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Predicting the sparticle spectrum from GUTs via SUSY threshold corrections with SusyTC

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Abstract: Grand Unified Theories (GUTs) can feature predictions for the ratios of quark and lepton Yukawa couplings at high energy, which can be tested with the increasingly precise results for the fermion masses, given at low energies. To perform such tests, the renormalization group (RG) running has to be performed with sufficient accuracy. In supersymmetric (SUSY) theories, the one-loop threshold corrections (TC) are of particular importance and, since they affect the quark-lepton mass relations, link a given GUT flavour model to the sparticle spectrum. To accurately study such predictions, we extend and generalize various formulas in the literature which are needed for a precision analysis of SUSY flavour GUT models. We introduce the new software tool `SusyTC`, a major extension to the Mathematica package `REAP` [1], where these formulas are implemented. `SusyTC` extends the functionality of `REAP` by a full inclusion of the (complex) MSSM SUSY sector and a careful calculation of the one-loop SUSY threshold corrections for the full down-type quark, up-type quark and charged lepton Yukawa coupling matrices in the electroweak-unbroken phase. Among other useful features, `SusyTC` calculates the one-loop corrected pole mass of the charged (or the CP-odd) Higgs boson as well as provides output in SLHA conventions, i.e. the necessary input for external software, e.g. for performing a two-loop Higgs mass calculation. We apply `SusyTC` to study the predictions for the parameters of the CMSSM (mSUGRA) SUSY scenario from the set of GUT scale Yukawa relations $\frac{y_e}{y_d} = -\frac{1}{2}$, $\frac{y_\mu}{y_s} = 6$, and $\frac{y_\tau}{y_b} = -\frac{3}{2}$, which has been proposed recently in the context of SUSY GUT flavour models.

Keywords: Beyond Standard Model, GUT, Renormalization Group, Supersymmetric Standard Model

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Contents

1 Introduction

Supersymmetric (SUSY) Grand Unified Theory (GUT) models of flavour are promising candidates towards solving the open questions of the Standard Model (SM) of Particle Physics. They embrace the unification of the SM gauge couplings, a dark matter candidate, a solution to the gauge hierarchy problem and an explanation of the hierarchies of masses and mixing angles in the flavour sector. Whether a flavour GUT model can successfully explain the observations in the flavour sector, depends on the renormalization group (RG) evolution of the Yukawa matrices from the GUT scale to lower energies. Furthermore, it is known that $\tan\beta$ enhanced supersymmetric threshold corrections (see e.g. [2–6]) are essential in the investigation of mass (or Yukawa coupling) ratios predicted at the GUT scale. Interesting well-known GUT predictions in this context are $b-\tau$ and $b-\tau-t$ unification (for early work see e.g. [7–10]) and $y_\mu = 3y_s$ [11].[1] Other promising quark-lepton mass relations at the GUT scale have been discussed in [12, 13], e.g. $y_\tau = \pm\frac{3}{2}y_b$, $y_\mu = 6y_s$ or $y_\mu = \frac{9}{2}y_s$. Various aspects regarding the impact of such GUT relations for phenomenology have been studied in the literature, see e.g. [14–31] for recent works.

[1] y_i are the Yukawa couplings in the diagonal basis.

With the discovery of the Higgs boson at the LHC [32, 33] and the possible discovery of sparticles in the near future, the question whether a set of SUSY soft-breaking parameters can be in agreement with both, specific SUSY threshold corrections as required for realizing the flavour structure of a GUT model, and constraints from the Higgs boson mass and results on the sparticle spectrum, gains importance. To accurately study this question, we introduce the new software tool `SusyTC`, a major extension to the Mathematica package `REAP` [1].

`REAP`, which is designed to run the SM and neutrino parameters in seesaw scenarios with a proper treatment of the right-handed neutrino thresholds, is a convenient tool for top-down analyses of flavour (GUT) models, with the advantage of a user-friendly Wolfram Mathematica front-end. However, the SUSY sector is not included. In order to take supersymmetric threshold corrections into account in the analyses, for example of the A_4 flavour GUT models in [34, 35], the following procedure was undertaken: first `REAP` was used to run the Yukawa matrices in the MSSM from the GUT-scale to a user-defined "SUSY"-scale. At this scale, the SUSY threshold corrections were incorporated as mere model parameters, in a simplified treatment assuming, e.g., degenerate first and second generation sparticle masses (cf. [36]), without specializing any details on the SUSY sector. Finally, the Yukawa matrices, corrected by these $\tan\beta$-enhanced thresholds, were taken as input for a second run of `REAP`, evolving the parameters in the SM from the "SUSY"-scale to the top-mass scale.[2] Although this procedure is quite SUSY model-independent, it only allows to study the constraints on the SUSY sector indirectly (i.e. via the introduced additional parameters and with simplifying assumptions), and it is unclear whether an explicit SUSY scenario with these assumptions and requirements can be realised.

The aim of this work is to make full use of the SUSY threshold corrections to gain information on the SUSY model parameters from GUTs. Towards this goal we extend and generalize various formulas in the literature which are needed for a precision analysis of SUSY flavour GUT models and implement them in `SusyTC`. For example, `SusyTC` includes the full (CP violating) MSSM SUSY sector, sparticle spectrum calculation, a careful calculation of the one-loop SUSY threshold corrections for the full down-type quark, up-type quark and charged lepton Yukawa coupling matrices in the electroweak-unbroken phase, and automatically performs the matching of the MSSM to the SM, including $\overline{\text{DR}}$ to $\overline{\text{MS}}$ conversion. Among other useful features, `SusyTC` calculates the one-loop corrected pole mass of the charged (or the CP-odd) Higgs boson as well as provides output in SLHA conventions, i.e. the necessary input for external software, e.g. for performing a two-loop Higgs mass calculation.

`SusyTC` is specifically developed to perform top-down analyses of SUSY flavour GUT models. This is a major difference to other well-known SUSY spectrum generators (e.g. [37–41], see e.g. [42] for a comparison), which run experimental constraints from low energies to high energies, apply GUT-scale boundary conditions, run back to low energies and repeat this procedure iteratively. `SusyTC` instead starts directly from the GUT-scale, allowing the

[2]Such a treatment of threshold corrections as additional model parameters is now implemented in the latest version of `RGEMSSMON.m` of `REAP` 11.1.2.

user to define general (complex) Yukawa, trilinear, and soft-breaking matrices, as well as non-universal gaugino masses, as input. These parameters are then run to low energies, thereby enabling an investigation whether the GUT-scale Yukawa matrix structures of a given SUSY flavour GUT model are in agreement with experimental data.

We apply `SusyTC` to study the predictions for the parameters of the Constrained MSSM (mSUGRA) SUSY scenario from the GUT-scale Yukawa relations $\frac{y_e}{y_d} = -\frac{1}{2}$, $\frac{y_\mu}{y_s} = 6$, and $\frac{y_\tau}{y_b} = -\frac{3}{2}$, which have been proposed recently in the context of SUSY GUT flavour models. With a Markov Chain Monte Carlo analysis we find a "best-fit" benchmark point as well as the 1σ ranges for the sparticle masses and the correlations between the SUSY parameters. Without applying any constraints from LHC SUSY searches or dark matter, we find that the considered GUT scenario predicts a CMSSM sparticle spectrum above past LHC sensitivities, but within reach of the current LHC run or a future high-luminosity upgrade. Furthermore, the scenario generically features a bino-like neutralino LSP and a stop NLSP with a mass that can be close to the present bounds.

This paper is organized as follows: in section 2 we review GUT predictions for Yukawa coupling ratios. In section 3, we describe the numerical procedure in `SusyTC` and present the main used formulas. We give a short introduction to the new features `SusyTC` adds to `REAP` in section 4. In section 5 we study the predictions for the parameters of the CMSSM SUSY scenario from the above mentioned GUT-scale Yukawa relations with `SusyTC`. In the appendices we present other relevant formulas and a detailed documentation of `SusyTC`.

2 Predictions for Yukawa coupling ratios from GUTs

GUTs not only contain a unification of the SM forces, they also unify fermions into joint representations. After the GUT gauge group is broken to the SM gauge group, this can lead to predictions for the ratios of down-type quark and charged lepton Yukawa couplings which result from group theoretical Clebsch Gordan (CG) factors. To confront such predictions of GUT models with the experimental data, the RG evolution of the Yukawa couplings from high to low energies has to be performed, including (SUSY) threshold corrections.

In SU(5) GUTs, for example, the right-handed down-type quarks and the lepton doublets are unified in five-dimensional representations of SU(5) and the quark doublets plus right-handed up-type quarks and right-handed charged leptons are unified in a ten-dimensional SU(5) representation. The Higgs doublets are supplemented by $\mathrm{SU}(3)_c$ triplets and embedded into five-dimensional representations of SU(5). Using only these fields and a single renormalizable operator to generate the Yukawa couplings for the down-type quarks and charged leptons, so-called minimal SU(5) predicts $Y_e = Y_d^T$ for the Yukawa matrices at the GUT scale. To correct this experimentally challenged scenario, SU(5) GUT flavour models often introduce a 45-dimensional Higgs representation, which can lead to the Georgi-Jarlskog relations $y_\mu = -3y_s$ and $y_e = \frac{1}{3}y_d$ [11].

It was pointed out in [12] (see also [13]) that other promising Yukawa coupling GUT ratios can emerge in SU(5), e.g. $y_\tau = \pm\frac{3}{2}y_b$, $y_\mu = 6y_s$ or $y_\mu = \frac{9}{2}y_s$, and $y_e = -\frac{1}{2}y_d$ from higher dimensional GUT operators containing for instance a GUT breaking 24-dimensional Higgs representation. A convenient test whether GUT predictions for the first two families can be consistent with the experimental data is provided by the — RG invariant and SUSY threshold correction invariant[3] — double ratio [36]

$$\left|\frac{y_\mu}{y_s}\frac{y_d}{y_e}\right| = 10.7^{+1.8}_{-0.8}\,. \tag{2.1}$$

While the Georgi-Jarlskog relations [11] imply a double ratio of 9, disfavoured by more than 2σ, other combinations of CG factors [12, 13], e.g. $y_\mu = 6y_s$ and $y_e = -\frac{1}{2}y_d$ can be in better agreement (here with a double ratio of 12, within the 1σ experimental range).

The combination of GUT-scale Yukawa relations $y_e = -\frac{1}{2}y_d$, $y_\mu = 6y_s$, and $y_\tau = -\frac{3}{2}y_b$ (as direct result of CG factors, cf. table 1 and figure 1, or as approximate relation after diagonalization of the GUT-scale Yukawa matrices Y_d and Y_e) has been used to construct SU(5) SUSY GUT flavour models in refs. [34, 35, 43, 44]. A subset of these relation, $y_\mu = 6y_s$, and $y_\tau = -\frac{3}{2}y_b$, has been used in [45]. In addition to providing viable quark and charged lepton masses, the GUT CG factors $(Y_e)_{ji}/(Y_d)_{ij} = -\frac{1}{2}$ and 6 can also be applied to realize the promising relation between the lepton mixing angle $\theta_{13}^{\text{PMNS}}$ and the Cabibbo angle, $\theta_{13}^{\text{PMNS}} \simeq \theta_C \sin\theta_{23}^{\text{PMNS}}$, in flavour models, as discussed in [46].

As mentioned above, in supersymmetric GUT models the SUSY threshold corrections can have an important influence on the Yukawa coupling ratios. When the MSSM is matched to the SM, integrating out the sparticles at loop-level leads to the emergence of effective operators, which can contribute sizeably to the Yukawa couplings, depending on the values of the sparticle masses, $\tan\beta$, and the soft-breaking trilinear couplings. Thereby, via the SUSY threshold corrections, a given set of GUT predictions for the ratios $\frac{y_\tau}{y_b}$, $\frac{y_\mu}{y_s}$ and $\frac{y_e}{y_d}$ imposes important constraints on the SUSY spectrum.

The SUSY threshold corrections can be subdivided into three classes: while at tree-level the down-type quarks only couple to the Higgs field H_d, via exchange of sparticles at one-loop level they can also couple to H_u, as shown in figure 2 in section 3. When the sparticles are integrated out the emerging effective operator is enhanced for large $\tan\beta$ (i.e. "$\tan\beta$-enhanced"). Analogously, there are also $\tan\beta$-enhanced threshold corrections to the charged lepton Yukawa couplings. For Y_u, however, the threshold effects emerging from effective couplings to H_d are $\tan\beta$-suppressed. The second class of threshold corrections emerges from the supersymmetric loops shown in figure 3. While some of them are strongly suppressed, others lead to the emergence of effective operators proportional to the soft SUSY-breaking trilinear couplings. For large trilinear couplings, they too can become important. Finally there are threshold corrections from exchange of heavy Higgs doublets and canonical normalization of external fields (i.e. from wavefunction renormalization diagrams), as shown in figures 4 and 5–7, respectively. Given the importance of the SUSY threshold corrections, we will discuss them and their implementation in `SusyTC` in detail in the next section.

[3]The invariance under SUSY threhold corrections holds under some generic conditions, cf. [36].

$A\,B$	$C\,D$	R	$(Y_e)_{ji}/(Y_d)_{ij}$
$H_{24}\,F$	$T\,\bar{H}_{45}$	$\overline{\mathbf{45}}$	$-\frac{1}{2}$
$H_{24}\,F$	$T\,\bar{H}_{5}$	$\bar{\mathbf{5}}$	$-\frac{3}{2}$
$H_{24}\,T$	$F\,\bar{H}_{5}$	$\mathbf{10}$	6

Table 1. Examples for dimension 5 effective operators $(AB)_R(CD)_{\bar{R}}$ and CG factors emerging from the supergraphs of figure 1 when R and $\bar{R}$ are integrated out [12].

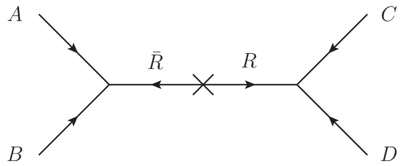

Figure 1. Supergraphs corresponding to the operators of table 1 generating effectively Yukawa couplings when the pair of messengers fields R and $\bar{R}$ is integrated out.

3 SUSY threshold corrections & numerical procedure

We follow the notation of `REAP` [1] (see also [47]) and use a RL convention for the Yukawa matrices. The MSSM superpotential extended by a type-I seesaw mechanism [48–54] is thus given by

$$\begin{aligned} W_{\text{MSSM}} &= Y_{e_{ij}} E^c_i H_d \cdot L_j + Y_{\nu_{ij}} N^c_i H_u \cdot L_j \\ &\quad + Y_{d_{ij}} D^c_i H_d \cdot Q_j - Y_{u_{ij}} U^c_i H_u \cdot Q_j + \frac{1}{2} M_{n_{ij}} N^c_i N^c_j + \mu H_u \cdot H_d \,, \end{aligned} \tag{3.1}$$

where the left-chiral superfields Φ^c contain the charge conjugated fields $\psi^\dagger$ and $\tilde{\phi}^*_R$. We use the totally antisymmetric SU(2) tensor $\epsilon_{12} = -\epsilon_{21} = 1$ for the product $\Phi \cdot \Psi \equiv \epsilon_{ab} \Phi^a \Psi^b$. The soft-breaking Lagrangian is given by

$$\begin{aligned} -\mathcal{L}_{\text{soft}} &= -\frac{1}{2} M_a \lambda^a \lambda^a + h.c. \\ &\quad + T_{e_{ij}} \tilde{e}^*_{R_i} H_d \cdot \tilde{L}_j + T_{\nu_{ij}} \tilde{\nu}^*_{R_i} H_u \cdot \tilde{L}_j + T_{d_{ij}} \tilde{d}^*_{R_i} H_d \cdot \tilde{Q}_j - T_{u_{ij}} \tilde{u}^*_{R_i} H_u \cdot \tilde{Q}_j + h.c. \\ &\quad + \tilde{Q}^\dagger_i m^2_{\tilde{Q}_{ij}} \tilde{Q}_j + \tilde{L}^\dagger_i m^2_{\tilde{L}_{ij}} \tilde{L}_j + \tilde{u}^*_{R_i} m^2_{\tilde{u}_{ij}} \tilde{u}_{R_j} + \tilde{d}^*_{R_i} m^2_{\tilde{d}_{ij}} \tilde{d}_{R_j} + \tilde{e}^*_{R_i} m^2_{\tilde{e}_{ij}} \tilde{e}_{R_j} + \tilde{\nu}^*_{R_i} m^2_{\tilde{\nu}_{ij}} \tilde{\nu}_{R_j} \\ &\quad + m^2_{H_u} |H_u|^2 + m^2_{H_d} |H_d|^2 + (m^2_3 H_u \cdot H_d + h.c.) \,. \end{aligned} \tag{3.2}$$

Note that these conventions differ from SUSY Les Houches Accord 2 [55]. They can easily be translated by

`REAP` & `SusyTC`	Y_u	Y_d	Y_e	T_u	T_d	T_e	$m^2_{\tilde{u}}$	$m^2_{\tilde{d}}$	$m^2_{\tilde{e}}$
SLHA 2	Y^T_u	Y^T_d	Y^T_e	T^T_u	T^T_d	T^T_e	$(m^2_{\tilde{u}})^T$	$(m^2_{\tilde{d}})^T$	$(m^2_{\tilde{e}})^T$

Since `REAP` includes the RG running in the type-I seesaw extension of the MSSM (with the $\overline{\text{DR}}$ two-loop β-functions for the MSSM parameters and the neutrino mass operator given in [47]), we have calculated the $\overline{\text{DR}}$ two-loop β-functions of the gaugino mass parameters M_a, the trilinear couplings T_f, the sfermion squared mass matrices $m^2_{\tilde f}$, and soft-breaking Higgs mass parameters $m^2_{H_u}$ and $m^2_{H_d}$ in the presence of Y_ν, M_n and $m^2_{\tilde\nu}$ (using the general formulas of [56]). We list these β-functions in appendix A.

The Yukawa matrices and soft-breaking parameters are evolved to the SUSY scale

$$Q = \sqrt{m_{\tilde t_1} m_{\tilde t_2}}\ , \tag{3.3}$$

where the stop masses are defined by the up-type squark mass eigenstates $\tilde u_i$ with the largest mixing to $\tilde t_1$ and $\tilde t_2$.[4] `REAP` automatically integrates out the right-handed neutrinos, as described in [1]. We assume that M_n is much larger than the SUSY scale Q. `REAP` also features the possibility to add one-loop right-handed neutrino thresholds for the SM parameters, following [57].

At the SUSY scale Q the tree-level sparticle masses and mixings are calculated. Considering heavy sparticles and large $Q \gtrsim$ TeV the SUSY threshold corrections are calculated in the electroweak (EW) unbroken phase. In the EW unbroken phase there are in total twelve types of loop diagrams contributing to the SUSY threshold corrections for Y_d (cf. figures 2 and 3).

The SUSY threshold corrections to Y_d are calculated in the basis of diagonal squark masses and are given by (in terms of $\overline{\text{DR}}$ quantities)

$$\tilde Y_d^{\text{SM}} = P_d \tilde Y_d' P_Q^T P_h\ , \tag{3.4}$$

where P_d, P_Q, and P_h are the threshold corrections (3.12), (3.19), (3.20), and (3.22) due to canonical normalization. $\tilde Y'_{d_{ij}}$ is given by

$$\begin{aligned}\tilde Y'_{d_{ij}} = \tilde Y^{\text{MSSM}}_{d_{ij}} \cos\beta \left(1 + \frac{1}{16\pi^2}\tan\beta\left(\eta^G_{ij} + \eta^W_{ij} + \eta^B_{ij} + \eta^T_{ij}\right) + \frac{1}{16\pi^2}\left(\epsilon^W_{ij} + \epsilon^B_{ij} + \epsilon^T_{ij} + \epsilon^H_{ij}\right)\right) \\ + \tilde T^{\text{MSSM}}_{d_{ij}} \cos\beta \frac{1}{16\pi^2}\left(\zeta^G_{ij} + \zeta^B_{ij}\right)\ ,\end{aligned} \tag{3.5}$$

where

$$\begin{aligned}\eta^G_{ij} &= -\frac{8}{3} g_3^2 \frac{\mu^*}{M_3} H_2\left(\frac{m^2_{\tilde d_i}}{|M_3|^2}, \frac{m^2_{\tilde Q_j}}{|M_3|^2}\right)\ ,\\ \eta^W_{ij} &= \frac{3}{2} g_2^2 \frac{M_2^*}{\mu} H_2\left(\frac{M_2^2}{|\mu|^2}, \frac{m^2_{\tilde Q_j}}{|\mu|^2}\right)\ ,\\ \eta^B_{ij} &= \frac{3}{5} g_1^2 \left(\frac{1}{9}\frac{\mu^*}{M_1} H_2\left(\frac{m^2_{\tilde d_i}}{|M_1|^2}, \frac{m^2_{\tilde Q_j}}{|M_1|^2}\right) + \frac{1}{3}\frac{M_1^*}{\mu} H_2\left(\frac{m^2_{\tilde d_i}}{|\mu|^2}, \frac{|M_1|^2}{|\mu|^2}\right) + \frac{1}{6}\frac{M_1^*}{\mu} H_2\left(\frac{|M_1|^2}{|\mu|^2}, \frac{m^2_{\tilde Q_j}}{|\mu|^2}\right)\right)\ ,\\ \eta^T_{ij} &= -\frac{1}{\mu \tilde Y_{d_{ij}}} \sum_{n,m} \tilde Y_{d_{im}} \tilde T^\dagger_{u_{mn}} \tilde Y_{u_{nj}} H_2\left(\frac{m^2_{\tilde u_n}}{|\mu|^2}, \frac{m^2_{\tilde Q_m}}{|\mu|^2}\right)\ ,\end{aligned} \tag{3.6}$$

[4] `SusyTC` can also be set to use the convention $Q = \sqrt{m_{\tilde u_1} m_{\tilde u_6}}$ or a user-defined SUSY scale, as described in appendix C.

correspond to the tan β-enhanced loops of figure 2, and

$$\begin{aligned}
\epsilon^W_{ij} &= 12 g_2^2 C_{00}\left(\frac{|\mu|^2}{Q^2}, \frac{|M_2|^2}{|\mu|^2}, \frac{m^2_{\tilde{Q}_j}}{|\mu|^2}\right) , \\
\epsilon^B_{ij} &= \frac{3}{5} g_1^2 \left(\frac{4}{3} C_{00}\left(\frac{|\mu|^2}{Q^2}, \frac{|M_1|^2}{|\mu|^2}, \frac{m^2_{\tilde{d}_i}}{|\mu|^2}\right) + \frac{2}{3} C_{00}\left(\frac{|\mu|^2}{Q^2}, \frac{|M_1|^2}{|\mu|^2}, \frac{m^2_{\tilde{Q}_j}}{|\mu|^2}\right)\right) , \\
\epsilon^T_{ij} &= \frac{1}{\tilde{Y}_{d_{ij}}} \sum_{n,m} \tilde{Y}_{d_{im}} \tilde{Y}^\dagger_{u_{mn}} \tilde{Y}_{u_{nj}} H_2\left(\frac{m^2_{\tilde{u}_n}}{|\mu|^2}, \frac{m^2_{\tilde{Q}_m}}{|\mu|^2}\right) , \\
\epsilon^H_{ij} &= \frac{2\sin^2\beta}{\tilde{Y}_{d_{ij}}} \sum_{n,m} \tilde{Y}_{d_{im}} \tilde{Y}^\dagger_{u_{mn}} \tilde{Y}_{u_{nj}} B_1\left(\frac{m^2_H}{Q^2}\right) , \\
\zeta^G_{ij} &= \frac{8}{3} g_3^2 \frac{1}{M_3} H_2\left(\frac{m^2_{\tilde{d}_i}}{|M_3|^2}, \frac{m^2_{\tilde{Q}_j}}{|M_3|^2}\right) , \\
\zeta^B_{ij} &= -\frac{3}{5} g_1^2 \frac{1}{9}\frac{1}{M_1} H_2\left(\frac{m^2_{\tilde{d}_i}}{|M_1|^2}, \frac{m^2_{\tilde{Q}_j}}{|M_1|^2}\right) ,
\end{aligned} \tag{3.7}$$

correspond to the loops in figures 3 and 4, respectively, where the contributions ζ^G_{ij} and ζ^B_{ij} can become important in cases of small tan β and large trilinear couplings. The loop functions H_2, C_{00}, and B_1 are defined as

$$H_2(x, y) \equiv \frac{x\log(x)}{(1-x)(x-y)} + \frac{y\log(y)}{(1-y)(y-x)} , \tag{3.8}$$

$$C_{00}(q, x, y) \equiv \frac{1}{4}\left(\frac{3}{2} - \log(q) + \frac{x^2\log(x)}{(1-x)(x-y)} + \frac{y^2\log(y)}{(1-y)(y-x)}\right) , \tag{3.9}$$

$$B_1(q) \equiv -\frac{3}{4} + \frac{1}{2}\log q . \tag{3.10}$$

$\tilde{Y}$, $\tilde{T}$ are the Yukawa- and trilinear coupling matrices rotated into the basis where the squark mass matrices are diagonal, using the transformations

$$\begin{aligned}
Y_u &= \tilde{W}_{\tilde{u}} \tilde{Y}_u \tilde{W}^\dagger_{\tilde{Q}} , \\
T_u &= \tilde{W}_{\tilde{u}} \tilde{T}_u \tilde{W}^\dagger_{\tilde{Q}} , \\
m^2_{\tilde{Q}} &= \tilde{W}_{\tilde{Q}} m^{2\ \mathrm{diag}}_{\tilde{Q}} \tilde{W}^\dagger_{\tilde{Q}} , \\
m^2_{\tilde{u}} &= \tilde{W}_{\tilde{u}} m^{2\ \mathrm{diag}}_{\tilde{u}} \tilde{W}^\dagger_{\tilde{u}} ,
\end{aligned} \tag{3.11}$$

and analogously for down-type (s)quarks and charged (s)leptons.

