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Loop operators in three-dimensional $\mathcal{N} = 2$ fishnet theories

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ABSTRACT: In this work, we study the line and loop operators in three-dimensional $\mathcal{N} = 2$ fishnet theories in detail. We construct the straight line and circular loop operators which are at least classically half-BPS. We develop a new regularization scheme at frame -1 which is suitable for the study of the fermionic BPS loops in general super-Chern-Simons-matter theories. We initialize the perturbative computation for the vacuum expectation values of the circular BPS loop operators based on this scheme. We construct the cusped line operators as well, and compute the vacuum expectation values of these cusped line operators up to two-loop order. We find that the universal cusp anomalous dimension vanishes, if we put aside the fact that the generalized potential has a double pole in the $1/\epsilon$ expansion.

KEYWORDS: Wilson, 't Hooft and Polyakov loops, Chern-Simons Theories

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1 Introduction

Wilson loop operators play an important role in the study of dynamics of gauge theory. The vacuum expectation value (VEV) of the Wilson loop provides the criteria for color confinement [1]. In supersymmetric gauge theory, it is natural to consider the Wilson loop operators preserving part of the supersymmetries, which have better ultraviolet behavior. The first kind of BPS Wilson loop operator was constructed [2, 3] in four-dimensional $\mathcal{N} = 4$ super Yang-Mills (SYM₄) theory. The obtained Maldacena-Wilson loop has a simple holographic description in terms of classical open string solution [2, 3] in the framework of the AdS/CFT correspondence. This opened a new window to study the AdS/CFT correspondence via the loop operators.

Due to its supersymmetric nature, the study of the Bogomol'nyi-Prasad-Sommerfield (BPS) Wilson loop operators benefits from new techniques in supersymmetric field theories. One typical example is the supersymmetric localization. Supersymmetric localization

reduces the path integral computing the VEV of the circular half-BPS Wilson loop in SYM_4 to a finite-dimensional integral [4]. And this finite-dimensional integral turns out to be a Gaussian matrix model. It was known before the localization computations that the results from this Gaussian matrix model are consistent with the prediction of the AdS/CFT correspondence [5]. Moreover the localization provides a very powerful tool to calculate the VEVs or certain correlators of the BPS Wilson loops even at finite N (for a collection of reviews, see [6]).

Besides the smooth line and loop operators, there are the line/loop operators with a cusp, possibly with the insertion of local composite operators. The cusp anomalous dimension is related to many important quantities like the ultraviolet divergences of cusped Wilson loops, the infrared divergences of gluon amplitudes and the anomalous dimension of twist-two operators in gauge theories. The study of the cusp anomaly in the BPS line operator could be helped by using the integrability in the AdS_5/CFT_4 duality [7, 8]. When we insert the composite operators into the Wilson loops, the computation of the anomalous dimensions leads to integrable open spin chains. This happens for both the BPS Maldacena-Wilson loops [9] and the usual Wilson loops [10]. The cusp anomalous dimension can be computed based on this open chain firstly using the open asymptotic Bethe ansatz equations and the boundary thermal Bethe ansatz [11, 12] and later using very powerful tools like the Y-system [13, 14] and quantum spectral curve [15, 16]. The obtained results are consistent with the ones from the localization [17, 18]. Further studies on comparing results from localization and integrability can be found in [19, 20]. Some details of the development on the Wilson loops and integrable structure can be found in a short review [21].

The supersymmetric localization has also been applied to the study of the BPS Wilson loop operators in the AdS_4/CFT_3 correspondence [22], which states that a three dimensional $\mathcal{N} = 6$ Chern-Simons-matter theory is dual to the IIA string theory on $AdS_4 \times CP^3$ background. In this Chern-Simons-matter theory proposed by Aharony, Bergman, Jafferis and Maldacena (ABJM), the construction of the BPS Wilson loops is subtle [23–25]. The half BPS one requires the introduction of superconnection [26]. The VEV's of both bosonic 1/6-BPS [23–25] and fermionic half-BPS Wilson loops [26] can be computed using the localization [27] which leads to the results [28] consistent with string theory prediction. Some fermionic BPS Wilson loops with less supersymmetries in the ABJM theory were constructed in [29–32]. The ABJM theory is also integrable in the planar limit [33, 34]. There is an interpolating function in the magnon dispersion relation [35]. The computation using the integrability of the cusp anomalous dimension of the Wilson loops was hoped to exactly confirm a conjecture [36] about this interpolating function [12]. For a recent progress report on various aspects on the BPS Wilson loops in Chern-Simons-matter theory, see [37] and [38].

It is certainly interesting to study the BPS Wilson loops, localizations and integrable structures in the theories with less supersymmetries. The BPS circular Wilson loops in four dimensional theories need that the theories have at least $\mathcal{N} = 2$ supersymmetries. The VEV of the BPS Wilson loop in the $\mathcal{N} = 2^*$ theory, which comes from a mass deformation of the $\mathcal{N} = 4$ SYM, was computed using the localizations and the results are consistent

with the predictions from the AdS/CFT correspondence [39]. This provided an important test of the holographic correspondence in the non-conformal case. However, the integrable β -deformed SYM_4 has only $\mathcal{N} = 1$ supersymmetry and it only admits the BPS Wilson line along a null straight line. The situation is different in three dimensions. The β -deformed ABJM theory has $\mathcal{N} = 2$ supersymmetries so it admits both bosonic half-BPS Wilson loops [40] and fermionic half-BPS Wilson loops [31, 32] beyond the lightlike case. Other studies of the BPS Wilson loops in three-dimensional Chern-Simons-matter theories with $2 \leq \mathcal{N} \leq 4$ supersymmetries can be found in [41–45]. It was raised in [42] that some classically BPS fermionic Wilson loops may not be truly supersymmetric at the quantum level. Though there were efforts to resolve this issue either by perturbative calculation [46–48] or by studying the precise map between the BPS Wilson loops and the probe M2-brane solutions in dual M-theory backgrounds [43], it is fair to say that this big question is still to be answered. It should be interesting to study this issue in some simple theories with key properties left.

Few years ago, Kazakov et al. proposed a new double scaling limit on the γ -twisted $\mathcal{N} = 4$ SYM theory and ABJM theories [49, 50]. The novel feature in this limit is to take the twisting parameters q_i to zero (or infinity) and the 't Hooft coupling λ to zero but keep the ratio q_i/λ (or the product $q_i\lambda$) of the parameters and the coupling to be finite such that the resulting actions with the gauge field (and gluinos in the 4D case) being decoupled include only the complex scalars and complex fermions. Under this limit, the theories are not unitary and generically break supersymmetry. However, the resulting new theories are still integrable in the planar limit, presenting the remarkable features in the fishnet theories [51, 52]. Various aspects [53–66] of these so-called conformal fishnet theories have been studied in the past few years.

In this work, we would like to study the BPS Wilson loop in the three-dimensional (3D) $\mathcal{N} = 2$ fishnet theories. A special double-scaling limit of the γ -deformed ABJM theory leads to $\mathcal{N} = 2$ fishnet theory with only scalars and fermions [50]. We construct the fermionic BPS line and loop operators in this $\mathcal{N} = 2$ fishnet theory and compute the VEV of circular loop operators perturbatively after developing a new regularization scheme. Our computation suggests that one-loop and two-loop contributions to the VEV are vanishing. Furthermore we investigate the aforementioned line operators with a cusp, which are not supersymmetric anymore. We find that the universal cusp anomalous dimension vanishes at two loop order.

In the perturbative computations of the BPS Wilson loops in the Chern-Simons (-matter) theory, one important issue is on the effects of the framing. The perturbative computations should be performed at framing -1 to compare with the prediction from the localization. In this work, our regularization scheme at framing -1 has taken into account of the effects on the spinors in the definition of the fermionic Wilson loops appropriately. Such improvement could be useful in other settings.

The remaining parts of this paper are organized as follows: in section 2 and 3, we briefly review the three-dimensional $\mathcal{N} = 2$ fishnet theories and construct the BPS Wilson loop operators in them respectively. In section 4, we present our detailed perturbative computation on the VEVs of the BPS circular loops operators. In section 5, we discuss

the line operator with a cusp. We conclude this paper with some discussions in section 6. We collect some technical details into the appendices.

2 Three-dimensional $\mathcal{N} = 2$ fishnet theories

Integrable field theories in higher than two dimensions are very rare. In three dimensional spacetime, the ABJM theory [22] was known to be integrable in the planar limit [33–35]. This theory admits various marginal deformations such as β - and γ -deformations [67], like the four dimensional $\mathcal{N} = 4$ SYM theory [68, 69]. The β -/ γ -deformed or twisted theories have less/no supersymmetries while the integrable structures are preserved [70–72].¹ Other studies on the spin chain from three dimensional Chern-Simons-matter theories with less or no supersymmetries include the ones in [73–75].

In [49], a double scaling limit of the β - and γ -deformed theories was proposed and the resulting theories were called as the fishnet theories due to their fishnet-like Feynman diagrams. One remarkable feature of the fishnet theories is that gauge fields and possible some matter fields in original theories are decoupled so that the resulted theories are substantially simplified. The fishnet structures of the Feynman diagrams indicate the integrability of these higher dimensional theories [51]. In this section we will review the construction of a three-dimensional fishnet theory with $\mathcal{N} = 2$ supersymmetries.

2.1 ABJM theory and γ -deformation

We start with the ABJM theory whose Lagrangian is shown in the appendix A. The ABJM theory is a $\mathcal{N} = 6$ supersymmetric Chern-Simons-matter theory with gauge group $U(N) \times U(N)$ and the Chern-Simons levels k and $-k$, respectively. The global symmetry group of this theory is $OSp(6|4) * U(1)_b$. In particular, the bosonic part of the symmetry group includes the three dimensional conformal group $Sp(4) \sim SO(2, 3)$, the R-symmetry group $SU(4) \sim SO(6)$ and an extra $U(1)_b$. The gauge sector consists of two gauge fields A_μ and B_μ associated with the first and the second $U(N)$. The matter sector contains the four complex scalars ϕ_I ($I = 1, \dots, 4$) and four fermions ψ^I in the bifundamental $(N, \bar{N})$ representation.

