Comments on observables for identity-based marginal solutions in Berkovits' superstring field theory

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ABSTRACT: We construct an analytic solution for tachyon condensation around identity-based marginal solutions in Berkovits' WZW-like open superstring field theory. Using this, which is a kind of wedge-based solution, the gauge invariant overlaps for the identity-based marginal solutions can be calculated analytically. This is a straightforward extension of a method in bosonic string field theory, which has been elaborated by the authors, to superstring. We also comment on a gauge equivalence relation between the tachyon vacuum solution and its marginally deformed one. From this viewpoint, we can find the vacuum energy of the identity-based marginal solutions to be zero, which agrees with the previous result as a consequence of ξ zero mode counting.

Contents

1 Introduction

Identity-based solutions corresponding to marginal deformations [1, 2] were constructed in Berkovits' Wess-Zumino-Witten (WZW) like open superstring field theory [3, 4]. They are characterized by a simple construction even for marginal currents with singular operator product expansions, namely for general current algebra. Since the contact term divergence does not appear, the solutions are easily handled compared with others [5] based on wedge states. Especially in the WZW-like theory, the vacuum energy of the identify-based marginal solutions can be found to be exactly zero, because the ξ zero mode is not contained in a correlation function in the calculation of the energy [1, 2]. As the vacuum energy was computed, a remaining problem is to evaluate gauge invariant overlaps of the identity-based marginal solutions in the superstring field theory.

In bosonic open string field theory, identity-based marginal solutions can be represented as a difference of wedge-based solutions plus an integration of a deformed BRST exact state [6]. This expression has made it possible to evaluate the gauge invariant overlaps for the identity-based solutions in the bosonic theory. In this calculation, the key ingredient is the "$K'Bc$ algebra" by which an analytic tachyon vacuum solution is constructed in the marginally deformed background [7]. This provides a useful technique in string field theory and, in fact, it is applied recently to construct a new simple analytic solution by Maccaferri [8]. Maccaferri's solution is gauge equivalent to the identity-based marginal solution but it is wedge-based, and so its physical observables are analytically calculable.

The main purpose of this paper is to present an analytic calculation of observables for the identity-based marginal solutions in the superstring field theory. To do this, we will

fully use the tachyon vacuum solution given by Erler [9] and a supersymmetric extension of the techniques used in the bosonic case.

In this paper, we will prove the following relation among the Erler solution Φ_T^{E}, the tachyon vacuum solution Φ_T in marginally deformed background, and the identity-based marginal solution Φ_J:

$$e^{-\Phi_J}\hat{Q}e^{\Phi_J} = e^{-\Phi_T^{\mathrm{E}}}\hat{Q}e^{\Phi_T^{\mathrm{E}}} - e^{-\Phi_T}\hat{Q}_{\Phi_J}e^{\Phi_T} + \int_0^1 \hat{Q}_{\tilde{\Phi}_T(t)}\Lambda_t\,dt, \tag{1.1}$$

where $\hat{Q}_\Phi$ is a BRST operator around a string field Φ and $\tilde{\Phi}_T(t)$ is a classical solution interpolating from Φ_T^{E} to Φ_T, and Λ_t is a state within the small Hilbert space. Thanks to this relation, we will be able to calculate the gauge invariant overlaps[1] for Φ_J. Here, we should note that the relation (1.1) implies a gauge equivalence relation between Φ_T^{E} and $\log\left(e^{\Phi_J}e^{\Phi_T}\right)$. Namely, there exist some group elements h and g such that $\hat{Q}h = 0$, $\hat{\eta}g = 0$ and $e^{\Phi_J}e^{\Phi_T} = h\,e^{\Phi_T^{\mathrm{E}}}g$. From this gauge equivalence, denoting the action with a BRST operator $\hat{Q}$ in the NS sector as $S[\Phi; \hat{Q}]$, we find that

$$S\left[\Phi_J; \hat{Q}\right] = S\left[\Phi_T^{\mathrm{E}}; \hat{Q}\right] - S\left[\Phi_T; \hat{Q}_{\Phi_J}\right]. \tag{1.2}$$

$-S\left[\Phi_T^{\mathrm{E}}; \hat{Q}\right]$ is the vacuum energy given by Erler and $S\left[\Phi_T; \hat{Q}_{\Phi_J}\right]$ will be calculated later. Accordingly, in addition to the gauge invariant overlaps, the vacuum energy of the identity-based marginal solutions can be evaluated analytically.

This paper is organized as follows. In section 2, we briefly illustrate the identity-based marginal solutions in the WZW-like theory and the theory expanded around the solutions, which corresponds to the theory in a marginally deformed background. Then we find a version of the extended KBc algebra in the expanded theory and construct a tachyon vacuum solution in a marginally deformed background as a variant of the Erler solution. For the solution, we find a homotopy operator and evaluate physical observables, the vacuum energy and the gauge invariant overlap, analytically. In section 3, we prove the gauge equivalence relation mentioned above and we calculate analytically the vacuum energy and the gauge invariant overlaps of the identity-based marginal solutions. In section 4, we give some concluding remarks. In appendix A, we comment on gauge equivalence relations.

2 Tachyon vacuum solution in the theory around the identity-based marginal solution

2.1 Identity-based marginal solutions

In [1], we have a type of identity-based marginal solutions in Berkovits' WZW-like superstring field theory:

$$\Phi_J = \tilde{V}_L^a(F_a)I, \qquad \tilde{V}_L^a(f) \equiv \int_{C_{\mathrm{L}}} \frac{dz}{2\pi i}f(z)\frac{1}{\sqrt{2}}c\gamma^{-1}\psi^a(z), \tag{2.1}$$

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[1]See [10], for example, in the framework of Berkovits' WZW-like superstring field theory.

where $F_a(z)$ is some function such as $F_a(-1/z) = z^2 F_a(z)$, C_L denotes a half unit circle: $|z| = 1$, $\mathrm{Re}\, z \geq 0$ and I is the identity state. We use a notation: $\gamma^{-1}(z) = e^{-\phi}\xi(z)$ as in [9], where β, γ-ghosts are expressed by ξ, η, ϕ, such as $\beta = e^{-\phi}\partial\xi$, $\gamma = \eta\, e^{\phi}$. As for ψ^a in Φ_J, we suppose that $\boldsymbol{J}^a(z, \theta) = \psi^a(z) + \theta J^a(z)$ gives a supercurrent associated with a Lie algebra, where a is its index, in the matter sector.[2]

The solution Φ_J, which is in the large Hilbert space, can be related to Ψ_J which is regarded as a marginal solution in the modified cubic superstring field theory [12, 13]:

$$\Psi_J \equiv -V_L^a(F_a)I + \frac{1}{8}\Omega^{ab}C_L(F_aF_b)I = e^{-\Phi_J}Q_{\mathrm{B}}e^{\Phi_J}, \tag{2.2}$$

$$V_L^a(f) \equiv \int_{C_{\mathrm{L}}} \frac{dz}{2\pi i}\frac{1}{\sqrt{2}}f(z)(cJ^a(z) + \gamma\psi^a(z)), \quad C_L(f) \equiv \int_{C_{\mathrm{L}}} \frac{dz}{2\pi i}f(z)c(z). \tag{2.3}$$

From the expression in (2.2), it can be easily found that Φ_J satisfies the equation of motion in the NS sector,

$$\eta_0\left(e^{-\Phi_J}Q_{\mathrm{B}}e^{\Phi_J}\right) = 0, \tag{2.4}$$

because Ψ_J is in the small Hilbert space.

