

Probing New Physics in $b \rightarrow d$ transitions

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ABSTRACT: Recent experimental data on several observables in semileptonic B -meson decays are found to be in tension with the corresponding Standard Model predictions. Most of these deviations are related to $b \rightarrow c$ and $b \rightarrow s$ flavour changing transitions. In this work, we estimate possible New Physics effects in $b \rightarrow d\mu^+\mu^-$ flavour changing neutral currents. We parametrize NP contributions in a model-independent way and determine the allowed ranges of corresponding Wilson coefficients from the data on the exclusive $B^\pm \rightarrow \pi^\pm\mu^+\mu^-$ decays measured recently by the LHCb collaboration. Afterwards, we investigate the impact of these results on other $b \rightarrow d$ processes such as the leptonic $B^0 \rightarrow \mu^+\mu^-$ decays and $B^0 - \bar{B}^0$ mixing. As an example, we consider a simplified Z' model that is found to be consistent with current $b \rightarrow d$ data in the certain regions of the NP parameter space. In addition, we estimate the correlations between the partial decay widths of $B \rightarrow \pi\mu^+\mu^-$ and $B \rightarrow K\mu^+\mu^-$ processes to be used for an independent determination of CKM matrix elements as well as for a combined New Physics analysis of both $b \rightarrow d$ and $b \rightarrow s$ transitions.

KEYWORDS: Beyond Standard Model, Heavy Quark Physics

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1 Introduction

Current tensions between experimental measurements and Standard Model (SM) predictions of some observables in B -hadron decays (so-called B -anomalies) have attracted a lot of attention. Most of these anomalies are related with $b \rightarrow c \ell \bar{\nu}_\ell$ and $b \rightarrow s \ell^+ \ell^-$ flavour changing transitions where ℓ denotes one of the charged leptons. To accommodate these tensions, several New Physics (NP) models have been proposed in the literature (e.g. with leptoquarks, Z' -boson, etc.) leading to the publication of large number of papers [1–42]. On the other side, if NP exists and is accessible at current energy level, it would be reasonable to expect such effects also in processes induced by $b \rightarrow d$ flavour changing neutral current (FCNC). Similar to $b \rightarrow s \ell^+ \ell^-$ transition, the $b \rightarrow d \ell^+ \ell^-$ FCNCs are forbidden at tree level in the SM and induced via loops, therefore they also might be sensitive to NP contributions. One of the specific features of the $b \rightarrow d$ transitions is an additional suppression compared to the $b \rightarrow s$ by ratio of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $|V_{td}/V_{ts}|^2$. The typical branching fraction of $b \rightarrow d \ell^+ \ell^-$ processes is $\mathcal{O}(10^{-8})$ which makes their measurements considerably more challenging. Additionally, the parts of the amplitude of the $b \rightarrow d \ell^+ \ell^-$ decays proportional to $V_{tb}V_{td}^*$, $V_{cb}V_{cd}^*$ and $V_{ub}V_{ud}^*$ are of the same order of the Wolfenstein parameter λ and, in addition to a relative CKM phase, they have different strong phases originating from the nonlocal hadronic amplitudes. This leads to non-vanishing direct CP -asymmetry in $b \rightarrow d \ell^+ \ell^-$ processes which is negligible in case of $b \rightarrow s \ell^+ \ell^-$ transition. Therefore $b \rightarrow d$ processes provide an even richer set of interesting observables to test the quark flavour sector of the Standard Model.

Up to now, only few semileptonic $b \rightarrow d \ell^+ \ell^-$ processes have been seen experimentally. The first measurement of the semileptonic $b \rightarrow d$ transition was done by the LHCb collaboration in 2012 providing an experimental value of the branching fraction of the exclusive $B^\pm \rightarrow \pi^\pm \mu^+ \mu^-$ decays [43]. In 2015, the LHCb collaboration has also measured

the differential decay distribution in the dimuon invariant mass squared and the total direct CP -asymmetry of the $B^\pm \rightarrow \pi^\pm \mu^+ \mu^-$ processes [44]. The LHCb data presented in ref. [44] is about 1.3σ away from the recent SM prediction including the computation of the corresponding nonlocal hadronic amplitudes and resonance contributions [45] (see figure 4 in ref. [44]). Curiously, this slight deviation of experiment and theory points in the same direction as the tensions found in the $b \rightarrow s \mu^+ \mu^-$ transitions (as it was also noticed in ref. [28]). Such a situation, together with current tensions in $b \rightarrow c$ and $b \rightarrow s$ motivated us to address the main goal of this paper, namely, to probe possible New Physics effects in $b \rightarrow d \ell^+ \ell^-$ processes.

Experimentally also the decays $B^0 \rightarrow \pi^+ \pi^- \mu^+ \mu^-$ [46] and $\Lambda_b^0 \rightarrow p \pi^- \mu^+ \mu^-$ [47] have been studied. The theoretical analysis of these processes is however quite challenging due to a poor knowledge of the underlying hadronic input including form factors and non-local hadronic amplitudes, therefore we do not include these decays in our NP analysis. In addition, the LHCb collaboration has recently found evidence of the $B_s^0 \rightarrow \bar{K}^{*0} \mu^+ \mu^-$ decay at the level of 3.4 standard deviations [48]. An analysis of this decay is of special interest in the light of existing anomalies in the $B \rightarrow K^* \mu^+ \mu^-$ processes.

