.41246$

$$\mathbf{T1416746} : V/g^2 = -1.41674647,\ \mathfrak{so}(6) \to \emptyset \tag{C.23}$$

$m^2/m_0^2[\psi] : 2.541_{\times 4}, 4.682_{\times 4}$

$$m^2/m_0^2[\phi]: -7.588, -6.331, -4.664, -3.637, -2.868, -2.381, -0.500, 0.000_{\times 18}, 2.018, 3.674, 5.078, 10.577, 11.395, 14.830, 15.208, 15.285, 17.454, 19.273, 19.969, 27.930, 27.963, 28.305, 28.585, 55.234, 55.252$$

$\Lambda^1{}_1 \approx -0.73037, \Lambda^2{}_2 \approx -0.46700, \Lambda^3{}_3 \approx -0.03604, \Lambda^4{}_4 \approx 0.22540, \Lambda^5{}_5 \approx 0.50269, \Lambda^6{}_6 \approx 0.50532$

$\Sigma_{+123;1+456;2} \approx -0.63728, \Sigma_{+126;1-345;2} \approx -0.22177, \Sigma_{+134;2-256;1} \approx 0.88862, \Sigma_{+135;2+246;1} \approx -0.10297, \Sigma_{+146;2+235;1} \approx 0.35261, \Sigma_{+156;2-234;1} \approx -0.20665$

$$\mathbf{T1417411}: V/g^2 = -1.41741118,\ \mathfrak{so}(6) \to \emptyset \tag{C.24}$$

$m^2/m_0^2[\psi]: 2.667_{\times 4}, 4.444_{\times 4}$

$$m^2/m_0^2[\phi]: -7.719, -6.560, -4.692, -3.535, -2.851, -2.492, 0.000_{\times 18}, 0.276, 1.895, 4.301, 5.567, 10.235, 11.353, 14.431, 15.458, 15.502, 17.694, 20.814, 22.838, 24.048, 25.549, 25.724, 25.766, 53.186, 53.213$$

$\Lambda^1{}_1 \approx -0.73052, \Lambda^2{}_2 \approx -0.47160, \Lambda^3{}_3 \approx -0.04027, \Lambda^4{}_4 \approx 0.24055, \Lambda^5{}_5 \approx 0.49948, \Lambda^6{}_6 \approx 0.50237$

$\Sigma_{+123;2-456;1} \approx -0.64026, \Sigma_{+126;2+345;1} \approx 0.23639, \Sigma_{+134;1+256;2} \approx -0.90336, \Sigma_{+146;1-235;2} \approx 0.36681$

$$\mathbf{T1460654}: V/g^2 = -1.46065435,\ \mathfrak{so}(6) \to \emptyset \tag{C.25}$$

$m^2/m_0^2[\psi]: 2.549_{\times 2}, 2.872_{\times 2}, 3.548_{\times 2}, 4.727_{\times 2}$

$$m^2/m_0^2[\phi]: -7.782, -7.781, -5.074, -3.077_{\times 2}, -2.429, -0.156, 0.000_{\times 18}, 5.812_{\times 2}, 8.206, 8.928, 9.049, 11.748, 15.607_{\times 2}, 18.569, 18.574, 18.983, 23.132, 23.187, 26.681, 26.684, 44.562_{\times 2}$$

$\Lambda^1{}_1 \approx -0.80626, \Lambda^2{}_2 \approx -0.17618, \Lambda^3{}_3 \approx 0.04003, \Lambda^4{}_4 \approx 0.04284, \Lambda^5{}_5 \approx 0.44978, \Lambda^6{}_6 \approx 0.44979$

$\Sigma_{+123;2-456;1} \approx -0.83799, \Sigma_{+124;1-356;2} \approx -0.83732, \Sigma_{+125;2-346;1} \approx -0.03678, \Sigma_{+126;1-345;2} \approx 0.03663, \Sigma_{+134;2-256;1} \approx -0.00494, \Sigma_{+135;1-246;2} \approx -0.22033, \Sigma_{+136;2-245;1} \approx -0.22065, \Sigma_{+145;2-236;1} \approx -0.21718, \Sigma_{+146;1-235;2} \approx 0.21663, \Sigma_{+156;2-234;1} \approx -0.13058$