By expanding the NS action $S[\Phi; Q_{\mathrm{B}}]$ of Berkovits' WZW-like superstring field theory around Φ_J as

$$e^{\Phi} = e^{\Phi_J}e^{\Phi'}, \tag{2.5}$$

we have $S[\Phi; Q_{\mathrm{B}}] = S[\Phi_J; Q_{\mathrm{B}}] + S[\Phi'; Q_{\Phi_J}]$, where $S[\Phi'; Q_{\Phi_J}]$ has the same form as the original action except the BRST operator Q_{Φ_J}:

$$Q_{\Phi_J} = Q_{\mathrm{B}} - V^a(F_a) + \frac{1}{8}\Omega^{ab}C(F_aF_b). \tag{2.6}$$

Here, $V^a(F_a)$ and $C(F_aF_b)$ are given by integrations along the whole unit circle, $|z| = 1$:

$$V^a(f) \equiv \oint \frac{dz}{2\pi i}\frac{1}{\sqrt{2}}f(z)(cJ^a(z) + \gamma\psi^a(z)), \qquad C(f) \equiv \oint \frac{dz}{2\pi i}f(z)c(z). \tag{2.7}$$

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[2]Its component fields satisfy following operator product expansions (OPE) [11]:

$$\psi^a(y)\psi^b(z) \sim \frac{1}{y-z}\frac{1}{2}\Omega^{ab}, \qquad J^a(y)\psi^b(z) \sim \frac{1}{y-z}f^{ab}{}_c\psi^c(z),$$

$$J^a(y)J^b(z) \sim \frac{1}{(y-z)^2}\frac{1}{2}\Omega^{ab} + \frac{1}{y-z}f^{ab}{}_cJ^c(z),$$

where constants $\Omega^{ab}, f^{ab}{}_c$ satisfy following relations

$$\Omega^{ab} = \Omega^{ba}, \qquad\qquad f^{ab}{}_c\Omega^{cd} + f^{ad}{}_c\Omega^{cb} = 0\,,$$

$$f^{ab}{}_c = -f^{ba}{}_c, \qquad f^{ab}{}_df^{cd}{}_e + f^{bc}{}_df^{ad}{}_e + f^{ca}{}_df^{bd}{}_e = 0\,.$$

2.2 Deformed algebra

Let us consider a version of the extended KBc algebra in [9], in which the BRST operator is replaced with a deformed BRST operator. First, we give the Virasoro generator, L'_n corresponding to $Q' \equiv Q_{\Phi_J}$ [1]:

$$L'_n = \{Q', b_n\} = L_n - \frac{1}{\sqrt{2}} \sum_{k \in \mathbb{Z}} F_{a,k} J^a_{n-k} + \frac{1}{8} \Omega^{ab} \sum_{k \in \mathbb{Z}} F_{a,n-k} F_{b,k}, \tag{2.8}$$

where we define the coefficients as $F_{a,n} \equiv \oint \frac{d\sigma}{2\pi} e^{i(n+1)\sigma} F_a(e^{i\sigma})$ and then the condition $F_a(-1/z) = z^2 F_a(z)$ imposes $F_{a,n} = -(-1)^n F_{a,-n}$. We note that only the matter sector is deformed and the central charge is not changed. Using this L'_n instead of the conventional Virasoro generators, we define a string field K' as in [16]. Then, among string fields such as K', B, c, γ, we have the following relations:[3]

$$B^2 = 0, \quad c^2 = 0, \quad Bc + cB = 1, BK' = K'B, K'c - cK' = Kc - cK \equiv \partial c, \tag{2.9}$$

$$\gamma B + B\gamma = 0, \quad c\gamma + \gamma c = 0, \quad K'\gamma - \gamma K' = K\gamma - \gamma K \equiv \partial \gamma, \tag{2.10}$$

$$\hat{Q}'B = K', \quad \hat{Q}'K' = 0, \quad \hat{Q}'c = cK'c - \gamma^2 = cKc - \gamma^2 = c\partial c - \gamma^2, \tag{2.11}$$

$$\hat{Q}'\gamma = \hat{Q}\gamma = c\partial \gamma - \frac{1}{2}(\partial c)\gamma, \tag{2.12}$$

where $\hat{Q}' \equiv Q'\sigma_3, \hat{Q} \equiv Q_B\sigma_3$ and σ_i $(i = 1, 2, 3)$ are Pauli matrices attached as the Chan-Paton factor in order to take into account the GSO($-$) sector.

Furthermore, for the string fields γ^{-1}, ζ and V, we see more relations:[4]

$$\gamma^{-1}\gamma = \gamma\gamma^{-1} = 1, \quad \gamma^{-1}B + B\gamma^{-1} = 0, \quad \gamma^{-1}c + c\gamma^{-1} = 0,$$
$$K'\gamma^{-1} - \gamma^{-1}K' = K\gamma^{-1} - \gamma^{-1}K \equiv \partial \gamma^{-1},$$
$$\hat{Q}'\gamma^{-1} = \hat{Q}\gamma^{-1} = c\partial \gamma^{-1} + \frac{1}{2}(\partial c)\gamma^{-1}, \qquad \hat{Q}'\zeta = \hat{Q}\zeta = cV + \gamma. \tag{2.13}$$

It is convenient to note further relations among string fields:

$$B\hat{Q}'\zeta + (\hat{Q}'\zeta)B = V, \quad B\hat{Q}'c - (\hat{Q}'c)B = \partial c, \qquad \zeta^2 = 0, c\zeta = \zeta c = 0,$$
$$\gamma\zeta = -\zeta\gamma = c, \quad (\hat{Q}'\zeta)\zeta = -\zeta\hat{Q}'\zeta = c, \qquad (\hat{Q}'c)\zeta = \zeta\hat{Q}'c = -\gamma c,$$
$$(\hat{Q}'\zeta)c = -c\hat{Q}'\zeta = \gamma c, \qquad (\hat{Q}'\zeta)^2 = -\hat{Q}'c, \tag{2.14}$$

for concrete computations.

It is noted that K', B, c, γ, γ^{-1}, ζ, V and $\hat{Q}'$ have the same algebraic structure as that of the extended KBc algebra [9] with $\hat{Q}$.

