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Unitarity bounds on extensions of Higgs sector

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Abstract: It is widely believed that extensions of the minimal Higgs sector is one of the promising directions for resolving many puzzles beyond the Standard Model (SM). In this work, we study the unitarity bounds on the models by extending the two-Higgs-doublet model with an additional real or complex Higgs triplet scalar. By noting that the SM gauge symmetries $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ are recovered at high energies, we can classify the two-body scattering states by decomposing the direct product of two scalar multiplets into their direct sum of irreducible representations of electroweak gauge groups. In such state bases, the s-wave amplitudes of two-body scalar scatterings can be written in the form of block-diagonalized scattering matrices. Then the application of the perturbative unitarity conditions on the eigenvalues of scattering matrices leads to the analytic constraints on the model parameters. Finally, we numerically investigate the complex triplet scalar extension of the two-Higgs-doublet model, finding that the perturbative unitarity places useful stringent bounds on the model parameter space.

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Contents

1 Introduction

Despite the overwhelming success of the Standard Model (SM) by discovering the 125 GeV Higgs boson at the Large Hadron Collider (LHC) in 2012 [1, 2], it has been widely believed that new physics is required to explain various phenomena beyond the SM, such as tiny neutrino masses [3], the nature of dark matter [4], and the origin of matter-antimatter asymmetry in the Universe [5]. One of the promising directions for resolving these puzzles is to extend the minimal SM Higgs sector by including additional scalars. Note that the shape of the SM Higgs potential is fully determined by the vacuum expectation value (VEV), v, and the quartic self-coupling, λ_H. However, in the non-minimal extension of the Higgs

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section, there will be inevitable deviations of the Higgs self-couplings with respect to the SM predictions. Therefore, the precise measurement of the Higgs self-couplings can help us to probe the new physics and to understand the electroweak symmetry breaking mechanism. Until now, the determination of the trilinear Higgs coupling has been performed at the LHC Run 2 and will be further searched for at the Run 3, by directly detecting the single and double Higgs boson productions [6] and other indirect probes [7–10]. In addition to the above experimental endeavors, there has already been considerable theoretical explorations in order to constrain the Higgs sector, such as the perturbative unitarity [11–14], vacuum stability and triviality [15, 16].

In this work, we shall focus on the systematic derivation of the perturbative unitarity bounds on the non-minimal Higgs sector with two or three Higgs multiplets. As early as 1977, Lee, Quigg, and Thacker [13, 14] made use of the perturbative unitarity and found the Higgs boson mass upper bound $m_h < 870\,\mathrm{GeV}$ in the minimal SM. The perturbative unitarity has recently been calculated in various extensions of the Higgs sector and been identified as a significant constraint on the new physics. One of the most popular extensions is the two-Higgs-doublet model (2HDM) (see refs. [17, 18] for recent reviews) whose perturbative unitarity was firstly calculated in refs. [19–23] with the assumptions of softly broken Z_2 symmetry and CP-conservation. The perturbative unitarity for the most general 2HDM was given in ref. [24] and the associated numerical investigationwas carried out in detail in ref. [25]. For other Beyond-SM theories, the unitarity bounds have been explored for the Georgi-Machacek model [26] in ref. [27], for the Type-II seesaw model [28–33] in ref. [34], for extended scalar sector with a real triplet scalar in ref. [35], for a complex triplet extension of the 2HDM with CP conservation and a softly broken Z_2 symmetry in ref. [36], and for the extension of SM with color-octet scalars in ref. [37], respectively. Some other applications of the unitarity bounds on new physics have been studied in refs. [38, 39].

In this paper, we systematically study the unitarity bounds in extensions of the 2HDM by including an additional real Higgs triplet Σ with hypercharge $Y = 0$ or a complex Higgs triplet scalar Δ with $Y = 2$ in the most general setup, in order to ensure the validity of perturbation theory. Here we only concentrate on the high-energy limit where the SM gauge symmetry effects can be ignored. Thus, we can classify the two-scalar-particle states according to their conserved isospin and hypercharge quantum numbers, and construct the associated 2-to-2 scattering amplitude matrices in terms of the bases of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ irreducible representations. Then we will consider the unitarity bounds in a few special cases, including the extensions of the SM or 2HDM by one additional real or complex scalar triplet, with or without a softly broken Z_2 symmetry. Finally, we will numerically apply our derived perturbative unitarity bounds to the complex triplet extension of the 2HDM, and show the corresponding constraints on model parameter spaces.

This work is organized as follows. In section 2, we provide the two-particle state eigenbasis according to the irreducible representations of given hypercharges and isospins. In sections 3 and 4, we present the scattering amplitude matrices in extensions of the 2HDM by including an additional real triplet and complex triplet, respectively. In section 5, we show the constraints of unitarity bounds on model parameters in the extension of 2HDM with a complex triplet scalar. The conclusions are summarized in section 6. In appendix A,

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we provide the analytical solutions for a dimensional-five scattering matrix appearing in the real or complex triplet augment of the 2HDM with a softly broken Z_2 symmetry. The analytic relations between parameters in the generic scalar basis and the Higgs basis in the complex triplet extension of 2HDM are provided in appendix B. In appendix C, we provide elements of the scalar mass matrices in this model. Finally, the trilinear couplings of a neutral scalar with two charged Higgs particles are summarized in appendix D.

2 Two-particle eigenstates and unitarity bounds on scattering matrices

The calculation of the unitarity bounds in the minimal SM was firstly investigated in refs. [13, 14] and has been applied to various extensions of the SM. It requires that the eigenvalues of this scattering matrix should be less than the unitarity limit [11, 12, 24], otherwise the perturbative calculation of scattering amplitudes at tree level is no more reliable. From another perspective, one can make a partial wave expansion of the scattering amplitudes for the interaction channels and put the unitarity bounds on the partial wave amplitudes. Concretely, the cross section of scalar scattering processes $s_1 s_2 \to s_3 s_4$ can be expressed in terms of the partial wave decomposition as

$$\sigma = \frac{16\pi}{s} \sum_{l=1}^{\infty} (2l+1)|a_l(s)|^2, \tag{2.1}$$

where s is the Mandelstam variable and a_l is the partial wave coefficients with the specific angular momenta l. Together with the optical theorem one finds the following bound of unitarity:

$$|\text{Re}(a_l)| < \frac{1}{2}, \quad \text{for all } l. \tag{2.2}$$

In the high energy limit, it is found that the s-wave amplitude $a_0(s)$ is dominated by the point vertex processes since the s-, t-, u-channel processes are suppressed by the scattering energy. Furthermore, the equivalence theorem [40–44] declares that at very high energy, the amplitudes of scattering processes involving longitudinal gauge bosons in the initial and final states are equivalent to those in which gauge bosons are replaced by the corresponding Nambu-Goldstone bosons. Thus, in the high energy limit $a_0(s)$ is fully determined by the quartic couplings of the scalar potential.

