Gauged Peccei-Quinn symmetry — A case of simultaneous breaking of SUSY and PQ symmetry

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ABSTRACT: Recently, a simple prescription to embed the global Peccei-Quinn (PQ) symmetry into a gauged U(1) symmetry has been proposed. There, explicit breaking of the global PQ symmetry expected in quantum gravity are highly suppressed due to the gauged PQ symmetry. In this paper, we apply the gauged PQ mechanism to models where the global PQ symmetry and supersymmetry (SUSY) are simultaneously broken at around $\mathcal{O}(10^{11-12})$ GeV. Such scenario is motivated by an intriguing coincidence between the supersymmetry breaking scale which explains the observed Higgs boson mass by the gravity mediated sfermion masses, and the PQ breaking scale which evades all the astrophysical and the cosmological constraints. As a concrete example, we construct a model which consists of a simultaneous supersymmetry/PQ symmetry breaking sector based on SU(2) dynamics and an additional PQ symmetry breaking sector based on SU(N) dynamics. We also show that new vector-like particles are predicted in the TeV range in the minimum model, which can be tested by the LHC experiments.

KEYWORDS: Beyond Standard Model, CP violation, Gauge Symmetry, Global Symmetries

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1 Introduction

The Peccei-Quinn (PQ) mechanism [1–4] provides us with a very successful solution to the strong CP problem. The effective θ -angle of QCD is canceled by the vacuum expectation value (VEV) of the pseudo-Nambu-Goldstone boson, the axion a , which results from spontaneous breaking of the global $U(1)$ Peccei-Quinn symmetry, $U(1)_{PQ}$.

The solution of the strong CP problem based on a global symmetry is, however, not on the very firm theoretical ground. As the QCD anomaly explicitly breaks the $U(1)_{PQ}$ symmetry, it cannot be an exact symmetry by definition. Besides, it is also argued that all global symmetries are broken by quantum gravity effects [5–10]. The explicit breaking of the PQ symmetry easily spoils the success of the PQ mechanism.

In ref. [11], a simple prescription has been proposed, with which the global $U(1)_{PQ}$ symmetry is embedded into a “gauged” $U(1)$ symmetry, $U(1)_{gPQ}$. There, the anomalies of the gauged PQ symmetry are canceled between the contributions from two (or more)

PQ charged sectors. With appropriate charge assignment of $U(1)_{\text{gPQ}}$, the PQ charged sectors are highly decoupled with each other, and a global $U(1)_{\text{PQ}}$ symmetry appears as an accidental symmetry. As a part of the gauge symmetry, the accidental $U(1)_{\text{PQ}}$ is also well protected from explicit breaking caused by quantum gravity effects. This prescription provides a concise generalization of previous attempts to achieve the PQ symmetry as an accidental symmetry resulting from (discrete) gauge symmetries [12–21].

In this paper, we apply the construction of the gauged PQ symmetry to a model in which the global PQ symmetry and supersymmetry are simultaneously broken at around $\mathcal{O}(10^{11-12})$ GeV [22]. Such scenario is motivated by an intriguing coincidence between the supersymmetry breaking scale which explains the observed Higgs boson mass by the gravity mediated sfermion masses in the hundreds to thousands TeV range [23] and the PQ breaking scale which evades all the astrophysical and the cosmological constraints.¹

The organization of the paper is as follows. In section 2, we summarize the supersymmetric version of the gauged PQ mechanism. In section 3, we construct a model in which supersymmetry and the PQ symmetry are broken simultaneously by $SU(2)$ strong dynamics. In section 4, we apply the gauged PQ mechanism to the model of simultaneous symmetry breaking. The final section is devoted to our conclusions.

2 General prescription of the gauged PQ mechanism

In this section, we briefly summarize a supersymmetric version of the gauged PQ mechanism [11].

2.1 Would-be Goldstone and axion superfields

As a simple example, let us consider two global PQ symmetries $U(1)_{\text{PQ}_1}$ and $U(1)_{\text{PQ}_2}$, which are broken by the VEVs of Φ_1 , $\bar{\Phi}_1$ and Φ_2 , $\bar{\Phi}_2$, respectively. For instance, such vacuum is achieved by the superpotential,

$$W = \lambda_1 X_1 (2\Phi_1 \bar{\Phi}_1 - \Lambda_1^2) + \lambda_2 X_2 (2\Phi_2 \bar{\Phi}_2 - \Lambda_2^2). \quad (2.1)$$

Here, Φ_i and $\bar{\Phi}_i$ ($i = 1, 2$) have charges ± 1 under $U(1)_{\text{PQ}_i}$ and have vanishing charges under $U(1)_{\text{PQ}_j}$ ($j \neq i$), respectively. The superfields, $X_{1,2}$, have vanishing charges under both the PQ symmetries. The parameters $\lambda_{1,2}$ are coupling constants, and $\Lambda_{1,2}$ are dimensionful parameters. After the spontaneous breaking of the PQ symmetries, Φ 's lead to the Goldstone superfields $A_{1,2}$,²

$$\Phi_1 = \frac{1}{\sqrt{2}} \Lambda_1 e^{A_1/\Lambda_1}, \quad \bar{\Phi}_1 = \frac{1}{\sqrt{2}} \Lambda_1 e^{-A_1/\Lambda_1}, \quad (2.2)$$

$$\Phi_2 = \frac{1}{\sqrt{2}} \Lambda_2 e^{A_2/\Lambda_2}, \quad \bar{\Phi}_2 = \frac{1}{\sqrt{2}} \Lambda_2 e^{-A_2/\Lambda_2}. \quad (2.3)$$

¹For correspondence between the sfermion mass scale and the Higgs boson mass, see also [24–26]. For constraints on the PQ breaking scale, see, e.g., [27–29].

²Here, we set the origins of A_i at which $\Phi_i = \bar{\Phi}_i$, while $\langle \Phi_i \rangle \neq \langle \bar{\Phi}_i \rangle$ for $\langle A_i \rangle \neq 0$.

By using the Goldstone superfields, the PQ symmetries are realized by,

$$A_1/\Lambda_1 \rightarrow A_1/\Lambda_1 + i\alpha_1, \quad (\alpha_1 = 0 - 2\pi), \quad (2.4)$$

$$A_2/\Lambda_2 \rightarrow A_2/\Lambda_2 + i\alpha_2, \quad (\alpha_2 = 0 - 2\pi). \quad (2.5)$$

The PQ symmetries are communicated to the supersymmetric Standard Model (SSM) sector by introducing extra quark multiplets as in the KSVZ axion model [30, 31]. Throughout this paper, we assume that the extra multiplets form $\mathbf{5}$ and $\bar{\mathbf{5}}$ representations of the SU(5) gauge group of the Grand Unified Theory (GUT). Let us suppose that $\Phi_{1,2}$ couple to N_1 and N_2 flavors of the KSVZ extra multiplets $\mathbf{5}_i, \bar{\mathbf{5}}_i$ ($i = 1, 2$), respectively,

$$W = \Phi_1 \mathbf{5}_1 \bar{\mathbf{5}}_1 + \bar{\Phi}_2 \mathbf{5}_2 \bar{\mathbf{5}}_2. \quad (2.6)$$

Through the above coupling, both the global PQ symmetries are broken by the Standard Model anomaly. The anomalous breaking of the global PQ symmetries lead to the anomalous coupling of the Goldstone superfields,

$$W_{\text{anom}} = \frac{1}{8\pi^2} \left(N_1 \frac{A_1}{\Lambda_1} - N_2 \frac{A_2}{\Lambda_2} \right) \sum_l W_l^\alpha W_{l\alpha}, \quad (2.7)$$

where, W_l^α ($l = 1, 2, 3$) denote the field strength superfields of the Standard Model gauge interactions.³ We normalize the gauge field strength so that the gauge kinetic functions are given by

$$\mathcal{L} = \frac{1}{2i} \left[\sum \tau_l W_l^\alpha W_{l\alpha} \right]_F + \text{h.c.}, \quad (2.8)$$

with

$$\tau_l = \frac{i}{g_l^2} + \frac{\theta_l}{8\pi^2}, \quad (2.9)$$

where g_l and θ_l are the gauge coupling constants and the vacuum angles of the corresponding gauge interactions.

An important observation here is that there is a linear combination of the PQ symmetries for which the Standard Model anomalies are absent. In fact, a U(1) symmetry under which $\Phi_{1,2}$ have charges q_1 and q_2 is free from the Standard Model anomaly for

$$q_1 N_1 - q_2 N_2 = 0. \quad (2.10)$$

In the gauged PQ mechanism, we identify the anomaly-free combination to be a gauge symmetry U(1)_{gPQ}. The gravitational anomaly and the self-anomaly of the U(1)_{gPQ} are canceled by adding U(1)_{gPQ} charged singlet fields. Hereafter, we take q_1 and q_2 are both positive and relatively prime numbers without loss of generality.