JHEP07(2020)240

$$\mathbf{T1460729}: V/g^2 = -1.46072960,\ \mathfrak{so}(6) \to \emptyset \tag{C.26}$$

$m^2/m_0^2[\psi]: 2.578_{\times 2}, 2.953_{\times 2}, 3.433_{\times 2}, 4.666_{\times 2}$

$m^2/m_0^2[\phi]: -7.837_{\times 2}, -5.119, -3.105_{\times 2}, -2.400, 0.000_{\times 17}, 0.081_{\times 2}, 5.873_{\times 2}, 8.042, 8.950_{\times 2}, 11.735, 16.355_{\times 2}, 17.388_{\times 2}, 19.322, 23.288_{\times 2}, 26.017_{\times 2}, 43.861_{\times 2}$

$\Lambda^1{}_1 \approx -0.80503, \Lambda^2{}_2 \approx -0.18004, \Lambda^3{}_3 = \Lambda^4{}_4 \approx 0.04349, \Lambda^5{}_5 = \Lambda^6{}_6 \approx 0.44904$

$\Sigma_{+123;1+456;2} \approx -0.81796, \Sigma_{+123;2-456;1} = \Sigma_{+124;1-356;2} \approx -0.19809,$
$\Sigma_{+124;2+356;1} \approx 0.81796, \Sigma_{+135;1-246;2} = \Sigma_{+136;2-245;1} = \Sigma_{+145;2-236;1} \approx -0.15970,$
$\Sigma_{+135;2+246;1} \approx 0.15381, \Sigma_{+136;1+245;2} = \Sigma_{+145;1+236;2} = \Sigma_{+146;2+235;1} \approx -0.15381,$
$\Sigma_{+146;1-235;2} \approx 0.15970$

$$\mathbf{T1497042}: V/g^2 = -1.49704248,\ \mathfrak{so}(6) \to \emptyset \tag{C.27}$$

$m^2/m_0^2[\psi]: 2.564_{\times 2}, 2.744_{\times 2}, 4.924_{\times 2}, 4.987_{\times 2}$

$m^2/m_0^2[\phi]: -7.876, -7.552, -6.086, -2.529, -1.033, -0.827, 0.000_{\times 18}, 3.290, 4.630, 5.745, 7.840, 8.291, 9.757, 11.599, 15.632, 18.415, 18.640, 21.703, 21.866, 28.340, 28.422, 33.168, 33.169, 60.129, 60.135$

$\Lambda^1{}_1 \approx -0.82463, \Lambda^2{}_2 \approx -0.19891, \Lambda^3{}_3 \approx 0.00718, \Lambda^4{}_4 \approx 0.00932, \Lambda^5{}_5 \approx 0.50354,$
$\Lambda^6{}_6 \approx 0.50351$

$\Sigma_{+123;2-456;1} \approx 0.82383, \Sigma_{+124;1-356;2} \approx 0.81305, \Sigma_{+125;1+346;2} \approx -0.13392,$
$\Sigma_{+136;1+245;2} \approx -0.28798, \Sigma_{+146;2+235;1} \approx -0.25309, \Sigma_{+156;2-234;1} \approx 0.38869$

$$\mathbf{T1499666}: V/g^2 = -1.49966681,\ \mathfrak{so}(6) \to \emptyset \tag{C.28}$$

$m^2/m_0^2[\psi]: 2.796_{\times 4}, 4.671_{\times 4}$

$m^2/m_0^2[\phi]: -8.146, -7.859, -6.341, -2.381, -0.858, 0.000_{\times 18}, 0.587, 3.588, 4.575, 6.215, 7.213, 8.309, 8.389, 11.542, 16.667, 18.400, 18.880, 22.379, 22.570, 25.467, 25.509, 31.179, 31.185, 57.469, 57.478$

