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Erratum: Towards a holographic quark matter crystal

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We have identified a sign in our paper that needed to be corrected. The main conclusion of our paper was the presence of instabilities that suggest the existence of crystalline phases in the system. This conclusion remains unchanged. However, this minus sign leads to a slight modification of some equations and plots, including the ones capturing the details of the instabilities of interest. Given that several (mostly minor) corrections need to be carried out throughout the paper, this erratum cannot be simply summarised in a few lines. We proceed to list all the changes here, following the same numeration as in the original paper. A final version of the paper, with all corrections implemented, is available at [arXiv:1707.06989](https://arxiv.org/abs/1707.06989).

1 Introduction and summary

- As a consequence of the corrected sign there is no critical value of the charge density. Thus, we rewrite the seventh paragraph of the introduction, which now reads:

Our results are pictorially summarised in figure 1, in which each solution on the gravity side, or equivalently each Renormalization Group (RG) flow on the gauge theory side, is represented by a curve running from top to bottom. Each curve corresponds to a different value of the charge density. A point on a curve corresponds to a given energy scale. At zero temperature a solution is described by an entire curve. Cutting off a curve at different points would correspond to different solutions with the same charge density and different non-zero values of the temperature. In the limit $T \gg N_q^{1/3}$ the solutions were constructed perturbatively in $N_q^{1/3}/T$ in [2, 3]. In this limit, however, all the relevant IR physics is hidden behind the horizon.

Figure 2. Phase diagram of the system as a function of temperature and charge density. The dimensionless quantities $\bar{N}_q$ and $\bar{T}$ are defined in (2.60) and (3.8), respectively. The different curves correspond to the locus where the indicated quantity crosses zero and thus changes sign (see table 1) and divide the diagram into six regions. Note that Region III includes a small area below and to the left of region IV.

- When describing the main results, two paragraphs below eq. (1.3), the text and figures change in several places. The new description until the end of this section now reads:

The main result of our analysis is the phase diagram of the system. This is schematically summarised in figure 2. We will refer to this figure as a “phase diagram” despite the fact that it is a slight abuse of terminology, since some of the solutions are unstable and we have not identified the putative stable solution that would be thermodynamically preferred. The signs of different physical quantities in each region of figure 2 are summarised in table 1, where C_Q is the specific heat at constant charge (5.4), χ is the charge susceptibility (5.2), c_s^2 is the speed of sound (5.8), and D is the charge diffusion constant (6.4). Note that the latter equation implies that the last column is the product of the first three.

	C_Q	c_s^2	χ	D
I	−	−	+	+
II	+	+	+	+
III	+	+	−	−
IV	+	−	−	+
V	+	−	+	−
VI	−	−	−	−

Table 1. Signs of several physical quantities in the different regions shown in figure 2.

Region I describes the high-temperature behavior of the system. In this region the solutions are locally thermodynamically unstable because the specific heat at constant charge, C_Q , is negative. The system is also afflicted by a dynamical instability associated to a negative speed of sound squared, $c_s^2 < 0$. The behavior of C_Q and c_s^2 in Region I is qualitatively analogous to the behavior in the high-temperature limit of the neutral solution of [1]. In the neutral case C_Q and c_s^2 become negative at exactly the same point because they are related through $c_s^2 = s/C_Q$, with s the entropy density, which is of course positive. It is remarkable that this feature extends to charged solutions, at least within our numerical precision. In other words, the top curve in figure 2 indicates the locus where both C_Q and c_s^2 change sign simultaneously. The behavior of C_Q and c_s^2 in region VI is the same as in Region I. In addition, in this region the charge susceptibility χ is negative. This indicates an additional thermodynamic instability towards charge clustering, and also results in an additional dynamical instability since the charge diffusion constant becomes negative. As explained in [1], the high-temperature behavior follows directly from the properties of the LP. Since our main interest is in IR features that are independent of the UV completion of the theory, we will not discuss Regions I and VI further.

Region III is both thermodynamically and dynamically unstable, since both χ and D are negative. In contrast, Region V is only dynamically unstable since c_s^2 and D are negative. However, while the existence of the other regions is robust, that of Region V may be a numerical artifact due to our finite resolution in parameter space.