In the gauged PQ mechanism, one of the linear combinations of $A_{1,2}$ is the would-be Goldstone supermultiplet, and the other combination corresponds to the physical axion

³Here, the gauge indices of SU(3)_c and SU(2)_L are suppressed, and the GUT normalization is used for U(1)_Y.

superfield. To see how the physical axion is extracted, let us consider the Kähler potential of Φ 's,

$$K = \Phi_1^\dagger e^{-2q_1 g V} \Phi_1 + \bar{\Phi}_1^\dagger e^{2q_1 g V} \bar{\Phi}_1 + \Phi_2^\dagger e^{-2q_2 g V} \Phi_2 + \bar{\Phi}_2^\dagger e^{2q_2 g V} \bar{\Phi}_2, \quad (2.11)$$

where V and g are the $U(1)_{\text{gPQ}}$ gauge supermultiplet and the gauge coupling constant, respectively. Under the $U(1)_{\text{gPQ}}$ gauge transformation, the gauge field is shifted by,

$$2gV \rightarrow 2gV' = 2gV - i\Theta + i\Theta^\dagger, \quad (2.12)$$

with Θ being the gauge parameter superfield. By substituting eqs. (2.2) and (2.3), the Kähler potential is reduced to

$$K = \Lambda_1^2 \cosh \left(2q_1 g V - \frac{A_1^\dagger + A_1}{\Lambda_1} \right) + \Lambda_2^2 \cosh \left(2q_2 g V - \frac{A_2^\dagger + A_2}{\Lambda_2} \right). \quad (2.13)$$

The physical axion and the would-be Goldstone superfields A and G are obtained by rearranging $A_{1,2}$ by

$$\begin{pmatrix} A^{(\dagger)} \\ G^{(\dagger)} \end{pmatrix} = \frac{1}{\sqrt{q_1^2 \Lambda_1^2 + q_2^2 \Lambda_2^2}} \begin{pmatrix} q_2 \Lambda_2 & -q_1 \Lambda_1 \\ q_1 \Lambda_1 & q_2 \Lambda_2 \end{pmatrix} \begin{pmatrix} A_1^{(\dagger)} \\ A_2^{(\dagger)} \end{pmatrix}. \quad (2.14)$$

By using A and G , the Kähler potential is rewritten by,

$$K = \Lambda_1^2 \cosh \left(2q_1 \tilde{V} - \frac{2q_2}{m_V} \frac{\Lambda_2}{\Lambda_1} (A^\dagger + A) \right) + \Lambda_2^2 \cosh \left(2q_2 \tilde{V} + \frac{2q_1}{m_V} \frac{\Lambda_1}{\Lambda_2} (A^\dagger + A) \right), \quad (2.15)$$

where

$$\tilde{V} = V - \frac{g}{m_V} (G^\dagger + G), \quad (2.16)$$

$$m_V = 2g \sqrt{q_1^2 \Lambda_1^2 + q_2^2 \Lambda_2^2}. \quad (2.17)$$

The final expression of eq. (2.15) shows there is no bi-linear term which mixes A and $\tilde{V}$. Therefore, we find that A corresponds to the physical axion superfield, while G is the would-be Goldstone superfield which is absorbed by V in the unitarity gauge. It should be noted that the physical axion A is invariant under the gauge $U(1)_{\text{gPQ}}$ transformation.

For a later purpose, let us discuss the domain and the effective decay constant of the axion. The domains of the imaginary parts of $A_{1,2}$ (corresponding to the phases of $\Phi_{1,2}$) are given by

$$\frac{\text{Im}[A_i]}{\Lambda_i} = \frac{a_i}{f_i} = [0, 2\pi), \quad (i = 1, 2), \quad (2.18)$$

where $a_i = \sqrt{2} \text{Im}[A_i]$ and $f_i = \sqrt{2} \Lambda_i$. When q_1 and q_2 are relatively prime integers, the gauge invariant axion interval is given by [11],

$$a = \sqrt{2} \text{Im}[A] = \left[0, \frac{2\pi f_1 f_2}{\sqrt{q_1^2 f_1^2 + q_2^2 f_2^2}} \right). \quad (2.19)$$

Accordingly, the global $U(1)_{\text{PQ}}$ symmetry is realized by

$$\frac{a}{F_a} \rightarrow \frac{a'}{F_a} = \frac{a}{F_a} + \delta_{\text{PQ}}, \quad (\delta_{\text{PQ}} = 0 - 2\pi), \quad (2.20)$$

where F_a is defined as an effective decay constant,

$$F_a = \frac{f_1 f_2}{\sqrt{q_1^2 f_1^2 + q_2^2 f_2^2}} = \frac{\sqrt{2} \Lambda_1 \Lambda_2}{\sqrt{q_1^2 \Lambda_1^2 + q_2^2 \Lambda_2^2}}. \quad (2.21)$$

2.2 Accidental global PQ symmetry

As argued in [5–10], global symmetries are expected to be broken by quantum gravity effects which are manifested by explicit breaking terms suppressed by the Planck scale. On the other hand, such explicit breaking does not appear for the $U(1)_{\text{gPQ}}$ as it is an exact gauge symmetry. Thus, question is how well the accidental global PQ symmetry (corresponding to the shift of A) is protected from explicit breaking by the $U(1)_{\text{gPQ}}$ symmetry. An important observation here is that the $U(1)_{\text{gPQ}}$ symmetry is not distinguishable from the global PQ symmetries in each sector. Thus, there are no explicit breaking terms of the global PQ symmetries which consist of the fields in each PQ symmetric sector. Therefore, the interaction terms which potentially ruin the global PQ symmetries are the gauge invariant operators consisting of the fields in multiple PQ symmetric sectors. In the following, we estimate how badly the accidental global PQ symmetry is broken.

In the present model, the lowest dimensional $U(1)_{\text{gPQ}}$ invariant operators which break the global PQ symmetries are given by,

$$W \sim \frac{1}{M_{\text{PL}}^{q_1+q_2-3}} (\Phi_1^{q_2} \bar{\Phi}_2^{q_1} + \bar{\Phi}_1^{q_2} \Phi_2^{q_1}), \quad (2.22)$$

where $M_{\text{PL}} = 2.4 \times 10^{18} \text{ GeV}$ denotes the reduced Planck scale. When supersymmetry is spontaneously broken in a separate sector, the above superpotential contributes to the axion potential through the supergravity effects,⁴

$$V \sim \frac{1}{2} m_a^2 F_a^2 \left(\frac{a}{F_a} \right)^2 + \frac{m_{3/2}}{M_{\text{PL}}^{q_1+q_2-3}} \Lambda_1^{q_2} \Lambda_2^{q_1} \frac{a}{F_a} + \text{h.c.} + \dots, \quad (2.24)$$

where $m_{3/2}$ denotes the gravitino mass. In the final expression, we use $\Phi_1^{q_2} \bar{\Phi}_2^{q_1} = \Lambda_1^{q_2} \Lambda_2^{q_1} / 2^{(q_2+q_1)/2} e^{ia/F_a}$, and the intrinsic θ angle of QCD is absorbed by the definition of the axion field. The first term represents the axion mass term due to the QCD effects [3],

$$m_a^2 \simeq \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{F_a^2}, \quad (2.25)$$

where $m_{u,d}$ are the u - and d -quark masses, m_π the pion mass, and $f_\pi \simeq 93 \text{ MeV}$ the pion decay constant.

⁴In supergravity, a superpotential term W_i directly appears in the scalar potential as

$$V = (n_i - 3) m_{3/2} \times W_i + \text{h.c.}, \quad (2.23)$$

with n_i being the mass dimension of W_i .

As a result, the effective θ angle at the vacuum of the axion is given by,

$$\begin{aligned}\theta_{\text{eff}} &\simeq \frac{1}{m_a^2 F_a^2} \frac{m_{3/2}}{M_{\text{PL}}^{q_1+q_2-3}} \Lambda_1^{q_2} \Lambda_2^{q_1} \\ &\sim 10^{66-6.4(q_1+q_2)} \times \left(\frac{m_{3/2}}{10^6 \text{ GeV}} \right)^2 \left(\frac{0.08 \text{ GeV}}{\sqrt{m_a F_a}} \right)^4 \left(\frac{\Lambda_1}{10^{12} \text{ GeV}} \right)^{q_2} \left(\frac{\Lambda_2}{10^{12} \text{ GeV}} \right)^{q_1}.\end{aligned}\quad (2.26)$$

Thus, for $q_1 + q_2 \gtrsim 12$, $m_{3/2} = \mathcal{O}(10^6) \text{ GeV}$, and $\Lambda_{1,2} = \mathcal{O}(10^{12}) \text{ GeV}$, the explicit breaking terms of the global PQ symmetries are small enough to be consistent with the measurement of the neutron EDM, i.e. $\theta_{\text{eff}} < 10^{-11}$ [32]. In this way, a *high quality* global PQ symmetry appears as an accidental symmetry in the gauged PQ mechanism.

2.3 Domain wall problem

Before closing this section, let us briefly discuss the domain wall problem. The anomaly cancelation condition in eq. (2.10) is generically solved by,

$$N_1 = N_{\text{GCD}} \times q_2, \quad N_2 = N_{\text{GCD}} \times q_1, \quad (2.27)$$

where $N_{\text{GCD}} \in \mathbb{N}$ is the greatest common divisor of $N_{1,2}$. Then, the anomalous coupling in eq. (2.7) is rewritten by,

$$W_{\text{anom}} = \frac{N_{\text{GCD}}}{8\pi^2} \frac{\sqrt{2}A}{F_a} \sum_l W_l^\alpha W_{l\alpha}, \quad (2.28)$$

and hence,

$$\mathcal{L}_{\text{anom}} = \frac{N_{\text{GCD}}}{32\pi^2} \frac{a}{F_a} \sum_l F_{\mu\nu}^l \tilde{F}^{l\mu\nu}. \quad (2.29)$$

It should be noted that the anomalous coupling of the axion respects a discrete symmetry, $\mathbb{Z}_{N_{\text{GCD}}}$,

$$\frac{a}{F_a} \rightarrow \frac{a}{F_a} + \frac{2\pi k}{N_{\text{GCD}}}, \quad k = 0, \dots, N_{\text{GCD}} - 1, \quad (2.30)$$

for $N_{\text{GCD}} > 1$.

The $\mathbb{Z}_{N_{\text{GCD}}}$ symmetry is eventually broken in the vacuum of the axion. Thus, the model with $N_{\text{GCD}} > 1$ suffers from the domain wall problem if the global PQ symmetry is broken after inflation since the average of the axion field value in each Hubble volume is randomly distributed. To avoid the domain wall problem, spontaneous breaking of the global PQ symmetry is required to take place before inflation, which in turn requires a rather small inflation scale to avoid the axion isocurvature problem (see, e.g. refs. [28, 33]).

