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Spectrum of the charmed and b-flavored mesons in the relativistic potential model

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Abstract: We study the bound states of heavy-light quark-antiquark system in the relativistic potential model, where the potential includes the long-distance confinement term, the short-distance Coulomb term and spin-dependent term. The spectrum of B, B^*, D, D^* and states with higher orbital quantum numbers are obtained. Compared with previous results predicted in the relativistic potential model, the predictions are improved and extended in this work, more theoretical masses are predicted which can be tested in experiment in the future.

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Contents

1 Introduction

Bound state of heavy-light quark-antiquark system $Q\bar{q}$ is of special interest. Weak decays of such heavy-light system can be used to determine the fundamental parameters such as the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements in the standard model (SM), and to explore the source of CP violation. Experimental data from B factories have confirmed the existence of CP violation in B meson weak decays [1, 2]. Theoretically to treat the weak decays of heavy-flavored mesons B and D, effects of strong interactions have to be considered. Strong interactions in B decays can be separated into two parts, one of which can be calculated perturbatively in QCD, while the other part is dynamically non-perturbative. The binding effect in the quark-antiquark system is one of the main source of the non-perturbative dynamics. How to treat the binding effect in QCD is still an open question at present. Before the non-perturbative problem in QCD being completely solved, using phenomenological method to treat the bound state is an effective way in practice.

The bound state of quark-antiquark can be described by the wave equation [3, 4] with an effective potential compatible with QCD. The potential shows a linear confining behavior at large distance and a Coulombic behavior at short distance. Since the light quark in the heavy-light system is relativistic, the wave equation is assumed to be with a relativistic kinematics.

The bound state effects in B and D mesons have been investigated with the relativistic potential model previously in refs. [4–10]. In the works of [5–10] the spin-dependent interactions are not included. For the heavy-light quark-antiquark system, the heavy quark can be viewed as a static color source in the rest frame of the meson, and the light quark is bound around the heavy quark by an effective potential. In the heavy quark limit the spin of the heavy quark decouples from the interaction [11–18]. For the realistic heavy-quark mass, the spin-dependent interactions can be treated as perturbative corrections. In ref. [4], the spin-dependent interactions were considered several decades ago. For most charmed and b-flavored mesons, the theoretical predictions are well consistent with experiment. Only

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some masses of states with orbital angular momentum $l \geq 1$ are approximately 100 MeV higher than experimental measurements. Currently, with more experimental data available, the prediction to the spectrum of charmed and b-flavored mesons in the relativistic potential model needs to be improved.

In this work we will revisit the bound state effect in the heavy-flavored mesons. The spectrum of B, B^*, D, D^* and other heavy-light bound states with higher orbital angular momentum and higher radial quantum number are studied. Comparing with the work of ref. [4], the details of the method of solving the relativistic wave equation are given, the mixing between more possible states are considered. The spin-dependent potential is slightly modified, the predictions for the masses of charmed and b-flavored mesons are more consistent with experimental measurements. We also give more predictions for the bound states with higher radial quantum number, which can be tested in experiment in the future.

The paper is organized as follows. In section 2, the relativistic wave equation for the heavy-light quark-antiquark system and the effective Hamiltonian are given. In section 3 the wave equation is solved. Section 4 is for the numerical result and discussion. Section 5 is a brief summary.

2 The wave equation for heavy-light system and the effective Hamiltonian

The heavy flavor mesons B and D contain light quarks, which requires the wave equation describing the heavy-light system include relativistic kinematics. The equation is a relativistic generalization of Schrödinger equation

$$H\psi(\vec{r}) = E\psi(\vec{r})\,. \tag{2.1}$$

The effective Hamiltonian can be written as

$$H = H_0 + H', \tag{2.2}$$

with

$$H_0 = \sqrt{-\hbar^2c^2\nabla_1^2 + m_1^2} + \sqrt{-\hbar^2c^2\nabla_2^2 + m_2^2} + V(r)\,, \tag{2.3}$$

where $\vec{r} = \vec{x}_2 - \vec{x}_1$, and $\vec{x}_1$ and $\vec{x}_2$ are the coordinates of the heavy and light quarks, respectively. The operators ∇_1^2 and ∇_2^2 involve partial derivatives relevant to the coordinates $\vec{x}_1$ and $\vec{x}_2$, respectively. m_1 is the mass of the heavy quark, and m_2 the mass of the light antiquark. $V(r)$ is the effective potential of the strong interaction between the heavy and light quarks. It can be taken as a combination of a Coulomb term and a linear confining term, whose behavior is compatible with QCD at both short- and long-distance [4, 19, 20]

$$V(r) = -\frac{4}{3}\frac{\alpha_s(r)}{r} + b\,r + c\,. \tag{2.4}$$

The first term is contributed by one-gluon-exchange diagram calculated in perturbative QCD. The Coulomb term dominates the behavior of the potential at short-distance. The

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second term is the linear confining term. The third term c is a phenomenological constant, which is adjusted to give the correct ground state energy level of the quark-antiquark system.

The running coupling constant in coordinate space $\alpha_s(r)$ can be obtained from the coupling constant in momentum space $\alpha_s(Q^2)$ by Fourior transformation. It can be written in the following form [4]

$$\alpha_s(r) = \sum_i \alpha_i \frac{2}{\sqrt{\pi}} \int_0^{\gamma_i r} e^{-x^2} dx \,, \tag{2.5}$$

where α_i are free parameters fitted to make the behavior of the running coupling constant at short distance be consistent with the coupling constant in momentum space predicted by QCD. The numerical values of these parameters fitted in this work are $\alpha_1 = 0.15$, $\alpha_2 = 0.15$, $\alpha_3 = 0.20$, and $\gamma_1 = 1/2$, $\gamma_2 = \sqrt{10}/2$, $\gamma_3 = \sqrt{1000}/2$.

The second term H' in eq. (2.2) is the spin-dependent part of the Hamiltonian

$$H' = H^{\rm hyp} + H^{\rm so}, \tag{2.6}$$

where $H^{\rm hyp}$ is the spin-spin hyperfine interaction, $H^{\rm so}$ is spin-orbit interaction.

The spin-spin hyperfine interaction used in this work is

$$H^{\rm hyp} = \frac{32\pi}{9m_1\tilde{m}_{2a}}\alpha_s(r)\delta_\sigma(r)\vec{s}_1\cdot\vec{s}_2 + \frac{4}{3}\frac{\alpha_s(r)}{m_1\tilde{m}_{2b}}\frac{1}{r^3}\left(\frac{3\vec{s}_1\cdot\vec{r}\vec{s}_2\cdot\vec{r}}{r^2} - \vec{s}_1\cdot\vec{s}_2\right) \tag{2.7}$$

with

$$\delta_\sigma(r) = \left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma^2 r^2}, \tag{2.8}$$

where the parameter σ is taken as quark mass-dependent [4]

$$\sigma = \sqrt{\sigma_0^2\left(\frac{1}{2} + \frac{1}{2}\left(\frac{4m_1m_2}{(m_1+m_2)^2}\right)^4\right) + s_0^2\left(\frac{2m_1m_2}{m_1+m_2}\right)^2}, \tag{2.9}$$

here σ_0 and s_0 are phenomenological parameters.