Region II is the only locally stable region in the phase diagram. Local thermodynamic stability is guaranteed by the fact that both C_Q and χ are positive. Our analysis shows no sign of a dynamical instability either, since c_s^2 and D are both positive.

Region IV is particularly interesting since it corresponds to the low-temperature, high-charge density regime. This region is locally thermodynamically unstable since $\chi < 0$ and also dynamically unstable since $c_s^2 < 0$.

The negative values of c_s^2 and/or D that we have encountered imply dynamical instabilities in the hydrodynamic sound and charge diffusion channels, respectively, towards the spontaneous breaking of translation invariance. This suggests

Figure 3. Relation between temperature and horizon position at fixed values of the charge density $\sqrt{2}\bar{N}_q$.

that the putative, stable phase in the corresponding regions may be a crystalline phase. We will come back to this point in sections 5 and 6.

2 Gravitational description

- Equations (2.48) and (2.49) now read

$$\mathcal{B} \equiv -\frac{2\kappa_5^2 L^4}{4} \frac{\delta S_5}{\delta C'_t} = -e^{\phi + \frac{4}{3}\sigma - 8w} \frac{L^4 \sqrt{g_{xx}^3}}{4\sqrt{-g_{tt} g_{rr}}} \left(C'_t + \frac{Q_f}{L} 2\pi \ell_s^2 \mathcal{A}'_t \right), \quad (2.48)$$

and

$$\mathcal{B}' = -\frac{2\kappa_5^2 L^4}{4} \frac{\delta S_5}{\delta C_t} = -2e^{\phi - 4\sigma + 4w} L^2 \frac{\sqrt{g_{xx}^3 g_{rr}}}{\sqrt{-g_{tt}}} C_t. \quad (2.49)$$

3 Numerical solutions

- Figure 3 changes slightly to the one provided in this erratum.
- Three paragraphs before eq. (3.14) we eliminate a reference to the critical value of the charge density, which is not present anymore. This happens again later in the section, but we will not mention it explicitly any further.
- The behaviour of the IR coefficients in figures 4 and 5, and the corresponding description of these plots in the two paragraphs below eq. (3.15), change to:

We see that for large values of r_h all the curves converge to a single one, reflecting the fact that the leading UV behavior is controlled by the neutral solution. In contrast, for sufficiently small values of r_h the charged curves deviate from the neutral one and approach straight lines with slopes that agree precisely with

Figure 4. IR coefficients ϕ_h and b_h for different values of the charge density $\sqrt{2}\bar{N}_q$ (given in the legend of the plot on the left-hand side). The dotted gray lines show the behavior determined by the IR Lifshitz solution.

Figure 5. Radial profile of the dilaton for solutions with the smallest value of r_h (where the curves end) for different values of $\sqrt{2}\bar{N}_q$ (see legend in figure 4).

those predicted by eqs. (3.14) and (3.15). For the dilaton the normalization also agrees with that in (3.15), whereas for the b_h parameter we have performed a fit to (3.14). As expected, the value of r_h at which the IR behavior sets in decreases as the charge density decreases. In particular, this means that solutions with small $\bar{N}_q$ develop an intermediate region controlled by the log-AdS geometry of the neutral solution, clearly visible in the left plot of figure 4, whereas solutions with large $\bar{N}_q$ transition directly from the UV controlled by the neutral solution to the IR controlled by the Lifshitz solution.

At a qualitative level, one would expect the value of the dilaton at the horizon for a solution with non-zero temperature to be approximately the same as the value of the dilaton at the position $r = r_h$ for a zero-temperature solution. In other words, the scalar fields in a solution with non-zero temperature should be similar to those in a solution at zero temperature cut off at the corresponding value of the radial coordinate. This is confirmed by the similarity between figure 4(left) and figure 5, where we plot the radial dependence of the dilaton for the coldest solution that we have constructed for each of the values Q_{st} .

Figure 6. Fit of the function b_h^0 defined in (3.14).

Figure 7. IR coefficients f_h, g_h for different values of $\sqrt{2} \overline{N}_q$ (see legend in figure 4).