Using the equivalence theorem, we can write down the two-particle state bases in terms of the components of the Higgs multiplets. Once given the scalar potential, we can determine the amplitudes for the $2 \to 2$ scattering processes with the bases. This largely simplifies the calculations for scattering amplitudes. In refs. [23–25, 27], the bases are further classified according to their electroweak (EW) charges, i.e., total hypercharge Y and total isospin I, since the EW $\text{SU}(2)_L \times \text{U}(1)_Y$ gauge symmetries are recovered at high energies so that their associated quantum numbers becomes conserved again. In this approach, we decompose the direct product of two Higgs multiplets into the direct sums of irreducible representations under EW gauge symmetries.

In this work, we adopt an intermediate route for the classification of the bases. Firstly, we classify the direct product of the two Higgs multiplets according to their total hypercharge,

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Field	Φ	$\tilde{\Phi}$	Σ	Δ	$\tilde{\Delta}$
$\mathrm{SU}(2)_L$ isospin	2	2	3	3	3
Hypercharge	1	-1	0	2	-2

Table 1. A summary of the quantum numbers of the Higgs multiplets. Φ, Σ, and Δ denotes the $\mathrm{SU}(2)_L$ doublet, real triplet, and complex triplet, respectively. We define $\tilde{\Phi} = i\tau_2\Phi^*$ and $\tilde{\Delta}_{ab} = (i\tau_2)_{ac}(i\tau_2)_{bd}(\Delta^\dagger)^{cd}$, which have negative hypercharge.

	$I=0$	$I=1$
$Y=0$	$\frac{1}{\sqrt{2}}\left(w_i^+ w_j^- + H_i^0 H_j^{0*}\right)$	$w_i^+ H_j^{0*}$ $\frac{1}{\sqrt{2}}\left(-w_i^+ w_j^- + H_i^0 H_j^{0*}\right)$ $-H_i^0 w_j^-$
$Y=2$	$\frac{1}{\sqrt{2}}\left(-w_i^+ H_j^0 + H_i^0 w_j^+\right)$	$w_i^+ w_j^+ \ \left(\times\frac{1}{\sqrt{2}} \text{ for } i=j\right)$ $\frac{1}{\sqrt{2}}\left(w_i^+ H_j^0 + H_i^0 w_j^+\right)$ $H_i^0 H_j^0 \ \left(\times\frac{1}{\sqrt{2}} \text{ for } i=j\right)$

Table 2. The bases of the irreducible representation for the two Higgs doublets direct product. The bases in the first and second row are corresponding to the direct product $\Phi_i \times \tilde{\Phi}_j$ ($Y=0$) and $\Phi_i \times \Phi_j$ ($Y=2$), respectively. Note that i and j indicate the Higgs doublet. We observe that the bases with ($Y=2,\ I=2$) vanish when the two Higgs doublets are identical, i.e., $i=j$.

where the isospins and hypercharges of the Higgs multiplets considered in the present work are summarized in table 1. Then we decompose the direct product into direct sums of the irreducible representations of the EW $\mathrm{SU}(2)_L$ symmetry. There are three types of direct products of Higgs multiplets we are concerned about, which are given as follows:

$$2\otimes 2 = 1\oplus 3,\ \ 2\otimes 3 = 2\oplus 4,\ \text{ and } 3\otimes 3 = 1\oplus 3\oplus 5. \tag{2.3}$$

In this way, we classify the two-particle bases according to their total isospins and hypercharges of the two Higgs multiplets. Furthermore, we express the bases of the irreducible representation in terms of components in the multiplets. The results are summarized in tables 2–7, in which the eigenstates are rescaled so that they are normalized. Moreover, due to the symmetry property when exchanging two identical bosons, some representations of the two-particle eigenstates vanish, e.g., the $(Y,I)=(2,0)$ state in the $\Phi_i\times\Phi_j$ when $i=j$ in table 2, the $(Y,I)=(0,1)$ state from $\Sigma\times\Sigma$ in table 5, and the $(Y,I)=(4,0)$ state from $\Delta\times\Delta$ in table 6.

Based on the above two-particle basis, we can determine the $2\to 2$ scattering amplitudes as follows [24]

$$S_{(Y,I)} = \left\langle (\phi\phi)^f_{Y,I}|\hat{S}|(\phi\phi)^i_{Y,I}\right\rangle, \tag{2.4}$$

in the sector with definite EW charges (Y,I) with Y and I as the total hypercharge and isospin, respectively. We do not distinguish states by the third components of isospin I_3,

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	$I=\frac{1}{2}$	$I=\frac{3}{2}$
$Y=1$	$\sqrt{\frac{2}{3}}\left(-\frac{i}{\sqrt{2}}w^+\sigma^0+H^0\sigma^+\right)$ $\sqrt{\frac{2}{3}}\left(-w^+\sigma^-+\frac{i}{\sqrt{2}}H^0\sigma^0\right)$	$w^+\sigma^+$ $\frac{1}{\sqrt{3}}\left(i\sqrt{2}w^+\sigma^0+H^0\sigma^+\right)$ $\frac{1}{\sqrt{3}}\left(w^+\sigma^-+i\sqrt{2}H^0\sigma^0\right)$ $H^0\sigma^-$

Table 3. The bases of the irreducible representation for the direct product of a Higgs doublet and a real Higgs triplet scalar, $\Phi\times\Sigma$.

	$I=\frac{1}{2}$	$I=\frac{3}{2}$
$Y=1$	$\sqrt{\frac{2}{3}}\left(-\frac{1}{\sqrt{2}}H^{0*}\delta^++w^-\delta^{++}\right)$ $\sqrt{\frac{2}{3}}\left(-H^{0*}\delta^0-\frac{1}{\sqrt{2}}w^-\delta^+\right)$	$-H^{0*}\delta^{++}$ $\frac{1}{\sqrt{3}}\left(\sqrt{2}H^{0*}\delta^++w^-\delta^{++}\right)$ $\frac{1}{\sqrt{3}}\left(H^{0*}\delta^0-\sqrt{2}w^-\delta^+\right)$ $-w^-\delta^0$
$Y=3$	$\sqrt{\frac{2}{3}}\left(-\frac{1}{\sqrt{2}}w^+\delta^+-H^0\delta^{++}\right)$ $\sqrt{\frac{2}{3}}\left(-w^+\delta^0+\frac{1}{\sqrt{2}}H^0\delta^+\right)$	$-w^+\delta^{++}$ $\frac{1}{\sqrt{3}}\left(\sqrt{2}w^+\delta^+-H^0\delta^{++}\right)$ $\frac{1}{\sqrt{3}}\left(w^+\delta^0+\sqrt{2}H^0\delta^+\right)$ $H^0\delta^0$

Table 4. The bases of the irreducible representation for the direct product of a Higgs doublet and a complex Higgs triplet scalar. The bases in the first and second row are corresponding to the direct product $\tilde{\Phi}\times\Delta$ ($Y=1$) and $\Phi\times\Delta$ ($Y=3$), respectively.

