# Correlation between $\boldsymbol{R}_{D^{(*)}}$ and top quark FCNC decays in leptoquark models 

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AbSTRACT: Some interpretations of $R_{D^{(*)}}$ anomaly in $B$ meson decay using leptoquark (LQ) models can also generate top quark decays through Flavor Changing Neutral Current (FCNC). In this work we focus on two LQs, i.e. scalar $S_{1}$ and vector $U_{1}$ which are both singlet under the $\operatorname{SU}(2)_{L}$ gauge group in the Standard Model (SM). We investigate their implications on the 3-body top FCNC decays $t \rightarrow c \ell_{i} \ell_{j}$ at tree level and the 2-body $t \rightarrow c V$ at one-loop level, with $\ell$ being the SM leptons and $V=\gamma, Z, g$ being the SM gauge bosons. We utilize the $2 \sigma$ parameter fitting ranges of the LQ models and find that $\operatorname{Br}\left(t \rightarrow c \ell_{i} \ell_{j}\right)$ at tree level can reach $\mathcal{O}\left(10^{-6}\right)$ and $\operatorname{Br}(t \rightarrow c V)$ at one-loop level can reach $\mathcal{O}\left(10^{-10}\right)$. Some quick collider search prospects are also analyzed.

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## 1 Introduction

The deviations of $B$ meson decays from the Standard Model (SM) predictions have attracted a lot of attention in the past several years [1-7]. Two significant processes are $R_{D^{(*)}}$ and $R_{K^{(*)}}$ which are defined through ratios of Branching Ratios (BRs) as follows:

$$
\begin{equation*}
R_{D^{(*)}}=\left.\frac{\operatorname{Br}\left(B \rightarrow D^{(*)} \tau \bar{\nu}\right)}{B r\left(B \rightarrow D^{(*)} l \bar{\nu}\right)}\right|_{l \in\{e, \mu\}} \quad, \quad R_{K^{(*)}}^{\left[q_{1}^{2}, q_{2}^{2}\right]}=\frac{\int_{q_{1}^{2}}^{q_{2}^{2}} d q^{2} \frac{\partial}{\partial q^{2}} \operatorname{Br}\left(B \rightarrow K^{(*)} \mu \mu\right)}{\int_{q_{1}^{2}}^{q_{2}^{2}} d q^{2} \frac{\partial}{\partial q^{2}} \operatorname{Br}\left(B \rightarrow K^{(*)} e e\right)}, \tag{1.1}
\end{equation*}
$$

with $q^{2}=\left(p_{l^{+}}+p_{l^{-}}\right)^{2}$ between $q_{1}^{2}$ and $q_{2}^{2}$ in units of $\mathrm{GeV}^{2}$. For $R_{D^{(*)}}$, the world-averaged results after the recent update from Belle Collaboration [8] are: ${ }^{1}$

$$
\begin{align*}
R_{D} & =0.334 \pm 0.031, & & R_{D^{*}}=0.297 \pm 0.015,  \tag{1.2}\\
R_{D}^{S M} & =0.299 \pm 0.003, & & R_{D^{*}}^{S M}=0.258 \pm 0.005, \tag{1.3}
\end{align*}
$$

which are larger than the SM predictions at about $3.1 \sigma$ [10-15].
The latest measurements of $R_{K}$ at LHCb [16] and $R_{K^{*}}$ at Belle [17] are: ${ }^{2}$

$$
\begin{equation*}
R_{K}^{[1.1,6]}=0.846_{-0.054}^{+0.060}(\text { stat. })_{-0.014}^{+0.016}(\text { sys. }), \quad R_{K^{*}}^{[1.1,6]}=0.96_{-0.29}^{+0.45}(\text { stat. })_{-0.11}^{+0.11}(\text { sys. }), \tag{1.4}
\end{equation*}
$$

which are smaller than the SM predictions shown below $[18,19]$ at around $2 \sigma$.

$$
\begin{equation*}
R_{K}^{[1.1,6], \mathrm{SM}}=1.00 \pm 0.01, R_{K^{*}}^{[1.16], \mathrm{SM}}=1.00 \pm 0.01 \tag{1.5}
\end{equation*}
$$

[^0]The observed deviations of $R_{D^{(*)}}^{\exp }>R_{D^{(*)}}^{\mathrm{SM}}$ and $R_{K^{(*)}}^{\exp }<R_{K^{(*)}}^{\mathrm{SM}}$ have motivated many interpretations by imposing physics beyond the SM (see a recent review in [20] and references therein). Many of the theoretical proposals introduce additional charged scalars [21-31] and/or vectors [32-41] to mediate the Charged Current (CC) in $R_{D^{(*)}}$ and Neutral Current (NC) in $R_{K^{(*)}}$, which can be realized in various UV-complete models. Recent discussions can be found in [42-60] and also recently in [61-97].

In this work we are not going to be ambitious to explain both deviations, but limit ourselves to $R_{D^{(*)}}$ interpretations in the leptoquark (LQ) models [98-118] and its interesting correlations to the top quark Flavor Changing Neutral Current (FCNC) decays. Recently, several studies investigated the implications of the six types of LQ models on $R_{D^{(*)}}$ and $R_{K^{(*)}}$, including three scalars $\left\{S_{1}, R_{2}, S_{3}\right\}$ and three vectors $\left\{U_{1}, V_{2}, U_{3}\right\}$ where the subscript denotes $2 T_{3}+1$ with $T_{3}$ being the LQ's weak isospin. Results show that three of them are still capable of accommodating $R_{D^{(*)}}$ excess while satisfying other flavor constraints, i.e. SM $\operatorname{SU}(2)_{L}$ singlet scalar $S_{1}$ and vector $U_{1}$, as well as $\operatorname{SU}(2)_{L}$ doublet scalar $R_{2}$.

In this work we concentrate on the two $\mathrm{SU}(2)_{L}$ singlet scenarios, i.e. $S_{1}$ and $U_{1}$, motivated by the simplicity and, as we will see later, the resulting clear correlation patterns between $R_{D^{(*)}}$ explanations and the top decays through FCNC. Note that the benchmark parameters we utilize in the numerical analysis may not be able to produce the observed $R_{K^{(*)}}$ anomaly. For example, requiring $S_{1}$ to explain $R_{K^{(*)}}$ appears to result in conflict with $R_{D}^{\mu / e}=\operatorname{Br}(B \rightarrow D \mu \nu) / \operatorname{Br}(B \rightarrow D e \nu)$ [119]. On the contrary, it has been shown that $U_{1}$ can still simultaneously generate the observed $R_{D^{(*)}}$ and $R_{K^{(*)}}$ [120]. Putting aside the complexities in accommodating both anomalies, in this work we will exclusively investigate the $R_{\left.D^{*}\right)}$ interpretation and the interesting correlations with the top quark FCNC when introducing LQ $S_{1}$ or $U_{1}$.

This paper is organized as follows. In section 2 we briefly capture the Lagrangian we consider for the scalar LQ $S_{1}$ and the vector LQ $U_{1}$, and the effective operators they generate in low-energy processes for $R_{D^{(*)}}$. In section 3 we present the results for top FCNC decays induced at both tree level and one-loop level. Collider search prospects are given in section 4 and Conclusion will be given in section 5. Appendix includes full expressions of one-loop Wilson coefficients of $t \rightarrow c V$ at one-loop level induced by the scalar LQ $S_{1}$.

## $2 \mathrm{LQ} S_{1}$ and $U_{1}$ for $R_{D^{(*)}}$

In this section we briefly capture the low-energy theory in terms of effective operators for $R_{D^{(*)}}$ and the Wilson coefficients generated by the scalar LQ $S_{1}$ and vector LQ $U_{1}$, respectively. Then we present the theoretical correlations between $R_{D^{(*)}}$ and BRs of top FCNC. We denote LQ as $\left(\mathrm{SU}(3)_{c}, \mathrm{SU}(2)_{L}\right)_{Y}$ which is its representation in the SM gauge group [120, 121]. Considering the misalignments between gauge and mass eigenstates in the quark sector, we define the left-handed quark doublet as $Q_{i}=\left[\left(V^{\dagger} u_{L}\right)_{i} d_{L i}\right]^{T}$ where $V$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

As mentioned earlier, we will focus on two LQs which are both singlet under the SM $\mathrm{SU}(2)_{L}$ group, i.e. scalar $S_{1} \equiv(\overline{\mathbf{3}}, \mathbf{1})_{1 / 3}$ and vector $U_{1}=(\mathbf{3}, \mathbf{1})_{2 / 3}$. Their interactions with
the SM fields we consider are

$$
\begin{align*}
& \mathcal{L}_{S_{1}}=g_{1 L}^{i j} \overline{Q_{i}^{C}} i \tau_{2} L_{j} S_{1}+g_{1 R}^{i j} \overline{u_{R i}^{C}} e_{R j} S_{1}+\text { h.c. },  \tag{2.1}\\
& \mathcal{L}_{U_{1}}=h_{1 L}^{i j} \bar{Q}_{i} \gamma_{\mu} L_{j} U_{1}^{\mu}+h_{1 R}^{i j} \bar{d}_{R i} \gamma_{\mu} \ell_{R j} U_{1}^{\mu}+\text { h.c. } \tag{2.2}
\end{align*}
$$

where $g_{1 L}^{i j}, g_{1 R}^{i j}$ and $h_{1 L}^{i j}, h_{1 R}^{i j}$ are matrices of new Yukawa interactions in the general case, and $\tau_{2}$ is the second Pauli matrix. We have neglected the terms of diquark couplings to LQ to ensure the stability of proton [121]. Note again that we have chosen the form of the left-handed quark doublet as $Q_{i}=\left[\left(V^{\dagger} u_{L}\right)_{i} d_{L i}\right]^{T}$ in which the down-type quarks are mass eigenstates. Therefore it will be $\left(V g_{1 L}\right)^{i j}$ and $\left(V h_{1 L}\right)^{i j}$ that enter the interactions involving up-type left handed quarks.