- Figures 6 and 7 change slightly, but the qualitative behaviour, and therefore our discussion of the plots, does not get altered.
- The description of figure 8 in the last two paragraphs of page 21, and the figure itself, change slightly:

Finally, figure 8(left) shows the horizon value $\mathcal{B}_h$. We see that at small r_h the slope is that determined by the leading term in (3.13), i.e. $\mathcal{B}_h \sim r_h^{10}$, whereas at large r_h the horizon value of $\mathcal{B}$ approaches the negative constant corresponding to the leading term in (3.6).

To summarise so far, we have seen that the large- and small- r_h values of the IR parameters of our numerical solutions match precisely those predicted by the LP and Lifshitz geometries, respectively. This strongly supports the fact that our configuration interpolates between the UV LP geometry of the neutral solution and the IR Lifshitz one. Moreover, the behavior of the dilaton at intermediate values of r_h for small values of Q_{st} supports that in this situation the transition

Figure 8. IR and UV coefficients associated to the function $\mathcal{B}$ for different values of $\sqrt{2}\overline{N}_q$ (see legend in figure 4). The dotted gray lines show the behavior determined by the IR Lifshitz solution.

Figure 9. UV coefficients κ_ϕ and κ_b for different values of $\sqrt{2}\overline{N}_q$ (see legend in figure 4).

Figure 10. UV coefficients κ_f and κ_{f_2} for different values of $\sqrt{2}\overline{N}_q$ (see legend in figure 4).

between the LP and the Lifshitz geometries proceeds through an intermediate log-AdS region.

- Figures 9–12 change to the ones provided here.

Figure 11. Dilaton scaling $\mathcal{R}_\phi$ as defined in (3.16).

Figure 12. Square of the Ricci tensor, $\text{Ric}^2 = R^{mn}R_{mn}$, for several zero-temperature solutions with different charge densities (see legend in figure 4).

4 Thermodynamics

- Figure 13 is modified, and the last paragraph of this section is changed accordingly to:

We close this section with plots in figure 13 of the energy density, E , the pressure, P , the enthalpy, $E + P$, and the chemical potential, μ . Note that these quantities are positive for all values of $\bar{T}$ and $\bar{N}_q$.

5 Instabilities

- The following two paragraphs right below eq. (5.7) receive minor modifications to accomodate the description of the new figures 14–16:

Figure 13. Dimensionless energy density, pressure, enthalpy and chemical potential, as defined in (4.26) and (4.28).

In figure 14(left) we show a contour plot of the specific heat of our solutions as a function of $\bar{T}$ and $\sqrt{2} \bar{N}_q$, while in figure 14(right) we show slices at several different values of the charge. We see that C_Q is positive at low temperature but becomes negative at high temperature. This property is also illustrated by figure 15, where the negative slope of the entropy density curves as a function of $\bar{T}$ is evident at high $\bar{T}$. In figures 14(right) and 15 we see that all curves converge to the same one at high $\bar{T}$. The common curve is the same as in the neutral case studied in [1]. As explained in that reference, the negativity of C_Q in this region is an UV effect associated to the presence of the LP. Since in this paper we are interested in IR physics that is safe from LP effects, we will not elaborate further on this instability.

In figure 16 we show a contour plot of the inverse charge susceptibility of our solutions as a function of $\bar{T}$ and $\sqrt{2} \bar{N}_q$. In contrast to the case of the specific

Figure 14. (Left) Contour plot of the dimensionless specific heat (4.25). The thick black curve indicates the locus where $C_Q = 0$. (Right) Dimensionless specific heat as a function of temperature for several fixed values of the charge density $\sqrt{2}\bar{N}_q$. Note that the black curve is hardly visible because it falls almost exactly behind the red curve.

Figure 15. Entropy density as a function of temperature for various values of the charge density $\sqrt{2}\bar{N}_q$.

heat, we see that χ^{-1} is negative at arbitrarily low $\bar{r}_h$ and $\bar{T}$ for charge densities $\sqrt{2}\bar{N}_q \gtrsim 4.5$. The negativity of χ signals an instability towards charge clustering, which suggests that the putative, stable phase in this region may break translational invariance spontaneously. We will come back to this point in section 6.

- Figure 17 is modified, and its description after equation (5.9) reads:

In figure 17(left) we show a contour plot of the result, and in figure 17(right) several slices at constant charge. As for C_Q , the region where c_s^2 is negative includes a UV region sensitive to LP effects that is of no interest to us. However,

Figure 16. Contour plot of the inverse charge susceptibility. The thick, black, continuous curve indicates the locus where $\overline{\chi}^{-1}$ changes sign.