	$I=0$	$I=1$	$I=2$
$Y=0$	$\sqrt{\frac{2}{3}}\left(\sigma^+\sigma^-+\frac{1}{2}\sigma^0\sigma^0\right)$	0	$\frac{1}{\sqrt{2}}\sigma^+\sigma^+$ $i\sigma^+\sigma^0$ $\frac{1}{\sqrt{3}}\left(\sigma^+\sigma^--\sigma^0\sigma^0\right)$ $i\sigma^-\sigma^0$ $\frac{1}{\sqrt{2}}\sigma^-\sigma^-$

Table 5. The bases of the irreducible representation for the two real Higgs triplets direct product, $\Sigma\times\Sigma$. We only consider the case of the direct product of two identical triplet scalar (i.e., $i=j$). We find the bases with $I=1$ vanish in this case.

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	$I=0$	$I=1$	$I=2$
$Y=0$	$\frac{1}{\sqrt{3}}(\delta^{++}\delta^{--}+\delta^{+}\delta^{-}+\delta^{0}\delta^{0*})$	$\frac{1}{\sqrt{2}}(-\delta^{++}\delta^{-}+\delta^{+}\delta^{0*})$ $-\frac{1}{\sqrt{2}}(\delta^{++}\delta^{--}-\delta^{0}\delta^{0*})$ $-\frac{1}{\sqrt{2}}(-\delta^{+}\delta^{--}+\delta^{-}\delta^{0})$	$-\delta^{++}\delta^{0*}$ $\frac{1}{\sqrt{2}}(\delta^{++}\delta^{-}+\delta^{+}\delta^{0*})$ $\frac{1}{\sqrt{6}}(-2\delta^{+}\delta^{-}+\delta^{++}\delta^{--}+\delta^{0}\delta^{0*})$ $\frac{1}{\sqrt{2}}(-\delta^{+}\delta^{--}-\delta^{0}\delta^{-})$ $-\delta^{--}\delta^{0}$
$Y=4$	$\sqrt{\frac{2}{3}}\left(-\delta^{++}\delta^{0}-\frac{1}{2}\delta^{+}\delta^{+}\right)$	0	$\frac{1}{\sqrt{2}}\delta^{++}\delta^{++}$ $-\delta^{++}\delta^{+}$ $\frac{1}{\sqrt{3}}(\delta^{+}\delta^{+}-\delta^{++}\delta^{0})$ $\delta^{0}\delta^{+}$ $\frac{1}{\sqrt{2}}\delta^{0}\delta^{0}$

Table 6. The bases of the irreducible representation for the direct product of two complex Higgs triplet scalars. The bases in the first and second row are corresponding to the direct product $\Delta \times \tilde{\Delta}$ $(Y=0)$ and $\Delta \times \Delta$ $(Y=4)$, respectively. We observe again that the bases with $(Y=4,\ I=1)$ vanish because the two triplets are identical.

	$I=0$	$I=1$	$I=2$
$Y=2$	$\frac{1}{\sqrt{3}}(\sigma^{+}\delta^{0}-i\sigma^{0}\delta^{+}-\sigma^{-}\delta^{++})$	$-\frac{1}{\sqrt{2}}(\sigma^{+}\delta^{+}+i\sigma^{0}\delta^{++})$ $-\frac{1}{\sqrt{2}}(\sigma^{+}\delta^{0}+\sigma^{-}\delta^{++})$ $-\frac{1}{\sqrt{2}}(i\sigma^{0}\delta^{0}-\sigma^{-}\delta^{+})$	$\sigma^{-}\delta^{++}$ $\frac{1}{\sqrt{2}}(\sigma^{+}\delta^{+}-i\sigma^{0}\delta^{++})$ $\frac{1}{\sqrt{6}}(\sigma^{+}\delta^{0}+i2\sigma^{0}\delta^{+}-\sigma^{-}\delta^{++})$ $\frac{1}{\sqrt{2}}(i\sigma^{0}\delta^{0}+\sigma^{-}\delta^{+})$ $\sigma^{-}\sigma^{0}$

Table 7. The bases of the irreducible representation for the direct product of a real Higgs triplet and a complex Higgs triplet, $\Sigma \times \Delta$.

since the states with same (Y, I) but different I_3 would lead to exactly the same scattering matrix. In the tree-level approximation, the elements of the scattering matrix among scalars are determined by the quartic couplings in the scalar potential. Here we do not decompose a complex scalar field into its real and imaginary parts either in the external state basis or in the scalar potential due to the recovered $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ symmetry at high energies.

As argued before, the unitarity of scattering amplitudes requires that the s-wave amplitude $a_0(s)$ in the partial-wave expansion should fulfill the bound in eq. (2.2). Note that the amplitude of two scalar scatterings is dominated by the s-wave one at the tree level, so that the unitarity bounds can be transformed into the following condition on the eigenvalues $\Lambda_{(Y,I)}$ of the scattering matrices $16\pi S_{(Y,I)}$ as follows

$$|\Lambda_{(Y,I)}| \leq 8\pi\,. \tag{2.5}$$

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3 Two-Higgs-doublet model plus a real triplet

In this section, we will focus on the model that contains two Higgs doublets and a real Higgs triplet scalar. The scattering matrix for the scalar potential is provided in section 3.2. Using these results, we consider the perturbative unitarity constraints on two simplified cases: the model with a softly broken Z_2 symmetry and the ΣSM model [45].