The paper is organised as follows. In section 2 we determine allowed intervals of the NP coefficients in a model-independent way from data on the differential branching fraction of the $B^\pm \rightarrow \pi^\pm \mu^+ \mu^-$ decays. In section 3 we consider the impact on the leptonic $B^0 \rightarrow \mu^+ \mu^-$ decays. Section 4 is devoted to an analysis of NP effects in $B^0 - \bar{B}^0$ mixing: as an example we study a simplified NP model with Z' -boson. We conclude in section 5 and in appendix A we present the correlation matrix between different hadronic parts of the partial decays width in $B \rightarrow \pi \ell^+ \ell^-$ and $B \rightarrow K \ell^+ \ell^-$ processes in the SM.

2 New Physics effects in $B^\pm \rightarrow \pi^\pm \mu^+ \mu^-$ decays

We perform an analysis of possible NP effects in the $B^\pm \rightarrow \pi^\pm \mu^+ \mu^-$ decays in a model-independent way based on an assumption that these effects are induced at a large energy scale (by heavy particles e.g. Z' -boson, leptoquarks, etc.). After integrating out their contributions are described by an effective Lagrangian

$$\mathcal{L}_{\text{eff}}^{\text{NP}} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{td}^* (C_9^\mu \mathcal{O}_9^\mu + C_{10}^\mu \mathcal{O}_{10}^\mu + C_9^{\prime\mu} \mathcal{O}_9^{\prime\mu} + C_{10}^{\prime\mu} \mathcal{O}_{10}^{\prime\mu}) + \text{h.c.}, \quad (2.1)$$

where G_F is the Fermi constant, $C_{9,10}^{(\prime)\mu}$ denote the short-distance NP Wilson coefficients, and the effective dimension-6 semileptonic operators are defined as

$$\mathcal{O}_9^\mu = \frac{\alpha_{\text{em}}}{4\pi} (\bar{d} \gamma_\rho P_L b) (\bar{\mu} \gamma^\rho \mu), \quad \mathcal{O}_{10}^\mu = \frac{\alpha_{\text{em}}}{4\pi} (\bar{d} \gamma_\rho P_L b) (\bar{\mu} \gamma^\rho \gamma_5 \mu), \quad (2.2)$$

$$\mathcal{O}_9^{\prime\mu} = \frac{\alpha_{\text{em}}}{4\pi} (\bar{d} \gamma_\rho P_R b) (\bar{\mu} \gamma^\rho \mu), \quad \mathcal{O}_{10}^{\prime\mu} = \frac{\alpha_{\text{em}}}{4\pi} (\bar{d} \gamma_\rho P_R b) (\bar{\mu} \gamma^\rho \gamma_5 \mu), \quad (2.3)$$

with α_{em} denoting the fine structure constant, and $P_{L,R} = (1 \mp \gamma_5)/2$. Here we make several comments regarding the NP ansatz by eq. (2.1) used in our analysis. First, we consider the effective NP operators with muons only since the measurements of $b \rightarrow d \ell^+ \ell^-$ modes with electrons or τ -leptons are absent at present time. Therefore, currently Lepton Flavour

Universality (LFU) cannot be tested in $b \rightarrow d\ell^+\ell^-$ processes. Furthermore, we emphasize that this is just an initial study and due to insufficient current data in $b \rightarrow d$ transition we restrict ourselves by a simplified ansatz in eq. (2.1) without considering the (pseudo)scalar and tensor as well as electromagnetic, chromomagnetic and four-quark effective operators. The choice of the NP effective Lagrangian (2.1) is motivated by the $b \rightarrow s\ell^+\ell^-$ case where a better agreement with data is achieved from the fit by using the vector NP operators (see e.g. ref. [35]).

The effective NP Hamiltonian (2.1) modifies the expression for the dilepton invariant mass distribution of the $B^- \rightarrow \pi^- \mu^+ \mu^-$ decay [45]¹

$$\frac{d\text{BR}^{\text{NP}}(B^- \rightarrow \pi^- \mu^+ \mu^-)}{dq^2} = \tau_{B^-} \frac{G_F^2 \alpha_{\text{em}}^2 |V_{tb} V_{td}^*|^2}{1536 \pi^5 m_B^3} |f_{B\pi}^+(q^2)|^2 \lambda^{3/2}(m_B^2, m_\pi^2, q^2) \times \left\{ \left| C_9^{\text{SM}} + C_9^{\text{NP}} + \Delta C_9^{B\pi}(q^2) + \frac{2m_b}{m_B + m_\pi} C_7^{\text{SM}} \frac{f_{B\pi}^T(q^2)}{f_{B\pi}^+(q^2)} \right|^2 + |C_{10}^{\text{SM}} + C_{10}^{\text{NP}}|^2 \right\}, \quad (2.4)$$

where the following notations are introduced:

$$C_9^{\text{NP}} \equiv C_9^\mu + C_9'^\mu, \quad C_{10}^{\text{NP}} \equiv C_{10}^\mu + C_{10}'^\mu. \quad (2.5)$$

