Differential decay rates of CP-even and CP-odd Higgs bosons to top and bottom quarks at NNLO QCD

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ABSTRACT: We consider the decay of a neutral Higgs boson of arbitrary CP nature to a massive quark antiquark pair at next-to-next-to-leading order in perturbative QCD. Our analysis is made at the differential level using the antenna subtraction framework. We apply our general set-up to the decays of a CP-even and CP-odd heavy Higgs boson to a top-quark top-antiquark pair and to the decay of the 125 GeV Higgs boson to a massive bottom-quark bottom-antiquark pair. In the latter case we calculate, in particular, the two-jet, three-jet, and four-jet decay rates and, for two-jet events, the energy distribution of the leading jet.

KEYWORDS: Higgs Physics, Beyond Standard Model, Heavy Quark Physics

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Contents

1	Introduction	1
2	The differential decay rate at NNLO QCD	3
2.1	Double real radiation corrections	4
2.2	Real-virtual corrections	5
2.3	Double virtual corrections	6
3	Decay rate in terms of the $\overline{\text{MS}}$ Yukawa coupling	6
4	Decays of non-standard Higgs bosons to $t\bar{t}X$	11
5	Decay of $h(125)$ to $b\bar{b}X$	13
5.1	Inclusive decay width	14
5.2	Two-jet, three-jet, and four-jet rates	15
6	Conclusions	17

1 Introduction

The detailed investigation of the production and decay modes of the 125 GeV Higgs boson and the search for new Higgs resonances remain among the major research topics at the Large Hadron Collider (LHC). The interactions of the 125 GeV Higgs boson, as investigated by the ATLAS and CMS experiments, are in accord with the predictions of the Standard Model (SM), and considerable improvement of the experimental precision is expected with the future high luminosity LHC program. (For a recent overview see, for instance, [1].) Precision Higgs physics and searches for new spin-zero resonances are also key issues for putting forward plans for future linear or circular e^+e^- colliders [2–7].

A plethora of theoretical investigations have been made on non-standard Higgs bosons and their interactions within SM extensions and, specifically, on the production and the decay modes of the SM and of non-SM Higgs bosons including higher-order QCD and electroweak radiative corrections. In this paper we are concerned with the decays of heavy neutral non-SM Higgs bosons and of the 125 GeV Higgs boson — these states will be generically denoted by h in the following — into a massive quark antiquark pair ($Q\bar{Q}$) at next-to-next-to-leading (NNLO) order in the QCD coupling. Let us briefly recapitulate previous work on neutral Higgs-boson decays to quarks. The order α_s QCD corrections to the decay width of $h \rightarrow Q\bar{Q}$ and several distributions including the full quark-mass dependence were computed both for scalar and pseudoscalar Higgs bosons by [8–14]. Higher order QCD corrections to the hadronic decay widths of neutral Higgs bosons, in particular the decay widths into quark antiquark pairs, were determined from the imaginary part

of respective two-point current correlation functions. The corrections to order α_s^2 were computed in [15] for massless quarks, while quadratic and higher-order quark-mass corrections in the m_Q/m_h expansion were calculated both for scalar and pseudoscalar Higgs bosons by [16–19] and by [20–22]. The order α_s^2 scalar and pseudoscalar current correlators were determined by [23] for arbitrary values of m_Q/m_h using Padé approximations. The hadronic decay width of a scalar Higgs boson was computed for massless quarks to order α_s^3 and α_s^4 in [24, 25] and [26–29], respectively. Partial results for the decay of a pseudoscalar Higgs boson to three massless partons at NNLO QCD were presented in [30]. Electroweak corrections were determined in [31, 32]. For Higgs-boson decays into quark antiquark pairs, differential distributions were computed by [33, 34] for the decay of a scalar Higgs boson into massless b quarks at order α_s^2 .

We analyze in this paper the decay of a neutral Higgs boson h into a massive quark antiquark pair,

$$h \rightarrow Q\bar{Q}X, \quad Q = t, b, \quad (1.1)$$

at order α_s^2 , i.e. at NNLO QCD, and to lowest order in the Yukawa and electroweak interactions. The quark-mass dependence of the matrix elements is taken into account without any approximation.¹ We consider the general case of a neutral Higgs boson with both scalar and pseudoscalar couplings to quarks. Its Yukawa interaction is

$$\mathcal{L}_{Y,Q} = -y_{0,Q} [a_Q \bar{Q}Q + b_Q \bar{Q}i\gamma_5 Q] h, \quad (1.2)$$

where Q and h denote bare fields, $y_{0,Q}$ is the bare SM Yukawa coupling,

$$y_{0,Q} = \frac{m_{0,Q}}{v}, \quad (1.3)$$

$m_{0,Q}$ is the bare mass of Q , $v = (\sqrt{2}G_F)^{-1/2}$ is the SM Higgs vacuum expectation value with G_F being Fermi's constant, and a_Q, b_Q are dimensionless reduced Yukawa couplings whose values depend on the parameters of the specific model under consideration. If h is identified with the SM Higgs boson then

$$\text{SM:} \quad a_Q = 1, \quad b_Q = 0. \quad (1.4)$$

Because we consider in this work only QCD corrections, the reduced Yukawa couplings a_Q, b_Q do not get renormalized.

Our set-up applies to the general case (1.2) where h is not a CP eigenstate. For the decay of a heavy Higgs boson into $t\bar{t}$ we analyze the cases where h is CP-even (i.e., a scalar where $b_t = 0$) and CP-odd (a pseudoscalar where $a_t = 0$), respectively. Because we consider unpolarized $Q, \bar{Q}$ our NNLO QCD results can be combined and applied to the case of h being a CP mixture (see below). In our analysis of $h(125\text{GeV}) \rightarrow b\bar{b}X$ we assume that h is CP-even.

Our paper is organized as follows. We outline in the next section the calculation of the differential decay rate and distributions of IR-safe observables of (1.1) at order α_s^2 using the

¹With the exception of the top-quark triangle contribution to the two-loop $h \rightarrow b\bar{b}$ amplitude, see section 3.

antenna subtraction method [35–38]. First we use renormalized matrix elements where the QCD coupling is defined in the $\overline{\text{MS}}$ scheme and the quark masses and the Yukawa coupling are defined in the on-shell scheme. In section 3 we express the on-shell Yukawa coupling in terms of the $\overline{\text{MS}}$ Yukawa coupling and derive a generic formula for the resulting differential decay widths of (1.1) to order α_s^2 . In section 4 we apply this set-up to the decays of a scalar and a pseudoscalar Higgs boson $h \rightarrow t\bar{t}X$. We calculate the respective decay widths for a sequence of Higgs-boson masses and we compare with results that are obtained by expansion in $(m_t/m_h)^2$ to fourth order [21]. In order to demonstrate the usefulness of our differential approach we compute the top-quark energy distribution in the Higgs-boson rest frame and the $t\bar{t}$ invariant mass distribution at NNLO QCD both for a scalar and a pseudoscalar Higgs boson of mass $m_h = 500$ GeV. In section 5 we analyze $h(125) \rightarrow b\bar{b}X$ at NNLO QCD for massive b quarks, using the formulas of sections 2 and 3, where we assume the 125 GeV Higgs boson to be purely CP-even. We compute the inclusive decay width and the decay rates into two, three, and four jets using the Durham cluster algorithm. Moreover, we compute for two-jet events the distribution of the energy of the leading jet. In addition we compare with results from the literature obtained at order α_s^2 for massless b quarks. We conclude in section 6.

2 The differential decay rate at NNLO QCD

In this section we briefly outline the salient features of computing the decay rate of $h \rightarrow Q\bar{Q}X$ to order α_s^2 at the differential level using the antenna subtraction method. Here Q denotes a massive quark, for instance, the b or t quark. The antenna method with massive quarks in the final state that we use has been described in detail in [39] for the process $e^+e^- \rightarrow Q\bar{Q}X$. Because the strategy and formulas presented in [39] can be applied in analogous fashion to the reaction at hand we will delineate in the following only the most important features of computing the various infrared-finite contributions to the differential Higgs-boson decay rate to a pair of massive quarks at order α_s^2 and refer for details to [39].