3.1 The scalar potential

The scalar potential for the extension of the 2HDM with an additional real Higgs triplet field Σ is given by

$$V_r = V(\Phi_1, \Phi_2) + V(\Sigma) + V(\Phi_1, \Phi_2, \Sigma), \tag{3.1}$$

where the Higgs doublet and real triplet scalar are

$$\Phi_i = \begin{pmatrix} w_i^+ \\ H_i^0 \end{pmatrix}, \text{ and } \Sigma = \begin{pmatrix} \sigma^0/\sqrt{2} & \sigma^+ \\ \sigma^- & -\sigma^0/\sqrt{2} \end{pmatrix}, \tag{3.2}$$

where we can further expand the $H_i^0 = \frac{1}{\sqrt{2}}(\varphi_i + i z_i)$. The most general renormalizable scalar potential for the 2HDM in the generic basis $\{\Phi_1, \Phi_2\}$ is commonly written as [100]

$$\begin{aligned} V(\Phi_1, \Phi_2) = {} & m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 - \left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{H.c.}\right) + \frac{1}{2}\lambda_1 \left(\Phi_1^\dagger \Phi_1\right)^2 \\ & + \frac{1}{2}\lambda_2 \left(\Phi_2^\dagger \Phi_2\right)^2 + \lambda_3 \Phi_1^\dagger \Phi_1 \Phi_2^\dagger \Phi_2 + \lambda_4 \Phi_1^\dagger \Phi_2 \Phi_2^\dagger \Phi_1 + \left[\frac{1}{2}\lambda_5 \left(\Phi_1^\dagger \Phi_2\right)^2\right. \\ & \left. + \lambda_6 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_1\right) + \lambda_7 \left(\Phi_2^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) + \text{H.c.}\right]. \end{aligned} \tag{3.3}$$

The parameters m_{12}^2, λ_5, λ_6, and λ_7 should be real if we impose the CP conservation on the potential. If the Z_2 symmetry with $\Phi_1 \to \Phi_1$ and $\Phi_2 \to -\Phi_2$ is only softly broken by the term proportional to m_{12}^2, we should require $\lambda_6 = \lambda_7 = 0$. The potential for the real Higgs triplet scalar is given by

$$V(\Sigma) = \frac{1}{2} m_\Sigma^2 \,\text{Tr}\,\Sigma^2 + \frac{1}{4}\lambda_\Sigma \,\text{Tr}\,\Sigma^4 \,, \tag{3.4}$$

while the interactions between the Higgs doublets and the real triplet read as follows

$$\begin{aligned} V(\Phi_1, \Phi_2, \Sigma) = {} & \frac{1}{\sqrt{2}}\left[a_1 \Phi_1^\dagger \Sigma \Phi_1 + a_2 \Phi_2^\dagger \Sigma \Phi_2 + \left(a_{12} \Phi_1^\dagger \Sigma \Phi_2 + \text{H.c.}\right)\right] \\ & + \frac{1}{2}\,\text{Tr}\,\Sigma^2 \left[\lambda_8 \Phi_1^\dagger \Phi_1 + \lambda_9 \Phi_2^\dagger \Phi_2 + \left(\lambda_{10} \Phi_1^\dagger \Phi_2 + \text{H.c.}\right)\right]. \end{aligned} \tag{3.5}$$

For a real triplet, the possible terms $\text{Tr}(\Sigma^4)$ and $\Phi^\dagger \Sigma^2 \Phi$ are not independent since they can be expressed as the combination of $\left[\text{Tr}(\Sigma^2)\right]^2$ and $\text{Tr}(\Sigma^2)\Phi^\dagger\Phi$. Also, the potential cubic terms in the first line of eq. (3.5) break the Z_2^Σ symmetry: $\Sigma \to -\Sigma$, and are negligible for the $2 \to 2$ scalar scattering in the high energy limit. Therefore, these terms play no roles in deriving the perturbative unitarity bounds. Furthermore, λ_{10} can be a complex parameter and should vanish when the Z_2 symmetry involving the two Higgs doublets, softly-broken or not, is imposed.

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3.2 Scattering matrix

Based on the two-particle bases given in tables 2, 3, and 5 classified according to the conserved quantum numbers (Y, I), we can expand the general potential in eq. (3.1) with the scalar components defined in eq. (3.2) and obtain the following scattering matrices of given (Y, I):

$$16\pi S_{(0,0)} = \begin{pmatrix} 3\lambda_1 & 2\lambda_3+\lambda_4 & 3\lambda_6 & 3\lambda_6^* & \sqrt{3}\lambda_8 \\ 2\lambda_3+\lambda_4 & 3\lambda_2 & 3\lambda_7 & 3\lambda_7^* & \sqrt{3}\lambda_9 \\ 3\lambda_6^* & 3\lambda_7^* & \lambda_3+2\lambda_4 & 3\lambda_5^* & \sqrt{3}\lambda_{10}^* \\ 3\lambda_6 & 3\lambda_7 & 3\lambda_5 & \lambda_3+2\lambda_4 & \sqrt{3}\lambda_{10} \\ \sqrt{3}\lambda_8 & \sqrt{3}\lambda_9 & \sqrt{3}\lambda_{10} & \sqrt{3}\lambda_{10}^* & 5\lambda_\Sigma \end{pmatrix} \tag{3.6}$$

$$16\pi S_{(0,1)} = \begin{pmatrix} \lambda_1 & \lambda_4 & \lambda_6 & \lambda_6^* \\ \lambda_4 & \lambda_2 & \lambda_7 & \lambda_7^* \\ \lambda_6^* & \lambda_7^* & \lambda_3 & \lambda_5^* \\ \lambda_6 & \lambda_7 & \lambda_5 & \lambda_3 \end{pmatrix} \tag{3.7}$$

$$16\pi S_{(0,2)} = 2\lambda_\Sigma \tag{3.8}$$

$$16\pi S_{(1,\frac{1}{2})} = 16\pi S_{(1,\frac{3}{2})} = \begin{pmatrix} \lambda_8 & \lambda_{10}^* \\ \lambda_{10} & \lambda_9 \end{pmatrix} \tag{3.9}$$

$$16\pi S_{(2,0)} = \lambda_3 - \lambda_4 \tag{3.10}$$

$$16\pi S_{(2,1)} = \begin{pmatrix} \lambda_1 & \lambda_5^* & \sqrt{2}\lambda_6^* \\ \lambda_5 & \lambda_2 & \sqrt{2}\lambda_7 \\ \sqrt{2}\lambda_6 & \sqrt{2}\lambda_7^* & \lambda_3+\lambda_4 \end{pmatrix} \tag{3.11}$$

Comparing with the 2HDM results in ref. [24], the scattering matrix $16\pi S_{(0,0)}$ now becomes 5-dimensional, since there is an additional state with $(Y, I) = (0, 0)$ composed solely by components in the triplet Σ. Furthermore, the scattering processes in the sectors with $(Y, I) = (0, 2)$, $(1, \frac{1}{2})$, and $(1, \frac{3}{2})$ take place only between two scalar triplets.