In eq. (2.4), $f_{B\pi}^+(q^2)$ and $f_{B\pi}^T(q^2)$ are the vector and tensor $B \rightarrow \pi$ transition form factors, respectively, $\lambda(m_B^2, m_\pi^2, q^2)$ is the Källén function, and $\Delta C_9^{B\pi}(q^2)$ denotes the q^2 -dependent effective Wilson coefficient accumulating contributions from the non-local hadronic amplitudes. The definitions of above mentioned quantities and functions are given in ref. [45]. The non-perturbative input include the form factors and non-local hadronic amplitudes. The former were determined using the Light-Cone Sum Rules (LCSR) method while the latter were obtained using combination of the QCD factorisation and LCSR methods with hadronic dispersion relations (see refs. [45, 49] for details). In the numerical analysis we use the same input as in ref. [49]. We note that due to parity conservation in QCD the hadronic matrix element

$$\langle \pi(p) | \bar{d} \gamma_\mu \gamma_5 b | B(p+q) \rangle = 0 \quad (2.6)$$

vanishes and therefore it is not possible to resolve the contributions from left- and right-handed quark operators in the $B \rightarrow \pi \mu^+ \mu^-$ decays.

We define the CP-averaged bin of the dilepton invariant mass distribution as

$$\mathcal{B}[q_1^2, q_2^2] \equiv \frac{1}{2} \frac{1}{q_2^2 - q_1^2} \int_{q_1^2}^{q_2^2} dq^2 \left[\frac{d\text{BR}(B^- \rightarrow \pi^- \ell^+ \ell^-)}{dq^2} + \frac{d\text{BR}(B^+ \rightarrow \pi^+ \ell^+ \ell^-)}{dq^2} \right]. \quad (2.7)$$

The SM prediction for this observable in the bin $[1 - 6] \text{ GeV}^2$ presented in table 5 of [49] is about 1.3σ above the corresponding experimental measurement by the LHCb collaboration [44]. Experimental values of $\mathcal{B}[q_1^2, q_2^2]$ for three bins $[2 - 4] \text{ GeV}^2$, $[4 - 6] \text{ GeV}^2$ and $[6 - 8] \text{ GeV}^2$ [44] are also not directly overlapping with the SM predictions. Using these experimental data we perform a fit of the NP coefficients C_9^{NP} and C_{10}^{NP} assuming them to

¹We denote explicitly by $C_{9,10}^{\text{SM}}$ the SM Wilson coefficients, $C_9^{\text{SM}}(m_b) \approx -C_{10}^{\text{SM}}(m_b) \approx 4.1$.

One bin			
Scenario	Best-fit values	1σ interval	Pull
C_9^{NP} only	-2.3; -4.5	[-6.2, -0.6]	1.5
C_{10}^{NP} only	+1.5; +6.7	[+0.4, +7.8]	1.5
$C_9^{\text{NP}} = -C_{10}^{\text{NP}}$	-0.8, -6.7	[-1.4, -0.2] \cup [-7.3, -6.1]	1.5
both C_9^{NP} and C_{10}^{NP}	see figure 1		

Three bins			
Scenario	Best-fit value(s)	1σ interval	Pull
C_9^{NP} only	-3.6	[-5.2, -1.9]	2.6
C_{10}^{NP} only	+2.8; +5.4	[+1.4, +6.8]	2.7
$C_9^{\text{NP}} = -C_{10}^{\text{NP}}$	-1.2; -6.4	[-1.8, -0.7] \cup [-7.0, -5.8]	2.7
both C_9^{NP} and C_{10}^{NP}	see figure 1		

Table 1. Estimated 1σ ranges of the NP coefficients C_9^{NP} and C_{10}^{NP} in different scenarios. “One bin” refers to the bin [1 – 6] GeV², and “three bins” includes [2 – 4] GeV², [4 – 6] GeV² and [6 – 8] GeV². Pull is defined as a square root of the difference of χ^2 values between the best-fit and SM points: $\text{pull} = \sqrt{\chi_{\text{SM}}^2 - \chi_{\text{min}}^2}$.

be real. Note that we do not include the bin [0.1 – 2] GeV² near the ρ - and ω -resonances due to large hadronic uncertainties arising in their theoretical description. The fit is performed by using the method of least squares introducing the χ^2 function

$$\chi^2 = \sum_{i=1}^{N_b} \frac{(\mathcal{B}_i^{\text{NP}} - \mathcal{B}_i^{\text{exp}})^2}{\sigma_i^2}, \tag{2.8}$$

where N_b is the number of bins, $\mathcal{B}_i^{\text{NP}}$ denotes the theoretical expression for the bin of the dimuon invariant mass distribution depending on the NP Wilson coefficients, and $\mathcal{B}_i^{\text{exp}}$ is the corresponding experimental measurement. Both theoretical and experimental uncertainties are assumed to be Gaussian distributed, no correlations between experimental values of the bins are quoted in ref. [44]. The theory predictions for the bins are in general correlated between each other but we neglect these effects in our analysis, since the uncertainty of fit is mostly dominated by the experimental errors. In the future, the analysis can be improved by including the correlations between bins when more accurate data will be available. The standard deviation σ_i in eq. (2.8) includes both experimental and theoretical uncertainties in quadrature. Note that theory uncertainties are determined only for vanishing NP Wilson coefficients. In our analysis we consider the following scenarios: (1) only C_9^{NP} ; (2) only C_{10}^{NP} ; (3) both C_9^{NP} and C_{10}^{NP} as independent from each other; and (4) $C_9^{\text{NP}} = -C_{10}^{\text{NP}}$. The results obtained are presented in table 1 and figure 1.

