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Higher derivative extension of $6D$ chiral gauged supergravity

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Abstract: Six-dimensional $(1, 0)$ supersymmetric gauged Einstein-Maxwell supergravity is extended by the inclusion of a supersymmetric Riemann tensor squared invariant. Both the original model as well as the Riemann tensor squared invariant are formulated off-shell and consequently the total action is off-shell invariant without modification of the supersymmetry transformation rules. In this formulation, superconformal techniques, in which the dilaton Weyl multiplet plays a crucial role, are used. It is found that the gauging of the U(1) R-symmetry in the presence of the higher-order derivative terms does not modify the positive exponential in the dilaton potential. Moreover, the supersymmetric Minkowski$_4 \times S^2$ compactification of the original model, without the higher-order derivatives, is remarkably left intact. It is shown that the model also admits non-supersymmetric vacuum solutions that are direct product spaces involving de Sitter spacetimes and negative curvature internal spaces.

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Contents

1 Introduction

Higher-order curvature terms in supergravity theories are of considerable importance for different reasons. They can be considered as higher-order correction terms (in α') to an effective supergravity Lagrangian of a (compactified) string theory (see, e.g., [1]). These Lagrangians are supersymmetric only order by order in the perturbation parameter α'. On the other hand off-shell formulations for different curvature squared invariants in 4, 5 and 6 dimensions have been constructed in [2–7]. These invariants, added to a pure off-shell supergravity theory, are *exactly* supersymmetric and can be considered in their own right. The off-shell nature of these theories implies that they contain auxiliary fields. It is well-known that, when adding higher derivative terms to the Lagrangian, the auxiliary fields become propagating. Hence, the elimination of these auxiliary fields becomes much harder since their field equations are not algebraic anymore. Assuming that the dimensionful parameter in front of the higher derivative part of the Lagrangian is very small, one can solve the auxiliary field equations *perturbatively* and eliminate these fields order by order in the small parameter. It remains an open question if and how the on-shell Lagrangian obtained in this way is related to the compactified string Lagrangian, which does not contain any auxiliary fields to begin with.[1]

Theories containing higher-order curvature terms can provide corrections to black hole entropies [9–11] and can source higher-order effects in the AdS/CFT correspondence [12, 13]. When considering these theories as toy models on their own they can be compactified to lower dimensions. A particular case to consider is the compactification to three dimensions [8]. A particular feature of three dimensions is that $D = 3$ gravitons are non-propagating when only considering 2-derivative Lagrangians. Instead, the addition of higher-derivative terms can turn these non-propagating modes into propagating *massive* graviton modes, see, e.g., [14] and references therein. These theories can then be regarded as simple toy models to study quantum gravity.

In this paper we study higher-order corrections to a six-dimensional $(1, 0)$ supersymmetric $\mathrm{U}(1)_R$ gauged Einstein-Maxwell supergravity theory, usually referred to as the Salam-Sezgin model [15], which is a special case of a $\mathrm{Sp}(n) \times \mathrm{Sp}(1)_R$ gauged matter-coupled supergravity theory that was first obtained in [16]. We shall refer to this more general case as $6D$ chiral gauged supergravity as well. An intriguing feature of the Salam-Sezgin model is that it allows a compactification over S^2 to a four-dimensional Minkowski spacetime while retaining half of the supersymmetry [15]. One of the purposes of this work is to investigate whether this feature survives after the addition of higher-order derivative corrections. To facilitate the addition of such higher-order corrections to the model we will first construct its off-shell formulation. It turns out that this is only possible for the dual formulation of the model where the 2-form potential $\tilde{B}$ has been replaced by a dual 2-form potential B [17, 18]. This has the effect that the curvature of the original 2-form potential no longer contains a Maxwell-Chern-Simons term, but that instead a term of the form $B \wedge F \wedge F$, where F is the Maxwell field strength, appears in the Lagrangian.

To construct the off-shell formulation we will make use of the superconformal tensor calculus. As a first step we will review the construction of off-shell minimal $D = 6$ supergravity [19, 20]. In this construction one makes use of the dilaton Weyl multiplet (obtained by coupling the regular Weyl multiplet to a tensor multiplet) coupled to a linear multiplet as compensator. After fixing the conformal symmetries, this theory still has a remaining $\mathrm{U}(1)$ R-symmetry which is gauged by an auxiliary vector $\mathcal{V}_\mu$. We will couple this 'pure' theory to an Abelian vector multiplet and show that after solving for the auxiliary $\mathcal{V}_\mu$, the gauging proceeds via the vector W_μ of the Abelian vector multiplet.

After constructing the off-shell formulation of the gauged $(1, 0)$ supergravity theory, we investigate its deformation by an off-shell curvature squared invariant [2, 3]. To construct this invariant it is essential to make use of the dilaton Weyl multiplet. We review the construction of this higher-derivative term and add it to the off-shell $(1, 0)$ supergravity theory. Next, we study the gauging procedure in the presence of the Riemann tensor squared invariant.

[1]The elimination of auxiliary fields in higher derivative theories has been discussed in [4]. A conjectured duality between a supergravity Lagrangian with the auxiliary fields eliminated perturbatively and a compactified string Lagrangian, without auxiliary fields, can be found in section 5 of [8].

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As a first step towards understanding the properties of the higher-derivative extension of the model we perform a systematic search for vacuum solutions. We construct both supersymmetric as well as non-supersymmetric solutions. For one particular supersymmetric solution, namely six-dimensional Minkowski spacetime, we calculate the fluctuations around this background and show how these fluctuations fit into supermultiplets.

This paper is organized as follows. In section 2 we review the off-shell version of the $(1,0)$ supergravity model [19, 20] and describe its gauging. In section 3, we introduce an alternative off-shell formulation of the model in view of the fact that it is best suited for the addition of the Riemann tensor squared invariant [2]. In section 4 we discuss the construction of the Riemann tensor squared invariant and arrive at the total Lagrangian for the higher-derivative extended $6D$ chiral gauged supergravity theory. In section 5, we investigate the vacuum solutions of this model. We summarize and comment further on our results and on some interesting open problems in the Conclusions section. Throughout the paper we follow the notation given in appendix A of [20].

2 Off-shell gauged $(1,0)$ supergravity

In this section we present an off-shell version of the dual formulation [17, 18] of the Salam-Sezgin model [15, 16]. In the first subsection we give the off-shell Lagrangian of pure supergravity plus a tensor multiplet as constructed in [19, 20]. In the next subsection we couple a vector multiplet to this theory and show that the resulting Einstein-Maxwell model leads to a non-trivial U(1) gauge symmetry that is not gauged by an auxiliary vector field. In the last subsection we show that after eliminating the auxiliary fields one ends up with a Lagrangian in which the U(1) gauge symmetry is effectively gauged by the physical vector of the vector multiplet. We furthermore show that, after dualizing the 2-form potential into a dual 2-form potential, this Einstein-Maxwell model is nothing else than the original Salam-Sezgin model.

2.1 Off-shell Poincaré action

The off-shell $(1,0)$ supergravity action has been constructed by means of a superconformal tensor calculus in which the off-shell so-called dilaton Weyl multiplet with independent fields

$$\{\, e_\mu{}^a\,, \psi^i_\mu, B_{\mu\nu}\,, \mathcal{V}^{ij}_\mu\,, b_\mu\,, \psi^i\,, \sigma\,\} \tag{2.1}$$

and Weyl weights $(-1,-1/2,0,0,0,5/2,2)$, respectively, is coupled to an off-shell linear multiplet consisting of the fields

$$\{\, E_{\mu\nu\rho\sigma}\,, L^{ij}\,, \varphi^i\,\}\,, \tag{2.2}$$

with Weyl weights $(0,4,9/2)$, respectively. The fields $(\psi^i_\mu, \psi^i, \varphi^i)$ are symplectic Majorana-Weyl spinors labelled by a $\mathrm{Sp}(1)_R$ doublet index, the fields B and E are two- and four-forms with tensor gauge symmetries, respectively, b_μ is the dilatation gauge field and L_{ij} are three real scalars. An appropriate set of gauge choices for obtaining off-shell supergravity with the Einstein-Hilbert term, namely $\mathcal{L} = eR + \cdots$, is given by

$$L_{ij} = \frac{1}{\sqrt{2}}\,\delta_{ij}\,, \quad \varphi^i = 0\,, \quad b_\mu = 0 \tag{2.3}$$

which fixes the dilatations, conformal boost and special supersymmetry transformations. Moreover, the first of the gauge choices in (2.3) breaks $\mathrm{Sp}(1)_R$ down to $\mathrm{U}(1)_R$. This set of gauge choices leads to an off-shell multiplet containing 48+48 degrees of freedom described by the fields [19] (see table 5 of [20])

$$e_\mu{}^a\ (15)\,,\quad \mathcal{V}'^{\,ij}_\mu\ (12)\,,\quad \mathcal{V}_\mu\ (5)\,,\quad B_{\mu\nu}\ (10)\,,\quad \sigma\ (1)\,,\quad E_{\mu\nu\rho\sigma}\ (5)\,;\quad \psi_\mu{}^i\ (40)\,,\quad \psi^i\ (8)\,. \tag{2.4}$$

The field $\mathcal{V}_\mu$ is the gauge field of the surviving $\mathrm{U}(1)_R$ gauge symmetry. It arises in the decomposition

$$\mathcal{V}^{ij}_\mu = \mathcal{V}'^{ij}_\mu + \frac{1}{2}\delta^{ij}\mathcal{V}_\mu\,, \qquad \mathcal{V}'^{ij}_\mu\,\delta_{ij} = 0\,, \tag{2.5}$$

where the traceless part $\mathcal{V}'^{ij}_\mu$ has no gauge symmetry. A superconformal tensor calculus method was employed in [19] where the bosonic action was given, and a procedure for obtaining the full action was provided. This full action, including the quartic terms, was constructed in [20]. The Lagrangian up to quartic fermion terms is given by [19, 20] [2]

$$\begin{aligned}
e^{-1}\mathcal{L}_R\big|_{L=1} = {} & \frac{1}{2}R - \frac{1}{2}\sigma^{-2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{24}\sigma^{-2}F_{\mu\nu\rho}(B)F^{\mu\nu\rho}(B) + \mathcal{V}'_{\mu ij}\mathcal{V}'^{\mu ij}\\
& -\frac{1}{4}E^\mu E_\mu + \frac{1}{\sqrt{2}}E^\mu\mathcal{V}_\mu - \frac{1}{4\sqrt{2}}E_\rho\bar\psi^i_\mu\gamma^{\rho\mu\nu}\psi^j_\nu\delta_{ij}\\
& -\frac{1}{2}\bar\psi_\mu\gamma^{\mu\nu\rho}D_\nu(\omega)\psi_\rho - 2\sigma^{-2}\bar\psi\gamma^\mu D'_\mu(\omega)\psi + \sigma^{-2}\bar\psi_\nu\gamma^\mu\gamma^\nu\psi\,\partial_\mu\sigma\\
& -\frac{1}{48}\sigma^{-1}F_{\mu\nu\rho}(B)\left(\bar\psi^\lambda\gamma_{[\lambda}\gamma^{\mu\nu\rho}\gamma_{\tau]}\psi^\tau + 4\sigma^{-1}\bar\psi_\lambda\gamma^{\mu\nu\rho}\gamma^\lambda\psi - 4\sigma^{-2}\bar\psi\gamma^{\mu\nu\rho}\psi\right)\,.
\end{aligned} \tag{2.6}$$

The indication $L = 1$ in the left-hand side indicates all the gauge choices (2.3). Here we have defined the field strength for the 2-form potential and the dual of the field strength for the 4-form potentials as follows[3]

$$F_{\mu\nu\rho}(B) = 3\partial_{[\mu}B_{\nu\rho]}\,, \tag{2.7}$$

$$E^\mu = \frac{1}{24}e^{-1}\varepsilon^{\mu\nu_1\cdots\nu_5}\partial_{[\nu_1}E_{\nu_2\cdots\nu_5]}\,. \tag{2.8}$$

The $\mathrm{U}(1)_R$ covariant derivatives $D_\mu(\omega)$ and the full $\mathrm{SU}(2)$ covariant derivatives $D'_\mu(\omega)$ are given by

$$D_\mu(\omega)\psi^i_\nu = \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}\right)\psi^i_\nu - \frac{1}{2}\mathcal{V}_\mu\delta^{ij}\psi_{\nu j}\,, \tag{2.9}$$

$$D'_\mu(\omega)\psi^i = \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}\right)\psi^i - \frac{1}{2}\mathcal{V}_\mu\delta^{ij}\psi_j + \mathcal{V}_\mu{}'^i{}_j\psi^j\,, \tag{2.10}$$

[2] We use the conventions of [20]. In particular, the spacetime signature is $(-++++\,+)$, $\gamma_{a_1\cdots a_6} = \varepsilon_{a_1\cdots a_6}\gamma_*$, $\gamma_*\epsilon = \epsilon$, $\bar\psi_i\psi_j = -\bar\psi_j\psi_i$ and $\bar\psi_i\gamma_\mu\psi_j = \bar\psi_j\gamma_\mu\psi_i$. These conventions differ from those in [19] in using signature $(-+\ldots+)$ rather than the Pauli convention $(++\ldots+)$, in rescaling $\mathcal{V}^i_{\mu j}$ by a factor of $-1/2$, and the minus sign in the definition of the Ricci tensor. The signature change merely results in rescaling $\varepsilon^{\mu_1\cdots\mu_6}$ by a factor of i.