3.3 Z_2 symmetry

Now we simplify our discussion by imposing the softly broken Z_2 symmetry with $\Phi_1 \to \Phi_1$ and $\Phi_2 \to -\Phi_2$ on the scalar potential (3.1), so that we have $\lambda_6 = \lambda_7 = \lambda_{10} = 0$ but leaving a nonzero m_{12}^2. Such a model is phenomenologically important because it protects the theory from flavor changing neutral currents at tree level. Using the results given in appendix A, the matrix (3.6) can be block diagonalized into a 2×2 matrix $16\pi S^{(2)}_{(0,0)}$ and a 3×3 one $16\pi S^{(3)}_{(0,0)}$ as follows

$$16\pi S^{(2)}_{(0,0)} = \begin{pmatrix} \lambda_3+2\lambda_4 & 3\lambda_5^* \\ 3\lambda_5 & \lambda_3+2\lambda_4 \end{pmatrix}, \quad 16\pi S^{(3)}_{(0,0)} = \begin{pmatrix} 3\lambda_1 & 2\lambda_3+\lambda_4 & \sqrt{3}\lambda_8 \\ 2\lambda_3+\lambda_4 & 3\lambda_2 & \sqrt{3}\lambda_9 \\ \sqrt{3}\lambda_8 & \sqrt{3}\lambda_9 & 5\lambda_\Sigma \end{pmatrix}. \tag{3.12}$$

The eigenvalues for $16\pi S^{(3)}_{(0,0)}$ can be found numerically or analytically by applying eq. (A.8). Furthermore, the matrix $16\pi S_{(0,1)}$ in eq. (3.7) can also be decomposed into the following

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two matrices,

$$16\pi S^{\rm u}_{(0,1)} = \begin{pmatrix} \lambda_1 & \lambda_4 \\ \lambda_4 & \lambda_1 \end{pmatrix}, \quad 16\pi S^{\rm d}_{(0,1)} = \begin{pmatrix} \lambda_3 & \lambda_5^* \\ \lambda_5 & \lambda_3 \end{pmatrix}. \tag{3.13}$$

Apart from $16\pi S^{(3)}_{(0,0)}$, the eigenvalues for the scattering matrices are summarized as follows:

$$\begin{aligned}
\Lambda^{(2)\pm}_{(0,0)} &= \lambda_3 + 2\lambda_4 \pm 3|\lambda_5|,\\
\Lambda^{\rm u\pm}_{(0,1)} &= \frac{1}{2}\Big(\lambda_1 + \lambda_2 \pm \sqrt{(\lambda_1-\lambda_2)^2 + 4\lambda_4^2}\Big),\\
\Lambda^{\rm d\pm}_{(0,1)} &= \lambda_3 \pm |\lambda_5|,\\
\Lambda_{(2,0)} &= \lambda_3 - \lambda_4, \quad \Lambda_{(2,1)} = \lambda_3 + \lambda_4,\\
\Lambda^{\pm}_{(2,1)} &= \frac{1}{2}\Big(\lambda_1 + \lambda_2 \pm \sqrt{(\lambda_1-\lambda_2)^2 + 4|\lambda_5|^2}\Big),\\
\Lambda_{(0,2)} &= 2\lambda_\Sigma, \quad \Lambda^1_{(1,\frac{1}{2})} = \Lambda^1_{(1,\frac{3}{2})} = \lambda_8, \quad \Lambda^2_{(1,\frac{1}{2})} = \Lambda^2_{(1,\frac{3}{2})} = \lambda_9,
\end{aligned} \tag{3.14}$$

where $\Lambda^{2\pm}_{(0,0)}$, $\Lambda^{\rm u\pm}_{(0,1)}$, and $\Lambda^{\rm d\pm}_{(0,1)}$ are the eigenvalues for $16\pi S^{(2)}_{(0,0)}$, $16\pi S^{\rm u}_{(0,1)}$, and $16\pi S^{\rm d}_{(0,1)}$. By further assuming $\lambda_8 = \lambda_9 = \lambda_\Sigma = 0$ in the matrix $S^{(3)}_{(0,0)}$, we can reproduce the eigenvalues $\Lambda^{\rm even}_{00\pm}$ in eq. (10) of ref. [24]. The last line of eq. (3.14) gives the eigenvalues for the scattering matrices involving only components of the real triplet. Together with numerical eigenvalues of $16\pi S^{(3)}_{(0,0)}$, we have provided all eigenvalues for the model (3.1) with the softly broken Z_2 symmetry.

3.4 The ΣSM model

The ΣSM model is a simple extension of the SM by a real triplet scalar, many aspects of which has been extensively investigated in the literature, such as the dark matter phenomenology [45–47], the LHC searches [48, 49], and the strongly first-order EW phase transition [50, 51]. The potential of ΣSM can be reproduced by setting all the couplings in the 2HDM scalar potential eq. (3.1) involving the second Higgs doublet Φ_2 to vanish, which is given by

$$V_{\Sigma\rm SM} = V(\Phi) + V(\Sigma) + V(\Phi,\Sigma)\,, \tag{3.15}$$

where Φ is the SM Higgs doublet, $V(\Sigma)$ is given in eq. (3.4), and

$$V(\Phi) = \mu^2\Phi^\dagger\Phi + \lambda_\Phi\Big(\Phi^\dagger\Phi\Big)^2\,, \tag{3.16}$$

$$V(\Phi,\Sigma) = \frac{1}{\sqrt{2}}a_1\Phi^\dagger\Sigma\Phi + \frac{\lambda_8}{2}\Big({\rm Tr}\Sigma^2\Big)\Phi^\dagger\Phi\,. \tag{3.17}$$

By using eqs. (3.12) and (3.14), the unitarity bounds on the ΣSM are then found to be

$$\begin{aligned}
&|\lambda_\Phi| \le 4\pi, \ |\lambda_\Sigma| \le 4\pi, \ |\lambda_8| \le 8\pi,\\
&|6\lambda_\Phi + 5\lambda_\Sigma \pm \sqrt{(6\lambda_\Phi - 5\lambda_\Sigma)^2 + 12\lambda_8}| \le 16\pi\,,
\end{aligned} \tag{3.18}$$

which confirm the unitarity bounds provided in ref. [35].

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4 Two-Higgs-doublet model plus a complex triplet

In this section, we will consider the extension of 2HDM by a complex Higgs triplet Δ with $Y = 2$ [52]. By using the state bases provided in section 2, we shall calculate the scattering matrix for the most general case of the model. We shall then impose the perturbative unitarity constraints on the eigenvalues of the scattering matrix for several simplified models, such as the one with a softly broken Z_2 symmetry and the Type-II seesaw model [28–33, 53].