The spin-orbit interaction is

$$H^{\rm so} = \frac{4}{3}\frac{\alpha_s(r)}{r^3}\left(\frac{1}{m_1} + \frac{1}{\tilde{m}_{2c}}\right)\left(\frac{\vec{s}_1\cdot\vec{L}}{m_1} + \frac{\vec{s}_2\cdot\vec{L}}{\tilde{m}_{2c}}\right) - \frac{1}{2r}\frac{\partial V(r)}{\partial r}\left(\frac{\vec{s}_1\cdot\vec{L}}{m_1^2} + \frac{\vec{s}_2\cdot\vec{L}}{(\tilde{m}_{2d})^2}\right), \tag{2.10}$$

where $\vec{L} = \vec{r}\times\vec{P}$ is the relative orbital angular momentum between the quark and antiquark.

The spin-dependent interactions can be predicted by one-gluon-exchange forces in QCD [3, 4]. The exact form of $\propto \delta(\vec{r})$ for the spin-spin contact term $\vec{s}_1\cdot\vec{s}_2$ and $1/m_q$ for the tensor term in $H^{\rm hyp}$ and spin-orbit interaction $H^{\rm so}$ are the predictions of one-gluon-exchange calculation in the non-relativistic approximation. It is reasonable that there might be contributions of non-perturbative dynamics in the bound state system and relativistic corrections for the light quark. In this work the form of the spin-spin contact hyperfine interaction is replaced by an interaction with the behavior of exponential suppression $e^{-\sigma^2 r^2}$ as in ref. [21], the mass of the light quark m_2's in the denominators are replaced

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by a set of new parameters $\tilde{m}_{2i}$, $i = a, b, c, d$, which include the relativistic corrections and the bound-state effect in the heavy meson. Originally in the potential calculated with one-gluon-exchange diagram [3, 4], the light quark mass m_2's are in the places of the new parameters $\tilde{m}_{2i}$'s in eqs. (2.7) and (2.10), which are obtained with the approximation that both the heavy and light quarks are viewed as non-relativistic, the momenta of the quarks are dropped. However, the light quark in the heavy meson should be highly relativistic, dropping the momentum of the light quark is not a good approximation. In addition, there may also be bound-state effect in the spin-spin and spin-orbit interaction terms, which can not be treated by one-gluon-exchange diagram. To include the relativistic effect for the light quark and the nonperturbative bound-state effect, we assume these effects can be effectively described by introducing a set of new parameters which replace the light quark mass in the denominator of the spin-dependent interaction terms. These new parameters are to be determined by fitting the experimental data on the spectrum of the charmed and b-flavored meson states. In the section of numerical treatment, one can find that this assumption does work. All the masses measured in experiment can be accommodated well.

3 The solution of the wave equation

Without the spin-dependent interaction, the solutions of the wave equation for pseudoscalar and vector states of the quark-antiquark system shall be degenerate. The prediction to the masses of B and B^*, D and D^* will be the same. For the heavy quark and light antiquark system, the interaction decouples to the heavy quark spin in the heavy quark limit [11–18]. Quark-spin dependent interaction can be treated as perturbation. The masses of B and B^*, D and D^* measured in experiment support this treatment, the mass-differences of B and B^*, D and D^* are only at the order of a few percent [22].

We solve the eigen equation of H_0 at first, then treat the spin-dependent Hamiltonian H' in the perturbation theory. The effect of H' will be considered to the first order in the perturbative expansion. Denote the eigenfunction and eigenvalue of the Hamiltonian H_0 by $\psi^{(0)}(\vec{r})$ and $E^{(0)}$, respectively, then the eigen equation of H_0 is

$$\left[\sqrt{-\hbar^2c^2\nabla_1^2+m_1^2}+\sqrt{-\hbar^2c^2\nabla_2^2+m_2^2}+V(r)\right]\psi^{(0)}(\vec{r})=E^{(0)}\psi^{(0)}(\vec{r})\,. \tag{3.1}$$

To solve the above equation, we express the wave function in terms of spectrum integration

$$\begin{aligned}\psi^{(0)}(\vec{r}) &= \int d^3r'\delta^3(\vec{r}-\vec{r}\,')\psi^{(0)}(\vec{r}\,') \\ &= \int d^3r' \int \frac{d^3k}{(2\pi\hbar c)^3} e^{i\vec{k}\cdot(\vec{r}-\vec{r}\,')/\hbar c}\psi^{(0)}(\vec{r}\,')\,.\end{aligned} \tag{3.2}$$

With the above expression, the wave equation becomes

$$\int \frac{d^3k}{(2\pi\hbar c)^3}d^3r'\left(\sqrt{k^2+m_1^2}+\sqrt{k^2+m_2^2}\right)e^{i\vec{k}\cdot(\vec{r}-\vec{r}\,')/\hbar c}\psi^{(0)}(\vec{r}\,')=\left(E^{(0)}-V(r)\right)\psi^{(0)}(\vec{r})\,. \tag{3.3}$$

The exponential $e^{i\vec{k}\cdot\vec{r}/\hbar c}$ can be decomposed in terms of spherical harmonics

$$e^{i\vec{k}\cdot\vec{r}/\hbar c}=4\pi\sum_{ln} i^l j_l\left(\frac{kr}{\hbar c}\right)Y_{ln}^*(\hat{k})Y_{ln}(\hat{r})\,, \tag{3.4}$$

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where j_l is the spherical Bessel function, $Y_{ln}(\hat{r})$ is the spherical harmonics, and $\hat{r}$ the unit vector along the direction of $\vec{r}$. The spherical harmonics satisfies the normalization condition

$$\int d\Omega Y_{l_1 n_1}(\hat{r}) Y_{l_2 n_2}(\hat{r}) = \delta_{l_1 l_2}\delta_{n_1 n_2}\,. \tag{3.5}$$

Using eq. (3.4) and factorize the wave function into radial and angular parts

$$\psi^{(0)}(\vec{r}) = \Phi_l(r) Y_{ln}(\hat{r})\,, \tag{3.6}$$

eq. (3.3) is transformed to be

$$\begin{aligned} &V(r)\Phi_l(r) + \frac{2}{\pi(\hbar c)^3}\int dk k^2 \int dr' r'^2 \left(\sqrt{k^2+m_1^2}+\sqrt{k^2+m_2^2}\right) j_l\left(\frac{kr}{\hbar c}\right) j_l\left(\frac{kr'}{\hbar c}\right)\Phi_l(r') \\ &\quad = E^{(0)}\Phi_l(r)\,. \end{aligned} \tag{3.7}$$