We work in QCD with n_f massless quarks q and one massive quark² Q . All matrix elements in this and in the following sections refer to renormalized matrix elements. We define the renormalized mass of Q in the on-shell scheme and denote it by m_Q , while the QCD coupling α_s is defined in the $\overline{\text{MS}}$ scheme. Dimensional regularization is used to handle IR singularities that appear in intermediate steps of our calculation. As to the renormalization of the Yukawa coupling $y_{0,Q}$, we proceed as follows. First, we renormalize it in the on-shell scheme and then convert it to the $\overline{\text{MS}}$ scheme. Details will be given in section 3.

We use the following notation for the differential rate of the decay (1.1):

$$d\Gamma = d\Gamma_{LO} + d\Gamma_1 + d\Gamma_2 + \mathcal{O}(\alpha_s^3), \quad (2.1)$$

where $d\Gamma_{LO}$ is the leading-order contribution and $d\Gamma_1, d\Gamma_2$ are the contributions of order α_s and α_s^2 . The first order correction receives contributions from the interference of the Born and one-loop amplitude of $h \rightarrow Q\bar{Q}$ and the squared Born amplitude of $h \rightarrow Q\bar{Q}g$.

²In section 5 both the b quark and, of course, also the top quark are taken to be massive.

In any subtraction scheme for handling the IR divergences the NLO QCD correction can be written as follows:

$$\int d\Gamma_1 = \int_{\Phi_3} \left[\left(d\Gamma_{Q\bar{Q}g}^R \right)_{\epsilon=0} - \left(d\Gamma_{Q\bar{Q}g}^S \right)_{\epsilon=0} \right] + \int_{\Phi_2} \left[d\Gamma_{Q\bar{Q}}^V + \int_1 d\Gamma_{Q\bar{Q}g}^S \right]_{\epsilon=0}, \quad (2.2)$$

where $\epsilon = (4 - D)/2$ is the dimensional regularization parameter and the subscripts Φ_n denote n -particle phase-space integrals. The second term in the first and second square bracket of (2.2) is the unintegrated and integrated subtraction term that renders the difference, respectively the sum of the terms in the square brackets finite in $D = 4$ dimensions. Here and in the following the symbol $\int_n$ indicates the analytic integration over the phase space of n unresolved partons in $D \neq 4$ dimensions. The NLO subtraction terms labeled with the superscript S in (2.2) were computed within the antenna framework in [40].

The second-order term $d\Gamma_2$ in the expansion (2.1) of the differential decay rate receives the following contributions: i) the double real radiation contribution $d\Gamma_{\text{NNLO}}^{RR}$ associated with the squared Born amplitudes of $h \rightarrow Q\bar{Q}gg$ and $h \rightarrow Q\bar{Q}q\bar{q}$ (where q denotes a massless quark), and above the $4Q$ threshold, the squared Born matrix element of $h \rightarrow Q\bar{Q}Q\bar{Q}$, ii) the real-virtual cross section $d\Gamma_{\text{NNLO}}^{RV}$ associated with the matrix element of $h \rightarrow Q\bar{Q}g$ to order α_s^2 (1-loop times Born), and iii) the double virtual correction $d\Gamma_{\text{NNLO}}^{VV}$ associated with the matrix element of $h \rightarrow Q\bar{Q}$ to order α_s^2 (i.e., 2-loop times Born and 1-loop squared).

Apart from the $Q\bar{Q}Q\bar{Q}$ contribution, which is IR finite, the terms i), ii), iii) are IR divergent. When a subtraction method is used the second order correction $d\Gamma_2$, where the different pieces are separately finite in $D = 4$ dimensions, is constructed schematically as follows:

$$\begin{aligned} \int d\Gamma_2 = & \int_{\Phi_4} (d\Gamma_{\text{NNLO}}^{RR} - d\Gamma_{\text{NNLO}}^S) + \int_{\Phi_3} (d\Gamma_{\text{NNLO}}^{RV} - d\Gamma_{\text{NNLO}}^T) \\ & + \left(\int_{\Phi_2} d\Gamma_{\text{NNLO}}^{VV} + \int_{\Phi_3} d\Gamma_{\text{NNLO}}^T + \int_{\Phi_4} d\Gamma_{\text{NNLO}}^S \right). \end{aligned} \quad (2.3)$$

The symbols $d\Gamma_{\text{NNLO}}^S$ and $d\Gamma_{\text{NNLO}}^T$ denote the double-real subtraction terms (for $h \rightarrow Q\bar{Q}q\bar{q}$ and $h \rightarrow Q\bar{Q}gg$) and the real-virtual subtraction term (for $h \rightarrow Q\bar{Q}g$), respectively. We shortly discuss in turn the various terms in (2.3).

2.1 Double real radiation corrections

The first term on the right-hand side of (2.3) receives the following contributions:

$$d\Gamma_{\text{NNLO}}^{RR} - d\Gamma_{\text{NNLO}}^S = d\Gamma_{\text{NNLO}}^{\text{sub}, Q\bar{Q}q\bar{q}} + d\Gamma_{\text{NNLO}}^{\text{sub}, Q\bar{Q}gg} + d\Gamma_{\text{NNLO}}^{Q\bar{Q}Q\bar{Q}}. \quad (2.4)$$

The last term on the right-hand side of this equation is the unsubtracted, IR-finite differential decay rate associated with the $4Q$ final state, while the first two terms denote the subtracted, IR-finite differential decay rates associated with the $Q\bar{Q}q\bar{q}$ and $Q\bar{Q}gg$ final states. Within the antenna subtraction framework they are defined as follows:

$$d\Gamma_{\text{NNLO}}^{\text{sub}, Q\bar{Q}ij} = d\Gamma_{\text{NNLO}}^{Q\bar{Q}ij} - d\Gamma_{\text{NNLO}}^{S,a, Q\bar{Q}ij} - d\Gamma_{\text{NNLO}}^{S,b,2, Q\bar{Q}ij} - d\Gamma_{\text{NNLO}}^{S,b,1, Q\bar{Q}ij}, \quad ij = q\bar{q}, gg. \quad (2.5)$$

In the case of $ij = q\bar{q}$ a sum over all massless quark-antiquark pairs is implicit on the right-hand side of (2.5). The term $d\Gamma_{\text{NNLO}}^{S,a, Q\bar{Q}ij}$ subtracts from $d\Gamma_{\text{NNLO}}^{Q\bar{Q}ij}$ the IR singularities

associated with the single unresolved parton configurations, while the subtraction term $d\Gamma_{\text{NNLO}}^{S,b,2,Q\bar{Q}ij}$ takes care of the singularities of $d\Gamma_{\text{NNLO}}^{Q\bar{Q}ij}$ associated with the double unresolved configurations. The latter subtraction term develops, however, singularities in the single unresolved limits that are subtracted by the term $d\Gamma_{\text{NNLO}}^{S,b,1,Q\bar{Q}ij}$.

The antenna subtraction terms required in (2.5) were determined in [41] and [42] for the $Q\bar{Q}q\bar{q}$ and $Q\bar{Q}gg$ final states, respectively. They involve redefined on-shell momenta constructed from the final-state parton momenta. For the construction in the case where two massive quarks are involved, see for instance [39].