The general low-energy effective dimension-six operators involved in $B \rightarrow D^{(*)} \tau \bar{\nu}$ are $[100,110]$

$$
\begin{equation*}
-\mathcal{L}_{\text {eff }}=\left(C_{\mathrm{SM}} \delta_{l \tau}+C_{V_{1}}^{l}\right) O_{V_{1}}^{l}+C_{V_{2}}^{l} O_{V_{2}}^{l}+C_{S_{1}}^{l} O_{S_{1}}^{l}+C_{S_{2}}^{l} O_{S_{2}}^{l}+C_{T}^{l} O_{T}^{l}, \tag{2.3}
\end{equation*}
$$

with $l=1,2,3$ being the neutrino generation index. $C_{\mathrm{SM}}=2 \sqrt{2} G_{F} V_{c b}$ is the SM contribution where $G_{F}$ is the Fermi constant. Operators above are defined as

$$
\begin{array}{rlrl}
O_{V_{1}}^{l} & =\left(\bar{c}_{L} \gamma^{\mu} b_{L}\right)\left(\bar{\tau}_{L} \gamma_{\mu} \nu_{l L}\right), & O_{V_{2}}^{l}=\left(\bar{c}_{R} \gamma^{\mu} b_{R}\right)\left(\bar{\tau}_{L} \gamma_{\mu} \nu_{l L}\right), \\
O_{S_{1}}^{l} & =\left(\bar{c}_{L} b_{R}\right)\left(\bar{\tau}_{R} \nu_{l L}\right), & O_{S_{2}}^{l}=\left(\bar{c}_{R} b_{L}\right)\left(\bar{\tau}_{R} \nu_{l L}\right), \\
O_{T}^{l} & =\left(\bar{c}_{R} \sigma^{\mu \nu} b_{L}\right)\left(\bar{\tau}_{R} \sigma_{\mu \nu} \nu_{l L}\right) & & \tag{2.6}
\end{array}
$$

The Wilson coefficients generated by $S_{1}$ and $U_{1}$ at the energy scale $\mu=M_{\mathrm{LQ}}$ are

$$
\begin{align*}
C_{V_{1}}^{l} & =\sum_{k=1}^{3} V_{k 3}\left(\frac{g_{1 L}^{k l} g_{1 L}^{23 *}}{2 M_{S_{1}}^{2}}+\frac{h_{1 L}^{2 l} h_{1 L}^{k 3 *}}{M_{U_{1}}^{2}}\right), & C_{V_{2}}^{l}=0,  \tag{2.7}\\
C_{S_{1}}^{l} & =\sum_{k=1}^{3} V_{k 3}\left(-\frac{2 h_{1 L}^{2 l} h_{1 R}^{k 3 *}}{M_{U_{1}}^{2}}\right), & C_{S_{2}}^{l}=\sum_{k=1}^{3} V_{k 3}\left(-\frac{g_{1 L}^{k l} g_{1 R}^{23 *}}{2 M_{S_{1}}^{2}}\right),  \tag{2.8}\\
C_{T}^{l} & =\sum_{k=1}^{3} V_{k 3}\left(\frac{g_{1 L}^{k l} g_{1 R}^{23 *}}{8 M_{S_{1}}^{2}}\right) & \tag{2.9}
\end{align*}
$$

For simplicity, in the following we only consider terms with $k=3$ and $V_{33} \approx 1$ for the LQ contributions. ${ }^{3}$ We note that [110] has provided the parameter ranges for various LQ models which can fit the $R_{D^{(*)}}$ data (see table.II therein), as well as how they confront other flavor constraints. For example, a small $g_{1 L}^{2 l}$ can help $S_{1}$ pass the constraints from $\bar{B} \rightarrow X_{s} \nu \bar{\nu}$ while having available $g_{1 L}^{3 l} g_{1 R}^{23 *}$ to interpret $R_{D^{(*)}}$. Note that there are only two parameters in our analysis when choosing a certain generation index $l$, i.e. $g_{1 L}^{3 l}, g_{1 R}^{23}$ for $S_{1}$ and $h_{1 L}^{2 l}, h_{1 L}^{k 3}$ for $U_{1}$, which is different from the more complex textures in other works, e.g. [120, 122]. Our choices can result in clear correlations between $R_{D^{(*)}}$ and top FCNC decays.

[^1]

Figure 1. Tree-level top FCNC decays considered in this work, induced by $\mathrm{SU}(2)$ singlet scalar LQ $S_{1}$ and vector LQ $U_{1}$.


Figure 2. One-loop top FCNC decays of $t \rightarrow c \gamma$ considered in this work, induced by $\mathrm{SU}(2)$ singlet scalar LQ $S_{1}$.


Figure 3. One-loop top FCNC decays of $t \rightarrow c \gamma$ induced by $\mathrm{SU}(2)$ singlet vector LQ $U_{1}$. Note that we do not calculate these diagrams in this work, due to the lack of ultraviolet completion for vector LQ $U_{1}$ in our phenomenological studies. See more discussions in section 3.2.

## 3 LQ $S_{1}$ and $U_{1}$ for top quark FCNC

Diagrams of $S_{1}, U_{1}$ contributions to top FCNC at tree level $t \rightarrow c \tau^{-} \ell_{i}^{+}$and $t \rightarrow c \nu_{\tau} \bar{\nu}_{i}$ are provided in figure 1 with $i$ denoting the lepton generation index. Square brackets indicate the chirality of couplings and replacement with particles in the round brackets generate processes involved in $R_{D^{(*)}}$. In figure 2 and figure 3 we also show the one-loop contributions to top FCNC $t \rightarrow c \gamma$ from $S_{1}$ and $U_{1}$, respectively, in which replacing external photon $\gamma$ with $Z$ boson or gluon $g$ with applicable vertices is straightforward.

In the numerical analysis, we utilize the parameter ranges in [110] for various LQ models which can fit the $R_{D^{(*)}}$ data at $2 \sigma$ level (see table.II therein). We remind ourselves that moderate differences in the $2 \sigma$ ranges of parameters presented in different papers do not affect the order of magnitude in top FCNC BRs we will discuss. To be more clear, the parameter ranges we take from [110] in the numerical studies are summarized in table 1. For simplicity, we assume all parameters are real in our analysis.

### 3.1 Tree level

One important feature in the top FCNC decay induced at tree level by LQ $S_{1}$ and $U_{1}$ is that heavy LQ can be reasonably integrated out into effective coefficients in the amplitude,

| LQ | $2 \sigma$ range for $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$ |
| :--- | :--- |
| $S_{1}$ | $g_{1 L}^{3 l} g_{1 R}^{23 *} \in\left(\frac{M_{S_{1}}}{1 \mathrm{TeV}}\right)^{2} \times \begin{cases}(1.64,1.81) & l=1,2 \\ (-0.87,-0.54) & l=3\end{cases}$ |
| $U_{1}$ | $h_{1 L}^{2 l} h_{1 L}^{33 *} \in\left(\frac{M_{U_{1}}}{1 \mathrm{TeV}}\right)^{2} \times \begin{cases}(0.52,0.84) & l=1,2 \\ (-2.94,-2.80) & l=3\end{cases}$ |

Table 1. Parameter ranges we utilize in numerical calculations, taken from table.II of [110]. For simplicity, we assume all parameters are real in our analysis.
i.e. $\frac{g_{1 L} g_{1 R}}{M_{S_{1}}^{2}}$ and $\frac{h_{1 L} h_{1 R}}{M_{U_{1}}^{2}}$, which contribute as a whole piece in both the top FCNC decay and the $R_{D^{(*)}}$. This infers an interesting correlation between the two processes despite the specific values of the couplings and LQ masses, as long as LQ masses are heavy enough to justify the effective coefficients as good approximations of the full calculations.

The top FCNC BRs in figure 1 can be approximated as follows.

$$
\begin{align*}
& S_{1}: \operatorname{Br}\left(t \rightarrow c \tau^{-} \ell_{l}^{+}\right) \approx \frac{1}{\Gamma_{t, \mathrm{SM}}}\left(\frac{m_{t}^{5}}{6144 \pi^{3}}\right)\left|\frac{\left.\right|_{1 L} ^{3 l} g_{1 R}^{23 *}}{M_{S_{1}}^{2}}\right|^{2}=10^{-6} \times \begin{cases}1.4 \sim 1.8 & l=1,2 \\
0.16 \sim 0.41 & l=3\end{cases}  \tag{3.1}\\
& U_{1}: \quad \operatorname{Br}\left(t \rightarrow c \nu_{\tau} \bar{\nu}_{l}\right) \approx \frac{1}{\Gamma_{t, \mathrm{SM}}}\left(\frac{m_{t}^{5}}{1536 \pi^{3}}\right)\left|\frac{h_{1 L}^{33} h_{1 L}^{2 l *}}{M_{U_{1}}^{2}}\right|^{2}=10^{-6} \times \begin{cases}0.58 \sim 1.5 & l=1,2 \\
17 \sim 19 & l=3\end{cases} \tag{3.2}
\end{align*}
$$

In the above, we take the SM parameters as $m_{c} \simeq m_{\tau} \simeq 0, m_{t}=172 \mathrm{GeV}$ and $\Gamma_{t, \mathrm{SM}}=$ 1.5 GeV , while $\frac{g_{1 L} g_{1 R}}{M_{S_{1}}^{2}}$ and $\frac{h_{1 L} h_{1 R}}{M_{U_{1}}^{2}}$ are taken from table 1. The analytic expressions are approximations by integrating out LQ propagators, while the numerical results are obtained from full calculations using MadGraph [123] with model files generated by FeynRules [124].

Note again that the connection between $R_{D^{(*)}}$ and top quark 3-body FCNC decays $\operatorname{Br}\left(t \rightarrow c \ell_{i} \ell_{j}\right) \sim 10^{-6}$ shown above do not depend directly on the specific values of couplings and LQ masses, but on the effective coefficients $\frac{g_{1 L} g_{1} R}{M_{S_{1}}}$ and $\frac{h_{1 L} h_{1 R}}{M_{U_{1}}}$ as a whole piece. It holds well for sufficiently heavy $M_{\mathrm{LQ}}(\gtrsim 1 \mathrm{TeV})$ which can justify the good approximations and suppress the high order terms $\propto \frac{m_{t}^{2}}{M_{\mathrm{LQ}}^{2}}$ in the full calculation.

### 3.2 One-loop level

For $t \rightarrow c V$ with $V=\gamma, g, Z$ at one-loop level, the amplitudes can be expressed in the following form:

$$
\begin{equation*}
i \mathcal{M}_{t c V}=\bar{u}\left(p_{2}\right) \Gamma_{t c V}^{\mu} u\left(p_{1}\right) \epsilon_{\mu}(k, \lambda), \tag{3.3}
\end{equation*}
$$

where $p_{1}, p_{2}$, and $k$ denote the 4 -momenta of the incoming top quark, outgoing charm quark and the outgoing gauge boson, respectively, and $\epsilon_{\mu}(k, \lambda)$ is the polarization vector of the outgoing gauge boson. The vertices $\Gamma^{\mu}$ can be decomposed as follows [125] when
external particles are on-shell:

$$
\begin{align*}
\Gamma_{t c Z}^{\mu} & =\gamma^{\mu}\left(P_{L} f_{V L}^{Z}+P_{R} f_{V R}^{Z}\right)+i \sigma^{\mu \nu} k_{\nu}\left(P_{L} f_{T L}^{Z}+P_{R} f_{T R}^{Z}\right)  \tag{3.4}\\
\Gamma_{t c \gamma}^{\mu} & =i \sigma^{\mu \nu} k_{\nu}\left(P_{L} f_{T L}^{\gamma}+P_{R} f_{T R}^{\gamma}\right)  \tag{3.5}\\
\Gamma_{t c g}^{\mu} & =T^{a} i \sigma^{\mu \nu} k_{\nu}\left(P_{L} f_{T L}^{g}+P_{R} f_{T L}^{g}\right) \tag{3.6}
\end{align*}
$$

with $P_{R, L}=\frac{1}{2}\left(1 \pm \gamma_{5}\right), \sigma^{\mu \nu}=\frac{i}{2}\left[\gamma^{\mu}, \gamma^{\nu}\right]$ and $T^{a}$ are the $\mathrm{SU}(3)$ color generators with $a=1, \ldots, 8$. The partial widths are