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[3]These string fields are given by

$$B = \frac{\pi}{2} B^L_1 I \sigma_3, \quad c = \frac{2}{\pi} \hat{U}_1 \tilde{c}(0)|0\rangle\sigma_3, \quad \gamma = \sqrt{\frac{2}{\pi}} \hat{U}_1 \tilde{\gamma}(0)|0\rangle\sigma_2 \quad K' = \frac{\pi}{2} K'^L_1 I, \quad K = \frac{\pi}{2} K^L_1 I,$$

where $K'^L_1 \equiv \{Q', B^L_1\}$. We use the conventions in [14] for the definitions of $K^L_1, B^L_1, \hat{U}_r \equiv U^\dagger_r U_r, U_r = (2/r)^{\mathcal{L}_0}, \cdots$ and $\tilde{c}(\tilde{z}), \tilde{\gamma}(\tilde{z}), \cdots$ denote worldsheet fields in the sliver frame.

[4]As in [9], they are defined by

$$\gamma^{-1} = \sqrt{\frac{\pi}{2}} \hat{U}_1 \tilde{\gamma}^{-1}(0)|0\rangle\sigma_2, \quad \zeta = \gamma^{-1}c = \sqrt{\frac{2}{\pi}} \hat{U}_1 \tilde{\gamma}^{-1}\tilde{c}(0)|0\rangle i\sigma_1, \quad V = \frac{1}{2}\gamma^{-1}\partial c = \sqrt{\frac{\pi}{2}} \hat{U}_1 \frac{1}{2}\tilde{\gamma}^{-1}\tilde{\partial}\tilde{c}(0)|0\rangle i\sigma_1.$$

2.3 Tachyon vacuum solution

From the result in [9] and the deformed algebra in the previous subsection, we can immediately construct a solution Φ_T in the theory with $\hat{Q}'$:

$$e^{\Phi_T} = (1 + q\zeta) \left(1 + (q^2 - 1)c \frac{B}{1 + K'} + q(\hat{Q}'\zeta) \frac{B}{1 + K'} \right)$$

$$= 1 - c\frac{B}{1 + K'} + q \left(\zeta + (\hat{Q}'\zeta) \frac{B}{1 + K'} \right), \tag{2.15}$$

where q is a nonzero constant. This solution has the form in which K of the Erler solution is replaced by K'. In the same way as [9], we find that Φ_T satisfies

$$e^{-\Phi_T} \hat{Q}' e^{\Phi_T} = c - (\hat{Q}'c) \frac{B}{1 + K'} = \left(c + \hat{Q}'(Bc) \right) \frac{1}{1 + K'}, \tag{2.16}$$

which is in the small Hilbert space, and therefore the equation of motion in the NS sector: $\hat{\eta} \left(e^{-\Phi_T} \hat{Q}' e^{\Phi_T} \right) = 0$, where $\hat{\eta} \equiv \eta_0 \sigma_3$, holds.

If we expand the action $S \left[\Phi'; \hat{Q}' \right]$ around the solution Φ_T as $e^{\Phi'} = e^{\Phi_T} e^{\Phi''}$, we have a new BRST operator $\hat{Q}'_{\Phi_T}$ as

$$\hat{Q}'_{\Phi_T} \Xi = \hat{Q}' \Xi + \left(e^{-\Phi_T} \hat{Q}' e^{\Phi_T} \right) \Xi - (-1)^{|\Xi|} \Xi \left(e^{-\Phi_T} \hat{Q}' e^{\Phi_T} \right), \tag{2.17}$$

where Ξ is an arbitrary string field. $|\Xi|$ denotes effective Grassmann parity of Ξ, which is defined by a sum of Grassmann parity and worldsheet spinor number of the vertex operator corresponding to Ξ [9, 15]. We note that the string field $e^{-\Phi_T} \hat{Q}' e^{\Phi_T}$ (2.16) is essentially the same as the tachyon vacuum solution on the marginally deformed background in the modified cubic superstring field theory [16]. Hence, we can find a homotopy operator $\hat{A}'$ for $\hat{Q}'_{\Phi_T}$:

$$\hat{A}' \Xi = \frac{1}{2} \left(A' \Xi + (-1)^{|\Xi|} \Xi A' \right), \tag{2.18}$$

for any string field Ξ, where $A' \equiv \frac{B}{1 + K'}$ is a homotopy state in the small Hilbert space. Actually, we find $\hat{Q}'_{\Phi_T} A' = 1$ and $(A')^2 = 0$, which imply the following relations:

$$\left\{ \hat{Q}'_{\Phi_T}, \hat{A}' \right\} = 1, \qquad \left(\hat{A}' \right)^2 = 0. \tag{2.19}$$

Accordingly, the existence of the homotopy operator $\hat{A}'$ indicates that there is no physical open string state around the solution Φ_T and therefore it is the tachyon vacuum solution in the marginally deformed background.

2.4 Energy and gauge invariant overlaps

The NS action $S \left[\Phi'; \hat{Q}_{\Phi_J} \right]$ around Φ_J is given by

$$S \left[\Phi'; \hat{Q}_{\Phi_J} \right] = -\int_0^1 dt \, \text{Tr} \left[\left(\hat{\eta} \left(g(t)^{-1} \partial_t g(t) \right) \right) \left(g(t)^{-1} \hat{Q}_{\Phi_J} g(t) \right) \right]. \tag{2.20}$$

Here we have used a formula in [17] instead of the conventional WZW-like form and $g(t)$ $(t \in [0,1])$ in the action is an interpolating string field such that $g(0) = 1$ and $g(1) = e^{\Phi'}$. The trace in the action implies $\mathrm{Tr}\, A \equiv \frac{1}{2}\mathrm{tr}\langle I|A\rangle$, where tr is the trace for 2×2 matrices corresponding to the Chan-Paton factor.

In order to evaluate the action for $\Phi' = \Phi_T$, which is given in (2.15), we take an interpolating string field:

$$g_T(t) = 1 + t\left(e^{\Phi_T} - 1\right)$$
$$= 1 - tc\frac{B}{1+K'} + qt\left(\zeta + (\hat{Q}'\zeta)\frac{B}{1+K'}\right), \tag{2.21}$$

after [9]. The integrand in the action for the solution: $S\left[\Phi_T; \hat{Q}_{\Phi_J}\right]$, can be manipulated in the same manner as the Erler solution. (Hence, we describe only an outline of our computations in the following.) Using graded cyclicity of the trace and $\mathrm{Tr}[\hat{\eta}A] = 0$, $\mathrm{Tr}\left[\hat{Q}'A\right] = 0$ for any string field A, we find