There is a subtlety associated with angular correlations in the unsubtracted squared matrix element of $Q\bar{Q}ij$, caused by the splitting into a pair of massless partons i, j of the virtual gluon radiated off the Q or $\bar{Q}$. The subtraction term $d\Gamma_{\text{NNLO}}^{S,b,2,Q\bar{Q}ij}$ contains these angular correlations, too, but the subtraction terms $d\Gamma_{\text{NNLO}}^{S,a,Q\bar{Q}ij}$ and $d\Gamma_{\text{NNLO}}^{S,b,1,Q\bar{Q}ij}$ do not [39]. This precludes a local cancellation of the IR singularities. A very efficient IR cancellation can be achieved by averaging out these angular correlations. This can be done as follows [43–46]. Denoting the final-state four-momenta by k_1, k_2, k_3, k_4 where k_3, k_4 are the momenta of the massless partons i, j , one evaluates the two terms in (2.5) that contain the angular correlations for each set of phase-space momenta k_1, k_2, k_3, k_4 and k_1, k_2, k_{3r}, k_{4r} and takes the average. The four-momenta k_{3r}, k_{4r} are obtained by rotating the spatial parts of k_3, k_4 by an angle $\pi/2$ around the collinear axis $\mathbf{k} = \mathbf{k}_3 + \mathbf{k}_4$. We will apply this procedure in our calculation of the inclusive cross sections $h \rightarrow Q\bar{Q}X$ ($Q = t, b$) and distributions associated with the massive quark. In section 5.2 we compute, in addition, the two-jet, three-jet, and four-jet rates for the decay $h \rightarrow b\bar{b}X$. In this case this averaging procedure is not applicable because the rotated momenta k_{3r}, k_{4r} are located, in general, outside of the respective n-jet phase space. Nevertheless, straightforward subtraction as prescribed in (2.5) leads to a numerically stable IR cancellation in the calculation of the n-jet cross sections.

2.2 Real-virtual corrections

Within the antenna subtraction framework, the second term on the right-hand side of (2.3) receives the following contributions:

$$\int_{\Phi_3} (d\Gamma_{\text{NNLO}}^{RV} - d\Gamma_{\text{NNLO}}^T) = \int_{\Phi_3} \left[d\sigma_{\text{NNLO}}^{RV,Q\bar{Q}g} - d\Gamma_{\text{NNLO}}^{T,a,Q\bar{Q}g} - d\Gamma_{\text{NNLO}}^{T,b,Q\bar{Q}g} - d\Gamma_{\text{NNLO}}^{T,c,Q\bar{Q}g} \right]_{\epsilon=0}. \quad (2.6)$$

The term $d\Gamma_{\text{NNLO}}^{RV}$ involves the interference of the tree-level and renormalized one-loop amplitude associated with $h \rightarrow Q\bar{Q}g$. This term contains explicit IR poles, i.e., single and doubles poles in ϵ . These poles are canceled by the explicit IR poles of the second term on the right-hand side of (2.6) that is determined by

$$d\Gamma_{\text{NNLO}}^{T,a,Q\bar{Q}g} = - \int_1 d\Gamma_{\text{NNLO}}^{S,a,Q\bar{Q}gg} - \sum_q \int_1 d\Gamma_{\text{NNLO}}^{S,a,Q\bar{Q}q\bar{q}}. \quad (2.7)$$

The integrals in (2.7) denote the analytic integration of the a-type subtraction terms appearing in (2.5) over the phase space of one unresolved massless parton in $D \neq 4$ dimensions. The result of the integration involves integrated tree-level antenna functions determined

in [40, 47]. Apart from its explicit IR poles $d\Gamma_{\text{NNLO}}^{RV}$ is IR-singular also in the limit where the external gluon becomes soft. The term $d\Gamma_{\text{NNLO}}^{T,b,Q\bar{Q}g}$ is constructed such that it cancels those terms of $d\Gamma_{\text{NNLO}}^{RV}$ which become singular in the limit $k_{\text{gluon}} \rightarrow 0$. This subtraction term involves massive one-loop antenna functions that were computed in [48].

In certain regions of phase space, the subtraction terms (2.7) and $d\Gamma_{\text{NNLO}}^{T,b,Q\bar{Q}g}$ develop IR singularities that do not coincide with the IR singularities of $d\Gamma_{\text{NNLO}}^{RV,Q\bar{Q}g}$. For removing these spurious singularities one must introduce an additional subtraction term given by

$$d\Gamma_{\text{NNLO}}^{T,c,Q\bar{Q}g} = - \int_1 d\Gamma_{\text{NNLO}}^{S,b,1,Q\bar{Q}gg} - \int_1 d\Gamma_{\text{NNLO}}^{S,b,1,Q\bar{Q}q\bar{q}} , \quad (2.8)$$

where the integrands are subtraction terms that appear in (2.5). The analytic integration in $D \neq 4$ dimensions over the phase space of one unresolved parton was performed in [48].

By construction the expression (2.6) is finite in $D = 4$ dimensions when integrated over the three-parton phase space, or arbitrary sections thereof when computing differential distributions. We recall that the terms $d\Gamma_{\text{NNLO}}^{T,a,Q\bar{Q}g}$ and $d\Gamma_{\text{NNLO}}^{T,c,Q\bar{Q}g}$ are counterbalanced by the double-real subtraction terms $d\Gamma_{\text{NNLO}}^{S,a,Q\bar{Q}ij}$ and $d\Gamma_{\text{NNLO}}^{S,b,1,Q\bar{Q}ij}$ ($ij = gg, q\bar{q}$), respectively, that were introduced in (2.5). Hence, only the integrated form of $d\Gamma_{\text{NNLO}}^{T,b,Q\bar{Q}g}$ has to be added back to the contribution from the double virtual corrections.

2.3 Double virtual corrections

Recalling the subtraction terms introduced in (2.5) and (2.6) we see that those that are not yet counterbalanced are $d\Gamma_{\text{NNLO}}^{S,b,2,Q\bar{Q}ij}$ ($ij = q\bar{q}, gg$) and $d\Gamma_{\text{NNLO}}^{T,b,Q\bar{Q}g}$. They have to be integrated over the unresolved two-parton, respectively unresolved one-parton phase space and then serve as counterterms for the subtraction of the IR poles of the double virtual correction. Thus the last term on the right-hand side of (2.3) is given by

$$\begin{aligned} \int_{\Phi_2} d\Gamma_{\text{NNLO}}^{VV} + \int_{\Phi_3} d\Gamma_{\text{NNLO}}^T + \int_{\Phi_4} d\Gamma_{\text{NNLO}}^S = \\ = \int_{\Phi_2} \left[d\Gamma_{\text{NNLO}}^{VV,Q\bar{Q}} + \int_1 d\Gamma_{\text{NNLO}}^{T,b,Q\bar{Q}g} + \int_2 d\Gamma_{\text{NNLO}}^{S,b,2,Q\bar{Q}gg} + \sum_q \int_2 d\Gamma_{\text{NNLO}}^{S,b,2,Q\bar{Q}q\bar{q}} \right]_{\epsilon=0} . \end{aligned} \quad (2.9)$$

The term $d\Gamma_{\text{NNLO}}^{VV,Q\bar{Q}}$ is the order α_s^2 correction computed from the interference of the renormalized two-loop and Born matrix element and the squared one-loop matrix element of $h \rightarrow Q\bar{Q}$. In the sum (2.9) all IR poles cancel in analytic fashion. One can then take the limit $\epsilon \rightarrow 0$ and perform the integration over the two-parton phase space in four dimensions.

3 Decay rate in terms of the $\overline{\text{MS}}$ Yukawa coupling

After having outlined our subtraction formalism we discuss a few details of our computation of the differential decay rate of (1.1) for general Yukawa couplings (1.2) and unpolarized $Q, \bar{Q}$. For constructing the subtraction terms we need the (un)integrated antenna functions determined at NLO QCD in [40, 47] and at NNLO QCD in [41, 42, 48] and, in addition, the squared tree level matrix elements of $h \rightarrow Q\bar{Q}$ and $h \rightarrow Q\bar{Q}g$, and the $\mathcal{O}(\alpha_s)$ interference

matrix element (tree level and renormalized one-loop amplitude) of $h \rightarrow Q\bar{Q}$. These matrix elements and the antenna subtraction terms [40, 47] determine also the differential decay rate $d\Gamma_1$ at NLO. Because we sum over spins, $d\Gamma_1$ does not contain a CP-odd term proportional to $a_Q b_Q$.