For $N_{\text{GCD}} = 1$, on the other hand, there is no discrete symmetry which is broken by the VEV of the axion field. Still, however, there can be domain wall problems when the global PQ breaking takes place after inflation. To see this problem, let us remember that there can be various types of cosmic string configurations formed at spontaneous symmetry breaking of the PQ symmetries. For example, when both the gauged and the

global PQ symmetries are broken spontaneously after inflation, there can be cosmic string configurations in which either the phase of Φ_1 or Φ_2 takes $0 - 2\pi$ around configurations. It should be noted that those configurations are the global strings and not the local string. Thus, the string tensions diverge in the limit of infinite volume which is cut off by the Hubble volume. The local string, on the other hand, corresponds to the configurations in which the phases of Φ_1 and Φ_2 wind q_1 times and q_2 times simultaneously. With the $U(1)_{\text{gPQ}}$ gauge field winding simultaneously, the tension of the local string is finite even in the limit of infinite volume for the local string.

A striking difference between the global strings and the local strings is how the axion field winds around the strings. Around the local strings, only the would-be-Goldstone field winds, while the axion winds around the global strings. Thus, when the axion potential is generated at around the QCD scale, the axion domain walls are formed only around the global strings, while they are not formed around the local strings. Once the domain walls are formed around the global strings, they immediately dominate over the energy density of the universe, which causes the domain wall problem. Therefore, for the domain wall problems not to occur, the local strings should be formed preferentially at the phase transition.

The string tensions of the global strings and the local strings, however, depend on model parameters. Thus, there is no guarantee that only the local strings preferentially survive in the course of the cosmic evolution. As an example, let us consider a case with $\langle \Phi_1 \rangle \gg \langle \Phi_2 \rangle$. In this case, the cosmic strings are formed at the first phase transition, i.e. $\langle \Phi_1 \rangle \neq 0$ with $\langle \Phi_2 \rangle = 0$. At this stage, strings around which the phase of Φ_1 winds just once are expected to be dominantly formed. They are *local* because we can take an appropriate charge normalization for the $U(1)_{\text{gPQ}}$. As the temperature of the universe decreases, the string networks follow the scaling solution where the number of the cosmic strings in each Hubble volume becomes constant (see, e.g., ref. [34]).

Once the temperature becomes lower than the scale of the second phase transition, i.e., $\langle \Phi_2 \rangle \neq 0$, the *local* strings formed at the first phase transition become no more the local strings.⁵ Besides, formations of the global strings of Φ_2 are also expected at the second phase transition in which the phase of Φ_2 winds just once. To form a genuine local string, it is required to bundle q_1 ex-local strings (formed by Φ_1) and q_2 global strings (formed by Φ_2) into a single string. However, the confluence of global strings into a local string is quite unlikely as there is no correlation between the nature of the cosmic strings in the adjacent Hubble volumes. Therefore, when $\langle \Phi_1 \rangle \gg \langle \Phi_2 \rangle$, the domain wall problem is expected to be not avoidable even if $N_{\text{GCD}} = 1$.⁶

In summary, let us list up possibilities to avoid the domain wall problem. The first possibility is a trivial one where both the gauged and the global PQ symmetries are broken before inflation. This solution does not require $N_{\text{GCD}} = 1$. In this possibility, there is a constraint on the Hubble scale during inflation from the axion isocurvature problem.

⁵The configuration of the gauge field formed at the first phase transition does not coincide with the one required for the local string with $\langle \Phi \rangle_2 \neq 0$.

⁶As there is no corresponding discrete symmetry, the domain wall is not stable completely. For $\langle \Phi_1 \rangle \gg \langle \Phi_2 \rangle$, however, the decay rate (i.e., the puncture rate and/or the rate of the breaking off) is highly suppressed.

The next possibility is only applicable for $N_{\text{GCD}} = 1$ with $q_1 = 1$ and $q_2 = N(> 1)$. Here, it is assumed that the first phase transition (i.e. $\langle \Phi_1 \rangle \neq 0$) takes place before inflation while the second phase transition (i.e. $\langle \Phi_2 \rangle \neq 0$) occurs after inflation. In this second possibility, the *local* strings formed at the first phase transition are inflated away. The global strings formed at the second phase transition, on the other hand, do not cause the domain wall problem as each of the global string is attached to only one domain wall [35, 36].

In addition to these two possibilities, there can be another possibility which is applicable for $N_{\text{GCD}} = 1$ with $\langle \Phi_1 \rangle \sim \langle \Phi_2 \rangle$. In this case, there can be a possibility where the local strings are preferentially formed at the phase transition. Besides, the axion domain wall attached to the global strings may have very short lifetime for $N_{\text{DW}} = 1$ even if they are formed. To confirm this possibility, detailed numerical simulations are required, which goes beyond the scope of this paper.

It should be noted that the second possibility (and the third possibility if numerically confirmed) is one of the advantages of the gauged PQ mechanism over the models in which the global PQ symmetry results from an exact discrete symmetry, such as $\mathbb{Z}_N$. In such models, the axion potential also respects the $\mathbb{Z}_N$ symmetry, and hence, the domain wall problem is not avoidable when the global PQ symmetry is spontaneously broken after inflation. In the gauged PQ models, on the other hand, it is possible that the global PQ symmetry is broken after inflation without causing the domain wall problem nor the axion isocurvature problem.

3 Dynamical supersymmetry/PQ symmetry breaking

In this section, we discuss a model of a simultaneous breaking of supersymmetry and the *global* PQ symmetry. As we are interested in solutions to the strong *CP*-problem without severe fine-tuning, it is natural to seek models in which the PQ breaking scale is generated by dynamical transmutation. Thus, in the following, we construct a model of a simultaneous supersymmetry/PQ symmetry breaking sector based on a strong dynamics. For now, we do not consider the gauged PQ mechanism which will be implemented in the next section.

3.1 Simultaneous breaking of supersymmetry and global PQ symmetry

As the simplest example of the dynamical supersymmetry breaking models, we consider a model of supersymmetry breaking based on $\text{SU}(2)$ gauge dynamics (the IYIT model) [37, 38]. The advantage of this model is that the nature of dynamical supersymmetry breaking is calculable by using effective composite states.

The model consists of four $\text{SU}(2)$ doublets, Q_i ($i = 1 - 4$), and six singlets, $Z_{ij} = -Z_{ji}$ ($i, j = 1 - 4$). Those superfields couple via the superpotential

$$W_{\text{IYIT}} = \sum \lambda_{ij}^{kl} Z^{ij} Q_k Q_l \quad (3.1)$$

where λ_{ij}^{kl} denote coupling constants with $\lambda_{ij}^{kl} = -\lambda_{ji}^{kl} = -\lambda_{ij}^{lk}$. The maximal non-abelian global symmetry of the IYIT model is $\text{SU}(4)$ flavor symmetry, $\text{SU}(4)_f$, which is broken by λ_{ij}^{kl} .

The superpotential eq. (3.1) respects a global $U(1)_A$ symmetry with charges, Z 's(+2), Q 's(-1), and a continuous R -symmetry, $U(1)_R$, with Z 's(+2), Q 's(0) (table 1). The former is broken down to the discrete subgroup, $\mathbb{Z}_4$, by the $SU(2)$ anomaly, while the latter is free from the $SU(2)$ anomaly. As we seek a solution to the strong CP problem not relying on global symmetries, we consider that the $\mathbb{Z}_4$ and $U(1)_R$ symmetries are accidental symmetries and are broken by Planck suppressed operators.

It should be noted, however, that a discrete subgroup of $U(1)_R$, $\mathbb{Z}_{NR}$ ($N > 2$), plays crucial roles in constructing the SSM. Without $\mathbb{Z}_{NR}$ ($N > 2$) symmetry, the VEV of the superpotential is expected to be of the order of the Planck scale. Such a large VEV of the superpotential, in turn, does not allow a supersymmetry breaking scale lower than the Planck scale due to the condition for the flat present universe. In addition, it is also known that R -symmetry (or at least an approximate R -symmetry) is relevant for supersymmetry breaking vacua to be stable [39, 40]. Given its importance, we assume that the $\mathbb{Z}_{NR}$ ($N > 2$) symmetry is an exact discrete gauge symmetry [41–47].⁷ In this paper, we take the simplest possibility, $\mathbb{Z}_{4R}$, assuming a presence of an extra multiplet of the **5**, $\bar{\mathbf{5}}$ representations of the $SU(5)$ GUT. The $\mathbb{Z}_{4R}$ symmetry is free from the Standard Model anomaly when the R -charges of the bilinear term of the Higgs doublets and that of the extra multiplets are vanishing [48–51].⁸

In this model, we identify the global PQ symmetry with a $U(1)$ subgroup of $SU(4)_f$ (table 1). As it is a subgroup of $SU(4)_f$, the PQ symmetry is free from the $SU(2)$ anomaly. Under the global $U(1)_{PQ}$ symmetry, the superpotential is reduced to

$$W_{\text{IYIT}} = \lambda_{12}^{12} Z^{12} Q_1 Q_2 + \lambda_{34}^{34} Z^{34} Q_3 Q_4 + \sum \tilde{\lambda}_{ij}^{kl} Z^{ij} Q_k Q_l \quad (3.2)$$

where λ 's are dimensionless coupling constants with $\tilde{\lambda}_{ij}^{kl} = 0$ for $ij = 12, 34$ or $kl = 12, 34$. Hereafter, we take $\lambda_{12}^{12} = \lambda_{34}^{34} = \lambda$ for simplicity, although it is straightforward to extend the following analysis for $\lambda_{12}^{12} \neq \lambda_{34}^{34}$. As we will see shortly, the PQ symmetry is spontaneously broken by the VEV of $Q_1 Q_2$ and $Q_3 Q_4$.

