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ABJ(M) chiral primary three-point function at two-loops

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Abstract: We compute the leading correction to the structure constant for the three-point function of three length-two chiral primary operators in planar ABJ(M) theory at weak 't Hooft coupling.

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Contents

1 Introduction

The $\mathrm{AdS}_4/\mathrm{CFT}_3$ correspondence between the $\mathcal{N} = 6$ superconformal Chern-Simons-matter theory of Aharony, Bergman, Jafferis, and Maldacena (ABJM) [1] and M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$ presents a fertile playground for further explorations of the gauge-gravity duality, beyond the well-trodden ground of the $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence between $\mathcal{N} = 4$ supersymmetric Yang-Mills in four dimensions and type-IIB strings on $AdS_5 \times S^5$. In many respects one expects a very similar picture to that found so far for $\mathcal{N} = 4$ SYM: planar integrability, localization formulae, supersymmetric Wilson loops, scattering amplitudes, etc. have been found on both sides of this $\mathrm{AdS}_4/\mathrm{CFT}_3$ correspondence. In many cases ABJM simply presents an added complication, as in the case of perturbation theory where often the analogue to one-loop corrections in $\mathcal{N} = 4$ SYM are two-loop corrections in ABJM.

A point of departure is found however, in the three-point correlation functions of chiral primary operators (CPO's) of dimension J

$$\mathcal{O}_A^J = \frac{1}{\sqrt{J/2}} \left(\frac{k}{4\pi\sqrt{NM}} \right)^{J/2} (\mathcal{C}_A)^{A_1 \ldots A_{J/2}}_{B_1 \ldots B_{J/2}} \operatorname{Tr} \left(Y^{B_1} Y^\dagger_{A_1} \cdots Y^{B_{J/2}} Y^\dagger_{A_{J/2}} \right) , \tag{1.1}$$

built from the $\mathrm{U}(N) \times \mathrm{U}(M)$ bifundamental scalar fields Y^A of the ABJ(M) [2] theory using the tensors $\mathcal{C}_A$ which are symmetric in upper and in lower indices and are traceless in any pair consisting of one upper and one lower index.[1] These operators have vanishing anomalous dimension. Unlike in $\mathcal{N}=4$ SYM, where chiral primary operators have protected three-point functions (cf. [3] and references therein), in ABJM we know from supergravity [4] that they scale as $\lambda^{1/4}/N$ where $\lambda = N/k$ is the 't Hooft coupling.[2] Given the conformally-fixed form

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle = \frac{1}{(4\pi)^\gamma}\frac{C_{123}(\lambda)}{|x_{12}|^{\gamma_3}|x_{23}|^{\gamma_1}|x_{31}|^{\gamma_2}}, \tag{1.2}$$

where $\gamma_i = (\sum_j J_j - 2J_i)/2$, $\gamma = \gamma_1+\gamma_2+\gamma_3$, and $x_{ij} = x_i - x_j$, the specific pattern of structure constants at strong coupling and for large $N = M$ was given in [5] (we take $J_3 \geq J_2 \geq J_1$)

$$\begin{aligned} &C_{123}(\lambda \gg 1) = \\ &\frac{1}{N}\left(\frac{\lambda}{2\pi^2}\right)^{1/4}\frac{\prod_{i=1}^3\sqrt{J_i+1}\,(J_i/2)!\,\Gamma(\gamma_i/2+1)}{\Gamma(\gamma/2+1)} \\ &\sum_{p=0}^{\gamma_3}\frac{(\mathcal{C}_1)^{I_1\ldots I_p I_{p+1}\ldots I_{J_1/2}}_{K_1\ldots K_{\gamma_3-p}K_{\gamma_3-p+1}\ldots K_{J_1/2}}\,(\mathcal{C}_2)^{K_1\ldots K_{\gamma_3-p}L_1\ldots L_{\gamma_1-J_2/2+p}}_{I_1\ldots I_p M_1\ldots M_{J_2/2-p}}\,(\mathcal{C}_3)^{K_{\gamma_3-p+1}\ldots K_{J_1/2}M_1\ldots M_{J_2/2-p}}_{I_{p+1}\ldots I_{J_1/2}L_1\ldots L_{\gamma_1-J_2/2+p}}}{p!(\gamma_3-p)!(\gamma_1-J_2/2+p)!(J_2/2-p)!(\gamma_2-J_1/2+p)!(J_1/2-p)!}, \end{aligned} \tag{1.3}$$

in terms of the three tensors $\mathcal{C}$ defining the three CPO's. This is a remarkable difference from $\mathcal{N}=4$ SYM: not only do the structure constants C_{123} depend on the coupling λ, they consist of a host of interpolating functions — one for each γ_3+1 ways[3] the $\mathcal{C}$ tensors can be contracted. These interpolating functions depend on the dimensions of the operators J_i in a non-trivial way. Why should one associate an interpolating function to each possible $\mathcal{C}$ tensor contraction? Because at tree-level in the planar theory there is a single way[4] in which the $\mathcal{C}$ tensors contract, which is included in the sum over p above and which we will label as $\langle \mathcal{C}_1\,\mathcal{C}_2\,\mathcal{C}_3\rangle_{\text{tree}}$ (the colour factor C_F is given in footnote 4)

$$C_{123}(\lambda \ll 1) = C_F\sqrt{(J_1/2)(J_2/2)(J_3/2)}\,\langle \mathcal{C}_1\,\mathcal{C}_2\,\mathcal{C}_3\rangle_{\text{tree}} + \text{loop corrections}. \tag{1.4}$$