[3] Note that the definition of E^μ here is purely bosonic, and it differs from the definition used in [19, 20], where it is a superconformal covariant expression with fermionic bilinear terms.

where $\omega_{\mu ab}$ is the standard torsion-free connection. Note that the symmetric traceless field $\mathcal{V}'_\mu{}^{ij}$, occurring in the decomposition (2.5), is absent in the covariant derivative of the gravitino [20]. This is a consequence of having broken the SU(2) symmetry present in the dilaton Weyl multiplet by the gauge choices (2.3). In the above formula, and throughout the paper the spin connection $\omega_{\mu ab}$ is the standard one associated with the Christoffel symbol, and as such, it does not depend on fermionic or bosonic torsion. The supersymmetry transformations, up to cubic fermion terms, are obtained from section 2 of [20]:

$$
\begin{aligned}
\delta e_\mu{}^a &= \frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu\,,\\
\delta\psi^i_\mu &= D_\mu(\omega)\epsilon^i + \frac{1}{48}\sigma^{-1}\gamma\cdot F(B)\gamma_\mu\epsilon^i - \mathcal{V}'^{ij}_\mu\epsilon_j + \gamma_\mu\eta^i\,,\\
\delta B_{\mu\nu} &= -\sigma\bar{\epsilon}\gamma_{[\mu}\psi_{\nu]} - \bar{\epsilon}\gamma_{\mu\nu}\psi\,,\\
\delta\psi^i &= \frac{1}{48}\gamma\cdot F(B)\epsilon^i + \frac{1}{4}\partial\!\!\!/\sigma\epsilon^i - \sigma\eta^i\,,\\
\delta\sigma &= \bar{\epsilon}\psi\,,\\
\delta E_{\mu\nu\rho\sigma} &= 2\sqrt{2}\bar{\psi}_{[\mu}{}^i\gamma_{\nu\rho\sigma]}\epsilon^j\delta_{ij}\,,\\
\delta\mathcal{V}^{ij}_\mu &= \frac{1}{2}\bar{\epsilon}^{(i}\gamma^\nu R_{\mu\nu}{}^{j)}(Q) + \frac{1}{8}\sigma^{-1}\bar{\epsilon}^{(i}\gamma^\nu\left(F_{[\mu}{}^{ab}(B)\gamma_{ab}\psi^{j)}_{\nu]}\right) + \frac{1}{24}\sigma^{-1}\bar{\epsilon}^{(i}\gamma\cdot F(B)\psi^{j)}_\mu\\
&\quad+\frac{1}{2}\sigma^{-1}\bar{\epsilon}^{(i}\gamma_\mu D\!\!\!\!/\,'(\omega)\psi^{j)} - \frac{1}{8}\sigma^{-1}\bar{\epsilon}^{(i}\gamma_\mu\gamma^\rho\partial\!\!\!/\sigma\psi_\rho{}^{j)} - \frac{1}{48}\sigma^{-2}\bar{\epsilon}^{(i}\gamma_\mu\gamma\cdot F(B)\psi^{j)} + 2\bar{\eta}^{(i}\psi^{j)}_\mu\,,
\end{aligned}
\tag{2.11}
$$

where $D_\mu(\omega)\epsilon^i$ is defined as in (2.9), $R_{\mu\nu}{}^i(Q)$ is the gravitino curvature and η_i is the effective contribution from the S-supersymmetry in the superconformal algebra:

$$
\begin{aligned}
D_\mu(\omega)\epsilon^i &= \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}\right)\epsilon^i - \frac{1}{2}\mathcal{V}_\mu\delta^{ij}\epsilon_j\,,\\
R_{\mu\nu}{}^i(Q) &= 2D_{[\mu}(\omega)\psi^i_{\nu]} - 2\mathcal{V}'_{[\mu}{}^{ij}\psi_{\nu]j}\,, \qquad (2.12)\\
\eta_k &= \frac{1}{4}\Big(\gamma^\mu\mathcal{V}_\mu{}'^{(i}{}_l\delta^{j)l}\epsilon_j - \frac{1}{2\sqrt{2}}E_\mu\gamma^\mu\epsilon^i\Big)\delta_{ik}\,. \qquad (2.13)
\end{aligned}
$$

The latter equation gives the compensating special supersymmetry transformation parameter in the gauge $\varphi^i = 0$, as can be read off from eq. (3.37) of [19]. Note that the $\mathrm{U}(1)_R$ part of $\mathcal{V}^{ij}_\mu$ has dropped out in this expression. The surviving $\mathrm{U}(1)_R$ symmetry of the Lagrangian $\mathcal{L}_R$ is gauged by the auxiliary gauge field $\mathcal{V}_\mu$, which acts as follows[4]

$$
\delta(\lambda)\mathcal{V}_\mu = \partial_\mu\lambda\,,\qquad \delta(\lambda)\psi_\mu{}^i = \frac{1}{2}\delta^{ij}\lambda\psi_{\mu j}\,,\qquad \delta(\lambda)\psi^i = \frac{1}{2}\delta^{ij}\lambda\psi_j\,,
\tag{2.14}
$$

with λ being the parameter of the gauged symmetry.

[4]The $\mathrm{U}(1)_R$ is the subgroup of the full SU(2), under which the gravitino transforms as

$$
\delta\psi_\mu{}^i = -\lambda^i{}_j\psi_\mu{}^j = \left(\lambda'^{ij} + \frac{1}{2}\lambda\delta^{ij}\right)\psi_{\mu j}\,,
$$

where λ'^{ij} is traceless. A similar formula holds for ψ^i.

2.2 Coupling to an off-shell vector multiplet

We now wish to introduce a gauge multiplet, whose vector is *not* auxiliary, to gauge the U(1) R-symmetry. The present gauging by $\mathcal{V}_\mu$, discussed in the previous subsection, is undesirable since $\mathcal{V}_\mu$ has no standard kinetic term. In fact, we will show in subsection 2.3 that the gauge symmetry becomes trivial after solving the 4-form potential in terms of a scalar field.

To obtain this non-trivial gauging we follow [19] and add to $\mathcal{L}_\mathrm{R}$ the kinetic terms for an abelian vector multiplet $\mathcal{L}_\mathrm{V}$. The multiplet consists of the fields $(W_\mu, Y_{ij}, \Omega_i)$, being a physical gauge field, an auxiliary SU(2) triplet, and a physical fermion. They transform under dilatations with Weyl weights $(0, 2, 3/2)$, respectively. We add the coupling $g\mathcal{L}_\mathrm{VL}$ of the vector multiplet to the compensating linear multiplet. Prior to fixing any of the conformal symmetries, these Lagrangians, up to quartic fermion terms, are given by [19]

$$\begin{aligned} e^{-1}\mathcal{L}_\mathrm{V} &= \sigma\Big(-\frac{1}{4}F_{\mu\nu}(W)F^{\mu\nu}(W) - 2\bar{\Omega}\gamma^\mu D'_\mu(\omega)\Omega + Y^{ij}Y_{ij}\Big) \\ &\quad -\frac{1}{16}e^{-1}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}F_{\rho\sigma}(W)F_{\lambda\tau}(W) - 4\bar{\Omega}^i\psi^j Y_{ij} \\ &\quad +\frac{1}{2}\left(\sigma\bar{\Omega}\gamma^\mu\gamma\cdot F(W)\psi_\mu + 2\bar{\Omega}\gamma\cdot F(W)\psi\right) + \frac{1}{12}\bar{\Omega}\gamma\cdot F(B)\Omega\,, \end{aligned} \tag{2.15}$$

$$e^{-1}\mathcal{L}_\mathrm{VL} = Y_{ij}L^{ij} + 2\bar{\Omega}\varphi - L^{ij}\bar{\psi}_{\mu i}\gamma^\mu\Omega_j + \frac{1}{2}W_\mu E^\mu\,, \tag{2.16}$$

where $D'_\mu(\omega)\Omega^i$ is defined as in (2.10). This action has the full SU(2) symmetry.

The coupling of the vector multiplet to supergravity is then achieved by considering the Lagrangian

$$\mathcal{L}_1 = \Big(\mathcal{L}_R + \mathcal{L}_V + g\mathcal{L}_{VL}\Big)\Big|_{L=1}\,, \tag{2.17}$$

where as before '$L = 1$' refers to the set of gauges given in (2.3). This formula, up to quartic fermion terms, yields the result

$$\begin{aligned} e^{-1}\mathcal{L}_1 &= \frac{1}{2}R - \frac{1}{2}\sigma^{-2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{\sqrt{2}}g\delta^{ij}Y_{ij} - \frac{1}{24}\sigma^{-2}F_{\mu\nu\rho}(B)F^{\mu\nu\rho}(B) \\ &\quad +\mathcal{V}'^{\,ij}_\mu\mathcal{V}'^\mu{}_{ij} - \frac{1}{4}E^\mu E_\mu + \frac{1}{\sqrt{2}}E^\mu\left(\mathcal{V}_\mu + \frac{1}{\sqrt{2}}gW_\mu\right) \\ &\quad +\sigma Y^{ij}Y_{ij} - \frac{1}{4}\sigma F_{\mu\nu}(W)F^{\mu\nu}(W) - \frac{1}{16}e^{-1}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}F_{\rho\sigma}(W)F_{\lambda\tau}(W) \\ &\quad -\frac{1}{2}\bar{\psi}_\rho\gamma^{\mu\nu\rho}D_\mu(\omega)\psi_\nu - 2\sigma^{-2}\bar{\psi}\gamma^\mu D'_\mu(\omega)\psi + \sigma^{-2}\bar{\psi}_\nu\gamma^\mu\gamma^\nu\psi\partial_\mu\sigma \\ &\quad -\frac{1}{48}\sigma^{-1}F_{\mu\nu\rho}(B)\Big(\bar{\psi}^\lambda\gamma_{[\lambda}\gamma^{\mu\nu\rho}\gamma_{\tau]}\psi^\tau + 4\sigma^{-1}\bar{\psi}_\lambda\gamma^{\mu\nu\rho}\gamma^\lambda\psi - 4\sigma^{-2}\bar{\psi}\gamma^{\mu\nu\rho}\psi\Big) \\ &\quad -\frac{1}{4\sqrt{2}}E_\rho\psi^i_\mu\gamma^{\rho\mu\nu}\psi^j_\nu\delta_{ij} - \frac{1}{\sqrt{2}}g\delta^{ij}\bar{\Omega}_i\gamma^\mu\psi_{\mu j} - 2\sigma\bar{\Omega}\gamma^\mu D'_\mu(\omega)\Omega - 4Y^{ij}\bar{\Omega}_i\psi_j \\ &\quad +\frac{1}{2}F_{\mu\nu}(W)\left(\sigma\bar{\Omega}\gamma^\lambda\gamma^{\mu\nu}\psi_\lambda + 2\bar{\Omega}\gamma^{\mu\nu}\psi\right) + \frac{1}{12}F_{\mu\nu\rho}(B)\bar{\Omega}\gamma^{\mu\nu\rho}\Omega\,. \end{aligned} \tag{2.18}$$

The action corresponding to the Lagrangian $\mathcal{L}_1$ is invariant under the supersymmetry transformations (2.11) supplemented by the supersymmetry transformations of the components of the off-shell vector multiplet. The transformations of the latter are given up to cubic fermion terms by [19]

$$
\begin{aligned}
\delta W_\mu &= -\bar{\epsilon}\gamma_\mu\Omega\,, \\
\delta\Omega^i &= \frac{1}{8}\gamma\cdot F(W)\epsilon^i - \frac{1}{2}Y^{ij}\epsilon_j\,, \\
\delta Y^{ij} &= -\frac{1}{2}\bar{\epsilon}^i\gamma^\mu\left(D'_\mu(\omega)\Omega^j - \frac{1}{8}\gamma\cdot F(W)\psi^j_\mu + \frac{1}{2}Y^{jk}\psi_{\mu k}\right) + \bar{\eta}^i\Omega^j + (i\leftrightarrow j)\,, \qquad (2.19)
\end{aligned}
$$

where η is as defined in (2.13). The Lagrangian $\mathcal{L}_1$ also has a manifest $\mathrm{U}(1)_R\times\mathrm{U}(1)$ symmetry with transformations parametrized by λ and η

$$
\begin{aligned}
&\delta\mathcal{V}_\mu = \partial_\mu\lambda\,, && \delta W_\mu = \partial_\mu\eta, && \\
&\delta\psi^i_\mu = \frac{1}{2}\lambda\delta^{ij}\psi_{\mu j}\,, && \delta\psi^i = \frac{1}{2}\lambda\delta^{ij}\psi_{\mu j}\,, && \delta\Omega^i = \frac{1}{2}\lambda\delta^{ij}\Omega_j\,, \qquad (2.20)
\end{aligned}
$$

where $(\lambda\,,\eta)$ are the parameters of the $\big(\mathrm{U}(1)_R\,,\mathrm{U}(1)\big)$ symmetry, respectively.

2.3 Elimination of auxiliary fields

We consider Lagrangian $\mathcal{L}_1$ given in (2.18), and begin by writing down the field equations for the auxiliary fields $Y_{ij}, \mathcal{V}'^{ij}_\mu, \mathcal{V}_\mu, E_{\mu\nu\rho\sigma}$:

$$
0 = \sigma Y_{ij} + \frac{1}{2\sqrt{2}}g\delta_{ij} - 2\bar{\Omega}_{(i}\psi_{j)}\,, \qquad (2.21)
$$

$$
0 = \mathcal{V}'^{ij}_\mu + \left(\sigma^{-2}\bar{\psi}^i\gamma_\mu\psi^j + \sigma\bar{\Omega}^i\gamma_\mu\Omega^j - \text{trace}\right)\,, \qquad (2.22)
$$

$$
0 = E^\mu + \sqrt{2}\delta^{ij}\left(\frac{1}{4}\bar{\psi}_{\nu i}\gamma^{\mu\nu\rho}\psi_{\rho j} - \sigma^{-2}\bar{\psi}_i\gamma^\mu\psi_j - \sigma\bar{\Omega}_i\gamma^\mu\Omega_j\right)\,, \qquad (2.23)
$$

$$
0 = \varepsilon^{\lambda\tau\rho\sigma\mu\nu}\partial_\mu\left(E_\nu - \sqrt{2}\mathcal{V}_\nu - gW_\nu + \frac{1}{2\sqrt{2}}\bar{\psi}^{\alpha i}\gamma_{\nu\alpha\beta}\psi^{\beta j}\delta_{ij}\right)\,. \qquad (2.24)
$$

The elimination of Y_{ij} in (2.18) by means of (2.21) gives a positive definite potential $\frac{1}{4}g^2\sigma^{-1}$ and the elimination of $\mathcal{V}'^{ij}_\mu$ by means of (2.22) gives only quartic fermion terms in the action. Next, (2.24) implies that locally we can write

$$
E_\mu - \sqrt{2}\mathcal{V}_\mu - gW_\mu + \frac{1}{2\sqrt{2}}\bar{\psi}^{\nu i}\gamma_{\mu\nu\rho}\psi^{\rho j}\delta_{ij} = \partial_\mu\phi\,, \qquad (2.25)
$$

for some scalar field ϕ transforming under the $\mathrm{U}(1)_R\times\mathrm{U}(1)$ transformations (2.20) as

$$
\delta\phi = -g\eta - \sqrt{2}\lambda\,. \qquad (2.26)
$$

The terms in (2.25) can be rearranged to write

$$
E_\mu = D_\mu\phi - \frac{1}{2\sqrt{2}}\bar{\psi}^{\nu i}\gamma_{\mu\nu\rho}\psi^{\rho j}\,\delta_{ij}\,, \qquad (2.27)
$$

with the covariant derivative of the scalar field defined as

$$
D_\mu\phi = \partial_\mu\phi + \sqrt{2}\mathcal{V}_\mu + gW_\mu\,. \qquad (2.28)
$$