By assuming the KSVZ axion model, the PQ symmetry is communicated to the SSM sector through couplings to the KSVZ extra multiplets in **5** and $\bar{\mathbf{5}}$ representations of the $SU(5)$ GUT,⁹

$$W = \frac{1}{M_{\text{PL}}} Q_1 Q_2 \mathbf{5} \bar{\mathbf{5}}. \quad (3.3)$$

The PQ charges of the KSVZ extra multiplets are given in table 1. Hereafter, we assume that there are N_f flavors of the KSVZ extra multiplets. Once $\langle Q_3 Q_4 \rangle$ spontaneously breaks the PQ symmetry, the axion couples to the SSM sector via eq. (3.3) and the extra multiplets obtain masses of $\mathcal{O}(\langle Q_3 Q_4 \rangle / M_{\text{PL}})$.

⁷In ref. [18], it is proposed to achieve the global PQ symmetry as an accidental symmetry protected by the exact discrete R -symmetry without relying on the gauged PQ mechanism.

⁸For GUT models which are consistent with the $\mathbb{Z}_{4R}$ symmetry, see, e.g., [52, 53].

⁹The KSVZ extra multiplets should be distinguished the extra multiplets required to cancel the Standard Model anomaly of the $\mathbb{Z}_{4R}$ symmetry.

	$Q_{1,2}$	$Q_{3,4}$	$Z_{12}(Z_-)$	$Z_{34}(Z_+)$	$Z_{13,14,23,24}(Z_0^a (a = 1 - 4))$	5	$\bar{\mathbf{5}}$
SU(2)	2	2	1	1	1	1	1
U(1) _{PQ}	+1	−1	−2	+2	0	−2	0
U(1) _R	0	0	+2	+2	+2	r_5	$r_{\bar{5}}$
U(1) _A	−1	−1	+2	+2	+2	+2	+0

Table 1. Charge assignment of the simultaneous symmetry breaking model. The chiral superfields, Q 's, and Z 's, are the SU(2) doublets and singlets of the IYIT model, respectively. The U(1)_{PQ} symmetry is a subgroup of the maximum flavor symmetry of the IYIT model. The KSVZ extra multiplets consist of the **5** and $\bar{\mathbf{5}}$ representations of the SU(5) GUT group. The U(1)_R and U(1)_A symmetries are accidental symmetries of the IYIT model. A discrete subgroup of U(1)_R, i.e. $\mathbb{Z}_{4R}$ is assumed to be an exact symmetry. The R -charges of the KSVZ extra multiplets are taken to be $r_5 + r_{\bar{5}} = 2$.

Now, let us discuss how supersymmetry and the PQ symmetry are broken spontaneously. Below the dynamical scale of SU(2) dynamics, Λ , the IYIT model is well described by using the composite fields, $M_{ij} \sim Q_i Q_j$, with an effective superpotential,

$$W_{\text{eff}} \sim \lambda \Lambda Z_- M_+ + \lambda \Lambda Z_+ M_- + \tilde{\lambda}_{ab} \Lambda Z_0^a M_0^b + \mathcal{X} (2M_+ M_- + M_0^a M_0^a - \Lambda^2). \quad (3.4)$$

Here, $M_+ \sim Q_1 Q_2 / \Lambda$ and $M_- \sim Q_3 Q_4 / \Lambda$ denote the PQ charged mesons, while M_0^a ($a = 1 - 4$) are the PQ neutral mesons. The coupling constants $\tilde{\lambda}$ and the singlets Z_0 's are also rearranged accordingly. In the effective superpotential, the quantum modified constraint $2M_+ M_- + M_0 M_0 - \Lambda^2 = 0$ [54] is implemented by a Lagrange multiplier field $\mathcal{X}$.

By assuming that λ 's are perturbative, and $\lambda_{\pm} (= \lambda)$ are smaller than $\tilde{\lambda}$'s, the VEVs of $M_{\pm}$ are given by

$$\langle M_+ \rangle = \frac{1}{\sqrt{2}} \Lambda, \quad \langle M_- \rangle = \frac{1}{\sqrt{2}} \Lambda. \quad (3.5)$$

Other fields do not obtain VEVs of $\mathcal{O}(\Lambda)$.¹⁰ At this vacuum, the PQ symmetry is spontaneously broken by $\langle M_{\pm} \rangle$ while supersymmetry is broken by the VEVs of the F -components of $Z_{\pm}$, i.e.,

$$F_{Z_{\pm}} \sim \frac{1}{\sqrt{2}} \lambda \Lambda^2, \quad (3.6)$$

simultaneously.

Here, let us comment that the $\mathbb{Z}_{4R}$ is not enough to restrict the superpotential in the form of eq. (3.1). In fact, there can be superpotential terms such as Z_0^3 or $Z_0 Z_+ Z_-$ without the U(1)_A (or $\mathbb{Z}_4$) symmetry. As those terms make the supersymmetry breaking vacuum in eqs. (3.5) and (3.6) metastable, the coefficients of those terms should be rather suppressed to make the vacuum long lived. Such suppression can be achieved, for example, by assuming that a subgroup of $\mathbb{Z}_4$ and U(1)_{PQ} is an exact symmetry where Z_0 's are charged but $Z_{\pm}$

¹⁰The scalar components of $Z_{\pm}$ and $\mathcal{X}$ obtain small VEVs of $\mathcal{O}(m_{3/2})$.

are neutral.¹¹ It is also possible to suppress the unwanted terms by extending the SU(2) dynamics of the IYIT sector into a conformal window by adding extra doublets [55–57].

3.2 Axion supermultiplet

The degeneracy due to the PQ symmetry breaking is parametrized by the axion superfield A ,

$$M_+ = \frac{1}{\sqrt{2}}\Lambda e^{A/\Lambda}, \quad M_- = \frac{1}{\sqrt{2}}\Lambda e^{-A/\Lambda}, \quad (3.7)$$

with which the PQ symmetry is realized by

$$A/\Lambda \rightarrow A/\Lambda + i\alpha, \quad (\alpha = 0 - 2\pi). \quad (3.8)$$

Here, we reduce the domain of the U(1)_{PQ} rotation parameter from $\alpha = 0 - 4\pi$ to $\alpha = 0 - 2\pi$, since all the SU(2) gauge invariant fields have the PQ charge of ± 2 (see table 1). In other words, the sign changes of Q 's by a phase rotation with $\alpha = 2\pi$ can be absorbed by a part of SU(2) transformation.

The effective Kähler potential and superpotential of $M_{\pm}$ and $Z_{\pm}$ are given by,

$$K_{\text{eff}} \sim |Z_+|^2 + |Z_-|^2 + |M_+|^2 + |M_-|^2 + \dots, \quad (3.9)$$

$$W_{\text{eff}} \sim \lambda\Lambda M_+ Z_- + \lambda\Lambda M_- Z_+, \quad (3.10)$$

where the ellipses denote the higher dimensional operators. By substituting the axion superfield, the effective theory is reduced to

$$K_{\text{eff}} \sim |Z_+|^2 + |Z_-|^2 + \frac{1}{2}(A^\dagger + A)^2 + \dots, \quad (3.11)$$

$$W_{\text{eff}} \sim \frac{1}{\sqrt{2}}\lambda\Lambda^2(Z_- e^{A/\Lambda} + Z_+ e^{-A/\Lambda}), \quad (3.12)$$

with some irrelevant holomorphic terms omitted in the Kähler potential. The scalar potential is accordingly given by,¹²

$$V \sim \lambda^2\Lambda^4 \cosh\left(\frac{A^\dagger + A}{\Lambda}\right) + \frac{1}{2}\lambda^2\Lambda^4 \left|Z_+ e^{-A/\Lambda} - Z_- e^{A/\Lambda}\right|^2 \quad (3.13)$$

$$\sim \lambda^2\Lambda^4 \cosh\left(\frac{A^\dagger + A}{\Lambda}\right) + \lambda^2\Lambda^4 |T|^2. \quad (3.14)$$

In the final expression, we rearranged the scalar fields by introducing complex scalar fields S and T ,

$$Z_+ = \frac{1}{\sqrt{2}}(S + T)e^{-A/\Lambda}, \quad (3.15)$$

$$Z_- = \frac{1}{\sqrt{2}}(S - T)e^{A/\Lambda}, \quad (3.16)$$

so that the PQ symmetry is manifest in the scalar potential.

¹¹As this symmetry is not broken spontaneously at the vacuum, and hence, Z_0 's and M_0 's are predicted to be stable. Thus, the simultaneous breaking of the IYIT sector should take place before inflation to avoid the production of those stable particles if we assume the above symmetry.

¹²Throughout the paper, we use the same symbols to describe the superfields and their scalar components.

The above scalar potential shows that the complex scalar T and the real component of A (the saxion) obtain masses of $\lambda\Lambda$, around their origins. The complex scalar field S (the pseudo-flat direction) and the imaginary part of A (the axion a), on the other hand, remain massless. The pseudo-flat direction eventually obtains a mass from the higher order terms in the Kähler potential. For perturbative λ and $\tilde{\lambda}$, the mass is dominated by the one-loop contributions [58–60],

$$m_S^2 \simeq \frac{1}{32\pi^2} \left(\lambda^2(2\log 2 - 1) + \frac{4\lambda^4}{3\tilde{\lambda}^2} \right) \frac{F_S^2}{\Lambda^2}, \quad (F_S = \lambda\Lambda^2), \quad (3.17)$$

with which the pseudo-flat direction is stabilized at its origin.¹³

The superpotential in eq. (3.12) also shows that the fermion partners of A (the axino) and T obtain a Dirac mass of $\lambda\Lambda$, with each other. The fermion partner of S corresponds to the goldstino which is absorbed into the gravitino by the super-Higgs mechanism.