Define a reduced radial wave function $u_l(r)$ by

$$\Phi_l(r) = \frac{u_l(r)}{r}\,, \tag{3.8}$$

then the wave equation becomes

$$\begin{aligned} &V(r)u_l(r) + \frac{2}{\pi(\hbar c)^3}\int dk k^2 \int dr' r r' \left(\sqrt{k^2+m_1^2}+\sqrt{k^2+m_2^2}\right) j_l\left(\frac{kr}{\hbar c}\right) j_l\left(\frac{kr'}{\hbar c}\right) u_l(r') \\ &\quad = E^{(0)}u_l(r)\,. \end{aligned} \tag{3.9}$$

As explained in ref. [10], for a bound state of quark and antiquark, when the distance between them is large enough, the wave function will drop seriously. Eventually the wave function will effectively vanish at a typically large distance. We assume such a typical distance is L, then the quark and antiquark in bound state can be viewed as if they are restricted in a limited space, $0 < r < L$. In the limited space the reduced wave function $u_l(r)$ for angular momentum l can be expanded in terms of the spherical Bessel function

$$u_l(r) = \sum_{n=1}^{\infty} c_n \frac{a_n r}{L} j_l\left(\frac{a_n r}{L}\right)\,, \tag{3.10}$$

where c_n's are the expansion coefficients, a_n the n-th root of the spherical Bessel function $j_l(a_n) = 0$. In practice the above summation can be truncated to a large enough integer N

$$u_l(r) = \sum_{n=1}^{N} c_n \frac{a_n r}{L} j_l\left(\frac{a_n r}{L}\right)\,. \tag{3.11}$$

In the limited space, the momentum k will be discrete. From the argument of $j_l\left(\frac{a_n r}{L}\right)$ in eq. (3.10), one can see the relevance

$$\frac{a_n r}{L} \Longleftrightarrow \frac{kr}{\hbar c}\,. \tag{3.12}$$

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Then the momentum is discretized, and the integration over k in eq. (3.9) should be replaced by a summation

$$\frac{k}{\hbar c} \to \frac{a_n}{L}\,, \qquad \int \frac{dk}{\hbar c} \to \sum_n \frac{\Delta a_n}{L}\,, \tag{3.13}$$

where $\Delta a_n = a_n - a_{n-1}$.

Considering the limited space $0 < r,\, r' < L$, the discrete momentum of eq. (3.13), and substituting eq. (3.11) into eq. (3.9), and simplify it, one can finally obtain the equation for the coefficients c_n's

$$\sum_{n=1}^{N} \frac{a_n}{N_m^2 a_m} \int_0^L dr V(r) r^2 j_l\left(\frac{a_m r}{L}\right) j_l\left(\frac{a_n r}{L}\right) c_n + \frac{2}{\pi L^3}\Delta a_m \cdot a_m^2 N_m^2 \left(\sqrt{\left(\frac{a_m \hbar c}{L}\right)^2 + m_1^2} + \sqrt{\left(\frac{a_m \hbar c}{L}\right)^2 + m_2^2}\right) c_m = E^{(0)} c_m\,, \tag{3.14}$$

where N_m is the module of the spherical Bessel function

$$N_m^2 = \int_0^L dr' r'^2 j_l\left(\frac{a_m r'}{L}\right)^2. \tag{3.15}$$

Eq. (3.14) is the eigenstate equation in the matrix form. It can be reduced to eq. (17) in ref. [10] for the case $l = 0$. It is not difficult to solve this equation numerically. The solution only slightly depends on the values of N and L if they are large enough. We find that when $N > 50$, $L > 5\,\mathrm{fm}$, the solution of the wave equation will be stationary.

Next we shall discuss the contribution of the spin-dependent interaction.

The spin-dependent interaction is considered perturbatively in the basis of the $|JM, sl\rangle$ sectors. $|JM, sl\rangle$ is the eigenvector of spin-independent Hamiltonian H_0, where J is the total angular momentum of the bound state, M the magnetic quantum number, s the total spin of the quark and antiquark, l the relevant orbital angular momentum between them. The tensor part of the hyperfine interaction H^{hyp} in eq. (2.7) does not conserve the orbital angular momentum, it causes mixing between the states with different orbital angular momenta ${}^3L_J \leftrightarrow {}^3L'_J$, while the spin-orbit interaction H^{so} in eq. (2.10) does not conserve the total quark and antiquark spin, it can cause mixing between the states with different total spin quantum numbers ${}^1L_J \leftrightarrow {}^3L_J$. The mass matrix elements are calculated perturbatively in the basis of $|JM, sl\rangle$. The matrix is then diagonalized to get the mixing eigenstates. The perturbative contribution of the spin-dependent Hamiltonian H' to the eigenvalues of the bound states are given below.

(1) The eigenvalue of pseudoscalar state

The quantum number of the pseudoscalar state is $J^P = 0^-$, the total spin and orbital angular momentum are $s = 0$, $l = 0$, i.e., it is ${}^1S_0$ state. The eigenvalue of the pseudoscalar state is calculated to be

$$m(0^-) = E^{(0)}_{l=0} - \frac{3}{4}\langle \psi^{(0)}_{l=0}(r)|f(r)|\psi^{(0)}_{l=0}(r)\rangle\,. \tag{3.16}$$

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(2) The mass matrix of the vector state, $J^P = 1^-$

Both $s = 1$, $l = 0$ and $s = 1$, $l = 2$ can construct $J^P = 1^-$ state. The $^3S_1$ and $^3D_1$ states can mix through the spin-orbit interaction. The basis for the mixing is denoted to be $|\psi_1\rangle = |^3S_1\rangle$, and $|\psi_2\rangle = |^3D_1\rangle$. The mass matrix can be written as

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}, \tag{3.17}$$

the results of the matrix elements are

$$H_{11} = E^{(0)}_{l=0} + \frac{1}{4}\langle\psi^{(0)}_{l=0}(r)|f(r)|\psi^{(0)}_{l=0}(r)\rangle\,, \tag{3.18}$$

$$H_{12} = \frac{1}{\sqrt{2}}\langle\psi^{(0)}_{l=0}(r)|g(r)|\psi^{(0)}_{l=2}(r)\rangle\,, \tag{3.19}$$

$$H_{21} = H^*_{12}\,, \tag{3.20}$$

$$H_{22} = E^{(0)}_{l=2} + \langle\psi^{(0)}_{l=2}(r)|\left[\frac{1}{4}f(r) - \frac{1}{2}g(r) - \frac{3}{2}h_1(r) - \frac{3}{2}h_2(r)\right]|\psi^{(0)}_{l=2}(r)\rangle\,. \tag{3.21}$$

Diagonalizing the matrix H, one can get the eigenvalues of the two mixing states and the mixing angle. With the matrix elements given in eqs. (3.18)–(3.21), the above mixing matrix (3.17) can be easily extended to the cases with more $|^3S_1\rangle$ and $|^3D_1\rangle$ states mixing.