4.1 The general scalar potential

The general scalar potential for the model with two Higgs doublets and a complex Higgs triplet scalar Δ is given by

$$V_c = V(\Phi_1, \Phi_2) + V(\Delta) + V(\Phi_1, \Phi_2, \Delta), \tag{4.1}$$

where the complex Higgs triplet is written as

$$\Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}. \tag{4.2}$$

Note that the neutral component δ^0 is a complex scalar. The 2HDM potential $V(\Phi_1, \Phi_2)$ has been provided in eq. (3.3), while the part related to the self-interactions of the complex Higgs triplet is given by

$$V(\Delta) = m_\Delta^2 \operatorname{Tr} \Delta^\dagger \Delta + \lambda_{\Delta 1} \Big(\operatorname{Tr} \Delta^\dagger \Delta\Big)^2 + \lambda_{\Delta 2} \operatorname{Tr} \Big(\Delta^\dagger \Delta\Big)^2. \tag{4.3}$$

The third part in eq. (4.1) gives the interactions among the Higgs doublets and the triplet [52]

$$\begin{aligned} V(\Phi_1, \Phi_2, \Delta) = & \Big(\mu_1 \Phi_1^T i\tau_2 \Delta^\dagger \Phi_1 + \mu_2 \Phi_2^T i\tau_2 \Delta^\dagger \Phi_2 + \mu_3 \Phi_1^T i\tau_2 \Delta^\dagger \Phi_2 + \text{H.c.}\Big) \\ & + \Big[\lambda_8 \Phi_1^\dagger \Phi_1 + \lambda_9 \Phi_2^\dagger \Phi_2 + \Big(\lambda_{10} \Phi_1^\dagger \Phi_2 + \text{H.c.}\Big)\Big] \operatorname{Tr} \Delta^\dagger \Delta \\ & + \lambda_{11} \Phi_1^\dagger \Delta \Delta^\dagger \Phi_1 + \lambda_{12} \Phi_2^\dagger \Delta \Delta^\dagger \Phi_2 + \Big(\lambda_{13} \Phi_1^\dagger \Delta \Delta^\dagger \Phi_2 + \text{H.c.}\Big). \end{aligned} \tag{4.4}$$

The Z_2^Δ symmetry of the transformation $\Delta \to -\Delta$ is only softly broken by the cubic terms in the first line of eq. (4.4), which are negligible for the $2 \to 2$ scalar scatterings in the high energy limit, so that they cannot be constrained by the unitarity bounds. Note that there are possibly additional cubic interactions like $\Delta^a_b \Delta^b_c \Delta^c_a$. But we ignore them since they do not contribute to the unitarity bounds. The parameters λ_{10} and λ_{13} can be complex, and the associated interactions explicitly break the Z_2 symmetry involved in the two Higgs doublets.

4.2 Scattering matrix

We expand the scalar potential (4.1) in terms of components in the two doublets and the triplet. With the two-particle eigenstates given in tables. 2, 4, and 6, we can determine the

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scattering matrices for different conserved quantum numbers (Y, I), which are summarized as follows:

$$16\pi S_{(0,0)} = \begin{pmatrix} 3\lambda_1 & 2\lambda_3+\lambda_4 & 3\lambda_6 & 3\lambda_6^* & \lambda_a \\ 2\lambda_3+\lambda_4 & 3\lambda_2 & 3\lambda_7 & 3\lambda_7^* & \lambda_b \\ 3\lambda_6^* & 3\lambda_7^* & \lambda_3+2\lambda_4 & 3\lambda_5^* & \lambda_c^* \\ 3\lambda_6 & 3\lambda_7 & 3\lambda_5 & \lambda_3+2\lambda_4 & \lambda_c \\ \lambda_a & \lambda_b & \lambda_c & \lambda_c^* & \lambda_\Delta \end{pmatrix} \text{ with } \begin{cases} \lambda_a = \sqrt{\frac{3}{2}}(2\lambda_8+\lambda_{11}) \\ \lambda_b = \sqrt{\frac{3}{2}}(2\lambda_9+\lambda_{12}) \\ \lambda_c = \sqrt{\frac{3}{2}}(2\lambda_{10}+\lambda_{13}) \\ \lambda_\Delta = 2(4\lambda_{\Delta 1}+3\lambda_{\Delta 2}) \end{cases} \tag{4.5}$$

$$16\pi S_{(0,1)} = \begin{pmatrix} \lambda_1 & \lambda_4 & \lambda_6 & \lambda_6^* & \lambda_{11} \\ \lambda_4 & \lambda_2 & \lambda_7 & \lambda_7^* & \lambda_{12} \\ \lambda_6^* & \lambda_7^* & \lambda_3 & \lambda_5^* & \lambda_{13}^* \\ \lambda_6 & \lambda_7 & \lambda_5 & \lambda_3 & \lambda_{13} \\ \lambda_{11} & \lambda_{12} & \lambda_{13} & \lambda_{13}^* & 2\lambda_{\Delta 1}+4\lambda_{\Delta 2} \end{pmatrix} \tag{4.6}$$

$$16\pi S_{(0,2)} = 2\lambda_{\Delta 1} \tag{4.7}$$

$$16\pi S_{(1,\frac{1}{2})} = \begin{pmatrix} \lambda_8+3\lambda_{11}/2 & \lambda_{10}+3\lambda_{13}/2 \\ \lambda_{10}^*+3\lambda_{13}^*/2 & \lambda_9+3\lambda_{12}/2 \end{pmatrix} \tag{4.8}$$

$$16\pi S_{(1,\frac{3}{2})} = \begin{pmatrix} \lambda_8 & \lambda_{10} \\ \lambda_{10}^* & \lambda_9 \end{pmatrix} \tag{4.9}$$

$$16\pi S_{(2,0)} = \lambda_3 - \lambda_4 \tag{4.10}$$

$$16\pi S_{2,1} = \begin{pmatrix} \lambda_1 & \lambda_5^* & \sqrt{2}\lambda_6 \\ \lambda_5 & \lambda_2 & \sqrt{2}\lambda_7^* \\ \sqrt{2}\lambda_6^* & \sqrt{2}\lambda_7 & \lambda_3+\lambda_4 \end{pmatrix} \tag{4.11}$$

$$16\pi S_{(3,\frac{1}{2})} = \begin{pmatrix} \lambda_8-\lambda_{11}/2 & \lambda_{10}^*-\lambda_{13}^*/2 \\ \lambda_{10}-\lambda_{13}/2 & \lambda_9-\lambda_{12}/2 \end{pmatrix} \tag{4.12}$$

$$16\pi S_{(3,\frac{3}{2})} = \begin{pmatrix} \lambda_8+\lambda_{11} & \lambda_{10}^*+\lambda_{13}^* \\ \lambda_{10}+\lambda_{13} & \lambda_9+\lambda_{12} \end{pmatrix} \tag{4.13}$$

$$16\pi S_{(4,0)} = 2\lambda_{\Delta 1} - \lambda_{\Delta 2} \tag{4.14}$$

$$16\pi S_{(4,2)} = 2(\lambda_{\Delta 1} + \lambda_{\Delta 2}) \tag{4.15}$$

We observe that the scattering matrices $16\pi S_{(0,0)}$ and $16\pi S_{(0,1)}$ are now five-dimensional, which is compared with the four-dimensional matrices in the 2HDM. This is caused by the fact that irreducible representations of the product of two Higgs triplets contain the states with $(Y, I) = (0, 0)$ and $(0, 1)$, which can scatter into two components of Higgs doublets with the same quantum numbers. On the other hand, the scattering processes with $(Y, I) = (0, 2)$, $(1, \frac{1}{2})$, $(1, \frac{3}{2})$, $(3, \frac{1}{2})$, $(3, \frac{3}{2})$, $(4, 0)$, and $(4, 2)$ take place only among the complex Higgs triplets.