The threshold correction P_h due to canonical normalization of external Higgs doublets is given by

$$P_h = \sqrt{(U \mathcal{K}_h U^\dagger)^{-1}_{11}} \tag{3.12}$$

with U defined by

$$\begin{pmatrix} h^a \\ H^a \end{pmatrix} \equiv U \begin{pmatrix} -\epsilon_{ab} H^{a*}_d \\ H^a_u \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} -\epsilon_{ab} H^{a*}_d \\ H^a_u \end{pmatrix} , \tag{3.13}$$

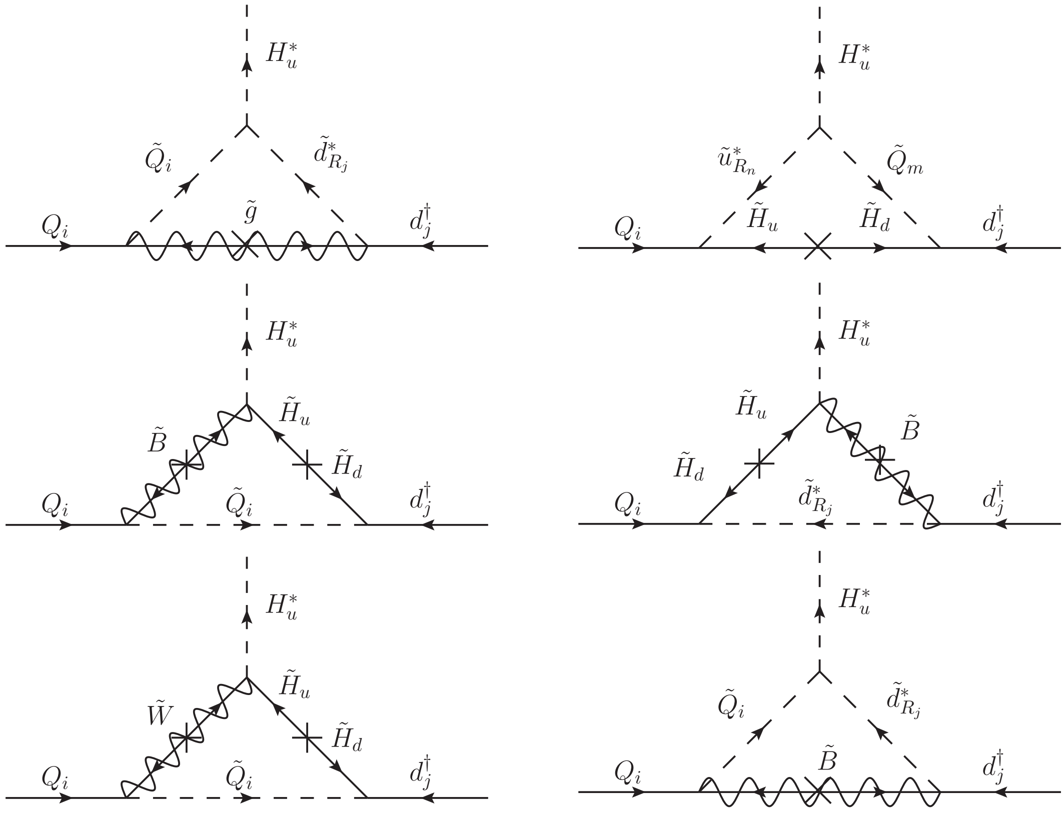

Figure 2. $\tan\beta$-enhanced SUSY threshold corrections to Y_d.

where h and H are the mass eigenstates doublets of the MSSM Higgs scalar fields and

$$\mathcal{K}_h \equiv \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} + \frac{1}{16\pi^2} \begin{pmatrix} X_{11}+Y_{11}+\tilde{Y}_{11}+A_{11}+B_{11} & X_{12}+Z_{12}+\tilde{Z}_{12}+A_{12}+B_{12} \\ X_{12}^*+Z_{12}^*+\tilde{Z}_{12}^*+A_{12}^*+B_{12}^* & X_{22}+Y_{22}+\tilde{X}_{22}+A_{11}+B_{11} \end{pmatrix} , \tag{3.14}$$

with

$$\begin{aligned}
X_{11} &= \sum_{i,j} |\mu|^2 |\tilde{Y}_{u_{ij}}|^2 B_2\left(m^2_{\tilde{u}_i}, m^2_{\tilde{Q}_j}\right) , \\
Y_{11} &= \sum_{i,j} |\tilde{T}_{d_{ij}}|^2 B_2\left(m^2_{\tilde{d}_i}, m^2_{\tilde{Q}_j}\right) , \\
\tilde{Y}_{11} &= \sum_{i,j} |\tilde{T}_{e_{ij}}|^2 B_2\left(m^2_{\tilde{e}_i}, m^2_{\tilde{L}_j}\right) , \\
X_{12} &= \sum_{i,j} -\mu \tilde{T}_{u_{ij}} \tilde{Y}^*_{u_{ij}} B_2\left(m^2_{\tilde{u}_i}, m^2_{\tilde{Q}_j}\right) , \\
Z_{12} &= \sum_{i,j} -\mu \tilde{T}_{d_{ij}} \tilde{Y}^*_{d_{ij}} B_2\left(m^2_{\tilde{d}_i}, m^2_{\tilde{Q}_j}\right) , \\
\tilde{Z}_{12} &= \sum_{i,j} -\mu \tilde{T}_{e_{ij}} \tilde{Y}^*_{e_{ij}} B_2\left(m^2_{\tilde{e}_i}, m^2_{\tilde{L}_j}\right) ,
\end{aligned}$$

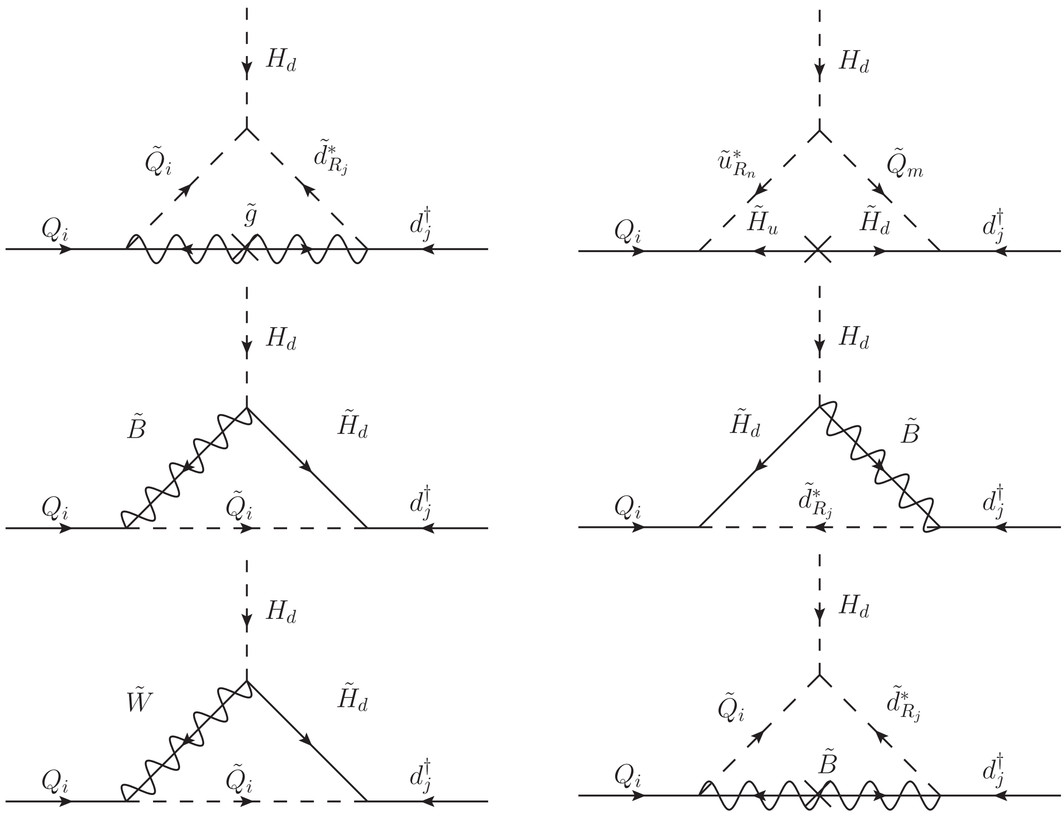

Figure 3. None $\tan\beta$-enhanced SUSY threshold corrections to Y_d.

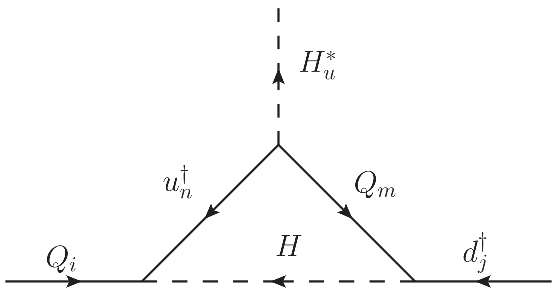

Figure 4. Additional diagram with heavy Higgs doublet exchanged.

$$
\begin{aligned}
X_{22} &= \sum_{i,j} |\mu|^2 |\tilde{Y}_{d_{ij}}|^2 B_2\left(m^2_{\tilde{d}_i}, m^2_{\tilde{Q}_j}\right) \ , \\
\tilde{X}_{22} &= \sum_{i,j} |\mu|^2 |\tilde{Y}_{e_{ij}}|^2 B_2\left(m^2_{\tilde{e}_i}, m^2_{\tilde{L}_j}\right) \ , \\
Y_{22} &= \sum_{i,j} |\tilde{T}_{u_{ij}}|^2 B_2\left(m^2_{\tilde{u}_i}, m^2_{\tilde{Q}_j}\right) \ , \\
A_{11} &= \frac{3}{5} g_1^2 \frac{1}{12\left(|\mu|^2 - |M_1|^2\right)^3} \tilde{B}_2\left(Q^2, |M_1|^2, |\mu|^2\right) \ ,
\end{aligned} \tag{3.15}
$$

$$
\begin{aligned}
B_{11} &= g_2^2 \frac{1}{4\left(|\mu|^2-|M_2|^2\right)^3} \tilde{B}_2\left(Q^2,|M_2|^2,|\mu|^2\right) \ , \\
A_{12} &= \frac{3}{5} g_1^2 \mu^* M_1^* B_2\left(|\mu|^2,|M_1|^2\right) \ , \\
B_{12} &= g_2^2 3 \mu^* M_2^* B_2\left(|\mu|^2,|M_2|^2\right) \ ,
\end{aligned} \tag{3.16}
$$

which correspond to the loops shown in figures 5 and 6. The loop functions B_2 and $\tilde{B}_2$ are given by

$$
B_2(x,y) \equiv \frac{x^2-y^2}{2(x-y)^3} + \frac{x\,y}{(x-y)^3}\log\left(\frac{x}{y}\right) \ , \tag{3.17}
$$

$$
\tilde{B}_2(q,x,y) \equiv 5\left(x^3-y^3\right)+27x\,y\,(y-x)+6y^2(y-3x)\log\left(\frac{y}{x}\right)+6(y-x)^3\log\left(\frac{x}{q}\right) \tag{3.18}
$$

The threshold corrections P_f due to canonical normalization of the external fermion fields are given by

$$
P_f \equiv \sqrt{\left(\mathcal{K}_f^{\text{diag}}\right)^{-1}} U_f \quad \text{with} \quad \mathcal{K}_f^{\text{diag}} = U_f \mathcal{K}_f U_f^\dagger \ . \tag{3.19}
$$

For the threshold corrections of the down-type Yukawa matrix the $\mathcal{K}_f$ are given by

$$
\mathcal{K}_{Q_{ij}} = \delta_{ij} + \frac{1}{16\pi^2}\left(\left(\rho_3+\rho_2+\rho_1\right)\delta_{ij} + \rho^d_{ij} + \tilde{\rho}^d_{ij} + \rho^u_{ij} + \tilde{\rho}^u_{ij}\right) \ , \tag{3.20}
$$

with

$$
\begin{aligned}
\rho_3 &= \frac{8}{3} g_3^2 B_3\left(Q^2,|M_3|^2,m^2_{\tilde{Q}_j}\right) \ , \\
\rho_2 &= \frac{3}{2} g_2^2 B_3\left(Q^2,|M_2|^2,m^2_{\tilde{Q}_j}\right) \ , \\
\rho_1 &= \frac{1}{18}\frac{3}{5} g_1^2 B_3\left(Q^2,|M_1|^2,m^2_{\tilde{Q}_j}\right) \ , \\
\rho^d_{ij} &= \sum_k \tilde{Y}^\dagger_{d_{jk}} \tilde{Y}_{d_{ki}} B_3\left(Q^2,|\mu|^2,m^2_{\tilde{d}_k}\right) \ , \\
\tilde{\rho}^d_{ij} &= \sum_k \tilde{Y}^\dagger_{d_{jk}} \tilde{Y}_{d_{ki}} \tilde{B}_1\left(\frac{m_H^2}{Q^2}\right)\sin^2\beta \ , \\
\rho^u_{ij} &= \sum_k \tilde{Y}^\dagger_{u_{jk}} \tilde{Y}_{u_{ki}} B_3\left(Q^2,|\mu|^2,m^2_{\tilde{u}_k}\right) \ , \\
\tilde{\rho}^u_{ij} &= \sum_k \tilde{Y}^\dagger_{u_{jk}} \tilde{Y}_{u_{ki}} \tilde{B}_1\left(\frac{m_H^2}{Q^2}\right)\cos^2\beta \ ,
\end{aligned} \tag{3.21}
$$

corresponding to the loops in figure 7, and

$$
\mathcal{K}_{d_{ij}} = \delta_{ij} + \frac{1}{16\pi^2}\left(\left(\chi_3+\chi_1\right)\delta_{ij} + \chi^d_{ij} + \tilde{\chi}^d_{ij}\right) \ , \tag{3.22}
$$

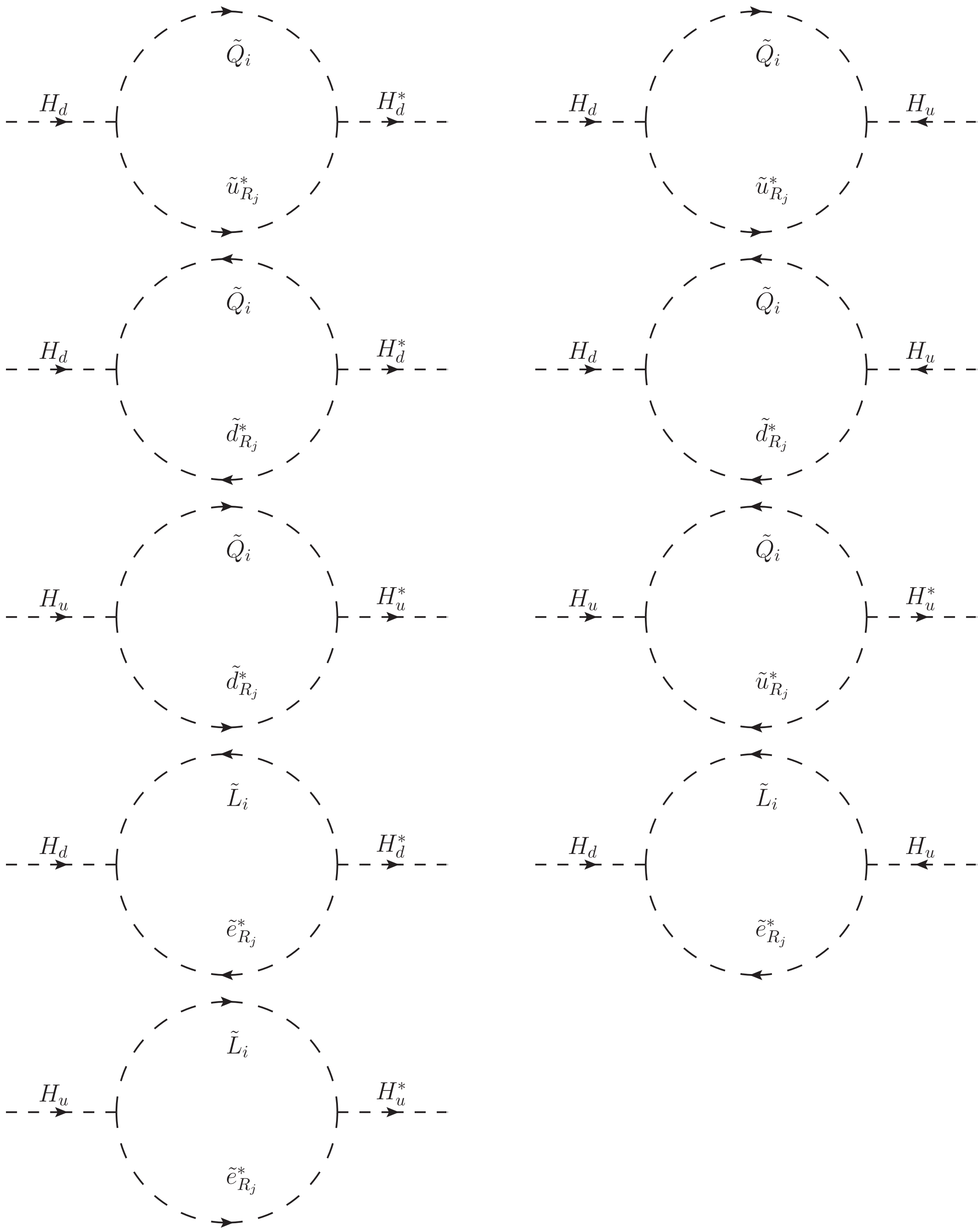

Figure 5. Scalar loops contributing to threshold corrections from canonical normalization of external Higgs fields.

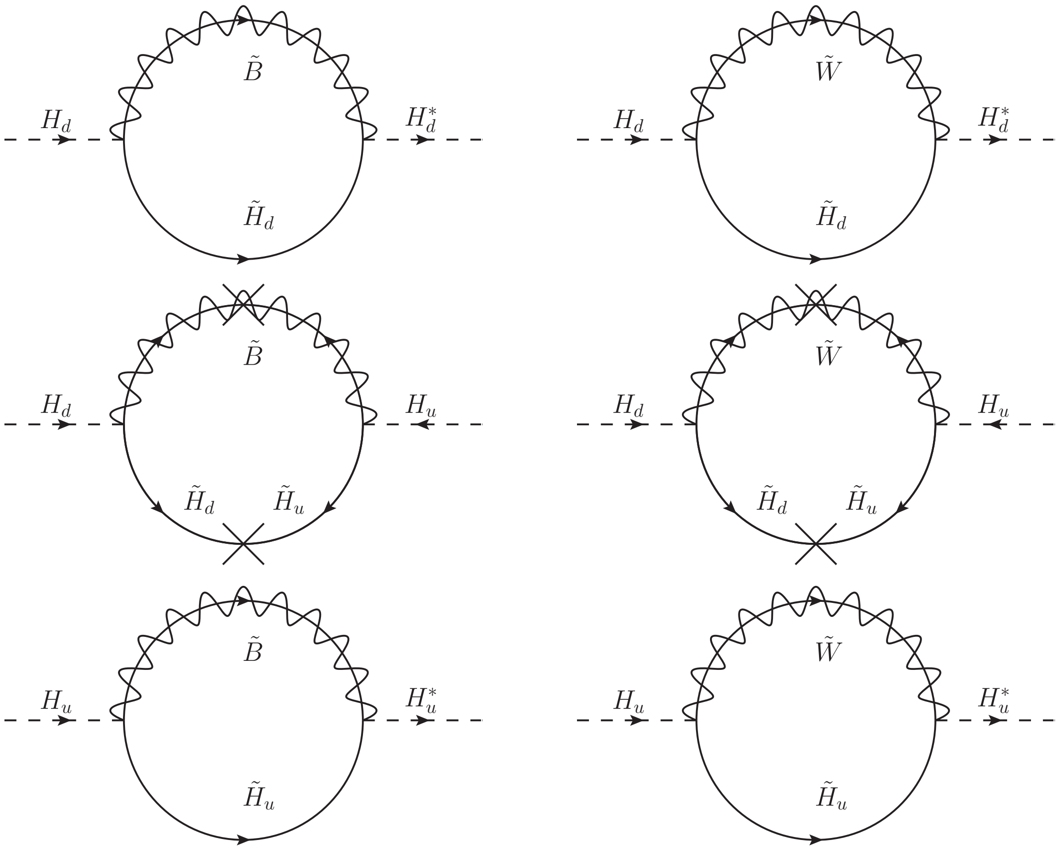

Figure 6. Fermion loops contributing to threshold corrections from canonical normalization of external Higgs fields.

with

$$
\begin{aligned}
\chi_3 &= \frac{8}{3} g_3^2 \left(Q^2, |M_3|^2, m_{\tilde{d}_j}^2 \right) \ , \\
\chi_1 &= \frac{2}{9}\frac{3}{5} g_1^2 \left(Q^2, |M_1|^2, m_{\tilde{d}_j}^2 \right) \ , \\
\chi_{ij}^d &= \sum_k 2\, \tilde{Y}_{d_{ik}} \tilde{Y}_{d_{kj}}^\dagger B_3 \left(Q^2, |\mu|^2, m_{\tilde{Q}_k}^2 \right) \ , \\
\tilde{\chi}_{ij}^d &= \sum_k 2\, \tilde{Y}_{d_{ik}} \tilde{Y}_{d_{kj}}^\dagger \tilde{B}_1 \left(\frac{m_H^2}{Q^2} \right) \sin^2\beta \ .
\end{aligned}
\tag{3.23}
$$

The diagrams for $\mathcal{K}_d$ are similar to the ones for $\mathcal{K}_Q$, when Q is replaced with d, with the exception of the wino loop, which does not exist for external right-handed fermions. The loop-functions $\tilde{B}_1$ and B_3 are given by

$$\tilde{B}_1(q) \equiv -\frac{1}{4} + \frac{1}{2}\log(q) \ , \tag{3.24}$$

$$B_3(q,x,y) \equiv -\frac{1}{4} - \frac{x}{2(x-y)} + \frac{x^2}{2(x-y)^2}\log\left(\frac{x}{y}\right) + \frac{1}{2}\log\left(\frac{y}{q}\right) \ . \tag{3.25}$$

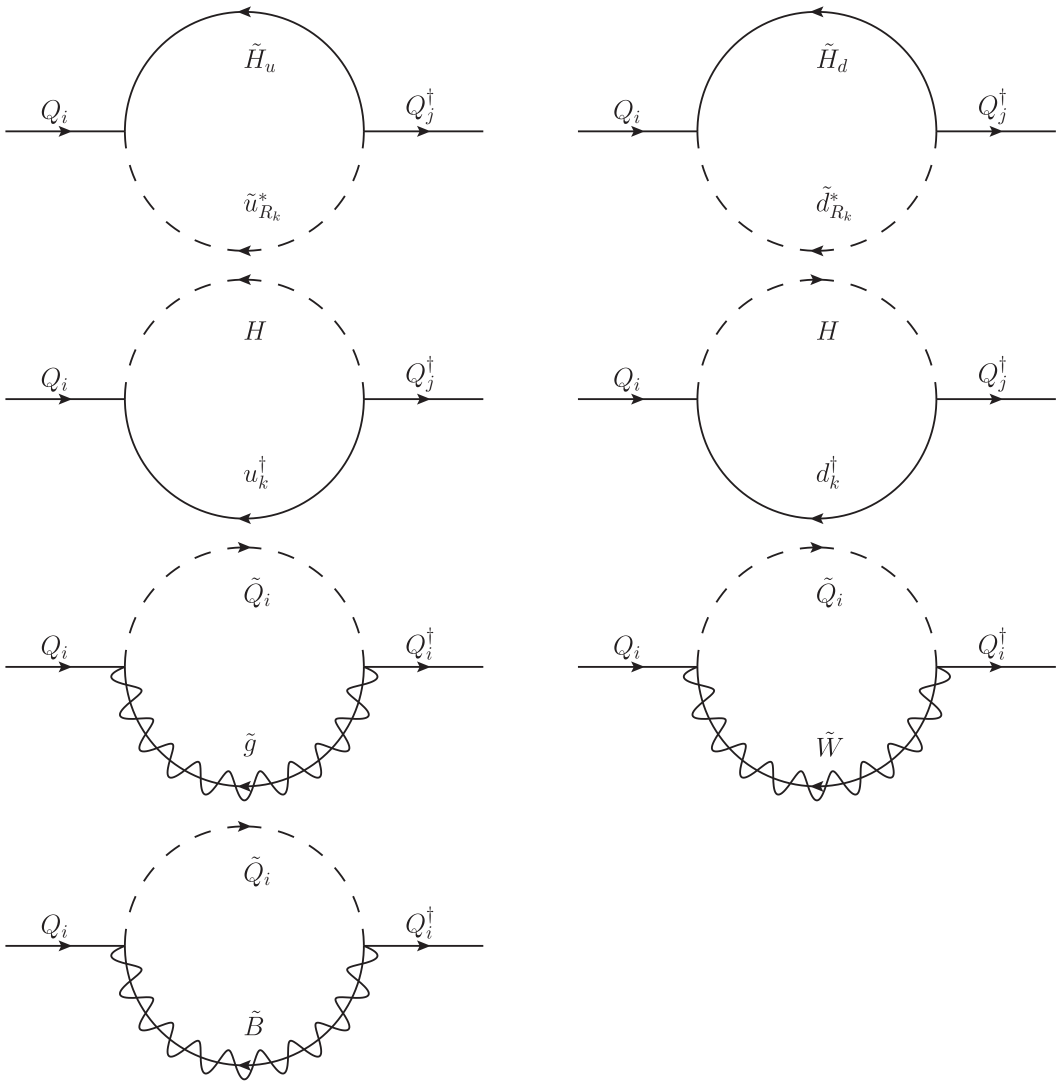

Figure 7. Loops contributing to threshold corrections from canonical normalization of external quark doublets.

For the charged leptons, $Y_e^{\rm SM}$ is given by

$$\tilde{Y}_e^{\rm SM} = P_e \tilde{Y}_e' P_L^T P_h \,, \tag{3.26}$$

where P_e and P_L are threshold corrections due to canonical normalization of external charged lepton fields and

$$\begin{aligned} \tilde{Y}'_{e_{ij}} = \tilde{Y}^{\rm MSSM}_{e_{ij}} \cos\beta \left(1 + \frac{1}{16\pi^2} \tan\beta \left(\tau^W_{ij} + \tau^B_{ij}\right) + \frac{1}{16\pi^2}\left(\delta^W_{ij} + \delta^B_{ij}\right)\right) \\ + \tilde{T}^{\rm MSSM}_{e_{ij}} \cos\beta \frac{1}{16\pi^2} \xi^B_{ij} \,, \end{aligned} \tag{3.27}$$

with the $\tan\beta$-enhanced contributions

$$\begin{aligned}
\tau_{ij}^W &= \frac{3}{2} g_2^2 \frac{M_2^*}{\mu} H_2\left(\frac{|M_2|^2}{|\mu|^2}, \frac{m_{\tilde{L}_j}^2}{|\mu|^2}\right) , \qquad (3.28)\\
\tau_{ij}^B &= \frac{3}{5} g_1^2 \left(-\frac{\mu^*}{M_1} H_2\left(\frac{m_{\tilde{e}_i}^2}{|M_1|^2}, \frac{m_{\tilde{L}_j}^2}{|M_1|^2}\right) + \frac{M_1^*}{\mu} H_2\left(\frac{m_{\tilde{e}_i}^2}{|\mu|^2}, \frac{|M_1|^2}{|\mu|^2}\right) - \frac{1}{2}\frac{M_1^*}{\mu} H_2\left(\frac{|M_1|^2}{|\mu|^2}, \frac{m_{\tilde{L}_j}^2}{|\mu|^2}\right)\right) ,
\end{aligned}$$

and

$$\begin{aligned}
\delta_{ij}^W &= 12 g_2^2 C_{00}\left(\frac{|\mu|^2}{Q^2}, \frac{|M_2|^2}{|\mu|^2}, \frac{m_{\tilde{L}_j}^2}{|\mu|^2}\right) ,\\
\delta_{ij}^B &= \frac{3}{5} g_1^2 \left(-2C_{00}\left(\frac{|\mu|^2}{Q^2}, \frac{|M_1|^2}{|\mu|^2}, \frac{m_{\tilde{L}_j}^2}{|\mu|^2}\right) + 4C_{00}\left(\frac{|\mu|^2}{Q^2}, \frac{|M_1|^2}{|\mu|^2}, \frac{m_{\tilde{e}_i}^2}{|\mu|^2}\right)\right) ,\\
\xi_{ij}^B &= \frac{3}{5} g_1^2 \frac{1}{M_1} H_2\left(\frac{m_{\tilde{e}_i}^2}{|M_1|^2}, \frac{m_{\tilde{L}_j}^2}{|M_1|^2}\right) . \qquad (3.29)
\end{aligned}$$

The diagrams for the Y_e SUSY threshold corrections are analogous to the ones in figures 2 and 3, with the exception that the loop diagrams shown in the top rows do not exist. The diagram of figure 4 also doesn't have an analogue for Y_e. P_e and P_L are calculated from (3.19) with

$$\mathcal{K}_L = \delta_{ij} + \frac{1}{16\pi^2}\left((\tilde{\rho}_2 + \tilde{\rho}_1)\,\delta_{ij} + \rho_{ij}^e + \tilde{\rho}_{ij}^e\right) , \qquad (3.30)$$

where

$$\begin{aligned}
\tilde{\rho}_2 &= \frac{3}{2} g_2^2 B_3\left(Q^2, |M_2|^2, m_{\tilde{L}_j}^2\right) ,\\
\tilde{\rho}_1 &= \frac{1}{2}\frac{3}{5} g_1^2 B_3\left(Q^2, |M_1|^2, m_{\tilde{L}_j}^2\right) ,\\
\rho_{ij}^e &= \sum_k \tilde{Y}_{e_{jk}}^\dagger \tilde{Y}_{e_{ki}} B_3\left(Q^2, |\mu|^2, m_{\tilde{e}_k}^2\right) ,\\
\tilde{\rho}_{ij}^e &= \sum_k \tilde{Y}_{e_{jk}}^\dagger \tilde{Y}_{e_{ki}} \tilde{B}_1\left(\frac{m_H^2}{Q^2}\right) \sin^2\beta , \qquad (3.31)
\end{aligned}$$

and

$$\mathcal{K}_e = \delta_{ij} + \frac{1}{16\pi^2}\left(\tilde{\chi}_1 \delta_{ij} + \chi_{ij}^e + \tilde{\chi}_{ij}^e\right) , \qquad (3.32)$$

with

$$\begin{aligned}
\tilde{\chi}_1 &= 2\frac{3}{5} g_1^2 B_3\left(Q^2, |M_1|^2, m_{\tilde{e}_i}^2\right) ,\\
\chi_{ij}^e &= \sum_k 2\, \tilde{Y}_{e_{ik}} \tilde{Y}_{e_{kj}}^\dagger B_3\left(Q^2, |\mu|^2, m_{\tilde{L}_k}^2\right) ,\\
\tilde{\chi}_{ij}^e &= \sum_k \tilde{2}\, Y_{e_{ik}} \tilde{Y}_{e_{kj}}^\dagger \tilde{B}_1\left(\frac{m_H^2}{Q^2}\right) \sin^2\beta . \qquad (3.33)
\end{aligned}$$

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Again, the loop diagrams for $\mathcal{K}_L$ and $\mathcal{K}_e$ can be easily obtained from the diagrams for $\mathcal{K}_Q$ by suitable exchange of labels and indices and dropping non-existent gaugino loops.