(3) The eigenvalue of the scalar state, $J^P = 0^+$

For the scalar state, $J^P = 0^+$, the spin and orbital angular momentum are $s = 1$, $l = 1$. It is the $^3P_0$ state. The eigenvalue of the scalar state is

$$m(0^+) = E^{(0)}_{l=1} + \langle\psi^{(0)}_{l=1}(r)|\left[\frac{1}{4}f(r) - g(r) - h_1(r) - h_2(r)\right]|\psi^{(0)}_{l=1}(r)\rangle\,. \tag{3.22}$$

(4) The mass matrix of the axial-vector state, $J^P = 1^+$

The $J^P = 1^+$ state is mixture of $^1P_1$ and $^3P_1$ states, both states with $s = 0$, $l = 1$ and $s = 1$, $l = 1$ can construct the $J^P = 1^+$ state. The basis for the mixing is $|\psi_1\rangle = |^1P_1\rangle$, and $|\psi_2\rangle = |^3P_1\rangle$. The matrix elements of the mass matrix are

$$H_{11} = E^{(0)}_{l=1} - \frac{3}{4}\langle\psi^{(0)}_{l=1}(r)|f(r)|\psi^{(0)}_{l=1}(r)\rangle\,, \tag{3.23}$$

$$H_{12} = \frac{1}{\sqrt{2}}\langle\psi^{(0)}_{l=1}(r)|h_2(r) - h_1(r)|\psi^{(0)}_{l=1}(r)\rangle\,, \tag{3.24}$$

$$H_{21} = H^*_{12}\,, \tag{3.25}$$

$$H_{22} = E^{(0)}_{l=1} + \langle\psi^{(0)}_{l=1}(r)|\left[\frac{1}{4}f(r) + \frac{1}{2}g(r) - \frac{1}{2}h_1(r) - \frac{1}{2}h_2(r)\right]|\psi^{(0)}_{l=1}(r)\rangle\,. \tag{3.26}$$

With eqs. (3.23)–(3.26), the cases with more $|^1P_1\rangle$ and $|^3P_1\rangle$ mixing states can be obtained.

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(5) The mass matrix of the tensor state, $J^P = 2^+$

The $J^P = 2^+$ state is mixture of $^3P_2$ and $^3F_2$ states, both states with $s = 1$, $l = 1$ and $s = 1$, $l = 3$ can construct the $J^P = 2^+$ state. The basis for the mixing is $|\psi_1\rangle = |^3P_2\rangle$, and $|\psi_2\rangle = |^3F_2\rangle$. The matrix elements of the mass matrix are

$$H_{11} = E^{(0)}_{l=1} + \langle\psi^{(0)}_{l=1}(r)|\left[\frac{1}{4}f(r) - \frac{1}{10}g(r) + \frac{1}{2}h_1(r) + \frac{1}{2}h_2(r)\right]|\psi^{(0)}_{l=1}(r)\rangle\,, \tag{3.27}$$

$$H_{12} = \frac{3}{5}\sqrt{\frac{3}{2}}\langle\psi^{(0)}_{l=1}(r)|g(r)|\psi^{(0)}_{l=3}(r)\rangle\,, \tag{3.28}$$

$$H_{21} = H^*_{12}\,, \tag{3.29}$$

$$H_{22} = E^{(0)}_{l=3} + \langle\psi^{(0)}_{l=3}(r)|\left[\frac{1}{4}f(r) - \frac{2}{5}g(r) - 2h_1(r) - 2h_2(r)\right]|\psi^{(0)}_{l=3}(r)\rangle\,. \tag{3.30}$$

With eqs. (3.27)–(3.30), the cases with more $|^3P_2\rangle$ and $|^3F_2\rangle$ mixing states can be obtained.

In the above equations, the functions $f(r)$, $g(r)$, $h_1(r)$ and $h_2(r)$ are defined as

$$f(r) = \frac{32\pi}{9m_1\tilde{m}_{2a}}\alpha_s(r)\delta_\sigma(r)\,, \tag{3.31}$$

$$g(r) = \frac{4}{3}\frac{\alpha_s(r)}{m_1\tilde{m}_{2b}}\frac{1}{r^3}\,, \tag{3.32}$$

$$h_1(r) = \left[\frac{4}{3}\frac{\alpha_s(r)}{r^3}\left(\frac{1}{m_1} + \frac{1}{\tilde{m}_{2c}}\right) - \frac{1}{2r}\frac{\partial V(r)}{\partial r}\frac{1}{m_1}\right]\frac{1}{m_1}\,, \tag{3.33}$$

$$h_2(r) = \left[\frac{4}{3}\frac{\alpha_s(r)}{r^3}\left(\frac{1}{m_1} + \frac{1}{\tilde{m}_{2c}}\right)\frac{1}{\tilde{m}_{2c}} - \frac{1}{2r}\frac{\partial V(r)}{\partial r}\frac{1}{(\tilde{m}_{2d})^2}\right]. \tag{3.34}$$

4 Numerical result and discussion

The parameters used in this work include the quark masses, the potential parameters b, c, $\tilde{m}_{2i}$, σ_0 and s_0. They are selected to fit the masses of the quark-antiquark bound states. The values we obtain by fitting are

$$\begin{aligned} m_b &= 4.99\,\text{GeV}, & m_c &= 1.59\,\text{GeV}, \\ m_s &= 0.30\,\text{GeV}, & m_u &= m_d = 0.06\,\text{GeV}, \\ b &= 0.16\,\text{GeV}^2, & c &= -0.28\,\text{GeV}, \\ \sigma_0 &= 1.80\,\text{GeV}, & s_0 &= 1.55\,. \end{aligned} \tag{4.1}$$

The values of $\tilde{m}_{2a}$, $\tilde{m}_{2b}$, $\tilde{m}_{2c}$ and $\tilde{m}_{2d}$ depend on the quark-antiquark system, they can be written as

$$\tilde{m}_{2i} = \epsilon_i\tilde{m}_2\,, \quad i = a, b, c, d\,. \tag{4.2}$$

We find the values of ϵ_i's and $\tilde{m}_2$ are

$$(\epsilon_a, \epsilon_b, \epsilon_c, \epsilon_d) = (1.00, 1.30, 1.30, 1.32) \tag{4.3}$$

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for $(b\bar{q})$ and $(c\bar{q})$ systems,

$$(\epsilon_a, \epsilon_b, \epsilon_c, \epsilon_d) = (1.00, 1.10, 1.10, 1.31) \tag{4.4}$$

for $(b\bar{s})$ and $(c\bar{s})$ systems, and

$$\tilde{m}_2 = \begin{cases} 0.562\,\text{GeV} & \text{for } (b\bar{q}) \text{ system}\,, \\ 0.679\,\text{GeV} & \text{for } (b\bar{s}) \text{ system}\,, \\ 0.412\,\text{GeV} & \text{for } (c\bar{q}) \text{ system}\,, \\ 0.488\,\text{GeV} & \text{for } (c\bar{s}) \text{ system}\,, \end{cases} \tag{4.5}$$

here q is the light quark u or d.