Putting together, the model achieves dynamical breaking of supersymmetry and the PQ breaking simultaneously. The axion supermultiplet splits into a massless axion and massive saxion/axino with masses of the supersymmetry/PQ breaking scale. The axion couples to the SSM sector via the coupling in eq. (3.3), i.e.,

$$W \sim \frac{\Lambda^2}{\sqrt{2}M_{\text{PL}}} e^{A/\Lambda} \bar{\mathbf{5}} \mathbf{5} \sim \frac{\Lambda^2}{\sqrt{2}M_{\text{PL}}} e^{ia/f_a} \bar{\mathbf{5}} \mathbf{5}, \quad (3.18)$$

where $a = \sqrt{2} \text{Im}[A]$ denotes the axion field and $f_a = \sqrt{2}\Lambda$. After integrating out the extra KSVZ multiplets, the axion couples to the SM gauge fields through

$$W_{\text{anom}} = \frac{N_f}{8\pi^2} \frac{A}{\Lambda} \sum_l W_l^\alpha W_{l\alpha}, \quad (3.19)$$

with which the strong CP problem is solved

3.3 Explicit breaking of the PQ symmetry

Now, let us discuss explicit breaking of the global PQ symmetry expected in quantum gravity. In this model, the most relevant terms which break the global PQ symmetry are given by,¹⁴

$$W \sim \frac{\kappa}{M_{\text{PL}}^2} Z_+ (Q_1 Q_2)^2 + \frac{\kappa}{M_{\text{PL}}^2} Z_- (Q_3 Q_4)^2 \sim \frac{\kappa \Lambda^2}{M_{\text{PL}}^2} Z_\pm M_\pm^2, \quad (3.20)$$

with κ being a dimensionless coupling constant.¹⁵ The corresponding symmetry breaking terms in the scalar potentials are given by,

$$V \sim \frac{1}{2} m_a^2 f_a^2 \left(\frac{a}{f_a} \right)^2 + \lambda \kappa \left(\frac{\Lambda}{M_{\text{PL}}} \right)^2 \Lambda^4 e^{i\frac{a}{f_a}} + \text{h.c.} \quad (3.21)$$

¹³Here, we neglect the one-loop contributions from the $U(1)_{\text{gPQ}}$ gauge interaction by assuming that the gauge coupling constant is small. The contributions from the gauge interaction, in fact, destabilize the origin of the pseudo flat direction [59–61].

¹⁴Here, we require that $U(1)_{\text{PQ}}$ is not broken by renormalizable interactions as a part of definition of the global symmetry.

¹⁵Lower dimensional operators which break the PQ symmetry, such as Z_+^4/M_{PL} , are forbidden by the $\mathbb{Z}_{4R}$ symmetry.

Here, we inserted the VEVs of $M_{\pm}$ and those of F -terms of $Z_{\pm}$. Due to the explicit breaking, the VEV of the axion, and hence, the effective θ angle is shifted to

$$\theta_{\text{eff}} = \frac{\langle a \rangle}{f_a} \simeq \text{Im}[\kappa\lambda] \left(\frac{\Lambda}{M_{\text{PL}}} \right)^2 \left(\frac{\Lambda^4}{m_a^2 f_a^2} \right) \Big|_{\text{mod } 2\pi} \quad (3.22)$$

$$\simeq 10^{40} \times \text{Im}[\kappa\lambda] \left(\frac{0.08 \text{ GeV}}{\sqrt{m_a f_a}} \right)^4 \left(\frac{\Lambda}{10^{12} \text{ GeV}} \right)^6 \Big|_{\text{mod } 2\pi}. \quad (3.23)$$

Thus, unless $\text{Im}[\kappa\lambda]$ is finely tuned to be smaller than $\mathcal{O}(10^{-11})$, the effective θ angle is too large to be consistent with the measurement of the neutron electric dipole moment (EDM) [32].

4 Gauged PQ extension of simultaneous breaking model

Let us now implement the gauged PQ mechanism to the model of the simultaneous breaking of supersymmetry and the PQ symmetry in section 3. For that purpose, we introduce an additional sector based on $\text{SU}(3)$ dynamics which breaks a PQ symmetry spontaneously. In the following, we call this model the $\text{SU}(3)'$ model, and put primes on the superfields and the symmetry groups in this sector.

4.1 $\text{SU}(3)'$ PQ symmetry breaking model

The $\text{SU}(3)'$ model consists of three flavors of the (anti-)fundamental representation of $\text{SU}(3)$, Q' , $\bar{Q}'$, and nine $\text{SU}(3)$ singlets, Z' . The charge assignment of the global symmetries is given in table 2. Under these symmetries, they couple via the superpotential

$$W_{\text{PQ}} = \lambda_{ij}^{\prime kl} Z'^{ij} Q'_k \bar{Q}'_l, \quad (4.1)$$

where $\lambda_{ij}^{\prime kl}$ denote coupling constants with $(i, j, k, l = 1 - 3)$. The baryon symmetry, $\text{U}(1)_B$, is identified with the global PQ symmetry, $\text{U}(1)'_{\text{PQ}}$, while the maximal flavor symmetry, $\text{SU}(3)_L \times \text{SU}(3)_R$, is completely broken by λ' 's.

In addition to the global $\text{U}(1)'_{\text{PQ}}$ symmetry, the superpotential possesses a continuous R -symmetry and a $\text{U}(1)'_A$ symmetry (broken down to a $\mathbb{Z}_6$ symmetry by the $\text{SU}(3)'$ anomaly) in table 2. As discussed previously, however, we consider that only $\mathbb{Z}_{4R}$ is an exact symmetry, and assume that $\text{U}(1)_R$ and $\text{U}(1)'_A$ are accidental symmetries broken by higher dimensional operators.¹⁶

Below the dynamical scale of $\text{SU}(3)'$, Λ' , the $\text{SU}(3)'$ sector is well described by the composite mesons and baryons,

$$M' \sim Q' \bar{Q}' / \Lambda', \quad B'_+ \sim Q' Q' Q' / \Lambda'^2, \quad B'_- \sim \bar{Q}' \bar{Q}' \bar{Q}' / \Lambda'^2, \quad (4.2)$$

with an effective superpotential,

$$W'_{\text{eff}} = \lambda' \Lambda' Z' M' + \mathcal{X}' (B'_+ B'_- + \det(M') / \Lambda' - \Lambda'^2). \quad (4.3)$$

¹⁶Without $\text{U}(1)'_A$ (or $\mathbb{Z}_6$), the superpotential terms such as Z'^3 are allowed even if we assume the $\mathbb{Z}_{4R}$ symmetry. Such terms, however, do not change the following discussion.

	Q'	$\bar{Q}'$	Z'	$\mathbf{5}'$	$\bar{\mathbf{5}}'$
$\text{SU}(3)'$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\text{U}(1)'_{\text{PQ}}$	+1	−1	0	3	0
$\text{U}(1)_R$	0	0	+2	r'_5	$r'_{\bar{5}}$
$\text{U}(1)'_A$	+1	+1	−2	−3	0

Table 2. Charge assignment of the dynamical PQ symmetry breaking sector. The chiral superfields, Q' 's, and Z' 's, are the $\text{SU}(3)'$ triplets and singlets, respectively. The $\text{U}(1)'_{\text{PQ}}$ symmetry corresponds to $\text{U}(1)_B$ symmetry in the $\text{SU}(3)'$ sector. The KSVZ extra multiplets are denoted by $\mathbf{5}'$ and $\bar{\mathbf{5}}'$. The $\text{U}(1)_R$ and $\text{U}(1)'_A$ symmetries are accidental symmetries, with $\mathbb{Z}_{4R}$ being an exact symmetry. The R -charges of the KSVZ extra multiplets are taken to be $r'_5 + r'_{\bar{5}} = 2$.

Here, the second term implements the deformed moduli constraint by a Lagrange multiplier field $\mathcal{X}'$ [54]. The mesons are neutral under $\text{U}(1)'_{\text{PQ}}$ while the baryons have charges ± 3 .

From the superpotential in eq. (4.3), we find that the PQ symmetry is spontaneously broken by the VEVs of $B_{\pm}$. Accordingly, the vacuum is parametrized by the Goldstone superfield A' ,¹⁷

$$B'_+ = \frac{1}{\sqrt{2}} \Lambda_2 e^{A'/\Lambda_2}, \quad (4.4)$$

$$B'_- = \frac{1}{\sqrt{2}} \Lambda_2 e^{-A'/\Lambda_2}, \quad (4.5)$$

with $\Lambda_2 = \sqrt{2}\Lambda'$. By using A' , the PQ symmetry is non-linearly realized by

$$\frac{A'}{\Lambda_2} \rightarrow \frac{A'}{\Lambda_2} + i\alpha', \quad (\alpha' = 0 - 2\pi). \quad (4.6)$$

As in the case of the IYIT sector, the domain of the PQ symmetry is reduced from $\alpha' = 0 - 6\pi$ to $\alpha' = 0 - 2\pi$ as the $\text{SU}(3)'$ invariant fields have the PQ charges of ± 3 .

The $\text{U}(1)'_{\text{PQ}}$ symmetry in this sector is also communicated to the SSM sector through the couplings to N'_f flavors of the KSVZ extra multiplets, $\mathbf{5}'$ and $\bar{\mathbf{5}}'$. With the charge assignment in table 2, the baryons couple to the extra multiplets in the superpotential,

$$W = \frac{1}{M_{\text{PL}}^2} \bar{Q}' \bar{Q}' \bar{Q}' \mathbf{5}' \bar{\mathbf{5}}' \sim \frac{\Lambda'^2}{M_{\text{PL}}^2} B'_- \mathbf{5}' \bar{\mathbf{5}}'. \quad (4.7)$$

Once $\text{U}(1)'_{\text{PQ}}$ is broken, the axion obtains the anomalous coupling to the SSM gauge fields, while the extra multiplets obtain masses of $\mathcal{O}(\Lambda'^3/M_{\text{PL}}^2)$.

4.2 Gauged PQ symmetry

Now, we are ready to find out a model of the gauged PQ symmetry by combining the simultaneous supersymmetry and the PQ symmetry breaking model in section 3 and the PQ symmetry breaking model in subsection 4.1. To apply the prescription in section 2,

¹⁷The origin of A' is set at which $B'_+ = B'_-$, and $\langle B'_+ \rangle \neq \langle B'_- \rangle$ for $\langle A' \rangle \neq 0$, accordingly.