$$
\begin{align*}
\Gamma(t \rightarrow c Z)= & \frac{m_{t}^{3}}{32 \pi m_{Z}^{2}}\left(1-\frac{m_{Z}^{2}}{m_{t}^{2}}\right)^{2}\left[\left(1+2 \frac{m_{Z}^{2}}{m_{t}^{2}}\right)\left(\left|f_{V L}^{Z}\right|^{2}+\left|f_{V R}^{Z}\right|^{2}\right)\right. \\
& \left.-6 \frac{m_{Z}^{2}}{m_{t}} R e\left(f_{V L}^{Z} f_{T R}^{Z *}+f_{T L}^{Z} f_{V R}^{Z *}\right)+m_{Z}^{2}\left(2+\frac{m_{Z}^{2}}{m_{t}^{2}}\right)\left(\left|f_{T L}^{Z}\right|^{2}+\left|f_{T R}^{Z}\right|^{2}\right)\right]  \tag{3.7}\\
\Gamma(t \rightarrow c \gamma)= & \frac{m_{t}^{3}}{16 \pi}\left(\left|f_{T L}^{\gamma}\right|^{2}+\left|f_{T R}^{\gamma}\right|^{2}\right)  \tag{3.8}\\
\Gamma(t \rightarrow c g)= & C_{F} \frac{m_{t}^{3}}{16 \pi}\left(\left|f_{T L}^{g}\right|^{2}+\left|f_{T R}^{g}\right|^{2}\right) \tag{3.9}
\end{align*}
$$

where $C_{F}=\left(N_{c}^{2}-1\right) / 2 N_{c}$ with $N_{c}=3$ is the Casimir factor of $\mathrm{SU}(N)$ and we set $m_{c}=0$ for simplicity. We use FeynArts/FormCalc $[126,127]$ to perform the one-loop calculations which is then linked to LoopTools [127] to obtain numerical results.

First of all, we note that in the case of vector LQ $U_{1}$, the model is non-renormalizable by introducing a single vector LQ $U_{1}$. This results in a divergent $U_{1}$ contribution to $t \rightarrow c V$ at one-loop level, unless the ultraviolet (UV) completion is established to generate the $U_{1}$ mass (see e.g. [128-134]). The approach will be model-dependent and we will not address it further in this work. More discussions on effects of $U_{1}$ at one-loop level can be found in, e.g. [118].

In the full calculations we include both the SM and the LQ contributions to take into account the interference effects. For the LQ $S_{1}$ contributions, we present the full expressions of Wilson coefficients at one-loop level in the appendix. In the heavy mass range $M_{S_{1}} \simeq 1 \mathrm{TeV}$ which indicates $M_{S_{1}} \gg m_{t}, m_{c}, m_{\tau}$, one can have approximated results, especially for the massless gauge bosons $V=\gamma, g$. By setting $m_{c}=0$ and taking $x_{\tau}=$ $m_{\tau}^{2} / M_{S_{1}}^{2}, x_{t}=m_{t}^{2} / M_{S_{1}}^{2}$, we have:

$$
\begin{align*}
f_{T L}^{g} \simeq & \frac{1}{16 \pi^{2}} g_{s} m_{\tau} \frac{g_{1 L}^{33} g_{1 R}^{23 *}}{M_{S_{1}}^{2}} \\
& \times \frac{1}{12}\left(-6\left(22 x_{\tau} x_{t}+3 x_{t}+16 x_{\tau}+6\right) \log x_{\tau}-49 x_{t}-48\right)  \tag{3.10}\\
f_{T L}^{\gamma} \simeq & -\frac{1}{16 \pi^{2}} e m_{\tau} \frac{g_{1 L}^{33} g_{1 R}^{23 *}}{M_{S_{1}}^{2}} \\
& \times \frac{1}{3}\left(\frac{1}{6}\left(14 x_{t} x_{\tau}+x_{t}+9 x_{\tau}+3\right)+\left(x_{t}+1\right) x_{\tau} \log x_{\tau}\right),  \tag{3.11}\\
f_{T R}^{g}= & f_{T R}^{\gamma}=f_{T R}^{Z}=0 \tag{3.12}
\end{align*}
$$

where $g_{s}$ is the coupling of strong interaction. In the above, we have utilized PackageX $[135,136]$ to perform the loop function reductions. Note that the absence of right-handed


Figure 4. $\operatorname{Br}(t \rightarrow c V), V=\gamma, g, Z$ at one-loop level induced by $\mathrm{SU}(2)$ singlet scalar LQ $S_{1}$. In the left panel we choose $g_{1 L}^{33} g_{1 R}^{23 *}=1$ as an ordinary coupling benchmark to show the decoupling behavior of LQ $S_{1}$ contribution with respect to $M_{S_{1}}$. In the right panel, we fix $\frac{g_{1 L}^{33} g_{1 R}^{23 *}}{M_{S_{1}}^{2}}=0.87$ which is the upper bound value of numerical fitting for LQ models to explain $R_{D^{(*)}}$ at $2 \sigma$ (see table 1 ). Solid lines include both the SM and the LQ contribution, while dashed lines are the SM predictions with the CKM matrix values taken from Particle Data Group [138].
dipole current is because of the coupling textures we considered in table 1. In the case of massive $Z$ boson, the loop function approximations are tediously long [137] and we keep the full expressions in the appendix.

In figure 4 we show the numerical results of $\operatorname{Br}(t \rightarrow c V)$ with colors of red, green, blue indicating $V=\gamma, g, Z$, respectively. Solid lines include both the SM and the LQ contribution, while dashed lines are the SM predictions with the CKM matrix values taken from Particle Data Group [138]. In the left panel we choose $g_{1 L}^{33} g_{1 R}^{23 *}=1$ as an ordinary coupling benchmark to show the decoupling behavior of LQ $S_{1}$ contribution with respect to (w.r.t.) $M_{S_{1}}$. We see that when including the LQ $S_{1}$ contributions with $M_{S_{1}} \simeq 1 \mathrm{TeV}$, $B r(t \rightarrow c \gamma)(B r(t \rightarrow c Z))$ are increased by a factor of about 2000 (400) from the SM predictions $5 \times 10^{-14}\left(1 \times 10^{-14}\right)$ to around $1 \times 10^{-10}\left(4 \times 10^{-12}\right)$. However, there is only a mild enhancement by a factor of around 3 for $\operatorname{Br}(t \rightarrow c g)$, which is from $6 \times 10^{-12}$ in the SM to around $2 \times 10^{-11}$ when including the $S_{1}$ contributions. However, with sufficiently heavy $M_{S_{1}}$ all values of $\operatorname{Br}(t \rightarrow c V)$ will reduce to the SM predictions.

In the right panel, we fix $\frac{g_{1 L}^{33} g_{1}^{23 *}}{M_{S_{1}}^{2}}=0.87$ which is the upper bound value of numerical fitting for LQ models to explain $R_{D^{(*)}}$ at $2 \sigma$ (see table 1). To keep $g_{1 L}^{33}, g_{1 R}^{23 *}$ perturbative in this set-up, $M_{S_{1}}$ should not be too heavy. In the region of $M_{S_{1}} \simeq \mathcal{O}(1) \mathrm{TeV}$, one can learn from eq. (3.10) and eq. (3.11) that we have small but almost stable $x_{\tau}, x_{t} \ll \mathcal{O}(1)$. When combined with the SM contributions, $\operatorname{Br}(t \rightarrow c V)$ are also fairly stable for $M_{S_{1}} \gtrsim 2 \mathrm{TeV}$ with values around $1 \times 10^{-10}, 1 \times 10^{-11}, 5 \times 10^{-13}$ for $V=\gamma, g, Z$, respectively.

## 4 Collider search prospects

The 2-body top quark FCNC decays have been searched intensively at the LHC. The current constraints on the $\operatorname{Br}(t \rightarrow c \gamma), \operatorname{Br}(t \rightarrow c g)$ and $\operatorname{Br}(t \rightarrow c Z)$ are found to be $2 \times 10^{-3}$ [139], $2 \times 10^{-4}[140,141]$ and $2 \times 10^{-4}$ [142, 143], respectively. These are about six
orders of magnitude above the predicted BR values $\mathcal{O}\left(10^{-10}\right)$ at one-loop level induced by our LQ scenarios of explaining $R_{D^{(*)}}$, as presented in figure 4 . Therefore, with such small BRs there is basically no hope to detect the signals of the 2-body top quark FCNC decays induced by $S_{1}$ explanation of $R_{D^{(*)}}$.

As for the 3-body top quark FCNC decays at tree level, our discussions in section 3.1 show that the LQ explanation of the $R_{D^{(*)}}$ can induce $t \rightarrow c \mu \tau, t \rightarrow c \tau \tau$ and $t \rightarrow c \nu \nu$ with BRs $\sim 10^{-6}$. In the following we perform some assessments of the search prospects for the 3-body top FCNC decays at the future upgraded LHC with integrated luminosity of $3000 \mathrm{fb}^{-1}$ and collision energy at 13 TeV . We will first consider the cut-and-count analysis which turns out to be not effective, then we proceed with further studies using multi-variate analysis techniques of Boosted Decision Tree (BDT) method.

### 4.1 Cut-and-count analysis

The LHC is a top quark factory. With integrated luminosity of $3000 \mathrm{fb}^{-1}$ and collision energy at 13 TeV , about $2.5 \times 10^{9}$ top quark pair events will be produced. If the LQ explanation of the $R_{D^{(*)}}$ can induce $t \rightarrow c \mu \tau, t \rightarrow c \tau \tau$ and $t \rightarrow c \nu \nu$ with BRs $\sim 10^{-6}$, there will be $\sim 2500$ events which include at least one top quark decaying in these 3 -body FCNC modes. In order to suppress the multi-jet events and trigger the signal events, we require the other top quark in the top quark pair event to decay leptonically $(t \rightarrow b W, W \rightarrow \ell \nu)$. This requirement still gives $\sim 500$ 3-body top quark FCNC events induced by our LQ scenarios of explaining $R_{D^{(*)}}$. The dominant SM backgrounds are $t \bar{t}$ with both top quarks decay through $t \rightarrow b W$, diboson production (VV), Drell-Yan process (DY) and $W+$ jets events.

The following preselections will be applied to pick out the final state for each one of $t \rightarrow c \mu \tau, t \rightarrow c \tau \tau$ and $t \rightarrow c \nu \nu$.

- Selection 1: Exactly one lepton, at least three jets including exactly one $b$ jet and two $\tau$ jets.
- Selection 2: Exactly two leptons, at least one muon, at least 2 jets including exactly one $b$ jet and one $\tau$ jet.
- Selection 3: Exactly one lepton and more than two jets in the final state, where one of the jet is $b$-tagged, the missing transverse energy $E_{T}^{\text {miss }}>80 \mathrm{GeV}$.