As to the unsubtracted matrix elements that contribute to $d\Gamma_2$. Computation of the squared tree-level matrix elements of $h \rightarrow Q\bar{Q}gg$, $h \rightarrow Q\bar{Q}q\bar{q}$, and $h \rightarrow Q\bar{Q}Q\bar{Q}$ is straightforward. Summation over the spins of the partons in the final state eliminates the CP-odd terms proportional to $a_Q b_Q$. That is, these squared matrix elements are given by the incoherent sum of the respective squared matrix elements for the decay of a CP-even and CP-odd Higgs boson h . This holds true also for the contributions to $d\Gamma_2$ associated with three-parton and two-parton final states.

The renormalized one-loop amplitude for $h \rightarrow Q\bar{Q}g$ required in (2.6) is computed with standard one-loop perturbation theory methods. As to the $\mathcal{O}(\alpha_s^2)$ contribution to $h \rightarrow Q\bar{Q}$. The renormalized (but IR-divergent) non-singlet two-loop scalar and pseudoscalar $hQ\bar{Q}$ form factors (and the one-loop form factors including the terms of order ϵ) that enter the respective matrix element were computed in [49]. The two-loop singlet scalar form factor is contained in the results of [49] while the singlet pseudoscalar form factor was determined in [50]. The singlet form factors are UV- and IR-finite. The scalar and pseudoscalar two-loop form factors were recently computed also in [51]. While the unrenormalized non-singlet form factors of [51] agree with those of [49] the renormalized ones differ by terms proportional to $\zeta(2) = \pi^2/6$. This is because the renormalization constant for the QCD coupling in the $\overline{\text{MS}}$ scheme used in [51] is not identical to the one used³ in [49]; they differ at order ϵ^2 by a term proportional to $\zeta(2)$. This does not affect the UV renormalization of Green functions with at most a single UV pole per loop, but it affects the two-loop quark wave function and mass renormalization constants in the on-shell scheme that have double IR ϵ -poles and the renormalized non-singlet form factors. However, we emphasize that these differences are unphysical infrared artefacts. Our NNLO subtraction antenna subtraction terms [48] were computed with the same $\overline{\text{MS}}$ coupling renormalization convention as used in [49, 52, 53]. Therefore these IR artefacts cancel in the computation of observables.

A further remark pertains to the order α_s^2 flavor-singlet contributions to the scalar and pseudoscalar $h \rightarrow Q\bar{Q}$ and $h \rightarrow Q\bar{Q}g$ amplitude, depicted in figure 1a and 1b, respectively. These amplitudes are ultraviolet- and infrared-finite. Let us first discuss $h \rightarrow t\bar{t}$. The case where h has a scalar coupling to the internal top-quark triangle (that is, equal masses associated with the inner and outer fermion line) has been taken into account in [49], while the amplitude associated with a pseudoscalar coupling of h to the internal top-quark loop was determined in [50]. Here we neglect the masses of the five quarks $b, \dots, u$ in the propagators of the respective fermion triangle. Therefore, they do not contribute to the flavor-singlet scalar and pseudoscalar amplitudes $h \rightarrow t\bar{t}$ and $h \rightarrow t\bar{t}g$ because the respective massless quark triangle is proportional to the trace of the product of an odd number of γ matrices. The contribution to $h \rightarrow t\bar{t}g$ with a top quark circulating in the triangle is computed with standard methods.

³The $\overline{\text{MS}}$ renormalization of the QCD coupling used in [49] is explicitly defined in [52, 53].

Figure 1. The flavor-singlet contributions to $h \rightarrow Q\bar{Q}$ and $h \rightarrow Q\bar{Q}g$, where $Q, Q' = t, b$ and h denotes a scalar or pseudoscalar Higgs boson. If the quark in a triangle (a) or (b) is taken to be massless then the contribution is zero. Diagrams with reversed fermion flow in the triangles are not shown.

In our calculation of $h(125\text{GeV}) \rightarrow b\bar{b}X$ we assume h to be CP-even, take the b quark to be massive and u, d, c, s to be massless. As just mentioned, concerning the $h \rightarrow b\bar{b}$ amplitude, the flavor-singlet scalar equal-mass case with an internal and external massive b quark is contained in the results given in [49]. The massive b -quark triangle contribution to $h \rightarrow b\bar{b}g$ is again computed with standard methods. The scalar flavor-singlet contribution to $h \rightarrow b\bar{b}X$ with a top quark circulating in the triangle — which does not decouple — and an external massive b quark, that is, the sum of the contributions shown in figure 1a and 1b, where $Q' = t$ and $Q = b$, was computed in [20] to leading order in the b -quark mass as an expansion in inverse powers of the top-quark mass. (The first four terms of this expansion are given in [20].) For our computation of the two-jet and three-jet rates for $h \rightarrow b\bar{b}X$ in section we will need the contributions figure 1a and 1b with a top quark (and b quark) circulating in the respective quark triangle separately. We compute the contribution to the differential decay rate of $h \rightarrow b\bar{b}g$ with a top-quark in the triangle, and by subtraction of this term from the result of [20] we obtain the top-quark triangle contribution to $h \rightarrow b\bar{b}$.

As already mentioned above, we first compute the differential decay width in terms of the on-shell mass m_Q of Q and in terms of the on-shell Yukawa coupling $y_Q = m_Q/v$. It reads schematically to order α_s^2 :

$$d\Gamma^{Q\bar{Q}} = y_Q^2 \left[d\hat{\Gamma}_0^{Q\bar{Q}} + \frac{\alpha_s(\mu)}{\pi} d\hat{\Gamma}_1^{Q\bar{Q}} + \left(\frac{\alpha_s(\mu)}{\pi} \right)^2 d\hat{\Gamma}_2^{Q\bar{Q}} \right], \quad (3.1)$$

where the $\overline{\text{MS}}$ coupling $\alpha_s(\mu)$ is defined in QCD with n_l massless and one massive quark, μ is the renormalization scale,

$$d\hat{\Gamma}_i^{Q\bar{Q}} = d\hat{\Gamma}_i^{Q\bar{Q}}(m_Q, m_h, a_Q, b_Q, \mu), \quad (3.2)$$

and m_h denotes the Higgs-boson mass. To lowest order in α_s we have

$$\hat{\Gamma}_0^{Q\bar{Q}} = N_c \frac{\beta_Q}{8\pi m_h} [a_Q^2(m_h^2 - 4m_Q^2) + b_Q^2 m_h^2], \quad (3.3)$$

where $\beta_Q = \sqrt{1 - 4m_Q^2/m_h^2}$ and N_c denotes the number of colors.

It has been known for a long time [8] that it is not a good idea to express the decay rate in terms of the on-shell Yukawa coupling when $m_Q/m_h \ll 1$, because then the decay rate contains already at NLO a large logarithm in m_Q/m_h that arises from the on-shell renormalization. This logarithm can be absorbed by using the $\overline{\text{MS}}$ Yukawa coupling $\bar{y}_Q(\mu) = \bar{m}_Q(\mu)/v$, where $\bar{m}_Q(\mu)$ is the running $\overline{\text{MS}}$ mass, and choosing $\mu = m_h$. In the following we convert in (3.1) the on-shell Yukawa coupling y_Q to $\bar{y}_Q(\mu)$ using the relation between these couplings to order α_s^2 . One may express the coefficients $d\hat{\Gamma}_i^{Q\bar{Q}}$ defined in (3.2) also in terms of the $\overline{\text{MS}}$ mass $\bar{m}_Q(\mu)$. However, we keep their dependence on the on-shell mass m_Q because one may argue that this yields a somewhat more realistic description of the decay kinematics when computing distributions. We notice that this hybrid renormalization prescription also avoids large logarithms in the case $m_Q/m_h \ll 1$. In our analysis of $h \rightarrow t\bar{t}X$ in section 4 we consider a range of Higgs-boson masses for which the ratio m_t/m_h is not very small. Nevertheless it turns out that using the $\overline{\text{MS}}$ Yukawa coupling $\bar{y}_t$ is appropriate also in this case, see section 4.