Turning to Y_u, the types of diagrams which were $\tan\beta$-enhanced for Y_d and Y_e are now $\tan\beta$-suppressed. However, there also exist SUSY threshold corrections which are independent of $\tan\beta$ and enhanced by large trilinear couplings. These SUSY threshold corrections to Y_u can have important effects. For example the SUSY threshold corrections to the top Yukawa coupling y_t can be of significance in analyses of the Higgs mass and vacuum stability. The expression for the Y_u SUSY threshold corrections can be readily obtained from the SUSY threshold corrections to Y_d (3.4)–(3.7) and (3.19)–(3.23) by the replacement[5]

$$\begin{aligned} d &\to u\,, \\ \cos\beta &\to \sin\beta\,, \end{aligned} \tag{3.34}$$

with the exception of the bino-loops, whose contribution become

$$\begin{aligned}
\eta^B_{ij} &= \frac{3}{5}g_1^2\left(-\frac{2}{9}\frac{\mu^*}{M_1}H_2\left(\frac{m^2_{\tilde{d}_i}}{|M_1|^2},\frac{m^2_{\tilde{Q}_j}}{|M_1|^2}\right)+\frac{2}{3}\frac{M_1^*}{\mu}H_2\left(\frac{m^2_{\tilde{u}_i}}{|\mu|^2},\frac{|M_1|^2}{|\mu|^2}\right)-\frac{1}{6}\frac{M_1^*}{\mu}H_2\left(\frac{|M_1|^2}{|\mu|^2},\frac{m^2_{\tilde{Q}_j}}{|\mu|^2}\right)\right)\,,\\
\epsilon^B_{ij} &= \frac{3}{5}g_1^2\left(\frac{8}{3}C_{00}\left(\frac{|\mu|^2}{Q^2},\frac{|M_1|^2}{|\mu|^2},\frac{m^2_{\tilde{u}_i}}{|\mu|^2}\right)-\frac{2}{3}C_{00}\left(\frac{|\mu|^2}{Q^2},\frac{|M_1|^2}{|\mu|^2},\frac{m^2_{\tilde{Q}_j}}{|\mu|^2}\right)\right)\,,\\
\zeta^B_{ij} &= \frac{3}{5}g_1^2\frac{2}{9}\frac{1}{M_1}H_2\left(\frac{m^2_{\tilde{u}_i}}{|M_1|^2},\frac{m^2_{\tilde{Q}_j}}{|M_1|^2}\right)\,,\\
\chi_1 &= \frac{8}{9}\frac{3}{5}g_1^2\left(Q^2,|M_1|^2,m^2_{\tilde{u}_i}\right)\,,
\end{aligned} \tag{3.35}$$

due to the different U(1) hypercharges of the (s)particles in the loop. The loop diagrams are identical to the ones of figures 2, 3, and 4 with u and d exchanged.

After the SUSY threshold corrections are incorporated in the $\overline{\text{DR}}$ scheme, `REAP` converts the Yukawa and gauge couplings to the $\overline{\text{MS}}$ scheme following [58].

Finally `SusyTC` calculates the value of $|\mu|$ and m_3 from $m^2_{H_u}$, $m^2_{H_d}$, $\tan\beta$ and M_Z by requiring the existence of spontaneously broken EW vacuum, which is equivalent to vanishing one-loop corrected tad-pole equations of H_u and H_d

$$\mu = \mathrm{e}^{i\phi_\mu}\sqrt{\frac{1}{2}\left(\tan(2\beta)\left(\bar{m}^2_{H_u}\tan\beta-\bar{m}^2_{H_d}\cot\beta\right)-M_Z^2-Re\left(\Pi^T_{ZZ}\left(M_Z^2\right)\right)\right)}\,, \tag{3.36}$$

with $\bar{m}^2_{H_u}\equiv m^2_{H_u}-t_u$ and $\bar{m}^2_{H_d}\equiv m^2_{H_d}-t_d$. In the real (CP conserving) MSSM the phase ϕ_μ is restricted to 0 and π. The expressions for the one-loop tadpoles t_u, t_d and the transverse Z-boson self energy Π^T_{ZZ} are based on [59], but extended to include inter-generational mixing, and are presented in appendix B. Because μ enters the one-loop formulas for the threshold corrections, treating t_u, t_d and Π^T_{ZZ} as functions of tree-level parameters is sufficiently accurate. The one-loop expression of the soft-breaking mass m_3 is calculated as

$$m_3 = \sqrt{\frac{1}{2}\left(\tan(2\beta)\left(\bar{m}^2_{H_u}-\bar{m}^2_{H_d}\right)-\left(M_Z^2+Re\left(\Pi^T_{ZZ}\left(M_Z^2\right)\right)\right)\sin(2\beta)\right)}\,. \tag{3.37}$$

[5]Note that (3.12)–(3.15) stay invariant.

If desired, SusyTC allows to outsource a two-loop Higgs mass calculation to external software, e.g. FeynHiggs [60–67], by calculating the pole mass m_{H^+} (m_A) as input for the complex (real) MSSM

$$m^2_{H^+} = \frac{1}{\cos(2\beta)}\Bigg(\bar{m}^2_{H_u} - \bar{m}^2_{H_d} - M_Z^2 - Re\left(\Pi^T_{ZZ}\left(M_Z^2\right)\right) + \hat{M}_W^2 \\ - Re\left(\Pi_{H^+H^-}\left(m_{H^+}\right)\right) + t_d \sin^2\beta + t_u \cos^2\beta\Bigg) \ , \tag{3.38}$$

$$m^2_{A} = \frac{1}{\cos(2\beta)}\Bigg(\bar{m}^2_{H_u} - \bar{m}^2_{H_d} - M_Z^2 - Re\left(\Pi^T_{ZZ}\left(M_Z^2\right)\right) \\ \times Re\left(\Pi_{AA}\left(m_{A}\right)\right) + t_d \sin^2\beta + t_u \cos^2\beta\Bigg) \ , \tag{3.39}$$

where $\hat{M}_W$ is the $\overline{\text{DR}}$ W-boson mass given as

$$\hat{M}_W^2(Q) = M_W^2 + Re\left(\Pi^T_{WW}\left(M_W^2\right)\right) = g_2 \frac{\hat{v}(Q)}{2} \ , \tag{3.40}$$

with M_Z and M_W pole masses and the $\overline{\text{DR}}$ vacuum expectation value $\hat{v}(Q)$ given by

$$\hat{v}^2(Q) = 4\frac{M_Z^2 + Re\left(\Pi^T_{ZZ}\left(M_Z^2\right)\right)}{\frac{3}{5}g_1^2(Q) + g_2^2(Q)} \ . \tag{3.41}$$

As in the previous formulas, the self energies $\Pi_{H^+H^-}$ and Π_{AA} are based on [59], but extended to include inter-generational mixing, and are understood as functions of tree-level parameters. They are given in appendix B.

4 The REAP extension SusyTC

In this section we provide a "Getting Started" calculation for SusyTC. A full documentation of all features is included in appendix C. Since SusyTC is an extension to REAP, an up-to-date version of REAP-MPT [1] (available at http://reapmpt.hepforge.org) needs to be installed on your system. SusyTC consists out of the REAP model file RGEMSSMsoftbroken.m, which is based on the model file RGEMSSM.m of REAP 1.11.2 and additionally contains, among other things, the RGEs of the MSSM soft-breaking parameters and the matching to the SM, and the file SusyTC.m, which includes the formulas for the sparticle spectrum and SUSY threshold correction calculations. Both files can be downloaded from http://particlesandcosmology.unibas.ch/pages/SusyTC.htm and have to be copied into the REAP directory.

To begin a calculation with SusyTC, one first needs to import RGEMSSMsoftbroken.m:

```
Needs["REAP`RGEMSSMsoftbroken`"];
```

The model MSSMsoftbroken is then defined by RGEAdd, including additional options such as RGEtanβ:

```
RGEAdd["MSSMsoftbroken",RGEtanβ → 30];
```

In MSSMsoftbroken all REAP options of the model MSSM are available. The options additionally available in SusyTC are given in appendix C. The input is given by RGESetInitial. Let us illustrate some features of SusyTC: to test for example the GUT scale prediction for the Yukawa coupling ratio $\frac{y_\mu}{y_s} = 6$, considering a given example parameter point in the Constrained MSSM, one can type:

```
RGESetInitial[2·10^16,
    RGEYd → DiagonalMatrix[{1.2·10^-3, 2.2·10^-3, 0.16}],
    RGEYe → DiagonalMatrix[{1.2·10^-3, 6·2.2·10^-3, 0.16}],
    RGEM12 → 2000, RGEA0 → 1000, RGEm0 → 3000];
```

Of course, any general matrices can be used as input for the Yukawa, trilinear, and soft-breaking matrices, as given by the specific SUSY flavour GUT model under consideration. Also, non-universal gaugino masses can be specified. As in REAP, dimensional quantities are given in units of GeV. The RGEs are then solved from the GUT scale to the Z-boson mass scale by

```
RGESsolve[91,2·10^16];
```

The ratio of the μ and strange quark Yukawa couplings at the Z-boson mass scale can now be obtained with RGEGetSolution, CKMParameters, and MNSParameters:

```
Yu = RGEGetSolution[91, RGEYu];
Yd = RGEGetSolution[91, RGEYd];
Ye = RGEGetSolution[91, RGEYe];
Mν = RGEGetSolution[91, RGEMν];
MNSParameters[Mν,Ye][[3, 2]]/CKMParameters[Yu, Yd][[3, 2]]
```

Repeating this calculation with all SU(5) CG factors listed in table 2 of [12], one obtains the results shown in figure 8.

As described in appendix C, SusyTC can also read and write "Les Houches" files [55, 58] as input and output.

5 The sparticle spectrum predicted from CG factors

In this section we apply SusyTC to investigate the constraints on the sparticle spectrum which arise from a set of GUT scale predictions for the quark-lepton Yukawa coupling ratios $\frac{y_e}{y_d}$, $\frac{y_\mu}{y_s}$, and $\frac{y_\tau}{y_b}$. As GUT scale boundary conditions for the SUSY-breaking terms we take the Constrained MSSM. The experimental constraints are given by the Higgs boson mass $m_{h^0} = 125.09 \pm 0.21 \pm 0.11$ GeV [68] as well as the charged fermion masses (and the quark mixing matrix). We use the experimental constraints for the running $\overline{\text{MS}}$ Yukawa couplings at the Z-boson mass scale calculated in [36], where we set the uncertainty of the charged lepton Yukawa couplings to one percent to account for the estimated theoretical uncertainty (which here exceeds the experimental uncertainty). When applying the measured Higgs mass as constraint, we use a 1σ interval of ± 3 GeV, including the estimated theoretical uncertainty.

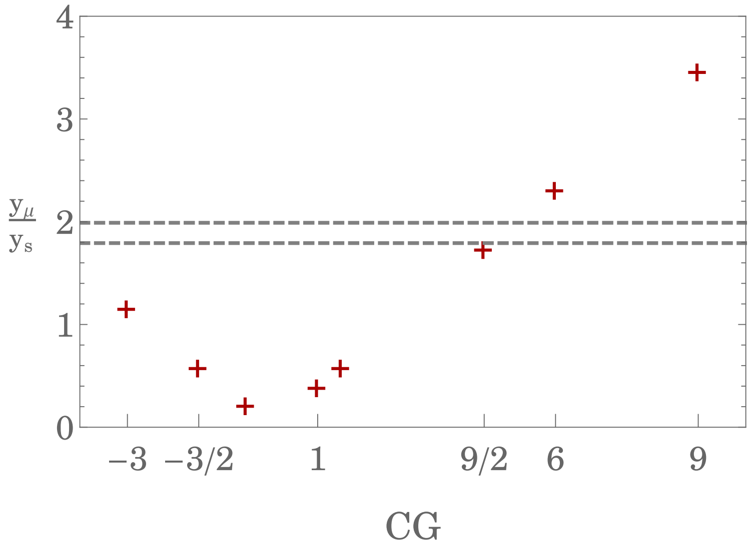

Figure 8. Example results for $\frac{y_\mu}{y_s}$ at the electroweak scale, considering the SU(5) GUT-scale CG factors from table 2 of [12], i.e. the GUT predictions $\frac{y_\mu}{y_s}$ = CG, for a given example Constrained MSSM parameter point with $\tan\beta = 30$, $m_{1/2} = 2000$ GeV, $A_0 = 1000$ GeV, and $m_0 = 3000$ GeV. The area between the dashed gray lines corresponds to the experimental 1σ range [36].

For our study, we consider GUT scale Yukawa coupling matrices which feature the GUT-scale Yukawa relations $\frac{y_e}{y_d} = -\frac{1}{2}$, $\frac{y_\mu}{y_s} = 6$, and $\frac{y_\tau}{y_b} = -\frac{3}{2}$ (cf. [12]):

$$Y_d = \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}, \quad Y_e = \begin{pmatrix} -\frac{1}{2}y_d & 0 & 0 \\ 0 & 6y_s & 0 \\ 0 & 0 & -\frac{3}{2}y_b \end{pmatrix},$$
$$Y_u = \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix} U_{\mathrm{CKM}}(\theta_{12}, \theta_{13}, \theta_{23}, \delta)\,, \tag{5.1}$$

These GUT relations can emerge as direct result of CG factors in SU(5) GUTs or as approximate relation after diagonalization of the GUT-scale Yukawa matrices Y_d and Y_e (cf. [34, 35, 43, 44]). For the soft-breaking parameters we restrict our analysis to the Constrained MSSM parameters m_0, $m_{1/2}$, A_0 and $\tan\beta$, with μ determined from requiring the breaking of electroweak symmetry as in (3.36) and set $\mathrm{sgn}(\mu) = +1$. We note that in specific models for the GUT Higgs potential, for instance in [43], μ can be realized as an effective parameter of the superpotential with a fixed phase, including the case that μ is real.

We note that we have also added a neutrino sector, i.e. a neutrino Yukawa matrix Y_ν and and a mass matrix M_n of the right-handed neutrinos, but we have set the entries of Y_ν to very small values below $\mathcal{O}(10^{-3})$, such that their effects on the RG evolution can be safely neglected, and the masses of the right-handed neutrinos to values many orders of magnitude higher than the expected SUSY scale. With these parameters, the neutrino sector is decoupled from the main analysis. Such small values of the neutrino Yukawa couplings are e.g. expected in the models [34, 35, 44], where they arise as effective operators.

Using one-loop RGEs, REAP 1.11.3 and SusyTC 1.1 we determine the soft-breaking parameters and μ at the SUSY scale, as well as the pole mass m_{H^+}. This output is then passed to FeynHiggs 2.11.3 [60–67] in order to calculate the two-loop corrected pole masses of the Higgs bosons in the complex MSSM. The MSSM is automatically matched to the SM and we compare the results for the Yukawa couplings at the Z-boson mass scale with the experimental values reported in [36].

When fitting the GUT-scale parameters to the experimental data, we found that our results for the up-type quark Yukawa couplings and CKM angles and CP-phase could be fitted to agree with observations to at least 10^{-3} relative precision, by adjusting the parameters of Y_u. The remaining six parameters are used to fit the Yukawa couplings of down-type quarks and charged leptons, as well as the mass of the SM-like Higgs boson.

We find a benchmark point with a $\chi^2 = 0.9$:

input GUT scale parameters			
y_d	y_s	y_b	
$8.92 \cdot 10^{-5}$	$1.57 \cdot 10^{-3}$	0.109	
m_0	A_0	$m_{1/2}$	$\tan\beta$
1629.48 GeV	−3152.70 GeV	1840.48 GeV	21.27

low energy results			
y_e	y_μ	y_τ	
$2.79 \cdot 10^{-6}$	$5.90 \cdot 10^{-4}$	$1.00 \cdot 10^{-2}$	
y_d	y_s	y_b	m_{h^0}
$1.75 \cdot 10^{-5}$	$3.07 \cdot 10^{-4}$	$1.64 \cdot 10^{-2}$	123.6 GeV

Looking at our results for the low-energy Yukawa coupling ratios, $\frac{y_e}{y_d} = 0.16$, $\frac{y_\mu}{y_s} = 1.92$, and $\frac{y_\tau}{y_b} = 0.61$, the importance of SUSY threshold corrections in evaluating the GUT-scale Yukawa ratios becomes evident. This can also be seen in figure 9. Additionally, as shown in figure 10, SUSY threshold corrections also affect the CKM mixing angles.

The SUSY spectrum obtained by SusyTC is shown in figure 11. The lightest supersymmetric particle (LSP) is a bino-like neutralino of about 827 GeV. The SUSY scale is obtained as $Q = 3014$ GeV. The μ parameter obtained from requiring spontaneous electroweak symmetry breaking is given by $\mu = 2634$ GeV. Note that the only experimental constraints we used were the results for quark and charged lepton masses as well as m_{h^0}. In particular, no bounds on the sparticle masses were applied as well as no restrictions from the neutralino relic density (which would require further assumptions on the cosmological evolution).[6]

[6]For example, the neutralino relic density may be diluted if additional entropy gets produced at late times. Therefore we do not use the neutralino relic density as a constraint here.

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Due to the large (absolute) values of the trilinear couplings, we find using the constraints from [69], that the vacuum of our benchmark point is meta-stable. The scalar potential possesses charge and colour breaking (CCB) vacua, as well as one "unbounded from below" (UFB) field direction in parameter space. However, estimating the stability of the vacuum via the Euclidean action of the "bounce" solution [70, 71] (following [72]) shows that the lifetime of the vacuum is many orders larger than the age of the universe.

Confidence intervals for the sparticle masses are obtained as Bayesian "highest posterior density" (HPD) intervals[7] from a Markov Chain Monte Carlo sample of 1.2 million points, using a Metropolis algorithm. We note that we did not compute the lifetime of the vacuum for each point in the MCMC analysis, which would take far too much computation time. This means that the obtained confidence intervals should be regarded as conservative, in the sense that including the lifetime constraints the upper bounds on the masses could become smaller. For some example points within the 1σ HPD regions we have checked that the lifetime constraints are satisfied. We applied the following priors to the MCMC analysis: $m_0 \in [0, 4]$ TeV, $A_0 \in [-10, 0]$ TeV, $m_{1/2} \in [0, 6]$ TeV and $\tan\beta \in [2, 40]$. As shown in figure 12 our results for the 1σ HPD intervals for the Constrained MSSM soft-breaking parameters are well within these intervals. The 1σ HPD results of the sparticle masses are shown in figure 13. Furthermore we find that for about 70% of the data points of the MCMC analysis, the lightest MSSM sparticle is a neutralino, while for the others it is the lightest charged slepton.[8] The HPD interval for the SUSY scale is obtained as $Q_{\text{HPD}} = [2048, 5108]$ GeV.

5.1 Comments and discussion

We would like to emphasize that the results described above have been obtained under specific assumptions for the input parameters at the GUT scale. In the following, we discuss these assumptions, how they may be obtained and/or generalized in fully worked out models, and also some limitations and uncertainties of our analysis.

To start with, we have chosen the specific GUT scale predictions $\frac{y_\tau}{y_b} = -\frac{3}{2}$, $\frac{y_\mu}{y_s} = 6$ and $\frac{y_e}{y_d} = -\frac{1}{2}$. This is indeed only one of the possible predictions that can arise from GUTs. Other possibilities can be found, e.g., in [11–13]. We have chosen the above set of GUT predictions since they are among the ones recently used successfully in GUT model building (see e.g. [34, 35, 43, 44]). In the future, it will of course be interesting to also test other combinations of promising Clebsch factors, and compare the predictions/constraints on the SUSY spectra. Of course, one can also construct GUT models which do not predict the quark-lepton Yukawa ratios. For such GUT models the constraints discussed here would not apply.

Furthermore, for our study we have assumed CMSSM boundary conditions for the soft breaking parameters, which is quite a strong assumption that will probably often be relaxed in realistic models. On the other hand, universal boundary conditions may also be a result of a specific SUSY breaking mechanism. Apart from the SUSY breaking

[7] An 1σ HPD interval is the interval $[\theta_L, \theta_H]$ such that $\int_{\theta_L}^{\theta_H} p(\theta) d\theta = 0.6826\ldots$ and the posterior probability density $p(\theta)$ inside the interval is higher than for any θ outside of the interval [73].

[8] Note that since in the latter case the LSP may be the gravitino, we do not exclude those points.

mechanism, GUTs themselves "unify" the soft breaking parameters since they unify the different types of SM fermions in GUT representations. In SU(5) GUTs, for example, one is left with only two soft breaking mass matrices at the GUT scale per family, one for the sfermions in the five-dimensional matter representation and one for the sfermions in the ten-dimensional representation. In SO(10) GUTs, there is only one unified sfermion mass matrix. In addition, the symmetries of GUT flavour models like [34, 35, 43, 44] include various (non-Abelian) "family symmetries", which lead to hierarchical Yukawa matrices and impose (partially) universal soft breaking mass matrices among different generations. The combination of these effects can indeed lead to GUT scale boundary conditions close to the CMSSM. Finally, we like to note that the absence of deviations from the SM in flavour physics processes leads to constraints on flavour non-universalities in the SUSY spectrum (if the sparticles are not too heavy) and can be seen as an experimental hint that, if SUSY exists at a comparably low scale, it should indeed be close to flavour-universal. In any case, it will be interesting to see how the constraints on the SUSY spectrum change when the assumption of an exact CMSSM at the GUT scale is relaxed.

On the other hand, although we have analyzed here a specific example only, some of the key effects that lead to a predicted sparticle spectrum seem rather general, as long as the quark-lepton Yukawa ratios are predicted at the GUT scale together with (close-to) universal soft-breaking parameters (which we assume for the remainder of this discussion):

- The main reason for the predictions/constraints on the SUSY spectrum is the fact that (to our knowledge) all the possible sets of GUT predictions for the quark-lepton Yukawa ratios require a certain amount of SUSY threshold corrections for each generation.[9] In general, to obtain the required size of the threshold corrections, one cannot have a sparticle spectrum which is too "split" (as e.g. in [74, 75]), since otherwise the loop functions (cf. section 3) get too suppressed. More specifically, the required threshold corrections constrain the ratios of trilinear couplings, gaugino masses, μ and sfermion masses. In a CMSSM-like scenario, this means the ratios between m_0, $m_{1/2}$ and A_0 are constrained. Furthermore, since the most relevant threshold corrections are the ones which are $\tan\beta$-enhanced, $\tan\beta$ cannot bee too small.

- Finally, with the ratios between m_0, $m_{1/2}$ and A_0 constrained and a moderate to large value of $\tan\beta$, the measured value of the mass m_h of the SM-like Higgs boson allows to constrain the SUSY scale. We note that this is an important ingredient, since the threshold corrections themselves depend only on the ratios of trilinear couplings, gaugino masses, μ and sfermion masses, and do not constrain the overall scale of the soft breaking parameters. The combination of the two effects leads to the result of a predicted sparticle spectrum from the assumed GUT boundary conditions.

Since m_h plays an important role, we would like to comment that it would be highly desirable to have a more precise computation of the Higgs mass available for the "large stop-mixing" regime. As discussed above, we use a theoretical uncertainty of ± 3 GeV, which

[9] One comment is in order here: in the CMSSM the SUSY threshold corrections are very similar for the first two families, and therefore the argument remains valid even if the quark-lepton Yukawa ratios are predicted for only two of the families, for the third family and either the second or the first family.

is dominating the 1σ interval for m_h entering our fit. This theoretical uncertainty should, strictly speaking, not be treated on the same footing as a pure statistical uncertainty (but the same of course also holds true for systematic experimental uncertainties). Furthermore, there are indications that the theoretical uncertainty in the m_h calculation in the most relevant regions of parameter space of our analysis may be larger, as recently discussed e.g. in [76], however there is no full agreement on this aspect. For our analysis we have used the external software `FeynHiggs` 2.11.3, the current version when our numerical analysis was performed, and the most commonly assumed estimate ± 3 GeV for the theoretical uncertainty.

One may also ask why we did not emphasize the aspect of gauge coupling unification. From a bottom-up perspective, one could indeed conclude that in order to make the gauge couplings meet at high energy, the SUSY scale cannot be too high. However, in realistic GUTs, such statements are strongly affected by GUT threshold corrections. They can be implemented easily with `REAP` once the specific GUT model is known, however they are very model-dependent. In particular, for more general GUT-Higgs potentials it is known that gauge coupling unification is not resulting in a relevant constraint on the SUSY scale. For our analysis, we have therefore simply set the GUT scale to 2×10^{16} GeV. In an explicit model, where the GUT threshold corrections can be computed and in particular if they significantly shift the GUT scale, we expect that the predictions for the SUSY spectrum will also be modified correspondingly, e.g. due to the increased amount of RG evolution.

Let us also comment on the fact that `SusyTC` is matching the MSSM to the SM at one common "SUSY scale". This is certainly a limitation of the current version of `SusyTC`. On the other hand, as discussed above, in the scenarios where SUSY threshold corrections play an important role, the sparticle spectrum cannot be too "split". This means that for the main applications of `SusyTC`, like the example presented in this section, one-step matching is sufficiently accurate.