One therefore sees that the various interpolating functions kick-in at higher orders as the 't Hooft coupling is increased. This is not true for the extremal correlators where $J_3 = J_1+J_2$ — they retain this form even at strong coupling because p is forced to zero

$$C^{\text{extremal}}_{123}(\lambda \gg 1) = \frac{1}{N}\left(\frac{\lambda}{2\pi^2}\right)^{1/4}\sqrt{(J_1+1)(J_2+1)(J_3+1)}\,\langle \mathcal{C}_1\mathcal{C}_2\mathcal{C}_3\rangle_{\text{tree}}. \tag{1.5}$$

[1] In this normalization the two-point function is given by $\langle \mathcal{O}^J_A(x)\mathcal{O}^J_B(0)\rangle = \delta_{AB}/(4\pi|x|)^J$.

[2] ABJM implies that $N = M$. When discussing results where N and M are distinct we will use $\lambda = N/k$ and $\hat{\lambda} = M/k$.

[3] Switching all upper and lower indices simultaneously on all $\mathcal{C}$ tensors is a symmetry of the three-point functions. This reduces the number $\gamma_3+1 \to (\gamma_3+1)/2$ if γ_3 is odd or $(\gamma_3+2)/2$ otherwise.

[4] This statement is true when the γ_i are even, which means that each $\mathcal{C}$ tensor shares as many up as down indices with each other $\mathcal{C}$ tensor. In this case C_{123} carries a colour factor of $C_F = (1/N + 1/M)$. In the case of odd γ_i there are two contractions of the $\mathcal{C}$ tensors which are related by switching all down and up indices simultaneously on all $\mathcal{C}$ tensors. In this case the colour factor is $C_F = 1/\sqrt{NM}$.

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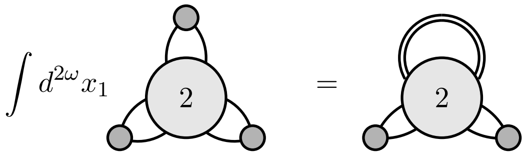

Figure 1. The main idea behind the method: two-loop corrections to the three-point function are gotten via integration over an external point thus transforming three-point integrals into three-loop propagator-type integrals. The line associated with the integrated operator has a doubled propagator, i.e. $1/p^4$ in momentum space.

In this paper we will take the first steps towards exploring the $C_{123}(\lambda)$ in perturbation theory. We will focus on one of the aforementioned interpolating functions arising from three length-two operators. In this setting there are just two possible ways of contracting the $\mathcal{C}$ tensors, but we will choose operators such that only one is non-zero.[5] The first correction appears at $\mathcal{O}(\lambda^2)$ or two-loops. This presents a challenge because three-loop integrals with three off-shell legs are difficult to work with and have not been widely considered in the literature. In order to overcome this obstacle, we integrate in configuration space over one of the operators' position using dimensional regularization, see figure 1. This reduces the problem to three-loop propagator diagrams which have been widely studied and are tractable. We believe that this trick works when dealing with protected operators since the three-point function is guaranteed to be finite *and* the coordinate dependence is fixed to a known form by conformal symmetry, where the powers of the coordinate differences are independent of the coupling.

We begin in section 2 with a presentation of the details of our method. In section 3 we give an exhibition of it in the setting of $\mathcal{N} = 4$ SYM at the one-loop level where we show that the three-point function indeed comes out uncorrected, i.e. independent of the coupling $g^2_{YM}N$. We continue in section 4 with our main result, the structure constant for three length-two CPO's in ABJ(M), given by (4.2). The calculational method is that employed originally in [6] and recently in an almost identical setting (two-loop form factors for the same operators) in [7]. Finally we end with a discussion in section 5 and give details of the calculation in two appendices.

2 Method

We consider the three CPO's

$$\mathcal{O}_1 = \frac{k}{4\pi\sqrt{MN}}\,\mathrm{Tr}(Y^1 Y_2^\dagger), \quad \mathcal{O}_2 = \frac{k}{4\pi\sqrt{MN}}\,\mathrm{Tr}(Y^2 Y_3^\dagger), \quad \mathcal{O}_3 = \frac{k}{4\pi\sqrt{MN}}\,\mathrm{Tr}(Y^3 Y_1^\dagger). \tag{2.1}$$

We know from conformal invariance that[6]

$$\big\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\big\rangle = \frac{1}{(4\pi)^3\sqrt{NM}}\frac{\hat{C}_{123}(\lambda)}{|x_{12}||x_{23}||x_{31}|}, \tag{2.2}$$

[5] In any case, by footnote 3 there is only one interpolating function at play.
[6] From now on we choose to factor the colour factor out of the structure constant.