For expressing y_Q in terms of $\bar{y}_Q$ we need the relation between the on-shell mass m_Q and the $\overline{\text{MS}}$ mass $\bar{m}_Q(\mu)$ to order α_s^2 . It reads in QCD with n_l massless and one massive quark [54, 55]:

$$m_Q = \bar{m}_Q(\mu) \left[1 + c_1(m_Q, \mu) \frac{\alpha_s(\mu)}{\pi} + c_2(m_Q, \mu) \left(\frac{\alpha_s(\mu)}{\pi} \right)^2 \right] + \mathcal{O}(\alpha_s^3), \quad (3.4)$$

where

$$c_1 = -u_1, \quad c_2 = u_1^2 - u_2, \quad (3.5)$$

and

$$\begin{aligned} u_1 &= -C_F \left(1 + \frac{3}{4} L_Q \right), \\ u_2 &= C_F^2 \left[\frac{7}{128} + 3\zeta(2) \left(-\frac{5}{8} + \ln 2 \right) - \frac{3}{4} \zeta(3) + \frac{21}{32} L_Q + \frac{9}{32} L_Q^2 \right] \\ &\quad + N_c C_F \left[-\frac{1111}{384} + \frac{\zeta(2)}{2} (1 - 3 \ln 2) + \frac{3}{8} \zeta(3) - \frac{185}{96} L_Q - \frac{11}{32} L_Q^2 \right] \\ &\quad + C_F T_F n_l \left[\frac{71}{96} + \frac{1}{2} \zeta(2) + \frac{13}{24} L_Q + \frac{1}{8} L_Q^2 \right] \\ &\quad + C_F T_F \left[\frac{143}{96} - \zeta(2) + \frac{13}{24} L_Q + \frac{1}{8} L_Q^2 \right]. \end{aligned} \quad (3.6)$$

Here

$$C_F = \frac{N_c^2 - 1}{2N_c}, \quad T_F = \frac{1}{2}, \quad L_Q = \ln \left(\frac{\mu^2}{m_Q^2} \right), \quad \zeta(2) = \frac{\pi^2}{6}, \quad \zeta(3) = 1.20205690 \dots \quad (3.8)$$

Inserting (3.4) into the squared Yukawa coupling in the on-shell scheme, $y_Q^2 = m_Q^2/v^2$, and expanding to order α_s^2 we get

$$y_Q^2 = \bar{y}_Q^2(\mu) \left[1 + r_1(m_Q, \mu) \frac{\alpha_s(\mu)}{\pi} + r_2(m_Q, \mu) \left(\frac{\alpha_s(\mu)}{\pi} \right)^2 \right], \quad (3.9)$$

where $\bar{y}_Q^2(\mu)$ is the squared Yukawa coupling in the $\overline{\text{MS}}$ scheme,

$$\bar{y}_Q^2(\mu) = \frac{\bar{m}_Q^2(\mu)}{v^2}, \quad r_1 = -2u_1, \quad r_2 = 3u_1^2 - 2u_2. \quad (3.10)$$

Let us now apply (3.9) to the differential decay width of $h \rightarrow Q\bar{Q}X$. Inserting (3.9) into (3.1) and expanding to order α_s^2 we obtain schematically:

$$d\bar{\Gamma}^{Q\bar{Q}} = \bar{y}_Q^2(\mu) \left[d\hat{\Gamma}_0^{Q\bar{Q}} + \frac{\alpha_s(\mu)}{\pi} \left(d\hat{\Gamma}_1^{Q\bar{Q}} + r_1 d\hat{\Gamma}_0^{Q\bar{Q}} \right) + \left(\frac{\alpha_s(\mu)}{\pi} \right)^2 \left(d\hat{\Gamma}_2^{Q\bar{Q}} + r_1 d\hat{\Gamma}_1^{Q\bar{Q}} + r_2 d\hat{\Gamma}_0^{Q\bar{Q}} \right) \right]. \quad (3.11)$$

Because of the truncation of the perturbation series, equation (3.11) is not identical to (3.1); therefore we use the bar notation. The decay width can be cast into the form

$$\bar{\Gamma}^{Q\bar{Q}} = \bar{y}_Q^2(\mu) \hat{\Gamma}_0^{Q\bar{Q}} \left[1 + \frac{\alpha_s(\mu)}{\pi} \left(\gamma_1^{Q\bar{Q}} + r_1 \right) + \left(\frac{\alpha_s(\mu)}{\pi} \right)^2 \left(\gamma_2^{Q\bar{Q}} + r_1 \gamma_1^{Q\bar{Q}} + r_2 \right) \right], \quad (3.12)$$

where

$$\gamma_1^{Q\bar{Q}} = \frac{\hat{\Gamma}_1^{Q\bar{Q}}}{\hat{\Gamma}_0^{Q\bar{Q}}}, \quad \gamma_2^{Q\bar{Q}} = \frac{\hat{\Gamma}_2^{Q\bar{Q}}}{\hat{\Gamma}_0^{Q\bar{Q}}}. \quad (3.13)$$

The $\overline{\text{MS}}$ Yukawa coupling makes the LO decay width $\bar{y}_Q^2(\mu) \hat{\Gamma}_0^{Q\bar{Q}}$ dependent on μ . In our application of (3.11) and (3.12) to $h \rightarrow t\bar{t}X$ we work in 6-flavor QCD with $\alpha_s = \alpha_s^{(6)}$ and $n_l = 5$ while in the application to $h \rightarrow b\bar{b}X$ we evaluate these formulas with $\alpha_s = \alpha_s^{(5)}$ and $n_l = 4$.

When computing the on-shell mass of Q from its $\overline{\text{MS}}$ mass the coefficients c_1, c_2 in (3.4) are expressed in terms of the $\overline{\text{MS}}$ mass instead of the on-shell mass. Iterating (3.4) we get

$$m_Q = \bar{m}_Q(\mu) \left[1 + \bar{c}_1(\bar{m}_Q, \mu) \frac{\alpha_s(\mu)}{\pi} + \bar{c}_2(\bar{m}_Q, \mu) \left(\frac{\alpha_s(\mu)}{\pi} \right)^2 \right], \quad (3.14)$$

where

$$\bar{c}_1 = -u_1(\bar{m}_Q, \mu), \quad \bar{c}_2 = u_1^2(\bar{m}_Q, \mu) - u_2(\bar{m}_Q, \mu) + \frac{3C_F}{2} u_1(\bar{m}_Q, \mu), \quad (3.15)$$

i.e., the logarithm L_Q in u_1, u_2 is replaced by

$$\bar{L}_Q = \ln \left(\frac{\mu^2}{\bar{m}_Q^2(\mu)} \right). \quad (3.16)$$

For computing the $\overline{\text{MS}}$ top and bottom Yukawa couplings at arbitrary scales μ we use the solution of the renormalization group equation for $\bar{m}_Q(\mu)$ at two-loops. It reads

$$\bar{m}_Q(\mu) = \bar{m}_Q(\mu_0) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\gamma_{m,0}/\beta_0} \left\{ 1 + \frac{(\gamma_{m,1}\beta_0 - \gamma_{m,0}\beta_1)}{\pi\beta_0^2} [\alpha_s(\mu) - \alpha_s(\mu_0)] + \mathcal{O}(\alpha_s^2) \right\}. \quad (3.17)$$

The coefficients are, putting $N_c = 3$ and $n_f = n_l + 1$:

$$\gamma_{m,0} = 1, \quad \gamma_{m,1} = \frac{303 - 10n_f}{72}, \quad (3.18)$$

$$\beta_0 = \frac{33 - 2n_f}{12}, \quad \beta_1 = \frac{153 - 19n_f}{24}, \quad (3.19)$$

and α_s in (3.17) is the coupling in n_f -flavor QCD.