Let us make several comments on these results. First, we note that rather broad intervals are still allowed for separately C_9^{NP} and C_{10}^{NP} , this is mainly due to large experimental

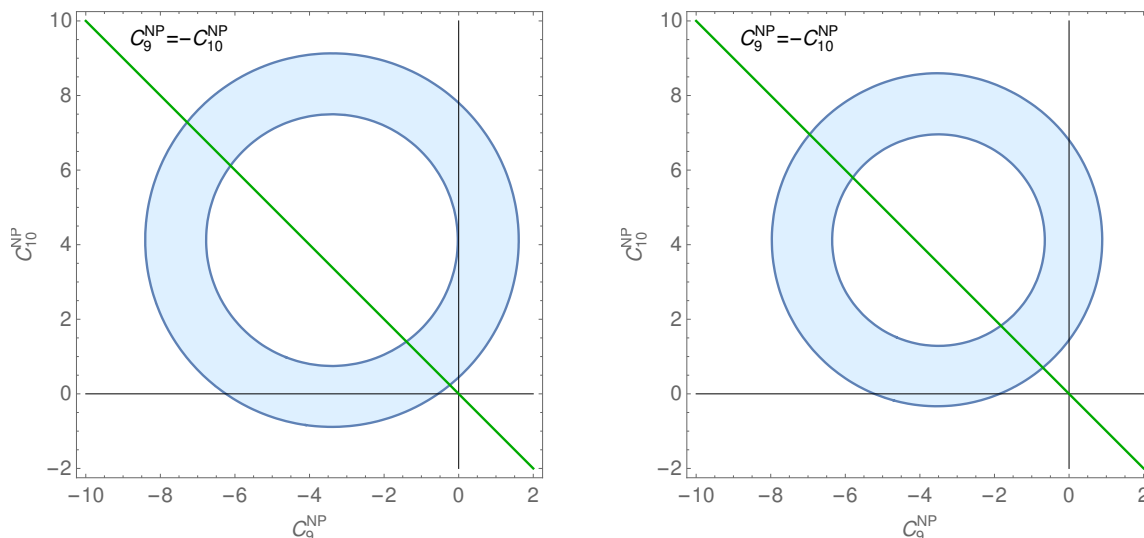


Figure 1. Estimated 1σ -regions of C_9^{NP} and C_{10}^{NP} from the fit using one bin (left plot) and three bins (right plot). The green lines correspond to the scenario $C_9^{\text{NP}} = -C_{10}^{\text{NP}}$.

errors. Second, considering scenario with $C_9^{\text{NP}} = -C_{10}^{\text{NP}}$ yields two separate solutions $C_9^{\text{NP}} = -C_{10}^{\text{NP}} \sim -6$ and $C_9^{\text{NP}} = -C_{10}^{\text{NP}} \sim -1$ (see figure 1 and table 1) where the latter one is quite similar² to the $b \rightarrow s\ell^+\ell^-$ case. Hereafter we focus on consideration the following solution

$$C_9^{\text{NP}} = -C_{10}^{\text{NP}} \simeq -1.2 \pm 0.6. \tag{2.9}$$

It is interesting to notice, that from the global fit of data on the $b \rightarrow s\ell^+\ell^-$ observables one also gets quite similar estimates (updated after Moriond 2019):

$$C_{9,bs}^\mu = -C_{10,bs}^\mu = -0.46 \pm 0.10, \tag{30} \tag{2.10}$$

$$C_{9,bs}^\mu = -C_{10,bs}^\mu = -0.41 \pm 0.10, \tag{36} \tag{2.11}$$

$$C_{9,bs}^\mu = -C_{10,bs}^\mu = -0.53 \pm 0.08, \tag{35} \tag{2.12}$$

assuming the LFU violation in μ - e sector. A comparison of the SM prediction [49], LHCb data [44] and the NP result (in the scenario of eq. (2.9)) for the binned differential branching fraction of the $B^\pm \rightarrow \pi^\pm \mu^+ \mu^-$ decays is presented in figure 2. We again emphasize that our estimate in eq. (2.9) still allows for both left- and right-handed quark operators $\mathcal{O}_{9,10}^\mu$ and $\mathcal{O}'_{9,10}^\mu$ as one can see from eq. (2.5). This ambiguity can be resolved by considering the leptonic $B^0 \rightarrow \mu^+ \mu^-$ decay which is sensitive to another combination of the effective operators. This question is discussed in the next section.

²Note, that “similar” in this context just refers to the similar values of NP Wilson coefficients in $b \rightarrow d$ and $b \rightarrow s$ transitions, respectively, as it follows from the normalisation of corresponding NP Lagrangian. In fact, the NP effects in $b \rightarrow d$ are not of similar size as in $b \rightarrow s$ and are suppressed by the ratio $|V_{td}/V_{ts}|$, for instance as a consequence of minimally broken U(2)-flavour symmetry, see e.g. ref. [37].