$$\mathrm{Tr}\left[\left(\hat{\eta}\left(g_T(t)^{-1}\partial_t g_T(t)\right)\right)\left(g_T(t)^{-1}\hat{Q}_{\Phi_J}g_T(t)\right)\right]$$
$$= q(1-t)\mathrm{Tr}\left[\left(\hat{\eta}\hat{Q}'\zeta\right)\left(\frac{2q^2t^2 - t}{\mathcal{D}'}Bc\frac{1}{\mathcal{D}'} + \frac{qt}{\mathcal{D}'}B(\hat{Q}'\zeta)\frac{1}{\mathcal{D}'}\right)\right], \tag{2.22}$$

where $\mathcal{D}' \equiv 1 - t + q^2t^2 + K' + qtV$. Because the trace becomes zero except for terms with bc-ghost number 3, we expand $\frac{1}{\mathcal{D}'}$ as $\frac{1}{\mathcal{D}'} = \frac{1}{\mathcal{D}''}\sum_{n=0}^{\infty}(-1)^n\left(qtV\frac{1}{\mathcal{D}''}\right)^n$ ($\mathcal{D}'' \equiv 1 - t + q^2t^2 + K'$) and the above trace can be computed as

$$\mathrm{Tr}\left[\left(\hat{\eta}\left(g_T(t)^{-1}\partial_t g_T(t)\right)\right)\left(g_T(t)^{-1}\hat{Q}_{\Phi_J}g_T(t)\right)\right] = q^2t^2(1-t)(2q^2t - 1)\mathcal{X}_1 + q^2t(1-t)\mathcal{X}_2, \tag{2.23}$$

$$\mathcal{X}_1 = \mathrm{Tr}\left[\left(\hat{\eta}\hat{Q}'\zeta\right)\frac{1}{\mathcal{D}''}cV\frac{B}{(\mathcal{D}'')^2}\right], \qquad \mathcal{X}_2 = \mathrm{Tr}\left[\left(\hat{\eta}\hat{Q}'\zeta\right)B\frac{1}{\mathcal{D}''}cV\frac{1}{\mathcal{D}''}\right]. \tag{2.24}$$

In order to evaluate $\mathcal{X}_1$ and $\mathcal{X}_2$, we note the following equations, related to a derivation $\mathcal{L}_0' - \mathcal{L}_0'^{\dagger} = \left\{Q', \mathcal{B}_0 - \mathcal{B}_0^{\dagger}\right\}$ with respect to the star product among string fields, as in [9, 16]. Namely,

$$\mathrm{Tr}\left[\left(\mathcal{L}_0' - \mathcal{L}_0'^{\dagger}\right)A\right] = 0, \tag{2.25}$$

for any string field A and

$$\frac{1}{2}\left(\mathcal{L}_0' - \mathcal{L}_0'^{\dagger}\right)c = -c, \quad \frac{1}{2}\left(\mathcal{L}_0' - \mathcal{L}_0'^{\dagger}\right)B = B, \quad \frac{1}{2}\left(\mathcal{L}_0' - \mathcal{L}_0'^{\dagger}\right)\gamma = -\frac{1}{2}\gamma, \quad \frac{1}{2}\left(\mathcal{L}_0' - \mathcal{L}_0'^{\dagger}\right)K' = K',$$
$$\frac{1}{2}\left(\mathcal{L}_0' - \mathcal{L}_0'^{\dagger}\right)\gamma^{-1} = \frac{1}{2}\gamma^{-1}, \quad \frac{1}{2}\left(\mathcal{L}_0' - \mathcal{L}_0'^{\dagger}\right)V = \frac{1}{2}V, \quad \frac{1}{2}\left(\mathcal{L}_0' - \mathcal{L}_0'^{\dagger}\right)\zeta = -\frac{1}{2}\zeta. \tag{2.26}$$

In particular, using

$$e^{\frac{1}{2}\left(\mathcal{L}_0' - \mathcal{L}_0'^{\dagger}\right)\log\left(1 - t + q^2t^2\right)}\frac{1}{\mathcal{D}''} = \left(1 - t + q^2t^2\right)^{-1}\frac{1}{1+K'}, \tag{2.27}$$

the traces are simplified as

$$\mathcal{X}_1 = -\frac{1}{(1 - t + q^2t^2)^3}\mathrm{Tr}\left[B(\hat{\eta}(cV))\frac{1}{1+K'}cV\frac{1}{(1+K')^2}\right], \tag{2.28}$$

$$\mathcal{X}_2 = \frac{1}{(1 - t + q^2t^2)^2}\mathrm{Tr}\left[B(\hat{\eta}(cV))\frac{1}{1+K'}cV\frac{1}{1+K'}\right]. \tag{2.29}$$

In order to calculate these traces, we use an expression with the Schwinger parameter for the inverse of $1 + K'$:

$$\frac{1}{1 + K'} = \int_0^\infty d\alpha \, e^{-\alpha} e^{-\alpha K'}. \tag{2.30}$$

Performing some change of variables in the integrations and inserting $e^{\frac{1}{2}\left(\mathcal{L}_0' - \mathcal{L}_0'^\dagger\right)(-\log \ell)}$ in the trace appropriately, we reach the following expressions:

$$\mathcal{X}_1 = \frac{-2}{(1 - t + q^2 t^2)^3} \int_0^1 d\theta \, (1 - \theta) \text{Tr}\Big[B(\hat{\eta}(cV)) e^{-\theta K'} cV e^{-(1-\theta)K'}\Big], \tag{2.31}$$

$$\mathcal{X}_2 = \frac{1}{(1 - t + q^2 t^2)^2} \int_0^1 d\theta \, \text{Tr}\Big[B(\hat{\eta}(cV)) e^{-\theta K'} cV e^{-(1-\theta)K'}\Big]. \tag{2.32}$$

For both of them, we need to calculate the trace $\text{Tr}\Big[B(\hat{\eta}(cV)) e^{-\theta K'} cV e^{-(1-\theta)K'}\Big]$. Unlike the case of [9], $e^{-\alpha K'}$ appears in the above trace instead of $e^{-\alpha K}$. Here, we use the result for the modified cubic superstring field theory in the marginally deformed background [16]:

$$e^{-\alpha K'} = e^{-\alpha \frac{\pi}{2} \mathcal{C}} \hat{U}_{\alpha+1} \mathbf{T} \exp\left(\frac{\pi}{4} \int_{-\alpha}^\alpha du \int_{-\infty}^\infty dv f_a(v) \tilde{J}^a \left(iv + \frac{\pi}{4} u\right)\right) |0\rangle, \tag{2.33}$$

$$f_a(v) \equiv \frac{F_a\left(\tan\left(iv + \frac{\pi}{4}\right)\right)}{2\pi\sqrt{2}\cos^2\left(it + \frac{\pi}{4}\right)}, \qquad \mathcal{C} \equiv \frac{\pi}{2} \int_{-\infty}^\infty dv \, \Omega^{ab} f_a(v) f_b(v), \tag{2.34}$$

where $\mathbf{T}$ is an ordering symbol with respect to the real part of the argument of $\tilde{J}^a$. The star products in the trace can be calculated as

$$B(\hat{\eta}(cV)) e^{-\theta K'} cV e^{-(1-\theta)K'} = \frac{e^{-\frac{\pi}{2}\mathcal{C}}}{4} B_1^L \left(\eta_0 \tilde{\gamma}^{-1}\right) \tilde{c}\tilde{\partial}\tilde{c}\left(\frac{\pi}{4}\right) \tilde{\gamma}^{-1} \tilde{c}\tilde{\partial}\tilde{c}\left(\frac{\pi}{4}(1 - 2\theta)\right)$$

$$\times \mathbf{T} \exp\left(\frac{\pi}{4} \int_{-1}^1 du \int_{-\infty}^\infty dv f_a(v) \tilde{J}^a \left(iv + \frac{\pi}{4} u\right)\right) |0\rangle. \tag{2.35}$$