$\Lambda^1{}_1 \approx -0.81651, \Lambda^2{}_2 \approx -0.22484, \Lambda^3{}_3 \approx -0.00212, \Lambda^4{}_4 \approx 0.03391, \Lambda^5{}_5 \approx 0.50465,$
$\Lambda^6{}_6 \approx 0.50490$

$\Sigma_{+123;2-456;1} \approx 0.83606, \Sigma_{+124;1-356;2} \approx 0.83978, \Sigma_{+126;2+345;1} \approx -0.09844,$
$\Sigma_{+135;2+246;1} \approx -0.29598, \Sigma_{+145;1+236;2} \approx 0.27814, \Sigma_{+156;2-234;1} \approx -0.26863$

JHEP07(2020)240

$$\mathbf{T1501862}: V/g^2 = -1.50186250,\ \mathfrak{so}(6) \to \emptyset \tag{C.29}$$
$m^2/m_0^2[\psi]: 2.958_{\times 4}, 4.361_{\times 4}$
$m^2/m_0^2[\phi]: -8.390, -8.149, -6.547, -2.243, 0.000_{\times 18}, 0.534_{\times 2}, 3.715, 4.659, 6.331, 6.591, 8.036_{\times 2}, 11.530, 17.908, 19.043, 19.546, 22.488_{\times 2}, 22.827_{\times 2}, 28.910, 28.950, 54.551, 54.564$
$\Lambda^1{}_1 \approx -0.81048, \Lambda^2{}_2 \approx -0.24631, \Lambda^3{}_3 = \Lambda^4{}_4 \approx 0.02461, \Lambda^5{}_5 \approx 0.50349, \Lambda^6{}_6 \approx 0.50407$
$\Sigma_{+123;1+456;2} \approx 0.85496, \Sigma_{+124;2+356;1} \approx -0.85496, \Sigma_{+135;1-246;2} = \Sigma_{+145;2-236;1} \approx 0.30382$

$$\mathbf{T1510900}: V/g^2 = -1.51090053,\ \mathfrak{so}(6) \to \emptyset \tag{C.30}$$
$m^2/m_0^2[\psi]: 2.341_{\times 2}, 2.515_{\times 2}, 5.252_{\times 2}, 5.535_{\times 2}$
$m^2/m_0^2[\phi]: -7.096, -6.092, -5.664, -3.912, -3.831, -1.977, 0.000_{\times 18}, 4.992, 5.327, 5.471, 8.192, 10.978, 11.323, 12.154, 15.682, 18.760, 19.789, 23.119, 24.250, 32.619, 32.622, 38.729, 38.866, 54.809, 54.833$
$\Lambda^1{}_1 \approx -0.80362, \Lambda^2{}_2 \approx -0.54087, \Lambda^3{}_3 \approx 0.11891, \Lambda^4{}_4 \approx 0.28645, \Lambda^5{}_5 \approx 0.46652, \Lambda^6{}_6 \approx 0.47261$
$\Sigma_{+123;2-456;1} \approx -0.65751, \Sigma_{+126;1-345;2} \approx -0.15110, \Sigma_{+134;1+256;2} \approx -0.55854, \Sigma_{+135;2+246;1} \approx 0.32344, \Sigma_{+146;2+235;1} \approx 0.35501, \Sigma_{+156;1+234;2} \approx -0.74695$

$$\mathbf{T1547778}: V/g^2 = -1.54777840,\ \mathfrak{so}(6) \to \emptyset \tag{C.31}$$
$m^2/m_0^2[\psi]: 2.919_{\times 2}, 3.511_{\times 4}, 4.696_{\times 2}$
$m^2/m_0^2[\phi]: -7.690, -7.121, -5.196, -3.768, -3.229, 0.000_{\times 18}, 2.491, 5.758, 5.882, 7.906_{\times 2}, 11.876, 15.646, 16.511, 16.875_{\times 2}, 18.734, 19.897, 21.602_{\times 2}, 25.828, 34.397, 37.884_{\times 2}, 40.118$
$\Lambda^1{}_1 = \Lambda^2{}_2 \approx -0.71461, \Lambda^3{}_3 \approx 0.13652, \Lambda^4{}_4 = \Lambda^5{}_5 \approx 0.41599, \Lambda^6{}_6 \approx 0.46074$
$\Sigma_{+123;1+456;2} \approx 0.70763, \Sigma_{+126;2+345;1} \approx -0.25491, \Sigma_{+134;1+256;2} = \Sigma_{+146;2+235;1} \approx 0.45958, \Sigma_{+135;2+246;1} = \Sigma_{+156;1+234;2} \approx -0.51427$