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Using (2.27) to eliminate E_μ in the Lagrangian (2.18) amounts to dualization of the 4-form potential $E_{\mu\nu\rho\sigma}$ related to E_μ as in (2.8).[5]

The shift symmetry (2.26) can be used to eliminate the scalar field ϕ, by setting it to a constant ϕ_0. This in turn implies a compensating $\lambda = -g\eta/\sqrt{2}$ transformation, leading to an unbroken U(1) symmetry. Eliminating ϕ in this way, (2.23) and (2.25) imply

$$\mathcal{V}_\mu + \frac{1}{\sqrt{2}} g W_\mu = \left(\sigma^{-2}\bar{\psi}^i\gamma_\mu\psi^j + \sigma\bar{\Omega}^i\gamma_\mu\Omega^j\right)\delta_{ij}\,, \tag{2.29}$$

Using this equation and (2.23) in the terms involving E_μ in the action gives rise to only quartic fermion terms. The use of (2.25) in the fermionic kinetic terms, however, has the effect of replacing $\mathcal{V}_\mu$ by $-gW_\mu/\sqrt{2}$, up to quartic fermion terms in the action. Thus, altogether, the elimination of all the auxiliary fields yields, up to quartic fermion terms, the following Lagrangian:

$$\begin{aligned}
e^{-1}\mathcal{L}_{NS} =\ & \frac{1}{2}R - \frac{1}{2}\sigma^{-2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{4}g^2\sigma^{-1} - \frac{1}{24}\sigma^{-2}F_{\mu\nu\rho}(B)F^{\mu\nu\rho}(B) \\
& - \frac{1}{4}\sigma F_{\mu\nu}(W)F^{\mu\nu}(W) + \frac{1}{24}e^{-1}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}F_{\mu\nu\rho}(B)F_{\lambda\tau}(W)W_\sigma \\
& - \frac{1}{2}\bar{\psi}_\rho\gamma^{\mu\nu\rho}\mathcal{D}_\mu\psi_\nu - 2\sigma^{-2}\bar{\psi}\gamma^\mu\mathcal{D}_\mu\psi - 2\sigma\bar{\Omega}\gamma^\mu\mathcal{D}_\mu\Omega \\
& + \sigma^{-2}\bar{\psi}_\nu\gamma^\mu\gamma^\nu\psi\partial_\mu\sigma + \frac{g}{2\sqrt{2}}\delta^{ij}\left(\bar{\psi}_{\mu i}\gamma^\mu\Omega_j + 4\sigma^{-1}\bar{\Omega}_i\psi_j\right) \\
& + \frac{1}{2}F_{\mu\nu}(W)\left(\sigma\bar{\Omega}\gamma^\rho\gamma^{\mu\nu}\psi_\rho + 2\bar{\Omega}\gamma^{\mu\nu}\psi\right) + \frac{1}{12}F_{\mu\nu\rho}(B)\bar{\Omega}\gamma^{\mu\nu\rho}\Omega \\
& - \frac{1}{48}\sigma^{-1}F_{\mu\nu\rho}(B)\Big(\bar{\psi}^\lambda\gamma_{[\lambda}\gamma^{\mu\nu\rho}\gamma_{\tau]}\psi^\tau + 4\sigma^{-1}\bar{\psi}_\lambda\gamma^{\mu\nu\rho}\gamma^\lambda\psi - 4\sigma^{-2}\bar{\psi}\gamma^{\mu\nu\rho}\psi\Big)\,,
\end{aligned} \tag{2.30}$$

where

$$\begin{aligned}
\mathcal{D}_\mu\psi^i_\nu &= \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}\right)\psi^i_\nu + \frac{1}{2\sqrt{2}}gW_\mu\delta^{ij}\psi_{\nu j}\,, \\
\mathcal{D}_\mu\psi^i &= \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}\right)\psi^i + \frac{1}{2\sqrt{2}}gW_\mu\delta^{ij}\psi_j\,, \\
\mathcal{D}_\mu\Omega^i &= \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}\right)\Omega^i + \frac{1}{2\sqrt{2}}gW_\mu\delta^{ij}\Omega_j\,.
\end{aligned} \tag{2.31}$$

This Lagrangian has the on-shell supersymmetry given, up to cubic fermion terms, by the transformation rules for $(e^a_\mu, \psi^i_\mu, B_{\mu\nu}, \psi_i, \sigma)$ in (2.11), and for (W_μ, Ω^i) in (2.19), with the replacements

$$Y^{ij} \to -\frac{1}{2\sqrt{2}}g\sigma^{-1}\delta^{ij}\,, \qquad \mathcal{V}_\mu \to -\frac{1}{\sqrt{2}}gW_\mu\,, \qquad \mathcal{V}'^{ij}_\mu \to 0\,, \qquad \eta^i \to 0\,. \tag{2.32}$$

The last substitution is due to the fact that the elimination of $\mathcal{V}'^{ij}_\mu$ and E_μ in (2.13) gives rise to quadratic fermion terms only. These results agree with the Lagrangian obtained in [17]

[5] The same result is obtained by adding a total derivative Lagrange multiplier term $eE^\mu\partial_\mu\phi$ to the Lagrangian (2.18) and integrating over E^μ.

by direct application of the Noether procedure based on the on-shell closed supersymmetry transformations.

A dual formulation in which the field equation and Bianchi identity for the 2-form potential are interchanged is easily obtained by adding a Lagrange multiplier term

$$\Delta\mathcal{L} = \frac{1}{24}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}F_{\mu\nu\rho}(B)\partial_\sigma\widetilde{B}_{\lambda\tau}\,. \tag{2.33}$$

Treating $F_{\mu\nu\rho}(B)$ as an independent field in $\mathcal{L}+\Delta\mathcal{L}$, its field equation can be used back in the action, yielding

$$\begin{aligned}
e^{-1}\mathcal{L}_{SS} = {} & \frac{1}{2}R - \frac{1}{2}\sigma^{-2}\partial_a\sigma\partial^a\sigma - \frac{1}{4}g^2\sigma^{-1} - \frac{1}{24}\sigma^2 G_{\mu\nu\rho}G^{\mu\nu\rho} - \frac{1}{4}\sigma F_{\mu\nu}(W)F^{\mu\nu}(W)\\
& -\frac{1}{2}\bar\psi_\rho\gamma^{\mu\nu\rho}\mathcal{D}_\mu\psi_\nu - 2\sigma^{-2}\bar\psi\gamma^\mu\mathcal{D}_\mu\psi - 2\sigma\bar\Omega\gamma^\mu\mathcal{D}_\mu\Omega\\
& +\sigma^{-2}\bar\psi_\nu\gamma^\mu\gamma^\nu\psi\partial_\mu\sigma + \frac{g}{2\sqrt{2}}\delta^{ij}\left(\bar\psi_{\mu i}\gamma^\mu\Omega_j + 4\sigma^{-1}\bar\Omega_i\psi_j\right)\\
& +\frac{1}{2}F_{\mu\nu}(W)\left(\sigma\bar\Omega\gamma^\rho\gamma^{\mu\nu}\psi_\rho + 2\bar\Omega\gamma^{\mu\nu}\psi\right) - \frac{1}{2}\sigma^2 G_{\mu\nu\rho}\bar\Omega\gamma^{\mu\nu\rho}\Omega\\
& +\frac{1}{8}\sigma G_{\mu\nu\rho}\Big(\bar\psi^\lambda\gamma_{[\lambda}\gamma^{\mu\nu\rho}\gamma_{\tau]}\psi^\tau - 4\sigma^{-1}\bar\psi_\lambda\gamma^{\mu\nu\rho}\gamma^\lambda\psi - 4\sigma^{-2}\bar\psi\gamma^{\mu\nu\rho}\psi\Big)\,,
\end{aligned} \tag{2.34}$$

where

$$G_{\mu\nu\rho} = 3\partial_{[\mu}\widetilde{B}_{\nu\rho]} + 3F_{[\mu\nu}(W)W_{\rho]}\,. \tag{2.35}$$

This Lagrangian has the on-shell supersymmetry given, up to cubic fermion terms, by the transformation rules for $(e_\mu^a, \psi_\mu^i, \widetilde{B}_{\mu\nu}, \psi_i, \sigma)$ in (2.11), and for (W_μ, Ω^i) in (2.19), with the replacements

$$\begin{aligned}
Y^{ij} &\to -\frac{1}{2\sqrt{2}}g\sigma^{-1}\delta^{ij}\,, & \mathcal{V}_\mu &\to -\frac{1}{\sqrt{2}}gW_\mu\,, & \mathcal{V}'^{ij}_\mu &\to 0\,,\\
B_{\mu\nu} &\to \widetilde{B}_{\mu\nu}\,, & F_{\mu\nu\rho}(B) &\to \frac{1}{3!}\sigma^2 e\varepsilon_{\mu\nu\rho\sigma\lambda\tau}G^{\sigma\lambda\tau}\,, & \eta^i &\to 0\,.
\end{aligned} \tag{2.36}$$

These results agree with [15–17], after taking into account the fact that some of the fermions are to be redefined by scaling them with a suitable power of the scalar field σ.

3 An alternative off-shell formulation

Starting from a superconformal coupling of the dilaton Weyl multiplet to the compensating linear multiplet, we made the set of gauge choices (2.3) which led to an off-shell Poincaré supergravity with field content (2.4). If we do not insist on the canonical Einstein-Hilbert term in the action, there exists a natural alternative set of gauge choices given by

$$\sigma = 1\,,\quad L_{ij} = \frac{1}{\sqrt{2}}\delta_{ij}L\,,\quad \psi^i = 0\,,\quad b_\mu = 0 \tag{3.1}$$

which fix the dilatations, conformal boost and special supersymmetry, and lead to an alternative off-shell Poincaré multiplet consisting of the fields

$$e_\mu{}^a\ (15)\,,\quad \mathcal{V}'^{\,ij}_\mu\ (12)\,,\quad \mathcal{V}_\mu\ (5)\,,\quad B_{\mu\nu}\ (10)\,,\quad L\ (1)\,,\quad E_{\mu\nu\rho\sigma}\ (5)\,;\quad \psi_\mu{}^i\ (40)\,,\quad \varphi^i\ (8)\,. \tag{3.2}$$

Compared to the previous multiplet given in (2.4) σ and ψ^i are replaced by L and φ^i, and therefore this multiplet again has $48+48$ off-shell degrees of freedom. It turns out that this formulation of the off-shell Poincaré multiplet is very convenient in the construction of the only known off-shell higher derivative invariant in $D = 6$, which is a supersymmetric completion of the Riemann tensor squared [2]. What makes the gauge choice (3.1) very useful in this construction is that it furnishes a map between the off-shell supersymmetry transformations of the Yang-Mills and Poincaré multiplets. We shall review this construction in the next section. Here we shall focus on coupling a vector multiplet to this alternative Poincaré supermultiplet. This amounts to seeking an expression for $\mathcal{L} = \mathcal{L}_R + \mathcal{L}_V + g\mathcal{L}_{VL}$ in the gauge (3.1).

Starting from (2.15) and (2.16), it is straightforward to obtain $\mathcal{L}_V$ and $g\mathcal{L}_{VL}$ in the gauge (3.1). To construct the Einstein-Hilbert Lagrangian in this gauge, on the other hand, we first restore superconformal invariance[6] by performing suitable field redefinitions in (2.6). This is achieved by replacing the fields that transform under dilatations and special supersymmetry by

$$\begin{aligned}
\widetilde{e}_\mu{}^a &= L^{1/4} e_\mu{}^a \,, \\
\widetilde{\psi}^i_\mu &= L^{1/8}\left(\psi^i_\mu - \frac{1}{2\sqrt{2}} L^{-1}\delta^{ij}\gamma_\mu\varphi_j\right) \,, \\
\widetilde{\mathcal{V}}_\mu{}^{ij} &= \mathcal{V}_\mu{}^{ij} - \frac{1}{\sqrt{2}} L^{-1}\delta^{k(i}\bar{\varphi}_k\psi_\mu{}^{j)} + \frac{1}{8}L^{-2}\delta^{li}\delta^{jk}\bar{\varphi}_l\gamma_\mu\varphi_k \,, \\
\widetilde{\sigma} &= L^{-1/2}\sigma \,, \\
\widetilde{\psi}^i &= L^{-5/8}\left(\psi^i + \frac{1}{2\sqrt{2}} L^{-1}\sigma\delta^{ij}\varphi_j\right) \,, \\
\widetilde{E}_a &= L^{-5/4} E_a \,, \\
\widetilde{\epsilon}^i &= L^{1/8}\epsilon^i \,,
\end{aligned} \tag{3.3}$$

which are invariant under dilatations and special supersymmetry, as can be checked by using the transformation rules given in [19]. Next, we impose the gauge choices (3.1). Thus, we construct the Lagrangian

$$\mathcal{L}_2 = \Big(\mathcal{L}_R + \mathcal{L}_V + g\mathcal{L}_{VL}\Big)\Big|_{\sigma=1} \,, \tag{3.4}$$

where $\mathcal{L}_R$ is the Lagrangian given in (2.6) with the field redefinitions (3.3) performed, such that the superconformal invariance is restored, and $\sigma = 1$ refers to all the gauge choices of (3.1). A summary of the different gauge conditions and what parts of the superconformal Lagrangian they affect can be found in table 1.

[6] To be precise, we restore superconformal invariance partially since we do not restore the K-symmetry.