The solution of the wave equation does not depend on the values of L and N if they are taken large enough. Numerical calculation shows that the solution is stable when $L > 5\,\text{fm}$, $N > 50$. Here we take $L = 10\,\text{fm}$, $N = 100$.

The numerical results for $(b\bar{q})$, $(b\bar{s})$, $(c\bar{q})$ and $(c\bar{s})$ bound states with the component of radial quantum number $n = 1$ dominant are given in table 1. Mixings between states with appropriate quantum numbers are considered in our calculation. We find that the theoretical calculation can accommodate the experimental data well. In addition to the masses, the mixing states relevant to each meson is also given in this table. The vector meson states are generally mixing states of $|^3S_1\rangle$ and $|^3D_1\rangle$. The components of $|1^3S_1\rangle$ in B^*, B_s^*, D^* and D_s^* are overwhelmingly dominant, while the components of $|^3D_1\rangle$ states are tiny. The masses of vector states with $|1^3D_1\rangle$ component dominant have also been predicted, which are shown in table 1.

The axial vector states with $J^P = 1^+$ found in experiment, such as $B_1(5721)$, $B_{s1}(5830)$, $D_1(2420)$, can be explained as mixing states of $|^1P_1\rangle$ and $|^3P_1\rangle$ states. For $(b\bar{q})$ and $(c\bar{q})$ systems, we predict two almost degenerate states, respectively. The mass difference of the two mixing states in each system is very tiny. For $B_1(5721)$, we predict two states with masses 5.72 GeV and 5.74 GeV. For $D_1(2420)$, the predicted masses of the two mixing states are 2.40 GeV and 2.41 GeV, which are very close. For the $J^P = 1^+$ state of $(c\bar{q})$ system, the component of state with higher radial quantum number $n = 2$ is not so small, the amplitude of $|2^3P_1\rangle$ can be as large as 0.116.

The 2^+ states can be explained as mixing states of $|^3P_2\rangle$ and $|^3F_2\rangle$. The details can be found in table 1. Also the mixing from state of higher radial quantum number with $n = 2$ can not be completely neglected, it can be as large as 10%.

Compared with the theoretical predictions given in ref. [4], mixings between more quantum states are considered in this work. For bound states of $(b\bar{q})$ and $(b\bar{s})$, the masses obtained in this work are approximately consistent with the theoretical masses given in ref. [4], but more predicted masses are presented in this work. For 0^+ state of $(c\bar{q})$ and 0^+, 1^+ states of $(c\bar{s})$, the masses predicted in this work are about 100 MeV smaller than the relevant predictions given in ref. [4], our results are more consistent with the experimental data now. The other states predicted in this work are also consistent with experiment well.

The radial excited states with the quantum number up to $n = 2$ are also predicted, they are given in table 2. In general, our results for $n = 2$ are slightly smaller than the