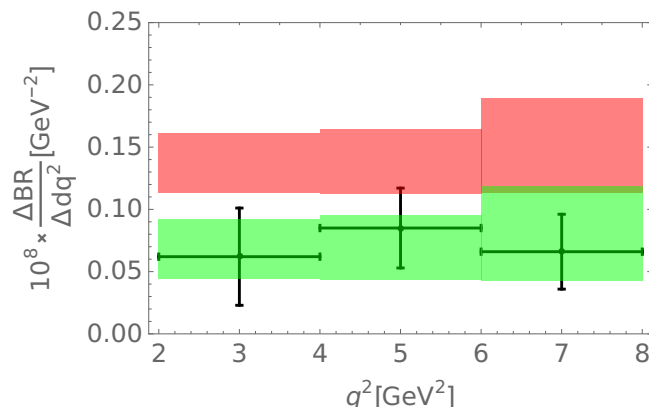


Figure 2. Theoretical predictions for differential branching fraction of the $B^\pm \rightarrow \pi^\pm \mu^+ \mu^-$ decays in comparison with the data (black) by the LHCb collaboration [44]. The red bands correspond to the SM prediction [49] and green ones indicate the NP result (for the solution in eq. (2.9)).

3 Impact on the $B^0 \rightarrow \mu^+ \mu^-$ decay

There are several experimental analyses of the $B^0 \rightarrow \mu^+ \mu^-$ decay performed by the ATLAS, CMS and LHCb collaborations [50–55]. No significant evidence of $B^0 \rightarrow \mu^+ \mu^-$ decay was found so far, and only upper limits are set up. The most recent bounds are

$$\text{BR}(B^0 \rightarrow \mu^+ \mu^-) < 2.1 \times 10^{-10}, \quad 95\% \text{ CL}, [53] \quad (\text{ATLAS}) \quad (3.1)$$

$$\text{BR}(B^0 \rightarrow \mu^+ \mu^-) < 3.6 \times 10^{-10}, \quad 95\% \text{ CL}, [54] \quad (\text{CMS}) \quad (3.2)$$

$$\text{BR}(B^0 \rightarrow \mu^+ \mu^-) < 3.4 \times 10^{-10}, \quad 95\% \text{ CL}, [55] \quad (\text{LHCb}) \quad (3.3)$$

The Particle Data Group quotes the average value [56] (online update) based on combination of the results in refs. [51, 53, 55]

$$\text{BR}(B^0 \rightarrow \mu^+ \mu^-) = (1.4^{+1.6}_{-1.4}) \times 10^{-10} \quad (3.4)$$

that is consistent with zero.

The SM prediction for the $B^0 \rightarrow \mu^+ \mu^-$ decay width is known up to $\mathcal{O}(\alpha_{em})$ and $\mathcal{O}(\alpha_s^2)$ corrections [57–59]. Using values for the decay constant from Lattice QCD [60] the most recent SM value for the $B^0 \rightarrow \mu^+ \mu^-$ decay branching fraction is obtained in ref. [59]

$$\text{BR}^{\text{SM}}(B^0 \rightarrow \mu^+ \mu^-) = (1.027 \pm 0.051) \times 10^{-10}. \quad (3.5)$$

This result is consistent with experimental upper limits in eqs. (3.1), (3.2), (3.3) as well as with the average in eq. (3.4). The NP Hamiltonian (2.1) leads to a modification of the branching fraction of the $B^0 \rightarrow \mu^+ \mu^-$ decay also induced at quark level by the $b \rightarrow d \mu^+ \mu^-$ transition. The modified expression for the $B^0 \rightarrow \mu^+ \mu^-$ branching fraction reads:

$$\text{BR}^{\text{NP}}(B^0 \rightarrow \mu^+ \mu^-) = \tau_{B^0} \frac{G_F^2 \alpha_{em}^2 |V_{tb}^* V_{td}|^2}{16\pi^3} m_{B^0} f_B^2 m_\mu^2 \sqrt{1 - \frac{4m_\mu^2}{m_{B^0}^2}} |C_{10}^{\text{SM}} + C_{10}^\mu - C_{10}'^\mu|^2. \quad (3.6)$$

As one can see from eq. (3.6), the left- and right-handed quark current operators \mathcal{O}_{10} and \mathcal{O}'_{10} give opposite sign contributions to the $B^0 \rightarrow \mu^+\mu^-$ branching fraction. Keeping in mind the relation (2.5) and considering two cases with left- and right-handed operators separately, using the value in eq. (2.9) we get the following NP estimates for the $B^0 \rightarrow \mu^+\mu^-$ decay branching fraction, respectively:

$$\text{BR}^{\text{NP}}(B^0 \rightarrow \mu^+\mu^-) \simeq (0.6 \pm 0.2) \times 10^{-10}, \quad \text{if } C_{10}^{\text{NP}} = C_{10}, \quad (3.7)$$

$$\text{BR}^{\text{NP}}(B^0 \rightarrow \mu^+\mu^-) \simeq (1.8 \pm 0.4) \times 10^{-10}, \quad \text{if } C_{10}^{\text{NP}} = C'_{10}. \quad (3.8)$$

Both values above are consistent with the experimental bounds (3.1), (3.2), (3.3) while the value in eq. (3.8) is quite close to the upper limit by ATLAS collaboration (3.1). Therefore, currently we are not able to make an unambiguous conclusion regarding preference of left- or right-handed quark currents in $b \rightarrow d\mu^+\mu^-$ transition. Nevertheless, future more precise data on the $B^0 \rightarrow \mu^+\mu^-$ decay would clarify this situation.