Gauge choices	$\mathcal{L}_R\ (L,\varphi,\sigma,\psi)$	$\mathcal{L}_{R^2}\ (\sigma,\psi)$	$\mathcal{L}_{\rm V}\ (\sigma,\psi)$	$\mathcal{L}_{\rm VL}\ (L,\varphi)$
$L=1,\ \varphi^i=0$	breaks SC	SC	SC	breaks SC
$\sigma=1,\ \psi^i=0$	breaks SC	breaks SC	breaks SC	SC

Table 1. This table shows which gauge conditions leave which parts of the total Lagrangian superconformal (SC) invariant and which parts not. In the top row we have indicated on which fields the different parts of the superconformal Lagrangian depend.

Formula (3.4), up to quartic fermion terms, gives rise to the following expression:

$$\begin{aligned}
e^{-1}\mathcal{L}_2 = {} & \frac{1}{2}LR + \frac{1}{2}L^{-1}\partial_\mu L\partial^\mu L + \frac{1}{\sqrt{2}}gL\delta^{ij}Y_{ij} - \frac{1}{24}LF_{\mu\nu\rho}(B)F^{\mu\nu\rho}(B) \\
& + L\mathcal{V}'^{\,ij}_{\mu}\mathcal{V}'^{\mu}{}_{ij} - \frac{1}{4}L^{-1}E^\mu E_\mu + \frac{1}{\sqrt{2}}E^\mu\left(\mathcal{V}_\mu + \frac{1}{\sqrt{2}}gW_\mu\right) \\
& + Y^{ij}Y_{ij} - \frac{1}{4}F_{\mu\nu}(W)F^{\mu\nu}(W) - \frac{1}{16}e^{-1}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}F_{\rho\sigma}(W)F_{\lambda\tau}(W) \\
& - \frac{1}{2}L\bar\psi_\rho\gamma^{\mu\nu\rho}D_\mu(\omega)\psi_\nu - \sqrt{2}\bar\varphi_i\gamma^{\mu\nu}D_\mu(\omega)\psi_{\nu j}\delta^{ij} + L^{-1}\bar\varphi\not{D}'(\omega)\varphi - 2\bar\Omega\not{D}'(\omega)\Omega \\
& - \frac{1}{2}\left(L\bar\psi^\mu\gamma^\nu\psi_\nu + \sqrt{2}\delta_{ij}\bar\psi^i_\nu\gamma^\mu\gamma^\nu\varphi^j\right)L^{-1}\partial_\mu L - \frac{1}{\sqrt{2}}gL\bar\Omega_i\gamma^\mu\psi_{\mu j}\delta^{ij} \\
& + 2g\bar\Omega\varphi + \frac{1}{2}\bar\Omega\gamma^\mu\gamma\cdot F(W)\psi_\mu + \frac{1}{12}\bar\Omega\gamma\cdot F(B)\Omega + \frac{1}{24}L^{-1}\bar\varphi\gamma\cdot F(B)\varphi \\
& - \frac{1}{48}LF_{\mu\nu\rho}(B)\left(\bar\psi^\lambda\gamma_{[\lambda}\gamma^{\mu\nu\rho}\gamma_{\tau]}\psi^\tau + 2\sqrt{2}L^{-1}\bar\psi_{\lambda i}\gamma^{\lambda\mu\nu\rho}\varphi_j\delta^{ij}\right) \\
& - \frac{1}{4\sqrt{2}}E_\rho\left(\bar\psi^i_\mu\gamma^{\rho\mu\nu}\psi^j_\nu\delta_{ij} - 2\sqrt{2}L^{-1}\bar\psi_\sigma\gamma^\rho\gamma^\sigma\varphi + 2L^{-2}\bar\varphi_i\gamma^\rho\varphi_j\delta^{ij}\right) \\
& + \frac{1}{2}\mathcal{V}'^{\mu ij}\left(2\sqrt{2}\bar\varphi^k\psi_{\mu i}\delta_{jk} - 3L^{-1}\bar\varphi_i\gamma_\mu\varphi_j\right)\,, \qquad (3.5)
\end{aligned}$$

where E_μ is not an independent field but rather the dual of the field strength for the four-form potential, see (2.8), the derivative $D_\mu(\omega)\psi_\nu$ is U(1) covariant as in (2.9), and the derivatives $D'_\mu(\omega)\varphi$ and $D'_\mu(\omega)\Omega$ are SU(2) covariant as in (2.10).

The off-shell supersymmetry transformations for this Lagrangian are to be obtained from those of the dilaton Weyl multiplet upon fixing the gauges (3.1). It is important to note that the field redefinitions (3.3) are not to be performed in this process since these transformations are independent of the linear multiplet fields that were used to impose the gauge choices (2.3). In obtaining these transformations, the compensating transformations required to maintain the gauge (3.1) must also be incorporated. These are a compensating special supersymmetry transformation and a compensating (traceless) SU(2) transformation with parameters given by (up to cubic fermion terms)

$$\begin{aligned}
\eta^i &= \frac{1}{48}\gamma\cdot F(B)\epsilon^i\,, \\
\lambda'^{ij} &= -\frac{1}{\sqrt{2}L}\left(S'^{k(i}\delta^{j)l}\epsilon_{kl}\right), \qquad (3.6)
\end{aligned}$$

where[7]

$$S'^{ij} \equiv \bar{\varepsilon}^{(i}\varphi^{j)} - \frac{1}{2}\delta^{ij}\bar{\varepsilon}^{k}\varphi^{\ell}\delta_{k\ell} \tag{3.7}$$

is the supersymmetry transformation of the traceless part of L^{ij}. Note that the prime stands for 'traceless', i.e. $S'^{ij}\delta_{ij} = 0$. These compensating transformations can be obtained from the transformation rules for ψ^i and L^{ij} given in [19].

Thus, using the supersymmetry transformation rules for the dilaton Weyl multiplet provided in [19, 20], the gauge conditions (3.1) and the compensating transformations with parameters given in (3.6), we find that the supersymmetry transformations of the off-shell Poincaré multiplet, up to cubic fermion terms, take the form

$$\begin{aligned}
\delta e_\mu{}^a &= \frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu\,,\\
\delta\psi_\mu{}^i &= (\partial_\mu + \frac{1}{4}\omega_{\mu ab}\gamma^{ab})\epsilon^i + \mathcal{V}_\mu{}^i{}_j\epsilon^j + \frac{1}{8}F_{\mu\nu\rho}(B)\gamma^{\nu\rho}\epsilon^i\,,\\
\delta B_{\mu\nu} &= -\bar{\epsilon}\gamma_{[\mu}\psi_{\nu]}\,,\\
\delta\varphi^i &= \frac{1}{2\sqrt{2}}\gamma^\mu\delta^{ij}\partial_\mu L\epsilon_j - \frac{1}{4}\gamma^\mu E_\mu\epsilon^i + \frac{1}{\sqrt{2}}\gamma^\mu\mathcal{V}'^{(i}_{\mu\,k}\delta^{j)k}L\epsilon_j - \frac{1}{12\sqrt{2}}L\delta^{ij}\gamma\cdot F(B)\epsilon_j\;,\\
\delta L &= \frac{1}{\sqrt{2}}\bar{\epsilon}^i\varphi^j\delta_{ij}\,,\\
\delta E_{\mu\nu\rho\sigma} &= L\bar{\epsilon}^i\gamma_{[\mu\nu\rho}\psi^j_{\sigma]}\delta_{ij} - \frac{1}{2\sqrt{2}}\bar{\epsilon}\gamma_{\mu\nu\rho\sigma}\varphi\,,\\
\delta\mathcal{V}_\mu &= \frac{1}{2}\bar{\epsilon}^i\gamma^\nu\widehat{R}_{\mu\nu}{}^j(Q)\delta_{ij} + \frac{1}{12}\bar{\epsilon}^i\gamma\cdot F(B)\psi_\mu{}^j\delta_{ij} - 2\lambda'^i{}_k\mathcal{V}'^{\,jk}_\mu\delta_{ij}\,,\\
\delta\mathcal{V}'^{\,ij}_\mu &= \frac{1}{2}\bar{\epsilon}^{(i}\gamma^\nu\widehat{R}_{\mu\nu}{}^{j)}(Q) + \frac{1}{12}\bar{\epsilon}^{(i}\gamma\cdot F(B)\psi_\mu{}^{j)} - \frac{1}{4}\bar{\epsilon}^k\gamma^\nu\widehat{R}_{\mu\nu}{}^\ell(Q)\delta_{k\ell}\delta^{ij}\\
&\quad -\frac{1}{24}\bar{\epsilon}^k\gamma\cdot F(B)\psi_\mu{}^\ell\delta_{k\ell}\delta^{ij} + \partial_\mu\lambda'^{ij} - \lambda'^{(i}{}_k\delta^{j)k}\mathcal{V}_\mu\,,
\end{aligned}\tag{3.8}$$

where

$$\widehat{R}_{\mu\nu}{}^i(Q) = 2D_{[\mu}(\omega)\psi^i_{\nu]} - 2\mathcal{V}'^{\,ij}_{[\mu}\psi_{\nu]j} + \frac{1}{4}\gamma^{ab}\psi_{[\nu}F_{\mu]ab}\,. \tag{3.9}$$

The supersymmetry transformations of the off-shell vector multiplet are (up to cubic fermion terms)

$$\begin{aligned}
\delta W_\mu &= -\bar{\epsilon}\gamma_\mu\Omega\,,\\
\delta\Omega^i &= \frac{1}{8}\gamma^{\mu\nu}F_{\mu\nu}\epsilon^i - \frac{1}{2}Y^{ij}\epsilon_j\,,\\
\delta Y^{ij} &= -\bar{\epsilon}^{(i}\gamma^\mu D'_\mu(\omega)\Omega^{j)} + \frac{1}{8}\bar{\epsilon}^{(i}\gamma^\mu\gamma\cdot F(B)\psi^{j)}_\mu - \frac{1}{24}\bar{\epsilon}^{(i}\gamma\cdot F(B)\Omega^{j)}\\
&\quad -\frac{1}{2}Y^{k(i}\bar{\epsilon}^{j)}\gamma^\mu\psi_{\mu k} - 2\lambda'^{(i}{}_kY^{j)k}\;.
\end{aligned}\tag{3.10}$$

To keep the notation relatively simple we did not use the explicit expression for λ'^{ij} in the above transformation rules. Remember that it is given in (3.6).

[7]It is instructive to write out the λ' parameter in components:

$$\lambda'^{11} = -\lambda'^{22} = \frac{1}{\sqrt{2}L}S'^{21}\,,\qquad \lambda'^{12} = -\frac{1}{\sqrt{2}L}S'^{11}\,.$$

Considering the Lagrangian (3.5) by itself, that is, without any higher derivative extension, all the auxiliary fields, namely $(\mathcal{V}_\mu^{ij}, E_{\mu\nu\rho\sigma}, Y^{ij})$ can be eliminated, thereby arriving at the on-shell formulation. Computations similar to those described in detail in section 2.3 imply that the on-shell Lagrangian, up to quartic fermion terms, is obtained from (3.5) by the following substitutions:

$$Y^{ij} \;\to\; -\frac{1}{2\sqrt{2}} g \delta^{ij} L\,, \quad \mathcal{V}_\mu \;\to\; -\frac{1}{\sqrt{2}} g W_\mu\,, \quad \mathcal{V}_\mu^{\prime ij} \;\to\; 0\,, \quad E_\mu \;\to\; 0\,. \tag{3.11}$$

The on-shell supersymmetry transformations, up to cubic fermion terms, are obtained from (3.8) and (3.10) by making these substitutions, and dropping the transformation rules for the auxiliary fields $(E_{\mu\nu\rho\sigma}, \mathcal{V}_\mu^{ij}, Y^{ij})$.

4 Inclusion of the $R_{\mu\nu ab}R^{\mu\nu ab}$ invariant

In this section we add an off-shell supersymmetric Riemann tensor squared term to the Lagrangian $\mathcal{L}_2$, defined in (3.4), which we constructed in the gauge (3.1). This gauge gave rise to an alternative off-shell formulation of the Poincaré multiplet. In the first subsection we begin with a review of the construction of the Riemann squared invariant [2]. In the second subsection we consider the total Lagrangian and briefly discuss the gauging procedure and the elimination of auxiliary fields.

4.1 Construction of the $R_{\mu\nu ab}R^{\mu\nu ab}$ invariant

To begin with, we shall review a map between the Yang-Mills supermultiplet and a set of fields in the alternative Poincaré multiplet discussed in the previous section. We follow the discussion in [3]. This map can be used, together with an expression for the superconformal action for the Yang-Mills multiplet given in [19], to write down a supersymmetric Riemann tensor squared action. We will describe this in detail below.

In establishing the map between the Yang-Mills and Poincaré multiplets, it is important to consider the full supersymmetry transformations, including the cubic fermion terms which have been omitted so far. In particular, this means that we need to keep track of the complete spin connection, containing the fermionic torsion terms. This is due to the fact that, while the fermionic torsion gave rise to only quartic fermion terms in the Lagrangians considered above, in the case of the Riemann tensor square invariant under consideration in this section, the same fermionic torsion will contribute to terms that are bilinear in the fermion terms. We shall show this explicitly below. In the following, we shall need the (full) supersymmetry transformation rules only for the fields $(e_\mu^a, \psi_\mu, \mathcal{V}_\mu^{ij}, B_{\mu\nu})$, and the Yang-Mills multiplet fields $(W_\mu^I, \Omega^I, Y^{ijI})$, where I labels the adjoint representation of the Yang-Mills gauge group.