6 Summary and conclusion

In this work we discussed how predictions for the sparticle spectrum can arise from GUTs, which feature predictions for the ratios of quark and lepton Yukawa couplings at high energy. To test them by comparing with the experimental data, the RG running between high and low energy has to be performed with sufficient accuracy, including threshold corrections. In SUSY theories, the one-loop threshold corrections when matching the SUSY model to the SM are of particular importance, since they can be enhanced by $\tan\beta$ or large trilinear couplings, and thus have the potential to strongly affect the quark-lepton mass relations. Since the SUSY threshold corrections depend on the SUSY parameters, they link a given GUT flavour model to the SUSY model. In other words, via the SUSY threshold corrections, GUT models can predict properties of the sparticle spectrum from the pattern of quark-lepton mass ratios at the GUT scale.

To accurately study such predictions, we extend and generalize various formulas in the literature which are needed for a precision analysis of SUSY flavour GUT models: for example, we extend the RGEs for the MSSM soft breaking parameters at two-loop by the

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additional terms in the seesaw type-I extension (cf. appendix A). We generalize the one-loop calculation of μ and pole mass calculation of m_A and m_{H^+} to include inter-generational mixing in the self energies (cf. appendix B). Furthermore, we calculate the full one-loop SUSY threshold corrections for the down-type quark, up-type quark and charged lepton Yukawa coupling matrices in the electroweak unbroken phase (cf. section 3).

We introduce the new software tool `SusyTC`, a major extension to the Mathematica package `REAP`, where these formulas are implemented. In addition, `SusyTC` calculates the $\overline{\mathrm{DR}}$ sparticle spectrum and the SUSY scale Q, and can provide output in SLHA "Les Houches" files which are the necessary input for external software, e.g. for performing a two-loop Higgs mass calculation. `REAP` extended by `SusyTC` accepts general GUT scale Yukawa, trilinear and soft breaking mass matrices as well as non-universal gaugino masses as input, performs the RG evolution (integrating out the right-handed neutrinos at their mass thresholds in the type I seesaw extension of the MSSM) and automatically matches the MSSM to the SM, making it a convenient tool for top-down analyses of SUSY flavour GUT models.

We applied `SusyTC` to study the predictions for the parameters of the Constrained MSSM SUSY scenario from the set of GUT-scale Yukawa relations $\frac{y_e}{y_d} = -\frac{1}{2}$, $\frac{y_\mu}{y_s} = 6$, and $\frac{y_\tau}{y_b} = -\frac{3}{2}$, which has been proposed recently in the context of GUT flavour models. With a Markov Chain Monte Carlo analysis we find a "best-fit" benchmark point as well as the 1σ Bayesian confidence intervals for the sparticle masses. Without applying any constraints from LHC SUSY searches or dark matter, we find that the considered GUT scenario predicts a sparticle spectrum above past LHC sensitivities, but partly within reach of (a high-luminosity upgrade of) the LHC, and possibly fully testable at a future O(100 TeV) pp collider like the FCC-hh [80] or the SPPC [81].

Acknowledgments

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A The β-functions in the seesaw type-I extension of the MSSM

In this appendix we list the β-functions of the SUSY soft-breaking parameters in the MSSM extended by the additional terms in the seesaw type-I extension (obtained using the general formulas of [56]). Our conventions for W and $\mathcal{L}_{\mathrm{soft}}$ are given in (3.1) and (3.2).

A.1 One-loop β-functions

$$16\pi^2\beta_{M_1} = \frac{66}{5}g_1^2 M_1\ , \tag{A.1}$$

$$16\pi^2\beta_{M_2} = 2g_2^2 M_2\ , \tag{A.2}$$

$$16\pi^2\beta_{M_3} = -6g_3^2 M_3\ , \tag{A.3}$$

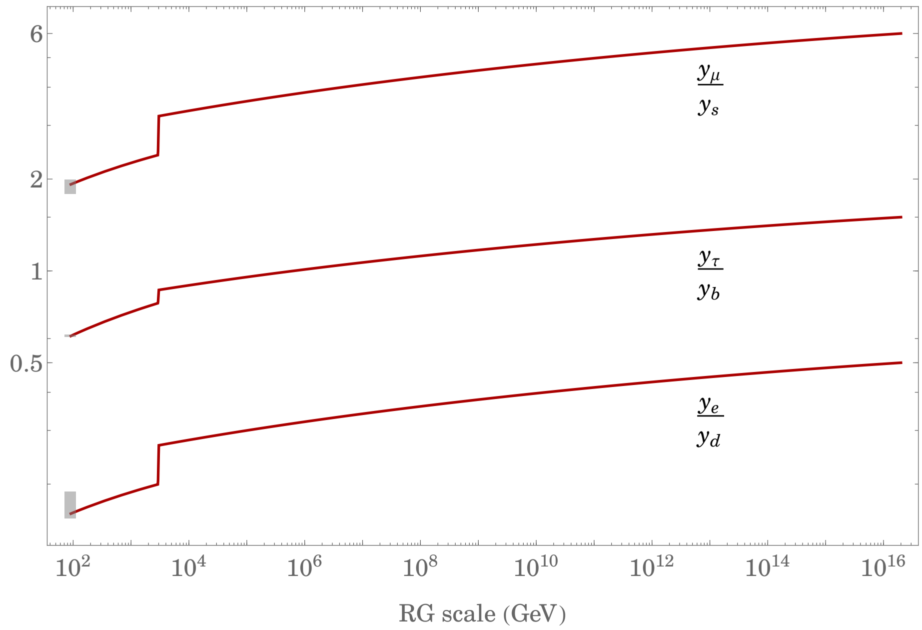

Figure 9. RG evolution of the Yukawa coupling ratios of the first, second and third family from the GUT-scale to the mass scale of the Z-boson. The GUT scale parameters correspond to our benchmark point from section 5. The effects of the threshold corrections are clearly visible at the SUSY scale $Q = 3015$ GeV. The light gray areas indicate the experimental Yukawa coupling ratios at M_Z, taken from [36].

$$\begin{aligned}16\pi^2\beta_{T_u} &= Y_u\left(2Y_d^\dagger T_d + 4Y_u^\dagger T_u\right) + T_u\left(Y_d^\dagger Y_d + 5Y_u^\dagger Y_u\right)\\ &\quad + Y_u\left(6\,\mathrm{Tr}(Y_u^\dagger T_u) + 2\,\mathrm{Tr}(Y_\nu^\dagger T_\nu) + \frac{26}{15}g_1^2 M_1 + 6g_2^2 M_2 + \frac{32}{3}g_3^2 M_3\right)\\ &\quad + T_u\left(3\,\mathrm{Tr}(Y_u^\dagger Y_u) + \mathrm{Tr}(Y_\nu^\dagger Y_\nu) - \frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2\right)\,, \end{aligned} \tag{A.4}$$

$$\begin{aligned}16\pi^2\beta_{T_d} &= Y_d\left(4Y_d^\dagger T_d + 2Y_u^\dagger T_u\right) + T_d\left(5Y_d^\dagger Y_d + Y_u^\dagger Y_u\right)\\ &\quad + Y_d\left(6\,\mathrm{Tr}(Y_d^\dagger T_d) + 2\,\mathrm{Tr}(Y_e^\dagger T_e) + \frac{14}{15}g_1^2 M_1 + 6g_2^2 M_2 + \frac{32}{3}g_3^2 M_3\right)\\ &\quad + T_d\left(3\,\mathrm{Tr}(Y_d^\dagger Y_d) + \mathrm{Tr}(Y_e^\dagger Y_e) - \frac{7}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2\right)\,, \end{aligned} \tag{A.5}$$

$$\begin{aligned}16\pi^2\beta_{T_e} &= Y_e\left(4Y_e^\dagger T_e + 2Y_\nu^\dagger T_\nu\right) + T_e\left(5Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu\right)\\ &\quad + Y_e\left(6\,\mathrm{Tr}(Y_d^\dagger T_d) + 2\,\mathrm{Tr}(Y_e^\dagger T_e) + \frac{18}{5}g_1^2 M_1 + 6g_2^2 M_2\right)\\ &\quad + T_e\left(3\,\mathrm{Tr}(Y_d^\dagger Y_d) + \mathrm{Tr}(Y_e^\dagger Y_e) - \frac{9}{5}g_1^2 - 3g_2^2\right)\,, \end{aligned} \tag{A.6}$$

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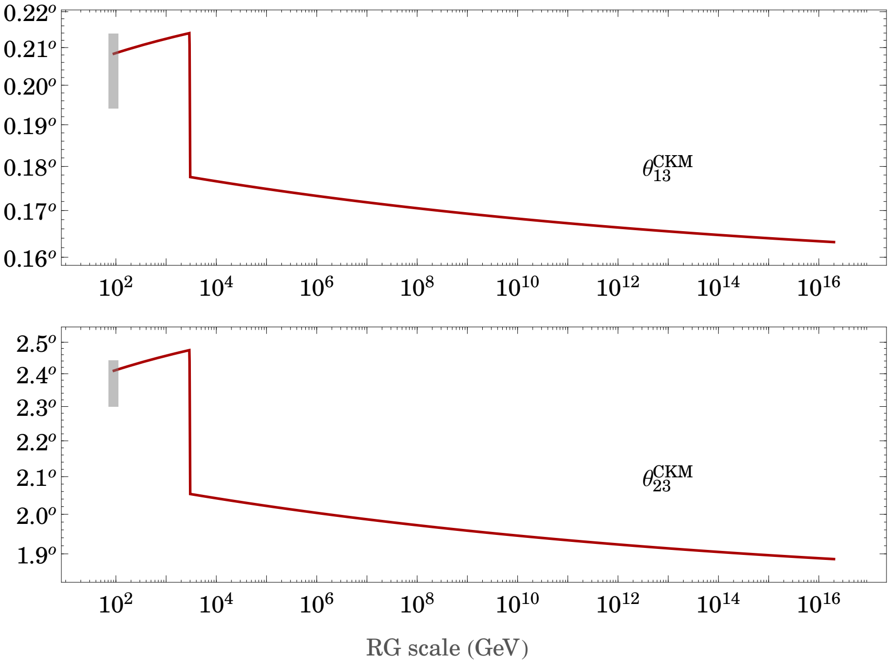

Figure 10. RG evolution of the CKM mixing angles $\theta_{13}^{\rm CKM}$ and $\theta_{23}^{\rm CKM}$ from the GUT-scale to the mass scale of the Z-boson. The GUT scale parameters correspond to our benchmark point from section 5. The effects of the threshold corrections are clearly visible at the SUSY scale $Q = 3015$ GeV. The light gray areas indicate the experimental values at M_Z.

$$
\begin{aligned}
16\pi^2 \beta_{T_\nu} =\, & Y_\nu \left(4 Y_\nu^\dagger T_\nu + 2 Y_e^\dagger T_e\right) + T_\nu \left(5 Y_\nu^\dagger Y_\nu + Y_e^\dagger Y_e\right) \\
& + Y_\nu \left(6\,\mathrm{Tr}(Y_u^\dagger T_u) + 2\,\mathrm{Tr}(Y_\nu^\dagger T_\nu) + \frac{6}{5} g_1^2 M_1 + 6 g_2^2 M_2\right) \\
& + T_\nu \left(3\,\mathrm{Tr}(Y_u^\dagger Y_u) + \mathrm{Tr}(Y_\nu^\dagger Y_\nu) - \frac{3}{5} g_1^2 - 3 g_2^2\right) \,, \qquad \text{(A.7)}
\end{aligned}
$$

$$
\begin{aligned}
16\pi^2 \beta_{m_{\tilde L}^2} =\, & 2 Y_e^\dagger m_{\tilde e}^2 Y_e + 2 Y_\nu^\dagger m_{\tilde \nu}^2 Y_\nu \\
& + m_{\tilde L}^2 \left(Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu\right) + \left(Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu\right) m_{\tilde L}^2 \\
& + 2 m_{H_u}^2 Y_\nu^\dagger Y_\nu + 2 m_{H_d}^2 Y_e^\dagger Y_e + 2 T_e^\dagger T_e + 2 T_\nu^\dagger T_\nu \\
& - \frac{6}{5} g_1^2 |M_1|^2\, \mathbf{1}_3 - 6 g_2^2 |M2|^2\, \mathbf{1}_3 - \frac{3}{5} g_1^2 S\, \mathbf{1}_3 \,, \qquad \text{(A.8)}
\end{aligned}
$$

$$
\begin{aligned}
16\pi^2 \beta_{m_{\tilde Q}^2} =\, & 2 Y_u^\dagger m_{\tilde u}^2 Y_u + 2 Y_d^\dagger m_{\tilde d}^2 Y_d \\
& + m_{\tilde Q}^2 \left(Y_u^\dagger Y_u + Y_d^\dagger Y_d\right) + \left(Y_u^\dagger Y_u + Y_d^\dagger Y_d\right) m_{\tilde Q}^2 \\
& + 2 m_{H_u}^2 Y_u^\dagger Y_u + 2 m_{H_d}^2 Y_d^\dagger Y_d + 2 T_u^\dagger T_u + 2 T_d^\dagger T_d \\
& - \frac{2}{15} g_1^2 |M_1|^2\, \mathbf{1}_3 - 6 g_2^2 |M2|^2\, \mathbf{1}_3 - \frac{32}{3} g_3^2 |M_3|^2\, \mathbf{1}_3 + \frac{1}{5} g_1^2 S\, \mathbf{1}_3 \,, \qquad \text{(A.9)}
\end{aligned}
$$

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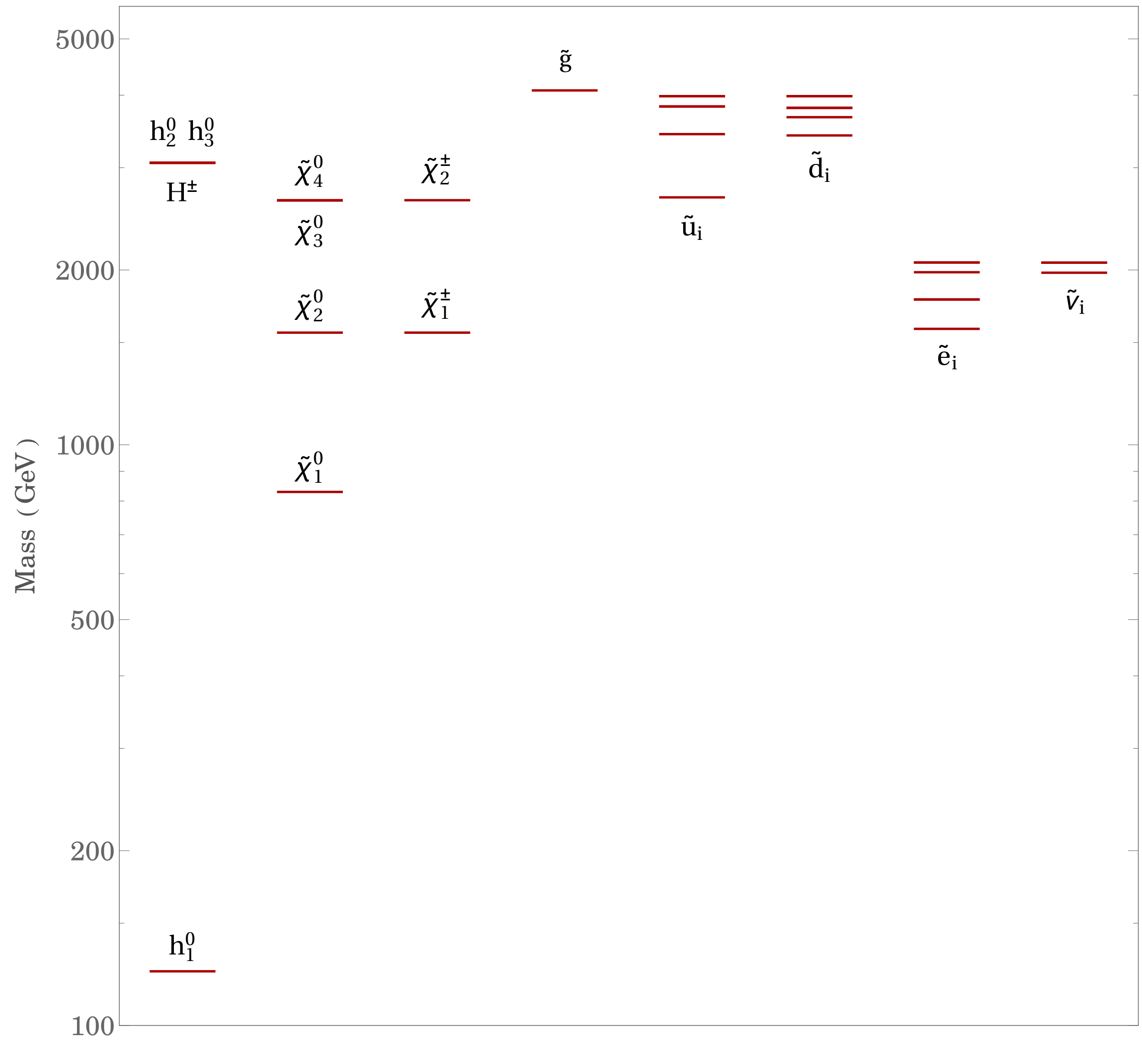

Figure 11. SUSY spectrum with SU(5) GUT scale boundary conditions $\frac{y_e}{y_d} = -\frac{1}{2}$, $\frac{y_\mu}{y_s} = 6$, and $\frac{y_\tau}{y_b} = -\frac{3}{2}$, corresponding to our benchmark point from section 5.

$$\begin{aligned}
16\pi^2 \beta_{m^2_{\tilde{u}}} &= 4 Y_u m^2_{\tilde{Q}} Y_u^\dagger + 2 Y_u Y_u^\dagger m^2_{\tilde{u}} + 2 m^2_{\tilde{u}} Y_u Y_u^\dagger \\
&\quad + 4 m^2_{H_u} Y_u Y_u^\dagger + 4 T_u T_u^\dagger \\
&\quad - \frac{32}{15} g_1^2 |M_1|^2 \, \mathbf{1}_3 - \frac{32}{3} g_3^2 |M_3|^2 \, \mathbf{1}_3 - \frac{4}{5} g_1^2 S \, \mathbf{1}_3 \, , \qquad \text{(A.10)} \\
16\pi^2 \beta_{m^2_{\tilde{d}}} &= 4 Y_d m^2_{\tilde{Q}} Y_d^\dagger + 2 Y_d Y_d^\dagger m^2_{\tilde{d}} + 2 m^2_{\tilde{d}} Y_d Y_d^\dagger \\
&\quad + 4 m^2_{H_d} Y_d Y_d^\dagger + 4 T_d T_d^\dagger \\
&\quad - \frac{8}{15} g_1^2 |M_1|^2 \, \mathbf{1}_3 - \frac{32}{3} g_3^2 |M_3|^2 \, \mathbf{1}_3 + \frac{2}{5} g_1^2 S \, \mathbf{1}_3 \, , \qquad \text{(A.11)}
\end{aligned}$$

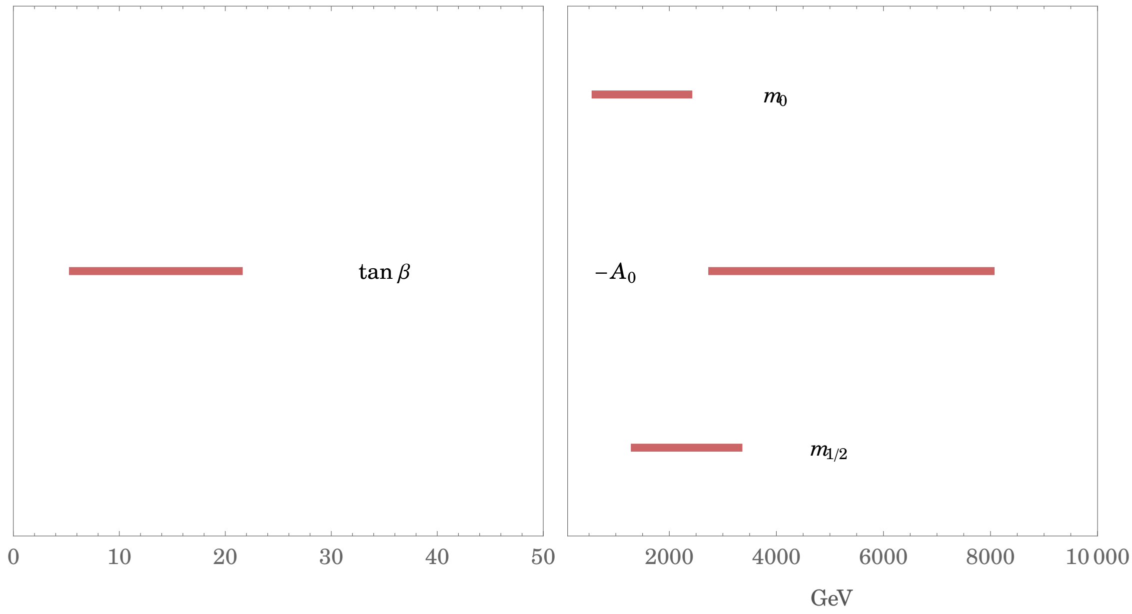

Figure 12. 1σ HPD intervals for the Constrained MSSM soft-breaking parameters.

$$
\begin{aligned}
16\pi^2 \beta_{m^2_{\tilde e}} &= 4 Y_e m^2_{\tilde L} Y_e^\dagger + 2 Y_e Y_e^\dagger m^2_{\tilde e} + 2 m^2_{\tilde e} Y_e Y_e^\dagger \\
&\quad + 4 m^2_{H_d} Y_e Y_e^\dagger + 4 T_e T_e^\dagger \\
&\quad - \frac{24}{5} g_1^2 |M_1|^2 \; \mathbf{1}_3 + \frac{6}{5} g_1^2 S \; \mathbf{1}_3 \,, \qquad \text{(A.12)} \\
16\pi^2 \beta_{m^2_{\tilde \nu}} &= 4 Y_\nu m^2_{\tilde L} Y_\nu^\dagger + 2 Y_\nu Y_\nu^\dagger m^2_{\tilde \nu} + 2 m^2_{\tilde \nu} Y_\nu Y_\nu^\dagger \\
&\quad + 4 m^2_{H_u} Y_\nu Y_\nu^\dagger + 4 T_\nu T_\nu^\dagger \,, \qquad \text{(A.13)} \\
16\pi^2 \beta_{m^2_{H_d}} &= 6\, \mathrm{Tr}(Y_d m^2_{\tilde Q} Y_d^\dagger + Y_d^\dagger m^2_{\tilde d} Y d) \\
&\quad + 2\, \mathrm{Tr}(Y_e m^2_{\tilde L} Y_e^\dagger + Y_e^\dagger m^2_{\tilde e} Y_e) \\
&\quad + 6 m^2_{H_d}\, \mathrm{Tr}(Y_d^\dagger Y_d) + 2 m^2_{H_d}\, \mathrm{Tr}(Y_e^\dagger Y_e) \\
&\quad + 6\, \mathrm{Tr}(T_d^\dagger T_d) + 2\, \mathrm{Tr}(T_e^\dagger T_e) \\
&\quad - \frac{6}{5} g_1^2 |M_1|^2 - 6 g_2^2 |M_2|^2 - \frac{3}{5} g_1^2 S \,, \qquad \text{(A.14)} \\
16\pi^2 \beta_{m^2_{H_u}} &= 6\, \mathrm{Tr}(Y_u m^2_{\tilde Q} Y_u^\dagger + Y_u^\dagger m^2_{\tilde u} Y u) \\
&\quad + 2\, \mathrm{Tr}(Y_\nu m^2_{\tilde L} Y_\nu^\dagger + Y_\nu^\dagger m^2_{\tilde \nu} Y_\nu) \\
&\quad + 6 m^2_{H_u}\, \mathrm{Tr}(Y_u^\dagger Y_u) + 2 m^2_{H_u}\, \mathrm{Tr}(Y_\nu^\dagger Y_\nu) \\
&\quad + 6\, \mathrm{Tr}(T_u^\dagger T_u) + 2\, \mathrm{Tr}(T_\nu^\dagger T_\nu) \\
&\quad - \frac{6}{5} g_1^2 |M_1|^2 - 6 g_2^2 |M_2|^2 + \frac{3}{5} g_1^2 S \,, \qquad \text{(A.15)}
\end{aligned}
$$

with

$$
S = m^2_{H_u} - m^2_{H_d} + \mathrm{Tr}\left(m^2_{\tilde Q} - m^2_{\tilde L} + m^2_{\tilde d} + m^2_{\tilde e} - 2 m^2_{\tilde u} \right) \,. \qquad \text{(A.16)}
$$

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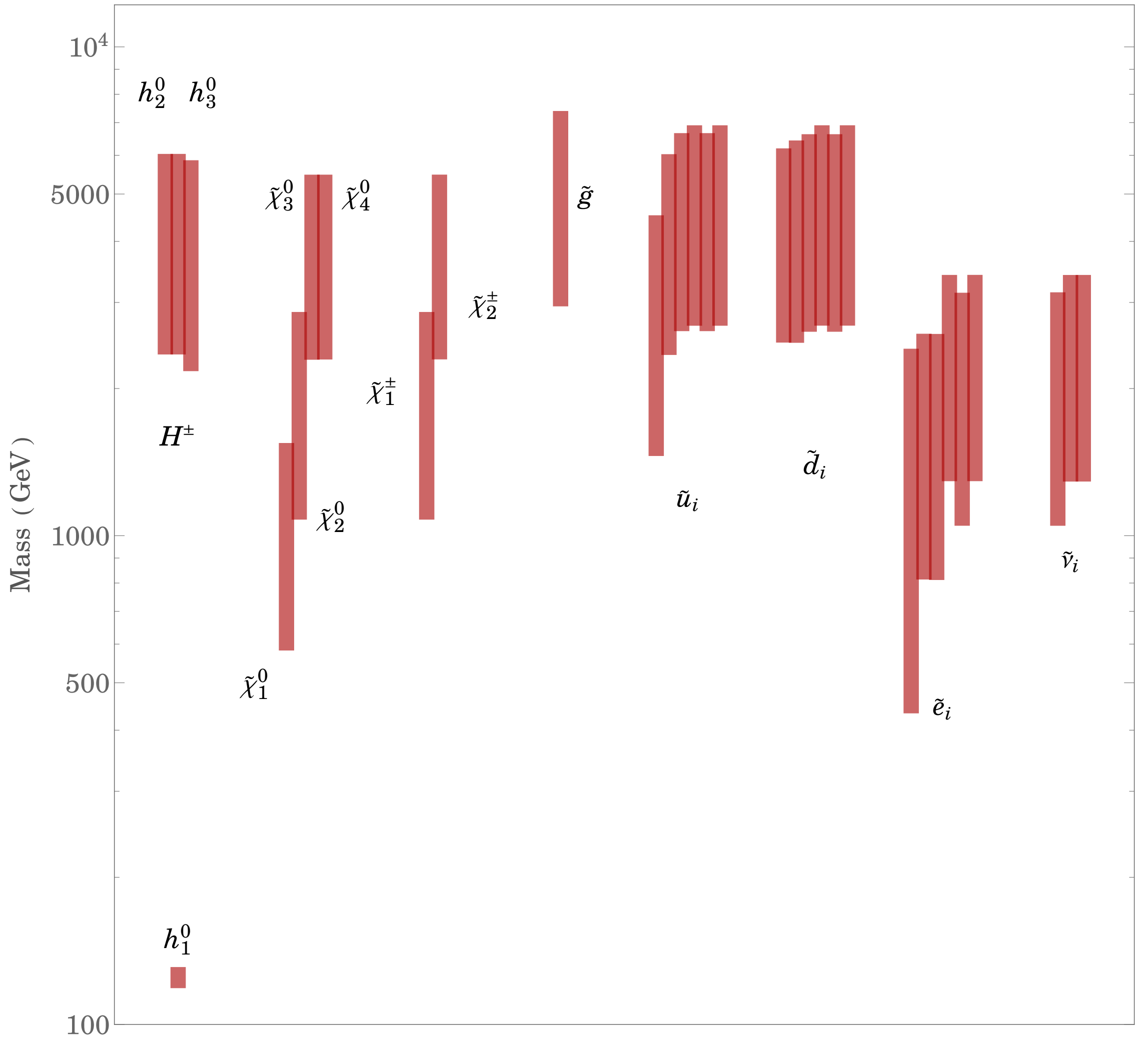

Figure 13. 1σ HPD intervals for the sparticle spectrum and Higgs boson masses with SU(5) GUT scale boundary conditions $\frac{y_e}{y_d} = -\frac{1}{2}$, $\frac{y_\mu}{y_s} = 6$, and $\frac{y_\tau}{y_b} = -\frac{3}{2}$. For about 70% of the data points, the LSP is the lightest neutralino $\tilde\chi_1^0$.