In the event selections, our requirements include:

- The muons are required to have $p_{T}>30(25) \mathrm{GeV}$ and $|\eta|<2.4$, while the electrons should have $p_{T}>35(30) \mathrm{GeV}$ and $|\eta|<2.4$ for leading (sub-leading) lepton.
- For both electrons and muons, the scalar sum of transverse momenta $H_{T}$ of all particles with $p_{T}>0.5 \mathrm{GeV}$ that lie within a cone of radius $R=0.3(0.4)$ around the $e(\mu)$ should be less than $25(15) \%$ of the transverse momentum of $e(\mu)$.
- The jets are reconstructed through anti- $k_{T}$ algorithm with radius parameter $R=0.4$. Each should have $p_{T}>30 \mathrm{GeV}$ and $|\eta|<2.4$.

|  | VV | DY | $W+$ jet | $t \bar{t}$ | $t \rightarrow c \mu \tau$ | $t \rightarrow c \tau \tau$ | $t \rightarrow c \nu \nu$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Selection 1 | 9559 | 108095 | - | 1189719 | 28 | 19 | 0.3 |
| Selection 2 | 5433 | 54047 | - | 839651 | 39 | 5 | 0.0 |
| Selection 3 | 296814 | 594522 | 16530371 | 64764862 | 140 | 94 | 102 |

Table 2. The number of signal and background events after selections at 13 TeV LHC with integrated luminosity of $3000 \mathrm{fb}^{-1}$. The signals are the $t \bar{t}$ events with one top quark decaying leptonically and the other one decaying though $t \rightarrow c \mu \tau, t \rightarrow c \tau \tau$ and $t \rightarrow c \nu \nu$ induced by our LQ scenarios of explaining $R_{\left.D^{*}\right)}$ with a universal benchmark $\mathrm{BR} \sim 10^{-6}$. The $W+\mathrm{jet}$ events in Selection 1 and 2 are negligible because of low statistics after the requirement of $b$ and $\tau$ jets.

- The $\tau$ jet tagging efficiency is $60 \%$ with a QCD jet mis-identification rate of $1 \%(5 \%)$ for $p_{T}<40 \mathrm{GeV}\left(p_{T}>40 \mathrm{GeV}\right)$. We set the $b$-tagging efficiency to be $68 \%$, and the corresponding mis-tagging rates for the charm and light flavor jets are 0.12 and 0.01 [144].

The number of signal and background events after selections at HL-LHC are provided in table 2 , in which the $W+$ jet events in Selection 1 and 2 are negligible because of low statistics after the requirement of $b$ and $\tau$ jets. We can see that after the preliminary selection, the $t \bar{t}$ events are the dominant SM background and they are about $10^{4} \sim 10^{5}$ times larger than signal events. We also found that with this simple cut-and-count method constructed above, the signal BRs above $\sim 5 \times 10^{-5}$ can be excluded at the $95 \%$ confidence level ( 2 standard deviations). Therefore, the simple cut-and-count analysis is not powerful enough for probing the 3-body top quark FCNC decay signals with BR $\sim 10^{-6}$ induced by our LQ scenarios of explaining $R_{D^{(*)}}$.

### 4.2 Multi-variate analysis

We proceed with some further studies using multi-variate analysis of BDT method, which is one of the machine learning techniques with the kinematic variables of the final objects, the angle distributions between leptons and $E_{T}^{\text {miss }}$, and so on. In our results shown in figure 5, the input variables we consider include:

- multiplicity of jets, $b$ jet, $c$ jet and $\tau$ jet
- $p_{T}, E_{T}^{\text {miss }}$ of the leading lepton
- $p_{T}$ of the leading $\tau$ jet
- $H_{T}$ which includes jets, leptons and $E_{T}^{\text {miss }}$
- $p_{T}$ of leptons $+E_{T}^{\text {miss }}, p_{T}$ of leptons $+\tau$ jet
- $\Delta R(\tau, \ell), \Delta \phi\left(\ell, E_{T}^{\text {miss }}\right), \Delta \phi\left(\tau, E_{T}^{\text {miss }}\right)$
- $\Delta \phi\left(\tau+\ell, E_{T}^{\text {miss }}\right)$
- $\Delta R(\ell$, leading $b$ jet $)$

Delphes Sim at $\sqrt{\mathbf{s}}=\mathbf{1 3} \mathbf{~ T e V}$


Figure 5. Normalized BDT output distribution for signal and background events. The signal events tend to be close to 1 and the background events are close to -1 . The training and testing samples are indicated by the dot and the filled histograms, respectively. The training and testing samples are compatible with each other which means there is no overfitting. We have utilized Delphes $[145,146]$ to simulate the detector responses.

In order to have a smooth distribution of the output BDT, we need to loosen the event selections. As an example, if we loosen the event Selection 1 to 1 lepton and 2 jets (one $b$ jet and one $\tau$ jet) for the signal of the $t \rightarrow c \mu \tau$ process, the signal significance defined by $^{4} S / \sqrt{B}$ goes down to 0.0138 . In figure 5 when focusing on the last 10 bins of signal and background in the MVA score distribution between $[0,1]$, we found that using BDT can help the cut-and-count method give an increased significance of 0.0185 . Therefore, using the shape of the final BDT distribution we are able to increase the significance by $\mathcal{O}(30 \%)$ compared to the cut-and-count method. However, to further increase the sensitivity in a more comprehensive analysis, we need to perform the shape analysis with more sophisticated statistical tools. As this involves dedicated data analysis with much more statistics, we would leave the technical improvement in future works.

## 5 Conclusion

In this work we studied the correlation between the interpretations of $R_{D^{(*)}}$ anomaly in $B$ meson decay using LQ models and the top quark FCNC decays, i.e. 3-body processes $t \rightarrow c \ell_{i} \ell_{j}$ at tree level and 2-body processes $t \rightarrow c V$ at one-loop level, with $\ell$ being the SM

[^2]leptons and $V=\gamma, Z, g$ being the SM gauge bosons. We focus on the scalar LQ $S_{1}$ and vector $\mathrm{LQ} U_{1}$ which are both singlet under the $\mathrm{SM} \operatorname{SU}(2)_{L}$ gauge group. Utilizing the $2 \sigma$ parameter fitting ranges of the LQ models, we find that 3-body processes $\operatorname{Br}\left(t \rightarrow c \ell_{i} \ell_{j}\right)$ at tree level can reach $\mathcal{O}\left(10^{-6}\right)$, and the 2 -body processes $\operatorname{Br}(t \rightarrow c V)$ induced by scalar LQ $S_{1}$ at one-loop level can reach $\mathcal{O}\left(10^{-10}\right)$. We also provided quick estimations of the collider search prospects for a benchmark scenario $\operatorname{Br}\left(t \rightarrow c \ell_{i} \ell_{j}\right) \sim 10^{-6}$ at 13 TeV HLLHC with integrated luminosity of $3000 \mathrm{fb}^{-1}$. We found that the simple cut-and-count method can not give promising collider signal significance, but multi-variate analysis using BDT technique can provide reasonable improvements. More refined collider analyses are desirable which are left for future dedicated works.

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## A One-loop Wilson coefficients of $t \rightarrow c V$ induced by $S_{1}$

Here we present the Wilson coefficients of 2-body top quark FCNC decays $t \rightarrow c V$ at one-loop level induced by the scalar LQ $S_{1}$ in section 3.2. For the dipole current we have:

$$
\begin{align*}
f_{T L}^{g}= & g_{1 L}^{33} g_{1 R}^{23 *} \frac{1}{16 \pi^{2}} g_{s} m_{\tau} \times C_{2}\left(0, m_{t}^{2}, m_{c}^{2}, M_{S_{1}}^{2}, M_{S_{1}}^{2}, m_{\tau}^{2}\right)  \tag{A.1}\\
f_{T L}^{\gamma}= & -g_{1 L}^{33} g_{1 R}^{23 *} \frac{1}{48 \pi^{2}} e m_{\tau} \\
& \times\left(3 C_{1}\left(m_{c}^{2}, 0, m_{t}^{2}, M_{S_{1}}^{2}, m_{\tau}^{2}, m_{\tau}^{2}\right)+C_{2}\left(0, m_{t}^{2}, m_{c}^{2}, M_{S_{1}}^{2}, M_{S_{1}}^{2}, m_{\tau}^{2}\right)\right. \\
& \left.+3 C_{2}\left(m_{c}^{2}, 0, m_{t}^{2}, M_{S_{1}}^{2}, m_{\tau}^{2}, m_{\tau}^{2}\right)\right)  \tag{A.2}\\
f_{T L}^{Z}= & g_{1 L}^{33} g_{1 R}^{23 *} \frac{1}{96 \pi^{2}} \frac{e}{c_{W} s_{W}} m_{\tau} \\
& \times\left(3\left(s_{W}^{2}-c_{W}^{2}\right) C_{1}\left(m_{c}^{2}, m_{Z}^{2}, m_{t}^{2}, M_{S_{1}}^{2}, m_{\tau}^{2}, m_{\tau}^{2}\right)\right.  \tag{A.3}\\
& \left.+2 s_{W}^{2}\left(3 C_{2}\left(m_{c}^{2}, m_{Z}^{2}, m_{t}^{2}, M_{S_{1}}^{2}, m_{\tau}^{2}, m_{\tau}^{2}\right)+C_{2}\left(m_{Z}^{2}, m_{t}^{2}, m_{c}^{2}, M_{S_{1}}^{2}, M_{S_{1}}^{2}, m_{\tau}^{2}\right)\right)\right), \\
f_{T R}^{g}= & f_{T R}^{\gamma}=f_{T R}^{Z}=0 . \tag{A.4}
\end{align*}
$$

Note that the absence of right-handed dipole current is because of the coupling textures we considered in table 1.