Figure 17. (Left) Contour plot of the speed of sound squared. The thick, black, continuous curve indicates the locus where c_s^2 changes sign. (Right) Plots of the speed of sound as a function of temperature for several fixed values of the charge density $\sqrt{2}\overline{N}_q$. The speed of sound diverges towards positive (negative) infinity as the upper black curve is approached from below (above). This is why the scale of the contour plot has such a large range.

Figure 18. Functions f and g' defined through eqs. (5.11) and (5.13), respectively.

Figure 19. Mass of a string stretching from the origin of a zero-temperature solution to the position of the LP as a function of the charge density, normalized to the value at $\bar{N}_q = 0$.

in contrast to C_Q and similarly to χ , c_s^2 is also negative in the IR region of arbitrarily low temperature and $\sqrt{2} \bar{N}_q \gtrsim 4.5$.

- Figure 18 changes slightly, with g' being monotonically increasing with the charge density.

6 Discussion

- Figure 19 changes slightly.
- The region where $c_s^2 < 0$, mentioned right before equation (6.3), changes in the new nomenclature of figure 2, most interestingly in Region IV.
- The paper ends after equation (6.4) with the following two paragraphs:

$$D = \frac{\sigma}{\chi} \frac{E + P}{c_s^2 T C_Q}. \quad (6.4)$$

In this expression σ is the electrical conductivity, which must be positive in order for the divergence of the entropy current of first-order hydrodynamics to be non-negative, i.e. in order for the second law of thermodynamics to hold. A negative value of D indicates a dynamical instability towards charge clustering (anti-diffusion), thus also generating inhomogeneities. Our numerical results show that this is the situation in several regions of the phase diagram 2, most interestingly in Regions III and V.

To conclude, we reiterate that some of the instabilities that we have identified suggest the possible existence of quark matter crystalline phases in our model, but establishing this definitively requires further analysis. We hope to report on these issues in the near future.

- Figure 20 is removed.

7 Appendices A–F

- In appendix B the last term of equation (B.2) gets a sign

$$\partial_y \left(\frac{\sqrt{-G_{tt}}}{\sqrt{G_{rr} G_f}} \frac{\mathcal{B}'}{\sqrt{G_{xx}^3}} \right) - 8 \frac{\sqrt{-G_{tt} G_{rr} G_f}}{\sqrt{G_{xx}^3 G_b^4}} \mathcal{B} - \frac{2e^\phi \sqrt{-G_{tt} G_{rr}} (\bar{N}_q + \mathcal{B})}{\sqrt{G_{xx}^3 G_b^2 G_f + e^{2\phi} (\bar{N}_q + \mathcal{B})^2}} = 0. \quad (\text{B.2})$$

The terms in (B.3) multiplying the A and E terms also switch sign

$$\mathcal{F}_{A,B,C,D,E} = \frac{e^{2\phi}}{G_{xx}^3 G_b^4 G_f} \left(-A G_b^2 \mathcal{B}'^2 + \frac{B}{2} G_{xx}^3 G_{rr} G_b^4 + \frac{C}{2} G_{xx}^3 G_{rr} \right. \\ \left. + 8D \bar{N}_q^2 G_{rr} G_b^4 G_f - 8E G_{rr} G_f \mathcal{B}^2 \right). \quad (\text{B.3})$$

- In appendix C equations (C.10) and (C.11) now read

$$\mathcal{B}'' + 2 \left(3 - 2e^{2(f-g)} \right) \mathcal{B}' - 2e^{2(f-g)} \left(Q_f e^\phi + 4e^{2(f-g)} \right) \mathcal{B} - \frac{Q_c Q_{st}}{2} e^{\phi+2(f-g)} = 0, \quad (\text{C.10})$$

and

$$\mathcal{B}'' + 6 \mathcal{B}' - 2e^{\phi+2(f-g)} \left(Q_f \mathcal{B} + \frac{Q_c Q_{st}}{4} \right) = \mathcal{O}(r^{-4}). \quad (\text{C.11})$$

- Appendix F is removed.

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