We perform the γ -deformation through replacing the usual product of any two fields A and B with the $\star$ product defined as

$$A \star B = e^{\frac{i}{2} \mathbf{q}_A \wedge \mathbf{q}_B} AB, \quad (2.1)$$

where the antisymmetric product of the two charge vectors $\mathbf{q}_A$ and $\mathbf{q}_B$ is given by

$$\mathbf{q}_A \wedge \mathbf{q}_B = \mathbf{q}_A^T \mathbf{C} \mathbf{q}_B, \quad \mathbf{C} = \begin{pmatrix} 0 & -\gamma_3 & \gamma_2 \\ \gamma_3 & 0 & -\gamma_1 \\ -\gamma_2 & \gamma_1 & 0 \end{pmatrix}. \quad (2.2)$$

¹In [71], a non-standard γ -deformation was considered. There it was mentioned that this deformation degenerates to β -deformation. This statement is incorrect since these two deformations lead to different actions. Nevertheless, the planar two-loop anomalous dimension matrices in the scalar sector are the same for these two deformations, and the Hamiltonians on the alternating spin chains are the same.

The U(1) charges of the fields are given by the table below:

f	ϕ_1	ϕ_2	ϕ_3	ϕ_4	$\bar{\psi}_1$	$\bar{\psi}_2$	$\bar{\psi}_3$	$\bar{\psi}_4$
q_1	−	+	+	−	−	+	+	−
q_2	+	−	+	−	+	−	+	−
q_3	+	+	−	−	+	+	−	−

(2.3)

where $\pm \equiv \pm \frac{1}{2}$. (Note that the gauge fields A_μ and B_μ are neutral under these three U(1)'s.) This replacement should be performed on every product appearing in the Lagrangian. The γ -deformed ABJM action in our convention is presented in the appendix A.

2.2 Double scaling limit and fishnet theory

Starting from the γ -deformed ABJM theory, there are several ways of taking double scaling limits [50] to get a new theory with only scalars and fermions. The one preserves supersymmetries is the following:

$$\gamma_1 = \gamma_2 = 0, \quad e^{-i\gamma_3/2} \rightarrow \infty, \quad \lambda \equiv N/k \rightarrow 0, \quad \xi \equiv e^{-i\gamma_3} \lambda^2 \text{ fixed.} \quad (2.4)$$

By setting $\gamma_1 = \gamma_2 = 0$ without taking the double scaling limit, the resulting theory is the well known β -deformed theory with $\mathcal{N} = 2$ supersymmetry [67]. After taking the double scaling limit we find that the $\mathcal{N} = 2$ supersymmetry is preserved. Under this limit, the gauge sector is decoupled and the Lagrangian reads

$$\mathcal{L} = \mathcal{L}_k + \mathcal{L}_Y + \mathcal{L}_{\text{scalar}}, \quad (2.5)$$

where the kinetic term is

$$\mathcal{L}_k = \text{Tr}(-\partial_\mu \bar{\phi}^I \partial^\mu \phi_I + i \bar{\psi}_I \gamma^\mu \partial_\mu \psi^I), \quad (2.6)$$

the scalar potential term is

$$\begin{aligned} \mathcal{L}_{\text{scalar}} = 16\pi^2 \frac{\xi}{N^2} \text{Tr}(&\bar{\phi}^1 \phi_3 \bar{\phi}^2 \phi_1 \bar{\phi}^3 \phi_2 + \bar{\phi}^1 \phi_3 \bar{\phi}^4 \phi_1 \bar{\phi}^3 \phi_4 \\ &\bar{\phi}^1 \phi_2 \bar{\phi}^4 \phi_1 \bar{\phi}^2 \phi_4 + \bar{\phi}^2 \phi_4 \bar{\phi}^3 \phi_2 \bar{\phi}^4 \phi_3), \end{aligned} \quad (2.7)$$

and the Yukawa-like terms involving the interactions among the scalars and the fermions are

$$\begin{aligned} \mathcal{L}_Y = -2\pi i \frac{\sqrt{\xi}}{N} \text{Tr}(&-2\bar{\phi}^1 \phi_3 \bar{\psi}_1 \psi^3 - 2\bar{\phi}^2 \phi_4 \bar{\psi}_2 \psi^4 \\ &-2\bar{\phi}^3 \phi_2 \bar{\psi}_3 \psi^2 - 2\bar{\phi}^4 \phi_1 \bar{\psi}_4 \psi^1 + 2\phi_1 \bar{\phi}^3 \psi^1 \bar{\psi}_3 + 2\phi_2 \bar{\phi}^4 \psi^2 \bar{\psi}_4 \\ &+ 2\phi_3 \bar{\phi}^2 \psi^3 \bar{\psi}_2 + 2\phi_4 \bar{\phi}^1 \psi^4 \bar{\psi}_1 + 2\bar{\phi}^1 \psi^4 \bar{\phi}^2 \psi^3 - 2\bar{\phi}^3 \psi^1 \bar{\phi}^4 \psi^2 \\ &- 2\phi_1 \bar{\psi}_4 \phi_2 \bar{\psi}_3 + 2\phi_3 \bar{\psi}_1 \phi_4 \bar{\psi}_2). \end{aligned} \quad (2.8)$$

This fishnet theory is invariant under the $\mathcal{N} = 2$ supersymmetries² whose transformation rules are summarized as follows,

$$\begin{aligned}
 \delta\phi_1 &= i\bar{\epsilon}_{12}\psi^2, & \delta\phi_2 &= -i\bar{\epsilon}_{12}\psi^1, & \delta\phi_3 &= i\bar{\epsilon}_{34}\psi^4, & \delta\phi_4 &= -i\bar{\epsilon}_{34}\psi^3, \\
 \delta\bar{\phi}^1 &= i\bar{\psi}_2\epsilon^{12}, & \delta\bar{\phi}^2 &= -i\bar{\psi}_1\epsilon^{12}, & \delta\bar{\phi}^3 &= i\bar{\psi}_4\epsilon^{34}, & \delta\bar{\phi}^4 &= -i\bar{\psi}_3\epsilon^{34}, \\
 \delta\psi^1 &= \gamma^\mu\epsilon^{12}\partial_\mu\phi_2 + \vartheta^{12}\phi_2 + \delta_3\psi^1, & \delta\psi^2 &= -\gamma^\mu\epsilon^{12}\partial_\mu\phi_1 - \vartheta^{12}\phi_1 + \delta_3\psi^2, \\
 \delta\psi^3 &= \gamma^\mu\epsilon^{34}\partial_\mu\phi_4 + \vartheta^{34}\phi_4 + \delta_3\psi^3, & \delta\psi^4 &= -\gamma^\mu\epsilon^{34}\partial_\mu\phi_3 - \vartheta^{34}\phi_3 + \delta_3\psi^4, \\
 \delta\bar{\psi}_1 &= -\bar{\epsilon}_{12}\gamma^\mu\partial_\mu\bar{\phi}^2 + \bar{\vartheta}_{12}\bar{\phi}^2 + \delta_3\bar{\psi}_1, & \delta\bar{\psi}_2 &= +\bar{\epsilon}_{12}\gamma^\mu\partial_\mu\bar{\phi}^1 - \bar{\vartheta}_{12}\bar{\phi}^1 + \delta_3\bar{\psi}_2, \\
 \delta\bar{\psi}_3 &= -\bar{\epsilon}_{34}\gamma^\mu\partial_\mu\bar{\phi}^4 + \bar{\vartheta}_{34}\bar{\phi}^4 + \delta_3\bar{\psi}_3, & \delta\bar{\psi}_4 &= \bar{\epsilon}_{34}\gamma^\mu\partial_\mu\bar{\phi}^3 - \bar{\vartheta}_{34}\bar{\phi}^3 + \delta_3\bar{\psi}_4,
 \end{aligned} \tag{2.9}$$

where

$$\begin{aligned}
 \delta_3\psi^1 &= -4\pi\frac{\sqrt{\xi}}{N}\epsilon^{34}(\phi_4\bar{\phi}^1\phi_3), & \delta_3\psi^2 &= 4\pi\frac{\sqrt{\xi}}{N}\epsilon^{34}(\phi_3\bar{\phi}^2\phi_4), \\
 \delta_3\psi^3 &= 4\pi\frac{\sqrt{\xi}}{N}\epsilon^{12}(\phi_1\bar{\phi}^3\phi_2), & \delta_3\psi^4 &= -4\pi\frac{\sqrt{\xi}}{N}\epsilon^{12}(\phi_2\bar{\phi}^4\phi_1), \\
 \delta_3\bar{\psi}_1 &= 4\pi\frac{\sqrt{\xi}}{N}\bar{\epsilon}_{34}(\bar{\phi}^4\phi_1\bar{\phi}^3), & \delta_3\bar{\psi}_2 &= -4\pi\frac{\sqrt{\xi}}{N}\bar{\epsilon}_{34}(\bar{\phi}^3\phi_2\bar{\phi}^4), \\
 \delta_3\bar{\psi}_3 &= -4\pi\frac{\sqrt{\xi}}{N}\bar{\epsilon}_{12}(\bar{\phi}^1\phi_3\bar{\phi}^2), & \delta_3\bar{\psi}_4 &= 4\pi\frac{\sqrt{\xi}}{N}\bar{\epsilon}_{12}(\bar{\phi}^2\phi_4\bar{\phi}^1).
 \end{aligned} \tag{2.10}$$

The supersymmetry parameters are $\epsilon^{IJ} = \theta^{IJ} + x^\mu\gamma_\mu\vartheta^{IJ}$ and $\bar{\epsilon}_{IJ} = \bar{\theta}_{IJ} - \bar{\vartheta}_{IJ}x^\mu\gamma_\mu$, (here IJ only take the values among 12, 21, 34, 43) with the constraints

$$\begin{aligned}
 \theta^{IJ} &= -\theta^{JI}, & \bar{\theta}_{IJ} &= -\bar{\theta}_{JI}, & (\theta^{IJ})^* &= \bar{\theta}_{IJ}, & \bar{\theta}_{12} &= \theta^{34}, & \bar{\theta}_{34} &= \theta^{12}, \\
 \vartheta^{IJ} &= -\vartheta^{JI}, & \bar{\vartheta}_{IJ} &= -\bar{\vartheta}_{JI}, & (\vartheta^{IJ})^* &= \bar{\vartheta}_{IJ}, & \bar{\vartheta}_{12} &= \vartheta^{34}, & \bar{\vartheta}_{34} &= \vartheta^{12}.
 \end{aligned} \tag{2.11}$$

In the Euclidean space, the constraints on the supersymmetry parameters are relaxed to

$$\begin{aligned}
 \theta^{IJ} &= -\theta^{JI}, & \bar{\theta}_{IJ} &= -\bar{\theta}_{JI}, & \bar{\theta}_{12} &= \theta^{34}, & \bar{\theta}_{34} &= \theta^{12}, \\
 \vartheta^{IJ} &= -\vartheta^{JI}, & \bar{\vartheta}_{IJ} &= -\bar{\vartheta}_{JI}, & \bar{\vartheta}_{12} &= \vartheta^{34}, & \bar{\vartheta}_{34} &= \vartheta^{12}.
 \end{aligned} \tag{2.12}$$

The Lagrangian of this fishnet theory has been derived in [50] as a particular example of the plethora of three dimensional fishnet theories. Even though this fishnet theory is not the simplest one which contains the least fields, the residue supersymmetries provide us another handle to study three dimensional integrable theory. In this note we focus on the supersymmetric (BPS) non-local operators such as the BPS line and loop operators in this fishnet theory.