A.2 Two-loop β-functions

$$\begin{aligned}(16\pi^2)^2\beta_{M_1}^{(2)} &= \frac{12}{5}g_1^2\,\mathrm{Tr}(T_\nu Y_\nu^\dagger) + \frac{28}{5}g_1^2\,\mathrm{Tr}(T_d Y_d^\dagger) + \frac{36}{5}g_1^2\,\mathrm{Tr}(T_e Y_e^\dagger) + \frac{52}{5}g_1^2\,\mathrm{Tr}(T_u Y_u^\dagger)\\ &\quad - \frac{12}{5}g_1^2 M_1\,\mathrm{Tr}(Y_\nu Y_\nu^\dagger) - \frac{28}{5}g_1^2 M_1\,\mathrm{Tr}(Y_d Y_d^\dagger)\\ &\quad - \frac{36}{5}g_1^2 M_1\,\mathrm{Tr}(Y_e Y_e^\dagger) - \frac{52}{5}g_1^2 M_1\,\mathrm{Tr}(Y_u Y_u^\dagger)\\ &\quad + \frac{176}{5}g_1^2 g_3^2 M_1 + \frac{176}{5}g_1^2 g_3^2 M_3 + \frac{54}{5}g_1^2 g_2^2 M_1 + \frac{54}{5}g_1^2 g_2^2 M_2 + \frac{796}{25}g_1^4 M_1\,, \qquad \text{(A.17)}\\ (16\pi^2)^2\beta_{M_2}^{(2)} &= 4g_2^2\,\mathrm{Tr}(T_\nu Y_\nu^\dagger) + 4g_2^2\,\mathrm{Tr}(T_e Y_e^\dagger) + 12g_2^2\,\mathrm{Tr}(T_d Y_d^\dagger) + 12g_2^2\,\mathrm{Tr}(T_u Y_u^\dagger)\\ &\quad - 4g_2^2 M_2\,\mathrm{Tr}(Y_\nu Y_\nu^\dagger) - 4g_2^2 M_2\,\mathrm{Tr}(Y_e Y_e^\dagger)\\ &\quad - 12g_2^2 M_2\,\mathrm{Tr}(Y_u Y_u^\dagger) - 12g_2^2 M_2\,\mathrm{Tr}(Y_d Y_d^\dagger)\end{aligned}$$

$$
\begin{aligned}
&+ \frac{18}{5} g_1^2 g_2^2 M_1 + \frac{18}{5} g_1^2 g_3^2 M_2 + 48 g_2^2 g_3^2 M_2 + 48 g_2^2 g_3^2 M_3 + 100 g_2^4 M_2 \,, && \text{(A.18)} \\
(16\pi^2)^2 \beta_{M_3}^{(2)} &= 8 g_3^2 \,\mathrm{Tr}(T_d Y_d^\dagger) + 8 g_3^2 \,\mathrm{Tr}(T_u Y_u^\dagger) - 8 g_3^2 M_3 \,\mathrm{Tr}(Y_d Y_d^\dagger) - 8 g_3^2 M_3 \,\mathrm{Tr}(Y_u Y_u^\dagger) \\
&+ \frac{22}{5} g_1^2 g_3^2 M_1 + \frac{22}{5} g_1^2 g_3^2 M_3 + 18 g_2^2 g_3^2 M_2 + 18 g_2^2 g_3^2 M_3 + 56 g_3^4 M_3 \,, && \text{(A.19)} \\
(16\pi^2)^2 \beta_{T_u}^{(2)} &= -2 T_u \left(Y_d^\dagger Y_d Y_d^\dagger Y_d + 2 Y_d^\dagger Y_d Y_u^\dagger Y_u + 3 Y_u^\dagger Y_u Y_u^\dagger Y_u \right) \\
&- 2 Y_u \Big(2 Y_d^\dagger T_d Y_d^\dagger Y_d + 2 Y_d^\dagger T_d Y_u^\dagger Y_u + 2 Y_d^\dagger Y_d Y_d^\dagger T_d \\
&+ Y_d^\dagger Y_d Y_u^\dagger T_u + 4 Y_u^\dagger T_u Y_u^\dagger Y_u + 3 Y_u^\dagger Y_u Y_u^\dagger T_u \Big) \\
&- T_u \left(Y_d^\dagger Y_d \,\mathrm{Tr}(Y_e Y_e^\dagger + 3 Y_d Y_d^\dagger) + 5 Y_u^\dagger Y_u \,\mathrm{Tr}(Y_\nu Y_\nu^\dagger + 3 Y_u Y_u^\dagger) \right) \\
&- 2 Y_u \Big(Y_d^\dagger T_d \,\mathrm{Tr}(Y_e Y_e^\dagger + 3 Y_d Y_d^\dagger) + Y_d^\dagger Y_d \,\mathrm{Tr}(T_e Y_e^\dagger + 3 T_d Y_d^\dagger) \\
&+ 2 Y_u^\dagger T_u \,\mathrm{Tr}(Y_\nu Y_\nu^\dagger + 3 Y_u Y_u^\dagger) + 3 Y_u^\dagger Y_u \,\mathrm{Tr}(T_\nu Y_\nu^\dagger + 3 T_u Y_u^\dagger) \Big) \\
&- T_u \left(\mathrm{Tr}(3 Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger + Y_\nu Y_e^\dagger Y_e Y_\nu^\dagger) + 3 \,\mathrm{Tr}(Y_u Y_d^\dagger Y_d Y_u^\dagger + 3 Y_u Y_u^\dagger Y_u Y_u^\dagger) \right) \\
&- 2 Y_u \Big(\mathrm{Tr}(6 T_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger + T_\nu Y_e^\dagger Y_e Y_\nu^\dagger + Y_\nu Y_e^\dagger T_e Y_\nu^\dagger) \\
&+ 3 \,\mathrm{Tr}(Y_u Y_d^\dagger T_d Y_u^\dagger + T_u Y_d^\dagger Y_d Y_u^\dagger + 6 T_u Y_u^\dagger Y_u Y_u^\dagger) \Big) \\
&+ \frac{2}{5} g_1^2 \left(T_u Y_d^\dagger Y_d + 2 Y_u Y_d^\dagger T_d + 3 Y_u Y_u^\dagger T_u - 2 M_1 (Y_u Y_d^\dagger Y_d + Y_u Y_u^\dagger Y_u) \right) \\
&+ 6 g_2^2 \left(Y_u Y_u^\dagger T_u + 2 T_u Y_u^\dagger Y_u - 2 M_2 Y_u Y_u^\dagger Y_u \right) \\
&+ \left(16 g_3^2 + \frac{4}{5} g_1^2 \right) \left(2 Y_u \,\mathrm{Tr}(Y_u^\dagger T_u) + T_u \,\mathrm{Tr}(Y_u^\dagger Y_u) \right) \\
&+ \frac{136}{45} g_1^2 g_3^2 T_u + \frac{15}{2} g_2^4 T_u - \frac{16}{9} g_3^4 T_u + \frac{2743}{450} g_1^4 T_u + g_1^2 g_2^2 T_u + 8 g_2^2 g_3^2 T_u \\
&- Y_u \,\mathrm{Tr}(Y_u^\dagger Y_u) \left(32 g_3^2 M_3 + \frac{8}{5} g_1^2 M_1 \right) - \frac{272}{45} g_1^2 g_3^2 Y_u (M_1 + M_3) \\
&- 2 g_1^2 g_2^2 Y_u (M_1 + M_2) - 16 g_2^2 g_3^2 Y_u (M_2 + M_3) \\
&- 30 g_2^4 M_2 Y_u - \frac{5486}{225} g_1^4 M_1 Y_u + \frac{64}{9} g_3^4 M_3 Y_u \,, && \text{(A.20)} \\
(16\pi^2)^2 \beta_{T_d}^{(2)} &= -2 T_d \left(3 Y_d^\dagger Y_d Y_d^\dagger Y_d + 2 Y_u^\dagger Y_u Y_d^\dagger Y_d + Y_u^\dagger Y_u Y_u^\dagger Y_u \right) \\
&- 2 Y_d \Big(4 Y_d^\dagger T_d Y_d^\dagger Y_d + 3 Y_d^\dagger Y_d Y_d^\dagger T_d + 2 Y_u^\dagger T_u Y_d^\dagger Y_d \\
&+ 2 Y_u^\dagger T_u Y_u^\dagger Y_u + Y_u^\dagger Y_u Y_d^\dagger T_d + 2 Y_u^\dagger Y_u Y_u^\dagger T_u \Big) \\
&- T_d \left(5 Y_d^\dagger Y_d \,\mathrm{Tr}(Y_e Y_e^\dagger + 3 Y_d Y_d^\dagger) + Y_u^\dagger Y_u \,\mathrm{Tr}(Y_\nu Y_\nu^\dagger + 3 Y_u Y_u^\dagger) \right) \\
&- 2 Y_d \Big(2 Y_d^\dagger T_d \,\mathrm{Tr}(Y_e Y_e^\dagger + 3 Y_d Y_d^\dagger) + 3 Y_d^\dagger Y_d \,\mathrm{Tr}(T_e Y_e^\dagger + 3 T_d Y_d^\dagger) \\
&+ Y_u^\dagger T_u \,\mathrm{Tr}(Y_\nu Y_\nu^\dagger + 3 Y_u Y_u^\dagger) + Y_u^\dagger Y_u \,\mathrm{Tr}(T_\nu Y_\nu^\dagger + 3 T_u Y_u^\dagger) \Big) \\
&- T_d \left(\mathrm{Tr}(Y_e Y_\nu^\dagger Y_\nu Y_e^\dagger + 3 Y_e Y_e^\dagger Y_e Y_e^\dagger) + 3 \,\mathrm{Tr}(Y_u Y_d^\dagger Y_d Y_u^\dagger + 3 Y_d Y_d^\dagger Y_d Y_d^\dagger) \right) \\
&- 2 Y_d \Big(\mathrm{Tr}(Y_e Y_\nu^\dagger T_\nu Y_e^\dagger + T_e Y_\nu^\dagger Y_\nu Y_e^\dagger + 6 T_e Y_e^\dagger Y_e Y_e^\dagger) \\
&+ 3 \,\mathrm{Tr}(T_d Y_u^\dagger Y_u Y_d^\dagger + T_u Y_d^\dagger Y_d Y_u^\dagger + 6 T_d Y_d^\dagger Y_d Y_d^\dagger) \Big)
\end{aligned}
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\begin{aligned}
&+\frac{2}{5}g_1^2\left(2T_dY_u^\dagger Y_u+3T_dY_d^\dagger Y_d+3Y_dY_d^\dagger T_d+4Y_dY_u^\dagger T_u-4M_1(Y_dY_d^\dagger Y_d+Y_dY_u^\dagger Y_u)\right)\\
&+6g_2^2\left(Y_dY_d^\dagger T_d+2T_dY_d^\dagger Y_d-2M_2Y_dY_d^\dagger Y_d\right)\\
&+\left(16g_3^2-\frac{2}{5}g_1^2\right)\left(2Y_d\,\mathrm{Tr}(Y_d^\dagger T_d)+T_d\,\mathrm{Tr}(Y_d^\dagger Y_d)\right)\\
&+\frac{6}{5}g_1^2\left(2Y_d\,\mathrm{Tr}(Y_e^\dagger T_e)+T_d\,\mathrm{Tr}(Y_e^\dagger Y_e)\right)\\
&+\frac{8}{9}g_1^2g_3^2T_d+\frac{15}{2}g_2^4T_d-\frac{16}{9}g_3^4T_d+\frac{287}{90}g_1^4T_d+g_1^2g_2^2T_d+8g_2^2g_3^2T_d\\
&-Y_d\,\mathrm{Tr}(Y_d^\dagger Y_d)\left(32g_3^2M_3-\frac{4}{5}g_1^2M_1\right)-\frac{12}{5}g_1^2M_1Y_d\,\mathrm{Tr}(Y_e^\dagger Y_e)\\
&-\frac{16}{9}g_1^2g_3^2Y_d(M_1+M_3)-2g_1^2g_2^2Y_d(M_1+M_2)-16g_2^2g_3^2Y_d(M_2+M_3)\\
&-30g_2^4M_2Y_d-\frac{574}{45}g_1^4M_1Y_d+\frac{64}{9}g_3^4M_3Y_d\,, \qquad\qquad \text{(A.21)}
\end{aligned}
$$

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\begin{aligned}
(16\pi^2)^2\beta_{T_e}^{(2)}=&-2T_e\left(3Y_e^\dagger Y_eY_e^\dagger Y_e+2Y_\nu^\dagger Y_\nu Y_e^\dagger Y_e+Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu\right)\\
&-2Y_e\Big(4Y_e^\dagger T_eY_e^\dagger Y_e+3Y_e^\dagger Y_eY_e^\dagger T_e+2Y_\nu^\dagger T_\nu Y_e^\dagger Y_e\\
&+2Y_\nu^\dagger T_\nu Y_\nu^\dagger Y_\nu+Y_\nu^\dagger Y_\nu Y_e^\dagger T_e+2Y_\nu^\dagger Y_\nu Y_\nu^\dagger T_\nu\Big)\\
&-T_e\left(5Y_e^\dagger Y_e\,\mathrm{Tr}(Y_eY_e^\dagger+3Y_dY_d^\dagger)+Y_\nu^\dagger Y_\nu\,\mathrm{Tr}(Y_\nu Y_\nu^\dagger+3Y_uY_u^\dagger)\right)\\
&-2Y_e\Big(2Y_e^\dagger T_e\,\mathrm{Tr}(Y_eY_e^\dagger+3Y_dY_d^\dagger)+3Y_e^\dagger Y_e\,\mathrm{Tr}(T_eY_e^\dagger+3T_dY_d^\dagger)\\
&+Y_\nu^\dagger T_\nu\,\mathrm{Tr}(Y_\nu Y_\nu^\dagger+3Y_uY_u^\dagger)+Y_\nu^\dagger Y_\nu\,\mathrm{Tr}(T_\nu Y_\nu^\dagger+3T_uY_u^\dagger)\Big)\\
&-T_e\left(\mathrm{Tr}(Y_eY_\nu^\dagger Y_\nu Y_e^\dagger+3Y_eY_e^\dagger Y_eY_e^\dagger)+3\,\mathrm{Tr}(Y_dY_u^\dagger Y_uY_d^\dagger+3Y_dY_d^\dagger Y_dY_d^\dagger)\right)\\
&-2Y_e\Big(\mathrm{Tr}(Y_eY_\nu^\dagger T_\nu Y_e^\dagger+T_eY_\nu^\dagger Y_\nu Y_e^\dagger+6T_eY_e^\dagger Y_eY_e^\dagger)\\
&+3\,\mathrm{Tr}(T_dY_u^\dagger Y_uY_d^\dagger+T_uY_d^\dagger Y_dY_u^\dagger+6T_dY_d^\dagger Y_dY_d^\dagger)\Big)\\
&+\frac{6}{5}g_1^2\left(Y_eY_e^\dagger T_e-T_eY_e^\dagger Y_e\right)\\
&+6g_2^2\left(Y_eY_e^\dagger T_e+2T_eY_e^\dagger Y_e-2M_2Y_eY_e^\dagger Y_e\right)\\
&+\left(16g_3^2-\frac{2}{5}g_1^2\right)\left(2Y_e\,\mathrm{Tr}(Y_d^\dagger T_d)+T_e\,\mathrm{Tr}(Y_d^\dagger Y_d)\right)\\
&+\frac{6}{5}g_1^2\left(2Y_e\,\mathrm{Tr}(Y_e^\dagger T_e)+T_e\,\mathrm{Tr}(Y_e^\dagger Y_e)\right)\\
&+\frac{15}{2}g_2^4T_e+\frac{27}{2}g_1^4T_e+\frac{9}{5}g_1^2g_2^2T_e\\
&-Y_e\,\mathrm{Tr}(Y_d^\dagger Y_d)\left(32g_3^2M_3-\frac{4}{5}g_1^2M_1\right)-\frac{12}{5}g_1^2M_1Y_e\,\mathrm{Tr}(Y_e^\dagger Y_e)\\
&-\frac{18}{5}g_1^2g_2^2Y_d(M_1+M_2)-30g_2^4M_2Y_e-54g_1^4M_1Y_e\,, \qquad\qquad \text{(A.22)}
\end{aligned}
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\begin{aligned}
(16\pi^2)^2\beta_{T_\nu}^{(2)}=&-2T_\nu\left(Y_e^\dagger Y_eY_e^\dagger Y_e+2Y_e^\dagger Y_eY_\nu^\dagger Y_\nu+3Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu\right)\\
&-2Y_\nu\Big(2Y_e^\dagger T_eY_e^\dagger Y_e+2Y_e^\dagger T_eY_\nu^\dagger Y_\nu+2Y_e^\dagger Y_eY_e^\dagger T_e\\
&+Y_e^\dagger Y_eY_\nu^\dagger T_\nu+4Y_\nu^\dagger T_\nu Y_\nu^\dagger Y_\nu+3Y_\nu^\dagger Y_\nu Y_\nu^\dagger T_\nu\Big)
\end{aligned}
$$

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\begin{aligned}
& - T_\nu \Big(Y_e^\dagger Y_e \operatorname{Tr}(Y_e Y_e^\dagger + 3Y_d Y_d^\dagger) + 5Y_\nu^\dagger Y_\nu \operatorname{Tr}(Y_\nu Y_\nu^\dagger + 3Y_u Y_u^\dagger)\Big) \\
& - 2Y_\nu \Big(Y_e^\dagger T_e \operatorname{Tr}(Y_e Y_e^\dagger + 3Y_d Y_d^\dagger) + Y_e^\dagger Y_e \operatorname{Tr}(T_e Y_e^\dagger + 3T_d Y_d^\dagger) \\
& + 2Y_\nu^\dagger T_\nu \operatorname{Tr}(Y_\nu Y_\nu^\dagger + 3Y_u Y_u^\dagger) + 3Y_\nu^\dagger Y_\nu \operatorname{Tr}(T_\nu Y_\nu^\dagger + 3T_u Y_u^\dagger)\Big) \\
& - T_\nu \Big(\operatorname{Tr}(3Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger + Y_\nu Y_e^\dagger Y_e Y_\nu^\dagger) + 3\operatorname{Tr}(Y_u Y_d^\dagger Y_d Y_u^\dagger + 3Y_u Y_u^\dagger Y_u Y_u^\dagger)\Big) \\
& - 2Y_\nu \Big(\operatorname{Tr}(T_\nu Y_e^\dagger Y_e Y_\nu^\dagger + Y_\nu Y_e^\dagger T_e Y_\nu^\dagger + 6T_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger) \\
& + 3\operatorname{Tr}(T_d Y_u^\dagger Y_u Y_d^\dagger + T_u Y_d^\dagger Y_d Y_u^\dagger + 6T_u Y_u^\dagger Y_u Y_u^\dagger)\Big) \\
& + \frac{6}{5} g_1^2 \Big(2T_\nu Y_\nu^\dagger Y_\nu + 2Y_\nu Y_e^\dagger T_e + Y_\nu Y_\nu^\dagger T_\nu + T_\nu Y_e^\dagger Y_e - 2M_1(Y_\nu Y_\nu^\dagger Y_\nu + Y_\nu Y_e^\dagger Y_e)\Big) \\
& + 6g_2^2 \left(Y_\nu Y_\nu^\dagger T_\nu + 2T_\nu Y_\nu^\dagger Y_\nu - 2M_2 Y_\nu Y_\nu^\dagger Y_\nu\right) \\
& + \left(16g_3^2 + \frac{4}{5} g_1^2\right) \left(2Y_\nu \operatorname{Tr}(Y_u^\dagger T_u) + T_\nu \operatorname{Tr}(Y_u^\dagger Y_u)\right) \\
& + \frac{15}{2} g_2^4 T_\nu + \frac{207}{50} g_1^4 T_\nu + \frac{9}{5} g_1^2 g_2^2 T_\nu \\
& - Y_\nu \operatorname{Tr}(Y_u^\dagger Y_u) \left(32g_3^2 M_3 + \frac{8}{5} g_1^2 M_1\right) \\
& - \frac{18}{5} g_1^2 g_2^2 Y_\nu (M_1 + M_2) - 30g_2^4 M_2 Y_\nu - \frac{414}{25} g_1^4 M_1 Y_\nu \,, \qquad \text{(A.23)}
\end{aligned}
$$

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\begin{aligned}
(16\pi^2)^2 \beta^{(2)}_{m^2_{\tilde{L}}} = & -4T_e^\dagger \Big(T_e Y_e^\dagger Y_e + Y_e Y_e^\dagger T_e\Big) - 4T_\nu^\dagger \Big(T_\nu Y_\nu^\dagger Y_\nu + Y_\nu Y_\nu^\dagger T_\nu\Big) \\
& - 2Y_e^\dagger \Big(2m^2_{\tilde{e}} Y_e Y_e^\dagger Y_e + 2T_e T_e^\dagger Y_e + 2Y_e T_e^\dagger T_e \\
& + 2Y_e Y_e^\dagger m^2_{\tilde{e}} Y_e + Y_e Y_e^\dagger Y_e m^2_{\tilde{L}} + 2Y_e m^2_{\tilde{L}} Y_e^\dagger Y_e\Big) \\
& - 2Y_\nu^\dagger \Big(2m^2_{\tilde{\nu}} Y_\nu Y_\nu^\dagger Y_\nu + 2T_\nu T_\nu^\dagger Y_\nu + 2Y_\nu T_\nu^\dagger T_\nu \\
& + 2Y_\nu Y_\nu^\dagger m^2_{\tilde{\nu}} Y_\nu + Y_\nu Y_\nu^\dagger Y_\nu m^2_{\tilde{L}} + 2Y_\nu m^2_{\tilde{L}} Y_\nu^\dagger Y_\nu\Big) \\
& - 2\left(4m^2_{H_d} + m^2_{\tilde{L}}\right) Y_e^\dagger Y_e Y_e^\dagger Y_e - 2\left(4m^2_{H_u} + m^2_{\tilde{L}}\right) Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu \\
& - 2T_e^\dagger \Big(T_e \operatorname{Tr}(Y_e Y_e^\dagger + 3Y_d Y_d^\dagger) + Y_e \operatorname{Tr}(T_e Y_e^\dagger + 3T_d Y_d^\dagger)\Big) \\
& - 2T_\nu^\dagger \Big(T_\nu \operatorname{Tr}(Y_\nu Y_\nu^\dagger + 3Y_u Y_u^\dagger) + Y_\nu \operatorname{Tr}(T_\nu Y_\nu^\dagger + 3T_u Y_u^\dagger)\Big) \\
& - Y_e^\dagger \Big(2m^2_{\tilde{e}} Y_e \operatorname{Tr}(Y_e Y_e^\dagger + 3Y_d Y_d^\dagger) + 2T_e \operatorname{Tr}(Y_e T_e^\dagger + 3Y_d T_d^\dagger) \\
& + 2Y_e \operatorname{Tr}(T_e T_e^\dagger + 3T_d T_d^\dagger) + Y_e m^2_{\tilde{L}} \operatorname{Tr}(Y_e Y_e^\dagger + 3Y_d Y_d^\dagger)\Big) \\
& - Y_\nu^\dagger \Big(2m^2_{\tilde{\nu}} Y_\nu \operatorname{Tr}(Y_\nu Y_\nu^\dagger + 3Y_u Y_u^\dagger) + 2T_\nu \operatorname{Tr}(Y_\nu T_\nu^\dagger + 3Y_u T_u^\dagger) \\
& + 2Y_\nu \operatorname{Tr}(T_\nu T_\nu^\dagger + 3T_u T_u^\dagger) + Y_\nu m^2_{\tilde{L}} \operatorname{Tr}(Y_\nu Y_\nu^\dagger + 3Y_u Y_u^\dagger)) \\
& - \left(4m^2_{H_d} + m^2_{\tilde{L}}\right) Y_e^\dagger Y_e \operatorname{Tr}(Y_e Y_e^\dagger + 3Y_d Y_d^\dagger) \\
& - \left(4m^2_{H_u} + m^2_{\tilde{L}}\right) Y_\nu^\dagger Y_\nu \operatorname{Tr}(Y_\nu Y_\nu^\dagger + 3Y_u Y_u^\dagger) \\
& - 2Y_e^\dagger Y_e \operatorname{Tr}\left(Y_e m^2_{\tilde{L}} Y_e^\dagger + Y_e^\dagger m^2_{\tilde{e}} Y_e + 3Y_d m^2_{\tilde{Q}} Y_d^\dagger + 3Y_d^\dagger m^2_{\tilde{d}} Y_d\right) \\
& - 2Y_\nu^\dagger Y_\nu \operatorname{Tr}\left(Y_\nu m^2_{\tilde{L}} Y_\nu^\dagger + Y_\nu^\dagger m^2_{\tilde{\nu}} Y_\nu + 3Y_u m^2_{\tilde{Q}} Y_u^\dagger + 3Y_u^\dagger m^2_{\tilde{u}} Y_u\right) \\
& + \frac{6}{5} g_1^2 \Big(2m^2_{H_d} Y_e^\dagger Y_e + m^2_{\tilde{L}} Y_e^\dagger Y_e + Y_e^\dagger Y_e m^2_{\tilde{L}} + 2Y_e^\dagger m^2_{\tilde{e}} Y_e + 2T_e^\dagger T_e\Big)
\end{aligned}
$$