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where $\hat{C}_{123} = 1 + c_1\lambda^2 + c_2\hat{\lambda}^2 + c_3\lambda\hat{\lambda} + \ldots$. We continue this expression to $2\omega = d - 2\epsilon$ dimensions

$$\big\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\big\rangle = \frac{1}{\sqrt{NM}} \frac{\hat{C}_{123}(\lambda)}{\left(x_{12}^2 x_{23}^2 x_{31}^2\right)^{\omega-1}} \frac{\Gamma^3(\omega-1)}{(4\pi^\omega)^3}, \tag{2.3}$$

and integrate over x_1 using dimensional regularization. We obtain

$$\sqrt{MN}\int d^{2\omega}x_1 \, \langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle = \frac{\Gamma(\omega-1)\Gamma(\omega-2)}{2^6\pi^{2\omega}(x_{23}^2)^{2\omega-3}}\,\hat{C}_{123}(\lambda). \tag{2.4}$$

This integration is free from UV divergences in any dimension. It is however IR divergent in any dimension ≤ 4. Thus the method leaves unmolested the short distance physics determining the renormalization of the operators, but does introduce IR divergences into the loop integrations in momentum space. Since the method relies crucially on using one and the same renormalization parameter to regulate these two classes of divergences one may encounter the accidental cancellation of IR and UV poles in intermediate stages in the calculation, as we will see for example in section 3, see footnote 7. However, this is not a cause for concern: the r.h.s. of (2.3) is a guaranteed finite quantity. Thus the only true divergences are the IR divergences introduced by the integration over x_1 and these are regulated in a controlled way, i.e. by the r.h.s. of (2.4).

Using (2.4) we may express $\hat{C}_{123}(\lambda)$ via

$$\hat{C}_{123}(\lambda) = \sqrt{NM}\frac{2^6\pi^{2\omega}(x_{23}^2)^{2\omega-3}}{\Gamma(\omega-1)\Gamma(\omega-2)}\int d^{2\omega}x_1 \, \big\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\big\rangle. \tag{2.5}$$

In general $\hat{C}_{123}(\lambda)$ will be renormalized by the two-point function

$$\big\langle \mathcal{O}_i(x)\mathcal{O}_i(0)\big\rangle = \frac{\Gamma^2(\omega-1)}{(4\pi^\omega)^2}\frac{g_i(\lambda)}{(x^2)^{2(\omega-1)}}, \tag{2.6}$$

where $g_i = g = 1 + d_1\lambda^2 + d_2\hat{\lambda}^2 + d_3\lambda\hat{\lambda} + \ldots$, giving (cf. [8])

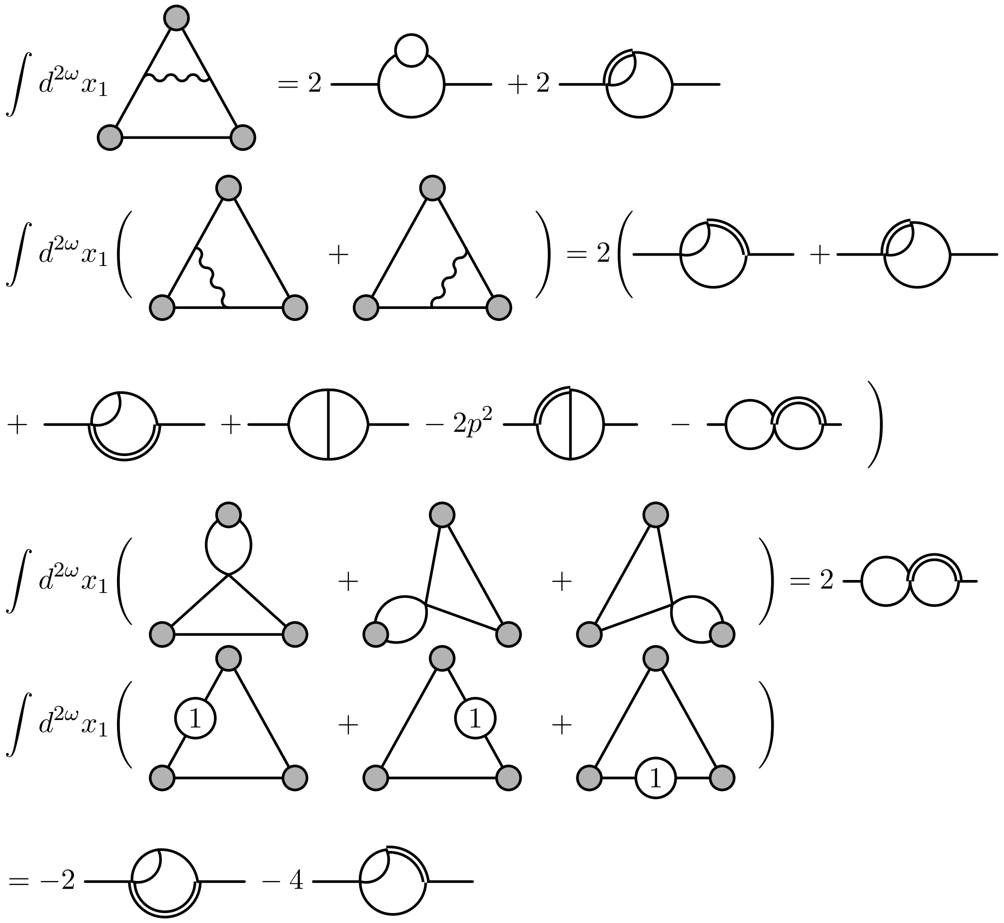
$$C_{123}\big|_{\mathcal{O}(\lambda^2)} = \left[\hat{C}_{123} - \frac{1}{2}\sum_{i=1}^{3} g_i\right]_{\mathcal{O}(\lambda^2)}. \tag{2.7}$$