The above discussions can be also applied to the ABJ theory [76] in which the only difference from the ABJM theory is that the gauge group is now chosen to be $U(N_1) \times U(N_2)$. Here we focus on the case where N_1 and N_2 are of the same order. Then the γ -deformation is performed as before. The fishnet limit preserving $\mathcal{N} = 2$ supersymmetries now becomes,

$$\begin{aligned}
 \gamma_1 &= \gamma_2 = 0, & e^{-i\gamma_3/2} &\rightarrow \infty, & \lambda_i &\equiv N_i/k \rightarrow 0, i = 1, 2, \\
 \xi_{(i)} &\equiv e^{-i\gamma_3}\lambda_i^2 & \text{being fixed, } & i = 1, 2.
 \end{aligned} \tag{2.13}$$

²We loosely call both Poincaré supersymmetry and superconformal symmetry as supersymmetry.

The above discussion on the ABJM theory can be carried through straightforwardly with $\frac{\sqrt{\xi}}{N}$ in eqs. (2.8) and (2.10) being replaced by $\frac{\xi^{1/4}\xi^{1/4}}{\sqrt{N_1N_2}}$ and $\frac{\xi}{N^2}$ in eq. (2.7) being replaced by $\frac{\sqrt{\xi_{(1)}\xi_{(2)}}}{N_1N_2}$.

3 BPS line/loop operators

The three-dimensional $\mathcal{N} = 2$ fishnet theory obtained from the ABJ(M) theory can be used as a prototype to study various problems in general super-Chern-Simons-matter theories. We start the investigation with the construction of the BPS line/loop operators in the fishnet theory. As in $\mathcal{N} \geq 2$ Chern-Simons-matter theories, the construction of the fermionic loop operators starts with a superconnection L which combines the bosonic and fermionic field contents together. The supersymmetric (BPS) condition is given by requiring that, under the preserved supercharges, the supersymmetry transformation of the superconnection can be written as

$$\delta L = \partial_\tau G + i[L, G] \quad (3.1)$$

for some G [26, 77]. The BPS conditions are relaxed comparing with $\delta L = 0$ for the bosonic loop operators. In the following, we still use W to denote the loop operators and reserve L for the superconnection.

3.1 Line operators along timelike straight line

Here and in the following, we will use the spinor conventions in [45]. For the Minkowski spacetime with the coordinates $x^\mu = (x^0, x^1, x^2)$ and the metric $\eta_{\mu\nu} = \text{diag}(-1, 1, 1)$, the γ matrices are chosen to be $\gamma_\alpha^\mu{}^\beta = (i\sigma^2, \sigma^1, \sigma^3)$. Following the discussions in [45], we can construct BPS line operator along the timelike straight line $x^\mu(\tau) = (\tau, 0, 0)$. The line operator, which is at least classically half-BPS, is³

$$W_{\text{line}} = \text{Tr} \left(\mathcal{P} \exp \left(-i \int d\tau L_{\text{line}}(\tau) \right) \right), \quad (3.2)$$

where

$$L_{\text{line}} = \begin{pmatrix} U^I_J \phi_I \bar{\phi}^J & \bar{\alpha}_I \psi^I_+ + \bar{\gamma}_I \psi^I_- \\ \bar{\psi}_{I-} \beta^I - \bar{\psi}_{I+} \delta^I & U^I_J \bar{\phi}^J \phi_I \end{pmatrix}, \quad (3.3)$$

with

$$U^I_J = \begin{pmatrix} \beta^2 \bar{\alpha}_2 & -\beta^1 \bar{\alpha}_2 \\ -\beta^2 \bar{\alpha}_1 & \beta^1 \bar{\alpha}_1 \\ & & -\delta^4 \bar{\gamma}_4 & \delta^3 \bar{\gamma}_4 \\ & & \delta^4 \bar{\gamma}_3 & -\delta^3 \bar{\gamma}_3 \end{pmatrix}, \quad (3.4)$$

$$\bar{\alpha}_I = (\bar{\alpha}_1, \bar{\alpha}_2, 0, 0), \quad \beta^I = (\beta^1, \beta^2, 0, 0), \quad \bar{\gamma}_I = (0, 0, \bar{\gamma}_3, \bar{\gamma}_4), \quad \delta^I = (0, 0, \delta^3, \delta^4). \quad (3.5)$$

³In this paper, we only consider taking trace in the fundamental representation of $U(N_1|N_2)$.

As in [45], we define the following bosonic spinors

$$u_{\pm\alpha} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \mp i \end{pmatrix}, \quad u_{\pm}^{\alpha} = \frac{1}{\sqrt{2}} (\mp i, -1), \quad (3.6)$$

and decompose a spinor as

$$\theta_{\alpha} = u_{+\alpha} \theta_{-} + u_{-\alpha} \theta_{+}. \quad (3.7)$$

For the above line operators to be BPS, the following constraints

$$\bar{\alpha}_{1,2} \delta^{3,4} = \bar{\gamma}_{3,4} \beta^{1,2} = 0, \quad (3.8)$$

should be satisfied. Then the corresponding preserved supercharges are $\theta_{+}^{12}, \theta_{-}^{34}, \vartheta_{+}^{12}, \vartheta_{-}^{34}$. There are two classes of solutions to the BPS constraints in eq. (3.8)

$$\text{Class I:} \quad \bar{\gamma}^I = \delta_I = 0, \quad (3.9)$$

$$\text{Class II:} \quad \bar{\gamma}^I = \beta_I = 0, \quad (3.10)$$

leading to nontrivial BPS line operators.

3.2 Circular loop operators

To study the circular loop operator, we put the theory on the Euclidean space⁴ with the coordinates $x^{\mu} = (x^1, x^2, x^3)$, the metric $\delta_{\mu\nu} = \text{diag}(1, 1, 1)$ and the γ matrices $\gamma_{\alpha}^{\mu\beta} = (-\sigma^2, \sigma^1, \sigma^3)$. We search for the BPS loop operators along the circle $x^{\mu}(\tau) = (\cos \tau, \sin \tau, 0)$.

The resulting operator, which is at least classically half-BPS, is,

$$W_{\text{cir.}} = \text{Tr} \left(\mathcal{P} \exp \left(-i \oint d\tau L_{\text{cir.}}(\tau) \right) \right), \quad (3.11)$$

where

$$L_{\text{cir.}} = \begin{pmatrix} -i U_J^I \phi_I \bar{\phi}^J & \bar{\alpha}_I \psi_+^I + \bar{\gamma}_I \psi_-^I \\ \bar{\psi}_{I+} \delta^I - \bar{\psi}_{I-} \beta^I & -i U_J^I \bar{\phi}^J \phi_I \end{pmatrix}, \quad (3.12)$$

with the same constants $U_J^I, \bar{\alpha}_I, \beta^I, \bar{\gamma}_I, \delta^I$ and the BPS constraints as the ones in the previous subsection. Then the corresponding preserved supercharges are the ones satisfying

$$\vartheta_{12} = -i \gamma_3 \theta_{12}, \quad \vartheta_{34} = i \gamma_3 \theta_{34}. \quad (3.13)$$

Notice that now the definition of $u_{\pm}$ is changed to

$$u_{+\alpha} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-\frac{i\tau}{2}} \\ e^{\frac{i\tau}{2}} \end{pmatrix}, \quad u_{-\alpha} = \frac{i}{\sqrt{2}} \begin{pmatrix} -e^{-\frac{i\tau}{2}} \\ e^{\frac{i\tau}{2}} \end{pmatrix}, \quad (3.14)$$

$$u_{+}^{\alpha} = \frac{1}{\sqrt{2}} \left(e^{\frac{i\tau}{2}}, -e^{-\frac{i\tau}{2}} \right), \quad u_{-}^{\alpha} = \frac{i}{\sqrt{2}} \left(e^{\frac{i\tau}{2}}, e^{-\frac{i\tau}{2}} \right). \quad (3.15)$$

⁴To construct BPS circular loop operators, the theory should be put in the Euclidean space [78]. This point was not taken into account in [24].

And for the circular loop operator, we use the decomposition of the spinor θ_α (3.7) with these $u_\pm$.

Based on the discussions in [26, 41, 45], we can show that classically

$$W_{\text{cir.}} = (N_1 + N_2) \quad (3.16)$$

is Q -exact, where the supercharge Q can be used to perform supersymmetric localization and N_1, N_2 are the ranks of the gauge groups. If this relation is preserved at quantum level, we will have

$$\langle W_{\text{cir.}} \rangle = N_1 + N_2, \quad (3.17)$$

exactly.

3.3 $\mathcal{N} = 2$ notations

Later on we will compute the expectation values of the circular and cusped loop/line operators. For the perturbative computations, it is convenient to switch from the ABJM notation to the $\mathcal{N} = 2$ notation [45],

$$\phi_I = (Z^1, Z^2, \bar{Z}_3, \bar{Z}_4), \quad \psi^I = (-\zeta^2, \zeta^1, -\bar{\zeta}_4, \bar{\zeta}_3). \quad (3.18)$$

As for the N_{ab} 's, we have

$$N_{12} = N_{21} = 2, \quad N_{11} = N_{22} = 0. \quad (3.19)$$

In the $\mathcal{N} = 2$ notations, the line operators are (3.2) with

$$\begin{aligned} L_{\text{line}} &= B + F, & B &= \bar{M}_Z N_{\bar{Z}} + N_{\bar{Z}} \bar{M}_Z, & F &= \bar{M}_\zeta + N_{\bar{\zeta}}, \\ [\bar{M}_Z]_{(ab)} &= \bar{m}_i^{ab} Z_{(ab)}^i, & [N_{\bar{Z}}]_{(ab)} &= n_{ab}^i \bar{Z}_i^{(ab)}, \\ [\bar{M}_\zeta]_{(ab)} &= \bar{m}_i^{ab} \zeta_{(ab)+}^i, & [N_{\bar{\zeta}}]_{(ab)} &= n_{ab}^i \bar{\zeta}_{i-}^{(ab)}. \end{aligned} \quad (3.20)$$

Note that $\zeta_{(ab)+}^i = i u_+ \zeta_{(ab)}^i$, $\bar{\zeta}_{i-}^{(ab)} = i \bar{\zeta}_i^{(ab)} u_-$ with $u_\pm$ given in eq. (3.6).