For the monopole current which appears for the massive gauge boson $Z$, we have:

$$
\begin{align*}
f_{V L}^{Z}= & g_{1 L}^{33} g_{1 R}^{23 *} \frac{1}{96 \pi^{2}} \frac{e}{c_{W} s_{W}} \frac{m_{\tau} m_{c}}{m_{c}^{2}-m_{t}^{2}} \\
& \times\left(\left(3 c_{W}^{2}-s_{W}^{2}\right)\left(B_{0}\left(m_{t}^{2}, m_{\tau}^{2}, M_{S_{1}}^{2}\right)-B_{0}\left(m_{c}^{2}, m_{\tau}^{2}, M_{S_{1}}^{2}\right)\right)\right. \\
& +2 s_{W}^{2}\left(m_{c}^{2}-m_{t}^{2}\right)\left(3 C_{0}\left(m_{c}^{2}, m_{Z}^{2}, m_{t}^{2}, M_{S_{1}}^{2}, m_{\tau}^{2}, m_{\tau}^{2}\right)+3 C_{1}\left(m_{c}^{2}, m_{Z}^{2}, m_{t}^{2}, M_{S_{1}}^{2}, m_{\tau}^{2}, m_{\tau}^{2}\right)\right. \\
& \left.\left.+3 C_{2}\left(m_{c}^{2}, m_{Z}^{2}, m_{t}^{2}, M_{S_{1}}^{2}, m_{\tau}^{2}, m_{\tau}^{2}\right)+C_{2}\left(m_{Z}^{2}, m_{t}^{2}, m_{c}^{2}, M_{S_{1}}^{2}, M_{S_{1}}^{2}, m_{\tau}^{2}\right)\right)\right)  \tag{A.5}\\
f_{V R}^{Z}= & g_{1 L}^{33} g_{1 R}^{23 *} \frac{1}{96 \pi^{2}} \frac{e}{c_{W} s_{W}} \frac{m_{\tau} m_{t}}{m_{c}^{2}-m_{t}^{2}} \\
& \times\left(4 s_{W}^{2}\left(B_{0}\left(m_{c}^{2}, m_{\tau}^{2}, M_{S_{1}}^{2}\right)-B_{0}\left(m_{t}^{2}, m_{\tau}^{2}, M_{S_{1}}^{2}\right)\right)\right. \\
& \quad+\left(m_{c}^{2}-m_{t}^{2}\right)\left(2 s_{W}^{2} C_{2}\left(m_{Z}^{2}, m_{t}^{2}, m_{c}^{2}, M_{S_{1}}^{2}, M_{S_{1}}^{2}, m_{\tau}^{2}\right)\right. \\
& \quad-3\left(c_{W}^{2}-s_{W}^{2}\right)\left(C_{0}\left(m_{c}^{2}, m_{Z}^{2}, m_{t}^{2}, M_{S_{1}}^{2}, m_{\tau}^{2}, m_{\tau}^{2}\right)\right. \\
& \left.\left.\left.\quad+C_{1}\left(m_{c}^{2}, m_{Z}^{2}, m_{t}^{2}, M_{S_{1}}^{2}, m_{\tau}^{2}, m_{\tau}^{2}\right)+C_{2}\left(m_{c}^{2}, m_{Z}^{2}, m_{t}^{2}, M_{S_{1}}^{2}, m_{\tau}^{2}, m_{\tau}^{2}\right)\right)\right)\right) . \tag{A.6}
\end{align*}
$$

Coefficients $B_{i}, C_{i}$ in the above are defined in the general one-loop tensor integral [127, 137]:

$$
\begin{equation*}
T_{\mu_{1} \ldots \mu_{P}}^{N}\left(p_{1}, \ldots, p_{N-1}, m_{0}, \ldots, m_{N-1}\right)=\frac{(2 \pi \mu)^{4-D}}{i \pi^{2}} \int d^{D} q \frac{q_{\mu_{1}} \cdots q_{\mu_{P}}}{D_{0} D_{1} \cdots D_{N-1}} \tag{A.7}
\end{equation*}
$$

with the following denominators:

$$
\begin{equation*}
D_{0}=q^{2}-m_{0}^{2}+i \varepsilon, \quad D_{i}=\left(q+p_{i}\right)^{2}-m_{i}^{2}+i \varepsilon, \quad i=1, \ldots, N-1 \tag{A.8}
\end{equation*}
$$

Then one can perform the decompositions as follows:

$$
\begin{align*}
& B_{\mu}=p_{1 \mu} B_{1}  \tag{A.9}\\
& C_{\mu}=p_{1 \mu} C_{1}+p_{2 \mu} C_{2} \tag{A.10}
\end{align*}
$$

To carry out the numerical calculations with the public package LoopTools, one needs to impose the parameter conventions of LoopTools by the following transformation:

$$
\begin{align*}
B_{i}\left(p_{1}, m_{0}, m_{1}\right) & \rightarrow B_{i}\left(p_{1}^{2}, m_{0}^{2}, m_{1}^{2}\right)  \tag{A.11}\\
C_{i}\left(p_{1}, p_{2}, m_{0}, m_{1}, m_{2}\right) & \rightarrow C_{i}\left(p_{1}^{2},\left(p_{1}-p_{2}\right)^{2}, p_{2}^{2}, m_{0}^{2}, m_{1}^{2}, m_{2}^{2}\right) \tag{A.12}
\end{align*}
$$

Note that the loop function coefficients presented in this appendix has been applied with the transformation.

In all of the numerical results presented in this work, we have used the complete expressions with loop functions. However, for $M_{S_{1}} \simeq \mathcal{O}(1) \mathrm{TeV}$ which indicates $M_{S_{1}} \gg$ $m_{t}, m_{c}, m_{\tau}$, one can have the following compact approximations by setting $m_{c}=0$ for the


Figure 6. The comparison of the approximations in eq. (3.10) and eq. (3.11) to the full results in eq. (A.1) and eq. (A.2).
massless cases of photon and gluon,

$$
\begin{align*}
f_{T L}^{g} \simeq & \frac{1}{16 \pi^{2}} g_{s} m_{\tau} \frac{g_{1 L}^{33} g_{1 R}^{23 *}}{M_{S_{1}}^{2}} \\
& \times \frac{1}{12}\left(-6\left(22 x_{\tau} x_{t}+3 x_{t}+16 x_{\tau}+6\right) \log x_{\tau}-49 x_{t}-48\right)  \tag{A.13}\\
f_{T L}^{\gamma} \simeq & -\frac{1}{16 \pi^{2}} e m_{\tau} \frac{g_{1 L}^{33} g_{1 R}^{23 *}}{M_{S_{1}}^{2}} \\
& \times \frac{1}{3}\left(\frac{1}{6}\left(14 x_{t} x_{\tau}+x_{t}+9 x_{\tau}+3\right)+\left(x_{t}+1\right) x_{\tau} \log x_{\tau}\right)  \tag{A.14}\\
f_{T R}^{g}= & f_{T R}^{\gamma}=f_{T R}^{Z}=0 \tag{A.15}
\end{align*}
$$

where $x_{\tau}=m_{\tau}^{2} / M_{S_{1}}^{2}, x_{t}=m_{t}^{2} / M_{S_{1}}^{2}$. Note that we have kept the first order effect of top quark mass $m_{t}$ in the expansion due to its relatively large value. To illustrate this, in figure 6 we show the comparison of the approximations to the full results. In the cases of massive $Z$ boson with two external massive particles in the 3 -point loop functions, even the approximated expressions are tediously long [137] and we do not proceed with the reductions but keeping the full expression.