3 $\mathcal{N} = 4$ SYM at one loop

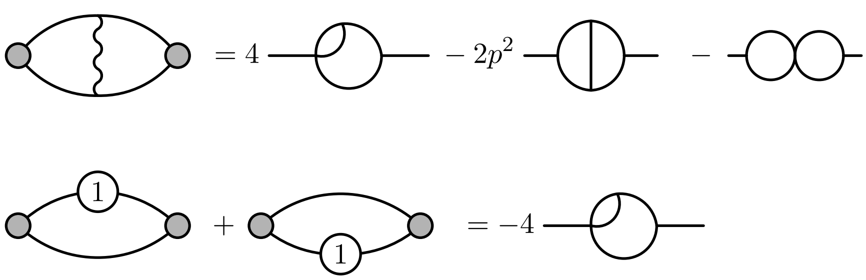
Before moving on to ABJM, it is instructive to see how the method described in section 2 works in the case of $\mathcal{N} = 4$ SYM at one-loop, where for CPO's we expect to find that there is no correction to C_{123}. In analogy with the ABJM case we take three length-two CPO's (here $\lambda = g_{YM}^2 N$, where g_{YM} is the Yang-Mills coupling constant)

$$\mathcal{O}_1 = \frac{1}{\sqrt{\lambda}}\,\mathrm{Tr}(\Phi^1\Phi^2), \quad \mathcal{O}_2 = \frac{1}{\sqrt{\lambda}}\,\mathrm{Tr}(\Phi^2\Phi^3), \quad \mathcal{O}_3 = \frac{1}{\sqrt{\lambda}}\,\mathrm{Tr}(\Phi^3\Phi^1), \tag{3.1}$$

which we represent by three grey blobs in the diagrams below. Here we use dimensional reduction in order to preserve supersymmetry. At one loop we find that the contributions to the three-point function are as follows, where we employ a double line to denote a propagator with doubled weight, i.e. $1/p^4$ instead of $1/p^2$. Some details of the calculation are provided in appendix B.[7]



We will also need to compute the normalization of the two-point functions, given by the following diagrams



[7] In the fourth line below we see that the bubble at the top of the first triangle integrates to zero. This is an example of the coincidental UV-IR pole cancellation discussed in the previous section.

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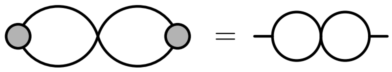
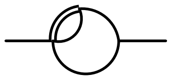
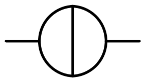
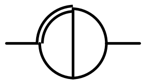
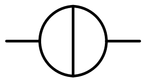


Summing-up these diagrams we find the following results

$$N\int d^{2\omega}x_1\langle\mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\rangle_{\mathcal{O}(\lambda)}=\int\frac{d^{2\omega}p}{(2\pi)^{2\omega}}e^{ip\cdot x_{23}}\Big(4$$

$$+\,2$$

$$-\,4p^2$$

$$\Big),$$

$$\langle\mathcal{O}_i(x)\mathcal{O}_i(0)\rangle_{\mathcal{O}(\lambda)}=\int\frac{d^{2\omega}p}{(2\pi)^{2\omega}}e^{ip\cdot x}\Big(-2p^2$$

$$\Big).$$

These expressions evaluate to

$$\begin{aligned}N\int d^{2\omega}x_1\langle\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle_{\mathcal{O}(\lambda)}&=\frac{1}{x_{23}^2}\frac{3\zeta(3)}{2^8\pi^6}+\mathcal{O}(\epsilon),\\ \langle\mathcal{O}_i(x)\mathcal{O}_i(0)\rangle_{\mathcal{O}(\lambda)}&=-\epsilon\frac{3\zeta(3)}{64\pi^6}\frac{1}{(x^2)^2}+\mathcal{O}(\epsilon^2).\end{aligned}\tag{3.2}$$

Therefore we have from (2.5) and (2.6) that

$$\hat{C}_{123}\big|_{\mathcal{O}(\lambda)}=g_i\big|_{\mathcal{O}(\lambda)}=-3\epsilon\frac{\zeta(3)}{4\pi^2},\qquad C_{123}\big|_{\mathcal{O}(\lambda)}=3\epsilon\frac{\zeta(3)}{8\pi^2}=\mathcal{O}(\epsilon),\tag{3.3}$$

and so both g_i and $\hat{C}_{123}$ are zero at the one-loop order and hence trivially so is $C_{123}\big|_{\mathcal{O}(\lambda)}=0$ on the physical dimension as expected.

4 Main calculation and results

In this section we summarize the results of the ABJ(M) calculation. The method used is that employed in [7]. Namely Feynman rules spelled-out in [6] are processed into master integrals using FIRE [9]. Note that for ABJM we cannot use dimensional reduction as there is no higher dimensional supersymmetric theory to reduce from. The scheme used here is that employed successfully in [6]: i.e. to reduce all numerators to scalar products in $d=3$ before integrating using $d=3-2\epsilon$. In figures 2 and 3 we list all non-zero Feynman diagrams. The results for these diagrams and further details of the calculation are found in appendix A. The results are as follows (below we quote the two-loop corrections to C_{123},