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\begin{aligned}
&- \frac{12}{5} g_1^2 \left(M_1^* Y_e^\dagger T_e - 2|M_1|^2 Y_e^\dagger Y_e + M_1 T_e^\dagger Y_e\right) \\
&+ \frac{621}{25} g_1^4 |M_1|^2 \ \mathbf{1}_3 + \frac{18}{5} g_1^2 g_2^2 (|M_1|^2 + |M_2|^2) \ \mathbf{1}_3 \\
&+ \frac{18}{5} g_1^2 g_2^2 Re(M_1 M_2^*) \ \mathbf{1}_3 + 33 g_2^4 |M_2|^2 \ \mathbf{1}_3 \\
&+ \frac{3}{5} g_1^2 \sigma_1 \ \mathbf{1}_3 + 3 g_2^2 \sigma_2 \ \mathbf{1}_3 - \frac{6}{5} g_1^2 S' \ \mathbf{1}_3 \,, \qquad \text{(A.24)}
\end{aligned}
$$

$$
\begin{aligned}
(16\pi^2)^2 \beta^{(2)}_{m^2_{\tilde{Q}}} = & -4 T_d^\dagger \left(T_d Y_d^\dagger Y_d + Y_d Y_d^\dagger T_d\right) - 4 T_u^\dagger \left(T_u Y_u^\dagger Y_u + Y_u Y_u^\dagger T_u\right) \\
&- 2 Y_d^\dagger \Big(2 m^2_{\tilde{d}} Y_d Y_d^\dagger Y_d + 2 T_d T_d^\dagger Y_d + 2 Y_d T_d^\dagger T_d \\
&+ 2 Y_d Y_d^\dagger m^2_{\tilde{d}} Y_d + Y_d Y_d^\dagger Y_d m^2_{\tilde{Q}} + 2 Y_d m^2_{\tilde{Q}} Y_d^\dagger Y_d\Big) \\
&- 2 Y_u^\dagger \Big(2 m^2_{\tilde{u}} Y_u Y_u^\dagger Y_u + 2 T_u T_u^\dagger Y_u + 2 Y_u T_u^\dagger T_u \\
&+ 2 Y_u Y_u^\dagger m^2_{\tilde{u}} Y_u + Y_u Y_u^\dagger Y_u m^2_{\tilde{Q}} + 2 Y_u m^2_{\tilde{Q}} Y_u^\dagger Y_u\Big) \\
&- 2 \left(4 m^2_{H_d} + m^2_{\tilde{Q}}\right) Y_d^\dagger Y_d Y_d^\dagger Y_d - 2 \left(4 m^2_{H_u} + m^2_{\tilde{Q}}\right) Y_u^\dagger Y_u Y_u^\dagger Y_u \\
&- 2 T_d^\dagger \left(T_d \operatorname{Tr}(Y_e Y_e^\dagger + 3 Y_d Y_d^\dagger) + Y_d \operatorname{Tr}(T_e Y_e^\dagger + 3 T_d Y_d^\dagger)\right) \\
&- 2 T_u^\dagger \left(T_u \operatorname{Tr}(Y_\nu Y_\nu^\dagger + 3 Y_u Y_u^\dagger) + Y_u \operatorname{Tr}(T_\nu Y_\nu^\dagger + 3 T_u Y_u^\dagger)\right) \\
&- Y_d^\dagger \Big(2 m^2_{\tilde{d}} Y_d \operatorname{Tr}(Y_e Y_e^\dagger + 3 Y_d Y_d^\dagger) + 2 T_d \operatorname{Tr}(Y_e T_e^\dagger + 3 Y_d T_d^\dagger) \\
&+ 2 Y_d \operatorname{Tr}(T_e T_e^\dagger + 3 T_d T_d^\dagger) + Y_d m^2_{\tilde{Q}} \operatorname{Tr}(Y_e Y_e^\dagger + 3 Y_d Y_d^\dagger)\Big) \\
&- Y_u^\dagger \Big(2 m^2_{\tilde{u}} Y_u \operatorname{Tr}(Y_\nu Y_\nu^\dagger + 3 Y_u Y_u^\dagger) + 2 T_u \operatorname{Tr}(Y_\nu T_\nu^\dagger + 3 Y_u T_u^\dagger) \\
&+ 2 Y_u \operatorname{Tr}(T_\nu T_\nu^\dagger + 3 T_u T_u^\dagger) + Y_u m^2_{\tilde{Q}} \operatorname{Tr}(Y_\nu Y_\nu^\dagger + 3 Y_u Y_u^\dagger)\Big) \\
&- \left(4 m^2_{H_d} + m^2_{\tilde{Q}}\right) Y_d^\dagger Y_d \operatorname{Tr}(Y_e Y_e^\dagger + 3 Y_d Y_d^\dagger) \\
&- \left(4 m^2_{H_u} + m^2_{\tilde{Q}}\right) Y_u^\dagger Y_u \operatorname{Tr}(Y_\nu Y_\nu^\dagger + 3 Y_u Y_u^\dagger) \\
&- 2 Y_d^\dagger Y_d \operatorname{Tr}\left(Y_e m^2_{\tilde{L}} Y_e^\dagger + Y_e^\dagger m^2_{\tilde{e}} Y_e + 3 Y_d m^2_{\tilde{Q}} Y_d^\dagger + 3 Y_d^\dagger m^2_{\tilde{d}} Y_d\right) \\
&- 2 Y_u^\dagger Y_u \operatorname{Tr}\left(Y_\nu m^2_{\tilde{L}} Y_\nu^\dagger + Y_\nu^\dagger m^2_{\tilde{\nu}} Y_\nu + 3 Y_u m^2_{\tilde{Q}} Y_u^\dagger + 3 Y_u^\dagger m^2_{\tilde{u}} Y_u\right) \\
&+ \frac{2}{5} g_1^2 \Big(2 m^2_{H_d} Y_d^\dagger Y_d + 4 m^2_{H_u} Y_u^\dagger Y_u + m^2_{\tilde{Q}} Y_d^\dagger Y_d + Y_d^\dagger Y_d m^2_{\tilde{Q}} + 2 Y_d^\dagger m^2_{\tilde{d}} Y_d \\
&+ 2 m^2_{\tilde{Q}} Y_u^\dagger Y_u + 2 Y_u^\dagger Y_u m^2_{\tilde{Q}} + 4 Y_u^\dagger m^2_{\tilde{u}} Y_u + 2 T_d^\dagger T_d + 4 T_u^\dagger T_u\Big) \\
&- \frac{4}{5} g_1^2 \Big(M_1^* Y_d^\dagger T_d + 2 M_1^* Y_u^\dagger T_u - 4 |M_1|^2 Y_u^\dagger Y_u \\
&+ M_1 T_d^\dagger Y_d + 2 M_1 T_u^\dagger Y_u - 2 |M_1|^2 Y_d^\dagger Y_d\Big) \\
&- \frac{128}{3} g_3^4 |M_3|^2 \ \mathbf{1}_3 + \frac{2}{5} g_1^2 g_2^2 Re\left(M_1 M_2^*\right) \ \mathbf{1}_3 + \frac{32}{45} g_1^2 g_3^2 Re\left(M_1 M_3^*\right) \ \mathbf{1}_3 \\
&+ \frac{199}{75} g_1^4 |M_1|^2 \ \mathbf{1}_3 + \frac{2}{5} g_1^2 g_2^2 (|M_1|^2 + |M_2|^2) \ \mathbf{1}_3 \\
&+ 32 g_2^2 g_3^2 (|M_2|^2 + Re(M_2 M_3^*) + |M_3|^2) \ \mathbf{1}_3 \\
&+ \frac{32}{45} g_1^2 g_3^2 (|M_1|^2 + |M_3|^2) \ \mathbf{1}_3 + 33 g_2^4 |M_2|^2 \ \mathbf{1}_3
\end{aligned}
$$

$$+\frac{1}{15}g_1^2\sigma_1\ \mathbf{1}_3+3g_2^2\sigma_2\ \mathbf{1}_3+\frac{16}{3}g_3^2\sigma_3\ \mathbf{1}_3+\frac{2}{5}g_1^2S'\ \mathbf{1}_3\ , \tag{A.25}$$

$$\begin{aligned}
(16\pi^2)^2\beta^{(2)}_{m^2_{\tilde u}} = & -2\left(2m^2_{H_d}+2m^2_{H_u}+m^2_{\tilde u}\right)Y_uY_d^\dagger Y_dY_u^\dagger-2\left(4m^2_{H_u}+m^2_{\tilde u}\right)Y_uY_u^\dagger Y_uY_u^\dagger\\
&-4T_u\left(T_d^\dagger Y_dY_u^\dagger+T_u^\dagger Y_uY_u^\dagger+Y_d^\dagger Y_dT_u^\dagger+Y_u^\dagger Y_uT_u^\dagger\right)\\
&-4Y_u\left(T_d^\dagger T_dY_u^\dagger+T_u^\dagger T_uY_u^\dagger+Y_d^\dagger T_dT_u^\dagger+Y_u^\dagger T_uT_u^\dagger\right)\\
&-2Y_u\left(2Y_d^\dagger m^2_{\tilde d}Y_dY_u^\dagger+Y_d^\dagger Y_dY_u^\dagger m^2_{\tilde u}+2Y_d^\dagger Y_dm^2_{\tilde Q}Y_u^\dagger+2Y_u^\dagger m^2_{\tilde u}Y_uY_u^\dagger\right.\\
&\left.+Y_u^\dagger Y_uY_u^\dagger m^2_{\tilde u}+2Y_u^\dagger Y_um^2_{\tilde Q}Y_u^\dagger+2m^2_{\tilde Q}Y_d^\dagger Y_dY_u^\dagger+2m^2_{\tilde Q}Y_u^\dagger Y_uY_u^\dagger\right)\\
&-2\left(4m^2_{H_u}+m^2_{\tilde u}\right)Y_uY_u^\dagger\,\mathrm{Tr}(Y_\nu Y_\nu^\dagger+3Y_uY_u^\dagger)\\
&-4T_u\left(T_u^\dagger\,\mathrm{Tr}(Y_\nu Y_\nu^\dagger+3Y_uY_u^\dagger)+Y_u^\dagger\,\mathrm{Tr}(Y_\nu T_\nu^\dagger+3Y_uT_u^\dagger)\right)\\
&-4Y_u\left(T_u^\dagger\,\mathrm{Tr}(T_\nu Y_\nu^\dagger+3T_uY_u^\dagger)+Y_u^\dagger\,\mathrm{Tr}(T_\nu T_\nu^\dagger+3T_uT_u^\dagger)\right)\\
&-2Y_u^\dagger\left(Y_u^\dagger m^2_{\tilde u}\,\mathrm{Tr}(Y_\nu Y_\nu^\dagger+3Y_uY_u^\dagger)+2m^2_{\tilde Q}Y_u^\dagger\,\mathrm{Tr}(Y_\nu Y_\nu^\dagger+3Y_uY_u^\dagger)\right.\\
&\left.+2Y_u^\dagger\,\mathrm{Tr}(Y_\nu m^2_{\tilde L}Y_\nu^\dagger+Y_\nu^\dagger m^2_{\tilde\nu}Y_\nu+3Y_um^2_{\tilde Q}Y_u^\dagger+3Y_u^\dagger m^2_{\tilde u}Y_u)\right)\\
&+\left(6g_2^2-\frac{2}{5}g_1^2\right)\left(m^2_{\tilde u}Y_uY_u^\dagger+Y_uY_u^\dagger m^2_{\tilde u}+2m^2_{H_u}Y_uY_u^\dagger+2Y_um^2_{\tilde Q}Y_u^\dagger+2T_uT_u^\dagger\right)\\
&-12g_2^2\left(M_2^*T_uY_u^\dagger-2|M_2|^2Y_uY_u^\dagger+M_2Y_uT_u^\dagger\right)\\
&+\frac{4}{5}g_1^2\left(M_1^*T_uY_u^\dagger-2|M_1|^2Y_uY_u^\dagger+M_1Y_uT_u^\dagger\right)\\
&-\frac{128}{3}g_3^4|M_3|^2\ \mathbf{1}_3+\frac{512}{45}g_1^2g_3^2\left(|M_1|^2+Re(M_1M_3^*)+|M_3|^2\right)\ \mathbf{1}_3\\
&+\frac{3424}{75}g_1^4|M_1|^2\ \mathbf{1}_3+\frac{16}{15}g_1^2\sigma_1\ \mathbf{1}_3+\frac{16}{3}g_3^2\sigma_3\ \mathbf{1}_3-\frac{8}{5}g_1^2S'\ \mathbf{1}_3\ ,
\end{aligned} \tag{A.26}$$

$$\begin{aligned}
(16\pi^2)^2\beta^{(2)}_{m^2_{\tilde d}} = & -2\left(2m^2_{H_d}+2m^2_{H_u}+m^2_{\tilde d}\right)Y_dY_u^\dagger Y_uY_d^\dagger-2\left(4m^2_{H_d}+m^2_{\tilde d}\right)Y_dY_d^\dagger Y_dY_d^\dagger\\
&-4T_d\left(T_d^\dagger Y_dY_d^\dagger+T_u^\dagger Y_uY_d^\dagger+Y_d^\dagger Y_dT_d^\dagger+Y_u^\dagger Y_uT_d^\dagger\right)\\
&-4Y_d\left(T_d^\dagger T_dY_d^\dagger+T_u^\dagger T_uY_d^\dagger+Y_d^\dagger T_dT_d^\dagger+Y_u^\dagger T_uT_d^\dagger\right)\\
&-2Y_d\left(2Y_d^\dagger m^2_{\tilde d}Y_dY_d^\dagger+Y_d^\dagger Y_dY_d^\dagger m^2_{\tilde d}+2Y_d^\dagger Y_dm^2_{\tilde Q}Y_d^\dagger+2Y_u^\dagger m^2_{\tilde u}Y_uY_d^\dagger\right.\\
&\left.+Y_u^\dagger Y_uY_d^\dagger m^2_{\tilde d}+2Y_u^\dagger Y_um^2_{\tilde Q}Y_d^\dagger+2m^2_{\tilde Q}Y_d^\dagger Y_dY_d^\dagger+2m^2_{\tilde Q}Y_u^\dagger Y_uY_d^\dagger\right)\\
&-2\left(4m^2_{H_d}+m^2_{\tilde d}\right)Y_dY_d^\dagger\,\mathrm{Tr}(Y_eY_e^\dagger+3Y_dY_d^\dagger)\\
&-4T_d\left(T_d^\dagger\,\mathrm{Tr}(Y_eY_e^\dagger+3Y_dY_d^\dagger)+Y_d^\dagger\,\mathrm{Tr}(Y_eT_e^\dagger+3Y_dT_d^\dagger)\right)\\
&-4Y_d\left(T_d^\dagger\,\mathrm{Tr}(T_eY_e^\dagger+3T_dY_d^\dagger)+Y_d^\dagger\,\mathrm{Tr}(T_eT_e^\dagger+3T_dT_d^\dagger)\right)\\
&-2Y_d^\dagger\left(Y_d^\dagger m^2_{\tilde d}\,\mathrm{Tr}(Y_eY_e^\dagger+3Y_dY_d^\dagger)+2m^2_{\tilde Q}Y_d^\dagger\,\mathrm{Tr}(Y_eY_e^\dagger+3Y_dY_d^\dagger)\right.\\
&\left.+2Y_d^\dagger\,\mathrm{Tr}(Y_em^2_{\tilde L}Y_e^\dagger+Y_e^\dagger m^2_{\tilde e}Y_e+3Y_dm^2_{\tilde Q}Y_d^\dagger+3Y_d^\dagger m^2_{\tilde d}Y_d)\right)\\
&+\left(6g_2^2+\frac{2}{5}g_1^2\right)\left(m^2_{\tilde d}Y_dY_d^\dagger+Y_dY_d^\dagger m^2_{\tilde d}+2m^2_{H_d}Y_dY_d^\dagger+2Y_dm^2_{\tilde Q}Y_d^\dagger+2T_dT_d^\dagger\right)\\
&-12g_2^2\left(M_2^*T_dY_d^\dagger-2|M_2|^2Y_dY_d^\dagger+M_2Y_dT_d^\dagger\right)
\end{aligned}$$

$$
\begin{aligned}
&- \frac{4}{5} g_1^2 \left(M_1^* T_d Y_d^\dagger - 2|M_1|^2 Y_d Y_d^\dagger + M_1 Y_d T_d^\dagger \right) \\
&- \frac{128}{3} g_3^4 |M_3|^2 \ \mathbf{1}_3 + \frac{128}{45} g_1^2 g_3^2 \left(|M_1|^2 + Re(M_1 M_3^*) + |M_3|^2 \right) \ \mathbf{1}_3 \\
&+ \frac{808}{75} g_1^4 |M_1|^2 \ \mathbf{1}_3 + \frac{4}{15} g_1^2 \sigma_1 \ \mathbf{1}_3 + \frac{16}{3} g_3^2 \sigma_3 \ \mathbf{1}_3 + \frac{4}{5} g_1^2 S' \ \mathbf{1}_3 \ , \qquad \text{(A.27)}
\end{aligned}
$$

$$
\begin{aligned}
(16\pi^2)^2 \beta^{(2)}_{m^2_{\tilde{e}}} = &-2 \left(2m^2_{H_d} + 2m^2_{H_u} + m^2_{\tilde{e}} \right) Y_e Y_\nu^\dagger Y_\nu Y_e^\dagger - 2 \left(4m^2_{H_d} + m^2_{\tilde{e}} \right) Y_e Y_e^\dagger Y_e Y_e^\dagger \\
&- 4 T_e \left(T_e^\dagger Y_e Y_e^\dagger + T_\nu^\dagger Y_\nu Y_e^\dagger + Y_e^\dagger Y_e T_e^\dagger + Y_\nu^\dagger Y_\nu T_e^\dagger \right) \\
&- 4 Y_e \left(T_e^\dagger T_e Y_e^\dagger + T_\nu^\dagger T_\nu Y_e^\dagger + Y_e^\dagger T_e T_e^\dagger + Y_\nu^\dagger T_\nu T_e^\dagger \right) \\
&- 2 Y_e \left(2 Y_e^\dagger m^2_{\tilde{e}} Y_e Y_e^\dagger + Y_e^\dagger Y_e Y_e^\dagger m^2_{\tilde{e}} + 2 Y_e^\dagger Y_e m^2_{\tilde{L}} Y_e^\dagger + 2 Y_\nu^\dagger m^2_{\tilde{\nu}} Y_\nu Y_e^\dagger \right. \\
&+ \left. Y_\nu^\dagger Y_\nu Y_e^\dagger m^2_{\tilde{e}} + 2 Y_\nu^\dagger Y_\nu m^2_{\tilde{L}} Y_e^\dagger + 2 m^2_{\tilde{L}} Y_e^\dagger Y_e Y_e^\dagger + 2 m^2_{\tilde{L}} Y_\nu^\dagger Y_\nu Y_e^\dagger \right) \\
&- 2 \left(4 m^2_{H_d} + m^2_{\tilde{e}} \right) Y_e Y_e^\dagger \operatorname{Tr}(Y_e Y_e^\dagger + 3 Y_d Y_d^\dagger) \\
&- 4 T_e \left(T_e^\dagger \operatorname{Tr}(Y_e Y_e^\dagger + 3 Y_d Y_d^\dagger) + Y_e^\dagger \operatorname{Tr}(Y_e T_e^\dagger + 3 Y_d T_d^\dagger) \right) \\
&- 4 Y_e \left(T_e^\dagger \operatorname{Tr}(T_e Y_e^\dagger + 3 T_d Y_d^\dagger) + Y_e^\dagger \operatorname{Tr}(T_e T_e^\dagger + 3 T_d T_d^\dagger) \right) \\
&- 2 Y_e^\dagger \left(Y_e^\dagger m^2_{\tilde{e}} \operatorname{Tr}(Y_e Y_e^\dagger + 3 Y_d Y_d^\dagger) + 2 m^2_{\tilde{L}} Y_e^\dagger \operatorname{Tr}(Y_e Y_e^\dagger + 3 Y_d Y_d^\dagger) \right. \\
&+ \left. 2 Y_e^\dagger \operatorname{Tr}(Y_e m^2_{\tilde{L}} Y_e^\dagger + Y_e^\dagger m^2_{\tilde{e}} Y_e + 3 Y_d m^2_{\tilde{Q}} Y_d^\dagger + 3 Y_d^\dagger m^2_{\tilde{d}} Y_d) \right) \\
&+ \left(6 g_2^2 - \frac{6}{5} g_1^2 \right) \left(m^2_{\tilde{e}} Y_e Y_e^\dagger + Y_e Y_e^\dagger m^2_{\tilde{e}} + 2 m^2_{H_d} Y_e Y_e^\dagger + 2 Y_e m^2_{\tilde{L}} Y_e^\dagger + 2 T_e T_e^\dagger \right) \\
&- 12 g_2^2 \left(M_2^* T_e Y_e^\dagger - 2|M_2|^2 Y_e Y_e^\dagger + M_2 Y_e T_e^\dagger \right) \\
&+ \frac{12}{5} g_1^2 \left(M_1^* T_e Y_e^\dagger - 2|M_1|^2 Y_e Y_e^\dagger + M_1 Y_e T_e^\dagger \right) \\
&+ \frac{2808}{25} g_1^4 |M_1|^2 \ \mathbf{1}_3 + \frac{12}{5} g_1^2 \sigma_1 \ \mathbf{1}_3 + \frac{12}{5} g_1^2 S' \ \mathbf{1}_3 \ , \qquad \text{(A.28)}
\end{aligned}
$$

$$
\begin{aligned}
(16\pi^2)^2 \beta^{(2)}_{m^2_{\tilde{\nu}}} = &-2 \left(2m^2_{H_d} + 2m^2_{H_u} + m^2_{\tilde{\nu}} \right) Y_\nu Y_e^\dagger Y_e Y_\nu^\dagger - 2 \left(4m^2_{H_u} + m^2_{\tilde{\nu}} \right) Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger \\
&- 4 T_\nu \left(T_e^\dagger Y_e Y_\nu^\dagger + T_\nu^\dagger Y_\nu Y_\nu^\dagger + Y_e^\dagger Y_e T_\nu^\dagger + Y_\nu^\dagger Y_\nu T_\nu^\dagger \right) \\
&- 4 Y_\nu \left(T_e^\dagger T_e Y_\nu^\dagger + T_\nu^\dagger T_\nu Y_\nu^\dagger + Y_e^\dagger T_e T_\nu^\dagger + Y_\nu^\dagger T_\nu T_\nu^\dagger \right) \\
&- 2 Y_\nu \left(2 Y_e^\dagger m^2_{\tilde{e}} Y_e Y_\nu^\dagger + Y_e^\dagger Y_e Y_\nu^\dagger m^2_{\tilde{\nu}} + 2 Y_e^\dagger Y_e m^2_{\tilde{L}} Y_\nu^\dagger + 2 Y_\nu^\dagger m^2_{\tilde{\nu}} Y_\nu Y_\nu^\dagger \right. \\
&+ \left. Y_\nu^\dagger Y_\nu Y_\nu^\dagger m^2_{\tilde{\nu}} + 2 Y_\nu^\dagger Y_\nu m^2_{\tilde{L}} Y_\nu^\dagger + 2 m^2_{\tilde{L}} Y_e^\dagger Y_e Y_\nu^\dagger + 2 m^2_{\tilde{L}} Y_\nu^\dagger Y_\nu Y_\nu^\dagger \right) \\
&- 2 \left(4 m^2_{H_u} + m^2_{\tilde{\nu}} \right) Y_\nu Y_\nu^\dagger \operatorname{Tr}(Y_\nu Y_\nu^\dagger + 3 Y_u Y_u^\dagger) \\
&- 4 T_\nu \left(T_\nu^\dagger \operatorname{Tr}(Y_\nu Y_\nu^\dagger + 3 Y_u Y_u^\dagger) + Y_\nu^\dagger \operatorname{Tr}(Y_\nu T_\nu^\dagger + 3 Y_u T_u^\dagger) \right) \\
&- 4 Y_\nu \left(T_\nu^\dagger \operatorname{Tr}(T_\nu Y_\nu^\dagger + 3 T_u Y_u^\dagger) + Y_\nu^\dagger \operatorname{Tr}(T_\nu T_\nu^\dagger + 3 T_u T_u^\dagger) \right) \\
&- 2 Y_\nu^\dagger \left(Y_\nu^\dagger m^2_{\tilde{\nu}} \operatorname{Tr}(Y_\nu Y_\nu^\dagger + 3 Y_u Y_u^\dagger) + 2 m^2_{\tilde{L}} Y_\nu^\dagger \operatorname{Tr}(Y_\nu Y_\nu^\dagger + 3 Y_u Y_u^\dagger) \right. \\
&+ \left. 2 Y_\nu^\dagger \operatorname{Tr}(Y_\nu m^2_{\tilde{L}} Y_\nu^\dagger + Y_\nu^\dagger m^2_{\tilde{\nu}} Y_\nu + 3 Y_u m^2_{\tilde{Q}} Y_u^\dagger + 3 Y_u^\dagger m^2_{\tilde{u}} Y_u) \right) \\
&+ \left(6 g_2^2 + \frac{6}{5} g_1^2 \right) \left(m^2_{\tilde{\nu}} Y_\nu Y_\nu^\dagger + Y_\nu Y_\nu^\dagger m^2_{\tilde{\nu}} + 2 m^2_{H_u} Y_\nu Y_\nu^\dagger + 2 Y_\nu m^2_{\tilde{L}} Y_\nu^\dagger + 2 T_\nu T_\nu^\dagger \right)
\end{aligned}
$$

$$
\begin{aligned}
&- 12g_2^2\left(M_2^* T_\nu Y_\nu^\dagger - 2|M_2|^2 Y_\nu Y_\nu^\dagger + M_2 Y_\nu T_\nu^\dagger\right)\\
&- \frac{12}{5}g_1^2\left(M_1^* T_\nu Y_\nu^\dagger - 2|M_1|^2 Y_\nu Y_\nu^\dagger + M_1 Y_\nu T_\nu^\dagger\right)\,, \qquad \text{(A.29)}
\end{aligned}
$$

$$
\begin{aligned}
(16\pi^2)^2\beta^{(2)}_{m^2_{H_d}} &= -2\left(m^2_{H_d} + m^2_{H_u}\right)\mathrm{Tr}(Y_e Y_\nu^\dagger Y_\nu Y_e^\dagger + 3Y_d Y_u^\dagger Y_u Y_d^\dagger)\\
&- 12 m^2_{H_d}\,\mathrm{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger + 3Y_d Y_d^\dagger Y_d Y_d^\dagger)\\
&- 12\,\mathrm{Tr}(T_e^\dagger T_e Y_e^\dagger Y_e + 3T_d^\dagger T_d Y_d^\dagger Y_d + T_e^\dagger Y_e Y_e^\dagger T_e + 3T_d^\dagger Y_d Y_d^\dagger T_d)\\
&- 2\,\mathrm{Tr}(T_e T_\nu^\dagger Y_\nu Y_e^\dagger + 3T_d T_u^\dagger Y_u Y_d^\dagger + T_e Y_\nu^\dagger Y_\nu T_e^\dagger + 3T_d Y_u^\dagger Y_u T_d^\dagger)\\
&- 2\,\mathrm{Tr}(Y_e T_\nu^\dagger T_\nu Y_e^\dagger + 3Y_d T_u^\dagger T_u Y_d^\dagger + Y_e Y_\nu^\dagger T_\nu T_e^\dagger + 3Y_d Y_u^\dagger T_u T_d^\dagger)\\
&- 36\,\mathrm{Tr}(Y_d Y_d^\dagger m^2_{\tilde d} Y_d Y_d^\dagger + Y_d^\dagger Y_d m^2_{\tilde Q} Y_d^\dagger Y_d)\\
&- 12\,\mathrm{Tr}(Y_e Y_e^\dagger m^2_{\tilde e} Y_e Y_e^\dagger + Y_e^\dagger Y_e m^2_{\tilde L} Y_e^\dagger Y_e)\\
&- 6\,\mathrm{Tr}(Y_u Y_d^\dagger m^2_{\tilde d} Y_d Y_u^\dagger + Y_d Y_u^\dagger m^2_{\tilde u} Y_u Y_d^\dagger + Y_u^\dagger Y_u m^2_{\tilde Q} Y_d^\dagger Y_d + Y_d^\dagger Y_d m^2_{\tilde Q} Y_u^\dagger Y_u)\\
&- 2\,\mathrm{Tr}(Y_\nu Y_e^\dagger m^2_{\tilde e} Y_e Y_\nu^\dagger + Y_e Y_\nu^\dagger m^2_{\tilde \nu} Y_\nu Y_e^\dagger + Y_\nu^\dagger Y_\nu m^2_{\tilde L} Y_e^\dagger Y_e + Y_e^\dagger Y_e m^2_{\tilde L} Y_\nu^\dagger Y_\nu)\\
&+ \left(32g_3^2 - \frac{4}{5}g_1^2\right)\mathrm{Tr}(T_d^\dagger T_d + m^2_{H_d} Y_d^\dagger Y_d + Y_d m^2_{\tilde Q} Y_d^\dagger + Y_d^\dagger m^2_{\tilde d} Y_d)\\
&+ \frac{12}{5}g_1^2\,\mathrm{Tr}(T_e^\dagger T_e + m^2_{H_d} Y_e^\dagger Y_e + Y_e m^2_{\tilde L} Y_e^\dagger + Y_e^\dagger m^2_{\tilde e} Y_e)\\
&- \frac{12}{5}g_1^2\,\mathrm{Tr}(M_1^* Y_e^\dagger T_e + M_1 T_e^\dagger Y_e - 2|M_1|^2 Y_e^\dagger Y_e)\\
&+ \frac{4}{5}g_1^2\,\mathrm{Tr}(M_1^* Y_d^\dagger T_d + M_1 T_d^\dagger Y_d - 2|M_1|^2 Y_d^\dagger Y_d)\\
&- 32g_3^2\,\mathrm{Tr}(M_3^* Y_d^\dagger T_d + M_3 T_d^\dagger Y_d - 2|M_3|^2 Y_d^\dagger Y_d)\\
&+ \frac{18}{5}g_1^2 g_2^2\left(|M_1|^2 + Re(M_1 M_2^*) + |M_2|^2\right)\\
&+ \frac{621}{25}g_1^4|M_1|^2 + 33g_2^4|M_2|^2 + \frac{3}{5}g_1^2\sigma_1 + 3g_2^2\sigma_2 - \frac{6}{5}g_1^2 S'\,, \qquad \text{(A.30)}
\end{aligned}
$$