We begin with the supersymmetry transformation rules of $(e_\mu^a, \psi_\mu, \mathcal{V}_\mu^{ij}, B_{\mu\nu})$ in the gauge (3.1). Up to cubic fermions the transformation rules are already given in (3.8). In this section we will, however, keep the complete SU(2) symmetry, i.e. we do not impose $L^{ij} = \frac{1}{\sqrt{2}} L\delta^{ij}$. In this way we do not need to accommodate for the compensating

SU(2) transformations proportional to λ' in (3.8).[8] The full version of the supersymmetry transformations is given by [2]

$$
\begin{aligned}
\delta e_\mu{}^a &= \frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu\,,\\
\delta\psi_\mu{}^i &= \partial_\mu\epsilon^i + \frac{1}{4}\widehat{\omega}_{+\mu}{}^{ab}\gamma_{ab}\epsilon^i + \mathcal{V}_\mu{}^i{}_j\epsilon^j \;\equiv\; D_\mu(\widehat{\omega}_+)\epsilon^i + \mathcal{V}'_\mu{}^i{}_j\epsilon^j\,,\\
\delta\mathcal{V}_\mu{}^{ij} &= -\frac{1}{2}\bar{\epsilon}^{(i}\gamma^\lambda\widehat{R}_{\lambda\mu}{}^{j)}(Q) + \frac{1}{12}\bar{\epsilon}^{(i}\gamma\cdot\widehat{F}(B)\psi_\mu{}^{j)}\,,\\
\delta B_{\mu\nu} &= -\bar{\epsilon}\gamma_{[\mu}\psi_{\nu]}\,,
\end{aligned}
\tag{4.1}
$$

where the fermionic torsion and the different supercovariant objects are defined as

$$
\begin{aligned}
\widehat{\omega}_{\mu\pm}{}^{ab} &= \widehat{\omega}_\mu{}^{ab} \pm \frac{1}{2}\widehat{F}_\mu{}^{ab}(B)\,,\\
\widehat{\omega}_\mu{}^{ab} &= 2e^{\nu[a}\partial_{[\mu}e_{\nu]}{}^{b]} - e^{\rho[a}e^{b]\sigma}e_\mu{}^c\partial_\rho e_{\sigma c} + K_\mu{}^{ab}\,,\\
K_\mu{}^{ab} &= \frac{1}{4}\big(2\bar{\psi}_\mu\gamma^{[a}\psi^{b]} + \bar{\psi}^a\gamma_\mu\psi^b\big)\,,\\
\widehat{F}_{\mu\nu\rho}(B) &= 3\partial_{[\mu}B_{\nu\rho]} + \frac{3}{2}\bar{\psi}_{[\mu}\gamma_\nu\psi_{\rho]}\,,\\
\widehat{R}_{\mu\nu}{}^i(Q) &= 2\left(\partial_{[\mu} + \frac{1}{4}\widehat{\omega}_{+[\mu}{}^{ab}\gamma_{ab}\right)\psi_{\nu]}{}^i + 2\mathcal{V}_{[\mu}{}^i{}_j\psi_{\nu]}{}^j\,.
\end{aligned}
\tag{4.2}
$$

Next, we consider the following transformations [3]

$$
\begin{aligned}
\delta\widehat{\omega}_{-\mu}{}^{ab} &= -\frac{1}{2}\bar{\epsilon}\gamma_\mu\widehat{R}^{ab}(Q)\,,\\
\delta\widehat{R}^{abi}(Q) &= \frac{1}{4}\gamma^{cd}\epsilon^i\widehat{R}_{cd}{}^{ab}(\widehat{\omega}_-) - \widehat{F}^{abij}(\mathcal{V})\epsilon_j\,,\\
\delta\widehat{F}^{abij}(\mathcal{V}) &= -\frac{1}{2}\bar{\epsilon}^{(i}\gamma^\mu\widehat{D}_\mu\widehat{R}^{abj)}(Q) + \frac{1}{48}\bar{\epsilon}^{(i}\gamma\cdot\widehat{F}(B)\widehat{R}^{abj)}(Q)\,,
\end{aligned}
\tag{4.3}
$$

where $\widehat{F}_{\mu\nu}{}^{ij}(\mathcal{V})$ and $\widehat{R}_{\mu\nu}{}^{ab}(\widehat{\omega}_-)$ are the supercovariant curvatures of $\mathcal{V}_\mu{}^{ij}$ and $\widehat{\omega}_{-\mu}{}^{ab}$, respectively:

$$
\begin{aligned}
\widehat{F}_{\mu\nu}{}^{ij}(\mathcal{V}) &= F_{\mu\nu}{}^{ij}(\mathcal{V}) - \bar{\psi}_{[\mu}{}^{(i}\gamma^\rho\widehat{R}_{\nu]\rho}{}^{j)}(Q) - \frac{1}{12}\bar{\psi}_{[\mu}{}^{(i}\gamma\cdot\widehat{F}(B)\psi_{\nu]}{}^{j)}\,,\\
\widehat{R}_{\mu\nu}{}^{ab}(\widehat{\omega}^-) &= R_{\mu\nu}{}^{ab}(\widehat{\omega}^-) + \bar{\psi}_{[\mu}\gamma_{\nu]}\widehat{R}^{ab}(Q)\,,\\
\widehat{D}_\mu\widehat{R}^{abi}(Q) &= \partial_\mu\widehat{R}^{abi}(Q) + \frac{1}{4}\widehat{\omega}_\mu{}^{cd}\gamma_{cd}\widehat{R}^{abi}(Q) + \mathcal{V}_\mu{}^i{}_j\widehat{R}^{abj}(Q)\\
&\quad - \frac{1}{4}\gamma^{cd}\psi_\mu{}^i\widehat{R}_{cd}{}^{ab}(\widehat{\omega}_-) + \widehat{F}^{abij}(\mathcal{V})\psi_{\mu j} + 2\widehat{\omega}_{-\mu}{}^{[ac}\widehat{R}_c{}^{b]i}(Q)\,.
\end{aligned}
\tag{4.4}
$$

[8]In this section we only want to establish a map between the Poincaré multiplet and the Yang-Mills multiplet and propose an R^2-invariant based on the action for the Yang-Mills multiplet. Both actions are invariant under the SU(2) R-symmetry. To prove the validity of this map, we need the full nonlinear SUSY transformation rules. After we construct the action we can still impose the gauge $L^{ij} = \frac{1}{\sqrt{2}}L\delta^{ij}$. This will not affect the R^2-invariant. It modifies the supersymmetry transformation rules with SU(2) compensating transformations, which leave the action separately invariant. The resulting transformations are those given already in (3.8).

We now compare the above transformation rules with those of the $\mathcal{N}=(1,0)$, $D=6$ vector multiplet [19]

$$\begin{aligned}\delta W_\mu{}^I &= -\bar\epsilon\gamma_\mu\Omega^I\,,\\ \delta\Omega^{Ii} &= \frac{1}{8}\gamma\cdot\widehat{F}^I(W)\epsilon^i-\frac{1}{2}Y^{Iij}\epsilon_j\,,\\ \delta Y^{Iij} &= -\bar\epsilon^{(i}\gamma^\mu\widehat{D}_\mu\Omega^{j)I}+\frac{1}{24}\bar\epsilon^{(i}\gamma\cdot\widehat{F}(B)\Omega^{j)I}\,,\end{aligned}\tag{4.5}$$

where

$$\begin{aligned}\widehat{F}_{\mu\nu}{}^I(W) &= F_{\mu\nu}{}^I(W)+2\bar\psi_{[\mu}\gamma_{\nu]}\Omega^I\,,\\ \widehat{D}_\mu\Omega^{Ii} &= \partial_\mu\Omega^{Ii}+\frac{1}{4}\widehat\omega_\mu{}^{ab}\gamma_{ab}\Omega^{Ii}+V_\mu{}^i{}_j\Omega^{Ij}\\ &\quad -\frac{1}{8}\gamma\cdot\widehat{F}^I(W)\psi_\mu{}^i+\frac{1}{2}Y^{I\,ij}\psi_{\mu j}-f_{KL}{}^IW_\mu{}^K\Omega^{Li}\,.\end{aligned}\tag{4.6}$$

We observe that the transformation rules (4.3) and (4.5) become identical by making the following identifications:

$$\Big(-2\widehat\omega_{-\mu}{}^{ab},-\widehat{R}^{abi}(Q),-2\widehat{F}^{abij}(\mathcal{V})\Big)\longrightarrow\Big(W_\mu{}^I,\Omega^{Ii},Y^{Iij}\Big)\,.\tag{4.7}$$

Using this observation we can now easily write down a supersymmetric R^2-action using the superconformal invariant exact action formula for the Yang-Mills multiplet constructed in [19]. In the gauge (3.1) and up to quartic fermions, the Lagrangian becomes

$$\begin{aligned}e^{-1}\mathcal{L}_{\rm YM}\big|_{\sigma=1} &= -\frac{1}{4}F_{\mu\nu}{}^I(W)F^{\mu\nu I}(W)-2\bar\Omega^I\gamma^\mu D'_\mu(\omega)\Omega^I+Y^{Iij}Y^I_{ij}+\frac{1}{12}F_{\mu\nu\rho}(B)\bar\Omega^I\gamma^{\mu\nu\rho}\Omega^I\\ &\quad -\frac{1}{16}e^{-1}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}F^I_{\rho\sigma}(W)F^I_{\lambda\tau}(W)+\frac{1}{2}F_{\nu\rho}{}^I\bar\Omega^I\gamma^\mu\gamma^{\nu\rho}\psi_\mu\,.\end{aligned}\tag{4.8}$$

Using the map (4.7) in this formula produces the result for the supersymmetrized Riemann tensor squared action. In presenting the results up to quartic fermion terms, it is useful to note the following simplification in the torsionful spin connection

$$\begin{aligned}\widehat\omega_{\mu-}{}^{ab} &= \omega_{\mu+}{}^{ab}+\frac{1}{2}\bar\psi^a\gamma_\mu\psi^b\,,\\ \omega_{\mu\pm}{}^{ab} &\equiv \omega_\mu{}^{ab}\pm\frac{1}{2}F_\mu{}^{ab}(B)\,,\end{aligned}\tag{4.9}$$

where $\omega_\mu{}^{ab}$ is the standard torsion-free connection. The map (4.7) applied to the action

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formula (4.8) then yields, up to quartic fermion terms, the result[9]

$$\begin{aligned}
e^{-1}\mathcal{L}_{\mathrm{R}^2}\big|_{\sigma=1} =\; & R_{\mu\nu}{}^{ab}(\omega_-)R^{\mu\nu}{}_{ab}(\omega_-) - 2F^{ab}(\mathcal{V})F_{ab}(\mathcal{V}) - 4F'^{abij}(\mathcal{V})F'_{abij}(\mathcal{V}) \\
& + \frac{1}{4}e^{-1}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}R_{\rho\sigma}{}^{ab}(\omega_-)R_{\lambda\tau ab}(\omega_-) \\
& + 2\bar{R}_{+ab}(Q)\gamma^\mu D_\mu(\omega,\omega_-)R_+^{ab}(Q) - R_{\nu\rho}{}^{ab}(\omega_-)\bar{R}_{+ab}(Q)\gamma^\mu\gamma^{\nu\rho}\psi_\mu \\
& - 8F'_{\mu\nu}{}^{ij}(\mathcal{V})\left(\bar{\psi}_i^\mu\gamma_\lambda R_{+j}^{\lambda\nu}(Q) + \frac{1}{6}\bar{\psi}_i^\mu\gamma\cdot F(B)\psi_j^\nu\right) \\
& - \frac{1}{12}\bar{R}_+^{ab}(Q)\gamma\cdot F(B)R_{+ab}(Q) \\
& - \frac{1}{2}\Big[D_\mu(\omega_-,\Gamma_+)R^{\mu\rho ab}(\omega_-) - 2F_{\mu\nu}{}^\rho(B)R^{\mu\nu ab}(\omega_-)\Big]\bar{\psi}_a\gamma_\rho\psi_b\,, \qquad (4.10)
\end{aligned}$$

where

$$\begin{aligned}
D_\mu(\omega,\omega_-)R_+^{abi}(Q) &= \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{cd}\gamma_{cd}\right)R_+^{abi}(Q) - 2\omega_{\mu-}{}^{c[a}R_{+c}{}^{b]i}(Q) + \mathcal{V}_\mu{}^i{}_j R_+^{ab\,j}(Q)\,, \\
R_{+\mu\nu}{}^i(Q) &= 2D_{[\mu}(\omega_+)\psi_{\nu]}^i - 2\mathcal{V}'_{[\mu}{}^{ij}\psi_{\nu]j}\,, \qquad (4.11)
\end{aligned}$$

and the torsionful modification of the Christoffel symbol $\Gamma^\rho_{\mu\nu\pm}$ is defined as

$$\Gamma^\rho_{\mu\nu\pm} \equiv \Gamma^\rho_{\mu\nu} \pm \frac{1}{2}F_{\mu\nu}{}^\rho(B)\,. \qquad (4.12)$$

This completes the construction of the supersymmetric R^2-invariant.

4.2 The total gauged $R + R^2$ supergravity lagrangian

We now want to discuss what the influence is of these R^2-terms on the gauging procedure described in section 2.2. The Lagrangian we consider is the following

$$\mathcal{L}_{\mathrm{total}} = \mathcal{L}_2 - \frac{1}{8M^2}\mathcal{L}_{R^2}\Big|_{\sigma=1}\,, \qquad (4.13)$$

with $\mathcal{L}_2$ given in (3.5) and $\mathcal{L}_{R^2}$ given in (4.10) and with M an arbitrary mass parameter. Recall that $\mathcal{L}_2$ has been obtained as a sum of off-shell supersymmetric Lagrangians $\mathcal{L}_R, \mathcal{L}_V$ and $\mathcal{L}_{VL}$ and that $\mathcal{L}_{R^2}$ is off-shell supersymmetric as well. Thus all four parts of the total Lagrangian we consider are *completely off-shell supersymmetric.* So their sum, the total Lagrangian, is still off-shell supersymmetric. In particular, the bosonic part of this total Lagrangian, which will be the starting point of the next section, takes the form

$$\begin{aligned}
e^{-1}\mathcal{L}^{\mathrm{tot}}_{\mathrm{bos}} =\; & \frac{1}{2}LR + \frac{1}{\sqrt{2}}gL\delta^{ij}Y_{ij} + Y^{ij}Y_{ij} + \frac{1}{2}L^{-1}\partial_\mu L\partial^\mu L - \frac{1}{24}LF_{\mu\nu\rho}(B)F^{\mu\nu\rho}(B) \\
& + 2LZ_\mu Z^{*\mu} - \frac{1}{4}L^{-1}E_\mu E^\mu + \frac{1}{\sqrt{2}}E^\mu\left(\mathcal{V}_\mu + \frac{1}{\sqrt{2}}gW_\mu\right) \\
& - \frac{1}{4}F_{\mu\nu}(W)F^{\mu\nu}(W) - \frac{1}{16}e^{-1}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}F_{\rho\sigma}(W)F_{\lambda\tau}(W) \\
& - \frac{1}{8M^2}\Big[R_{\mu\nu}{}^{ab}(\omega_-)R^{\mu\nu}{}_{ab}(\omega_-) - 2F^{\mu\nu}(\mathcal{V})F_{\mu\nu}(\mathcal{V}) - 8F^{\mu\nu}(Z)F^*_{\mu\nu}(Z) \\
& + \frac{1}{4}e^{-1}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}R_{\rho\sigma}{}^{ab}(\omega_-)R_{\lambda\tau ab}(\omega_-)\Big]\,, \qquad (4.14)
\end{aligned}$$