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## References

[1] BABAR collaboration, Evidence for an excess of $\bar{B} \rightarrow D^{(*)} \tau^{-} \bar{\nu}_{\tau}$ decays, Phys. Rev. Lett. 109 (2012) 101802 [arXiv:1205.5442] [inSPIRE].
[2] BABAR collaboration, Measurement of an Excess of $\bar{B} \rightarrow D^{(*)} \tau^{-} \bar{\nu}_{\tau}$ Decays and Implications for Charged Higgs Bosons, Phys. Rev. D 88 (2013) 072012 [arXiv:1303.0571] [INSPIRE].
[3] Belle collaboration, Measurement of the branching ratio of $\bar{B} \rightarrow D^{(*)} \tau^{-} \bar{\nu}_{\tau}$ relative to $\bar{B} \rightarrow D^{(*)} \ell^{-} \bar{\nu}_{\ell}$ decays with hadronic tagging at Belle, Phys. Rev. D 92 (2015) 072014 [arXiv:1507.03233] [INSPIRE].
[4] LHCb collaboration, Measurement of the ratio of branching fractions $\mathcal{B}\left(\bar{B}^{0} \rightarrow D^{*+} \tau^{-} \bar{\nu}_{\tau}\right) / \mathcal{B}\left(\bar{B}^{0} \rightarrow D^{*+} \mu^{-} \bar{\nu}_{\mu}\right)$, Phys. Rev. Lett. 115 (2015) 111803 [Erratum ibid. 115 (2015) 159901] [arXiv:1506.08614] [inSPIRE].
[5] Belle collaboration, Measurement of the $\tau$ lepton polarization and $R\left(D^{*}\right)$ in the decay $\bar{B} \rightarrow D^{*} \tau^{-} \bar{\nu}_{\tau}$, Phys. Rev. Lett. 118 (2017) 211801 [arXiv:1612.00529] [inSPIRE].
[6] Belle collaboration, Measurement of the branching ratio of $\bar{B}^{0} \rightarrow D^{*+} \tau^{-} \bar{\nu}_{\tau}$ relative to $\bar{B}^{0} \rightarrow D^{*+} \ell^{-} \bar{\nu}_{\ell}$ decays with a semileptonic tagging method, Phys. Rev. D 94 (2016) 072007 [arXiv:1607.07923] [INSPIRE].
[7] BELLE collaboration, Measurement of the branching ratio of $\bar{B}^{0} \rightarrow D^{*+} \tau^{-} \bar{\nu}_{\tau}$ relative to $\bar{B}^{0} \rightarrow D^{*+} \ell^{-} \bar{\nu}_{\ell}$ decays with a semileptonic tagging method, in Proceedings, 51st Rencontres de Moriond on Electroweak Interactions and Unified Theories: La Thuile, Italy, March 12-19, 2016, arXiv:1603.06711 [INSPIRE].
[8] Belle collaboration, Measurement of $\mathcal{R}(D)$ and $\mathcal{R}\left(D^{*}\right)$ with a semileptonic tagging method, arXiv:1904.08794 [inSPIRE].
[9] HFLAV collaboration, Averages of b-hadron, c-hadron and $\tau$-lepton properties as of summer 2016, Eur. Phys. J. C 77 (2017) 895 [arXiv:1612.07233] [INSPIRE].
[10] MILC collaboration, $B \rightarrow$ D $\ell$ form factors at nonzero recoil and $\left|V_{c b}\right|$ from 2+1-flavor lattice $Q C D$, Phys. Rev. D 92 (2015) 034506 [arXiv:1503.07237] [InSPIRE].
[11] HPQCD collaboration, $B \rightarrow$ Dl form factors at nonzero recoil and extraction of $\left|V_{c b}\right|$, Phys. Rev. D 92 (2015) 054510 [Erratum ibid. D 93 (2016) 119906] [arXiv:1505.03925] [inSPIRE].
[12] S. Aoki et al., Review of lattice results concerning low-energy particle physics, Eur. Phys. J. C 77 (2017) 112 [arXiv:1607.00299] [INSPIRE].
[13] D. Bigi, P. Gambino and S. Schacht, $R\left(D^{*}\right),\left|V_{c b}\right|$, and the Heavy Quark Symmetry relations between form factors, JHEP 11 (2017) 061 [arXiv:1707.09509] [inSPIRE].
[14] F.U. Bernlochner, Z. Ligeti, M. Papucci and D.J. Robinson, Combined analysis of semileptonic $B$ decays to $D$ and $D^{*}: R\left(D^{(*)}\right),\left|V_{c b}\right|$, and new physics, Phys. Rev. D 95 (2017) 115008 [arXiv:1703.05330] [InSPIRE].
[15] S. Jaiswal, S. Nandi and S.K. Patra, Extraction of $\left|V_{c b}\right|$ from $B \rightarrow D^{(*)} \ell \nu_{\ell}$ and the Standard Model predictions of $R\left(D^{(*)}\right)$, JHEP 12 (2017) 060 [arXiv:1707.09977] [INSPIRE].
[16] LHCb collaboration, Search for lepton-universality violation in $B^{+} \rightarrow K^{+} \ell^{+} \ell^{-}$decays, Phys. Rev. Lett. 122 (2019) 191801 [arXiv:1903.09252] [INSPIRE].
[17] Belle collaboration, Test of lepton flavor universality in $B \rightarrow K^{*} \ell^{+} \ell^{-}$decays at Belle, arXiv:1904. 02440 [INSPIRE].
[18] M. Bordone, G. Isidori and A. Pattori, On the Standard Model predictions for $R_{K}$ and $R_{K^{*}}$, Eur. Phys. J. C 76 (2016) 440 [arXiv:1605.07633] [INSPIRE].
[19] G. Hiller and F. Krüger, More model-independent analysis of $b \rightarrow s$ processes, Phys. Rev. D 69 (2004) 074020 [hep-ph/0310219] [inSPIRE].
[20] S. Bifani, S. Descotes-Genon, A. Romero Vidal and M.-H. Schune, Review of Lepton Universality tests in B decays, J. Phys. G 46 (2019) 023001 [arXiv:1809.06229] [InSPIRE].
[21] A. Crivellin, C. Greub and A. Kokulu, Explaining $B \rightarrow D \tau \nu, B \rightarrow D^{*} \tau \nu$ and $B \rightarrow \tau \nu$ in a 2HDM of type-III, Phys. Rev. D 86 (2012) 054014 [arXiv:1206.2634] [InSPIRE].
[22] M. Tanaka and R. Watanabe, New physics in the weak interaction of $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$, Phys. Rev. D 87 (2013) 034028 [arXiv:1212.1878] [INSPIRE].
[23] A. Celis, M. Jung, X.-Q. Li and A. Pich, Sensitivity to charged scalars in $\boldsymbol{B} \rightarrow \boldsymbol{D}^{(*)} \boldsymbol{\tau} \boldsymbol{\nu}_{\boldsymbol{\tau}}$ and $\boldsymbol{B} \rightarrow \boldsymbol{\tau} \boldsymbol{\nu}_{\boldsymbol{\tau}}$ decays, JHEP 01 (2013) 054 [arXiv:1210.8443] [INSPIRE].
[24] A. Crivellin, A. Kokulu and C. Greub, Flavor-phenomenology of two-Higgs-doublet models with generic Yukawa structure, Phys. Rev. D 87 (2013) 094031 [arXiv:1303.5877] [INSPIRE].
[25] A. Crivellin, J. Heeck and P. Stoffer, A perturbed lepton-specific two-Higgs-doublet model facing experimental hints for physics beyond the Standard Model, Phys. Rev. Lett. 116 (2016) 081801 [arXiv:1507.07567] [INSPIRE].
[26] C.-H. Chen and T. Nomura, Charged-Higgs on $R_{D^{(*)}}, \tau$ polarization, and FBA, Eur. Phys. J. C 77 (2017) 631 [arXiv:1703.03646] [inSPIRE].
[27] S. Iguro and K. Tobe, $R\left(D^{(*)}\right)$ in a general two Higgs doublet model, Nucl. Phys. B 925 (2017) 560 [arXiv: 1708.06176] [inSPIRE].
[28] C.-H. Chen and T. Nomura, Charged Higgs boson contribution to $B_{q}^{-} \rightarrow \ell \bar{\nu}$ and $\bar{B} \rightarrow(P, V) \ell \bar{\nu}$ in a generic two-Higgs doublet model, Phys. Rev. D 98 (2018) 095007 [arXiv:1803.00171] [inSPIRE].
[29] P. Ko, Y. Omura, Y. Shigekami and C. Yu, LHCb anomaly and B physics in flavored $Z^{\prime}$ models with flavored Higgs doublets, Phys. Rev. D 95 (2017) 115040 [arXiv:1702.08666] [INSPIRE].
[30] P. Ko, Y. Omura and C. Yu, $B \rightarrow D^{(*)} \tau \nu$ and $B \rightarrow \tau \nu$ in chiral $\mathrm{U}(1)^{\prime}$ models with flavored multi Higgs doublets, JHEP 03 (2013) 151 [arXiv:1212.4607] [INSPIRE].
[31] K. Cheung, T. Nomura and H. Okada, Testable radiative neutrino mass model without additional symmetries and explanation for the $b \rightarrow s \ell^{+} \ell^{-}$anomaly, Phys. Rev. D 94 (2016) 115024 [arXiv: 1610.02322] [INSPIRE].
[32] A. Greljo, G. Isidori and D. Marzocca, On the breaking of Lepton Flavor Universality in $B$ decays, JHEP 07 (2015) 142 [arXiv:1506.01705] [inSPIRE].
[33] S.M. Boucenna, A. Celis, J. Fuentes-Martin, A. Vicente and J. Virto, Non-abelian gauge extensions for B-decay anomalies, Phys. Lett. B 760 (2016) 214 [arXiv:1604.03088] [INSPIRE].
[34] S.M. Boucenna, A. Celis, J. Fuentes-Martin, A. Vicente and J. Virto, Phenomenology of an $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ model with lepton-flavour non-universality, JHEP 12 (2016) 059 [arXiv:1608.01349] [inSPIRE].
[35] E. Megias, M. Quirós and L. Salas, Lepton-flavor universality violation in $R_{K}$ and $R_{D^{(*)}}$ from warped space, JHEP 07 (2017) 102 [arXiv:1703.06019] [INSPIRE].
[36] R. Alonso, P. Cox, C. Han and T.T. Yanagida, Flavoured B - L local symmetry and anomalous rare $B$ decays, Phys. Lett. B 774 (2017) 643 [arXiv:1705.03858] [inSPIRE].
[37] R. Alonso, P. Cox, C. Han and T.T. Yanagida, Anomaly-free local horizontal symmetry and anomaly-full rare B-decays, Phys. Rev. D 96 (2017) 071701 [arXiv:1704.08158] [INSPIRE].
[38] G.H. Duan, X. Fan, M. Frank, C. Han and J.M. Yang, A minimal U(1)' extension of MSSM in light of the $B$ decay anomaly, Phys. Lett. B 789 (2019) 54 [arXiv:1808.04116] [INSPIRE].
[39] L. Bian, H.M. Lee and C.B. Park, B-meson anomalies and Higgs physics in flavored $\mathrm{U}(1)^{\prime}$ model, Eur. Phys. J. C 78 (2018) 306 [arXiv:1711.08930] [inSPIRE].
[40] L. Bian, S.-M. Choi, Y.-J. Kang and H.M. Lee, A minimal flavored U(1)' for B-meson anomalies, Phys. Rev. D 96 (2017) 075038 [arXiv:1707.04811] [inSPIRE].
[41] P. Ko, T. Nomura and H. Okada, Explaining $B \rightarrow K^{(*)} \ell^{+} \ell^{-}$anomaly by radiatively induced coupling in $\mathrm{U}(1)_{\mu-\tau}$ gauge symmetry, Phys. Rev. D 95 (2017) 111701 [arXiv:1702.02699] [inSPIRE].
[42] A. Biswas, D. Kumar Ghosh, N. Ghosh, A. Shaw and A.K. Swain, Novel collider signature of $U_{1}$ Leptoquark and $B \rightarrow \pi$ observables, arXiv:1808.04169 [INSPIRE].
[43] M. Blanke et al., Impact of polarization observables and $B_{c} \rightarrow \tau \nu$ on new physics explanations of the $b \rightarrow c \tau \nu$ anomaly, Phys. Rev. D 99 (2019) 075006 [arXiv:1811.09603] [INSPIRE].
[44] A. Biswas, A. Shaw and A.K. Swain, Collider signature of $V_{2}$ Leptoquark with $b \rightarrow s$ flavour observables, LHEP 2 (2019) 126 [arXiv:1811.08887] [INSPIRE].
[45] S. Iguro, T. Kitahara, Y. Omura, R. Watanabe and K. Yamamoto, $D^{*}$ polarization vs. $R_{D^{(*)}}$ anomalies in the leptoquark models, JHEP 02 (2019) 194 [arXiv:1811.08899] [inSPIRE].
[46] J. Aebischer, A. Crivellin and C. Greub, QCD improved matching for semileptonic $B$ decays with leptoquarks, Phys. Rev. D 99 (2019) 055002 [arXiv:1811.08907] [inSPIRE].