The circular loop operators now in the $\mathcal{N} = 2$ notations read (3.11) with

$$\begin{aligned} L_{\text{cir.}} &= B + F, & B &= -i(\bar{M}_Z N_{\bar{Z}} + N_{\bar{Z}} \bar{M}_Z), & F &= \bar{M}_\zeta - N_{\bar{\zeta}}, \\ [\bar{M}_Z]_{(ab)} &= \bar{m}_i^{ab} Z_{(ab)}^i, & [N_{\bar{Z}}]_{(ab)} &= n_{ab}^i \bar{Z}_i^{(ab)}, \\ [\bar{M}_\zeta]_{(ab)} &= \bar{m}_i^{ab} \zeta_{(ab)+}^i, & [N_{\bar{\zeta}}]_{(ab)} &= n_{ab}^i \bar{\zeta}_{i-}^{(ab)}. \end{aligned} \quad (3.21)$$

Note that $\zeta_{(ab)+}^i = i u_+ \zeta_{(ab)}^i$, $\bar{\zeta}_{i-}^{(ab)} = i \bar{\zeta}_i^{(ab)} u_-$ with $u_\pm$ in eq. (3.14).

For both the line and circular loop operators, we have the constraints that

$$\bar{m}_i^{ac} \bar{m}_j^{cb} = n_{ac}^i n_{cb}^j = 0, \quad (3.22)$$

with no summations over c in the above equation. There are two classes of solutions of such constraints,

$$\begin{aligned} \text{Class I:} & \quad \bar{m}_i^{21} = n_{12}^i = 0, \\ \text{Class II:} & \quad \bar{m}_i^{12} = n_{21}^i = 0, \end{aligned} \quad (3.23)$$

which lead to nontrivial line/loop operators.

We can easily switch between the ABJM notations and the $\mathcal{N} = 2$ notations by just finding possibly nonzero components⁵ of $\bar{m}_i^{ab}, n_{ab}^i$. For the line operators, they are

$$\begin{aligned} \bar{m}_1^{12} &= \bar{\alpha}_2, & \bar{m}_2^{12} &= -\bar{\alpha}_1, & \bar{m}_3^{21} &= -\delta^4, & \bar{m}_4^{21} &= \delta^3, \\ n_{21}^1 &= \beta^2, & n_{21}^2 &= -\beta^1, & n_{12}^3 &= \bar{\gamma}_4, & n_{12}^4 &= -\bar{\gamma}_3, \end{aligned} \quad (3.24)$$

And for the circular loop operators, they are

$$\begin{aligned} \bar{m}_1^{12} &= \bar{\alpha}_2, & \bar{m}_2^{12} &= -\bar{\alpha}_1, & \bar{m}_3^{21} &= \delta^4, & \bar{m}_4^{21} &= -\delta^3, \\ n_{21}^1 &= \beta^2, & n_{21}^2 &= -\beta^1, & n_{12}^3 &= -\bar{\gamma}_4, & n_{12}^4 &= \bar{\gamma}_3. \end{aligned} \quad (3.25)$$

It is then easy to see that the solutions to the BPS constraints in these two notations map to each other.

4 Perturbative computations

In this section we will evaluate the vacuum expectation value (VEV) of the circular loop operator constructed in the last section up to two-loop level. As we have discussed, the difference between this operator and $N_1 + N_2$ are Q -exact classically. If this relation is still valid at the quantum level, the vacuum expectation value of this operator will be just $N_1 + N_2$, independent of the coupling constants and the parameters in the definition of the loop operators. It will be of great interest to compute this VEV perturbatively to probe whether the above statement about Q -exactness is valid at the quantum level or not.

Let us first recall some issues arising from the perturbative computations of the BPS Wilson loops in the Chern-Simons-matter theory. When one tries to compare the perturbative data with the prediction from supersymmetric localization, a subtle issue of framing dependence rises. The framing dependence of pure Chern-Simons theory has been clearly addressed in [79, 80]. In the perturbation theory, it comes from the point-splitting regularization and there it is just a phase factor determined by the linking number of the Wilson loop contour and the auxiliary contour used for the point-splitting. For the BPS Wilson loops in the super-Chern-Simons theories, the result from the supersymmetric localization is at framing -1 [27]. The reason is that the localization is performed by putting the theory on the round S^3 and the Wilson loop contour is along a Hopf fiber. The auxiliary contours used for the point-splitting should be put on nearby Hopf fibers. After doing stereographic projection to $\mathbb{R}^3$, the link number between the Wilson loop contour and any such auxiliary contour is -1 [27]. This fact makes the comparison between the results from the localization and the ones from the perturbative computations complicated since the perturbative calculation is usually done at framing 0 . So we need to modify the perturbative result by suitably including the effects of framing -1 which can be done at lower loop order, or we need directly perform the perturbative calculations at framing -1 . In the Chern-Simons-matter theories, the framing dependence is more complicated due to the facts that the framing factors arising from the non-trivial quantum corrections to the vector

⁵The index i in $\bar{m}_i^{21}$ and n_{12}^i only takes the value 3 or 4, in order to be consistent with eq. (3.18).

propagator may obtain the contributions from the diagrams with vertices as well [47]. The perturbative calculations at such supersymmetric framing for the fermionic BPS Wilson loops were performed in [45] based on the same regularization scheme for the bosonic BPS Wilson loops.

Notice that the definition of the fermionic BPS Wilson loop involves certain constant bosonic spinors. These spinors are the solutions to the Killing spinor equations determined by the Wilson loop contour. When we perform the point-splitting regularization, we move some points from the Wilson loop to the auxiliary contours as mentioned above. It is reasonable to modify the regularization scheme by demanding that the spinors corresponding to these points should also be changed to the solutions of the Killing spinor equations on these contours. This is the key of our proposal for a new regularization scheme for the computations of the fermionic BPS Wilson loops.

In the following subsection, we will first give the details of our new regularization scheme and later we will use this scheme to compute the VEV of the circular loop operator in the three-dimensional $\mathcal{N} = 2$ fishnet theory.

4.1 Regularization scheme for fermionic loop operators at framing -1

The supersymmetrical localization is first performed on a round S^3 [27]. We parametrize this round $S^3 = \{X^i \in \mathbb{R}^4 | X^i X_i = 1\}$ as

$$X^i = (\cos \eta \cos(\tau - \phi), \cos \eta \sin(\tau - \phi), \sin \eta \sin(\tau + \phi), \sin \eta \cos(\tau + \phi)). \quad (4.1)$$

Here (ϕ, η) parametrize a S^2 and for each fixed pair (ϕ, η) , the τ -circle is the Hopf fiber. We use the following stereographic projection

$$x^\mu(X^i) = \left(\frac{X^1}{1 - X^4}, \frac{X^2}{1 - X^4}, \frac{X^3}{1 - X^4} \right), \quad (4.2)$$

to map $S^3 \setminus \{(0, 0, 0, 1)\}$ to $\mathbb{R}^3$. This gives the following parametrization for $\mathbb{R}^3$,

$$x^\mu = \left(\frac{\cos \eta \cos(\tau - \phi)}{1 - \sin \eta \cos(\tau + \phi)}, \frac{\cos \eta \sin(\tau - \phi)}{1 - \sin \eta \cos(\tau + \phi)}, \frac{\sin \eta \sin(\tau + \phi)}{1 - \sin \eta \cos(\tau + \phi)} \right). \quad (4.3)$$

Obviously the τ -circle with $\eta = \phi = 0$ gives the Wilson loop contour

$$x_{\text{WL}}^\mu(\tau) = (\cos \tau, \sin \tau, 0). \quad (4.4)$$

For the auxiliary contour, we can choose $\phi = 0, \eta \rightarrow 0$ and keep the terms up to the linear order of η . The result is

$$x_\eta^\mu(\tau) = (\cos \tau, \sin \tau, 0) + \eta(\cos^2 \tau, \cos \tau \sin \tau, \sin \tau), \quad (4.5)$$

which is just the auxiliary contour used in [47]. The BPS conditions for the spinors $u_\pm$ along the auxiliary contour are [45]

$$\gamma_\mu \dot{x}_\eta^\mu u_{\eta\pm} = \pm |\dot{x}_\eta| u_{\eta\pm}, \quad u_{\eta+} u_{\eta-} = -i, \quad u_{\eta\pm} \partial_\tau u_{\eta\mp} = 0. \quad (4.6)$$

The first equation is a Killing spinor equation along the auxiliary contour. Demanding that when $\eta \rightarrow 0$ these spinors go back to the spinors in eq. (3.14), we get that

$$u_\eta(\tau)_{+\alpha} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-\frac{i\tau}{2}} (1 - \frac{i}{2}\eta \sin \tau) + \frac{\eta}{2} e^{\frac{i\tau}{2}} \\ e^{\frac{i\tau}{2}} (1 + \frac{i}{2}\eta \sin \tau) - \frac{\eta}{2} e^{-\frac{i\tau}{2}} \end{pmatrix} + \mathcal{O}(\eta^2), \quad (4.7)$$

$$u_\eta(\tau)_{-\alpha} = \frac{i}{\sqrt{2}} \begin{pmatrix} -e^{-\frac{i\tau}{2}} (1 - \frac{i}{2}\eta \sin \tau) + \frac{\eta}{2} e^{\frac{i\tau}{2}} \\ e^{\frac{i\tau}{2}} (1 + \frac{i}{2}\eta \sin \tau) + \frac{\eta}{2} e^{-\frac{i\tau}{2}} \end{pmatrix} + \mathcal{O}(\eta^2). \quad (4.8)$$

These spinors can be also obtained without directly solving the BPS conditions. Notice that the auxiliary contour $x_\eta^\mu(\tau)$ can be got from the Wilson loop contour x_{WL}^μ by an affine transformation

$$\tilde{x}^\mu(\tau) = \Lambda_\nu^\mu x_{\text{WL}}^\nu(\tau) + a^\mu, \quad (4.9)$$

followed by a reparametrization of $\tau = \tau(\tau')$

$$x_\eta^\mu(\tau') = \tilde{x}^\mu(\tau(\tau')) + \mathcal{O}(\eta^2), \quad (4.10)$$

where

$$\Lambda_\nu^\mu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \eta & -\sin \eta \\ 0 & \sin \eta & \cos \eta \end{pmatrix}, \quad (4.11)$$

is a rotation matrix,

$$a^\mu = (\eta, 0, 0), \quad (4.12)$$

and

$$\tau(\tau') = \tau' + \eta \sin \tau' + \mathcal{O}(\eta^2). \quad (4.13)$$

From

$$u_\eta(\tau')_\pm = \exp\left(-\frac{i}{2}\eta\gamma_1\right) u(\tau(\tau'))_\pm, \quad (4.14)$$

with $u(\tau)_\pm$ being the ones given in eq. (3.14) and by keeping only the terms up to the linear order of η , we find

$$u_\eta(\tau')_{+\alpha} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-\frac{i\tau'}{2}} (1 - \frac{i}{2}\eta \sin \tau') + \frac{\eta}{2} e^{\frac{i\tau'}{2}} \\ e^{\frac{i\tau'}{2}} (1 + \frac{i}{2}\eta \sin \tau') - \frac{\eta}{2} e^{-\frac{i\tau'}{2}} \end{pmatrix} + \mathcal{O}(\eta^2), \quad (4.15)$$

$$u_\eta(\tau')_{-\alpha} = \frac{i}{\sqrt{2}} \begin{pmatrix} -e^{-\frac{i\tau'}{2}} (1 - \frac{i}{2}\eta \sin \tau') + \frac{\eta}{2} e^{\frac{i\tau'}{2}} \\ e^{\frac{i\tau'}{2}} (1 + \frac{i}{2}\eta \sin \tau') + \frac{\eta}{2} e^{-\frac{i\tau'}{2}} \end{pmatrix} + \mathcal{O}(\eta^2), \quad (4.16)$$

as claimed in eq. (4.8). This procedure can be performed for a finite η with a finite ϕ as well, but we would not give the details here.