4 Impact on $B^0 - \bar{B}^0$ mixing

We would like to emphasize that in general the model-independent Lagrangian (2.1) does not necessarily give a sizeable impact in $B^0 - \bar{B}^0$ mixing. Indeed, the NP operators in the form (2.3) can give contribution to the mixing via muonic loops that are suppressed compared to the tree-level contribution of the SM dimension-6 operator $Q_1 = \bar{d}\gamma_\mu(1 - \gamma_5)b \times \bar{d}\gamma^\mu(1 - \gamma_5)b$. However, depending on a specific model, the $B^0 - \bar{B}^0$ mixing might be strongly affected by NP in $b \rightarrow d$ transition.

The mass difference of the mass eigenstates in $B^0 - \bar{B}^0$ system is given by (see e.g. refs. [61, 62]):

$$\Delta M_d = 2|M_{12}^d| = \frac{G_F^2}{6\pi^2} |V_{tb}V_{td}^*|^2 m_W^2 S_0(x_t) \hat{\eta}_B m_B f_B^2 B, \quad (4.1)$$

where $S(x_t)$ ($x_t = m_t^2/m_W^2$) is the Inami-Lim function [63], $\hat{\eta}_B$ encodes perturbative QCD corrections [64], and B denotes the Bag parameter characterising the matrix element of the dimension-6 operator Q_1 . Note that due to parity conservation of QCD the matrix element of the operator with right-handed currents $Q'_1 = \bar{d}\gamma_\mu(1 + \gamma_5)b \times \bar{d}\gamma^\mu(1 + \gamma_5)b$ is described by the same Bag parameter $B' = B$.

The mass difference ΔM_d is measured very precisely, the value quoted by HFLAV in 2019 [65]

$$\Delta M_d^{\text{exp}} = (0.5064 \pm 0.0019) \text{ ps}^{-1} \quad (4.2)$$

is in agreement with the average value [66] obtained using a combination of HQET Sum Rule [67–71] and Lattice QCD results [60, 72, 73]:

$$\Delta M_d^{\text{average}} = (0.533_{-0.036}^{+0.022}) \text{ ps}^{-1}. \quad (4.3)$$

As an example we will investigate a simplified NP model with Z' -boson that couples with left-handed quarks and leptons in order to find a preferred range of NP parameters

consistent with current $b \rightarrow d\mu^+\mu^-$ data and $B^0 - \bar{B}^0$ mixing. In the framework of this model we consider the following interaction Lagrangian [66]:

$$\mathcal{L}_{Z'} = \left[g_{ij}^Q (\bar{d}_L^i \gamma^\mu d_L^j) + g_{ij}^L (\bar{\ell}_L^i \gamma^\mu \ell_L^j) \right] Z'_\mu + \text{h.c.} \quad (4.4)$$

Integrating out the heavy Z' -boson yields the following effective Lagrangian

$$\mathcal{L}_{Z'}^{\text{eff}} = -\frac{1}{2m_{Z'}^2} \left[g_{ij}^Q (\bar{d}_L^i \gamma^\mu d_L^j) + g_{ij}^L (\bar{\ell}_L^i \gamma^\mu \ell_L^j) \right]^2 + \text{h.c.} \quad (4.5)$$

In the above, we hereafter consider only terms relevant for the $b \rightarrow d\mu^+\mu^-$ transition and $B^0 - \bar{B}^0$ -mixing:

$$\mathcal{L}_{Z'}^{\text{eff}} = -\frac{1}{2m_{Z'}^2} \left[\left(g_{13}^Q \right)^2 (\bar{d}_L \gamma^\mu b_L)(\bar{d}_L \gamma_\mu b_L) + 2g_{13}^Q g_{22}^L (\bar{d}_L \gamma^\mu b_L)(\bar{\mu}_L \gamma_\mu \mu_L) \right] + \dots \quad (4.6)$$

Parametrising NP effects in mass difference ΔM_d as

$$\frac{\Delta M_d^{\text{exp}}}{\Delta M_d^{\text{SM}}} = \left| 1 + \frac{C_{bd}^{LL}}{R} \right|, \quad (4.7)$$

where $R = \sqrt{2}G_F m_W^2 S_0(x_t) \hat{\eta}_B / (16\pi^2) \approx 1.34 \times 10^{-3}$, and taking into account the expression for the effective NP Lagrangians (2.1) and (4.6), one gets the following relation between the NP coefficients and parameters of the simplified Z' model [61, 66]:

$$C_9^\mu = -C_{10}^\mu = -\frac{\sqrt{2}\pi}{2G_F m_{Z'}^2 \alpha_{\text{em}}} \left(\frac{g_{13}^Q g_{22}^L}{V_{tb} V_{td}^*} \right), \quad (4.8)$$