4.3 Special case with a softly broken Z_2 symmetry

Now we consider some simplified models in the above complex triplet extension of the 2HDM, which may allow us to obtain the eigenvalues of scattering matrices analytically.

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The first example is to impose a softly broken Z_2 symmetry on the potential of the two Higgs doublets in eq. (4.1) so that we have the condition $\lambda_6 = \lambda_7 = \lambda_{10} = \lambda_{13} = 0$ for the potential. In this case, the 5×5 scattering matrix (4.5) and (4.6) can be decomposed into a 2-dimensional (which is correspondingly denoted as $16\pi S^{(2)}_{(0,0)}$ and $16\pi S^{(2)}_{(0,1)}$) and a 3-dimensional matrices. The corresponding 2-dimensional and 3-dimensional matrices are given by

$$16\pi S^{(2)}_{(0,0)} = \begin{pmatrix} \lambda_3 + 2\lambda_4 & 3\lambda_5^* \\ 3\lambda_5 & \lambda_3 + 2\lambda_4 \end{pmatrix}, \qquad 16\pi S^{(2)}_{(0,1)} = \begin{pmatrix} \lambda_3 & \lambda_5^* \\ \lambda_5 & \lambda_3 \end{pmatrix}. \tag{4.16}$$

$$16\pi S^{(3)}_{(0,0)} = \begin{pmatrix} 3\lambda_1 & 2\lambda_3 + \lambda_4 & \lambda_a \\ 2\lambda_3 + \lambda_4 & 3\lambda_2 & \lambda_b \\ \lambda_a & \lambda_b & \lambda_\Delta \end{pmatrix}, \quad 16\pi S^{(3)}_{(0,1)} = \begin{pmatrix} \lambda_1 & \lambda_4 & \lambda_{11} \\ \lambda_4 & \lambda_2 & \lambda_{12} \\ \lambda_{11} & \lambda_{12} & 2\lambda_{\Delta 1} + 4\lambda_{\Delta 2} \end{pmatrix}. \tag{4.17}$$

It is convenient to find the eigenvalues for $16\pi S^{(3)}_{(0,0)}$ and $16\pi S^{(3)}_{(0,1)}$ numerically, we also provide the analytical solutions in eq. (A.8). We collect the eigenvalues for the remaining scattering matrices as follows:

$$\begin{aligned}
&\Lambda^{2\pm}_{(0,0)} = \lambda_3 + 2\lambda_4 \pm 3|\lambda_5|, \\
&\Lambda^{2\pm}_{(0,1)} = \lambda_3 \pm |\lambda_5|, \\
&\Lambda_{(0,2)} = 2\lambda_{\Delta 1}, \\
&\Lambda^{1}_{(1,\frac{1}{2})} = \lambda_8 + 3\lambda_{11}/2, \qquad \Lambda^{2}_{(1,\frac{1}{2})} = \lambda_9 + 3\lambda_{12}/2, \\
&\Lambda^{1}_{(1,\frac{3}{2})} = \lambda_8, \qquad \Lambda^{2}_{(1,\frac{3}{2})} = \lambda_9, \\
&\Lambda_{(2,0)} = \lambda_3 - \lambda_4, \qquad \Lambda^{1}_{(2,1)} = \lambda_3 + \lambda_4, \\
&\Lambda^{2\pm}_{(2,1)} = \frac{1}{2}\left(\lambda_1 + \lambda_2 \pm \sqrt{(\lambda_1 - \lambda_2)^2 + 4|\lambda_5|^2}\right), \\
&\Lambda^{1}_{(3,\frac{1}{2})} = \lambda_8 - \lambda_{11}/2, \qquad \Lambda^{2}_{(3,\frac{1}{2})} = \lambda_9 - \lambda_{12}/2, \\
&\Lambda^{1}_{(3,\frac{3}{2})} = \lambda_8 + \lambda_{11}, \qquad \Lambda^{2}_{(3,\frac{3}{2})} = \lambda_9 + \lambda_{12}, \\
&\Lambda_{(4,0)} = 2\lambda_{\Delta 1} - \lambda_{\Delta 2}, \qquad \Lambda_{(4,2)} = 2(\lambda_{\Delta 1} + \lambda_{\Delta 2}),
\end{aligned} \tag{4.18}$$

where $\Lambda^{2\pm}_{(0,0)}$ and $\Lambda^{2\pm}_{(0,1)}$ are the eigenvalues for $16\pi S^{(2)}_{(0,0)}$ and $16\pi S^{(2)}_{(0,1)}$, respectively. The remaining results in eq. (4.18) represent the eigenvalues for the matrices (4.7)–(4.15). Combining with numerical eigenvalues of the matrices $16\pi S^{(3)}_{(0,0)}$ and $16\pi S^{(3)}_{(0,1)}$, we have provided all eigenvalues for the scattering matrices in the model (4.1) with a softly broken Z_2 symmetry. Note that the unitarity bounds of the same model was already considered in ref. [36]. Although we have used different notations to parametrize the scalar potential from ref. [36], a careful comparison of their eigenvalues of the scattering matrices given in eqs. (40) and (41) of ref. [36] with the counterparts in eqs. (4.17) and (4.18) shows that most of them are actually exactly the same. The only exception is that authors of ref. [36] seemed to miss the cubic equation that corresponds to the first matrix in eq. (4.17) of our work.

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4.4 The ΔSM

Another simple example belonging the present class is to consider a model with only one Higgs doublet and one complex triplet, the so-called ΔSM, which has been widely employed to explain the tiny neutrino masses by the type-II seesaw mechanism [28–33, 53–56]. The investigations of this model are extended to the searches at colliders [57–62], dark matter [63] and electroweak phase transition (EWPT) phenomena [34, 64, 65]. The scalar potential for ΔSM is given by

$$V_{\Delta \mathrm{SM}} = V(\Phi) + V(\Delta) + V(\Phi, \Delta)\,, \tag{4.19}$$

where $V(\Phi)$ is the SM Higgs potential in eq. (3.16), $V(\Delta)$ is given by eq. (4.3), and

$$V(\Phi, \Delta) = \left(\mu_1 \Phi^T i\tau_2 \Delta^\dagger \Phi + \mathrm{H.c.}\right) + \lambda_8 \Phi^\dagger \Phi \,\mathrm{Tr}\, \Delta^\dagger \Delta + \lambda_{11} \Phi^\dagger \Delta \Delta^\dagger \Phi. \tag{4.20}$$