Now, consider an unregularized integral from the computations of VEV of the loop operator,

$$\oint d\tau_{1>\dots>n} I(x(\tau_m), u(\tau_m)), \quad (4.17)$$

Figure 1. One-loop Feynman diagram.

where $x(\tau_m) = (\cos \tau_m, \sin \tau_m, 0)$ and $u(\tau_m)$ being given in eq. (3.14), and in this section $\oint d\tau_{1>\dots>n}$ means

$$\int_{2\pi > \tau_1 > \dots > \tau_n > 0} \prod_{i=1}^n d\tau_i. \quad (4.18)$$

To regularize it, we replace $x(\tau_m), u(\tau_m)$ with

$$x_m(\tau_m) = (\cos \tau_m + (m-1)\delta \cos^2 \tau_m, \sin \tau_m + (m-1)\delta \sin \tau_m \cos \tau_m, (m-1)\delta \sin \tau_m), \quad (4.19)$$

and

$$u_m(\tau_m)_{+\alpha} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-\frac{i\tau_m}{2}} (1 - \frac{i}{2}(m-1)\delta \sin \tau_m) + \frac{1}{2}(m-1)\delta e^{\frac{i\tau_m}{2}} \\ e^{\frac{i\tau_m}{2}} (1 + \frac{i}{2}(m-1)\delta \sin \tau_m) - \frac{1}{2}(m-1)\delta e^{-\frac{i\tau_m}{2}} \end{pmatrix}, \quad (4.20)$$

$$u_m(\tau_m)_{-\alpha} = \frac{i}{\sqrt{2}} \begin{pmatrix} -e^{-\frac{i\tau_m}{2}} (1 - \frac{i}{2}(m-1)\delta \sin \tau_m) + \frac{1}{2}(m-1)\delta e^{\frac{i\tau_m}{2}} \\ e^{\frac{i\tau_m}{2}} (1 + \frac{i}{2}(m-1)\delta \sin \tau_m) + \frac{1}{2}(m-1)\delta e^{-\frac{i\tau_m}{2}} \end{pmatrix}, \quad (4.21)$$

with δ the regularization parameter.⁶ This is our proposal of the regularization scheme for the circular BPS loop operators at framing -1 .

4.2 The vacuum expectation values of circular loop operators

In the perturbative computations, we assume⁷ $\bar{m}_i^{ab}$ and $n_{(ab)}^i$ to be of order $\mathcal{O}(\xi^{1/4}/\sqrt{N_1})$. We make this choice partly because this is similar to what happens for the half-BPS Wilson loops in the ABJM theory. Then up to two-loop order, there are no contributions from the diagrams with vertices. All the relevant diagrams have appeared in the previous computations [23–25, 45, 81–83], but here for the diagrams with fermionic propagators we would apply our new regularization scheme. The only contributing one-loop diagram is from exchanging one fermions (figure 1),

$$\begin{aligned} W_1 &= \sum_{a,b} \bar{m}_i^{ab} n_{ba}^i N_a N_b \mathcal{I}_1^{(-1)}, \\ &= \sum_{a \neq b} \bar{m}_i^{ab} n_{ba}^i \mathcal{I}_1^{(-1)}. \end{aligned} \quad (4.22)$$

⁶It is just the η in the previous discussions.

⁷Recall that we always assume that N_1 and N_2 are at the same order.

Figure 2. Two-loop Feynman diagrams.

with the integral being⁸

$$\mathcal{I}_1^{(-1)} = -i \frac{\Gamma(\frac{3}{2}-\epsilon)}{2\pi^{\frac{3}{2}-\epsilon}} \oint d\tau_{1>2} \left(\frac{|\dot{x}_1(\tau_1)| |\dot{x}_2(\tau_2)| u_1(\tau_1) + \gamma_\mu u_2(\tau_2) - (x_1^\mu(\tau_1) - x_2^\mu(\tau_2))}{|x_1(\tau_1) - x_2(\tau_2)|^{3-2\epsilon}} - (1 \leftrightarrow 2) \right). \quad (4.23)$$

There are numerical evidences that the results for the integral in the above equation is the same as the one from the dimensional regularization with dimensional reduction at framing 0 when $\epsilon > 1/2$. We take the continuation of the later to $\epsilon = 0$ as the results in our new supersymmetric regularization, then we are led to the claim that

$$\mathcal{I}_1^{(-1)} = 0. \quad (4.24)$$

One part of the two-loop contributions comes from the diagrams with two scalar propagators (figure 2a), which give

$$W_{2;1} = \sum_{a,b,c} (\bar{m}_i^{ab} n_{ba}^i \bar{m}_j^{ac} n_{ca}^j + \bar{m}_i^{ba} n_{ab}^i \bar{m}_j^{ca} n_{ac}^j) N_a N_b N_c \mathcal{I}_{2a}^{(-1)} \quad (4.25)$$

$$= N_1 N_2 (N_1 + N_2) \sum_{a \neq b} (\bar{m}_i^{ab} n_{ba}^i)^2 \mathcal{I}_{2a}^{(-1)}. \quad (4.26)$$

with

$$\mathcal{I}_{2a}^{(-1)} = \frac{\Gamma^2(\frac{1}{2}-\epsilon)}{16\pi^{3-2\epsilon}} \oint d\tau_{1>2} \frac{|\dot{x}_1(\tau_1)| |\dot{x}_2(\tau_2)|}{|x_1(\tau_1) - x_2(\tau_2)|^{2-4\epsilon}}. \quad (4.27)$$

It can be shown that it vanishes at any framing, not only at framing -1 .

The other part coming from the diagrams with two fermion propagators (figure 2b) gives

$$W_{2;2} = \sum_{a,b,c} N_a N_b N_c (\bar{m}_i^{ab} n_{ba}^i \bar{m}_j^{ac} n_{ca}^j \mathcal{I}_{2b}^{(-1)} + \bar{m}_i^{ba} n_{ab}^i \bar{m}_j^{ca} n_{ac}^j \mathcal{J}_{2b}^{(-1)}) \quad (4.28)$$

$$= N_1 N_2 (N_1 + N_2) \sum_{a \neq b} (\bar{m}_i^{ab} n_{ba}^i)^2 (\mathcal{I}_{2b}^{(-1)} + \mathcal{J}_{2b}^{(-1)}), \quad (4.29)$$

⁸The factor $|\dot{x}_i(\tau_i)|$ is introduced to make the integral be reparametrization invariant of τ_i . This could be understood as part of the regularization scheme at framing -1 . They do not appear in the construction of the loop operators since we have chosen τ such that we always have $|\dot{x}_i| = 1$ there.

where

$$\mathcal{I}_{2b}^{(-1)} = -\frac{\Gamma^2\left(\frac{3}{2}-\epsilon\right)}{4\pi^{3-2\epsilon}} \oint d\tau_1 > 2 > 3 > 4 \left(\frac{|\dot{x}_1(\tau_1)| |\dot{x}_2(\tau_2)| u_1(\tau_1) + \gamma_\mu u_2(\tau_2) - (x_1^\mu(\tau_1) - x_2^\mu(\tau_2))}{|x_1(\tau_1) - x_2(\tau_2)|^{3-2\epsilon}} \right. \\ \left. \times \frac{|\dot{x}_3(\tau_3)| |\dot{x}_4(\tau_4)| u_3(\tau_3) + \gamma_\mu u_4(\tau_4) - (x_3^\mu(\tau_3) - x_4^\mu(\tau_4))}{|x_3(\tau_3) - x_4(\tau_4)|^{3-2\epsilon}} - (1234 \rightarrow 2341) \right), \quad (4.30)$$

$$\mathcal{J}_{2b}^{(-1)} = -\frac{\Gamma^2\left(\frac{3}{2}-\epsilon\right)}{4\pi^{3-2\epsilon}} \oint d\tau_1 > 2 > 3 > 4 \left(\frac{|\dot{x}_1(\tau_1)| |\dot{x}_2(\tau_2)| u_2(\tau_2) + \gamma_\mu u_1(\tau_1) - (x_2^\mu(\tau_2) - x_1^\mu(\tau_1))}{|x_1(\tau_1) - x_2(\tau_2)|^{3-2\epsilon}} \right. \\ \left. \times \frac{|\dot{x}_3(\tau_3)| |\dot{x}_4(\tau_4)| u_4(\tau_4) + \gamma_\mu u_3(\tau_3) - (x_4^\mu(\tau_4) - x_3^\mu(\tau_3))}{|x_3(\tau_3) - x_4(\tau_4)|^{3-2\epsilon}} - (1234 \rightarrow 2341) \right). \quad (4.31)$$

It is easy to get that

$$\mathcal{I}_{2b}^{(-1)} = \mathcal{J}_{2b}^{(-1)}. \quad (4.32)$$

We expect them to be zero, but we have only weak numerical evidence for this. If it is the case, we have

$$\langle W \rangle = N_1 + N_2, \quad (4.33)$$

up to two-loop order.

5 Line operator with a cusp

The cusp anomalous dimension plays an important role in gauge theories. It is related to many important quantities like the ultraviolet divergences of the cusped Wilson loops, the infrared divergences of the gluon amplitudes and the anomalous dimension of the twist-two operators. The generalized Wilson loops in ABJ(M) have been constructed in [84] and their VEV's have been computed there up to two loops.⁹ Here we construct the cusped line operators in three-dimensional $\mathcal{N} = 2$ fishnet theory and compute their VEV's.