[9] To obtain (4.10) we used $-\mathcal{L}_{\mathrm{V}}$. Note also that $F_{\mu\nu}{}^{ij}(\mathcal{V}) = \frac{1}{2}F_{\mu\nu}(\mathcal{V})\delta^{ij} + F'_{\mu\nu}{}^{ij}(\mathcal{V})$ where $F_{\mu\nu}(\mathcal{V}) = 2\partial_{[\mu}\mathcal{V}_{\nu]} + 2\mathcal{V}'_\mu{}^i{}_k\mathcal{V}'_\nu{}^{jk}\,\delta_{ij}$ and $F'_{\mu\nu}{}^{ij}(\mathcal{V}) = 2\partial_{[\mu}\mathcal{V}'_{\nu]}{}^{ij} - 2\delta^{k(i}\,\mathcal{V}_{[\mu}\mathcal{V}'_{\nu]}{}^{j)}{}_k$.

where we have defined the complex vector fields

$$Z_\mu \equiv \mathcal{V}'^{11}_{\mu} + \mathrm{i}\mathcal{V}'^{12}_{\mu}\,, \qquad Z^*_\mu = \mathcal{V}'_{\mu 11} - \mathrm{i}\mathcal{V}'_{\mu 12} = -\mathcal{V}'^{11}_{\mu} + \mathrm{i}\mathcal{V}'^{12}_{\mu}\,, \tag{4.15}$$

and field strengths

$$F_{\mu\nu}(\mathcal{V}) = 2\partial_{[\mu}\mathcal{V}_{\nu]} - 4\mathrm{i}Z_{[\mu}Z^*_{\nu]}\,, \qquad F_{\mu\nu}(Z) = 2\partial_{[\mu}Z_{\nu]} - 2\mathrm{i}\mathcal{V}_{[\mu}Z_{\nu]}\,. \tag{4.16}$$

The part of the total Lagrangian containing the fermions is given in (3.5) and (4.10). None of the auxiliary fields have been eliminated so far, and the Lagrangian still possesses the $\mathrm{U}(1)_R \times \mathrm{U}(1)$ symmetry. The field equations for the auxiliary fields Z_μ and $\mathcal{V}_\mu$ are not algebraic anymore and therefore they become propagating. The auxiliary fields $(Y_{ij}, E_{\mu\nu\rho\sigma})$, on the other hand, still have algebraic field equations. Their elimination, as well as the breaking of $\mathrm{U}(1)_R \times \mathrm{U}(1)$ down to a single $\mathrm{U}(1)$ will be discussed in the next section.

At this point one may pursue two different lines of thought. The first is to consider the theory as a toy model in its own right and consider M^2 as an arbitrary (not necessarily large) parameter of the theory. The other is to think of $|M^2|$ as being large compared to a cut-off Λ in the momentum squared. In that case the theory is to be treated as an effective field theory that describes phenomena with external momenta not exceeding $\sqrt{\Lambda}$. Furthermore, the curvature-squared term is a correction term of order $\Lambda/|M^2|$.[10] In this case we can compare the theory with an effective (up to curvature squared terms) string theory Lagrangian compactified to 6 dimensions. In the next section we will only focus on the first line of thought. Let us however briefly comment on the elimination of the Z_μ and $\mathcal{V}_\mu$. For $\Lambda/|M^2| \ll 1$, one particular consequence of eliminating the auxiliary fields up to order $\Lambda/|M^2|$ is that

$$\mathcal{V}^\mu = -\frac{1}{\sqrt{2}}\left(gW^\mu + \frac{L^{-1}}{M^2}\nabla_\nu\Big(F^{\mu\nu}(\mathcal{V}) + \cdots\right) = 0\,, \tag{4.17}$$

which, upon substitution back into the Lagrangian (4.14), and trivial elimination of the other auxiliary fields, gives

$$\begin{aligned} e^{-1}\mathcal{L}^{\mathrm{tot}}_{\mathrm{bos}} =\ & \frac{1}{2}LR - \frac{1}{4}g^2L^2 + \frac{1}{2}L^{-1}\partial_\mu L\partial^\mu L - \frac{1}{24}LF_{\mu\nu\rho}(B)F^{\mu\nu\rho}(B) \\ & -\frac{1}{4}\left(1 - \frac{g^2}{2M^2}\right)F_{\mu\nu}(W)F^{\mu\nu}(W) - \frac{1}{16}e^{-1}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}F_{\rho\sigma}(W)F_{\lambda\tau}(W) \\ & -\frac{1}{8M^2}\Big[R_{\mu\nu}{}^{ab}(\omega_-)R^{\mu\nu}{}_{ab}(\omega_-) + \frac{1}{4}e^{-1}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}R_{\rho\sigma}{}^{ab}(\omega_-)R_{\lambda\tau ab}(\omega_-)\Big]\,. \end{aligned} \tag{4.18}$$

We observe that $g^2 = 2M^2$ is a critical coupling at which the Maxwell kinetic term drops out. However, this is a regime for large coupling constant, and as such it falls outside the regime of perturbative validity. We shall nonetheless examine further what happens for this coupling in the next section where we study the field equations in more detail. Another property of this Lagrangian is that the dualization of the 2-form potential by adding the

[10] In this case, the ghosts expected to arise in the spectrum will have masses of order $|M| \gg \Lambda$ which can be ignored in the effective theory valid up to the energy scale Λ.

Lagrange multiplier term (2.33) and integrating over $F(B)$, gives a dualized field strength of the form (2.35) which now contains also a Lorentz Chern-Simons term.

In the Lagrangian (4.14) presented above, the Einstein-Hilbert term is not in a canonical frame. The metric can be rescaled appropriately to obtain the canonical Einstein-Hilbert action, still remaining in the formulation in terms of the off-shell Poincaré supermultiplet displayed in (3.2). Alternatively, we can employ the off-shell Poincaré multiplet that results from the gauge choices (2.3) by following the following procedure. Since the Lagrangian $\mathcal{L}_1$ given in (2.18) is already formulated in the desired supermultiplet formulation, we need to only construct $\mathcal{L}_{\mathrm{R}^2}$ in the same gauge. This can be done as follows. Firstly, we restore the superconformal invariance (again modulo the conformal boosts which do not affect the final result) in (4.10) by going over to hatted fields defined by

$$\begin{aligned}
\widehat{e}_\mu{}^a &= \sigma^{1/2} e_\mu{}^a \,, \\
\widehat{\psi}_\mu{}^i &= \sigma^{1/4}\psi_\mu{}^i + \sigma^{-3/4}\gamma_\mu\psi^i \,, \\
\widehat{\mathcal{V}}_\mu{}^{ij} &= \mathcal{V}_\mu{}^{ij} - 4\sigma^{-1}\bar{\psi}^{(i}\psi_\mu{}^{j)} - 4\sigma^{-2}\bar{\psi}^{(i}\gamma_\mu\psi^{j)} \,, \\
\widehat{L} &= \sigma^{-2} L \,, \\
\widehat{\varphi}^i &= \sigma^{-9/4}\left(\varphi^i - 2\sqrt{2}\sigma^{-1} L\delta^{ij}\psi_j\right) \,, \\
\widehat{Y}_{ij} &= \sigma^{-1}\left(Y_{ij} + \frac{1}{3}\bar{\psi}^\mu_{(i}\gamma_\mu\Omega_{j)}\right) \,, \\
\widehat{\Omega}^i &= \sigma^{-3/4}\Omega^i \,, \\
\widehat{\epsilon}^i &= \sigma^{1/4}\epsilon^i \,.
\end{aligned} \tag{4.19}$$

Next, we impose the gauge conditions listed in (2.3) and add the result to (2.18) to obtain the full $R + R^2$ theory in this gauge. This straightforward computation will not be carried out here since we shall be working in the gauge (3.1) which leads to the result (4.13) for the total Lagrangian.

5 Vacuum solutions

The purpose of this section is to investigate the different supersymmetric and non-supersymmetric vacuum solutions of the R^2-extended Salam-Sezgin model discussed in the previous section. In the first subsection we present the bosonic field equations of this model. In the following three subsections we investigate vacuum solutions with no fluxes, 2-form fluxes and 3-form fluxes, respectively. In the last subsection we compute the spectrum of the theory around six dimensional Minkowski spacetime.

5.1 Bosonic field equations

For the purpose of finding the vacuum solutions, it is convenient to eliminate the auxiliary fields as much as possible. Prior to adding the Riemann tensor squared invariant, we saw that the auxiliary fields $(E_{\mu\nu\rho\sigma}, \mathcal{V}'^{ij}_\mu, \mathcal{V}_\mu, Y^{ij})$ can all be eliminated by using their field equations. However, upon the addition of the Riemann tensor squared invariant, while we can still eliminate $(Y^{ij}, E_{\mu\nu\rho\sigma})$, we can no longer eliminate $(\mathcal{V}'^{ij}_\mu, \mathcal{V}_\mu)$ since they acquire kinetic terms. Thus, we shall proceed with the elimination of $(Y^{ij}, E_{\mu\nu\rho\sigma})$ only. The relation

$$Y^{ij} = -\frac{1}{2\sqrt{2}} g L \delta^{ij}\,, \tag{5.1}$$

readily follows from (3.5), while the $E_{\mu\nu\rho\sigma}$ field equation gives

$$\varepsilon^{\lambda\tau\rho\sigma\mu\nu}\partial_\mu\left(L^{-1}E_\nu - \sqrt{2}\mathcal{V}_\nu - gW_\nu\right) = 0\ . \tag{5.2}$$

This implies that we can locally write

$$L^{-1}E_\mu - \sqrt{2}\mathcal{V}_\mu - gW_\mu = \partial_\mu\phi\,, \tag{5.3}$$

for some scalar ϕ, which inherits the shift gauge symmetry transformations (2.26). This symmetry is readily fixed by setting ϕ equal to a constant, thereby arriving at the field equation

$$E_\mu = \sqrt{2}L\left(\mathcal{V}_\mu + \frac{1}{\sqrt{2}}gW_\mu\right)\ . \tag{5.4}$$

Taking into account (5.1) and (5.4), we find the following bosonic field equations for the propagating fields in the theory (4.14):

$$\begin{aligned}
LR_{\mu\nu} &= \nabla_\mu\nabla_\nu L - L^{-1}\partial_\mu L\partial_\nu L + \frac{1}{4}g^2 g_{\mu\nu}L^2 + \frac{1}{4}LF_{\mu\rho\sigma}(B)F_\nu{}^{\rho\sigma}(B)\\
&\quad -4LZ_{(\mu}Z^*_{\nu)} - \frac{1}{2}L^{-1}E_\mu E_\nu + F_{\mu\rho}(W)F_\nu{}^\rho(W)\\
&\quad -\frac{1}{4}g_{\mu\nu}F_{\rho\sigma}(W)F^{\rho\sigma}(W) - \frac{1}{8M^2}S_{\mu\nu}\,, && (5.5)\\
R &= g^2L + 2L^{-1}\Box L - L^{-2}\partial_\mu L\partial^\mu L + \frac{1}{12}F_{\mu\nu\rho}(B)F^{\mu\nu\rho}(B)\\
&\quad -4Z_\mu Z^{*\mu} - \frac{1}{2}L^{-2}E_\mu E^\mu\,, && (5.6)\\
\nabla_\rho\left(LF^{\rho\mu\nu}(B)\right) &= \frac{1}{4}e^{-1}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}\left(F_{\rho\sigma}(W)F_{\lambda\tau}(W) + \frac{1}{2M^2}\widetilde{R}^{\alpha\beta}{}_{\rho\sigma}\widetilde{R}_{\alpha\beta\lambda\tau}\right)\\
&\quad +\frac{3}{M^2}\nabla_\alpha\widetilde{\nabla}_\beta\widetilde{R}^{[\mu\nu\alpha]\beta} + \frac{3}{M^2}\nabla_\alpha\left(F^{-\,\rho\sigma[\alpha}(B)\widetilde{R}^{\mu\nu]}{}_{\rho\sigma}\right)\,, && (5.7)\\
0 &= \nabla_\mu F^{\mu\nu}(W) + \frac{1}{2}gE^\nu + \frac{1}{2}\widetilde{F}^{\nu\rho\sigma}(B)F_{\rho\sigma}(W)\,, && (5.8)\\
0 &= \nabla_\nu F^{\mu\nu}(\mathcal{V}) + [2\mathrm{i}F^{\mu\nu}(Z)Z^*_\nu + h.c.] + \frac{1}{\sqrt{2}}M^2E^\mu\,, && (5.9)\\
0 &= (\partial_\mu - \mathrm{i}\mathcal{V}_\mu)F^{\mu\nu}(Z) - \mathrm{i}F^{\nu\rho}(\mathcal{V})Z_\rho - M^2LZ^\nu\,, && (5.10)
\end{aligned}$$

where E^μ is the U(1) invariant vector field determined in terms of the vector fields W_μ and $\mathcal{V}_\mu$ as in (5.4). The fact that E^μ is divergence free follows from (5.8), and separately from (5.9). We have also defined

$$\begin{aligned}
S_{\mu\nu} \equiv{} & 8F_{\mu\rho}(\mathcal{V})F_\nu{}^\rho(\mathcal{V}) - 2g_{\mu\nu}F_{\rho\sigma}(\mathcal{V})F^{\rho\sigma}(\mathcal{V}) - 32F_{\rho(\mu}(Z)F^*{}_{\nu)}{}^\rho(Z) - 8g_{\mu\nu}F_{\rho\sigma}(Z)F^{*\rho\sigma}(Z)\\
& -4\widetilde{R}^{\lambda\tau}{}_{\mu\rho}\widetilde{R}_{\lambda\tau\nu}{}^\rho + g_{\mu\nu}\widetilde{R}_{\lambda\tau\rho\sigma}\widetilde{R}^{\lambda\tau\rho\sigma} + 8\nabla^\alpha\widetilde{\nabla}^\beta\widetilde{R}_{\alpha(\mu\nu)\beta} + 8\nabla^\alpha\left(\widetilde{R}_{\alpha(\mu}{}^{\rho\sigma}F^-_{\nu)\rho\sigma}(B)\right)\\
& +4F^\alpha{}_{\lambda(\mu}(B)\widetilde{\nabla}^\beta\widetilde{R}^\lambda{}_{\nu)\alpha\beta} - 4\widetilde{R}_{\lambda(\mu}{}^{\alpha\beta}F_{\nu)}{}^{\lambda\tau}(B)F^-_{\tau\alpha\beta}(B)\,, \qquad (5.11)
\end{aligned}$$