Finally, the trace appeared in (2.31) and (2.32) can be evaluated as

$$\text{Tr}\Big[B(\hat{\eta}(cV)) e^{-\theta K'} cV e^{-(1-\theta)K'}\Big]$$

$$= -\frac{1}{4} \left\langle \left(\eta_0 \gamma^{-1}\left(\frac{\pi}{2}\right)\right) \gamma^{-1}\left(\frac{\pi}{2}(1 - 2\theta)\right)\right\rangle_{\xi\eta\phi} \left\langle B_1^L c\partial c\left(\frac{\pi}{2}\right) c\partial c\left(\frac{\pi}{2}(1 - 2\theta)\right)\right\rangle_{bc}$$

$$\times e^{-\frac{\pi}{2}\mathcal{C}} \left\langle \exp\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du \int_{-\infty}^\infty dv f_a(v) J^a(2iv + u)\right)\right\rangle_{\text{mat}}. \tag{2.36}$$

Here, the correlation functions are defined on a cylinder of circumference π. The last factor on the right hand side, which is a correlator in the matter sector, is 1 as proved in [16], and so the trace becomes the same result as the case of the Erler solution. Consequently, the vacuum energy of (2.15) is unchanged from the case in the original background without marginal deformations, namely, $E = -S\left[\Phi_T; \hat{Q}_{\Phi_J}\right] = -1/\left(2\pi^2\right)$.

Next, we evaluate the gauge invariant overlap (GIO) in Berkovits' WZW-like superstring field theory. We define the GIO $\langle \Phi \rangle_\mathcal{V}$ as

$$\langle \Phi \rangle_\mathcal{V} \equiv \text{Tr}[\mathcal{V}(i)\Phi] \tag{2.37}$$

after [10]. Here, $\mathcal{V}(i)$ denotes a midpoint insertion of a primary closed string vertex operator with the picture -1, the ghost number 2 and the conformal dimension $(0,0)$ and it should be BRST invariant and in the small Hilbert space, i.e., $[Q_\mathrm{B}, \mathcal{V}(i)] = 0$ and $[\eta_0, \mathcal{V}(i)] = 0$. It satisfies

$$\langle \hat{Q}\Lambda \rangle_\mathcal{V} = 0, \qquad \langle \hat{\eta}\Lambda \rangle_\mathcal{V} = 0, \qquad \langle \Lambda\Xi \rangle_\mathcal{V} = (-)^{|\Lambda||\Xi|}\langle \Xi\Lambda \rangle_\mathcal{V} \tag{2.38}$$

for any string fields Λ, Ξ. We note that an infinitesimal gauge transformation

$$\delta_\Lambda e^\Phi = (\hat{Q}\Lambda_0)e^\Phi + e^\Phi \hat{\eta}\Lambda_1, \tag{2.39}$$

(Λ_0 and Λ_1 are gauge parameter string field with the picture number 0 and 1, respectively.) which can be rewritten as[5]

$$\delta_\Lambda \Phi = \frac{\mathrm{ad}_\Phi}{e^{\mathrm{ad}_\Phi} - 1}\hat{Q}\Lambda_0 + \frac{-\mathrm{ad}_\Phi}{e^{-\mathrm{ad}_\Phi} - 1}\hat{\eta}\Lambda_1, \tag{2.41}$$

where $\mathrm{ad}_B(A) \equiv [B, A] = BA - AB$. Thanks to the properties (2.38), the GIO is invariant under the above transformation: $\langle \delta_\Lambda \Phi \rangle_\mathcal{V} = 0$. We note that the GIO for Φ' in the theory around a solution Φ_0 is also invariant under the gauge transformation because $\hat{Q}\Lambda_0$ is replaced with $\hat{Q}_{\Phi_0}\Lambda_0 \equiv \hat{Q}\Lambda_0 + \left[e^{-\Phi_0}\hat{Q}e^{\Phi_0}, \Lambda_0\right]$ in the above.

Let us evaluate the GIO for Φ_T. It is convenient to use $e^{-\Phi_T}\hat{Q}'e^{\Phi_T}$ (2.16) instead of Φ_T itself. Inserting $1 = \{Q_\mathrm{B}, \xi Y(i)\}$, where $Y(z) = c\partial\xi e^{-2\phi}(z)$ is the inverse picture changing operator, the GIO (2.37) can be rewritten as[6]

$$\langle \Phi \rangle_\mathcal{V} = \mathrm{Tr}\left[\mathcal{V}(i)\{Q_\mathrm{B}, \xi Y(i)\}\Phi\right] = \mathrm{Tr}\left[\xi Y\mathcal{V}(i)\sigma_3\hat{Q}\Phi\right] = \mathrm{Tr}\left[\xi Y\mathcal{V}(i)\sigma_3\hat{Q}'\Phi\right]$$
$$= \mathrm{Tr}\left[\xi Y\mathcal{V}(i)\sigma_3\, e^{-\Phi}\hat{Q}'e^\Phi\right]. \tag{2.43}$$

Substituting (2.16), we have

$$\langle \Phi_T \rangle_\mathcal{V} = \mathrm{Tr}\left[\xi Y\mathcal{V}(i)\sigma_3\, c\frac{1}{1 + K'}\right]. \tag{2.44}$$

In a similar way to the calculation of the vacuum energy, by using the Schwinger representation and the result of the correlation function in the modified cubic superstring field theory [16], we finally obtain the expression of the GIO in the marginally deformed

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[5]We have used

$$e^{-\Phi}\delta e^\Phi = \int_0^1 d\theta\, e^{-\theta\Phi}(\delta\Phi)e^{\theta\Phi} = \int_0^1 d\theta\, e^{-\theta\mathrm{ad}_\Phi}\delta\Phi = \frac{e^{-\mathrm{ad}_\Phi} - 1}{-\mathrm{ad}_\Phi}\delta\Phi. \tag{2.40}$$