	$Z_{\pm}$	$M_{\pm}$	B'_+	B'_-	$\mathbf{5}$	$\bar{\mathbf{5}}$	$\mathbf{5}'$	$\bar{\mathbf{5}}'$	Y	Y'
$U(1)_{\text{gPQ}}$	$\pm q_1$	$\pm q_1$	$-q_2$	q_2	$-q_1$	0	q_2	0	q_1	$-q_2$
$\mathbb{Z}_{4R}$	2	0	0	0	r_5	$r_{\bar{5}}$	r'_5	$r'_{\bar{5}}$	1	1

Table 3. The charge assignment of the gauged PQ symmetry and the $\mathbb{Z}_{4R}$ symmetry. The singlet fields Y 's and Y' 's are introduced to cancel the self-triangle and gravitational anomalies of $U(1)_{\text{gPQ}}$ (see subsection 4.7).

let us first identify Φ_1 with the meson operator M_+ in section 3 and Φ_2 with the baryon operator B'_+ , i.e.,

$$\Phi_1 = M_+, \quad (4.8)$$

$$\bar{\Phi}_2 = B'_-, \quad (4.9)$$

and assign $U(1)_{\text{gPQ}}$ charges of q_1 and $-q_2$ to them (table 3).¹⁸ Then, the anomaly-free condition of the $U(1)_{\text{gPQ}}$ symmetry in eq. (2.10) is given by,

$$q_1 N_f - q_2 N'_f = 0. \quad (4.10)$$

Once the two sectors are put together by the gauged PQ symmetry, spontaneous breaking of the PQ symmetries in the two sectors lead to the would-be Goldstone and the axion superfield. The would-be Goldstone is absorbed into the massive $U(1)_{\text{gPQ}}$ gauge multiplet, and the saxion and the axino in the axion supermultiplet obtain masses of the order of the supersymmetry breaking scale. As a result, the simultaneous breaking model with the gauged PQ mechanism leaves only a light axion which couples to the Standard Model gauge fields via eq. (2.29).

4.3 Accidental global PQ symmetry

As discussed in the previous section, the global PQ symmetry can be explicitly broken by the $U(1)_{\text{gPQ}}$ invariant operator consisting of the fields in the two sectors. Among the explicit breaking terms, the most relevant ones are given by,¹⁹

$$W \sim \frac{\kappa}{M_{\text{PL}}^{3q_1+2q_2-4}} (Z_+(Q_1 Q_2)^{q_2-1} (\bar{Q}' \bar{Q}' \bar{Q}')^{q_1} + Z_+(Q_3 Q_4)^{q_2-1} (Q' Q' Q')^{q_1}), \quad (4.11)$$

$$\sim \frac{\kappa \Lambda^{q_2-1} \Lambda'^{2q_1}}{M_{\text{PL}}^{3q_1+2q_2-4}} (Z_+ M_+^{q_2-1} B_-'^{q_1} + Z_- M_-^{q_2-1} B_+'^{q_1}), \quad (4.12)$$

with κ being a dimensionless coupling constant. It should be noted that these terms are consistent with the $\mathbb{Z}_{4R}$ symmetry, and hence, no factor of $m_{3/2}$ is required unlike the terms in eq. (2.22). These operators roughly contribute to the axion potential,

$$V \sim \frac{1}{2} m_a^2 F_a^2 \left(\frac{a}{F_a} \right)^2 + \frac{\text{Im}[\kappa \lambda]}{M_{\text{PL}}^{3q_1+2q_2-4}} \Lambda^{2q_2} \Lambda'^{3q_1} \frac{a}{F_a} + \text{h.c.} + \dots, \quad (4.13)$$

¹⁸The $U(1)_{\text{gPQ}}$ charges of $Q_{1,2}$ and $\bar{Q}'$'s corresponds to $q_1/2$ and $q_2/3$, respectively.

¹⁹There are lower dimensional operators which break the global PQ symmetry with $M_{\pm}$ replaced by $M_{\text{PL}} \times Z_{\pm}$ in eq. (4.11). The explicit breaking effects of those operators are comparable to the ones of eq. (4.11) due to suppressed A -term VEVs of $Z_{\pm} = \mathcal{O}(m_{3/2})$.

where the VEVs of $M_{\pm}$, $B'_{\pm}$, and those of the F -terms of $Z_{\pm}$ are inserted, Therefore, in the simultaneous breaking model with the gauged PQ mechanism, the effective θ angle at the vacuum is given by,

$$\theta_{\text{eff}} \simeq \frac{1}{m_a^2 F_a^2} \frac{\Lambda^{2q_2} \Lambda'^{3q_1}}{M_{\text{PL}}^{3q_1+2q_2-4}} \quad (4.14)$$

$$\sim 10^{77.5-6.4(3q_1+2q_2)} \times \left(\frac{0.08 \text{ GeV}}{\sqrt{m_a F_a}} \right)^4 \left(\frac{\Lambda}{10^{12} \text{ GeV}} \right)^{2q_2} \left(\frac{\Lambda'}{10^{12} \text{ GeV}} \right)^{3q_1}. \quad (4.15)$$

Thus, for $3q_1+2q_2 \gtrsim 14$, the explicit breaking of the global PQ symmetries are small enough to be consistent with the measurement of the neutron EDM, i.e., $\theta_{\text{eff}} < 10^{-11}$ [32].

4.4 Mass spectrum of the KSVZ multiplets

The KSVZ multiplets, $(\bar{\mathbf{5}}, \bar{\mathbf{5}})$ and $(\bar{\mathbf{5}}', \bar{\mathbf{5}}')$ were introduced to communicate the PQ symmetries to the SSM sector. After PQ symmetry breaking, those extra multiplets obtain supersymmetric masses of the order of

$$m_{\text{KSVZ}} \sim \frac{\Lambda^2}{M_{\text{PL}}}, \quad (4.16)$$

$$m'_{\text{KSVZ}} \sim \frac{\Lambda'^3}{M_{\text{PL}}^2}, \quad (4.17)$$

respectively (see eqs. (3.3) and (4.7)). The scalar components of the KSVZ multiplets also obtain masses of $\mathcal{O}(m_{3/2})$ through supergravity effects. Thus, most of the KSVZ multiplets become heavy and beyond the reach of the LHC experiments except for the fermion components of $(\bar{\mathbf{5}}', \bar{\mathbf{5}}')$.²⁰

The KSVZ extra multiplets are assumed to couple to the SSM particle via,

$$W \sim \frac{\epsilon}{M_{\text{PL}}} Q_1 Q_2 \bar{\mathbf{5}} \bar{\mathbf{5}}_{\text{SM}} + \frac{\epsilon'}{M_{\text{PL}}^2} \bar{Q}' \bar{Q}' \bar{Q}' \bar{\mathbf{5}}' \bar{\mathbf{5}}_{\text{SM}}, \quad (4.18)$$

$$\sim \epsilon m_{\text{KSVZ}} \bar{\mathbf{5}} \bar{\mathbf{5}}_{\text{SM}} + \epsilon' m'_{\text{KSVZ}} \bar{\mathbf{5}}' \bar{\mathbf{5}}_{\text{SM}}, \quad (4.19)$$

where $\bar{\mathbf{5}}_{\text{SM}}$ denotes the SSM matter multiplet, and $\epsilon^{(')}$ are coefficients. Here, we take $r_5 = r'_5 = 1$ so that $\bar{\mathbf{5}}$ and $\bar{\mathbf{5}}'$ have the same R -charges with $\bar{\mathbf{5}}_{\text{SM}}$. Through the mixing terms, the KSVZ extra multiplets decay immediately into the SSM particles.

Finally, let us note that there can be mixing terms between $(\bar{\mathbf{5}}, \bar{\mathbf{5}})$ and $(\bar{\mathbf{5}}', \bar{\mathbf{5}}')$ through,

$$W \sim \frac{1}{M_{\text{PL}}} Q_1 Q_2 \bar{\mathbf{5}} \bar{\mathbf{5}}' + \frac{1}{M_{\text{PL}}^2} \bar{Q}' \bar{Q}' \bar{Q}' \bar{\mathbf{5}}' \bar{\mathbf{5}}. \quad (4.20)$$

Although these operators consist of the fields in the two PQ symmetric sectors, they are invariant under not only the gauged PQ symmetry but also under the global PQ symmetries. Thus, these terms do not affect θ_{eff} . They do not affect the KSVZ mass spectrum significantly neither. From these reasons, we neglect these mixing terms throughout this paper.

²⁰The extra multiplet to achieve the $\mathbb{Z}_{4R}$ symmetry also obtains the mass of $\mathcal{O}(m_{3/2})$ from the R -symmetry breaking effects [62].

Figure 1. Constraints on charges q_1 and q_2 in the $SU(3)'$ model for $N_{\text{GCD}} = 1$. The allowed charges are colored by light blue and orange, although the orange colored charges are allowed only for $\Lambda' \gtrsim M_{\text{GUT}}$. The charges colored by blue lead to too large θ_{eff} or too light KSVZ extra multiplets. The gauge coupling constants of the SSM blow up below the GUT scale for the charges colored by red. The black colored charges are excluded as they are not relatively prime.