Finally, a remark concerning the relation between the on-shell and $\overline{\text{MS}}$ Yukawa coupling of Q . When the iterated relation (3.14) instead of (3.4) is used to express y_Q^2 in terms of $\overline{y}_Q^2$ to order α_s^2 , the coefficients r_1 and r_2 in (3.9) change; in particular, they depend then on the $\overline{\text{MS}}$ mass of Q . In cases where the ratio m_Q/m_h is not very small, one may consider it to be a matter of convention which one of these two variants one uses. In cases where m_Q/m_h tends to zero the formulas (3.11) and (3.12) are to be used if the matrix elements are computed in terms of the on-shell mass m_Q , as we do. This is because for small m_Q the $d\hat{\Gamma}_i^{QQ}$ ($i = 1, 2$) develop large logarithms in the on-shell mass which are compensated by the large logarithms in m_Q that appear in r_1, r_2 . We will show in section 5.1 that by choosing a small b -quark mass, eq. (3.12) allows to recover the known $h \rightarrow b\bar{b}X$ decay width at order α_s^2 for massless b quarks.

4 Decays of non-standard Higgs bosons to $t\bar{t}X$

In this section we analyze the decay of a non-standard heavy neutral Higgs boson of arbitrary CP nature into $t\bar{t}$ for a set of Higgs-boson masses. As emphasized before we consider unpolarized final states. Then, as mentioned in the previous section, the differential decay rate to order α_s^2 and to lowest order in the electroweak couplings decomposes into the incoherent sum

$$d\overline{\Gamma}^{t\bar{t}} = a_t^2 d\overline{\Gamma}_S^{t\bar{t}} + b_t^2 d\overline{\Gamma}_P^{t\bar{t}}, \quad (4.1)$$

where $d\overline{\Gamma}_S^{t\bar{t}}$ ($d\overline{\Gamma}_P^{t\bar{t}}$) denotes the differential decay rate of a CP-even (CP-odd) Higgs boson with reduced Yukawa couplings $a_t = 1, b_t = 0$ ($a_t = 0, b_t = 1$). In the following we compute $\overline{\Gamma}_S^{t\bar{t}}$, $\overline{\Gamma}_P^{t\bar{t}}$ and the respective top-quark energy distribution in the Higgs-boson rest frame and the $t\bar{t}$ invariant mass distribution using the NNLO antenna subtraction framework outlined in section 2 and the formulas (3.11), (3.12).

We use $m_t = 173.34$ GeV for the on-shell mass of the top quark. With the relation (3.4) we get $\overline{m}_t(\mu = m_t) = 163.46$ GeV for the $\overline{\text{MS}}$ mass at $\mu = m_t$. This value is used as an input for computing with (3.17) the running top-quark mass $\overline{m}_t(\mu)$ with which the $\overline{\text{MS}}$ Yukawa coupling $\overline{y}_t(\mu)$ is determined. The renormalization scale is chosen to be $\mu = m_h$. The effect of scale variations is assessed by varying μ between $\mu = m_h/2$ and $\mu = 2m_h$. For the QCD coupling we use $\alpha_s^{(5)}(m_Z) = 0.118$ and convert it to the $\overline{\text{MS}}$ coupling of $n_f = 6$ flavor QCD.

The left plot of figure 2 shows our result for the decay width $\overline{\Gamma}_S^{t\bar{t}}$ of a scalar Higgs boson with $a_t = 1$ at LO, NLO, and NNLO QCD for Higgs-boson masses between 500 GeV and below the four-top threshold. (For larger Higgs-boson masses the IR-finite $\mathcal{O}(\alpha_s^2)$ contribution from $h \rightarrow t\bar{t}t\bar{t}$ can be straightforwardly added.) The short-dashed, long-dashed, and solid lines correspond to the choice $\mu = m_h$ and the shaded bands correspond to scale variations between $\mu = m_h/2$ and $\mu = 2m_h$. The analogous result is depicted in the right plot of figure 2 for the decay width $\overline{\Gamma}_P^{t\bar{t}}$ of a pseudoscalar Higgs boson with $b_t = 1$. The plots in figure 2 show that inclusion of the order α_s^2 corrections reduce the scale uncertainties significantly. Moreover, the perturbation expansion in α_s of the decay widths is well behaved, although the order α_s^2 corrections are not negligible. For a scalar (pseudoscalar)

Figure 2. Left plot: the decay width into $t\bar{t}$ of a scalar Higgs boson with $a_t = 1$ at LO (short-dashed, green), NLO (long-dashed, blue), and NNLO (solid, red) QCD as a function of the Higgs-boson mass. The short-dashed, long-dashed, and solid lines correspond to the scale choice $\mu = m_h$ while the shaded bands show the effect of varying the renormalization scale between $m_h/2$ and $2m_h$. Right plot: same as left plot, but for a pseudoscalar Higgs boson with $b_t = 1$.

m_h [GeV]	$\bar{\Gamma}_S^{t\bar{t}}$ [GeV]	$\Gamma_S^{t\bar{t}}$ [GeV]	$\bar{\Gamma}_P^{t\bar{t}}$ [GeV]	$\Gamma_P^{t\bar{t}}$ [GeV]
500	$12.529^{+0.265}_{-0.314}$	$12.955^{+0.037}_{-0.046}$	$22.392^{+0.283}_{-0.411}$	$22.931^{+0.030}_{-0.062}$
680	$25.007^{+0.285}_{-0.408}$	$25.647^{+0.075}_{-0.101}$	$32.188^{+0.214}_{-0.397}$	$32.784^{+0.185}_{-0.225}$

Table 1. The $t\bar{t}$ decay widths at NNLO QCD, $\bar{\Gamma}_X^{t\bar{t}}$ and $\Gamma_X^{t\bar{t}}$ ($X = S, P$), of a scalar and pseudoscalar Higgs boson for two Higgs-boson masses computed with the $\overline{\text{MS}}$ and on-shell top-Yukawa couplings, respectively, cf. equations (3.12) and (3.1). The reduced Yukawa couplings are chosen to be $a_t = 1$ and $b_t = 1$, respectively. The central values correspond to the scale $\mu = m_h$ and the uncertainties to $\mu = m_h/2$ and $\mu = 2m_h$.

Higgs boson with mass $m_h = 500$ GeV and $\mu = m_h$ the α_s^2 correction increases the NLO width by 9.2% (5.8%) while for $m_h = 680$ GeV the increase amounts to 5.3% (3.4%).

If the ratio m_t/m_h is not significantly smaller than one, one may argue that the decay widths can also be computed using the on-shell top Yukawa coupling instead of the $\overline{\text{MS}}$ Yukawa coupling. In table 1 we list, for two Higgs-boson masses, our results for the $t\bar{t}$ decay widths at NNLO QCD obtained with (3.1) and (3.12). The decay widths $\bar{\Gamma}_X^{t\bar{t}}$ ($X = S, P$) are slightly smaller than the corresponding widths $\Gamma_X^{t\bar{t}}$ ($X = S, P$). Using the on-shell top Yukawa coupling we note that for $m_h \gtrsim 600$ GeV the order α_s^2 contribution to the respective decay width becomes of the same order of magnitude than the order α_s contribution, while figure 2 shows that the perturbation expansion in α_s works well when the $\overline{\text{MS}}$ top Yukawa coupling is used.

Moreover, we can compare our NNLO QCD results for the $t\bar{t}$ decay widths, which are exact in m_t , with those of⁴ [21] that were obtained by expanding the scalar and pseudoscalar current correlation functions to fourth order in $(m_t/m_h)^2$. Let us denote the order α_s^2 contribution to the respective $t\bar{t}$ decay width obtained with the results of [21] by $\Gamma_{2,X}^{t\bar{t},HS}$,

⁴We thank Robert Harlander for providing the results of [21] in electronic form.

Figure 3. Left plot: distribution $d\bar{\Gamma}_S^{t\bar{t}}/dx_t$ of the normalized top-quark energy $x_t = 2E_t/m_h$ for a 500 GeV scalar Higgs boson with $a_t = 1$ at LO (short-dashed, green), NLO (long-dashed, blue), and NNLO (solid, red) QCD. The short-dashed, long-dashed, and solid lines correspond to the scale choice $\mu = m_h$ while the shaded bands show the effect of varying the renormalization scale between $m_h/2$ and $2m_h$. Right plot: same as left plot, but for a pseudoscalar Higgs boson with $b_t = 1$.