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	Meson	J^P	Multiplet	Mass (GeV)	GI (GeV)	Exp. (MeV)
$(b\bar{q})$	B	0^-	$\|1^1S_0\rangle$	5.27	5.31	5279.25 ± 0.17
	B^*	1^-	$0.99958\|1^3S_1\rangle+0.011\|1^3D_1\rangle+0.021\|2^3S_1\rangle+0.011\|2^3D_1\rangle$ $-0.010\|3^3S_1\rangle-0.009\|3^3D_1\rangle$	5.32	5.32	5325.2 ± 0.4
			$-0.012\|1^3S_1\rangle+0.9963\|1^3D_1\rangle+0.018\|2^3S_1\rangle+0.074\|2^3D_1\rangle$ $+0.010\|3^3S_1\rangle-0.037\|3^3D_1\rangle$	6.05		
		0^+	$\|1^3P_0\rangle$	5.68		
	$B_1(5721)$	1^+	$-0.519\|1^1P_1\rangle+0.844\|1^3P_1\rangle+0.078\|2^1P_1\rangle-0.111\|2^3P_1\rangle$	5.72		5723.5 ± 2.0
			$0.851\|1^1P_1\rangle+0.524\|1^3P_1\rangle+0.030\|2^1P_1\rangle+0.029\|2^3P_1\rangle$	5.74		
	$B_2^*(5747)$	2^+	$0.995\|1^3P_2\rangle-0.005\|1^3F_2\rangle+0.086\|2^3P_2\rangle-0.004\|2^3F_2\rangle$ $-0.047\|3^3P_2\rangle-0.003\|3^3F_2\rangle$	5.76	5.8	5743 ± 5
			$0.006\|1^3P_2\rangle+0.9995\|1^3F_2\rangle-0.008\|2^3P_2\rangle+0.020\|2^3F_2\rangle$ $-0.005\|3^3P_2\rangle+0.020\|3^3F_2\rangle$	6.33		
$(b\bar{s})$	B_s	0^-	$\|1^1S_0\rangle$	5.35	5.39	5366.77 ± 0.24
	B_s^*	1^-	$0.9995\|1^3S_1\rangle-0.013\|1^3D_1\rangle+0.021\|2^3S_1\rangle-0.012\|2^3D_1\rangle$ $-0.011\|3^3S_1\rangle+0.010\|3^3D_1\rangle$	5.40	5.45	$5415.4^{+2.4}_{-2.1}$
			$0.015\|1^3S_1\rangle+0.992\|1^3D_1\rangle-0.026\|2^3S_1\rangle+0.113\|2^3D_1\rangle$ $-0.009\|3^3S_1\rangle-0.050\|3^3D_1\rangle$	6.09		
		0^+	$\|1^3P_0\rangle$	5.72		
		1^+	$-0.540\|1^1P_1\rangle+0.822\|1^3P_1\rangle+0.103\|2^1P_1\rangle-0.151\|2^3P_1\rangle$	5.75		
	$B_{s1}(5830)$		$0.834\|1^1P_1\rangle+0.548\|1^3P_1\rangle+0.053\|2^1P_1\rangle+0.039\|2^3P_1\rangle$	5.82		5829.4 ± 0.7
	$B_{s2}^*(5840)$	2^+	$0.993\|1^3P_2\rangle+0.006\|1^3F_2\rangle+0.107\|2^3P_2\rangle+0.005\|2^3F_2\rangle$ $-0.054\|3^3P_2\rangle+0.004\|3^3F_2\rangle$	5.84	5.88	5839.7 ± 0.6
			$-0.007\|1^3P_2\rangle+0.998\|1^3F_2\rangle+0.013\|2^3P_2\rangle+0.052\|2^3F_2\rangle$ $+0.007\|3^3P_2\rangle+0.030\|3^3F_2\rangle$	6.36		
$(c\bar{q})$	D	0^-	$\|1^1S_0\rangle$	1.87	1.88	1869.62 ± 0.15
	D^*	1^-	$0.997\|1^3S_1\rangle+0.033\|1^3D_1\rangle-0.056\|2^3S_1\rangle-0.031\|2^3D_1\rangle$ $-0.029\|3^3S_1\rangle-0.025\|3^3D_1\rangle$	2.01	2.04	2010.28 ± 0.13
			$-0.042\|1^3S_1\rangle+0.984\|1^3D_1\rangle-0.079\|2^3S_1\rangle-0.137\|2^3D_1\rangle$ $+0.037\|3^3S_1\rangle-0.067\|3^3D_1\rangle$	2.75	2.82	
	$D_0^*(2400)^0$	0^+	$\|1^3P_0\rangle$	2.30	2.40	2318 ± 29
		1^+	$-0.096\|1^1P_1\rangle+0.986\|1^3P_1\rangle-0.075\|2^1P_1\rangle+0.116\|2^3P_1\rangle$	2.40	2.44	
	$D_1(2420)$		$0.992\|1^1P_1\rangle+0.106\|1^3P_1\rangle+0.022\|2^1P_1\rangle-0.063\|2^3P_1\rangle$	2.41	2.49	2421.3 ± 0.6
	$D_2^*(2460)$	2^+	$0.989\|1^3P_2\rangle-0.013\|1^3F_2\rangle-0.130\|2^3P_2\rangle+0.011\|2^3F_2\rangle$ $-0.068\|3^3P_2\rangle+0.009\|3^3F_2\rangle$	2.45	2.50	2464.4 ± 1.9
			$0.016\|1^3P_2\rangle+0.998\|1^3F_2\rangle+0.024\|2^3P_2\rangle-0.045\|2^3F_2\rangle$ $-0.019\|3^3P_2\rangle-0.037\|3^3F_2\rangle$	3.07		
$(c\bar{s})$	$D_s^{\pm}$	0^-	$\|1^1S_0\rangle$	1.96	1.98	1968.49 ± 0.32
	$D_s^{*\pm}$	1^-	$0.996\|1^3S_1\rangle-0.041\|1^3D_1\rangle-0.056\|2^3S_1\rangle+0.037\|2^3D_1\rangle$ $-0.030\|3^3S_1\rangle-0.029\|3^3D_1\rangle$	2.10	2.13	2112.3 ± 0.5
			$0.057\|1^3S_1\rangle+0.963\|1^3D_1\rangle+0.157\|2^3S_1\rangle-0.187\|2^3D_1\rangle$ $-0.042\|3^3S_1\rangle+0.087\|3^3D_1\rangle$	2.77	2.90	
	$D_{s0}^*(2317)^0$	0^+	$\|1^3P_0\rangle$	2.31	2.48	2317.8 ± 0.6
	$D_{s1}(2460)$	1^+	$-0.480\|1^1P_1\rangle+0.850\|1^3P_1\rangle-0.111\|2^1P_1\rangle+0.184\|2^3P_1\rangle$	2.42	2.53	2459.6 ± 0.6
	$D_{s1}(2536)$		$0.869\|1^1P_1\rangle+0.493\|1^3P_1\rangle-0.037\|2^1P_1\rangle-0.031\|2^3P_1\rangle$	2.51	2.57	2535.12 ± 0.13
	$D_{s2}^*(2573)$	2^+	$0.984\|1^3P_2\rangle-0.016\|1^3F_2\rangle-0.157\|2^3P_2\rangle-0.012\|2^3F_2\rangle$ $-0.077\|3^3P_2\rangle-0.010\|3^3F_2\rangle$	2.55	2.59	2571.9 ± 0.8
			$0.025\|1^3P_2\rangle+0.992\|1^3F_2\rangle+0.057\|2^3P_2\rangle+0.091\|2^3F_2\rangle$ $-0.032\|3^3P_2\rangle+0.054\|3^3F_2\rangle$	3.09		

Table 1. Theoretical spectrum of $(b\bar{q})$, $(b\bar{s})$, $(c\bar{q})$ and $(c\bar{s})$ bound states mainly with the radial quantum number $n = 1$, and the comparison with the prediction of ref. [4] and experimental data. The numbers in the column labeled "GI" are theoretical masses from ref. [4]. The experimental masses in the last column are PDG averages [22].