$$
\begin{aligned}
(16\pi^2)^2\beta^{(2)}_{m^2_{H_u}} &= -2\left(m^2_{H_d} + m^2_{H_u}\right)\mathrm{Tr}(Y_e Y_\nu^\dagger Y_\nu Y_e^\dagger + 3Y_d Y_u^\dagger Y_u Y_d^\dagger)\\
&- 12 m^2_{H_u}\,\mathrm{Tr}(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger + 3Y_u Y_u^\dagger Y_u Y_u^\dagger)\\
&- 12\,\mathrm{Tr}(T_\nu^\dagger T_\nu Y_\nu^\dagger Y_\nu + 3T_u^\dagger T_u Y_u^\dagger Y_u + T_\nu^\dagger Y_\nu Y_\nu^\dagger T_\nu + 3T_u^\dagger Y_u Y_u^\dagger T_u)\\
&- 2\,\mathrm{Tr}(T_e T_\nu^\dagger Y_\nu Y_e^\dagger + 3T_d T_u^\dagger Y_u Y_d^\dagger + T_e Y_\nu^\dagger Y_\nu T_e^\dagger + 3T_d Y_u^\dagger Y_u T_d^\dagger)\\
&- 2\,\mathrm{Tr}(Y_e T_\nu^\dagger T_\nu Y_e^\dagger + 3Y_d T_u^\dagger T_u Y_d^\dagger + Y_e Y_\nu^\dagger T_\nu T_e^\dagger + 3Y_d Y_u^\dagger T_u T_d^\dagger)\\
&- 36\,\mathrm{Tr}(Y_u Y_u^\dagger m^2_{\tilde u} Y_u Y_u^\dagger + Y_u^\dagger Y_u m^2_{\tilde Q} Y_u^\dagger Y_u)\\
&- 12\,\mathrm{Tr}(Y_\nu Y_\nu^\dagger m^2_{\tilde \nu} Y_\nu Y_\nu^\dagger + Y_\nu^\dagger Y_\nu m^2_{\tilde L} Y_\nu^\dagger Y_\nu)\\
&- 6\,\mathrm{Tr}(Y_u Y_d^\dagger m^2_{\tilde d} Y_d Y_u^\dagger + Y_d Y_u^\dagger m^2_{\tilde u} Y_u Y_d^\dagger + Y_u^\dagger Y_u m^2_{\tilde Q} Y_d^\dagger Y_d + Y_d^\dagger Y_d m^2_{\tilde Q} Y_u^\dagger Y_u)\\
&- 2\,\mathrm{Tr}(Y_\nu Y_e^\dagger m^2_{\tilde e} Y_e Y_\nu^\dagger + Y_e Y_\nu^\dagger m^2_{\tilde \nu} Y_\nu Y_e^\dagger + Y_\nu^\dagger Y_\nu m^2_{\tilde L} Y_e^\dagger Y_e + Y_e^\dagger Y_e m^2_{\tilde L} Y_\nu^\dagger Y_\nu)\\
&+ \left(32g_3^2 + \frac{8}{5}g_1^2\right)\mathrm{Tr}(T_u^\dagger T_u + m^2_{H_u} Y_u^\dagger Y_u + Y_u m^2_{\tilde Q} Y_u^\dagger + Y_u^\dagger m^2_{\tilde u} Y_u)\\
&- \frac{8}{5}g_1^2\,\mathrm{Tr}(M_1^* Y_u^\dagger T_u + M_1 T_u^\dagger Y_u - 2|M_1|^2 Y_u^\dagger Y_u)\\
&- 32g_3^2\,\mathrm{Tr}(M_3^* Y_u^\dagger T_u + M_3 T_u^\dagger Y_u - 2|M_3|^2 Y_u^\dagger Y_u)
\end{aligned}
$$

$$+\frac{18}{5}g_1^2g_2^2\left(|M_1|^2+Re(M_1M_2^*)+|M_2|^2\right)$$
$$+\frac{621}{25}g_1^4|M_1|^2+33g_2^4|M_2|^2+\frac{3}{5}g_1^2\sigma_1+3g_2^2\sigma_2+\frac{6}{5}g_1^2S' , \tag{A.31}$$

with

$$\sigma_1=\frac{1}{5}g_1^2\left(3m_{H_u}^2+3m_{H_d}^2+\mathrm{Tr}\left(m_{\tilde{Q}}^2+3m_{\tilde{L}}^2+2m_{\tilde{d}}^2+6m_{\tilde{e}}^2+8m_{\tilde{u}}^2\right)\right) , \tag{A.32}$$
$$\sigma_2^2=g_2^2\left(m_{H_u}^2+m_{H_d}^2+\mathrm{Tr}\left(3m_{\tilde{Q}}^2+m_{\tilde{L}}^2\right)\right) , \tag{A.33}$$
$$\sigma_3=g_3^2\,\mathrm{Tr}\left(2m_{\tilde{Q}}^2+m_{\tilde{d}}^2+m_{\tilde{u}}^2\right) , \tag{A.34}$$

and

$$S'=m_{H_d}^2\,\mathrm{Tr}\left(Y_eY_e^\dagger+3Y_dY_d^\dagger\right)-m_{H_u}^2\,\mathrm{Tr}\left(Y_\nu Y_\nu^\dagger+3Y_uY_u^\dagger\right)$$
$$+\mathrm{Tr}\left(m_{\tilde{L}}^2Y_e^\dagger Y_e+m_{\tilde{L}}^2Y_\nu^\dagger Y_\nu\right)-\mathrm{Tr}\left(m_{\tilde{Q}}^2Y_d^\dagger Y_d+m_{\tilde{Q}}^2Y_u^\dagger Y_u\right)$$
$$-2\,\mathrm{Tr}\left(Y_dY_d^\dagger m_{\tilde{d}}^2+Y_eY_e^\dagger m_{\tilde{e}}^2-2Y_uY_u^\dagger m_{\tilde{u}}^2\right)$$
$$+\frac{1}{30}g_1^2\left(9m_{H_u}^2-9m_{H_d}^2+\mathrm{Tr}\left(m_{\tilde{Q}}^2-9m_{\tilde{L}}^2+4m_{\tilde{d}}^2+36m_{\tilde{e}}^2-32m_{\tilde{u}}^2\right)\right)$$
$$+\frac{3}{2}g_2^2\left(m_{H_u}^2-m_{H_d}^2+\mathrm{Tr}\left(m_{\tilde{Q}}^2-m_{\tilde{L}}^2\right)\right)$$
$$+\frac{8}{3}g_3^2\,\mathrm{Tr}\left(m_{\tilde{Q}}^2+m_{\tilde{d}}^2-2m_{\tilde{u}}^2\right) . \tag{A.35}$$

B Self-energies and one-loop tadpoles including inter-generational mixing

Here we present the used formulas for the self-energies Π_{ZZ}^T, $\Pi_{H^+H^-}$, Π_{AA}, and the one-loop tadpoles t_u, t_d, which are based on [59] but generalized to include inter-generational mixing. In this appendix we employ SLHA 2 conventions [55] in the Super-CKM and Super-PMNS basis, to agree with the convention of [59]. The soft-breaking mass matrices in the Super-CKM/Super-PMNS basis are obtained from our flavour basis conventions (3.1) and (3.2) by

$$\begin{aligned}
y_f &\equiv Y_f^{\rm diag}=U_f^\dagger Y_f V_f ,\\
\hat{T}_f &\equiv U_f^\dagger T_f V_f ,\\
\hat{m}_{\tilde{Q}}^2 &\equiv V_d^\dagger m_{\tilde{Q}}^2 V_d ,\\
\hat{m}_{\tilde{L}}^2 &\equiv V_e^\dagger m_{\tilde{L}}^2 V_e ,\\
\hat{m}_{\tilde{u}}^2 &\equiv U_u^\dagger m_{\tilde{u}}^2 U_u ,\\
\hat{m}_{\tilde{d}}^2 &\equiv U_d^\dagger m_{\tilde{d}}^2 U_d ,\\
\hat{m}_{\tilde{e}}^2 &\equiv U_e^\dagger m_{\tilde{e}}^2 U_e .
\end{aligned} \tag{B.1}$$

Let us briefly review our generalization to the sfermion mass matrices of [59]: we define the sfermion mixing matrices by[10]

$$M_{\tilde{f}}^2=W_{\tilde{f}}M_{\tilde{f}}^{2\ \rm diag}W_{\tilde{f}}^\dagger , \tag{B.2}$$

[10] The SLHA 2 convention sfermion mixing matrices $R_{\tilde{f}}$ can be obtained via $R_{\tilde{f}}=W_{\tilde{f}}^\dagger$.

with the sfermion mass matrices in the Super-CKM/Super-PMNS basis

$$
\begin{aligned}
M_{\tilde{u}}^2 &= \begin{pmatrix} V_{\rm CKM}\hat{m}_{\tilde{Q}}^2 V_{\rm CKM}^\dagger + \frac{v^2}{2}y_u^2\sin^2\beta + D_{u,L} & \frac{v}{\sqrt{2}}\hat{T}_u^\dagger\sin\beta - \mu\frac{v}{\sqrt{2}}y_u\cos\beta \\ \frac{v}{\sqrt{2}}\hat{T}_u\sin\beta - \mu^*\frac{v}{\sqrt{2}}y_u\cos\beta & \hat{m}_{\tilde{u}}^2 + \frac{v^2}{2}y_u^2\sin^2\beta + D_{u,R} \end{pmatrix} , \\
M_{\tilde{d}}^2 &= \begin{pmatrix} \hat{m}_{\tilde{Q}}^2 + \frac{v^2}{2}y_d^2\cos^2\beta + D_{d,L} & \frac{v}{\sqrt{2}}\hat{T}_d^\dagger\cos\beta - \mu\frac{v}{\sqrt{2}}y_d\sin\beta \\ \frac{v}{\sqrt{2}}\hat{T}_d\cos\beta - \mu^*\frac{v}{\sqrt{2}}y_d\sin\beta & \hat{m}_{\tilde{d}}^2 + \frac{v^2}{2}y_d^2\cos^2\beta + D_{d,R} \end{pmatrix} , \\
M_{\tilde{e}}^2 &= \begin{pmatrix} \hat{m}_{\tilde{L}}^2 + \frac{v^2}{2}y_e^2\cos^2\beta + D_{e,L} & \frac{v}{\sqrt{2}}\hat{T}_e^\dagger\cos\beta - \mu\frac{v}{\sqrt{2}}y_e\sin\beta \\ \frac{v}{\sqrt{2}}\hat{T}_e\cos\beta - \mu^*\frac{v}{\sqrt{2}}y_e\sin\beta & \hat{m}_{\tilde{e}}^2 + \frac{v^2}{2}y_e^2\cos^2\beta + D_{e,R} \end{pmatrix} , \\
M_{\tilde{\nu}}^2 &= V_{\rm PMNS}^\dagger \hat{m}_{\tilde{L}}^2 V_{\rm PMNS} + D_{\nu,L} \, .
\end{aligned} \tag{B.3}
$$

The D-terms are given by

$$
\begin{aligned}
D_{f,L} &= \hat{M}_Z^2(I_3 - Q_e\sin^2\theta_W)\cos(2\beta)\,\mathbf{1}_3 \, , \\
D_{f,R} &= \hat{M}_Z^2 Q_e\cos(2\beta)\sin^2\theta_W\,\mathbf{1}_3 \, ,
\end{aligned} \tag{B.4}
$$

where I_3 denotes the $\mathrm{SU}(2)_L$ isospin and Q_e the electric charge of the flavour f, and θ_W denotes the weak mixing angle. Note that our convention for μ differs by a sign from the convention in [59].

For the sake of completeness we also list the conventions for neutralino and chargino mass matrices and mixing matrices: the neutralino mixing matrix is defined by

$$
M_{\psi^0} = N^T M_{\psi^0}^{\rm diag} N \, , \tag{B.5}
$$

with

$$
M_{\psi^0} = \begin{pmatrix} M_1 & 0 & -M_Z\cos\beta\sin\theta_W & M_Z\sin\beta\sin\theta_W \\ 0 & M_2 & M_Z\cos\beta\cos\theta_W & -M_Z\sin\beta\cos\theta_W \\ -M_Z\cos\beta\sin\theta_W & M_Z\cos\beta\cos\theta_W & 0 & -\mu \\ M_Z\sin\beta\sin\theta_W & -M_Z\sin\beta\cos\theta_W & -\mu & 0 \end{pmatrix} . \tag{B.6}
$$

The chargino mixing matrix is defined by

$$
M_{\psi^+} = U^T M_{\psi^+}^{\rm diag} V \, , \tag{B.7}
$$

with

$$
M_{\psi^+} = \begin{pmatrix} M_2 & \sqrt{2}M_W\sin\beta \\ \sqrt{2}M_W\cos\beta & \mu \end{pmatrix} . \tag{B.8}
$$

We now present the generalization of Π_{ZZ}^T, $\Pi_{H^+H^-}$, Π_{AA}, t_u, and t_d of [59] to include inter-generational mixing. For all we have checked that our equations reduce to the corresponding equations in [59] when

$$
W_{\tilde{f}_{i\,i}} = W_{\tilde{f}_{i+3\,i+3}} = \cos\theta_{\tilde{f}_i}
$$

$$W_{\tilde{f}_{i\,i+3}} = -W_{\tilde{f}_{i+3\,i}} = -\sin\theta_{\tilde{f}_i}\ , \tag{B.9}$$

where $i = 1\dots 3$.

We keep the abbreviations of [59]:

$$s_{\alpha\beta} \equiv \sin(\alpha-\beta)\ , \tag{B.10}$$
$$c_{\alpha\beta} \equiv \cos(\alpha-\beta)\ , \tag{B.11}$$
$$g_{f_L} \equiv I_3 - Q_e \sin^2\theta_W\ , \tag{B.12}$$
$$g_{f_R} \equiv Q_e \sin^2\theta_W\ , \tag{B.13}$$
$$e \equiv g_2 \sin\theta_W\ , \tag{B.14}$$
$$N_C \equiv \begin{cases} 3 & \text{for (s)quarks} \\ 1 & \text{for (s)fermions} \end{cases}\ . \tag{B.15}$$

The conventions for the one-loop scalar functions A_0, B_{22}, $\tilde{B}_{22}$, H, G, and F [77, 78] are adopted from appendix B of [59]. Summations $\sum_f$ are over all fermions, whereas summations $\sum_{f_u}$, $\sum_{f_d}$ are restricted to up-type and down-type fermions, respectively. Summations $\sum_Q$, $\sum_{\tilde{Q}}$ are over SU(2) (s)quark doublets, and analogously for (s)leptons. In summations over sfermions the indices i, j, s, and t run from 1 to 6 for $\tilde{u}$, $\tilde{d}$, and $\tilde{e}$ and from 1 to 3 for $\tilde{\nu}$. In summations of neutralinos (charginos) the indices i, j run from 1 to 4 (2). The summations $\sum_{h^0}$ runs over all neutral Higgs- and Goldstone bosons, the summation $\sum_{h^+}$ over the charged ones.

$$\begin{aligned}
16\pi^2 \frac{\cos^2\theta_W}{g_2^2}\Pi^T_{ZZ}(p^2) = &-s^2_{\alpha\beta}\left(\tilde{B}_{22}(m_A,m_H) + \tilde{B}_{22}(M_Z,m_h) - M_Z^2 B_0(M_Z,m_h)\right) \\
&- c^2_{\alpha\beta}\left(\tilde{B}_{22}(M_Z,m_H) + \tilde{B}_{22}(m_A,m_h) - M_Z^2 B_0(M_Z,m_h)\right) \\
&- 2\cos^4\theta_W\left(2p^2 + M_W^2 - M_Z^2\frac{\sin^4\theta_W}{\cos^2\theta_W}\right)B_0(M_W,M_W) \\
&- \left(8\cos^4\theta_W + \cos^2(2\theta_W)\right)\tilde{B}_{22}(M_W,M_W) \\
&- \cos^2(2\theta_W)\tilde{B}_{22}(m_{H^+},m_{H^+}) \\
&- \sum_{\tilde{f}} N_C \sum_{s,t}\left| I_3 \sum_i W^*_{\tilde{f}is}W_{\tilde{f}it} - Q_e\sin^2\theta_W\delta_{st}\right|^2 4B_{22}(m_{\tilde{f}_s},m_{\tilde{f}_t}) \\
&+ \frac{1}{2}\sum_{\tilde{f}} N_C \sum_s\Bigg((1-8I_3Q_e\sin^2\theta_W)\sum_i W^*_{\tilde{f}is}W_{\tilde{f}is} \\
&+ 4Q_e^2\sin^4\theta_W\Bigg)A_0(m_{\tilde{f}_s}) \\
&+ \sum_f N_C\left((g^2_{f_L}+g^2_{f_R})H(m_f,m_f) - 4g_{f_L}g_{f_R}m_f^2 B_0(m_f,m_f)\right)
\end{aligned} \tag{B.16}$$

$$
+\frac{\cos^2\theta_W}{2g_2^2}\sum_{i,j} f^0_{ijZ} H(m_{\tilde\chi^0_i}, m_{\tilde\chi^0_j}) + 2g^0_{ijZ} m_{\tilde\chi^0_i} m_{\tilde\chi^0_j} B_0(m_{\tilde\chi^0_i}, m_{\tilde\chi^0_j})
$$
$$
+\frac{\cos^2\theta_W}{g_2^2}\sum_{i,j} f^+_{ijZ} H(m_{\tilde\chi^+_i}, m_{\tilde\chi^+_j}) + 2g^+_{ijZ} m_{\tilde\chi^+_i} m_{\tilde\chi^+_j} B_0(m_{\tilde\chi^+_i}, m_{\tilde\chi^+_j}) \,.
$$

The couplings f^0_Z, f^+_Z, g^0_Z, and g^+_Z are given in eqs. (A.7) and (D.5) of [59].

$$
\begin{aligned}
16\pi^2\Pi_{H^+H^-}(p^2) = &\sum_Q N_C \Bigg(\left(\cos^2\beta y_u^2 + \sin^2\beta y_d^2\right) G(m_u, m_d) \\
&- 2\sin(2\beta) y_u y_d m_u m_d B_0(m_u, m_d) \Bigg) \\
&+ \sum_L \sin^2\beta y_e^2 G(0, m_e) \\
&+ \sum_{\tilde Q} N_C \sum_{i,j} \left(\lambda_{H^+\tilde Q}\right)^2_{ij} B_0(m_{\tilde u_i}, m_{\tilde d_j}) \\
&+ \sum_{\tilde L}\sum_{i,j} \left(\lambda_{H^+\tilde L}\right)^2_{ij} B_0(m_{\tilde\nu_i}, m_{\tilde e_j}) \\
&+ \sum_{\tilde f}\sum_i \left(\lambda_{H^+H^-\tilde f}\right)_{ii} A_0(m_{\tilde f_i}) \\
&+ \frac{g_2^2}{4}\Bigg(s^2_{\alpha\beta} F(m_H, M_W) + c^2_{\alpha\beta} F(m_h, M_W) + F(m_A, M_W) \\
&+ \frac{\cos^2(2\theta_W)}{\cos^2\theta_W} F(m_{H^+}, M_Z) \Bigg) \\
&+ e^2 F(m_{H^+}, 0) + 2g_2^2 A_0(M_W) + g_2^2\cos^2(2\theta_W) A_0(M_Z) \\
&+ \sum_{h^+}\left(\sum_{h^0} \left(\lambda_{H^+h^0h^-}\right)^2 B_0(m_{h^0}, m_{h^+}) + \lambda_{H^+H^-h^+h^-} A_0(m_{h^+}) \right) \\
&+ g_2^2 \frac{M_W^2}{4} B_0(M_W, m_A) \\
&+ \frac{1}{2}\sum_{h^0} \lambda_{H^+H^-h^0h^0} A_0(m_{h^0}) \\
&+ \sum_{j,i} f_{ijH^+} G(m_{\tilde\chi^+_j}, m_{\tilde\chi^0_i}) - 2m_{\tilde\chi^+_j} m_{\tilde\chi^0_i} g_{ijH^+} B_0(m_{\tilde\chi^+_j}, m_{\tilde\chi^0_i}) \,.
\end{aligned} \tag{B.17}
$$

The couplings f_{H^+} and g_{H^+} are given in eq. (D.70) and eqs. (D.39)–(D.42) of [59]. The couplings $\lambda_{H^+H^-h^-}$, $\lambda_{H^+H^-h^+h^-}$, and $\lambda_{H^+H^-h^0h^0}$ are defined in eqs. (D.63)–(D.65) and eq. (D.67) of [59]. The couplings to sfermions in the case of inter-generational mixing are given by

$$
\lambda_{H^+\tilde Q} = W_{\tilde u}\begin{pmatrix} \frac{g_2}{\sqrt2}\hat M_W \sin(2\beta)\,\mathbf{1}_3 - \left(y_u^2+y_d^2\right)\cos\beta\frac{v\sin\beta}{\sqrt2} & -\hat T_d^\dagger \sin\beta - \mu y_d\cos\beta \\ -\hat T_u\cos\beta - \mu^* y_u \sin\beta & -y_u y_d \frac{v}{\sqrt2} \end{pmatrix} W_{\tilde d}\,, \tag{B.18}
$$
$$
\lambda_{H^+\tilde L} = W_{\tilde\nu}\left(\frac{g_2}{\sqrt2}\hat M_W \sin(2\beta)\,\mathbf{1}_3 - y_e^2\cos\beta\frac{v\sin\beta}{\sqrt2} \qquad -\hat T_e^\dagger\sin\beta - \mu y_e\cos\beta \right) W_{\tilde e}\,, \tag{B.19}
$$

$$\lambda_{H^+H^-\tilde{f}} = -W_{\tilde{f}}^\dagger \begin{pmatrix} -\frac{g_2^2}{2}\cos(2\beta)\left(\frac{\cos(2\theta_W)}{\cos^2\theta_W}I_3 + \tan^2\theta_W Q_e\right)\mathbf{1}_3 & 0 \\ 0 & \frac{g_2^2}{2}\cos(2\beta)\tan^2\theta_W Q_e\,\mathbf{1}_3 \end{pmatrix} W_{\tilde{f}}$$

$$+ \begin{cases} W_{\tilde{u}}^\dagger \begin{pmatrix} y_d^2\sin^2\beta & 0 \\ 0 & y_u^2\cos^2\beta \end{pmatrix} W_{\tilde{u}} & \tilde{f}=\tilde{u} \\ W_{\tilde{d}}^\dagger \begin{pmatrix} y_u^2\cos^2\beta & 0 \\ 0 & y_d^2\sin^2\beta \end{pmatrix} W_{\tilde{d}} & \tilde{f}=\tilde{d} \\ W_{\tilde{e}}^\dagger \begin{pmatrix} 0 & 0 \\ 0 & y_e^2\sin^2\beta \end{pmatrix} W_{\tilde{e}} & \tilde{f}=\tilde{e} \end{cases}, \tag{B.20}$$

$$\lambda_{H^+H^-\tilde{\nu}} = -W_{\tilde{\nu}}^\dagger\left(-\frac{g_2^2}{2}\cos(2\beta)\left(\frac{\cos(2\theta_W)}{\cos^2\theta_W}I_3 + \tan^2\theta_W Q_e\right)\mathbf{1}_3 - y_e^2\sin^2\beta\right)W_{\tilde{\nu}} \tag{B.21}$$

$$\begin{aligned} 16\pi^2\Pi_{AA}(p^2) = {}& \cos^2\beta\sum_{f_u} N_C y_{f_u}^2\left(p^2 B_0(m_{f_u},m_{f_u}) - 2A_0(m_{f_u})\right) \\ &+ \sin^2\beta\sum_{f_d} N_C y_{f_d}^2\left(p^2 B_0(m_{f_d},m_{f_d}) - 2A_0(m_{f_d})\right) \\ &+ \sum_{\tilde{f}}\sum_i N_C\left(\left(\lambda_{AA\tilde{f}}\right)_{ii}A_0(m_{\tilde{f}_i}) + \sum_j\left(\lambda_{A\tilde{f}}\right)_{ij}^2 B_0(m_{\tilde{f}_i},m_{\tilde{f}_j})\right) \\ &+ \frac{g_2^2}{4}\left(2F(m_{H^+},M_W) + \frac{s_{\alpha\beta}^2}{\cos^2\theta_W}F(m_H,M_Z) + \frac{c_{\alpha\beta}^2}{\cos^2\theta_W}F(m_h,M_Z)\right) \\ &+ \frac{1}{2}\sum_{h_n^0}\left(\sum_{h_m^0}\lambda_{Ah_n^0h_m^0}B_0(m_{h_n^0},m_{h_m^0}) + \lambda_{AAh_n^0h_n^0}A_0(m_{h_n^0})\right) \\ &+ g_2^2\left(\frac{M_W^2}{2}B_0(M_W,m_{H_p}) + 2A_0(M_W) + \frac{1}{\cos^2\theta_W}A_0(M_Z)\right) \\ &+ \sum_{h^+}\lambda_{AAh^+h^+}A_0(m_{h^+}) \\ &+ \frac{1}{2}\sum_{i,j} f_{ijA}^0 G(m_{\tilde{\chi}_i^0},m_{\tilde{\chi}_j^0}) - 2g_{ijA}^0 m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}B_0(m_{\tilde{\chi}_i^0},m_{\tilde{\chi}_j^0}) \\ &+ \sum_{i,j} f_{ijA}^+ G(m_{\tilde{\chi}_i^+},m_{\tilde{\chi}_j^+}) - 2g_{ijA}^+ m_{\tilde{\chi}_i^+}m_{\tilde{\chi}_j^+}B_0(m_{\tilde{\chi}_i^+},m_{\tilde{\chi}_j^+}) . \end{aligned} \tag{B.22}$$