5.1 The construction

After doing Wick rotation to the Euclidean space, the BPS line in the $\mathcal{N} = 2$ notations becomes (3.2) and (3.20) with the replacement

$$L_{\text{line}} = iB + F. \quad (5.1)$$

We consider a cusp which is parametrized by

$$x^1 = \tau \cos \phi, \quad x^2 = |\tau| \sin \phi, \quad x^3 = 0, \quad -L \leq \tau \leq L, \quad 0 \leq \phi < \frac{\pi}{2}. \quad (5.2)$$

The constant spinors in the definition of the cusped line operator is given by applying the proper rotation

$$S_R = e^{-\phi\gamma_2/2} = \begin{pmatrix} e^{-\frac{i\phi}{2}} & 0 \\ 0 & e^{\frac{i\phi}{2}} \end{pmatrix}, \quad S_L = e^{\phi\gamma_2/2} = \begin{pmatrix} e^{\frac{i\phi}{2}} & 0 \\ 0 & e^{-\frac{i\phi}{2}} \end{pmatrix} \quad (5.3)$$

⁹Studies on related Bremsstrahlung functions can be found in [85–88].

Figure 3. One-loop Feynman diagrams for cusp.

to the constant spinors $u_{\pm\alpha}$ in eq. (3.6). The results are

$$\text{Right-half :} \quad u_{+\alpha,R} = \frac{1}{\sqrt{2}} \begin{pmatrix} s_- \\ -is_+ \end{pmatrix}, \quad u_{-\alpha,R} = \frac{1}{\sqrt{2}} \begin{pmatrix} s_- \\ is_+ \end{pmatrix}, \quad (5.4)$$

$$\text{Left-half :} \quad u_{+\alpha,L} = \frac{1}{\sqrt{2}} \begin{pmatrix} s_+ \\ -is_- \end{pmatrix}, \quad u_{-\alpha,L} = \frac{1}{\sqrt{2}} \begin{pmatrix} s_+ \\ is_- \end{pmatrix}, \quad (5.5)$$

where we have defined $s_{\pm} = \exp(\pm i\phi/2)$. It is easy to check that along both the left and the right half the following BPS conditions are satisfied

$$\gamma_{\mu}\dot{x}^{\mu}u_{\pm} = \pm|\dot{x}|u_{\pm}, \quad u_+u_- = -i, \quad u_{\pm}\partial_{\tau}u_{\mp} = 0. \quad (5.6)$$

Finally, the cusp operators are defined by taking the contour in eq. (5.2) and the spinors in eq. (5.3). We also need to keep in mind the BPS constraints in eq. (3.23) whose solutions $\bar{m}_i^{21} = n_{12}^i = 0$ and $\bar{m}_i^{12} = n_{21}^i = 0$ lead to nontrivial cusp operators. Since the left and right part of the cusp operators preserves different supersymmetry the whole cusp configuration is not BPS. One may try to search for the BPS generalized cusp as in [84] in the fishnet theory. But we find that a direct generalization of the construction introduced in [84] does not lead to the BPS configuration. This is reasonable considering the fact that the fishnet theory has less supersymmetries than the ABJM theory.

5.2 Perturbative calculations

In this subsection, we compute the VEV's of the cusp operators constructed in the previous subsection. For simplicity, we do the computations in the framing 0 using the dimensional regularization with dimensional reduction and take the internal data $\bar{m}_i^{ab}, n_{ab}^i$ being the same along the cusp. As in the circular loop case, we assume $\bar{m}_i^{ab}$ and n_{ab}^i to be of order $\mathcal{O}(\xi^{1/4}/\sqrt{N})$. Almost all of the needed integrals have already been computed in [84], so we skip the details of the calculations for these integrals. Notices that we should keep the finite piece in the one-loop result. The one-loop contributions come from two kinds of diagrams with a single fermion propagator ending on the same line (figure 3a) or the different lines (figure 3b). The sum of their contributions is

$$W_1 = \sum_{a,b} \bar{m}_i^{ab} n_{ba}^i N_a N_b (2 \mathcal{I}_1 + \mathcal{I}_2), \quad (5.7)$$

Figure 4. Two-loop Feynman diagrams for cusp.

with

$$\mathcal{I}_1 = -\frac{\Gamma\left(\frac{3}{2}-\epsilon\right)}{\pi^{\frac{3}{2}-\epsilon}} \int_{-L < \tau_2 < \tau_1 < 0} \frac{d\tau_1 d\tau_2}{(\tau_1 - \tau_2)^{2-2\epsilon}} = \frac{1}{4\epsilon} \frac{\Gamma\left(\frac{1}{2}-\epsilon\right)}{\pi^{\frac{3}{2}-\epsilon}} L^{2\epsilon} \quad (5.8)$$

and

$$\begin{aligned} \mathcal{I}_2 &= -\frac{\Gamma\left(\frac{3}{2}-\epsilon\right) \cos \phi}{\pi^{\frac{3}{2}-\epsilon}} \int_0^L d\tau_1 \int_{-L}^0 d\tau_2 \frac{\tau_1 - \tau_2}{(\tau_1^2 + \tau_2^2 - 2\tau_1 \tau_2 \cos(2\phi))^{3/2-\epsilon}} \\ &= -\frac{\Gamma\left(\frac{1}{2}-\epsilon\right)}{\pi^{\frac{3}{2}-\epsilon}} L^{2\epsilon} \left[\frac{\sec \phi}{4\epsilon} - \frac{1}{2} \sec \phi \log(1 + \sec \phi) + \mathcal{O}(\epsilon) \right], \end{aligned} \quad (5.9)$$

Here we have used the identities

$$u_{+R(L)} \gamma_\mu u_{-R(L)} x_{12}^\mu = i\tau_{12}, \quad u_{+R(L)} \gamma_\mu u_{-L(R)} x_{12}^\mu = i\tau_{12} \cos(\phi). \quad (5.10)$$

The two-loop contributions are from the diagrams with two scalar propagators (figure 4a, figure 4b) and two fermion propagators (figures 4c–4h). The scalar contributions were not calculated in [84] separately and were combined with the contributions from the one-loop corrected vector propagators. The scalar contributions are given by

$$W_{2B} = \sum_{a,b,c} \left(\bar{m}_i^{ab} n_{ba}^i \bar{m}_j^{ac} n_{ca}^j + \bar{m}_i^{ba} n_{ab}^i \bar{m}_j^{ca} n_{ac}^j \right) N_a N_b N_c (2 \mathcal{I}_3 + \mathcal{I}_4), \quad (5.11)$$

with

$$\begin{aligned} \mathcal{I}_3 &= \frac{\Gamma^2\left(\frac{1}{2}-\epsilon\right)}{16\pi^{3-2\epsilon}} \int_{-L < \tau_2 < \tau_1 < 0} d\tau_1 d\tau_2 \frac{1}{(\tau_1 - \tau_2)^{2-4\epsilon}} \\ &= -\frac{\Gamma^2\left(\frac{1}{2}-\epsilon\right)}{16\pi^{3-2\epsilon}} L^{4\epsilon} \left(\frac{1}{4\epsilon} + \mathcal{O}(1) \right), \end{aligned} \quad (5.12)$$

and

$$\begin{aligned}
\mathcal{I}_4 &= \frac{\Gamma^2\left(\frac{1}{2}-\epsilon\right)}{16\pi^{3-2\epsilon}} \int_0^L d\tau_1 \int_{-L}^0 d\tau_2 \frac{1}{(\tau_1^2 + \tau_2^2 - 2\tau_1\tau_2 \cos 2\phi)^{1-2\epsilon}} \\
&= \frac{\Gamma^2\left(\frac{1}{2}-\epsilon\right)}{16\pi^{3-2\epsilon}} L^{4\epsilon} \int_0^1 dx \int_0^1 dy \frac{1}{(x^2 + y^2 + 2xya)^{1-2\epsilon}} \\
&= \frac{\Gamma^2\left(\frac{1}{2}-\epsilon\right)}{16\pi^{3-2\epsilon}} L^{4\epsilon} \left(\frac{\phi}{2\epsilon \sin 2\phi} + \mathcal{O}(1) \right).
\end{aligned} \tag{5.13}$$

Here we have defined a shorthand variable $a \equiv \cos(2\phi)$. For details we evaluate the last integral in the following way,

$$\begin{aligned}
\int_0^1 dx \int_0^1 dy \frac{1}{(x^2 + y^2 + 2xya)^{1-2\epsilon}} &= 2 \int_0^y dx \int_0^1 dy (x^2 + y^2 + 2axy)^{2\epsilon-1} \\
&= 2 \int_0^y \frac{dx}{y} \int_0^1 dy y^{4\epsilon-1} ((x/y)^2 + 1 + 2ax/y)^{2\epsilon-1} \\
&= 2 \int_0^1 d\alpha (\alpha^2 + 1 + 2a\alpha)^{2\epsilon-1} \int_0^1 dy y^{4\epsilon-1} = \frac{1}{2\epsilon} \frac{\phi}{\sin 2\phi} + \mathcal{O}(1).
\end{aligned} \tag{5.14}$$

Lastly we analyze the fermionic contributions which are

$$\begin{aligned}
W_{2F} &= \sum_{a,b,c} \left(\bar{m}_i^{ab} n_{ba}^i \bar{m}_j^{ac} n_{ca}^j + \bar{m}_i^{ba} n_{ab}^i \bar{m}_j^{ca} n_{ac}^j \right) N_a N_b N_c (I_s + I_t), \\
I_s &= 2\mathcal{I}_5 + 2\mathcal{I}_6 + \mathcal{I}_7, \\
I_t &= 2\mathcal{I}_8 + 2\mathcal{I}_9 + \mathcal{I}_{10},
\end{aligned} \tag{5.15}$$

with

$$\begin{aligned}
\mathcal{I}_5 &= \frac{\Gamma^2\left(\frac{3}{2}-\epsilon\right)}{4\pi^{3-2\epsilon}} \int_{0 < \tau_4 < \dots < \tau_1 < L} \frac{\prod_{i=1}^4 d\tau_i}{(\tau_1 - \tau_2)^{2-2\epsilon} (\tau_3 - \tau_4)^{2-2\epsilon}} \\
&= \frac{\Gamma\left(\frac{1}{2}-\epsilon\right)^2}{16\pi^{3-2\epsilon}} L^{4\epsilon} \left(\frac{1}{4\epsilon^2} + \mathcal{O}(1) \right).
\end{aligned} \tag{5.16}$$