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	J^P	Multiplet	Mass (GeV)	GI (GeV)
$(b\bar{q})$	0^-	$\|2^1S_0\rangle$	5.81	5.90
	1^-	$-0.020\|1^3S_1\rangle - 0.017\|1^3D_1\rangle + 0.999\|2^3S_1\rangle - 0.015\|2^3D_1\rangle$ $+0.024\|3^3S_1\rangle + 0.011\|3^3D_1\rangle$	5.85	5.93
		$-0.011\|1^3S_1\rangle - 0.077\|1^3D_1\rangle + 0.014\|2^3S_1\rangle + 0.994\|2^3D_1\rangle$ $-0.029\|3^3S_1\rangle - 0.071\|3^3D_1\rangle$	6.38	
	0^+	$\|2^3P_0\rangle$	6.04	
	1^+	$-0.077\|1^1P_1\rangle + 0.112\|1^3P_1\rangle - 0.557\|2^1P_1\rangle + 0.820\|2^3P_1\rangle$	6.10	
		$-0.034\|1^1P_1\rangle - 0.023\|1^3P_1\rangle + 0.826\|2^1P_1\rangle + 0.561\|2^3P_1\rangle$	6.16	
	2^+	$-0.077\|1^3P_2\rangle + 0.009\|1^3F_2\rangle + 0.984\|2^3P_2\rangle + 0.007\|2^3F_2\rangle$ $+0.161\|3^3P_2\rangle + 0.006\|3^3F_2\rangle$	6.18	
		$0.006\|1^3P_2\rangle - 0.021\|1^3F_2\rangle - 0.011\|2^3P_2\rangle + 0.998\|2^3F_2\rangle$ $+0.028\|3^3P_2\rangle + 0.042\|3^3F_2\rangle$	6.61	
$(b\bar{s})$	0^-	$\|2^1S_0\rangle$	5.89	5.98
	1^-	$-0.020\|1^3S_1\rangle + 0.024\|1^3D_1\rangle + 0.999\|2^3S_1\rangle + 0.017\|2^3D_1\rangle$ $+0.025\|3^3S_1\rangle - 0.012\|3^3D_1\rangle$	5.94	6.01
		$0.013\|1^3S_1\rangle - 0.117\|1^3D_1\rangle - 0.016\|2^3S_1\rangle + 0.987\|2^3D_1\rangle$ $+0.044\|3^3S_1\rangle - 0.097\|3^3D_1\rangle$	6.41	
	0^+	$\|2^3P_0\rangle$	6.08	
	1^+	$-0.103\|1^1P_1\rangle + 0.151\|1^3P_1\rangle - 0.550\|2^1P_1\rangle + 0.815\|2^3P_1\rangle$	6.15	
		$-0.055\|1^1P_1\rangle - 0.037\|1^3P_1\rangle + 0.827\|2^1P_1\rangle + 0.558\|2^3P_1\rangle$	6.24	
	2^+	$-0.095\|1^3P_2\rangle - 0.014\|1^3F_2\rangle + 0.977\|2^3P_2\rangle - 0.008\|2^3F_2\rangle$ $+0.192\|3^3P_2\rangle - 0.007\|3^3F_2\rangle$	6.26	
		$-0.016\|1^3P_2\rangle - 0.053\|1^3F_2\rangle + 0.036\|2^3P_2\rangle + 0.984\|2^3F_2\rangle$ $-0.148\|3^3P_2\rangle + 0.079\|3^3F_2\rangle$	6.64	
$(c\bar{q})$	0^-	$\|2^1S_0\rangle$	2.46	2.58
	1^-	$0.049\|1^3S_1\rangle + 0.075\|1^3D_1\rangle + 0.991\|2^3S_1\rangle - 0.050\|2^3D_1\rangle$ $-0.072\|3^3S_1\rangle - 0.034\|3^3D_1\rangle$	2.59	2.64
		$0.036\|1^3S_1\rangle + 0.142\|1^3D_1\rangle + 0.052\|2^3S_1\rangle + 0.970\|2^3D_1\rangle$ $+0.153\|3^3S_1\rangle + 0.111\|3^3D_1\rangle$	3.11	
	0^+	$\|2^3P_0\rangle$	2.67	
	1^+	$0.080\|1^1P_1\rangle - 0.130\|1^3P_1\rangle - 0.474\|2^1P_1\rangle + 0.867\|2^3P_1\rangle$	2.83	
		$0.010\|1^1P_1\rangle + 0.011\|1^3P_1\rangle + 0.877\|2^1P_1\rangle + 0.480\|2^3P_1\rangle$	2.87	
	2^+	$0.108\|1^3P_2\rangle - 0.028\|1^3F_2\rangle + 0.962\|2^3P_2\rangle + 0.019\|2^3F_2\rangle$ $-0.248\|3^3P_2\rangle + 0.015\|3^3F_2\rangle$	2.93	
		$-0.047\|1^3P_2\rangle + 0.040\|1^3F_2\rangle - 0.102\|2^3P_2\rangle + 0.929\|2^3F_2\rangle$ $-0.344\|3^3P_2\rangle + 0.062\|3^3F_2\rangle$	3.39	
$(c\bar{s})$	0^-	$\|2^1S_0\rangle$	2.55	2.67
	1^-	$0.043\|1^3S_1\rangle - 0.147\|1^3D_1\rangle + 0.982\|2^3S_1\rangle + 0.074\|2^3D_1\rangle$ $-0.070\|3^3S_1\rangle - 0.046\|3^3D_1\rangle$	2.68	2.73
		$-0.045\|1^3S_1\rangle + 0.183\|1^3D_1\rangle - 0.071\|2^3S_1\rangle + 0.893\|2^3D_1\rangle$ $-0.383\|3^3S_1\rangle - 0.125\|3^3D_1\rangle$	3.14	
	0^+	$\|2^3P_0\rangle$	2.69	
	1^+	$0.112\|1^1P_1\rangle - 0.184\|1^3P_1\rangle - 0.504\|2^1P_1\rangle + 0.837\|2^3P_1\rangle$	2.87	
		$0.042\|1^1P_1\rangle + 0.025\|1^3P_1\rangle + 0.856\|2^1P_1\rangle + 0.515\|2^3P_1\rangle$	2.97	
	2^+	$0.127\|1^3P_2\rangle - 0.063\|1^3F_2\rangle + 0.946\|2^3P_2\rangle - 0.028\|2^3F_2\rangle$ $-0.289\|3^3P_2\rangle - 0.020\|3^3F_2\rangle$	3.02	
		$0.000\|1^3P_2\rangle - 0.100\|1^3F_2\rangle - 0.013\|2^3P_2\rangle + 0.978\|2^3F_2\rangle$ $-0.123\|3^3P_2\rangle + 0.136\|3^3F_2\rangle$	3.41	

Table 2. Theoretical spectrum of $(b\bar{q})$, $(b\bar{s})$, $(c\bar{q})$ and $(c\bar{s})$ bound states with the radial quantum number mainly $n = 2$, and the comparison with the prediction of ref. [4]. The results in the column labeled "GI" are theoretical masses from ref. [4].

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theoretical masses given in ref. [4]. In addiction, more predicted masses are given in this work, which can be tested in experiment in the future.

Both Belle and BaBar collaborations found a D_{sJ} resonance in the analysis of DK mass distribution, $D_{sJ}(2700)$ denoted by Belle collaboration [23, 24], and $X(2690)$ denoted by BaBar collaboration [25]. The mass and width are

$$\begin{aligned} M &= 2708 \pm 9^{+11}_{-10}\,\text{MeV}, \\ \Gamma &= 108 \pm 23^{+36}_{-31}\,\text{MeV} \end{aligned} \tag{4.6}$$

measured by Belle collaboration [23, 24], and

$$\begin{aligned} M &= 2688 \pm 4 \pm 3\,\text{MeV}, \\ \Gamma &= 112 \pm 7 \pm 36\,\text{MeV} \end{aligned} \tag{4.7}$$

measured by Babar collaboration [25].

It is possible that $D_{sJ}(2700)$ and $X(2690)$ are the same resonance. Comparing the masses $D_{sJ}(2700)$ and $X(2690)$ measured by Belle and BaBar collaborations with the predicted mass for $J^P = 1^-$ $(c\bar{s})$ state given in table 2, one can find that the state with predicted mass 2.68 GeV is consisted with the D_{sJ} meson found in experiment. Therefore $D_{sJ}(2700)$ and/or $X(2690)$ can be identified as the first radial excitation of $D_s^*(2112)$, which agrees with ref. [26] analyzed due to heavy quark limit.