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where $F^{\pm}(B) = (F(B) \pm \tilde{F}(B))/2$ with $\tilde{F}^{\mu\nu\rho} = -\frac{1}{6}e^{-1}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}F_{\sigma\lambda\tau}$. We have simplified the Einstein equation by using (5.4) and the L field equation (5.6). We have also used the definitions

$$\widetilde{R}^{\alpha}{}_{\beta\mu\nu} = \partial_\mu \widetilde{\Gamma}^{\alpha}_{\nu\beta} + \cdots \,, \qquad \widetilde{\Gamma}^{\rho}{}_{\mu\nu} \equiv \Gamma^{\rho}{}_{+\mu\nu} = \Gamma^{\rho}{}_{\mu\nu} + \frac{1}{2}F^{\rho}{}_{\mu\nu}(B)\,. \tag{5.12}$$

Thus, we have

$$\widetilde{R}^{\alpha\beta}{}_{\mu\nu} = R^{\alpha\beta}{}_{\mu\nu} - \nabla_{[\mu}F_{\nu]}{}^{\alpha\beta}(B) - \frac{1}{2}F^{\alpha}{}_{\lambda[\mu}(B)F^{\beta\lambda}{}_{\nu]}(B)\,. \tag{5.13}$$

Given the vielbein postulate

$$\partial_\mu e^a_\nu + \omega_{\mu\pm}{}^{ab}e_{\nu b} - \Gamma^{\rho}_{\mp\mu\nu}\, e^a_\rho = 0 \tag{5.14}$$

with $\omega_{\mu\pm}{}^{ab}$ and $\Gamma^{\rho}{}_{\pm\mu\nu}$ defined in (4.9) and (4.12), respectively, it follows that

$$R_{\mu\nu}{}^{ab}(\omega_-)e^\lambda_a e_{\tau b} = R^{\lambda}{}_{\tau\mu\nu}(\Gamma_+) \equiv \widetilde{R}^{\lambda}{}_{\tau\mu\nu}\,. \tag{5.15}$$

The occurrence of covariant derivatives with and without bosonic torsion in the quantity $S_{\mu\nu}$ is due to the following manipulation:

$$\begin{aligned}
&\delta\int eR_{\mu\nu ab}(\omega_-)R^{\mu\nu ab}(\omega_-) = 4\int R^{\mu\nu}{}_{ab}(\omega_-)D_\mu(\omega_-)\delta\omega_{\nu-}{}^{ab} + \text{a term} \sim \delta(eg^{\mu\rho}g^{\nu\sigma})\\
&= 4\int R^{\mu\nu}{}_{ab}(\omega_-)\left[D_\mu(\omega_-,\Gamma_+)\delta\omega_{\nu-}{}^{ab} + \frac{1}{2}F_{\mu\nu}{}^{\rho}(B)\delta\omega_{\rho-}{}^{ab}\right] + \text{a term} \sim \delta(eg^{\mu\rho}g^{\nu\sigma})\,.
\end{aligned} \tag{5.16}$$

A partial integration in the first term is then responsible for the occurrence of $\widetilde{\nabla}$ in the expression for $S_{\mu\nu}$. Another useful variational formula takes the form

$$\begin{aligned}
&\delta\int \varepsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}R_{\rho\sigma}{}^{ab}(\omega_-)R_{\lambda\tau ab}(\omega_-)\\
&= \varepsilon^{\mu\nu\rho\sigma\lambda\tau}\left(\int(\delta B_{\mu\nu})R_{\rho\sigma}{}^{ab}(\omega_-)R_{\lambda\tau ab}(\omega_-) + 4B_{\mu\nu}\partial_\rho\left[R_{\lambda\tau ab}(\omega_-)\delta\omega_{\sigma-}{}^{ab}\right]\right)\,.
\end{aligned} \tag{5.17}$$

The field equations for the abelian vector fields W_μ and $\mathcal{V}_\mu$ have an intricate structure. Suitable combinations of these fields describe a gauge field X_μ and a gauge invariant Proca field Y_μ given by

$$X_\mu \equiv \mathcal{V}_\mu + \sqrt{2}g^{-1}M^2\,W_\mu\,, \qquad Y_\mu \equiv \mathcal{V}_\mu + \frac{g}{\sqrt{2}}\,W_\mu\,. \tag{5.18}$$

The field equations (5.8) and (5.9) can then be written as

$$\nabla_\mu X^{\mu\nu} = \frac{M^2}{g^2 - 2M^2}\,\tilde{F}^{\nu\rho\sigma}(B)(X_{\rho\sigma} - Y_{\rho\sigma})\,, \tag{5.19}$$

$$\nabla_\mu Y^{\mu\nu} + \frac{1}{2}(g^2 - 2M^2)LY^\nu = \frac{g^2}{2(g^2 - 2M^2)}\,\tilde{F}^{\nu\rho\sigma}(B)(X_{\rho\sigma} - Y_{\rho\sigma})\,, \tag{5.20}$$

Spacetime	n_1	n_2	n_3
$\mathrm{Mink}_4 \times S^2$	0	1	1
$\mathrm{dS}_4 \times T^2$	1	0	1/6
$\mathrm{dS}_4 \times S^2$	6/7	1/7	1/7
$\mathrm{Mink}_3 \times S^3$	0	1	1/3
$\mathrm{dS}_3 \times T^3$	1	0	1/3
$\mathrm{dS}_3 \times S^3$	1/2	1/2	1/6

Table 2. Solutions of the form $M_1 \times M_2$ in the absence of fluxes. The numbers (n_1, n_2, n_3) are defined in (5.22).

for $2M^2 - g^2 \neq 0$, and $X_{\mu\nu}$, $Y_{\mu\nu}$ given by

$$X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu \,, \qquad Y_{\mu\nu} = \partial_\mu Y_\nu - \partial_\nu Y_\mu \,. \tag{5.21}$$

In the special case that $M^2 = g^2/2$, the left hand side of the field equations (5.19) and (5.20) can no longer be diagonalized. As we saw earlier, this is a critical point at which the coefficient of the kinetic term for the Maxwell vector field vanishes to lowest order in $1/M^2$ when the auxiliary vector field $\mathcal{V}_\mu$ is eliminated to the same order.

5.2 Vacuum solutions without fluxes

If $g \neq 0$, the field equations do not admit a single constant curvature $6D$ spacetime solution for any value of the constant curvature, with or without supersymmetry. In particular, Minkowski spacetime is not a solution as can be readily seen from the equation $R = g^2 L_0$, where $L = L_0$ is a non-vanishing constant and all other fields are set equal to zero. If $g^2 = 0$, on the other hand, setting L equal to a constant and all the other fields equal to zero yields $\mathrm{Minkowski}_6$ as a supersymmetric solution.

Next, let the six dimensional spacetime be a direct product of constant curvature spaces $M_1 \times M_2$, with dimensions d_1 and d_2. We find that solutions exist with

$$\begin{aligned}
&R_{\mu\nu\rho\sigma} = \frac{n_1}{d_1(d_1-1)} g^2 L_0 \left(g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}\right), \quad R_{pqrs} = \frac{n_2}{d_2(d_2-1)} g^2 L_0 (g_{pr} g_{qs} - g_{ps} g_{qr}) \,, \\
&L = L_0 \,, \quad M^2 = \frac{1}{2} n_3 g^2 \,,
\end{aligned} \tag{5.22}$$

with all the other fields vanishing. Here L_0 is an arbitrary non-vanishing positive constant, and the numbers (n_1, n_2, n_3) are given in table 2. Note that here we are using the coordinates (x^μ, y^r).

There are also solutions involving a product of three 2-dimensional constant curvature spaces, whose curvature constants, allowed to vanish as well, are chosen properly. In all these solutions, and those tabulated above, M^2 is fixed in terms of g^2, and all solutions are non-supersymmetric.

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5.3 Vacuum solutions with 2-form flux

Next, let us consider a spacetime $M_4 \times M_2$, which is a direct product of two constant curvature spaces and turn on the fluxes produced by $F(W)$ and $F(\mathcal{V})$ on M_2. We set L equal to a positive non-vanishing constant and the remaining fields equal to zero. In particular, from (5.4) it follows that $\mathcal{V}_\mu = -gW_\mu/\sqrt{2}$. Using this information, we can make the following Ansatz for the non-vanishing fields:

$$\begin{aligned} R_{\mu\nu} &= 3a\, g_{\mu\nu}\,, & R_{rs} &= b\, g_{rs}\,, & L &= L_0\,, \\ F_{rs}(W) &= c\sqrt{g_2}\,\varepsilon_{rs}\,, & F_{rs}(\mathcal{V}) &= -\frac{g}{\sqrt{2}}\, c\sqrt{g_2}\,\varepsilon_{rs}\,, \end{aligned} \tag{5.23}$$

where a, b, c, L_0 are constants, $g_2 = \det g_{rs}$, we have used the coordinates (x^μ, y^r) and $\varepsilon_{12} = \varepsilon^{12} = 1$. Using this ansatz we find the following solutions. One of them is a direct product of $4D$ Minkowski spacetime with a 2-sphere, given by

$$\mathrm{Mink}_4 \times S^2 \;\; : \qquad a = 0\,, \qquad b = \frac{1}{2} g^2 L_0\,, \qquad c = \pm\frac{gL_0}{\sqrt{2}}\,. \tag{5.24}$$

Remarkably, this is precisely the supersymmetric Salam-Sezgin solution for any value of M^2! For this solution, the integrability condition for the Killing spinor equation $\delta_\epsilon \psi_{\hat\mu} = 0$ is

$$\left[R_{\hat\mu\hat\nu\hat a\hat b}\Gamma^{\hat a\hat b}\varepsilon^{ij} - 2F_{\hat\mu\hat\nu}(\mathcal{V})\delta^{ij}\right]\epsilon_j = 0\,, \tag{5.25}$$

where $\hat\mu, \hat a = 0, 1, \ldots, 5$. For the solution (5.24) this gives[11]

$$\mathrm{i}\,(\sigma_3)_A{}^B\, \delta_{ik}\varepsilon^{kj}\,\epsilon_{Bj} = \mp\epsilon_{Ai}\,. \tag{5.26}$$

The vanishing of $\delta_\epsilon\varphi^i$ follows trivially, and, using (5.24) and (5.26), it follows that $\delta_\epsilon\Omega^i = 0$ as well. So the only independent condition on the Killing spinor is given by (5.26). It implies $\mathcal{N} = 1$ supersymmetry in Minkowski$_4$. Indeed, using the Majorana spinors η_1 and η_2 defined in footnote 11, the condition (5.26) turns into $\mathrm{i}\gamma_*\eta_1 = \pm\eta_2$.

The other solutions are given by

$$\begin{aligned} a &= M^2 L_0\,, \qquad b = \frac{1}{2}(g^2 - 12M^2)L_0\,, \\ c &= \pm\, L_0\sqrt{\frac{(g^2-12M^2)(g^2-14M^2)}{2(g^2-2M^2)}}\,, \qquad M^2 \neq \frac{1}{2}g^2\,, \end{aligned} \tag{5.27}$$

and therefore they describe the following spaces:

$$AdS_4 \times S^2 \;\; : \qquad M^2 < 0\,, \tag{5.28}$$

$$dS_4 \times S^2 \;\; : \qquad \frac{1}{14}g^2 > M^2 > 0\,, \tag{5.29}$$

$$dS_4 \times H^2 \;\; : \qquad \frac{1}{2}g^2 > M^2 > \frac{1}{12}g^2\,, \tag{5.30}$$

[11] We decompose the $6D$ Dirac matrices as $\Gamma_\mu = \gamma_\mu \otimes 1, \Gamma_4 = \gamma_* \otimes \sigma_1$ and $\Gamma_5 = \gamma_* \otimes \sigma_2$. Then $\Gamma_* = \gamma_* \otimes \sigma_3$. This defines 4-dimensional spinors $\epsilon_{Ai} = \gamma_*(\sigma_3)_A{}^B\epsilon_{Bi}$, where the 4-dimensional spinor index is suppressed and $A, B = 1, 2$ labels the 2-dimensional spinors on S^2. The combinations $\eta_1 = \epsilon_{11} + \mathrm{i}\epsilon_{22}$ and $\eta_2 = \epsilon_{12} - \mathrm{i}\epsilon_{21}$ are 4-dimensional Majorana spinors.

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where H^2 is a 2-hyperboloid. For the special case of $M^2 = g^2/2$, there exists the following solution

$$\mathrm{Mink}_4 \times S^2 \ : \qquad a = 0\,, \qquad b = \frac{1}{2} g^2 L_0\,, \qquad M^2 = \frac{1}{2} g^2\,, \tag{5.31}$$

for *any value of* c, which contains as a special case the solution (5.24) for $c = \pm g L_0/\sqrt{2}$, and the first entry in table 2 for $c = 0$. Of all the solutions with the 2-flux turned on, the only supersymmetric one is the one given in (5.24).