$$\begin{aligned}
\mathcal{I}_6 &= \frac{\Gamma^2\left(\frac{3}{2}-\epsilon\right)}{4\pi^{3-2\epsilon}} \cos \phi \int_{-L}^0 d\tau_4 \int_{0 < \tau_3 < \tau_2 < \tau_1 < L} \prod_{i=1}^3 d\tau_i \frac{1}{(\tau_1 - \tau_2)^{2-2\epsilon}} \\
&\quad \times \frac{\tau_3 - \tau_4}{(\tau_3^2 + \tau_4^2 - 2\tau_3\tau_4 \cos 2\phi)^{3/2-\epsilon}} \\
&= \frac{\Gamma^2\left(\frac{1}{2}-\epsilon\right)}{32\pi^{3-2\epsilon}} L^{4\epsilon} \left[-\frac{\sec \phi}{2\epsilon^2} + \frac{\sec \phi \log(1 + \sec \phi)}{\epsilon} + \mathcal{O}(1) \right].
\end{aligned} \tag{5.17}$$

$$\mathcal{I}_7 = \mathcal{I}_1^2/4 = \frac{\Gamma\left(\frac{1}{2}-\epsilon\right)^2}{64\pi^{3-2\epsilon}\epsilon^2}. \tag{5.18}$$

$$\begin{aligned}
 \mathcal{I}_8 &= \frac{\Gamma^2\left(\frac{3}{2}-\epsilon\right)}{4\pi^{3-2\epsilon}} \int_{0<\tau_4\cdots<\tau_1<L} \frac{\prod_{i=1}^4 d\tau_i}{((\tau_1-\tau_4)(\tau_2-\tau_3))^{2-2\epsilon}} \\
 &= \frac{\Gamma^2\left(\frac{3}{2}-\epsilon\right)}{16\pi^{3-2\epsilon}} L^{4\epsilon} \left(\frac{1}{8\epsilon^2} + \frac{1}{4\epsilon} + \mathcal{O}(1) \right). \tag{5.19}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{I}_9 &= \frac{\Gamma^2\left(\frac{3}{2}-\epsilon\right)}{4\pi^{3-2\epsilon}} \cos\phi \int_{-L}^0 d\tau_4 \int_{0<\tau_3<\tau_2<\tau_1<L} \prod_{i=1}^3 d\tau_i \frac{1}{(\tau_2-\tau_3)^{2-2\epsilon}} \\
 &\quad \times \frac{\tau_1-\tau_4}{(\tau_1^2+\tau_4^2-2\tau_1\tau_4\cos 2\phi)^{3/2-\epsilon}} \\
 &= \frac{\Gamma^2\left(\frac{1}{2}-\epsilon\right)}{32\pi^{3-2\epsilon}} L^{4\epsilon} \left[-\frac{\sec\phi}{4\epsilon^2} + \frac{1}{\epsilon} \sec\phi \log(1+\sec\phi) + \frac{1}{2\epsilon} \sec\phi \log(\cos\phi) + \mathcal{O}(1) \right]. \tag{5.20}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{I}_{10} &= \frac{\Gamma^2\left(\frac{3}{2}-\epsilon\right)}{4\pi^{3-2\epsilon}} \frac{\cos^2\phi}{(1+\cos 2\phi)^2} \frac{L^{4\epsilon}}{(2\epsilon-1)^2} \int_{-L<\tau_4<\tau_3<0} d\tau_3 d\tau_4 \int_{0<\tau_2<\tau_1<L} d\tau_1 d\tau_2 \\
 &\quad s(\tau_1, \tau_4) s(\tau_2, \tau_3) \\
 &= \frac{\Gamma^2\left(\frac{1}{2}-\epsilon\right)}{32\pi^{3-2\epsilon}} \sec^2(\phi) L^{4\epsilon} \left[\frac{1}{4\epsilon^2} - \frac{1}{\epsilon} \log[1+\sec\phi] - \frac{1}{\epsilon} \frac{\phi \cos^2\phi}{\sin 2\phi} + \mathcal{O}(1) \right], \tag{5.21}
 \end{aligned}$$

where we have defined $s(x, y) = (\partial_x - \partial_y)(x^2 + y^2 - 2xy \cos(\phi))^{\epsilon-1/2}$.

Summing up all the contributions, we end up with the result up to two-loop level as¹⁰

$$\begin{aligned}
 \langle W \rangle &= N_1 + N_2 + (\mu L)^{2\epsilon} N_1 N_2 \sum_{a \neq b} (\bar{m}_i^{ab} n_{ba}^i) \frac{\Gamma\left(\frac{1}{2}-\epsilon\right)}{\pi^{\frac{3}{2}-\epsilon}} \left(\frac{1}{2\epsilon} - \frac{\sec\phi}{4\epsilon} \right. \\
 &\quad \left. + \frac{\sec\phi}{2} \log(1+\sec\phi) \right) + (\mu L)^{4\epsilon} N_1 N_2 (N_1 + N_2) \sum_{a \neq b} (\bar{m}_i^{ab} n_{ba}^i)^2 \left\{ \frac{\Gamma^2\left(\frac{1}{2}-\epsilon\right)}{16\pi^{3-2\epsilon}} \times \right. \\
 &\quad \times \left[\frac{1}{\epsilon^2} \left(1 - \frac{3\sec\phi}{4} + \frac{\sec^2\phi}{8} \right) + \frac{1}{\epsilon} \sec\phi \left(2\log(1+\sec\phi) + \frac{1}{2} \log\cos\phi \right. \right. \\
 &\quad \left. \left. - \frac{1}{2} \sec\phi \log(1+\sec\phi) + \mathcal{O}(1) \right) \right] \Bigg\}. \tag{5.22}
 \end{aligned}$$

Notice that the contributions from exchanging two scalars are cancelled by a part of the contributions from exchanging two fermions. As the case of the ABJM theory, there exists a double pole in the expression (5.22) which is from the exchanges of the fermions. For the case at hand, there are only such diagrams which contribute to the double pole. From this result we can extract a generalized potential V_N . Recall that the BPS constraints in eq. (3.23) admit the solutions with $\bar{m}_i^{21} = n_{12}^i = 0$ or $\bar{m}_i^{12} = n_{21}^i = 0$ which lead to

¹⁰We have replaced L with μL , where μ is some mass scale to make the quantity μL dimensionless.

nontrivial cusp operators. Without loss of generality, we consider the first case. From the definition of V_N [84],

$$\langle W \rangle = N_1 \exp(V_{N_2}) + N_2 \exp(V_{N_1}). \quad (5.23)$$

we can get the divergent part of V_N as

$$\begin{aligned} V_N = & N \bar{m}_i^{12} n_{21}^i (\mu L)^{2\epsilon} \frac{\Gamma(\frac{1}{2} - \epsilon)}{4\pi^{3/2-\epsilon}} \left(\frac{1}{\epsilon} - \frac{\sec \phi}{2\epsilon} + \sec \phi \log(1 + \sec \phi) \right) \\ & + N^2 (\bar{m}_i^{12} n_{21}^i)^2 \frac{\Gamma(\frac{1}{2} - \epsilon)^2}{16\pi^{3-2\epsilon}} (\mu L)^{4\epsilon} \left(\frac{1}{2\epsilon^2} - \frac{\sec \phi}{4\epsilon^2} + \frac{1}{\epsilon} \sec \phi \log(1 + \sec \phi) \right. \\ & \left. + \frac{1}{2\epsilon} \sec \phi \log \cos \phi \right). \end{aligned} \quad (5.24)$$

Notice that there are $1/\epsilon^2$ terms in $(\mu L)^{4\epsilon}$ even in the straight line limit. This is different from the ABJM case and is related to the fact that no diagrams with vertices appears at two loops in the fishnet theory. It indicates that there may be a better way to extract V_N from $\langle W \rangle$.¹¹ We leave this problem for further studies.

As discussed in [84], the suitable renormalization condition is that when $\phi = 0$ the renormalized V_N should vanish.¹² From this condition, we get the renormalized generalized potential

$$\begin{aligned} V_N^{\text{ren}} = & V_N - V_N|_{\phi=0} \\ = & N \bar{m}_i^{12} n_{21}^i (\mu L)^{2\epsilon} \frac{\Gamma(\frac{1}{2} - \epsilon)}{4\pi^{3/2-\epsilon}} \left(\frac{\sec \phi}{2\epsilon} - \frac{1}{2\epsilon} - \sec(\phi) \log(1 + \sec(\phi)) + \log 2 \right) \\ & + N^2 (\bar{m}_i^{12} n_{21}^i)^2 \frac{\Gamma(\frac{1}{2} - \epsilon)^2}{16\pi^{3-2\epsilon}} (\mu L)^{4\epsilon} \left(-\frac{\sec \phi}{4\epsilon^2} + \frac{1}{4\epsilon^2} + \frac{1}{\epsilon} \sec \phi \log(1 + \sec \phi) \right. \\ & \left. - \frac{1}{\epsilon} \log 2 + \frac{1}{2\epsilon} \sec \phi \log \cos \phi \right). \end{aligned} \quad (5.25)$$

The so-called universal cusp anomalous dimension, γ_{cusp} , is obtained from the large imaginary ϕ limit [90]:

$$\gamma_{\text{cusp}} = - \lim_{\phi \rightarrow \infty} \frac{2\epsilon(V_N^{\text{ren}}|_{\phi \rightarrow i\phi})}{\phi}. \quad (5.26)$$

If we still use prescription despite of the existence of $1/\epsilon^2$ terms, we get $\gamma_{\text{cusp}} = 0$ at two-loop order in our fishnet theory.

6 Conclusion and discussions

In this work, we studied the loop operators in the three-dimensional $\mathcal{N} = 2$ fishnet theories. This kind of theories was obtained by taking appropriate double scaling limit of the γ -deformed ABJM theory. The action of this 3D $\mathcal{N} = 2$ fishnet theory includes the kinetic

¹¹It was found in [89] that the prescription in [84] fails starting at three-loop order. An alternative prescription which works better at higher-loop order in the ladder limit was provided in [89]. This prescription is identical to the one in [84] up to two-loop order. We would like to thank Michelangelo Preti for discussions on this point.

¹²Recall that we take the same $\bar{m}_i^{ab}$'s and n_{ab}^i for the left and right part of the cusp.

terms and potential terms of the scalars and fermions, with the gauge sector got decoupled. It is not the simplest fishnet theory, but the presence of supersymmetries makes it attractive in studying the BPS loop operators in a setup simpler than the Chern-Simons-matter theories. We constructed the BPS line and loop operators, and computed the VEV of circular BPS operators perturbatively. We got some numerical evidence to suggest that the one-loop and two-loop contributions are vanishing. This is in consistency with the prediction from the localization, under the assumption that our loop operators are truly BPS at the quantum level.