4.5 PQ charges in the $SU(3)'$ model

For a given q_1 and q_2 , there are upper limits on Λ and Λ' to achieve a high-quality global PQ symmetry (see eq. (4.14)). The dynamical scales are also constrained from below for an appropriate supersymmetry breaking scale and for heavy enough KSVZ extra multiplets. As a lower limit on the supersymmetry breaking scale, i.e., Λ , we require

$$m_{3/2} \simeq \frac{\lambda \Lambda^2}{\sqrt{3} M_{\text{PL}}} \gtrsim 10 \text{ TeV}, \quad (4.21)$$

so that the observed Higgs boson mass, $m_H \simeq 125 \text{ GeV}$, is achieved by the gravity mediated sfermion masses of $\mathcal{O}(m_{3/2})$. As a lower limit on the KSVZ extra multiplets, we put

$$m'_{\text{KSVZ}} \simeq \frac{\Lambda'^3}{M_{\text{PL}}} \gtrsim 750 \text{ GeV}, \quad (4.22)$$

from the null results of the searches for a heavy b -type quark at the LHC experiments [63–66].

In figure 1, we show the charge choices for $SU(3)'$ model for $N_{\text{eff}} = 1$ and $\lambda = 1$. The charges colored by blue are excluded, with which θ_{eff} cannot be suppressed enough for $m_{3/2} \gtrsim \mathcal{O}(1) \text{ TeV}$ and $m'_{\text{KSVZ}} \gtrsim 750 \text{ GeV}$.²¹ In the figure, we require $\theta_{\text{eff}} \lesssim 10^{-10}$ given $\mathcal{O}(1)$ uncertainties of the coefficients of the explicit breaking terms. The figure shows that these constraints exclude relatively small charges as the suppression of the explicit breaking term relies on large PQ charges.

The perturbative coupling unification of the SSM gauge coupling constants also puts constraints on the charges. From the anomaly-free condition in eq. (4.10), N_f and N'_f are

²¹The charges colored by blue are not changed even if the lower limits on $m_{3/2}$ and m'_{KSVZ} are relaxed to $m_{3/2} \gtrsim 100 \text{ GeV}$ and $m'_{\text{KSVZ}} \gtrsim 100 \text{ GeV}$.

given by

$$N_f = N_{\text{GCD}} \times q_2, \quad N'_f = N_{\text{GCD}} \times q_1. \quad (4.23)$$

As the extra multiplets contribute to the renormalization group evolutions of the SSM gauge coupling constants and make them asymptotically non-free, the perturbative unification puts upper limits on N_f and N'_f , and hence, on q_1 and q_2 .

In figure 1, we color the charges by red, with which $\theta_{\text{eff}} \lesssim 10^{-10}$ is not compatible with the perturbative unification. Here, we use the renormalization group equation at the one-loop level and require that $g_{1,2,3} < 4\pi$ below the GUT scale, i.e., $M_{\text{GUT}} \simeq 10^{16}$ GeV. We also take the masses of the sfermions, the heavy charged/neutral Higgs boson, and the Higgsinos to be at the gravitino mass scale. The gaugino masses are assumed to be dominated by the anomaly mediation effects [67, 68] which are roughly given by (see, e.g. [69]),

$$m_{\text{bino}} \simeq 10^{-2} \times m_{3/2}, \quad (4.24)$$

$$m_{\text{wino}} \simeq 3 \times 10^{-3} \times m_{3/2}, \quad (4.25)$$

$$m_{\text{gluino}} \simeq 2.5 \times 10^{-2} \times m_{3/2}, \quad (4.26)$$

although the constraints do not depend on them significantly as long as they are in the TeV range. The gravitino mass is take to be within $10 \text{ TeV} \leq m_{3/2} \leq 10 \text{ PeV}$. These choices are motivated by the pure gravity mediation model in refs. [70–72] (see also refs. [73–76] for closely related models).²² In the renormalization group evolution, we also take into account an extra multiplet required for the anomaly free condition of the $\mathbb{Z}_{4R}$ symmetry, whose masses are also at the gravitino mass scale.

The figure shows that the requirement for perturbative unification excludes the charges with $q_2 > 7$ ($N_f > 7$). This is expected as N_f flavors of the KSVZ extra multiplets have masses of $10 \text{ TeV} \lesssim m_{\text{KSVZ}} \lesssim 10 \text{ PeV}$.²³ On the other hand, a large q_1 is allowed. This is because the explicit breaking terms are suppressed by $(\Lambda'/M_{\text{PL}})^{3q_1}$, and hence, a high-quality global PQ is possible even for a large Λ' as long as q_1 is large. For a large Λ' , m'_{KSVZ} also becomes large, with which the perturbative unification is possible even if $N'_f = q_1$ is large. It should be noted, however, that the effective field theory approach is no more reliable when Λ' is too close to the Planck scale. In the figure, we color the charges by orange if they require a large Λ' , i.e., $10^{16} \text{ GeV} \lesssim \Lambda' \lesssim 10^{17} \text{ GeV}$. For $N_{\text{GCD}} \geq 2$, there are no appropriate charges with which $\theta_{\text{eff}} < 10^{-10}$ and the perturbative unification are compatible.

4.6 Parameter regions in the $\text{SU}(3)'$ model

In figure 2, we show the parameter regions for a given q_1 and q_2 . In each panel, we take $m_{3/2} < 10 \text{ PeV}$ and $\lambda = 1, 10^{-1}, 10^{-2}$, respectively. The gray shaded region is excluded, as $\theta_{\text{eff}} < 10^{-10}$ is not satisfied (see eq. (4.14)). The perturbative unification is not achieved in the blue shaded region. The red shaded region is excluded for too light KSVZ extra multiplets, i.e., $m'_{\text{KSVZ}} \lesssim 750 \text{ GeV}$. The green dashed lines are contours of the effective decay constant in eq. (2.21).

²²Here, the Higgsino mediation effects neglected for simplicity. Besides, the gaugino spectrum is deflected from the anomaly mediation in the presence of the KSVZ extra multiplets [77].

²³If we restrict to $m_{3/2} < 1 \text{ PeV}$, the constraint becomes tighter and the charges with $q_2 > 5$ are excluded.

Figure 2. The constraints on the parameter regions for given PQ charges, q_1 and q_2 . The gray region is excluded as $\theta_{\text{eff}} < 10^{-10}$ is not satisfied. The perturbative unification is not achieved in the blue region. The red regions are excluded by $m'_{\text{KSVZ}} \gtrsim 750 \text{ GeV}$. The green lines are the contours of the effective decay constant F_a .

The figure shows that the dynamical scale Λ' is tightly constrained from above to achieve $\theta_{\text{eff}} < 10^{-10}$ for the minimum charge choice, i.e., $q_1 = 5$ and $q_2 = 1$. This is understood as the explicit breaking terms are not effectively suppressed for rather small charges. As a result, the PQ breaking scales are required to be low to avoid large explicit breaking effects. The upper limit on Λ' becomes tighter for a larger Λ as is expected from eq. (4.14). Furthermore, as the dynamical scale Λ becomes larger for a smaller λ , the upper limit becomes even tighter for a smaller λ for a given $m_{3/2}$. The constraints from

the perturbative unification are, on the contrary, weaker since m_{KSVZ} becomes larger for a smaller λ for a given $m_{3/2}$.

An interesting property of the minimum choice is that the model predicts the KSVZ extra multiplets ($\mathbf{5}'$, $\bar{\mathbf{5}}'$) in the TeV range. This feature reflects the suppressed fermion masses of the KSVZ extra multiplet in eq. (4.17) caused by the composite nature of the PQ breaking field, i.e., B'_- , with a tight upper limit on Λ' . Thus, the model with the minimum charge choice can be tested by searching for vector-like colored particles at the LHC experiments.

For $q_1 = 7$ and $q_2 = 1$, the upper limit on Λ' is weaker than for the minimum choice. This is because the suppression factor of the explicit breaking term, $(\Lambda'/M_{\text{PL}})^{3q_1}$, can be very small even for a rather large Λ' due to a large exponent. The constraint from the perturbative unification is, on the contrary, tighter for a large q_1 as N'_f is proportional to q_1 . For a large N'_f , the masses of the KSVZ extra multiplets, m'_{KSVZ} , is required to be high to avoid the blow-up of the gauge coupling constants below the GUT scale.

For $q_1 = 1$ and $q_2 = 7$, the upper limit on Λ' is also weaker than the minimum choice for $\lambda = 1$ due to a strong suppression of the explicit breaking terms by $(\Lambda/M_{\text{PL}})^{2q_2}$. As the suppression factor is sensitive to Λ , the upper limit on Λ' becomes very tight for a smaller λ for a given gravitino mass.

In all cases, we find that the gravitino mass is required to be in the hundreds TeV or larger, and hence, the model can be consistent with the observed Higgs boson mass achieved by the gravity mediated sfermion masses. It is also notable that the dynamical scale Λ' is larger than Λ in the allowed parameter region. Therefore, both the accidental global PQ symmetry and supersymmetry are broken by the IYIT sector while the gauged PQ symmetry is mainly broken by the $\text{SU}(3)'$ sector. This feature is attractive as it explains the coincidence between the global PQ breaking scale and the supersymmetry breaking scale.

Before closing this subsection, let us comment on the axion dark matter abundance. The axion starts coherent oscillation when the Hubble expansion rate becomes comparable to the axion mass, which leads to the present axion dark matter density [78],

$$\Omega_{\text{axion}} h^2 \simeq 0.2 \times \theta_i^2 \left(\frac{F_a}{10^{12} \text{ GeV}} \right)^{1.19}. \quad (4.27)$$

Here, θ_i is the initial misalignment angle of the axion field. Thus, the axion can be a dominant component for dark matter of $F_a = \mathcal{O}(10^{12})$ GeV, i.e., $\Omega_{\text{DM}} \simeq 0.12$ [79]. As the figures show, $F_a = \mathcal{O}(10^{12})$ GeV is possible in a wide range of the parameter space. Therefore, the model based on $\text{SU}(3)'$ can be consistent with the axion dark matter scenario.²⁴

4.7 Cancellation of self- and gravitational anomalies

As mentioned in section 2, the gravitational anomaly and the self-anomaly of $\text{U}(1)_{\text{gPQ}}$ are canceled by adding $\text{U}(1)_{\text{gPQ}}$ charged singlet fields. In this subsection, we show a concrete model of the anomaly cancellation.