The couplings f_A^0, g_A^0, f_A^+, and g_A^+ are given in eq. (D.70) and eqs. (D.34-D.38) of [59]. The couplings $\lambda_{Ah^0h^0}$, $\lambda_{AAh^0h^0}$, and $\lambda_{AAh^+h^+}$ are defined in eqs. (D.63-D.65) and eq. (D.67) of [59]. The couplings to sfermions in the case of inter-generational mixing are given by

$$\lambda_{A\tilde{u}} = W_{\tilde{u}}^\dagger \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}}\left(\hat{T}_u^\dagger\cos\beta + \mu y_u\sin\beta\right) \\ \frac{1}{\sqrt{2}}\left(\hat{T}_u\cos\beta + \mu^* y_u\sin\beta\right) & \end{pmatrix} W_{\tilde{u}}\,, \tag{B.23}$$

$$\lambda_{A\tilde{e}(\tilde{d})} = W_{\tilde{e}(\tilde{d})}^\dagger \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}}\left(\hat{T}_{e(d)}^\dagger\sin\beta + \mu y_{e(d)}\cos\beta\right) \\ \frac{1}{\sqrt{2}}\left(\hat{T}_{e(d)}\sin\beta + \mu^* y_{e(d)}\cos\beta\right) & \end{pmatrix} W_{\tilde{e}(\tilde{d})}\,, \tag{B.24}$$

$$\lambda_{A\tilde{\nu}} = 0\,, \tag{B.25}$$

$$\lambda_{AA\tilde{f}} = W_{\tilde{f}}^\dagger \begin{pmatrix} -\frac{g_2^2}{2}\cos(2\beta)\left(\frac{1}{\cos^2\theta_W}I_3 - \tan^2\theta_W Q_e\right) \mathbf{1}_3 & 0 \\ 0 & -\frac{g_2^2}{2}\cos(2\beta)\tan^2\theta_W Q_e \ \mathbf{1}_3 \end{pmatrix} W_{\tilde{f}}$$

$$+ \begin{cases} W_{\tilde{u}}^\dagger \begin{pmatrix} y_u^2\cos^2\beta & 0 \\ 0 & y_u^2\cos^2\beta \end{pmatrix} W_{\tilde{u}} & \tilde{f} = \tilde{u} \\ W_{\tilde{d}}^\dagger \begin{pmatrix} y_d^2\sin^2\beta & 0 \\ 0 & y_d^2\sin^2\beta \end{pmatrix} W_{\tilde{d}} & \tilde{f} = \tilde{d} \\ W_{\tilde{e}}^\dagger \begin{pmatrix} y_e^2\sin^2\beta & 0 \\ 0 & y_e^2\sin^2\beta \end{pmatrix} W_{\tilde{e}} & \tilde{f} = \tilde{e} \end{cases}, \tag{B.26}$$

$$\lambda_{AA\tilde{\nu}} = -\frac{g_2^2}{2}\cos(2\beta)\left(\frac{1}{\cos^2\theta_W}I_3 - \tan^2\theta_W Q_e\right) \mathbf{1}_3 \tag{B.27}$$

$$\begin{aligned} 16\pi^2 t_d = & -2\sum_{f_d} N_C y_{f_d}^2 A_0(m_{f_d}) \\ & + \sum_{\tilde{f}} N_C \sum_i \frac{g_2^2}{2M_W\cos\beta}\left(\lambda_{d\tilde{f}}\right)_{ii} A_0(m_{\tilde{f}_i}) \\ & - g_2^2\frac{\cos(2\beta)}{8\cos^2\theta_W}\left(A_0(m_A) + 2A_0(m_{H^+})\right) + \frac{g_2^2}{2}A_0(m_{H_p}) \\ & + \frac{g_2^2}{8\cos^2\theta_W}\left(3\sin^2\alpha - \cos^2\alpha + \sin(2\alpha)\tan\beta\right)A_0(m_h) \\ & + \frac{g_2^2}{8\cos^2\theta_W}\left(3\cos^2\alpha - \sin^2\alpha - \sin(2\alpha)\tan\beta\right)A_0(m_H) \\ & - g_2^2\sum_i \frac{m_{\tilde{\chi}_i^0}}{M_W\cos\beta} Re\left(N_{i3}(N_{i2} - N_{i,1}\tan\theta_W)\right) A_0(m_{\tilde{\chi}_i^0}) \\ & - \sqrt{2}g_2^2\sum_i \frac{m_{\tilde{\chi}_i^+}}{M_W\cos\beta} Re\left(V_{i1}U_{i2}\right) A_0(m_{\tilde{\chi}_i^+}) \\ & + \frac{3}{4}g_2^2\left(2A_0(M_W) + \frac{A_0(M_Z)}{\cos^2\theta_W}\right) \\ & + g_2^2\frac{\cos(2\beta)}{8\cos^2\theta_W}\left(2A_0(M_W) + A_0(M_Z)\right) . \end{aligned} \tag{B.28}$$

The couplings to sfermion in the case of inter-generational mixing are given by

$$\lambda_{d\tilde{u}} = W_{\tilde{u}}^\dagger \begin{pmatrix} g_2\frac{M_Z}{\cos\theta_W}g_{u_L}\cos\beta \ \mathbf{1}_3 & -y_u\frac{\mu}{\sqrt{2}} \\ -y_u\frac{\mu^*}{\sqrt{2}} & g_2\frac{M_Z}{\cos\theta_W}g_{u_R}\cos\beta \ \mathbf{1}_3 \end{pmatrix} W_{\tilde{u}} , \tag{B.29}$$

$$\lambda_{d\tilde{f}} = W_{\tilde{f}}^\dagger \begin{pmatrix} g_2\frac{M_Z}{\cos\theta_W}g_{f_L}\cos\beta \ \mathbf{1}_3 & \hat{T}_f^\dagger\frac{1}{\sqrt{2}} \\ \hat{T}_f\frac{1}{\sqrt{2}} & g_2\frac{M_Z}{\cos\theta_W}g_{f_R}\cos\beta \ \mathbf{1}_3 \end{pmatrix}$$

$$+ \begin{cases} W_{\tilde{d}}^\dagger \begin{pmatrix} Y_d^2 v\cos\beta & 0 \\ 0 & Y_d^2 v\cos\beta \end{pmatrix} W_{\tilde{d}} & \tilde{f} = \tilde{d} \\ W_{\tilde{e}}^\dagger \begin{pmatrix} Y_e^2 v\cos\beta & 0 \\ 0 & Y_e^2 v\cos\beta \end{pmatrix} W_{\tilde{e}} & \tilde{f} = \tilde{e} \end{cases}, \tag{B.30}$$

$$
\begin{aligned}
\lambda_{d\tilde{\nu}} &= g_2\frac{M_Z}{\cos\theta_W}g_{\nu_L}\cos\beta\;\mathbf{1}_3 && \text{(B.31)}\\
16\pi^2 t_u &= -2\sum_{f_u} N_C y_{f_u}^2 A_0(m_{f_u})\\
&\quad + \sum_{\tilde f} N_C \sum_i \frac{g_2^2}{2M_W\sin\beta}\left(\lambda_{u\tilde f}\right)_{ii} A_0(m_{\tilde f_i})\\
&\quad + g_2^2\frac{\cos(2\beta)}{8\cos^2\theta_W}\left(A_0(m_A)+2A_0(m_{H^+})\right)+\frac{g_2^2}{2}A_0(m_{H_p})\\
&\quad + \frac{g_2^2}{8\cos^2\theta_W}\left(3\cos^2\alpha-\sin^2\alpha+\sin(2\alpha)\cot\beta\right)A_0(m_h)\\
&\quad + \frac{g_2^2}{8\cos^2\theta_W}\left(3\sin^2\alpha-\cos^2\alpha-\sin(2\alpha)\cot\beta\right)A_0(m_H)\\
&\quad - g_2^2\sum_i \frac{m_{\tilde\chi_i^0}}{M_W\sin\beta}Re\left(N_{i4}(N_{i2}-N_{i,1}\tan\theta_W)\right)A_0(m_{\tilde\chi_i^0})\\
&\quad - \sqrt{2}g_2^2\sum_i \frac{m_{\tilde\chi_i^+}}{M_W\sin\beta}Re\left(V_{i2}U_{i1}\right)A_0(m_{\tilde\chi_i^+})\\
&\quad + \frac{3}{4}g_2^2\left(2A_0(M_W)+\frac{A_0(M_Z)}{\cos^2\theta_W}\right)\\
&\quad - g_2^2\frac{\cos(2\beta)}{8\cos^2\theta_W}\left(2A_0(M_W)+A_0(M_Z)\right)\;. && \text{(B.32)}
\end{aligned}
$$

The couplings to sfermion in the case of inter-generational mixing are given by

$$
\begin{aligned}
\lambda_{u\tilde u} &= W_{\tilde u}^\dagger\begin{pmatrix} -g_2\frac{M_Z}{\cos\theta_W}g_{u_L}\sin\beta\;\mathbf{1}_3+y_u^2 v\sin\beta & \hat{T}_u^\dagger\frac{1}{\sqrt{2}}\\ \hat{T}_u\frac{1}{\sqrt{2}} & -g_2\frac{M_Z}{\cos\theta_W}g_{u_R}\sin\beta\;\mathbf{1}_3+y_u^2 v\sin\beta\end{pmatrix}W_{\tilde u}\;, && \text{(B.33)}\\
\lambda_{u\tilde f} &= W_{\tilde f}^\dagger\begin{pmatrix} -g_2\frac{M_Z}{\cos\theta_W}g_{f_L}\sin\beta\;\mathbf{1}_3 & -y_{\tilde f}\frac{\mu}{\sqrt{2}}\\ -y_{\tilde f}\frac{\mu^*}{\sqrt{2}} & -g_2\frac{M_Z}{\cos\theta_W}g_{f_R}\sin\beta\;\mathbf{1}_3\end{pmatrix} \qquad \tilde f=\tilde e\,,\tilde d\,,\\
\lambda_{u\tilde\nu} &= -g_2\frac{M_Z}{\cos\theta_W}g_{\nu_L}\sin\beta\;\mathbf{1}_3\;. && \text{(B.34)}
\end{aligned}
$$

C `SusyTC` documentation

Here we present a documentation of the `REAP` extension `SusyTC`. To get started, please follow first the steps described in section 4.

We now describe that additional features of `SusyTC`: in addition to the features of `REAP` package `RGEMSSMsoftbroken.m` (described in the `REAP` documentation), `SusyTC` adds the following options to the command `RGEAdd`:

- `STCsign`μ is the general factor $e^{i\phi_\mu}$ in front of μ in (3.36). (default: +1)

- `STCcMSSM` is a switch to change between the CP-violating (complex) MSSM and CP-conserving (real) MSSM. (default: `True`)

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- `STCSusyScale` sets the SUSY scale Q (in GeV), where the MSSM is matched to the SM. If set to `"Automatic"`, `SusyTC` determines Q automatically from the sparticle spectrum. (default: `"Automatic"`)

- `STCSusyScaleFromStops` is a switch to choose whether `SusyTC` calculates the SUSY scale Q as geometric mean of the stop masses $Q = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$, where the stop masses are defined by the up-type squark mass eigenstates $\tilde{u}_i$ with the largest mixing to $\tilde{t}_1$ and $\tilde{t}_2$, or as geometric mean of the lightest and heaviest up-type squarks $Q = \sqrt{m_{\tilde{u}_1} m_{\tilde{u}_6}}$. Without effect if `STCSusyScale` is not set to `"Automatic"`. (default: `True`)

- `STCSearchSMTransition` is a switch to enable or disable the matching to the SM and the calculation of supersymmetric threshold corrections and sparticle spectrum. (default: `True`)

- `STCCCBConstraints` is a switch to enable or disable a warning message for potentially dangerous charge and colour breaking vacua, if large trilinear couplings violate the constraints of [79] at the SUSY scale Q

$$|T_{ij}|^2 \leq \left(m^2_{R_{ii}} + m^2_{L_{jj}} + m^2_{H_f} + |\mu|^2\right) \begin{cases} (y_i^2 + y_j^2) & i \neq j \\ \\ 3y_i^2 & i = j \end{cases} , \tag{C.1}$$

  where m_L, m_R and m_{H_f} denote the soft-breaking mass parameters of the scalar fields associated with the trilinear coupling T in the basis of diagonal Yukawa matrices. (default: `True`)

- `STCUFBConstraints` is a switch to enable or disable a warning message for possibly dangerous "unbounded from below" directions in the scalar potential, if the constraints of [69] are violated at the SUSY scale Q

$$m^2_{H_u} + |\mu|^2 + m^2_{\tilde{L}_i} - \frac{|m_3|^4}{m^2_{H_d} + |\mu|^2 - m^2_{\tilde{L}_i}} > 0 \qquad \text{UFB-2}\,, \tag{C.2}$$

$$m^2_{H_u} + m^2_{\tilde{L}_i} > 0 \qquad \text{UFB-3}\,, \tag{C.3}$$

  evaluated in the basis of (3.11). Note that the UFB-I constraint is automatically satisfied, since `SusyTC` calculates m_3^2 from $m^2_{H_u}$, $m^2_{H_d}$, M_Z, and $\tan\beta$ by requiring the existence of electroweak symmetry breaking. (default: `True`)

- `STCTachyonConstraints` is a switch to enable or disable the rejection of a parameter point with tachyonic running masses of sfermions at any renormalization scale above the SUSY scale Q. Regardless of this switch, tachyonic sfermion masses at Q are always rejected. (default: `False`)

Thus, a typical call of `RGEAdd[]` might look like

RGEAdd`["MSSMsoftbroken",RGEtan`β`->20,STCcMSSM->False,STCsign`μ`->-1]`;

In addition to the parameters known from the MSSM REAP model, the following soft-breaking parameters are available as input for RGESetInitial:

- RGETu, RGETd, RGETe, and RGETν are the soft-breaking trilinear coupling matrices. If given, the Constrained MSSM parameter RGEA0 for the corresponding trilinear coupling is overwritten. (default: Constrained MSSM)
- RGEM1, RGEM2, and RGEM3 are the soft-breaking gaugino mass parameters. If given, the Constrained MSSM parameter RGEM12 for the corresponding gaugino is overwritten. (default: Constrained MSSM)
- RGEm2Q, RGEm2L, RGEm2u, RGEm2d, RGEm2e, RGEm2ν are the soft-breaking squared mass matrices $m^2_{\tilde{f}}$ for the sfermions. If given, the Constrained MSSM parameter RGEm0 for the corresponding scalar masses is overwritten. (default: Constrained MSSM)
- RGEm2Hd and RGEm2Hu are the soft-breaking squared masses for H_d and H_u, respectively. If given, the Constrained MSSM parameter RGEm0 for the corresponding scalar mass is overwritten. (default: Constrained MSSM)
- RGEM12 is the Constrained MSSM parameter for gaugino mass parameters in GeV. (default: 750)
- RGEm0 is the Constrained MSSM parameter for all soft-breaking masses of scalars in GeV. (default: 1500)
- RGEA0 is the Constrained MSSM parameter A_0 for trilinear couplings, e.g. $T_f = A_0 Y_f$. (default: -500)

An example for an input at the GUT scale would be

```
RGESetInitial[2·10^16,RGEM1->863,RGEM2->131,
  RGEM3->-392,RGESuggestion->"GUT"];
```

The solution at a lower energy scale such as M_Z can now be obtained by the REAP command RGESolve:

```
RGESolve[91.19,2·10^16];
```

Some parameter points might lead to tachyonic sparticle masses. In such instances the evaluation of SusyTC is stopped and an error message is returned using the Mathematica command Throw. In order to properly catch such error messages, we therefore recommend to use instead

```
Catch[RGESolve[91.19,2·10^16],TachyonicMass];
```

In addition to the parameters known from the MSSM REAP model, the following soft-breaking parameters are available for RGEGetSolution at all energy scales higher than the SUSY scale Q:

- `RGETu`, `RGETd`, `RGETe`, and `RGET`ν are used to get the soft-breaking trilinear coupling matrices.
- `RawT`ν is used to get the raw (internal representation) of the soft-breaking trilinear matrix for sneutrinos.
- `RGEM1`, `RGEM2`, and `RGEM3` are used to get the soft-breaking gaugino mass parameters.
- `RGEm2Q`, `RGEm2L`, `RGEm2u`, `RGEm2d`, `RGEm2e`, `RGEm2`ν are used to get the soft-breaking squared mass matrices $m^2_{\tilde{f}}$ for the sfermions.
- `RGEm2Hd` and `RGEm2Hu` are used to get the soft-breaking squared masses for H_d and H_u, respectively.

To obtain the running $\overline{\text{DR}}$ gluino mass at a scale of two TeV for example, one uses

RGEGetSolution[`2000,RGEM3`];

With `SusyTC` the $\overline{\text{DR}}$ sparticle spectrum is automatically calculated. The following functions are included in `SusyTC`:

- `STCGetSUSYScale[]` returns the SUSY scale Q.
- `STCGetSUSYSpectrum[]` returns a list of replacement rules for the SUSY scale Q, the $\overline{\text{DR}}$ tree-level values of μ and m_3, and the $\overline{\text{DR}}$ sparticle masses and (tree-level) mixing matrices at the SUSY scale. In detail it contains
  - `"Q"` the SUSY scale Q.
  - `"`μ`","`m_3`"` the values of μ and m_3.
  - `"M1","M2","M3"` are the gaugino mass parameters.
  - `"Mh","MH","MA","MHp"` the (tree-level) masses of the MSSM Higgs bosons.[11]
  - `"m`χ`0"` a list of the four neutralino masses.
  - `"m`χ`p"` a list of the two chargino masses.
  - `"msude"` a 3×6 array of the six up-type quarks, down-type squarks and charged slepton masses, respectively.
  - `"ms`ν`"` a list of the three light sneutrino masses.
  - `"`θ`W"` the weak mixing angle.
  - `"tan`α`"` the mixing angle of the CP-even Higgs bosons.
  - `"N"` the mixing matrix of neutralinos.
  - `"U","V"` the mixing matrices for charginos.
  - `"Wude"` a list of the three sparticle mixing matrices for up-type squarks, down-type squarks and charged sleptons.

[11] Note that there is no CP-violation in the MSSM Higgs sector on tree-level.

– `"W`ν`"` the mixing matrix of the three light sneutrinos.

To obtain for example the SUSY scale and the tree-level masses of the charginos call

> `"Q"/.`**STCGetSUSYSpectrum**[];
> `"m`χ`p"/.`**STCGetSUSYSpectrum**[];

The squark masses and charged slepton masses are contained in a joint list as $\{m_{\tilde{u}}, m_{\tilde{d}}, m_{\tilde{e}}\}$, and analogously for the sfermion mixing matrices. To obtain for example the up-type squark masses, the charged slepton mixing matrix, and the sneutrino masses type

> `("msude"/.`**STCGetSUSYSpectrum**[]`)[[1]];`
> `("Wude"/.`**STCGetSUSYSpectrum**[]`)[[3]];`
> `"ms`ν`"/.`**STCGetSUSYSpectrum**[];

- `STCGetOneLoopValues[]` returns a list of replacement rules containing
  – `"`μ`","`m_3`"` the one-loop corrected $\overline{\text{DR}}$ μ-parameter and m_3 as in (3.36) and (3.37) at the SUSY scale Q.
  – `"vev"` the one-loop $\overline{\text{DR}}$ vev $\hat{v}$ as in (3.41) at the SUSY scale Q.
  – `"MHp" ("MA")` the pole-mass m_{H^+} (m_A) of the charged (CP-odd) Higgs boson for `STCcMSSM` = `True` (`False`).

  The value of μ can for example be obtained from

  > `"`μ`"/.`**STCGetOneLoopValues**[];

- `STCGetSCKMValues[]` returns a list of replacement rules with the soft-breaking mass squared and trilinear coupling matrices in the SCKM basis, where sparticles are rotated analogously with their corresponding superpartners.[12] Since they are used for the self-energies calculation as described in the previous appendix, they are returned in SLHA2 convention [55]! In detail, there are
  – `"VCKM"` the CKM mixing matrix.
  – `"VPMNS"` the PMNS mixing matrix.
  – `SCKMBasis["m2Q"], SCKMBasis["m2u"], SCKMBasis["m2d"]` the squark soft-breaking mass squared matrices in the super CKM basis with SLHA2 convetions.
  – `SCKMBasis["m2L"], SCKMBasis["m2e"]` the slepton soft-breaking mass squared matrices in the super PMNS basis with SLHA2 conventions.
  – `SCKMBasis["T"]` a list of the three trilinear coupling matrices for up-type squarks, down-type squarks and charged sleptons in the SCKM basis with SLHA2 conventions.

[12] We use the term "SCKM" for the super CKM and super PMNS basis.

- SCKMBasis["Y"] a 3×3 array of the Yukawa coupling singular values for up-type squarks, down-type squarks and charged sleptons.

To obtain the down-type trilinear coupling matrix and the mass squared matrix of the left-handed up-type squarks in the SLHA basis for example, type

```
(SCKMBasis["T"]/.STCGetSCKMValues[])[[2]];
SCKMBasis["mQ2u"]/.STCGetSCKMValues[];
```

- STCGetInternalValues[] returns everything that is internally used for the calculation of the threshold corrections and sparticle spectrum, i.e. the results from STCGetSCKMValues[] and STCGetSUSYSpectrum[] with the one-loop corrected parameters from STCGetOneLoopValues[] replacing tree-level ones if available. We recommend to the user to use those separate functions instead.

As additional feature, SusyTC optionally supports input and output as SLHA "Les Houches" files. These files follow SLHA conventions [55, 58]:

- STCSLHA2Input[*"Path"*] loads an "Les Houches" input file stored in *"Path"* and executes REAP and SusyTC. If no path is given, the default path is assumed as *"SusyTC.in"* in the Mathematica Notebook Directory. An important difference to other spectrum calculators is the pure "top-down" approach by SusyTC, i.e. there is no attempt of fitting SM inputs at a low scale or calculating a GUT-scale from gauge couplings unification. Instead, all input is given at a user-defined high energy scale, which is then evolved to lower scales. The input should be given in the flavour basis in SLHA 2 convention [55], with analogous convention for Y_ν and convention for M_n as in (3.1). The relation between the SusyTC conventions and the SLHA 2 conventions is given in section 3. In the following, we list all SLHA 2 input blocks, which are available in SusyTC:

  - Block MODSEL: the only available switch is:
    5 : (Default = 2) CP violation (STCcMSSM)
  - Block SusyTCInput: Switches (1=True, 0=False) are defined for RGEAdd[]:
    1 : (Default = 1) STCSusyScaleFromStops
    2 : (Default = 1) STCSearchSMTransition
    3 : (Default = 1) STCCCBConstraints
    4 : (Default = 1) STCUFBConstraints
    5 : (Default = 1) Print a Warning in case of Tachyonic masses
    6 : (Default = 1) One or Two Loop RGEs
  - Block MINPAR: Constrained MSSM parameters as defined in [55, 58]. Note however, that the input value of $\tan\beta$ is interpreted to be given at the SUSY scale.
  - Block IMMINPAR: reads the $\sin\phi_\mu$ in case of the complex MSSM:
    4 : $\sin\phi_\mu$

– Block EXTPAR:
0 : (Default $= 2 \cdot 10^{16}$): $M_{\rm input}$ Input scale
Note that with SusyTC an automatic calculation of the GUT scale is not possible. The remainder of the block works as usual, e.g. optionally one can overwrite common Constrained MSSM gaugino or Higgs soft-breaking parameters:
1 : $M_1(M_{\rm input})$ bino mass (real part)
2 : $M_2(M_{\rm input})$ wino mass (real part)
3 : $M_3(M_{\rm input})$ gluino mass (real part)
21 : $m^2_{H_d}(M_{\rm input})$
22 : $m^2_{H_u}(M_{\rm input})$
Imaginary components for the gaugino masses can be given in Block IMEXTPAR.

– Block IMEXTPAR: as defined in [55].

– Block QEXTPAR: low energy input:
0 : (Default = 91.1876): the low energy scale to which REAP evolves the SM RGEs.
23 : "SUSY scale" Q, where the MSSM is matched to the SM. If this entry is set, it overwrites the automatically calculated SUSY scale.

– Block GAUGE: the $\overline{\rm DR}$ gauge couplings at the input scale
1 : $g_1(M_{\rm input})$ U(1) gauge coupling
2 : $g_2(M_{\rm input})$ SU(2) gauge coupling
3 : $g_3(M_{\rm input})$ SU(3) gauge coupling

– Block YU, Block YD, Block YE, Block YN: the real parts of the Yukawa matrices Y_u, Y_d, Y_e, and Y_ν in the flavour basis [55]. They should be given in the FORTRAN format (1x,I2,1x,I2,3x,1P,E16.8,0P,3x,'#',1x,A), where the first two integers correspond to the indices and the double precision number to $Re(Y_{ij})$.

– Block IMYU, Block IMYD, Block IMYE, Block IMYN: the imaginary parts of the Yukawa matrices Y_u, Y_d, Y_e, and Y_ν in the flavour basis [55]. They are given in the same format as the real parts.

– Block MN: the real part of the symmetric Majorana mass matrix M_n of the right-handed neutrinos in the flavour basis (3.1). Only the "upper-triangle" entries should be given, the input format is as for the Yukawa matrices.

– Block IMMN: the imaginary part of the symmetric Majorana mass matrix M_n of the right-handed neutrinos in the flavour basis (3.1). Only the "upper-triangle" entries should be given, the input format is as for the Yukawa matrices.

The remaining blocks can be given optionally to overwrite Constrained MSSM input boundary conditions:

– Block TU, Block TD, Block TE, Block TN: the real parts of the trilinear soft-breaking matrices T_u, T_d, T_e, and T_ν in the flavour basis [55]. They should be given in the same format as the Yukawa matrices.

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– `Block IMTU, Block IMTD, Block IMTE, Block IMTN`: the imaginary parts of the trilinear soft-breaking matrices T_u, T_d, T_e, and T_ν in the flavour basis [55]. They should be given in the same format as the Yukawa matrices.

– `Block MSQ2, Block MSU2, Block MSD2, Block MSL2, Block MSE2, Block MSN2`: the real parts of the soft-breaking mass squared matrices $m^2_{\tilde{Q}}$, $m^2_{\tilde{u}}$, $m^2_{\tilde{d}}$, $m^2_{\tilde{L}}$, $m^2_{\tilde{e}}$, and $m^2_{\tilde{\nu}}$ in the flavour basis [55]. Only the "upper-triangle" entries should be given, the input format is as for the Yukawa matrices.

– `Block IMMSQ2, Block IMMSU2, Block IMMSD2,Block IMMSL2, Block IMMSE2, Block IMMSN2`: the imaginary parts of the soft-breaking mass squared matrices $m^2_{\tilde{Q}}$, $m^2_{\tilde{u}}$, $m^2_{\tilde{d}}$, $m^2_{\tilde{L}}$, $m^2_{\tilde{e}}$, and $m^2_{\tilde{\nu}}$ in the flavour basis [55]. Only the "upper-triangle" entries should be given, the input format is as for the Yukawa matrices.

- `STCWriteSLHA2Output[`*"Path"*`]` writes an "Les Houches" [55, 58] output file to *"Path"*. If no path is given, the output is saved in the Mathematica Notebook directory as *"SusyTC.out"*. The output follows SLHA conventions, with the following exceptions:

  – `Block MASS`: the mass spectrum is given as $\overline{\text{DR}}$ masses at the SUSY scale. The only exception is the pole mass M_{H^+} (M_A) for CP violation turned on (off).

  – `Block ALPHA`: the tree-level Higgs mixing angle α_{tree}.

  – `Block HMIX`: instead of M_A we give
  101 : m_3

  The other blocks follow the SLHA2 output conventions, e.g. $\overline{\text{DR}}$ values at the SUSY scale in the Super-CKM/Super-PMNS basis. To avoid confusion, the blocks `Block DSQMIX, Block USQMIX, Block SELMIX, Block SNUMIX` and the corresponding blocks for the imaginary entries, return the sfermion mixing matrices $R_{\tilde{f}}$ in SLHA 2 convention.

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