[6]Similarly, it can be expressed as [9]:

$$\langle \Phi \rangle_\mathcal{V} = \mathrm{Tr}\left[\xi Y\mathcal{V}(i)\sigma_3\, e^{-\Phi}\hat{Q}e^\Phi\right] = \mathrm{Tr}\left[\xi\mathcal{V}(i)\sigma_3\, e^{-\Phi}\hat{\eta}e^\Phi\right] = \int_0^1 dt\, \mathrm{Tr}\left[\mathcal{V}(i)\, e^{-\Phi(t)}\partial_t e^{\Phi(t)}\right], \tag{2.42}$$

where $\Phi(t)$ is an interpolation such as $\Phi(0) = 0, \Phi(1) = \Phi$.

background:[7]

$$\langle \Phi_T \rangle_{\mathcal{V}} = \frac{e^{-\pi \mathcal{C}}}{\pi} \left\langle \xi Y \mathcal{V}(i\infty) c \left(\frac{\pi}{2}\right) \exp \left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du \, \mathcal{J}(u) \right) \right\rangle_{C_\pi}, \tag{2.45}$$

$$\mathcal{J}(u) = \int_{-\infty}^{\infty} dv \, f_a(v) J^a(iv + u), \tag{2.46}$$

where $\langle \cdots \rangle_{C_\pi}$ denotes the correlation functions on a cylinder of circumference π.

3 Analytic evaluation of observables for identity-based marginal solutions

In this section, we consider the gauge equivalence relation mentioned in the introduction and we calculate two observables, the vacuum energy and the gauge invariant overlap, for the identity-based marginal solutions.

First of all, we take a particular interpolation $\Phi_J(t) = t\Phi_J$ ($t \in [0,1]$) such that $\Phi_J(0) = 0$ and $\Phi_J(1) = \Phi_J$. This interpolating string field $\Phi_J(t)$ is also an identity-based marginal solution to the equation of motion:

$$\hat{\eta} \left(e^{-\Phi_J(t)} \hat{Q} e^{\Phi_J(t)} \right) = 0, \tag{3.1}$$

because $\Phi_J(t)$ is given by a replacement of the weighting function: $F_a(z) \to tF_a(z)$ in (2.1). Hence, we have a new BRST operator $Q_{\Phi_J(t)}$:

$$Q_{\Phi_J(t)} = Q_{\mathrm{B}} - tV^a(F_a) + \frac{t^2}{8} \Omega^{ab} C(F_a F_b), \tag{3.2}$$

for the theory around a solution $\Phi_J(t)$. Following the same procedure in section 2, we define a string field $K'(t) \equiv \hat{Q}_{\Phi_J(t)} B$ ($\hat{Q}_{\Phi_J(t)} \equiv Q_{\Phi_J(t)} \sigma_3$). Then we can construct a tachyon vacuum solution $\Phi_T(t)$ as

$$e^{\Phi_T(t)} = 1 - c \frac{B}{1 + K'(t)} + q \left(\zeta + \left(\hat{Q}_{\Phi_J(t)} \zeta \right) \frac{B}{1 + K'(t)} \right), \tag{3.3}$$

and we can easily see that it satisfies the equation of motion around the identity-based solution $\Phi_J(t)$:

$$\hat{\eta} \left(e^{-\Phi_T(t)} \hat{Q}_{\Phi_J(t)} e^{\Phi_T(t)} \right) = 0. \tag{3.4}$$

We note that, in particular, $\Phi_T(t)$ satisfies $\Phi_T(1) = \Phi_T$ (the solution (2.15)) and $\Phi_T(0) = \Phi_T^{\mathrm{E}}$ (the Erler solution [9]) because $Q_{\Phi_J(1)} = Q_{\Phi_J}$ and $Q_{\Phi_J(0)} = Q_{\mathrm{B}}$.

Using the above string fields, we define a string field $\tilde{\Phi}_T(t)$ with the parameter t as

$$e^{\tilde{\Phi}_T(t)} \equiv e^{\Phi_J(t)} e^{\Phi_T(t)}, \tag{3.5}$$

and then we find a relation:

$$e^{-\tilde{\Phi}_T(t)} \hat{Q} e^{\tilde{\Phi}_T(t)} = e^{-\Phi_J(t)} \hat{Q} e^{\Phi_J(t)} + e^{-\Phi_T(t)} \hat{Q}_{\Phi_J(t)} e^{\Phi_T(t)}. \tag{3.6}$$

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[7]Here, we have used an expression in [6] rather than that in [16].

It indicates that, with (3.1) and (3.4), $\tilde{\Phi}_T(t)$ satisfies the equation of motion of the original theory:

$$\hat{\eta}\left(e^{-\tilde{\Phi}_T(t)}\hat{Q}e^{\tilde{\Phi}_T(t)}\right) = 0. \tag{3.7}$$

Expanding around the solution $\tilde{\Phi}_T(t)$ in the theory with $\hat{Q}$, we have the theory with the deformed BRST operator $\hat{Q}_{\tilde{\Phi}_T(t)}$ such that, for any string field Ξ,

$$\hat{Q}_{\tilde{\Phi}_T(t)}\Xi = \hat{Q}\Xi + \left(e^{-\tilde{\Phi}_T(t)}\hat{Q}e^{\tilde{\Phi}_T(t)}\right)\Xi - (-1)^{|\Xi|}\Xi\left(e^{-\tilde{\Phi}_T(t)}\hat{Q}e^{\tilde{\Phi}_T(t)}\right)$$
$$= \hat{Q}_{\Phi_J(t)}\Xi + \left(e^{-\Phi_T(t)}\hat{Q}_{\Phi_J(t)}e^{\Phi_T(t)}\right)\Xi - (-1)^{|\Xi|}\Xi\left(e^{-\Phi_T(t)}\hat{Q}_{\Phi_J(t)}e^{\Phi_T(t)}\right). \tag{3.8}$$

Comparing to (2.17), the last expression in the above implies that $\hat{Q}_{\tilde{\Phi}_T(t)}$ is the same as the BRST operator $\hat{Q}'_{\Phi_T(t)}$ in the theory around $\Phi_T(t)$, which is a tachyon vacuum solution in the theory around $\Phi_J(t)$. Following the previous results in section 2.3 with appropriate replacement, we find that there exists a homotopy state: $A'(t) \equiv \frac{B}{1+K'(t)}$ such as $\hat{Q}_{\tilde{\Phi}_T(t)}A'(t) = 1$, which implies that there is no cohomology for $\hat{Q}_{\tilde{\Phi}_T(t)}$ in the small Hilbert space. We also find the equation:[8]

$$\hat{Q}_{\tilde{\Phi}_T(t)}\frac{d}{dt}\left(e^{-\tilde{\Phi}_T(t)}\hat{Q}e^{\tilde{\Phi}_T(t)}\right) = 0. \tag{3.10}$$

Therefore, there exists a state Λ_t in the small Hilbert space such as

$$\frac{d}{dt}\left(e^{-\tilde{\Phi}_T(t)}\hat{Q}e^{\tilde{\Phi}_T(t)}\right) = \hat{Q}_{\tilde{\Phi}_T(t)}\Lambda_t. \tag{3.11}$$

Integrating (3.11), we have

$$e^{-\tilde{\Phi}_T(1)}\hat{Q}e^{\tilde{\Phi}_T(1)} = e^{-\Phi_T^{\mathrm{E}}}\hat{Q}e^{\Phi_T^{\mathrm{E}}} + \int_0^1 \hat{Q}_{\tilde{\Phi}_T(t)}\Lambda_t\,dt. \tag{3.12}$$

This is a gauge equivalence relation between the Erler solution Φ_T^{E} and $\tilde{\Phi}_T(1) = \log\left(e^{\Phi_J}e^{\Phi_T}\right)$. (See appendix A for details of gauge equivalence. This relation corresponds to (A.3).)