During the perturbative computations we proposed a new regularization scheme at framing -1 which takes good care of the spinors in the definition of the fermionic BPS loop operators. The new scheme fits better into the spirit that the point-splitting should be consistent with the Hopf fibration used in the localization. However this new scheme makes the already complicated computations at framing -1 more messy. It would be nice to further develop effective tools for such kind of computations in this scheme numerically or even analytically. We hope this new scheme will be helpful in doing perturbative calculations in the usual supersymmetric Chern-Simons-matter theories. It could be useful to probe whether these classically BPS loop operators and other similar Wilson loops in super-Chern-Simons-matter with less supersymmetries are truly supersymmetric at the quantum level.

Furthermore, we discussed the line operators with a cusp and studied their generalized cusp anomalous dimensions. We found that the two-loop generalized potential defined as in [84] has a double pole in the expansion of $1/\epsilon$. It will be interesting to see if there are other prescription for the generalized potential which leads to the results with only a simple pole. Put this issue aside, we found that the universal cusp anomalous dimension vanishes.

There has been little progress in the study of open chain from the (cusped) Wilson lines in the ABJM theory. It would be valuable to study this hard problem in the simple setup here. That means to insert composite operator in the (cusped) line operators in the 3D $\mathcal{N} = 2$ fishnet theory and then investigate whether the open spin chain obtained here is integrable or not. We leave this problem for future study.

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A The Lagrangian of ABJM theory and its γ -deformation

In our convention the Lagrangian of ABJM theory with gauge group $U(N) \times U(N)$ reads

$$\begin{aligned}
 \mathcal{L}_{\text{ABJM}} &= \mathcal{L}_{CS} + \mathcal{L}_k + \mathcal{L}_p + \mathcal{L}_Y, \\
 \mathcal{L}_{CS} &= \frac{k}{4\pi} \epsilon^{\mu\nu\rho} \text{Tr} \left(A_\mu \partial_\nu A_\rho + \frac{2i}{3} A_\mu A_\nu A_\rho - B_\mu \partial_\nu B_\rho - \frac{2i}{3} B_\mu B_\nu B_\rho \right), \\
 \mathcal{L}_k &= \text{Tr}(-D_\mu \bar{\phi}^I D^\mu \phi_I + i \bar{\psi}_I \gamma^\mu D_\mu \psi^I), \\
 \mathcal{L}_p &= \frac{4\pi^2}{3k^2} \text{Tr}(\phi_I \bar{\phi}^I \phi_J \bar{\phi}^J \phi_K \bar{\phi}^K + \phi_I \bar{\phi}^J \phi_J \bar{\phi}^K \phi_K \bar{\phi}^I + 4\phi_I \bar{\phi}^J \phi_K \bar{\phi}^I \phi_J \bar{\phi}^K \\
 &\quad - 6\phi_I \bar{\phi}^J \phi_J \bar{\phi}^I \phi_K \bar{\phi}^K), \\
 \mathcal{L}_Y &= \frac{2\pi i}{k} \text{Tr}(\phi_I \bar{\phi}^I \psi^J \bar{\psi}_J - 2\phi_I \bar{\phi}^J \psi^I \bar{\psi}_J - \bar{\phi}^I \phi_I \bar{\psi}_J \psi^J + 2\bar{\phi}^I \phi_J \bar{\psi}_I \psi^J \\
 &\quad + \epsilon^{IJKL} \phi_I \bar{\psi}_J \phi_K \bar{\psi}_L - \epsilon_{IJKL} \bar{\phi}^I \psi^J \bar{\phi}^K \psi^L). \tag{A.1}
 \end{aligned}$$

Here A_μ, B_μ are the gauge fields corresponding to the first and the second $U(N)$, respectively. ϕ_I and ψ^I are four scalars and four fermions in the $(N, \bar{N})$ representation of the $U(N) \times U(N)$ and their covariant derivatives are given by

$$\begin{aligned}
 D_\mu \phi_I &= \partial_\mu \phi_I + i A_\mu \phi_I - i \phi_I B_\mu, \\
 D_\mu \bar{\phi}^I &= \partial_\mu \bar{\phi}^I - i \bar{\phi}^I A_\mu + i B_\mu \bar{\phi}^I, \\
 D_\mu \psi^I &= \partial_\mu \psi^I + i A_\mu \psi^I - i \psi^I B_\mu, \\
 D_\mu \bar{\psi}_I &= \partial_\mu \bar{\psi}_I - i \bar{\psi}_I A_\mu + i B_\mu \bar{\psi}_I. \tag{A.2}
 \end{aligned}$$

After the γ -deformation, some of the interaction terms will acquire non-trivial phases. Among the bosonic potential terms, only the terms with $\phi_I \bar{\phi}^J \phi_K \bar{\phi}^I \phi_J \bar{\phi}^K \equiv (I, J, K)$ with I, J, K different from each other acquire non-trivial phases as follows,

$$\begin{aligned}
 &\sum_{\sigma \in S(123)} [e^{i \text{sign}(\sigma)(\gamma_1 + \gamma_2 + \gamma_3)}(\sigma(1), \sigma(2), \sigma(3))] \\
 &+ \sum_{\sigma \in S(124)} [e^{i \text{sign}(\sigma)(\gamma_1 + \gamma_2 - \gamma_3)}(\sigma(1), \sigma(2), \sigma(4))] \\
 &+ \sum_{\sigma \in S(234)} [e^{i \text{sign}(\sigma)(-\gamma_1 + \gamma_2 + \gamma_3)}(\sigma(2), \sigma(3), \sigma(4))] \\
 &+ \sum_{\sigma \in S(314)} [e^{i \text{sign}(\sigma)(\gamma_1 - \gamma_2 + \gamma_3)}(\sigma(3), \sigma(1), \sigma(4))], \tag{A.3}
 \end{aligned}$$

where $\text{sign}(\sigma) = 1 (-1)$ for the even (odd) permutation σ .

Among the Yukawa-like interaction only the following obtain non-trivial phase factors,

$$\begin{aligned}
 (I, J) \equiv \bar{\phi}^I \phi^J \bar{\psi}_I \psi^J \text{ (or } \phi_I \bar{\phi}^J \psi^I \bar{\psi}_J) \rightarrow \\
 e^{\frac{i}{2}(\gamma_1+\gamma_2)}(1, 2) + e^{-\frac{i}{2}(\gamma_1+\gamma_2)}(2, 1) + e^{\frac{i}{2}(\gamma_2+\gamma_3)}(2, 3) + e^{-\frac{i}{2}(\gamma_2+\gamma_3)}(3, 2) \\
 + e^{\frac{i}{2}(\gamma_1+\gamma_3)}(3, 1) + e^{-\frac{i}{2}(\gamma_1+\gamma_3)}(1, 3) + e^{\frac{i}{2}(\gamma_2-\gamma_3)}(4, 1) + e^{-\frac{i}{2}(\gamma_2-\gamma_3)}(1, 4) \\
 + e^{\frac{i}{2}(\gamma_3-\gamma_1)}(4, 2) + e^{-\frac{i}{2}(\gamma_3-\gamma_1)}(2, 4) + e^{\frac{i}{2}(\gamma_1-\gamma_2)}(4, 3) + e^{-\frac{i}{2}(\gamma_1-\gamma_2)}(3, 4), \quad (\text{A.4})
 \end{aligned}$$

and

$$\begin{aligned}
 [I, J, K, L] \equiv \epsilon_{IJKL} \bar{\phi}^I \psi^J \bar{\phi}^K \psi^L \text{ (or } \epsilon^{IJKL} \phi_I \bar{\psi}_J \phi_K \bar{\psi}_L) \rightarrow \\
 e^{\frac{i}{2}\gamma_1}([1, 3, 4, 2] + [4, 2, 1, 3] - [3, 4, 2, 1] - [2, 1, 3, 4]) \\
 + e^{-\frac{i}{2}\gamma_1}(-[1, 2, 4, 3] - [4, 3, 1, 2] + [2, 4, 3, 1] + [3, 1, 2, 4]) \\
 + e^{\frac{i}{2}\gamma_2}(-[1, 4, 3, 2] - [3, 2, 1, 4] + [2, 1, 4, 3] + [4, 3, 2, 1]) \\
 + e^{-\frac{i}{2}\gamma_2}([1, 2, 3, 4] + [3, 4, 1, 2] - [2, 3, 4, 1] - [4, 1, 2, 3]) \\
 + e^{\frac{i}{2}\gamma_3}(-[1, 3, 2, 4] - [2, 4, 1, 3] + [3, 2, 4, 1] + [4, 1, 3, 2]) \\
 + e^{-\frac{i}{2}\gamma_3}([1, 4, 2, 3] + [2, 3, 1, 4] - [3, 1, 4, 2] - [4, 2, 3, 1]). \quad (\text{A.5})
 \end{aligned}$$

Notice that terms like (I, I) are unchanged.

B The propagators of the scalars and fermions

In the Minkowski spacetime, we have the following kinematic terms for the scalars and the spinors in the $\mathcal{N} = 2$ notation

$$\mathcal{L}_k = \sum_{a,b} \text{Tr}(-\partial_\mu \bar{Z}_i^{(ba)} \partial^\mu Z_{(ab)}^i + i \bar{\zeta}_i^{(ba)} \gamma^\mu \partial_\mu \zeta_{(ab)}^i). \quad (\text{B.1})$$

By standard Wick rotation, these terms become

$$\mathcal{L}_k = \sum_{a,b} \text{Tr}(\partial_\mu \bar{Z}_i^{(ba)} \partial^\mu Z_{(ab)}^i - i \bar{\zeta}_i^{(ba)} \gamma^\mu \partial_\mu \zeta_{(ab)}^i). \quad (\text{B.2})$$

Then the explicit expressions for the propagators at tree level in position space are

$$\begin{aligned}
 \langle Z_{(ab)p}^i(x) \bar{Z}_j^{(cd)} r^s(y) \rangle &= \delta_a^d \delta_b^c \delta_j^i \delta_p^s \delta_r^q \frac{\Gamma(\frac{1}{2} - \epsilon)}{4\pi^{\frac{3}{2} - \epsilon}} \frac{1}{|x - y|^{1-2\epsilon}}, \\
 \langle \zeta_{(ab)p}^i(x) \bar{\zeta}_j^{(cd)} r^s(y) \rangle &= i \delta_a^d \delta_b^c \delta_j^i \delta_p^s \delta_r^q \frac{\Gamma(\frac{3}{2} - \epsilon)}{2\pi^{\frac{3}{2} - \epsilon}} \frac{\gamma_{\mu\alpha}^\beta (x - y)^\mu}{|x - y|^{3-2\epsilon}}, \quad (\text{B.3})
 \end{aligned}$$

where the dimensional regularization with $d = 3 - 2\epsilon$ has been used.

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