²⁴For $m_{3/2} \gg \mathcal{O}(1)$ PeV, the wino is expected to be heavier than $\mathcal{O}(1)$ TeV, whose relic abundance exceeds the observed dark matter density. In such parameter region, we need to assume either a dilution mechanism of dark matter or R -parity violation.

In the IYIT sector and the $SU(3)'$ sector, the $U(1)_{\text{gPQ}}$ charged fields are paired with fields with opposite charges. Thus, the fields in these sectors do not contribute to the self-anomaly nor the gravitational anomaly. The charges of the KSVZ extra multiplets are, on the other hand, not paired, and hence, they contribute to the anomalies,

$$\mathcal{A}_{\text{self}}^{\text{KSVZ}} = -5N_f q_1^3 + 5N'_f q_2^3, \quad (4.28)$$

$$\mathcal{A}_{\text{gravitational}}^{\text{KSVZ}} = -5N_f q_1 + 5N'_f q_2, \quad (4.29)$$

respectively. The easiest way to cancel the anomaly is to introduce $5N_f$ singlet superfields Y with a charge q_1 and $5N'_f$ singlet superfields Y' with a charge $-q_2$. The charges of Y 's and Y' 's are given in table 3.

As the singlet fields do not have mass partners with opposite charges, the supersymmetric masses of them are generated only after $U(1)_{\text{gPQ}}$ breaking. The mass terms of Y 's are given by

$$W \sim \frac{m_{3/2}}{M_{\text{PL}}^4} (Q_3 Q_4)^2 Y Y \sim m_{3/2} \frac{\Lambda^4}{M_{\text{PL}}^4} Y Y. \quad (4.30)$$

Here, we take the $\mathbb{Z}_{4R}$ charge of Y 's to be 1, so that their scalar and fermion components are odd and even under the R -parity, respectively. The factor $m_{3/2}$ encapsulates the effects of spontaneous breaking of the $\mathbb{Z}_{NR}$ symmetry. As a result, the fermionic components of Y 's obtain

$$m_Y \sim 3 \times 10^{-7} \text{ eV} \left(\frac{m_{3/2}}{10^6 \text{ GeV}} \right) \left(\frac{\Lambda}{10^{13} \text{ GeV}} \right)^4, \quad (4.31)$$

while the masses of scalar components are dominated by the gravity mediated soft masses of $\mathcal{O}(m_{3/2})$.

The supersymmetric masses of Y' 's are even smaller,

$$W \sim \frac{m_{3/2}}{M_{\text{PL}}^6} (\bar{Q}' \bar{Q}' \bar{Q}')^2 Y' Y' \sim m_{3/2} \left(\frac{\Lambda'}{M_{\text{pl}}} \right)^6 Y' Y'. \quad (4.32)$$

Here, we take the $\mathbb{Z}_{4R}$ charge of Y' 's to be 1, and the factor $m_{3/2}$ encapsulates the effects of spontaneous breaking of $\mathbb{Z}_{NR}$ again. As a result, the fermionic components of Y' 's obtain,

$$m_{Y'} \sim 5 \times 10^{-12} \text{ eV} \left(\frac{m_{3/2}}{10^6 \text{ GeV}} \right) \left(\frac{\Lambda'}{10^{14} \text{ GeV}} \right)^6 \quad (4.33)$$

while the masses of the scalar components of Y' 's are dominated by the gravity mediated soft masses as in the case of Y 's.²⁵

If the light fermions are abundantly produced in the early universe, they contribute to the dark radiation and result in an unacceptably large number of effective neutrino species, N_{eff} . To evade this problem, we assume that spontaneous breaking of $U(1)_{\text{gPQ}}$

²⁵There are mass terms proportional to $Y Y'$ which can be larger than eqs. (4.30) and (4.32) depending on the parameters. Even in such cases, there remain light fermions with masses either m_Y or $m'_{Y'}$ as the numbers of Y 's and Y' 's are different.

takes place before the end of inflation. We also assume that the gauge superfields of $U(1)_{\text{gPQ}}$ are heavier than the reheating temperature after inflation. Furthermore, it is also assumed that the branching fraction of the inflaton into Y 's and Y' 's are suppressed. With these assumptions, we can achieve cosmologically consistent models where the self- and the gravitational anomalies are canceled by the $U(1)_{\text{gPQ}}$ charged singlets.

4.8 $SU(N)'$ dynamical PQ symmetry breaking model

So far, we have considered the dynamical PQ breaking sector based on the $SU(3)$ gauge theory. There, the deformed moduli constraint plays an important role to break the global PQ symmetry (i.e., the baryon symmetry) spontaneously. In this subsection, we discuss the models of dynamical PQ breaking based on $SU(N)$ gauge theory other than $N = 3$. We call such models, the $SU(N)'$ dynamical PQ breaking model.

First, let us consider the $SU(2)'$ model. With four fundamental representations of $SU(2)'$, Q' , the model exhibits the deformed moduli constraint. In this model, there is no baryon symmetry, and the global PQ symmetry is identified with a subgroup of the maximal non-abelian group $SU(4)_f$ as in the case of the IYIT sector. Then, the global PQ symmetry breaking is achieved by introducing four PQ neutral singlet superfields, Z' .²⁶

In this model, the KSVZ extra multiplets coupling to the $SU(2)'$ sector obtain masses via,

$$W \sim \frac{1}{M_{\text{PL}}} Q'_1 Q'_2 \mathbf{5}' \bar{\mathbf{5}}', \quad (4.34)$$

leading to

$$m_{\text{KSVZ}'} \sim \frac{\Lambda'^2}{M_{\text{PL}}}. \quad (4.35)$$

Thus, the $SU(2)'$ model allows a rather small Λ' compared with the $SU(3)'$ model to achieve $m'_{\text{KSVZ}} \gtrsim 750 \text{ GeV}$. It is even possible to be $\Lambda' \ll \Lambda$. The possibility of $\Lambda' \ll \Lambda$ is, however, not very attractive as the model does not explain a coincidence between the PQ breaking scale and the supersymmetry breaking scale.

Next, let us consider the $SU(N)'$ ($N > 3$) model. In this case, the global PQ symmetry is identified with the baryon symmetry which is broken by the deformed moduli constraint as in the case of the $SU(3)'$ model. As the mass terms of the KSVZ extra multiplets, $\mathbf{5}'$ and $\bar{\mathbf{5}}'$, are given by,

$$W \sim \frac{1}{M_{\text{PL}}^{N-2}} (\bar{Q}' \cdots \bar{Q}') \mathbf{5}' \bar{\mathbf{5}}', \quad (4.36)$$

the dynamical scale Λ' should be much higher than Λ to satisfy $m'_{\text{KSVZ}} \gtrsim 750 \text{ GeV}$. Here, $\bar{Q} \cdots \bar{Q}$ denotes the baryon operators of the $SU(N)'$ sector. The $SU(N)'$ models are very similar to the $SU(3)'$ model except for the dynamical scale Λ' , although we do not discuss details of the $SU(N)'$ model further.

²⁶It is tempting to make the $SU(2)'$ sector also be the IYIT supersymmetry breaking sector by introducing six singlet fields, Z' 's, instead. In this case, however, supersymmetry and the gauged PQ symmetry are broken by the dynamics, while the global PQ symmetry is broken separately.

5 Conclusions

In this paper, we apply the gauged PQ mechanism to a model in which the global PQ symmetry and supersymmetry are broken simultaneously. As a concrete example, we considered models which consist of simultaneous supersymmetry/PQ symmetry breaking sector based on $SU(2)$ dynamics (the IYIT sector) and a dynamical PQ symmetry breaking sector based on $SU(N)$ dynamics (the $SU(N)'$ sector). As we have seen, the $SU(3)'$ model is particularly successful where the gauged PQ symmetry is mainly broken by the $SU(3)'$ sector while both the accidental global PQ symmetry and supersymmetry are broken by the IYIT sector. Thus, the model explains the coincidence between the favored values of the supersymmetry breaking scale and the PQ breaking scale.

Besides the model with the minimum charge choice, $q_1 = 5$ and $q_2 = 1$, predicts the KSVZ extra multiplets $(\mathbf{5}', \bar{\mathbf{5}}')$ in the TeV range due to a tight upper limit on Λ' . It should be noted that the light KSVZ fermions are due to composite nature of the PQ breaking field with a tight upper limit on Λ' in the minimal model. The model also predicts non-vanishing effective θ angle. Thus, the model with the minimum charge choice can be tested by combining the searches for vector-like colored particles at the LHC experiments and future measurements of the neutron EDM.

Finally, let us comment an advantage of the gauged PQ mechanism over the models in which the high-quality global PQ symmetry results from an exact discrete symmetry, such as $\mathbb{Z}_N$. As we have discussed briefly in subsection 2.3, the gauged PQ mechanism with $N_{\text{GCD}} = 1$, $q_1 = 1$ and $q_2 = N(> 1)$ allow models which are free from both the domain wall problem and the axion isocurvature problem. The assumption here is that the first stage of the phase transition (i.e. $\langle \Phi_1 \rangle \neq 0$) takes place before inflation while the second stage of the phase transition (i.e. $\langle \Phi_2 \rangle \neq 0$) occurs after inflation. Then, the local strings formed at the first phase transition are inflated away, while the global strings formed at the second phase transition do not cause the domain wall problem as Φ_2 couples to only one-flavor of the KSVZ extra multiplet.²⁷ As the global PQ symmetry is broken after inflation, the model does not suffer from the axion isocurvature problem. This option is not available in the models with an exact discrete symmetry where the axion potential is also symmetric under the discrete symmetry.

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²⁷In this case, the axion dark matter density is dominated by the axions produced by the decay of the string-domain wall networks, which requires $F_a = \mathcal{O}(10^{11})$ GeV [35, 36]. Such a rather low F_a is, for example, achieved in the $SU(2)'$ model.

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