[47] B. Döbrich, F. Ertas, F. Kahlhoefer and T. Spadaro, Model-independent bounds on light pseudoscalars from rare B-meson decays, Phys. Lett. B 790 (2019) 537 [arXiv:1810.11336] [INSPIRE].
[48] G. Faisel and J. Tandean, Rare nonleptonic $\bar{B}_{s}^{0}$ decays as probes of new physics behind $b \rightarrow s \mu \bar{\mu}$ anomalies, Phys. Rev. D 99 (2019) 075007 [arXiv:1810.11437] [InSPIRE].
[49] S. Bansal, R.M. Capdevilla and C. Kolda, Constraining the minimal flavor violating leptoquark explanation of the $R_{D^{(*)}}$ anomaly, Phys. Rev. D 99 (2019) 035047 [arXiv:1810.11588] [INSPIRE].
[50] JLQCD collaboration, $B \rightarrow D^{(*)} \ell \nu$ form factors from $N_{f}=2+1 Q C D$ with Möbius domain-wall quarks, PoS(LATTICE2018) 311 (2018) [arXiv:1811.00794] [INSPIRE].
[51] T. Mandal, S. Mitra and S. Raz, $R_{D^{(*)}}$ motivated $\mathcal{S}_{1}$ leptoquark scenarios: Impact of interference on the exclusion limits from LHC data, Phys. Rev. D 99 (2019) 055028 [arXiv:1811.03561] [INSPIRE].
[52] P. Maji, P. Nayek and S. Sahoo, Implication of family non-universal $Z^{\prime}$ model to rare exclusive $b \rightarrow s(l \bar{l}, \nu \bar{\nu})$ transitions, PTEP 2019 (2019) 033B06 [arXiv:1811.03869] [inSPIRE].
[53] K.S. Babu, B. Dutta and R.N. Mohapatra, A Theory of $R\left(D^{*}, D\right)$ Anomaly with Right-Handed Currents, JHEP 01 (2019) 168 [arXiv:1811.04496] [inSPIRE].
[54] R. Alonso, J. Martin Camalich and S. Westhoff, Tau Polarimetry in B Meson Decays, SciPost Phys. Proc. 1 (2019) 012 [arXiv:1811.05664] [INSPIRE].
[55] S. Kamali, New physics in inclusive semileptonic $B$ decays including nonperturbative corrections, Int. J. Mod. Phys. A 34 (2019) 1950036 [arXiv:1811.07393] [inSPIRE].
[56] D.A. Faroughy, LHC searches motivated by recent B-anomalies, SciPost Phys. Proc. 1 (2019) 021 [arXiv:1811.07582] [inSPIRE].
[57] J. Roy, Probing leptoquark chirality via top polarization at the Colliders, arXiv:1811.12058 [inSPIRE].
[58] C.-Q. Geng and H. Okada, Resolving B-meson anomalies by flavor-dependent gauged symmetries $\prod_{i=1}^{3} \mathrm{U}(1)_{B_{i}-L_{i}}$, arXiv: 1812.07918 [INSPIRE].
[59] P.T.P. Hutauruk, T. Nomura, H. Okada and Y. Orikasa, Dark matter and B-meson anomalies in a flavor dependent gauge symmetry, Phys. Rev. D 99 (2019) 055041 [arXiv:1901.03932] [INSPIRE].
[60] S. Baek, Scalar dark matter behind $b \rightarrow$ s $\mu \mu$ anomaly, JHEP 05 (2019) 104 [arXiv:1901.04761] [INSPIRE].
[61] LHCb collaboration, Search for the lepton-flavour-violating decays $B_{s}^{0} \rightarrow \tau^{ \pm} \mu^{\mp}$ and $B^{0} \rightarrow \tau^{ \pm} \mu^{\mp}$, arXiv:1905.06614 [inSPIRE].
[62] O. Popov, M.A. Schmidt and G. White, $R_{2}$ as a single leptoquark solution to $R_{D^{(*)}}$ and $R_{K^{(*)}}$, arXiv:1905. 06339 [inSPIRE].
[63] J. Davighi, Connecting neutral current $B$ anomalies with the heaviness of the third family, in 54 th Rencontres de Moriond on $Q C D$ and High Energy Interactions (Moriond QCD 2019) La Thuile, Italy, March 23-30, 2019, 2019, arXiv:1905. 06073 [INSPIRE].
[64] A. Kamada, M. Yamada and T.T. Yanagida, Unification of the Standard Model and Dark Matter Sectors in $[\mathrm{SU}(5) \times \mathrm{U}(1)]^{4}$, arXiv:1905.04245 [INSPIRE].
[65] A. Hryczuk et al., Testing dark matter with Cherenkov light - prospects of H.E.S.S. and CTA for exploring minimal supersymmetry, arXiv:1905.00315 [INSPIRE].
[66] S. Trifinopoulos, B-physics anomalies: The bridge between $R$-parity violating Supersymmetry and flavoured Dark Matter, arXiv:1904.12940 [inSPIRE].
[67] B.C. Allanach, J.M. Butterworth and T. Corbett, Collider Constraints on Z' Models for Neutral Current B-Anomalies, arXiv:1904.10954 [inSPIRE].
[68] C. Cornella, J. Fuentes-Martin and G. Isidori, Revisiting the vector leptoquark explanation of the B-physics anomalies, arXiv:1903.11517 [INSPIRE].
[69] L. Delle Rose, S. Khalil, S.J.D. King and S. Moretti, $R_{K}$ and $R_{K^{*}}$ in an Aligned 2HDM with Right-Handed Neutrinos, arXiv:1903.11146 [inSPIRE].
[70] K. Kowalska, D. Kumar and E.M. Sessolo, Implications for New Physics in $b \rightarrow s \mu \mu$ transitions after recent measurements by Belle and LHCb, arXiv:1903. 10932 [inSPIRE].
[71] V. Gherardi, D. Marzocca, M. Nardecchia and A. Romanino, Rank-One Flavor Violation and B-meson anomalies, arXiv:1903.10954 [INSPIRE].
[72] A. Shaw, Looking for $B \rightarrow X_{s} \ell^{+} \ell^{-}$in non-minimal Universal Extra Dimensional model, Phys. Rev. D 99 (2019) 115030 [arXiv:1903.10302] [INSPIRE].
[73] J. Aebischer, W. Altmannshofer, D. Guadagnoli, M. Reboud, P. Stangl and D.M. Straub, $B$-decay discrepancies after Moriond 2019, arXiv:1903.10434 [InSPIRE].
[74] A. Crivellin, D. Müller and C. Wiegand, $b \rightarrow s \ell^{+} \ell^{-}$Transitions in Two-Higgs-Doublet Models, JHEP 06 (2019) 119 [arXiv:1903.10440] [InSPIRE].
[75] A. Datta, J. Kumar and D. London, The B Anomalies and New Physics in $b \rightarrow s e^{+} e^{-}$, arXiv:1903. 10086 [INSPIRE].
[76] M. Ciuchini et al., New Physics in $b \rightarrow s \ell^{+} \ell^{-}$confronts new data on Lepton Universality, arXiv:1903.09632 [INSPIRE].
[77] R.-X. Shi, L.-S. Geng, B. Grinstein, S. Jäger and J. Martin Camalich, Revisiting the new-physics interpretation of the $b \rightarrow c \tau \nu$ data, arXiv:1905.08498 [INSPIRE].
[78] M. Blanke, A. Crivellin, T. Kitahara, M. Moscati, U. Nierste and I. Nišandžić, Addendum: "Impact of polarization observables and $B_{c} \rightarrow \tau \nu$ on new physics explanations of the $b \rightarrow c \tau \nu$ anomaly", arXiv:1905.08253 [INSPIRE].
[79] A. Crivellin and F. Saturnino, Correlating Tauonic B Decays to the Neutron EDM via a Scalar Leptoquark, arXiv:1905.08257 [INSPIRE].
[80] J. Heeck and D. Teresi, Pati-Salam and lepton universality in B decays, in 54th Rencontres de Moriond on Electroweak Interactions and Unified Theories (Moriond EW 2019) La Thuile, Italy, March 16-23, 2019, 2019, arXiv:1905.05211 [INSPIRE].
[81] P. Asadi and D. Shih, Maximizing the Impact of New Physics in $b \rightarrow c \tau \nu$ Anomalies, arXiv:1905.03311 [INSPIRE].
[82] S. Calí, S. Klaver, M. Rotondo and B. Sciascia, Impacts of radiative corrections on measurements of lepton flavour universality in $B \rightarrow D \ell \nu_{\ell}$ decays, arXiv:1905.02702 [inSPIRE].
[83] H. Yan, Y.-D. Yang and X.-B. Yuan, Phenomenology of $b \rightarrow c \tau \bar{\nu}$ decays in a scalar leptoquark model, arXiv:1905.01795 [INSPIRE].
[84] C. Murgui, A. Peñuelas, M. Jung and A. Pich, Global fit to $b \rightarrow c \tau \nu$ transitions, arXiv:1904.09311 [inSPIRE].
[85] W.-S. Hou, M. Kohda, T. Modak and G.-G. Wong, Enhanced B $\rightarrow \mu \bar{\nu}$ Decay at Tree Level as Probe of Extra Yukawa Couplings, arXiv:1903.03016 [INSPIRE].
[86] D. Choudhury, N. Kumar and A. Kundu, Search for opposite sign muon-tau pair and a $b$-jet at LHC in the context of flavor anomalies, arXiv:1905.07982 [INSPIRE].
[87] S. Kumbhakar and J. Saini, Flavor signatures of complex anomalous tcZ couplings, arXiv:1905. 07690 [INSPIRE].
[88] A. Datta, D. Sachdeva and J. Waite, A unified explanation of $b \rightarrow s \mu^{+} \mu^{-}$anomalies, neutrino masses and $B \rightarrow \pi K$ puzzle, arXiv:1905.04046 [INSPIRE].
[89] J. Zhang, C.-X. Yue, C.-H. Li and S. Yang, Constraints on scalar and vector leptoquarks from the LHC Higgs data, arXiv:1905. 04074 [inSPIRE].
[90] A. Arbey, T. Hurth, F. Mahmoudi, D.M. Santos and S. Neshatpour, Update on the $b \rightarrow s$ anomalies, arXiv:1904.08399 [INSPIRE].
[91] K. Azizi, Y. Sarac and H. Sundu, Lepton flavor universality violation in semileptonic tree level weak transitions, arXiv:1904.08267 [inSPIRE].
[92] P. Arnan, A. Crivellin, M. Fedele and F. Mescia, Generic Loop Effects of New Scalars and Fermions in $b \rightarrow s \ell^{+} \ell^{-}$and a Vector-like $4^{\text {th }}$ Generation, JHEP 06 (2019) 118 [arXiv:1904.05890] [INSPIRE].
[93] J. Kumar and D. London, New physics in $b \rightarrow s e^{+} e^{-}$?, Phys. Rev. D 99 (2019) 073008 [arXiv:1901.04516] [INSPIRE].
[94] D. Bardhan and D. Ghosh, B-meson charged current anomalies: the post-Moriond status, arXiv:1904.10432 [INSPIRE].
[95] M. Algueró et al., Emerging patterns of New Physics with and without Lepton Flavour Universal contributions, arXiv:1903.09578 [INSPIRE].
[96] A.K. Alok, A. Dighe, S. Gangal and D. Kumar, Continuing search for new physics in $b \rightarrow s \mu \mu$ decays: two operators at a time, JHEP 06 (2019) 089 [arXiv:1903.09617] [inSPIRE].
[97] A.K. Alok, D. Kumar, S. Kumbhakar and S. Uma Sankar, Impact of $D^{*}$ polarization measurement on solutions to $R_{D^{-}}-R_{D^{*}}$ anomalies, arXiv:1903.10486 [INSPIRE].
[98] S. Fajfer, J.F. Kamenik, I. Nišandžić and J. Zupan, Implications of Lepton Flavor Universality Violations in B Decays, Phys. Rev. Lett. 109 (2012) 161801 [arXiv:1206.1872] [INSPIRE].
[99] N.G. Deshpande and A. Menon, Hints of R-parity violation in B decays into $\tau \nu$, JHEP 01 (2013) 025 [arXiv:1208.4134] [inSPIRE].
[100] Y. Sakaki, M. Tanaka, A. Tayduganov and R. Watanabe, Testing leptoquark models in $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$, Phys. Rev. D 88 (2013) 094012 [arXiv:1309.0301] [inSPIRE].
[101] R. Alonso, B. Grinstein and J. Martin Camalich, Lepton universality violation and lepton flavor conservation in B-meson decays, JHEP 10 (2015) 184 [arXiv:1505.05164] [INSPIRE].