$X = S, P$. For $m_h = 500$ GeV and $\mu = m_h$ we get $\Gamma_{2,S}^{t\bar{t},HS}/\Gamma_{2,S}^{t\bar{t}} = 1.007$ and $\Gamma_{2,P}^{t\bar{t},HS}/\Gamma_{2,P}^{t\bar{t}} = 1.026$. As expected, for smaller ratios m_t/m_h the agreement becomes even better. For $m_h = 680$ GeV and $\mu = m_h$ we find $\Gamma_{2,S}^{t\bar{t},HS}/\Gamma_{2,S}^{t\bar{t}} = 0.9997$ and $\Gamma_{2,P}^{t\bar{t},HS}/\Gamma_{2,P}^{t\bar{t}} = 1.0003$.

Next we turn to differential distributions. We consider the distribution of the normalized top-quark energy $x_t = 2E_t/m_h$ in the rest frame of the Higgs boson and the $t\bar{t}$ invariant mass distribution. We recall that the distributions expressed in terms of the $\overline{\text{MS}}$ Yukawa coupling are computed with (3.11). For definiteness we choose $m_h = 500$ GeV.

The left plot of figure 3 shows $d\bar{\Gamma}_S^{t\bar{t}}/dx_t$ for a scalar Higgs boson at LO, NLO, and NNLO QCD, while the right plot of figure 3 displays the analogous result for a pseudoscalar Higgs boson. Again, the short-dashed, long-dashed, and solid lines correspond to the renormalization scale $\mu = m_h$ and the shaded bands correspond to scale variations between $\mu = m_h/2$ and $\mu = 2m_h$. The $t\bar{t}$ invariant mass distribution at LO, NLO, and NNLO QCD is displayed in the left and right plots of figure 4 for the $t\bar{t}$ decays of a scalar and pseudoscalar Higgs boson with mass $m_h = 500$ GeV. The same conventions are used as in figure 3. Figures 3 and 4 show that for a Higgs boson with mass of the order of 500 GeV most of the final-state events consist of $t, \bar{t}$ with additional soft or collinear massless partons. Events where $t, \bar{t}$ are accompanied by hard massless partons are significantly suppressed both at NLO and NNLO QCD. Inclusion of the order α_s^2 corrections significantly reduces the scale uncertainties of the distributions.

5 Decay of $h(125)$ to $b\bar{b}X$

In this section we analyze the decay of the 125 GeV Higgs boson $h(125)$ into $b\bar{b}$ at NNLO QCD, taking the b quark to be massive. The experimental data on the production and decays of this boson are compatible with SM predictions. Therefore, we identify it with

Figure 4. Left plot: distribution $d\bar{\Gamma}_S^{t\bar{t}}/dM_{t\bar{t}}$ of the $t\bar{t}$ invariant mass for a 500 GeV scalar Higgs boson with $a_t = 1$ at LO (short-dashed, green), NLO (long-dashed, blue), and NNLO (solid, red) QCD. The short-dashed, long-dashed, and solid lines correspond to the scale choice $\mu = m_h$ while the shaded bands show the effect of varying the renormalization scale between $m_h/2$ and $2m_h$. Right plot: same as left plot, but for a pseudoscalar Higgs boson with $b_t = 1$.

the SM Higgs boson, that is, we assume it to be CP-even with $a_b = 1$. Moreover, we use the Higgs-boson mass $m_h = 125.09$ GeV [56].

For the b -quark mass we use $\bar{m}_b(\mu = \bar{m}_b) = 4.18$ GeV [56] in $n_f = 5$ flavor QCD as input in (3.17) for computing the running $\overline{\text{MS}}$ b -quark mass and the Yukawa coupling $\bar{y}_b(\mu)$ at arbitrary scales μ . As in section 4 we use $\alpha_s^{(5)}(m_Z) = 0.118$. From the two-loop relation (3.14) we obtain $m_b = 4.78$ GeV. This value of the on-shell b -quark mass is used in computing the coefficients r_1, r_2 , the $d\hat{\Gamma}_i^{bb}$, and the γ_i^{bb} defined in eq. (3.10), (3.11), and (3.13), respectively. (Higher order corrections to (3.14) shift this value of m_b to a somewhat higher value [57], but for the sake of consistency of the order of the perturbative expansion, we stick to $m_b = 4.78$ GeV.) With the above value of $\bar{m}_b(\mu = \bar{m}_b)$ we get, using (3.17), the values $\bar{m}_b(\mu) = 2.97$ GeV, 2.80 GeV, and 2.65 GeV at $\mu = m_h/2, m_h$, and $2m_h$, respectively. With these values we compute the $\overline{\text{MS}}$ Yukawa couplings $\bar{y}_b(\mu)$. As in section 4 we use $\alpha_s^{(5)}(m_Z) = 0.118$. The displayed digits of our numerical results given below are not affected by our numerical integration errors.

5.1 Inclusive decay width

First, we determine the inclusive decay width of $h(125) \rightarrow b\bar{b}X$ at NNLO QCD using the antenna subtraction framework of section 2. We have to take into account also $h \rightarrow b\bar{b}b\bar{b}$ whose contribution is IR finite. We represent our result for the inclusive decay width at NNLO QCD in the form (3.12):

$$\bar{\Gamma}_{NNLO}^{bb} = \bar{\Gamma}_{LO}^{bb} \left[1 + g_1 \frac{\alpha_s^{(5)}}{\pi} + g_2 \left(\frac{\alpha_s^{(5)}}{\pi} \right)^2 \right], \quad (5.1)$$

where

$$\bar{\Gamma}_{LO}^{bb} = \bar{y}_b^2(\mu) \hat{\Gamma}_0^{bb}, \quad g_1 = \gamma_1^{bb} + r_1, \quad g_2 = \gamma_2^{bb} + r_1 \gamma_1^{bb} + r_2, \quad (5.2)$$

	$\mu = m_h/2$	$\mu = m_h$	$\mu = 2m_h$
g_1	3.024	5.796	8.569
g_2	3.685	37.371	86.112
$\bar{\Gamma}_{LO}^{bb}$ [MeV]	2.153	1.910	1.717
$\bar{\Gamma}_{NLO}^{bb}$ [MeV]	2.413	2.307	2.196
$\bar{\Gamma}_{NNLO}^{bb}$ [MeV]	2.425	2.399	2.353

Table 2. The coefficients g_1 , g_2 defined in eqs. (5.1) and (5.2) and the inclusive $h(125) \rightarrow b\bar{b}X$ decay width at LO, NLO, and NNLO QCD for three renormalization scales μ .

and $\hat{\Gamma}_0^{b\bar{b}}$ is defined in eq. (3.3). Our results for g_1 , g_2 , $\bar{\Gamma}_{LO}^{bb}$, $\bar{\Gamma}_{NLO}^{bb}$, and $\bar{\Gamma}_{NNLO}^{bb}$ are given in table 2 for $\mu = m_h/2, m_h$, and $2m_h$.

The numbers in table 2 show that the order α_s^2 correction increases the NLO $b\bar{b}$ decay width by 4% for $\mu = m_h$. Inclusion of these corrections reduces the renormalization scale uncertainties significantly. The values of the $\overline{\text{MS}}$ Yukawa couplings $\bar{y}_b(\mu)$, which are determined by the $\overline{\text{MS}}$ masses $\bar{m}_b(\mu)$ given above, have of course a decisive impact on the values of $\bar{\Gamma}_I^{bb}$ ($I=\text{LO}, \text{NLO}, \text{NNLO}$) given in table 2. Moreover, we point out that the top-quark triangle contribution to $h \rightarrow b\bar{b}$ and $h \rightarrow b\bar{b}g$ shown in figure 1 constitute an important part of the order α_s^2 corrections. This contribution, which is μ -independent, to g_2 listed in table 2 is $g_2(t) = 6.898$.