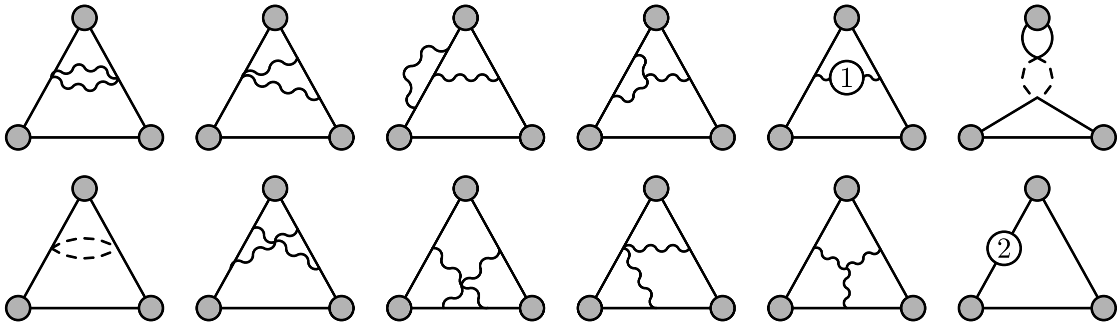


Figure 2. The Feynman diagrams which contribute to the three-point function in ABJ(M). Note that all unique diagrams obtained from these via rotations (by $2\pi/3$) and reflections about the perpendicular bisectors of the main triangle must also be considered.

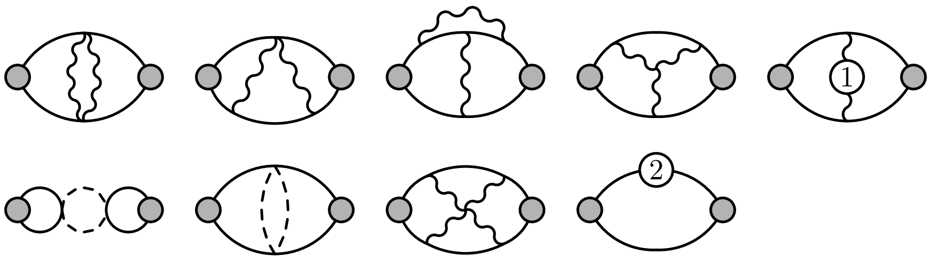


Figure 3. The Feynman diagrams which contribute to the two-point function in ABJ(M). Unique diagrams obtained through reflection about the horizontal axis must also be considered.

$\hat{C}_{123}$ and g; they are equal to unity at tree-level)

$$\begin{aligned}\sqrt{MN}\int d^{2\omega}x_1\langle\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle &= -\frac{\hat{C}_{123}}{32\pi^2} = (\lambda+\hat{\lambda})^2\frac{1}{2^{10}}\left(\frac{5}{3}+\frac{8}{\pi^2}\right)\\ &\quad+(\lambda-\hat{\lambda})^2\frac{1}{2^{10}}\left(-\frac{1507}{3}-\frac{24}{\pi^2}\right)+\mathcal{O}(\epsilon), \qquad (4.1)\\ \langle\mathcal{O}_i(x)\mathcal{O}_i(0)\rangle &= \frac{g}{16\pi^2x^2} = -\frac{(\lambda+\hat{\lambda})^2}{192}\frac{1}{x^2}+\frac{47}{48}\frac{(\lambda-\hat{\lambda})^2}{x^2}+\mathcal{O}(\epsilon).\end{aligned}$$

We notice here that the two-point function is finite, as expected, and that factors of Euler's constant γ and $\log\pi$ cancel out. This we interpret as confirmation of regularization scheme independence. We thus find

$$\begin{aligned}C_{123} &= -32\pi^2\sqrt{MN}\int d^{2\omega}x_1\langle\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle-\frac{3g}{2}\\ &= (\lambda+\hat{\lambda})^2\left(\frac{7\pi^2}{96}-\frac{1}{4}\right)+(\lambda-\hat{\lambda})^2\left(-\frac{749\pi^2}{96}+\frac{3}{4}\right).\end{aligned} \qquad (4.2)$$

5 Discussion

We have computed the structure constant for a CPO three-point function in ABJ(M) theory at leading order in perturbation theory (4.2). Concentrating on the ABJM case, we now have results for this quantity both at weak and at strong coupling in the planar limit; indeed using (1.3) we have

$$C_{123}(\lambda \ll 1) = 1 - \lambda^2 \left(1 - \frac{7\pi^2}{24}\right), \qquad C_{123}(\lambda \gg 1) = \left(\frac{\lambda}{2}\right)^{1/4} \frac{\sqrt{3\pi}}{2}. \tag{5.1}$$

It would be very interesting to find a way to compute this interpolating function for all values of the 't Hooft coupling. Although the machinery of integrability has been applied to computing three-point functions in $\mathcal{N} = 4$ SYM [10–13], here we have a rather different situation, in that three spin-chain vacuum states, i.e. states which are annihilated by the dilatation operator, nevertheless have a non-trivial structure function dependent upon the 't Hooft coupling. Indeed, because of the fact that at tree-level the tensors defining the CPO's contract in just one way, whereas at strong coupling they contract in all ways, it is clear that there are a host of interpolating functions, and each one likely begins at a different order in the 't Hooft coupling in the perturbative expansion. Understanding the connections between these functions remains a very interesting direction of future research.

We know that the M-theory dual of ABJ theory involves the appearance of a three-form in an $S^3/\mathbb{Z}_k \subset S^7/\mathbb{Z}_k$ [2]. It would be interesting to compute the fluctuation spectrum around this background and repeat the three-point function calculation in order to have a version of (1.3) with $N \neq M$.

Acknowledgments

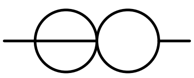
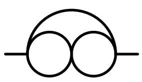
It is a pleasure to thank Andreas Brandhuber, Jan Plefka, Sanjaye Ramgoolam, Rodolfo Russo, Gordon Semenoff, Gabriele Travaglini, Brian Wecht, and Konstantin Zarembo for discussions.