Now that the gauge equivalence is established, we can analytically evaluate physical observables for the identity-based marginal solutions. First, the vacuum energy for the solution is calculated by using (1.2). We have seen in section 2.4 that $S\left[\Phi_T; \hat{Q}_{\Phi_J}\right]$ is equal to $S\left[\Phi_T^{\mathrm{E}}; \hat{Q}\right]$ and then we conclude that the vacuum energy of the identity-based marginal solution Φ_J is zero from (1.2): $E = -S\left[\Phi_J; \hat{Q}\right] = 0$. This result agrees with the previous one [1, 2] derived from zero mode counting of ξ.

Next, let us consider the GIO for the identity-based marginal solution: $\langle\Phi_J\rangle_{\mathcal{V}}$. Substituting the relation (3.6) with $t = 1$ into (3.12), we obtain the relation (1.1) mentioned

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[8]In general, $\Psi(t) \equiv e^{-\Phi(t)}\hat{Q}e^{\Phi(t)}$ satisfies $\hat{Q}\Psi(t) + \Psi(t)^2 = 0$. Differentiating it with respect to t, we find

$$\hat{Q}_{\Phi(t)}\frac{d}{dt}\left(e^{-\Phi(t)}\hat{Q}e^{\Phi(t)}\right) = 0. \tag{3.9}$$

in the introduction. Using the expressions for the GIOs, (2.42) and (2.43), it immediately leads to a relation for the GIOs:

$$\langle \Phi_J \rangle_{\mathcal{V}} = \langle \Phi_T^{\mathrm{E}} \rangle_{\mathcal{V}} - \langle \Phi_T \rangle_{\mathcal{V}} \,, \tag{3.13}$$

where the contribution of the last term in (1.1) vanishes thanks to (2.38). More explicitly, using the formula (2.45), the GIO for Φ_J can be represented by a correlation function:

$$\langle \Phi_J \rangle_{\mathcal{V}} = \frac{1}{\pi} \left\langle \xi Y \mathcal{V}(i\infty) c \left(\frac{\pi}{2} \right) \left\{ 1 - e^{-\pi \mathcal{C}} \exp \left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du \, \mathcal{J}(u) \right) \right\} \right\rangle_{C_\pi} . \tag{3.14}$$

As in [6], it can be rewritten as a difference between two disk amplitudes with the boundary deformation by taking

$$F_a(z; s) = \frac{2\lambda_a s (1 - s^2)}{\arctan \frac{2s}{1 - s^2}} \frac{1 + z^{-2}}{1 - s^2 (z^2 + z^{-2}) + s^4} \,, \tag{3.15}$$

for the function $F_a(z)$ in Φ_J, which satisfies

$$\int_{C_{\mathrm{L}}} \frac{dz}{2\pi i} F_a(z; s) = \frac{2\lambda_a}{\pi} \,, \tag{3.16}$$

$$F_a(z; s) \to 4\lambda_a \{ \delta(\theta) + \delta(\pi - \theta) \}, \qquad \left(s \to 1, \quad z = e^{i\theta} \right). \tag{3.17}$$

Namely for the limit $s \to 1$, the GIO becomes[9]

$$\langle \Phi_J \rangle_{\mathcal{V}} = \frac{1}{\pi} \left\langle \xi Y \mathcal{V}(i\infty) c \left(\frac{\pi}{2} \right) \left\{ 1 - e^{-\pi \mathcal{C}} \exp \left(\frac{\sqrt{2}}{\pi} \lambda_a \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du \, J^a(u) \right) \right\} \right\rangle_{C_\pi} . \tag{3.18}$$

This expression corresponds to the result in [18] for a wedge-based marginal solution [5].

4 Concluding remarks

In this paper, we have applied the method in [7, 16] to the Erler solution Φ_T^{E} [9]. We have constructed a tachyon vacuum solution Φ_T around the identity-based marginal solution Φ_J [1] in the framework of Berkovits' WZW-like superstring field theory. To get the solution, we have used an extended KBc algebra in the marginally deformed background. Around the solution, we have obtained a homotopy operator and we have evaluated the vacuum energy and the gauge invariant overlap (GIO) for it. The energy is the same value as that on the undeformed background but the GIO is deformed by the marginal operators.

Using the above, we have extended our computation for bosonic string field theory [6] to superstring. Then, we have evaluated the energy and the GIO for the identity-based marginal solution Φ_J. The energy for Φ_J vanishes and it is consistent with our previous result in [1]. The GIO for Φ_J is expressed by a difference of those of the tachyon vacuum solutions on the undeformed and deformed backgrounds. In the evaluation of the gauge invariants, the relation (1.1) is essential, which implies the gauge equivalence of Φ_T^{E} and $\log \left(e^{\Phi_J} e^{\Phi_T} \right)$.

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[9]In this expression, $\mathcal{C}$ becomes divergent for the function $\lim_{s \to 1} F_a(z; s)$ and then it cancels the contact term divergence due to singular OPE among the currents.

It should be pointed out that the general arguments of the gauge equivalence in appendix A is applicable to the bosonic open string field theory, where Ψ is regarded as the bosonic string field and the relations (A.2) and (A.3), without the η_0-constraint, are equivalent. Therefore, the calculation of the gauge invariant overlaps for the identity-based marginal solution in [6], where a version of (A.3) was used, implies that the vacuum energy for it is also evaluated analytically to be zero from gauge equivalence.