5.4 Vacuum solutions with 3-form flux

We shall take the $6D$ spacetime to be a direct product of two three-dimensional constant curvature spaces $M_1 \times M_2$ with coordinates (x^μ, y^r), set $L = L_0$, turn on the 3-form flux and set the remaining fields equal to zero. Thus we have

$$\begin{aligned} R_{\mu\nu}{}^{\rho\sigma} &= 2a\, \delta^\rho_{[\mu} \delta^\sigma_{\nu]}, & R_{pq}{}^{rs} &= 2b\, \delta^r_{[p} \delta^s_{q]}, \qquad L = L_0\,, \\ F_{\mu\nu\rho}(B) &= 2c_1 \sqrt{-g_1} \varepsilon_{\mu\nu\rho}\,, & F_{rst}(B) &= 2c_2 \sqrt{g_2} \varepsilon_{rst}\,, \end{aligned} \tag{5.32}$$

where $g_1 = \det g_{\mu\nu}$ and $g_2 = \det g_{rs}$. From (5.13) we get

$$\widetilde{R}_{\mu\nu}{}^{\rho\sigma} = 2(a + c_1^2)\, \delta^\rho_{[\mu} \delta^\sigma_{\nu]}\,, \qquad \widetilde{R}_{pq}{}^{rs} = 2(b - c_2^2)\, \delta^r_{[p} \delta^s_{q]}\ . \tag{5.33}$$

If we set $g^2 = 0$, then all the terms that depend on M^2 vanish since the curvatures defined above vanish due to the non-vanishing (parallelizing) torsion. This gives the known $AdS_3 \times S^3$ solution

$$AdS_3 \times S^3 \ : \qquad c_1^2 = c_2^2 = -a = b\ . \tag{5.34}$$

This solution preserves full supersymmetry. Indeed the integrability condition for the existence of Killing spinors requires that the torsionful curvatures vanish, and this is the case with the 3-form fluxes as given in (5.34). As a consequence, all the contributions of the Riemann tensor squared invariant to the field equations vanish in this case.

Next, we seek solutions with $g^2 \neq 0$ and nonvanishing 3-form flux. To bring the field equations to a manageable form, we shall supplement the Ansatz (5.32) with a further condition and introduce some notation

$$c_1 = -c_2 \equiv c\ . \tag{5.35}$$

Finding the most general such solution yields rather complicated relations among the parameters. However, we have managed to find the following relatively simple and intriguing solutions:

$$a = \frac{1}{6}(-6c^2 + g^2 L_0)\,, \quad b = c^2\,, \quad M^2 = \frac{g^2}{6}\,, \tag{5.36}$$

for arbitrary $c^2 > 0$. This solution corresponds to $dS_3 \times S^3$ for $0 < c^2 < \frac{g^2 L_0}{6}$ and to $AdS_3 \times S^3$ for $c^2 > \frac{g^2 L_0}{6}$. Another solution is given by

$$\begin{aligned} a_\pm &= \frac{1}{24}\Big(5g^2 L_0 - 24 L_0 M^2 \mp \sqrt{3}\sqrt{L_0^2(g^4 - 12 g^2 M^2 + 48 M^4)}\Big)\,, \\ b_\pm &= \frac{1}{24}\Big(-g^2 L_0 + 24 L_0 M^2 \pm \sqrt{3}\sqrt{L_0^2(g^4 - 12 g^2 M^2 + 48 M^4)}\Big)\,, \\ c_\pm^2 &= \frac{1}{24}\Big(g^2 L_0 \mp \sqrt{3}\sqrt{L_0^2(g^4 - 12 g^2 M^2 + 48 M^4)}\Big)\,, \end{aligned} \tag{5.37}$$

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where the + solution corresponds to $dS_3 \times S^3$ for $\frac{g^2}{12} < M^2 < \frac{g^2}{6}$ and the − solution corresponds to $AdS_3 \times S^3$ for $M^2 > \frac{11g^2}{36}$ and to $dS_3 \times H^3$ for $M^2 < \frac{g^2}{12}$. These solutions are non-supersymmetric.

5.5 Spectrum in Minkowski spacetime

Setting $g^2 = 0$, and expanding around $6D$ Minkowski spacetime, we define the linearized fluctuations

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\,, \qquad L = L_0 + \phi\,. \tag{5.38}$$

Since all the other background fields are vanishing, we find that the linearized Einstein and L field equations take the form

$$\begin{aligned} 0 &= L_0\left(\Box h_{\mu\nu} + \partial_\mu\partial_\nu h - 2\partial_{(\mu}\partial^\alpha h_{\nu)\alpha}\right) + 2\partial_\mu\partial_\nu\phi \\ &\quad -\frac{1}{M^2}\left(\Box\Box h_{\mu\nu} - 2\Box\partial_{(\mu}\partial^\alpha h_{\nu)\alpha} + \partial_\mu\partial_\nu\partial^\alpha\partial^\beta h_{\alpha\beta}\right)\,, \end{aligned} \tag{5.39}$$

$$0 = L_0\left(\Box h - \partial^\mu\partial^\nu h_{\mu\nu}\right) + 2\Box\phi\,. \tag{5.40}$$

Note that we have not imposed any gauge conditions yet. Using (5.40) in the trace of (5.39), we find

$$\Box\left(\Box - M^2 L_0\right)\phi = 0\,. \tag{5.41}$$

To simplify Einstein's equation, however, it is convenient to impose the gauge condition

$$\partial^\mu h_{\mu\nu} = \frac{1}{2}\partial_\nu h\,. \tag{5.42}$$

In this gauge, the trace of Einstein's equation and (5.40) give

$$\Box\left(\Box - M^2 L_0\right) h = 0\,, \tag{5.43}$$

$$\Box h = -4L_0^{-1}\Box\phi\,. \tag{5.44}$$

We shall assume that $M^2 \neq 0$. Then it follows from (5.41) that either $(\Box - M^2 L_0)\phi = 0$ or $\Box\phi = 0$. In the first case, defining $\chi \equiv \Box\phi$, it follows from (5.41), (5.43) and (5.44) that there is one massive scalar obeying $(\Box - M^2 L_0)\chi = 0$. In the latter case, $\Box\phi = 0$ and it follows from (5.44) that $\Box h = 0$ as well. However, the solution of $\Box h = 0$ can be gauged away by the residual general coordinate transformations that preserve the gauge condition (5.42). Thus, we are left with a massless scalar described by $\Box\phi = 0$.

Turning to Einstein's equation, using the gauge condition (5.42), and the field equations obeyed by h and ϕ, it becomes

$$\left(\Box - M^2 L_0\right)\Box h_{\mu\nu} = -2L_0^{-1}\left(\Box - M^2 L_0\right)\partial_\mu\partial_\nu\phi\,. \tag{5.45}$$

This equation, when $(\Box - M^2 L_0)\phi = 0$, reduces to $\left(\Box - M^2 L_0\right)\Box h_{\mu\nu} = 0$, describing a massless graviton and a massive graviton with mass $M\sqrt{L_0}$, one of which, depending on the overall sign in the action, has the wrong sign kinetic term. If $\Box\phi = 0$, then we have $\left(\Box - M^2 L_0\right)\Box h_{\mu\nu} = -2M^2\partial_\mu\partial_\nu\phi$. In this case, the solution of $\Box\phi = 0$ is to be substituted to the right hand side of this equation and treated as a given external source. Note that the gravitational field does not appear as a source in the field equation for the scalar ϕ, and there is no diagonalization problem here. Thus, the equation (5.45) again describes a massless and ghost massive graviton of mass $M\sqrt{L_0}$.

The remaining field equations in the usual Lorentz gauges take the form

$$\Box a_\mu = 0\,, \qquad (\Box - M^2 L_0)\begin{pmatrix} v_\mu \\ z_\mu \end{pmatrix} = 0\,, \qquad \Box(\Box - M^2 L_0) b_{\mu\nu} = 0\,, \tag{5.46}$$

where the notation for the fluctuations is self explanatory. These equations describe a massless vector and 2-form potential, a massive ghostly 2-form potential and three massive ghostly vectors.

Next, we examine the linearized fermion field equations. Imposing the gauge condition $\gamma^\mu \psi_\mu = 0$, and defining[12] $\psi^i \equiv \partial^\mu \psi^i_\mu$, a straightforward manipulation of the fermion field equations gives

$$\partial\!\!\!/(\Box - M^2 L_0)\psi'_\mu = 0\,, \qquad \partial\!\!\!/\Omega = 0\,, \tag{5.47}$$

$$\Box\psi_i = \sqrt{2}M^2 \partial\!\!\!/\varphi^j \delta_{ij}\,, \qquad \psi_i = \sqrt{2}L_0^{-1}\partial\!\!\!/\varphi^j \delta_{ij}\,, \tag{5.48}$$

where $\psi'_\mu \equiv \psi_\mu - \Box^{-1}\partial_\mu\partial_\nu\psi^\nu$, i.e. $\partial^\mu {\psi'}_\mu = 0$. From (5.48), it follows that $\partial\!\!\!/\left(\Box - M^2 L_0\right)\varphi = 0$. Therefore, altogether we have a massless gravitino, tensorino φ and gaugino together with a massive gravitino and tensorino, both with mass $M\sqrt{L_0}$.

In summary, the full spectrum consists of the massless Maxwell multiplet with fields $(a_\mu,\, \Omega)$, the (reducible) massless 16 + 16 supergravity multiplet with fields $(h_{\mu\nu},\, b_{\mu\nu},\, \phi,\, \psi_\mu,\, \varphi)$ and a massive 40 + 40 supergravity multiplet of ghosts with fields $(h_{\mu\nu},\, b_{\mu\nu},\, z_\mu,\, v_\mu,\, \phi,\, \psi_\mu,\, \varphi)$, all with the same mass, $M\sqrt{L_0}$, as expected.

6 Conclusions

Our main goal in this paper has been the study of the R-symmetry gauging in the presence of higher derivative corrections to Poincaré supergravity and its consequences for the vacuum solutions. To this end, we first studied the gauging of the U(1) R-symmetry of $\mathcal{N} = (1,0)$, $D = 6$ supergravity in the off-shell formulation. The off-shell Poincaré supergravity theory already has a local $\mathrm{U}(1)_R$ symmetry but it is gauged by an auxiliary vector field which is not dynamical. We performed the gauging that employs a dynamical gauge field by coupling the model to an off-shell vector multiplet equipped with its own U(1) symmetry. Then, we showed that this model has a shift symmetry which can be fixed, thereby breaking $\mathrm{U}(1)_R \times \mathrm{U}(1)$ down to a diagonal $\mathrm{U}(1)_R^{\rm diag}$. As a result the auxiliary vector gets related to the vector coming from the Maxwell multiplet, and the on-shell model obtained in this manner agrees with the dual formulation [16] of the gauged Einstein-Maxwell supergravity constructed long ago [15, 16].

Next, we added a curvature squared supersymmetric invariant, with the Riemann tensor squared as its leading term, to the off-shell model and studied its influence on the gauging procedure. This invariant causes the auxiliary fields to become 'propagating' and

[12] This ψ^i is unrelated to the ψ^i introduced in (2.1), which was eliminated by (3.1).

to mix with the physical fields. A particular combination of the physical vector and the auxiliary vector gauges the symmetry and another combination describes a massive vector field inert under U(1). We can, however, put a small parameter in front of the curvature squared part of the Lagrangian and consider it as a higher-order correction term. Then the auxiliary fields can be eliminated order by order and the gauging proceeds again via the vector field residing in the Maxwell multiplet. Treating the higher derivative extension either way, we have seen that the positive definite potential that arises in the minimal model does not get modified.

Chiral gauged supergravity in six dimensions is known to admit a (supersymmetric) chiral Minkowski$_4 \times S^2$ compactification, while it does not admit a six-dimensional Minkowski or (anti) de Sitter spacetime as a solution, regardless of supersymmetry [15]. We have shown that the inclusion of the Riemann tensor squared invariant remarkably leaves the supersymmetric Minkowski$_4 \times S^2$ solution intact. We have also found new solutions in which the spacetime and the internal spaces may have positive or negative curvature constants. It is noteworthy that de Sitter spacetime solutions exist, avoiding a no go theorem that exists for ten dimensional supergravities[13] [21, 22]. While the spectrum in the 2-sphere compactification remains to be determined, we have found that the spectrum of the ungauged theory in six dimensional Minkowski spacetime, not surprisingly, has a ghostly massive spin two multiplet in addition to a massless supergravity and a Maxwell vector multiplet.

Given that the $(1,0)$ supergravity theory in six dimensions is the most supersymmetric and highest-dimensional supergravity model that admits an off-shell formulation, and that it admits an exactly supersymmetric higher derivative extension, it is worthwhile to study this model further. The coupling of Yang-Mills and hypermultiplets would be useful. In particular, a possible modification of the quaternionic Kähler geometry, and consequences for the compactification would be interesting to determine. The model without such couplings harbors many anomalies. It is important to study the gravitational, gauge and mixed anomalies in the matter-coupled version of the higher derivative extended theory. The Green-Schwarz anomaly counterterm that involves the gravitational Chern-Simons term arises as part of the Riemann tensor squared invariant. However, the presence of the Riemann tensor squared term raises the question with regard to the presence of ghosts in the spectrum, defined in the presence of a suitable vacuum solution. Indeed, dealing with the ghost problem is of great importance for this model to have applications to model building, and it remains to be investigated. In particular, the consequences of the higher derivative extension for the braneworld scenarios put forward in [23] where 3-branes are inserted at singular points of the 2-dimensional internal space, would be worthwhile to explore.

Various properties of the model we have studied here would naturally be affected by the presence of an additional higher-derivative supersymmetric invariant. In five dimensions, for example, it is known that a Weyl tensor squared invariant exists, in addition to the Riemann tensor squared invariant, which can be obtained from a circle reduction of the one

[13]Note that a possible string theory embedding does not contradict the avoidance of the $10D$ no go theorem since this theorem no longer holds when higher derivative corrections are included.

studied here. However, whether the Weyl tensor squared or another combination of the curvature squared terms can be supersymmetrized in six dimensions is an open problem. If such invariants exist, not only would they be useful in avoiding the ghost problem, they would also play a significant role in a possible embedding of these theories, albeit in the ungauged setting, to the string theory low energy effective action. For a preliminary discussion of this problem, in the context of the Riemann tensor squared model we already have, see [8].

The embedding of the higher-derivative extended model to string theory might also provide new grounds for testing the conjectured connection between microscopic and macroscopic black hole entropy. The use of off-shell supersymmetric Riemann tensor squared extended $\mathcal{N} = 2$, $D = 4$ supergravity in this respect has been illustrated in [24]. The existence of static, rotationally symmetric black hole solutions that are $\mathcal{N} = 2$ supersymmetric and that approach Minkowski spacetime at spatial infinity and Bertotti-Robinson spacetime at the horizon play a significant role in the work of [24]. It is notoriously difficult to find exact black hole solutions of higher-derivative gravities. Black hole solutions of the ungauged $(1, 0)$ $6D$ supergravity have been found in [25] and there exists an exact string solution of the theory we have studied in this paper [26]. Nevertheless, black hole solutions in the presence of gauging and/or a higher-derivative extension remains an open and challenging problem.

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