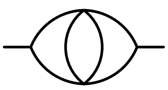
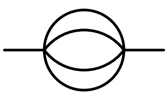
A Calculational details

We use the machinery developed in [6] and employed in a very similar setting, the calculation of the two-loop form factor of CPO's, in [7]. These references contain ample detail and we choose not to reprint the details of the action, Feynman rules, etc. here but rather refer the interested reader to these papers.

A.1 Master integrals

The three-loop master integrals required to complete the calculation are as follows

$P_1 =$  $= G^2(1,1)\, G(1, 2-\omega)$, $P_2 =$  $= G^2(1,1)\, G(1, 4-2\omega)$,

$P_3 =$  $= G(1,1)\, F_{2-\omega}$, $P_4 =$  $= G(1,1)\, G(1, 2-\omega)\, G(1, 3-2\omega)$,

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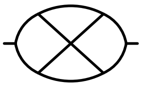
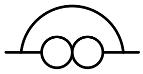
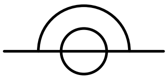
$$P_5 = $$  $$= \frac{1}{96} - \frac{13}{64\pi^2} + \mathcal{O}(\epsilon), \qquad P_7 = $$  $$= G^2(1,1)\, G(1, 6-2\omega),$$

$$P_6 = $$  $$= G(1,1)\, G(1, 2-\omega)\, G(1, 5-2\omega),$$

where [14, 15]

$$F_\lambda = \frac{2}{\pi^{2\omega}} \Gamma(\omega-1)\,\Gamma(\omega-\lambda-1)\,\Gamma(\lambda-2\omega+3)\Bigg(-\frac{\pi \cot\left(\pi(2\omega-\lambda)\right)}{\Gamma(2\omega-2)} + \frac{\Gamma(\omega-1)\,{}_3F_2\left(1, 2+\lambda-\omega, 2\omega-2; \lambda+1, \lambda-\omega+3; 1\right)}{(\omega-\lambda-2)\,\Gamma(1+\lambda)\,\Gamma(3\omega-\lambda-4)} \Bigg), \tag{A.1}$$

and where

$$G(\alpha,\beta) = \frac{1}{(4\pi)^\omega} \frac{\Gamma(\alpha+\beta-\omega)\,\Gamma(\omega-\alpha)\,\Gamma(\omega-\beta)}{\Gamma(\alpha)\,\Gamma(\beta)\,\Gamma(2\omega-\alpha-\beta)}. \tag{A.2}$$

The non-planar integral P_5 must be evaluated using the Gegenbauer polynomial technique [16], cf. [17]. The Fourier transform is given by

$$\int \frac{d^{2\omega}p}{(2\pi)^{2\omega}} \frac{e^{ip\cdot x}}{[p^2]^s} = \frac{\Gamma(\omega-s)}{4^s \pi^\omega \Gamma(s)} \frac{1}{[x^2]^{\omega-s}}. \tag{A.3}$$

A.2 Integral reduction for ABJ(M) diagrams

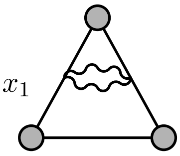
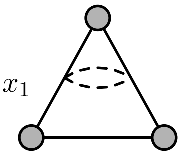
We give here the results for the integrated three-point and two-point diagrams in terms of basis integrals. Note that below a single triangle diagram represents all unique diagrams obtained through reflection and rotation, similarly a given two-point diagram represents all unique diagrams gotten through reflections. A factor of $(4\pi/k)^2$ is suppressed while the colour factors associated with the various diagrams are as follows

$$\begin{gathered} I_7, T_7 \to -2MN, \quad I_1, I_2, I_{10}, T_1, T_2 \to (N-M)^2 - 2MN, \\ I_3, I_5, I_8, I_9, T_3, T_5, T_8 \to 2MN, \quad I_4, I_{11}, T_4 \to N^2 + M^2, \\ I_6, T_6 \to (N-M)^2. \end{gathered} \tag{A.4}$$

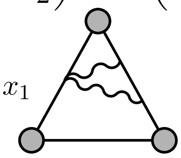
The two-loop self energy of the scalar field is given by [6]

$$\begin{aligned} Z_{\text{scalar}} = -\frac{1}{(4\pi)^2}\Bigg[& MN\left(\frac{3}{4(3-2\omega)} + \frac{1}{4}\left(-\frac{3\pi^2}{2} + 25 - 3\gamma + 3\log(4\pi)\right)\right) \\ & + \frac{(M-N)^2}{4}\left(\frac{1}{2(3-2\omega)} - \frac{\pi^2}{4} + \frac{1}{2}\left(3-\gamma+\log(4\pi)\right)\right)\Bigg] + \mathcal{O}(\epsilon). \end{aligned} \tag{A.5}$$

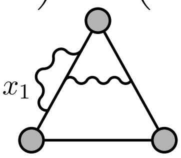
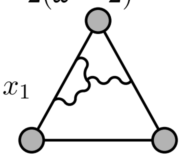
A.2.1 Three-point diagrams

$$I_1 = -\frac{1}{4} I_7 = \int d^{2\omega} x_1$$  $$= -\frac{1}{4}\int d^{2\omega} x_1$$  $$= -\frac{(\omega-1)^2}{2(\omega-2)} P_6$$

$$+ \left(\omega - \frac{3}{2}\right) P_3 + \left(-12\omega - \frac{3}{\omega-2} + 11\right) P_4$$

$$I_2 = \int d^{2\omega} x_1$$  $$= P_1\left(-5\omega + \frac{1}{\omega-2} + \frac{17}{2}\right) + P_2\left(2\omega + \frac{1}{4-2\omega} - 4\right)$$