$$C_{bd}^{LL} = \frac{\eta(m_{Z'})}{4\sqrt{2}G_F m_{Z'}^2} \left(\frac{g_{13}^Q}{V_{tb} V_{td}^*} \right)^2, \quad (4.9)$$

where $\eta(m_{Z'}) = (\alpha_s(m_{Z'})/\alpha_s(m_b))^{6/23}$ accounts for running from the $m_{Z'}$ scale down to the b -quark mass scale. Assuming that $C_9^{\text{NP}} = -C_{10}^{\text{NP}}$ in eq. (2.9) is given by left-handed quark currents and the coupling g_{13}^Q is real, taking into account eqs. (4.7), (4.8), and (4.9) and using the experimental (4.2) and average (4.3) values for ΔM_d we get constraints on parameters g_{13}^Q and $m_{Z'}$ presented in figure 3 (for three different reference values of $g_{22}^L = 0.2$, $g_{22}^L = 1$ and $g_{22}^L = \sqrt{4\pi}$). The red area corresponds to favored values from mixing and the blue one from the data on the $B^\pm \rightarrow \pi^\pm \mu^+ \mu^-$ decays, both at 1σ level (in the scenario given in eq. (2.9)). We notice that preferred area of g_{13}^Q and $m_{Z'}$ parameters shrinks with smaller values of g_{22}^L as one can see from comparing the plots in figure 3 from right to left.

So, we arrive to the conclusion that a simplified model with Z' -boson that couples with left-handed $b \rightarrow d$ quark current might potentially explain current data on $B^\pm \rightarrow \pi^\pm \mu^+ \mu^-$ and $B^0 \rightarrow \mu^+ \mu^-$ decays without spoiling B^0 -mixing only for relatively large values of g_{22}^L . On the other side, small coupling g_{22}^L disfavor large values of Z' mass, as one can see from the left plot of figure 3. Curiously, more or less the same picture is found in the case of $b \rightarrow s$ transition, see e.g. ref. [66].

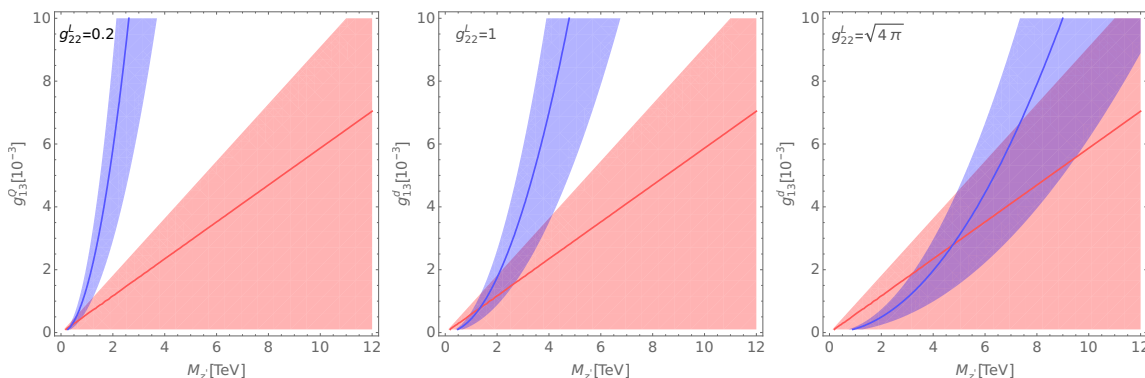


Figure 3. Bounds from $B^0 - \bar{B}^0$ mixing on the coupling g_{13}^Q and $m_{Z'}$ for fixed $g_{22}^L = 0.2$ (left), $g_{22}^L = 1$ (middle) and $g_{22}^L = \sqrt{4\pi}$ (right). The red area corresponds to preferred 1σ region from ΔM_d and the blue one to 1σ region from $B^\pm \rightarrow \pi^\pm \mu^+ \mu^-$ decays.

5 Conclusion and discussion

In contrast to the well studied and measured $b \rightarrow c$ and $b \rightarrow s$ flavour transitions where several anomalies have been found, the $b \rightarrow d\ell^+\ell^-$ processes are so far poorly investigated experimentally due to an additional suppression by CKM matrix elements. Nevertheless, recent experimental data by LHCb collaboration for the differential q^2 -distribution in $B^\pm \rightarrow \pi^\pm \mu^+ \mu^-$ decays deviate a bit more than 1σ from the recent Standard Model prediction. Interestingly, this slight tension points in the same direction as in $b \rightarrow s\mu^+ \mu^-$ decays. In this work, we performed a model-independent fit and obtained 1σ intervals for the NP Wilson coefficients C_9^{NP} and C_{10}^{NP} in different scenarios. Note that our results allow so far for both left- and -right-handed quark currents despite the latter is quite close to the experimental bound on $B^0 \rightarrow \mu^+ \mu^-$ decay. Considering a specific simplified model with a Z' -boson that couples with left-handed fermions ($b-d$ and $\mu-\mu$ currents) we found an 1σ range of NP parameters (the couplings g_{13}^Q , g_{22}^L and Z' -boson mass $m_{Z'}$) that is consistent with current experimental data on $B^\pm \rightarrow \pi^\pm \mu^+ \mu^-$ and $B^0 \rightarrow \mu^+ \mu^-$ decays and $B^0 - \bar{B}^0$ mixing.