As a check of our computational set-up in the case of $h \rightarrow b\bar{b}X$ we determine the QCD correction coefficients g_1 and g_2 in the limit of small b -quark mass and compare with the results for massless b quarks, $g_1(m_b = 0) = 5.6666$ and $g_2(m_b = 0) = 29.1467$ [15, 24]. We cannot perform the limit $m_b \rightarrow 0$ analytically. Therefore we choose a value for m_b that is very small compared to m_h , to wit, we choose $m_b = 0.5 \text{ GeV}$ and compute the resulting g_1 and g_2 for $\mu = m_h$. For the order α_s QCD coefficient we obtain $g_1(m_b = 0.5 \text{ GeV}) = 5.6685$. The α_s^2 QCD coefficient given in [15, 24] does not include the top-quark triangle contribution to $h \rightarrow b\bar{b}$ and $h \rightarrow b\bar{b}g$, respectively. In fact these contributions vanish for $m_b = 0$ because of helicity mismatch. Omitting these contributions we obtain $g_2(m_b = 0.5 \text{ GeV}) = 29.187$ which is very close to the number for $m_b = 0$. Using the same Yukawa coupling $\bar{y}_b(m_h)$ as in the massive case, the resulting NNLO decay width for $m_b = 0$ and $\mu = m_h$ is $\bar{\Gamma}_{NNLO}^{bb}(m_b = 0) = 2.391 \text{ MeV}$.

5.2 Two-jet, three-jet, and four-jet rates

Concerning the decay of $h(125)$ to b quarks, other interesting observables for future experimental analyses include the decay rates into two, three, and four jets. For definiteness we use here the Durham jet algorithm [58] (and the recombination scheme $k_{(ij)} = k_i + k_j$) with jet resolution parameters $y_{cut} = 0.01$ and $y_{cut} = 0.05$. The n -jet rates can be represented,

	$y_{cut} = 0.01$			$y_{cut} = 0.05$		
	$\mu = m_h/2$	$\mu = m_h$	$\mu = 2m_h$	$\mu = m_h/2$	$\mu = m_h$	$\mu = 2m_h$
$g_1(2 \text{ jet})$	−5.055	−2.283	0.490	0.291	3.063	5.836
$g_2(2 \text{ jet})$	−56.351	−66.532	−61.658	−19.496	−0.650	33.250
$g_1(3 \text{ jet})$	8.079	8.079	8.079	2.733	2.733	2.733
$g_2(3 \text{ jet})$	36.873	80.741	124.609	22.256	37.096	51.937
$g_2(4 \text{ jet})$	23.163	23.163	23.163	0.926	0.926	0.926

Table 3. The coefficients $g_i(\text{n jet})$ defined in eqs. (5.6)–(5.8) and computed with the Durham algorithm using $y_{cut} = 0.01$ and $y_{cut} = 0.05$ for three renormalization scales μ . They determine the n -jet rates (5.3)–(5.5).

in analogy to (3.11) and (3.12), to order α_s^2 as follows:

$$\bar{\Gamma}_{2\text{jet}}^{b\bar{b}} = \bar{\Gamma}_{LO}^{b\bar{b}} \left[1 + g_1(2\text{jet}) \frac{\alpha_s^{(5)}}{\pi} + g_2(2\text{jet}) \left(\frac{\alpha_s^{(5)}}{\pi} \right)^2 \right], \quad (5.3)$$

$$\bar{\Gamma}_{3\text{jet}}^{b\bar{b}} = \bar{\Gamma}_{LO}^{b\bar{b}} \left[g_1(3\text{jet}) \frac{\alpha_s^{(5)}}{\pi} + g_2(3\text{jet}) \left(\frac{\alpha_s^{(5)}}{\pi} \right)^2 \right], \quad (5.4)$$

$$\bar{\Gamma}_{4\text{jet}}^{b\bar{b}} = \bar{\Gamma}_{LO}^{b\bar{b}} \times g_2(4\text{jet}) \left(\frac{\alpha_s^{(5)}}{\pi} \right)^2, \quad (5.5)$$

where

$$g_1(2\text{jet}) = \gamma_1^{b\bar{b}}(2\text{jet}) + r_1, \quad g_2(2\text{jet}) = \gamma_2^{b\bar{b}}(2\text{jet}) + r_1 \gamma_1^{b\bar{b}}(2\text{jet}) + r_2, \quad (5.6)$$

$$g_1(3\text{jet}) = \gamma_1^{b\bar{b}}(3\text{jet}), \quad g_2(3\text{jet}) = \gamma_2^{b\bar{b}}(3\text{jet}) + r_1 \gamma_1^{b\bar{b}}(3\text{jet}), \quad (5.7)$$

$$g_2(4\text{jet}) = \gamma_2^{b\bar{b}}(4\text{jet}). \quad (5.8)$$

Our results for the coefficients $g_i(\text{n jet})$ are given in table 3 for $\mu = m_h/2, m_h$, and $2m_h$. With decreasing resolution parameter y_{cut} the two-jet rate decreases while the three- and four-jet rates increase. This is because a smaller y_{cut} resolves more and more gluon radiation respectively massless quark radiation. The three-jet rate has a maximum below $y_{cut} = 0.01$ and then decreases for smaller y_{cut} , while the four-jet rate still increases for these resolution parameters till its maximum is reached. Notice that for each value of y_{cut} and μ the sum of the $g_i(\text{n jet})$ listed in table 3 yields the inclusive coefficient g_i ($i = 1, 2$) given in table 2.

The coefficients $g_i(\text{n jet})$ were computed in [33] for $\mu = m_h$ and massless b quarks using the JADE algorithm and $y_{cut} = 0.01$.

Finally we analyze, for two-jet events, the distribution of the energy of the leading jet, i.e., the jet with the largest energy in the Higgs-boson rest frame. We use the dimensionless variable

$$x_{\max} = \max(E_{j_1}/m_h, E_{j_2}/m_h). \quad (5.9)$$

Figure 5. The distribution of the variable $x_{\max}$ defined in (5.9) for two-jet events using the Durham algorithm with $y_{\text{cut}} = 0.05$ at LO (short dashed, green), NLO (long dashed, blue), and NNLO (solid, red) QCD. The central lines correspond to $\mu = m_h$ and the shaded bands correspond to scale variations between $\mu = m_h/2$ and $(\mu = 2m_h)$.

Obviously, $x_{\max} \geq 0.5$. We choose the Durham algorithm with $y_{\text{cut}} = 0.05$ and the parameter set-up as specified at the beginning of this section. The distribution of this observable is shown in figure 5 at LO, NLO, and NNLO QCD. The figure shows that the perturbation series is well-behaved for this observable.

This distribution was presented before for massless b quarks in [33] (where the JADE algorithm with $y_{\text{cut}} = 0.1$ was used) and in [34] (where the Durham algorithm with $y_{\text{cut}} = 0.05$ was used). Comparing figure 5 with figure 2 of [34] shows that taking the non-zero b -quark mass into account has only a minor impact on the shape of this distribution.

6 Conclusions

We have presented, within the antenna subtraction framework, the set-up for calculating the fully differential decay rate of a scalar and pseudoscalar Higgs boson to a massive quark antiquark pair at NNLO in the perturbation series in α_s . Our approach is fully differential in the phase-space variables and can be used to compute any infrared-safe observable for these decays. So far, methods for calculating Higgs-boson decays at order α_s^2 in QCD at the differential level were available only for massless quarks. We have applied our set-up to the decay of a scalar and pseudoscalar Higgs boson to $t, \bar{t}$ and to the decay of the $h(125\text{GeV})$ Higgs boson to massive $b, \bar{b}$ quarks. Apart from computing the respective decay widths at NNLO accuracy we have determined also several distributions, in order to show the use of our approach. We have presented our results in terms of the $\overline{\text{MS}}$ Yukawa coupling $\bar{y}_Q$, while the on-shell mass m_Q is used in the matrix elements. For the NNLO decay width of $h(125\text{GeV}) \rightarrow b\bar{b}X$ we have shown numerically that this formulation recovers, for $m_b/m_h \rightarrow 0$, the known result for massless b quarks.

Our results can be used as a building block in a theoretical description of Higgs-boson production and decay to massive quarks at order α_s^2 at the differential level in hadron or electron-positron collisions.

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