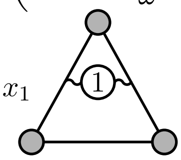
$$+ P_3\left(\frac{19}{4} - 3\omega\right) + P_4\left(24\omega - \frac{5}{\omega-2} - \frac{3}{2(\omega-2)^2} + \frac{1}{2\omega-3} - 32\right)$$

$$I_3 = 2I_4 = \int d^{2\omega} x_1$$  $$= 2\int d^{2\omega} x_1$$ 

$$= -\frac{2P_1(5\omega-8)(2(\omega-4)\omega+7)}{(\omega-2)^2}$$

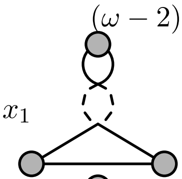
$$- P_2\left(-24\omega + \frac{6}{\omega-2} + \frac{2}{(\omega-2)^2} + 40\right) - P_3\left(8\omega + \frac{1}{\omega-2} - 10\right)$$

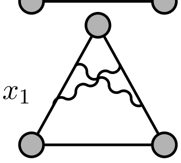
$$- 2P_4\left(-12\omega - \frac{6}{\omega-2} + \frac{1}{3-2\omega} + 5\right) - P_6\left(-4\omega - \frac{2}{\omega-2} + 2\right) - 2P_7(\omega-1)$$

$$I_5 = \int d^{2\omega} x_1$$ 

$$= P_3(8\omega-12) + P_6\left(8\omega + \frac{6}{\omega-2}\right)$$

$$- \frac{4P_4(2\omega-3)(3\omega-5)(4\omega-5)}{(\omega-2)^2}$$

$$I_6 = \int d^{2\omega} x_1$$  $$= -2\, G^2(1,1)\, G(2,1)$$

$$I_8 = \int d^{2\omega} x_1$$  $$= P_1\left(-74\omega - \frac{2(\omega(5\omega(5\omega+17)-529)+532)}{(\omega-2)^2(4\omega-7)}\right)$$

$$+ \frac{8P_2(2\omega-3)(\omega(9\omega-25)+17)}{(\omega-2)^2} + P_3\left(7\omega + \frac{5}{\omega-2} + \frac{5}{28-16\omega} - \frac{7}{4}\right)$$

$$+ 2P_4\left(-504\omega - \frac{1004}{\omega-2} - \frac{376}{(\omega-2)^2} - \frac{48}{(\omega-2)^3} + \frac{1}{3-2\omega} + \frac{50}{7-4\omega} - 181\right)$$

$$+ \frac{P_5(\omega-2)(2\omega-3)}{4\omega-7} + \frac{2P_6(\omega-1)\omega(2\omega-3)}{(\omega-2)^2} + P_7\left(\frac{1}{2-\omega} - 1\right)$$

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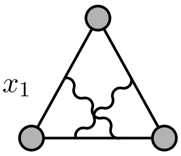
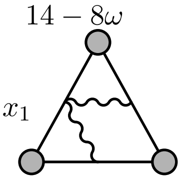
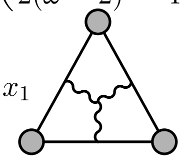
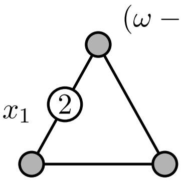
$$I_9 = \int d^{2\omega} x_1$$

$$= \frac{P_1(3\omega-4)(\omega(4(\omega-7)\omega+57)-35)}{(\omega-2)^2(4\omega-7)}$$
$$+ \frac{1}{8}P_3\left(36\omega + \frac{12}{\omega-2} + \frac{5}{4\omega-7} - 25\right)$$
$$+ 2P_4\left(174\omega + \frac{328}{\omega-2} + \frac{118}{(\omega-2)^2} + \frac{12}{(\omega-2)^3} + \frac{1}{2\omega-3} + \frac{25}{4\omega-7} + 53\right)$$
$$+ \frac{P_5(\omega-2)(2\omega-3)}{14-8\omega} - \frac{2P_2(2\omega-3)(\omega(8\omega-25)+19)}{(\omega-2)^2}$$

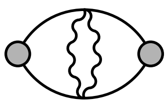
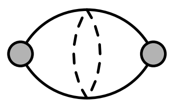
$$I_{10} = \int d^{2\omega} x_1$$

$$= -\frac{P_1(3\omega-4)(\omega(2\omega-5)+4)}{2(\omega-2)^2} + P_2\left(3\omega + \frac{\omega(\omega+5)-12}{2(\omega-2)^2}\right)$$
$$+ P_3\left(\frac{1}{2(\omega-2)} + \frac{3}{4}\right) + P_4\left(\frac{1}{2\omega-3} - \frac{6}{\omega-2} - 6\right)$$

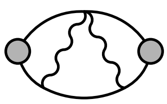
$$I_{11} = \int d^{2\omega} x_1$$

$$= -\frac{2P_4(3\omega-4)(4\omega-5)(\omega-1)^2}{(\omega-2)^2(2\omega-3)}$$
$$+ \frac{P_2(2\omega-3)(\omega(4\omega-13)+11)(\omega-1)}{(\omega-2)^3} + \frac{P_3(2\omega-3)(\omega-1)}{2(\omega-2)^2}$$
$$- \frac{P_1(3\omega-4)(2(\omega-3)\omega+5)(\omega-1)}{(\omega-2)^3}$$