Similarly, in the framework of the modified cubic superstring field theory, we can evaluate the energy and the GIO for the identity-based marginal solution $\Psi_J = e^{-\Phi_J}\hat{Q}e^{\Phi_J}$ with the result in [16]. From the gauge equivalence relations for the tachyon vacuum solution in the original theory [19, 20] and that in the marginally deformed background [16], which we denote as Ψ_T and Ψ'_T, respectively, the energy of Ψ_J vanishes because of $S\left[\Psi_J;\hat{Q}\right] = S\left[\Psi_T;\hat{Q}\right] - S\left[\Psi'_T;\hat{Q}'\right] = 1/\left(2\pi^2\right) - 1/\left(2\pi^2\right) = 0$. The GIO for Ψ_J, where $\langle\!\langle\Psi_J\rangle\!\rangle_\mathcal{V} \equiv \mathrm{Tr}[\mathcal{V}(i)\sigma_3\Psi_J]$, is computed by the relation $\langle\!\langle\Psi_J\rangle\!\rangle_\mathcal{V} = \langle\!\langle\Psi_T\rangle\!\rangle_\mathcal{V} - \langle\!\langle\Psi'_T\rangle\!\rangle_\mathcal{V}$. It is given by replacing $\xi Y\mathcal{V}$ with $\mathcal{V}$ in (3.14), which is evaluated using the correlator in the small Hilbert space. One might think that the value of the GIO for Ψ_J may become one half if the half brane solutions, Ψ_H and Ψ'_H, in the original theory [15] and the marginally deformed background [16], respectively, are used instead of the tachyon vacuum solutions due to the relations, $\langle\!\langle\Psi_H\rangle\!\rangle_\mathcal{V} = \frac{1}{2}\langle\!\langle\Psi_T\rangle\!\rangle_\mathcal{V}$ and $\langle\!\langle\Psi'_H\rangle\!\rangle_\mathcal{V} = \frac{1}{2}\langle\!\langle\Psi'_T\rangle\!\rangle_\mathcal{V}$. However, in the case of the half brane solutions, we expect that the BRST operators around them have non-trivial cohomology in contrast to vanishing cohomology around the tachyon vacuum. It implies that we cannot derive the equation corresponding to (3.11)[10] and therefore we cannot use $\langle\!\langle\Psi_H\rangle\!\rangle_\mathcal{V} - \langle\!\langle\Psi'_H\rangle\!\rangle_\mathcal{V}$ to evaluate $\langle\!\langle\Psi_J\rangle\!\rangle_\mathcal{V}$.

We comment on an evaluation of the GIO for the identity-based marginal solution Φ_J in the case that the current is given by the 9-th direction: $\boldsymbol{J}(z,\theta) = \psi^9(z) + \theta\frac{i}{\sqrt{2\alpha'}}\partial X^9(z)$. For the operator $\tilde{V}_L(F)$ in (2.1), we have a relation $\tilde{V}_L(F) = \{Q_\mathrm{B}, \Omega_L(F)\} + X_L(F)$, where $\Omega_L(F) \equiv \int_{C_\mathrm{L}} \frac{dz}{2\pi i}F(z)\frac{i}{2\sqrt{\alpha'}}\xi Y X^9(z)$ and $X_L \equiv \int_{C_\mathrm{L}} \frac{dz}{2\pi i}F(z)\frac{-i}{2\sqrt{\alpha'}}X^9(z)$. Therefore, the GIO might be computed such that $\langle\Phi_J\rangle_\mathcal{V} = \langle\tilde{V}_L(F)I\rangle_\mathcal{V} = \langle X_L(F)I\rangle_\mathcal{V}$, which formally vanishes[11] due to the lack of the ξ zero mode. However, it is not consistent with our result (3.14), which does not vanishes for $Y\mathcal{V}(i\infty) = ce^{-\phi}e^{\frac{i}{\sqrt{2\alpha'}}X^9}(i\infty)\,ce^{-\phi}e^{-\frac{i}{\sqrt{2\alpha'}}X^9}(-i\infty)$ as evaluated in [16]. More explicitly, we have $\langle\Phi_J\rangle_\mathcal{V} = \frac{1}{2\pi i}\left(1 - e^{i\pi\sqrt{2}f}\right)$, where $f \equiv \int_{C_\mathrm{L}} \frac{dz}{2\pi i}F(z)$. This apparent discrepancy seems to imply that $\langle X_L(F)I\rangle$ is ill-defined because of a singular property of the trace of identity-based states. In the case of $f = 0$, the solution Φ_J becomes pure gauge [1] and it is consistent with $\langle\Phi_J\rangle_\mathcal{V} = 0$.

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[10] In other words, the solutions, Ψ_T and $\Psi_J + \Psi'_T$, in the original theory are gauge equivalent, but the half brane solution Ψ_H seems to be gauge inequivalent to the solution $\Psi_J + \Psi'_H$.

[11] This argument for vanishing GIO corresponds to that in [21] for the bosonic case.

A On gauge equivalence relations

Here, we discuss some gauge equivalence relations in terms of the NS sector of Berkovits' WZW-like superstring field theory.

Let us consider a gauge transformation of the superstring field Φ by group elements $h(t)$ and $g(t)$ with one parameter t such that $g(0) = h(0) = 1$:

$$e^{\Phi(t)} = h(t)\, e^{\Phi}\, g(t), \qquad Q_{\mathrm{B}} h(t) = \eta_0 g(t) = 0. \tag{A.1}$$

For the string fields, Φ and $\Phi(t)$, "one-form" string fields are defined by $\Psi \equiv e^{-\Phi} Q_{\mathrm{B}} e^{\Phi}$ and $\Psi(t) \equiv e^{-\Phi(t)} Q_{\mathrm{B}} e^{\Phi(t)}$. From (A.1), these turn out to be related by a transformation:

$$\Psi(t) = g(t)^{-1} Q_{\mathrm{B}}\, g(t) + g(t)^{-1} \Psi\, g(t), \qquad \eta_0 g(t) = 0. \tag{A.2}$$

It is the same form with the gauge transformation in the NS sector of the modified cubic superstring field theory. Conversely, given the relation (A.2), we find that the relation (A.1) holds for $h(t) = e^{\Phi(t)} g(t)^{-1} e^{-\Phi}$. In fact, from (A.2), we immediately see that $Q_{\mathrm{B}}(e^{\Phi(t)} g(t) e^{-\Phi}) = 0$ holds.

Differentiating (A.2) with respect to t and integrating it again, we find another relation between Ψ and $\Psi(t)$:

$$\Psi(t) = \Psi + \int_0^t Q_{\Phi(t')} \Lambda(t')\, dt', \qquad \eta_0 \Lambda(t) = 0, \tag{A.3}$$

where $\Lambda(t) = g(t)^{-1} \frac{d}{dt} g(t)$ and Q_ϕ is a modified BRST operator associated with $\psi \equiv e^{-\phi} Q_{\mathrm{B}} e^{\phi}$: $Q_\phi \lambda = Q_{\mathrm{B}} \lambda + \psi \lambda - (-1)^{|\lambda|} \psi \lambda$ for any string field λ. Conversely, supposing that the equations (A.3) for a given $\Lambda(t)$ hold, we find the relations (A.2) hold for the group element $g(t)$ such as $g(0) = 1$:

$$g(t) = \mathrm{P} \exp\left(\int_0^t \Lambda(t') dt' \right), \tag{A.4}$$

where $\mathrm{P} \exp$ means a t-ordered exponent.

Consequently, the above relations (A.1), (A.2) and (A.3) are all equivalent. It is noted that we can easily include internal Chan-Paton factors in these relations.

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