To make more robust statements concerning New Physics in the $b \rightarrow d$ sector more experimental data on semileptonic and leptonic $b \rightarrow d$ processes will be necessary, including (1) a more precise measurement of $B \rightarrow \pi\mu^+ \mu^-$ decays, (2) an upcoming measurement of $\bar{B}_s^0 \rightarrow K^{0*} \mu^+ \mu^-$, (3) a first measurement of $B \rightarrow \rho\mu^+ \mu^-$, (4) a more accurate measurement of $B^0 \rightarrow \mu^+ \mu^-$. Additionally, measurements of the semileptonic $b \rightarrow d\ell^+\ell^-$ processes with electrons or τ -leptons will provide an additional test of the Lepton Flavour Universality in the SM.

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A Correlation between $B \rightarrow K\ell^+\ell^-$ and $B \rightarrow \pi\ell^+\ell^-$ decays

We consider the ratio of the partially integrated branching fraction of $B^- \rightarrow K^-\ell^+\ell^-$ and $B^- \rightarrow \pi^-\ell^+\ell^-$ decays, which can be written as [49]

$$\frac{\mathcal{B}(B^- \rightarrow \pi^-\ell^+\ell^-[q_1^2, q_2^2])}{\mathcal{B}(B^- \rightarrow K^-\ell^+\ell^-[q_1^2, q_2^2])} = \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{\mathcal{F}_{B\pi}[q_1^2, q_2^2]}{\mathcal{F}_{BK}[q_1^2, q_2^2]} \left\{ 1 + \kappa_d^2 \frac{\mathcal{D}_{B\pi}[q_1^2, q_2^2]}{\mathcal{F}_{B\pi}[q_1^2, q_2^2]} + 2\kappa_d \left(\cos \xi_d \frac{\mathcal{C}_{B\pi}[q_1^2, q_2^2]}{\mathcal{F}_{B\pi}[q_1^2, q_2^2]} - \sin \xi_d \frac{\mathcal{S}_{B\pi}[q_1^2, q_2^2]}{\mathcal{F}_{B\pi}[q_1^2, q_2^2]} \right) \right\}, \quad (\text{A.1})$$

where $\kappa_d e^{i\xi_d} = (V_{ub}V_{ud}^*)/(V_{tb}V_{td}^*)$. In the above expression, the CKM matrix elements are explicitly isolated and the quantities \mathcal{F}_{BK} , $\mathcal{F}_{B\pi}$, $\mathcal{D}_{B\pi}$, $\mathcal{C}_{B\pi}$ and $\mathcal{S}_{B\pi}$ are CKM independent and accumulate contributions from Wilson coefficients, form factors, non-local hadronic amplitudes and phase space integration. Explicit expressions of the above quantities can be found in ref. [49] and their numerical values for the bin $[1 - 6] \text{ GeV}^2$ are quoted in table 4 in ref. [49] where no correlations were taken into account. Nevertheless, the $B \rightarrow K$ and $B \rightarrow \pi$ form factors have been determined using the LCSR method, and due to common input involved in both sum rules the $B \rightarrow K$ and $B \rightarrow \pi$ form factors are actually correlated to each other. To fill this gap, we improve the numerical analysis by accounting the correlation between both LCSRs for vector $B \rightarrow \pi$ and $B \rightarrow K$ form factors and as a consequence we calculate the correlations between the quantities \mathcal{F}_{BK} , $\mathcal{F}_{B\pi}$, $\mathcal{D}_{B\pi}$, $\mathcal{C}_{B\pi}$, and $\mathcal{S}_{B\pi}$. In the corresponding statistical simulation we use the same input as in ref. [49]. The resulting correlation matrix for the bin $[1 - 6] \text{ GeV}^2$ is

$$\left(\begin{array}{c|ccccc} & \mathcal{F}_{BK} & \mathcal{F}_{B\pi} & \mathcal{D}_{B\pi} & \mathcal{C}_{B\pi} & \mathcal{S}_{B\pi} \\ \hline \mathcal{F}_{BK} & 1 & 0.53 & 0.02 & 0.08 & -0.09 \\ \mathcal{F}_{B\pi} & 0.53 & 1 & 0.07 & 0.24 & -0.15 \\ \mathcal{D}_{B\pi} & 0.02 & 0.07 & 1 & 0.83 & -0.34 \\ \mathcal{C}_{B\pi} & 0.08 & 0.24 & 0.83 & 1 & 0.03 \\ \mathcal{S}_{B\pi} & -0.09 & -0.15 & -0.34 & 0.03 & 1 \end{array} \right). \quad (\text{A.2})$$

The above matrix represents an addition to our numerical result presented in table 4 in ref. [49] and to be used in any further analysis, e.g. in determination of CKM matrix elements from the observables in the $B^\pm \rightarrow \pi^\pm\ell^-\ell^-$ and $B^\pm \rightarrow K^\pm\ell^-\ell^-$ decays (see ref. [49] for more details) or for testing some BSM scenarios where one needs to account for possible correlation by considering both $b \rightarrow s$ and $b \rightarrow d$ transitions.

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