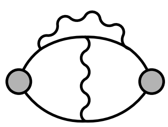
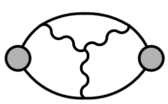
$$I_{12} = \int d^{2\omega} x_1$$


$$= (2\,G(1, 5-2\omega) + G(2, 4-2\omega))\, Z_{\text{scalar}}$$

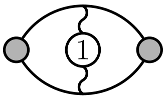
A.2.2 Two-point diagrams

$$T_1 = -\frac{1}{4}T_7 =$$

$$= -\frac{1}{4}$$

$$= \frac{P_3(3-2\omega)}{4(3\omega-4)}$$
$$+ \frac{1}{6}P_4\left(\frac{1}{3\omega-4} + \frac{3}{2\omega-3} + 10\right)$$

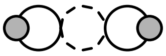
$$T_2 =$$

$$= \frac{P_3(9-6\omega)}{16-12\omega} + P_4\left(\frac{1}{8-6\omega} + \frac{1}{3-2\omega} - 4\right) + P_1 - \frac{P_2}{2}$$

$$T_3 = 2T_4 =$$

$$= 2$$

$$= P_1\left(\frac{2}{2\omega-3} + 4\right) + \frac{P_3(5-4\omega)}{4-3\omega}$$

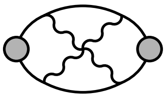
$$- \frac{2P_4(\omega-1)(4\omega-5)(8\omega-11)}{(3-2\omega)^2(3\omega-4)} - 4P_2$$

$T_5 =$ 

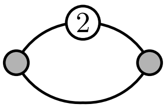
$$= \frac{P_3(5-4\omega)}{3\omega-4} + P_4\left(\frac{2}{12-9\omega} + \frac{12}{\omega-2} + \frac{40}{3}\right)$$

$T_6 =$ 

$$= -G^3(1,1)$$

$T_8 =$ 

$$\begin{aligned} &= P_1\left(\frac{7}{(\omega-2)^2} + \frac{35}{4(4\omega-7)} + \frac{3}{3-2\omega} + \frac{35}{2(\omega-2)} + \frac{43}{4}\right) \\ &+ \frac{P_2(4(14-5\omega)\omega-38)}{(\omega-2)^2} + \frac{1}{24}P_3\left(\frac{15}{4\omega-7} + \frac{16}{4-3\omega} - \frac{30}{\omega-2} - 37\right) - \frac{P_5(\omega-2)}{4(4\omega-7)} \\ &+ P_4\left(\frac{140}{(\omega-2)^2} + \frac{24}{(\omega-2)^3} + \frac{11}{2\omega-3} + \frac{50}{4\omega-7}\right. \\ &\qquad \left. + \frac{3}{(3-2\omega)^2} + \frac{4}{12-9\omega} + \frac{222}{\omega-2} + \frac{401}{3}\right) \end{aligned}$$

$T_9 =$ 

$$= \quad 2\,G(1,4-2\omega)\,Z_{\text{scalar}}$$

B $\mathcal{N}=4$ SYM details

We follow the conventions given in [18]. The relevant terms in the Euclidean action arise from the scalar potential and gauge coupling and are given by

$$S = \int d^{2\omega}x\left(f^{abc}\partial_\mu\Phi^{Ia}A^{b\mu}\Phi^{Ic} + \frac{1}{4}f^{abe}f^{cde}\Phi^{Ia}\Phi^{Jb}\Phi^{Ic}\Phi^{Jd}\right) + \text{not relevant}. \tag{B.1}$$

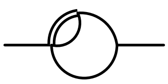
We use Feynman gauge where the gauge field propagator is

$$\langle A^a_\mu A^b_\nu\rangle = g^2_{YM}\frac{\delta_{\mu\nu}\delta^{ab}}{p^2}. \tag{B.2}$$

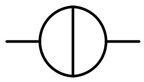
The one-loop correction to the scalar field is [18]

$$\langle\Phi^{Ia}\Phi^{Jb}\rangle = g^2_{YM}\frac{\delta^{IJ}\delta^{ab}}{p^2} - 2\,g^4_{YM}N\,G(1,1)\,\frac{\delta^{IJ}\delta^{ab}}{p^{6-2\omega}}. \tag{B.3}$$

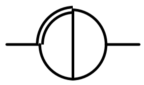
The integrals we need to evaluate for the $\mathcal{N}=4$ SYM example are as follows (cf. appendix J of [6])



$$= G(1,2)\,G(1,4-\omega),$$



$$= G(1,1)\,[\Delta(1,1) + 2C(1,1)\,G(3-\omega,2)]\,,$$



$$= G(1,1)\,[\Delta(2,1) + C(2,1)\,G(3-\omega,3) + C(1,2)\,G(4-\omega,2)]\,,$$

where

$$C(\alpha,\beta) = \frac{\alpha}{\alpha+\beta+2-2\omega}, \quad \Delta(\alpha,\beta) = -\frac{\alpha\,G(\alpha+1,\beta)+\beta\,G(\alpha,\beta+1)}{\alpha+\beta+2-2\omega}, \tag{B.4}$$

and where $G(\alpha,\beta)$ is given in (A.2).

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