$$\mathbf{T1738407}: V/g^2 = -1.73840792,\ \mathfrak{so}(6) \to \emptyset \tag{C.32}$$
$m^2/m_0^2[\psi]: 2.380_{\times 2}, 3.106_{\times 2}, 5.205_{\times 2}, 5.322_{\times 2}$
$m^2/m_0^2[\phi]: -6.504_{\times 2}, -6.250_{\times 2}, -3.019, 0.000_{\times 17}, 3.819_{\times 2}, 11.175, 11.214_{\times 2}, 13.297, 17.275_{\times 2}, 21.000_{\times 2}, 25.218_{\times 2}, 26.699_{\times 2}, 28.635, 30.545_{\times 2}, 34.291_{\times 2}, 35.846$
$\Lambda^1{}_1 \approx -1.11580, \Lambda^2{}_2 \approx 0.20510, \Lambda^3{}_3 = \Lambda^4{}_4 \approx 0.22634, \Lambda^5{}_5 = \Lambda^6{}_6 \approx 0.22901$
$\Sigma_{+125;1+346;2} = \Sigma_{+126;2+345;1} \approx -0.80729,$
$\Sigma_{+135;1-246;2} = \Sigma_{+145;2-236;1} = \Sigma_{+146;1-235;2} \approx 0.42803, \Sigma_{+136;2-245;1} \approx -0.42803$

JHEP07(2020)240

Degeneracy	m^2L^2	BF Stability
2	0	S
20	-3	S
20	-4	S

Table 6. The SO(6) point.

Degeneracy	m^2L^2	Stability
1	8	S
7	0	S
20	-2	S
14	$-16/3$	U

Table 7. The SO(5) point.

Degeneracy	m^2L^2	Stability
1	8	S
17	0	S
12	$-16/9$	S
12	$-40/9$	U

Table 8. The SU(3) point.

Degeneracy	m^2L^2	BF stability
1	$4+2\sqrt{7}$	S
2	3	S
13	0	S
1	$4-2\sqrt{7}$	S
4	$-39/16$	S
2	-3	S
4	$-55/16$	S
12	$-15/4$	S
3	-4	S

Table 9. The SU(2) $\times$ U(1) point.

Degeneracy	m^2L^2	Stability
2	52/5	S
1	48/5	S
2	84/25	S
13	0	S
6	$-12/5$	S
8	$-64/25$	S
4	-4	S
6	$-136/25$	U

Table 10. The SU(2) $\times$ U(1)^2 point.

D Scalar mass spectra for the classic vacua

In this appendix we collect results for the spectrum of scalar fluctuations around each of the five classic critical points found in [4].

The spectrum of scalar masses at the SO(6) point, `T075000`, in table 6 follows from $\mathcal{N} = 8$ supersymmetry. At the SU(2) $\times$ U(1) point, `T0839947`, the full spectrum was computed and organized into multiplets of $\mathcal{N} = 2$ supersymmetry in [14] with the scalar masses given in table 9. Both points are perturbatively stable. The BF instability of the SU(3) point, `T0843750`, was established in [56] and subsequently confirmed in [12, 57]. The full scalar spectrum at this point, see also [13], is given in table 8. Finally, the scalar spectra at the SO(5) point, `T0780031`, and the SU(2) $\times$ U(1)^2 point, `T0870298`, were computed in 1999 [58] and are given in tables 7 and 10.

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JHEP07(2020)240

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