[102] L. Calibbi, A. Crivellin and T. Ota, Effective Field Theory Approach to $b \rightarrow$ sl( $\left.{ }^{( }{ }^{( }\right)$, $B \rightarrow K^{(*)} \nu \bar{\nu}$ and $B \rightarrow D^{(*)} \tau \nu$ with Third Generation Couplings, Phys. Rev. Lett. 115 (2015) 181801 [arXiv:1506.02661] [inSPIRE].
[103] M. Bauer and M. Neubert, Minimal Leptoquark Explanation for the $R_{D^{(*)}}, R_{K}$, and $(g-2)_{g}$ Anomalies, Phys. Rev. Lett. 116 (2016) 141802 [arXiv:1511.01900] [inSPIRE].
[104] S. Fajfer and N. Košnik, Vector leptoquark resolution of $R_{K}$ and $R_{D^{(*)}}$ puzzles, Phys. Lett. B 755 (2016) 270 [arXiv:1511.06024] [inSPIRE].
[105] R. Barbieri, G. Isidori, A. Pattori and F. Senia, Anomalies in B-decays and $\mathrm{U}(2)$ flavour symmetry, Eur. Phys. J. C 76 (2016) 67 [arXiv:1512.01560] [inSPIRE].
[106] N.G. Deshpande and X.-G. He, Consequences of $R$-parity violating interactions for anomalies in $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$ and $b \rightarrow s \mu^{+} \mu^{-}$, Eur. Phys. J. C 77 (2017) 134 [arXiv:1608.04817] [INSPIRE].
[107] X.-Q. Li, Y.-D. Yang and X. Zhang, Revisiting the one leptoquark solution to the $R\left(D^{(*)}\right)$ anomalies and its phenomenological implications, JHEP 08 (2016) 054 [arXiv:1605.09308] [INSPIRE].
[108] S. Sahoo, R. Mohanta and A.K. Giri, Explaining the $R_{K}$ and $R_{D^{(*)}}$ anomalies with vector leptoquarks, Phys. Rev. D 95 (2017) 035027 [arXiv:1609.04367] [inSPIRE].
[109] D. Bečirević, S. Fajfer, N. Košnik and O. Sumensari, Leptoquark model to explain the B-physics anomalies, $R_{K}$ and $R_{D}$, Phys. Rev. D 94 (2016) 115021 [arXiv:1608.08501] [inSPIRE].
[110] B. Dumont, K. Nishiwaki and R. Watanabe, LHC constraints and prospects for $S_{1}$ scalar leptoquark explaining the $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$ anomaly, Phys. Rev. D 94 (2016) 034001 [arXiv:1603.05248] [inSPIRE].
[111] D. Das, C. Hati, G. Kumar and N. Mahajan, Towards a unified explanation of $R_{D^{(*)}}, R_{K}$ and $(g-2)_{\mu}$ anomalies in a left-right model with leptoquarks, Phys. Rev. D 94 (2016) 055034 [arXiv:1605.06313] [INSPIRE].
[112] R. Barbieri, C.W. Murphy and F. Senia, B-decay Anomalies in a Composite Leptoquark Model, Eur. Phys. J. C 77 (2017) 8 [arXiv:1611.04930] [inSPIRE].
[113] C.-H. Chen, T. Nomura and H. Okada, Excesses of muon $g-2, R_{D^{(*)}}$, and $R_{K}$ in a leptoquark model, Phys. Lett. B 774 (2017) 456 [arXiv:1703.03251] [INSPIRE].
[114] W. Altmannshofer, P.S. Bhupal Dev and A. Soni, $R_{D^{(*)}}$ anomaly: A possible hint for natural supersymmetry with R-parity violation, Phys. Rev. D 96 (2017) 095010 [arXiv:1704.06659] [INSPIRE].
[115] A.K. Alok, B. Bhattacharya, A. Datta, D. Kumar, J. Kumar and D. London, New Physics in $b \rightarrow s \mu^{+} \mu^{-}$after the Measurement of $R_{K^{*}}$, Phys. Rev. D 96 (2017) 095009 [arXiv:1704.07397] [inSPIRE].
[116] A.K. Alok, B. Bhattacharya, D. Kumar, J. Kumar, D. London and S.U. Sankar, New physics in $b \rightarrow s \mu^{+} \mu^{-}$: Distinguishing models through CP-violating effects, Phys. Rev. D 96 (2017) 015034 [arXiv:1703.09247] [InSPIRE].
[117] A.K. Alok, J. Kumar, D. Kumar and R. Sharma, Lepton flavor non-universality in the B-sector: a global analyses of various new physics models, arXiv:1704.07347 [INSPIRE].
[118] A. Crivellin, C. Greub, D. Müller and F. Saturnino, Importance of Loop Effects in Explaining the Accumulated Evidence for New Physics in B Decays with a Vector Leptoquark, Phys. Rev. Lett. 122 (2019) 011805 [arXiv: 1807.02068] [INSPIRE].
[119] D. Bečirević, N. Košnik, O. Sumensari and R. Zukanovich Funchal, Palatable Leptoquark Scenarios for Lepton Flavor Violation in Exclusive $b \rightarrow s \ell_{1} \ell_{2}$ modes, JHEP 11 (2016) 035 [arXiv:1608.07583] [INSPIRE].
[120] A. Angelescu, D. Bečirević, D.A. Faroughy and O. Sumensari, Closing the window on single leptoquark solutions to the B-physics anomalies, JHEP 10 (2018) 183 [arXiv:1808.08179] [inSPIRE].
[121] I. Doršner, S. Fajfer, A. Greljo, J.F. Kamenik and N. Košnik, Physics of leptoquarks in precision experiments and at particle colliders, Phys. Rept. 641 (2016) 1 [arXiv:1603.04993] [INSPIRE].
[122] Y. Cai, J. Gargalionis, M.A. Schmidt and R.R. Volkas, Reconsidering the One Leptoquark solution: flavor anomalies and neutrino mass, JHEP 10 (2017) 047 [arXiv:1704.05849] [inSPIRE].
[123] J. Alwall et al., The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations, JHEP 07 (2014) 079 [arXiv:1405.0301] [INSPIRE].
[124] A. Alloul, N.D. Christensen, C. Degrande, C. Duhr and B. Fuks, FeynRules 2.0 - A complete toolbox for tree-level phenomenology, Comput. Phys. Commun. 185 (2014) 2250 [arXiv:1310.1921] [INSPIRE].
[125] J.L. Lopez, D.V. Nanopoulos and R. Rangarajan, New supersymmetric contributions to $t \rightarrow c V$, Phys. Rev. D 56 (1997) 3100 [hep-ph/9702350] [inSPIRE].
[126] T. Hahn, Generating Feynman diagrams and amplitudes with FeynArts 3, Comput. Phys. Commun. 140 (2001) 418 [hep-ph/0012260] [INSPIRE].
[127] T. Hahn and M. Pérez-Victoria, Automatized one loop calculations in four-dimensions and D-dimensions, Comput. Phys. Commun. 118 (1999) 153 [hep-ph/9807565] [inSPIRE].
[128] N. Assad, B. Fornal and B. Grinstein, Baryon Number and Lepton Universality Violation in Leptoquark and Diquark Models, Phys. Lett. B 777 (2018) 324 [arXiv:1708.06350] [inSPIRE].
[129] L. Di Luzio, A. Greljo and M. Nardecchia, Gauge leptoquark as the origin of B-physics anomalies, Phys. Rev. D 96 (2017) 115011 [arXiv:1708.08450] [INSPIRE].
[130] M. Bordone, C. Cornella, J. Fuentes-Martin and G. Isidori, A three-site gauge model for flavor hierarchies and flavor anomalies, Phys. Lett. B 779 (2018) 317 [arXiv:1712.01368] [inSPIRE].
[131] L. Calibbi, A. Crivellin and T. Li, Model of vector leptoquarks in view of the B-physics anomalies, Phys. Rev. D 98 (2018) 115002 [arXiv:1709.00692] [INSPIRE].
[132] M. Blanke and A. Crivellin, B Meson Anomalies in a Pati-Salam Model within the Randall-Sundrum Background, Phys. Rev. Lett. 121 (2018) 011801 [arXiv:1801.07256] [inSPIRE].
[133] R. Barbieri and A. Tesi, B-decay anomalies in Pati-Salam SU(4), Eur. Phys. J. C 78 (2018) 193 [arXiv:1712.06844] [INSPIRE].
[134] A. Greljo and B.A. Stefanek, Third family quark-lepton unification at the TeV scale, Phys. Lett. B 782 (2018) 131 [arXiv:1802.04274] [INSPIRE].
[135] H.H. Patel, Package-X: A Mathematica package for the analytic calculation of one-loop integrals, Comput. Phys. Commun. 197 (2015) 276 [arXiv:1503.01469] [inSPIRE].
[136] H.H. Patel, Package-X 2.0: A Mathematica package for the analytic calculation of one-loop integrals, Comput. Phys. Commun. 218 (2017) 66 [arXiv:1612.00009] [inSPIRE].
[137] A. Denner, Techniques for calculation of electroweak radiative corrections at the one loop level and results for $W$ physics at LEP-200, Fortsch. Phys. 41 (1993) 307 [arXiv:0709.1075] [inSPIRE].
[138] Particle Data Group collaboration, Review of particle physics, Phys. Rev. D 98 (2018) 030001 [INSPIRE].
[139] CMS collaboration, Search for Anomalous Single Top Quark Production in Association with a Photon in pp Collisions at $\sqrt{s}=8$ TeV, JHEP 04 (2016) 035 [arXiv:1511.03951] [inSPIRE].
[140] ATLAS collaboration, Search for single top-quark production via flavour-changing neutral currents at 8 TeV with the ATLAS detector, Eur. Phys. J. C 76 (2016) 55 [arXiv:1509.00294] [INSPIRE].
[141] CMS collaboration, Search for anomalous Wtb couplings and flavour-changing neutral currents in t-channel single top quark production in pp collisions at $\sqrt{s}=7$ and 8 TeV , JHEP 02 (2017) 028 [arXiv:1610.03545] [InSPIRE].
[142] CMS collaboration, Search for flavour changing neutral currents in top quark production and decays with three-lepton final state using the data collected at $\sqrt{s}=13 \mathrm{TeV}$, CMS-PAS-TOP-17-017 (2017).
[143] ATLAS collaboration, Search for flavour-changing neutral current top-quark decays $t \rightarrow q Z$ in proton-proton collisions at $\sqrt{s}=13 \mathrm{TeV}$ with the ATLAS detector, JHEP 07 (2018) 176 [arXiv:1803.09923] [INSPIRE].
[144] CMS collaboration, Identification of heavy-flavour jets with the CMS detector in pp collisions at $13 \mathrm{TeV}, 2018$ JINST 13 P05011 [arXiv:1712.07158] [INSPIRE].
[145] S. Ovyn, X. Rouby and V. Lemaître, DELPHES, a framework for fast simulation of a generic collider experiment, arXiv:0903. 2225 [InSPIRE].
[146] DELPHES 3 collaboration, DELPHES 3, A modular framework for fast simulation of a generic collider experiment, JHEP 02 (2014) 057 [arXiv:1307.6346] [INSPIRE].


[^0]:    ${ }^{1}$ These updates do not change significantly from the previous ones when considering uncertainties, i.e. $R_{D}=0.407 \pm 0.046$ and $R_{D^{*}}=0.306 \pm 0.015$ [9]. Our main conclusions in this work are not affected much, especially for the order of magnitude in our numerical results.
    ${ }^{2} R_{K^{*}}^{[1.16]}$ cited are using the combined charged and neutral channels in Belle's measurement. $R_{K^{(*)}}$ in other energy bins can be found in [17].

[^1]:    ${ }^{3}$ To make it consistent, in the following calculations we also ignored terms in LQ-quark-lepton couplings that are induced by non-diagonal CKM elements, i.e. from the up-type quark mixings defined in $Q_{i}=$ $\left[\left(V^{\dagger} u_{L}\right)_{i} d_{L i}\right]^{T}$. We checked that the dropped terms are negligibly small.

[^2]:    ${ }^{4}$ Systematic uncertainties are not included in this quick multi-variate analysis given the limited statistics.