It is interesting to discuss the properties of the heavy-light quark-antiquark bound states from the point view of heavy quark symmetry. The spectroscopy of mesons with open charm and beauty flavors was analyzed in the heavy quark limit recently in ref. [26–28], where the mesons are classified in heavy quark doublets and the quantum numbers are assigned to the heavy flavored mesons. Here we would like to discuss how the properties of the heavy flavored mesons implied by the heavy quark limit are reproduced in the calculation in the potential model. In the heavy quark limit, the spin of the heavy quark decouples from the light degrees of freedom. The heavy quark spin s_Q and the total angular momentum of the light antiquark s_l conserve separately in the bound state of the heavy-light system. Therefore heavy-flavored mesons can be classified according to the values of the angular momentum of the light antiquark s_l. The total spins of the mesons are $J = s_l \pm \frac{1}{2}$, according to which the mesons can be collected into doublets. For any value of the orbital angular momentum of the light antiquark l, the parity of the meson is $P = (-1)^{l+1}$, and the total angular momentum of the light antiquark $\vec{s}_l = \vec{s}_q + \vec{l}$, where s_q is the spin of the light antiquark. Since the properties of the hadronic states do not depend on the spin and flavor of the heavy quark due to the heavy quark symmetry, the mesons within the same doublet degenerate in the heavy quark limit. For $l = 0$, the total angular momentum of the light antiquark is $s_l = \frac{1}{2}$, then the total spin of the $Q\bar{q}$ meson could be 0 and 1. These two states form a doublet with $J^P_{s_l} = (0^-, 1^-)_{1/2}$. For $l = 1$, the possible angular momenta of light antiquark are $s_l = \frac{1}{2}$ and $s_l = \frac{3}{2}$. There are two doublets in this case, $J^P_{s_l} = (0^+, 1^+)_{1/2}$ and $J^P_{s_l} = (1^+, 2^+)_{3/2}$. For each meson state with specified quantum numbers classified in the heavy quark limit, we can denote them by $|J^P\rangle^l_{s_l}$. By analyzing angular momentum addition with the help of Clebsch-Gordan

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coefficients, we can decompose the states in the heavy quark limit into combination of states with definite quantum numbers l, S and J, i.e., the state $|^{2S+1}L_J\rangle$. For states in the doublet $J^P_{s_l} = (0^-, 1^-)_{1/2}$, we get

$$|0^-\rangle^{l=0}_{s_l=1/2} = |^1S_0\rangle\,, \tag{4.8}$$

$$|1^-\rangle^{l=0}_{s_l=1/2} = |^3S_1\rangle\,. \tag{4.9}$$

For states in the doublet $J^P_{s_l} = (0^+, 1^+)_{1/2}$, we can obtain

$$|0^+\rangle^{l=1}_{s_l=1/2} = |^3P_0\rangle\,, \tag{4.10}$$

$$|1^+\rangle^{l=1}_{s_l=1/2} = -\sqrt{\frac{1}{3}}|^1P_1\rangle + \sqrt{\frac{2}{3}}|^3P_1\rangle\,. \tag{4.11}$$

For states in the doublet $J^P_{s_l} = (1^+, 2^+)_{3/2}$, the results are

$$|1^+\rangle^{l=1}_{s_l=3/2} = \sqrt{\frac{2}{3}}|^1P_1\rangle + \sqrt{\frac{1}{3}}|^3P_1\rangle\,, \tag{4.12}$$

$$|2^+\rangle^{l=1}_{s_l=3/2} = |^3P_2\rangle\,. \tag{4.13}$$

For the 1^- state in the doublet $J^P_{s_l} = (1^-, 2^-)_{3/2}$, the result is

$$|1^-\rangle^{l=2}_{s_l=3/2} = |^3D_1\rangle\,. \tag{4.14}$$

The result of 2^+ state in the doublet $J^P_{s_l} = (2^+, 3^+)_{5/2}$ is

$$|2^+\rangle^{l=3}_{s_l=5/2} = |^3F_2\rangle\,. \tag{4.15}$$

The above eqs. (4.8)–(4.15) give the results that the meson states with definite J^P quantum number expanded as states of $|^{2S+1}L_J\rangle$ in the heavy quark limit. Comparing these results with the column "Multiplet" in table 1 and 2 for each meson, one can see that the results are consistent with the heavy quark limit, there are only small deviation from the heavy quark limit for most mesonic states. The small deviation is due to the masses of the heavy quarks b and c used here are realistic values, not infinity. Only the component for the $J^P = 1^+$ state of non-strange $c\bar{q}$ meson is greatly different from the case in the heavy quark limit. The mixing of $|1^1P_1\rangle$ and $|1^3P_1\rangle$ in 1^+ state of $c\bar{q}$ is very small, the mixing angle is only about -0.10 rad, while the mixing angle in the heavy quark limit should be $-\,\mathrm{ArcSin}\,\sqrt{1/3} = -0.615\,\mathrm{rad}$. We checked the reason and find that without considering the contribution of states with $n = 2$, the mixing angle is indeed very close to the heavy quark limit. In addition, the eigenvalues of the two mixing states is very near, they almost degenerate, and the gap between the energy levels with radial numbers $n = 1$ and $n = 2$ is not large, the mixing effect of $n = 2$ state will not be negligible. After adding the mixing effect of $n = 2$ state, the mixing angle between $|1^1P_1\rangle$ and $|1^3P_1\rangle$ is seriously affected, which makes it very small, i.e., the mixing angle is $\theta = -0.10\,\mathrm{rad}$. The result with small mixing angle is consistent with experiment. The Belle collaboration determined the possible mixing angle between the two 1^+ non-strange charmed meson states. They obtain

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a small mixing angle: $\theta = -0.10 \pm 0.03 \pm 0.02 \pm 0.02\,\text{rad}$ [29]. The theoretical prediction for the mixing angle in this work is in good agreement with experiment.

Finally the wave function of each bound state can be obtained simultaneously when solving the wave equation, which is not given here explicitly. But it is easy to get the wave function when it is needed.

5 Summary

The bound states of heavy-light quark and antiquark system are studied in the relativistic potential model. The dynamics of the light quark in the system requires the wave equation describing the bound state include relativistic kinematics. The potential is compatible with QCD, it shows the behavior of Coulomb potential at short distance, and a linear confining behavior at large distance. The spin-dependent interactions are also considered. The spectrums of B and D system are obtained. Compared with the results obtained in the relativistic potential model previously, the predictions to the spectrum are improved. The masses of the bound states with the radial quantum number $n = 1$ are well consistent with the experimental measurement. In addition, the masses of more meson states are predicted, which can be tested in experiment in the future. The wave function of each bound state can be also obtained by solving the wave equation.

Note added. After this work is finished, we find the experimental data newly presented by LHCb collaboration [30], where several new resonances are observed in the mass region between 2500 and 3000 MeV. Comparing with the experimental data, we find that the resonance $D_J(2580)$ can be assigned as dominantly $|2^3S_1\rangle$ state of $(c\bar{q})$ with $J^P = 1^-$, $D_J(3000)$ assigned as $|2^3P_2\rangle$ state with $J^P = 2^+$, which can be seen by comparing the theoretical prediction in table 2 with experimental data in [30].

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