By using the matrices in eq. (4.17) and the eigenvalues in eq. (4.18), the unitarity bounds for the ΔSM are given by

$$\begin{aligned}
&|\lambda_\Phi| \leq 4\pi,\ |\lambda_{\Delta 1}| \leq 4\pi,\ |\lambda_8| \leq 8\pi \\
&|2\lambda_{\Delta 1} - \lambda_{\Delta 2}| \leq 8\pi,\ |\lambda_{\Delta 1} + \lambda_{\Delta 2}| \leq 4\pi, \\
&|\lambda_8 + 3\lambda_{11}/2| \leq 8\pi,\ |\lambda_8 - \lambda_{11}/2| \leq 8\pi,\ |\lambda_8 + \lambda_{11}| \leq 8\pi, \\
&|\lambda_\Phi + \lambda_{\Delta 1} + 2\lambda_{\Delta 2} \pm \sqrt{(\lambda_\Phi - \lambda_{\Delta 1} - 2\lambda_{\Delta 2}) + \lambda_{11}^2}| \leq 8\pi, \\
&|6\lambda_\Phi + 8\lambda_{\Delta 1} + 6\lambda_{\Delta 2} \pm \sqrt{(6\lambda_\Phi - 8\lambda_{\Delta 1} - 6\lambda_{\Delta 2})^2 + 6(2\lambda_8 + \lambda_{11})^2}| \leq 16\pi\,,
\end{aligned} \tag{4.21}$$

which are in agreement with the results given in ref. [34].

5 Applications

The unitarity constraint on the quartic couplings can be translated into the upper bounds on the Higgs boson masses if $\sqrt{\lambda_i}v$ dominates the masses of the associated Higgs bosons. The unitarity bound as well as other constraints on the 2HDM have been fully explored in previous literature (see e.g. ref. [17] for a review and references there in). Moreover, we would like to mention that it is valuable to investigate the phenomenology of the real triplet extension of the 2HDM, which has been somewhat less explored in the literature. However, the careful study requires not only the unitarity bounds derived in section 3 but also many other theoretical and experimental constraints, which are obviously beyond the scope of the present work. Thus, in this section, we shall focus on the perturbative unitarity bounds to the complex triplet extension of the 2HDM, and show the quantitative constraints on the model parameters. Note that this model was recently proposed to explain the muon $g-2$ anomaly in ref. [52].

The muon anomalous magnetic dipole moment (denoted by $(g-2)_\mu$) is one of the long-standing anomalies in the particle physics. This discrepancy has further been confirmed by the recent muon $g-2$ measurement performed by the Muon experiment at Fermilab, which has yielded the most precise experimental muon $g-2$ value $a_\mu^{\mathrm{Exp}} = (116\,592\,061 \pm 41) \times 10^{-11}$ [66] by combining the Brookhaven data [67]. On the other hand, the state-of-the-art

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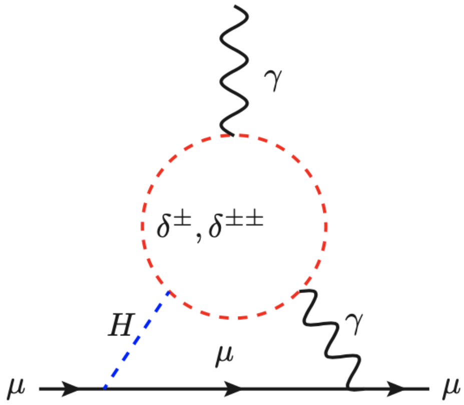

Figure 1. The Barr-Zee type Feynman diagrams for the muon $g-2$, with charged scalars $\delta^{\pm}$ and $\delta^{\pm\pm}$ running in the loops.

calculations of various SM contributions [68–87] predict $a_\mu^{\rm SM} = (116\,591\,810 \pm 43) \times 10^{-11}$ (see e.g. ref. [88] for a recent review and reference therein). As a result, the discrepancy between the SM and experimental values of a_μ is given by [66]

$$\Delta a_\mu = a_\mu^{\rm Exp} - a_\mu^{\rm SM} = (251 \pm 59) \times 10^{-11}\,, \tag{5.1}$$

with the significance reaching 4.25σ. Possible solutions to the muon $g-2$ anomaly has been widely discussed in the 2HDM content in refs. [89–94]. At one-loop level, both the charged and neutral Higgs bosons in the 2HDM contribute to the muon $g-2$, but it is found that these corrections are too small to explain the observed deviation. On the other hand, the two-loop Barr-Zee diagrams can give rise to the dominant contribution to the muon $g-2$ in some parameter space. However, it has been shown that the explanation of the muon $g-2$ anomaly with the Barr-Zee mechanism requires a light pseudo-scalar mass with $m_A \lesssim 100\,\text{GeV}$ and $t_\beta \sim 50$ when various constraints are imposed [90, 95, 96]. Note that one class of the strictest constraints is provided by the unitarity bounds in the theory. In particular, ref. [95] has shown that most of the parameter space with $m_A \gtrsim 100\,\text{GeV}$ in the typical 2HDM is already excluded by the unitarity alone.

The recent work in ref. [52] shows that if a complex Higgs triplet is added to the 2HDM, the charged components of the Higgs triplet can induce new Barr-Zee-type contributions illustrated in figure 1, which may explain the muon $g-2$ while easily evading other experimental constraints. From these Feynman diagrams, it is clear that the new contribution to muon $g-2$ is proportional to the trilinear scalar couplings $\lambda_i v$ which might be well constrained by the perturbative unitarity. However, ref. [52] only applied the approximate unitarity bounds from the aligned two-Higgs doublet model (A2HDM) [97] to constrain the parameter space. It is more appropriate to apply the exact unitarity bounds derived in the previous section for this complex triplet extension of the 2HDM, which is the main motivation for this section.

Following ref. [52], we will consider the decoupling limit of the model. By using the minimization conditions for the scalar potential in eq. (4.1) and the mass matrices provided in appendix C, we find that the decoupling between components in Higgs doublets and those in the triplet can be achieved when $v_\Delta,\ \tilde{\mu}_3 \ll 1\,\text{GeV}$. In the two-Higgs-doublet sector,

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we consider the case with a softly broken Z_2 symmetry and CP conservation, in which all parameters in the scalar potential are real. Also, following ref. [52], we shall consider the aligned limit of the two Higgs doublets, i.e., $c_{\beta-\alpha} \approx 0$. In this case, the mass eigenstates h and H are almost h_1^0 and h_2^0, so that the trilinear scalar couplings are given by eq. (D.1) in appendix D.

To calculate the Barr-Zee diagram shown in figure 1, we need to know the coupling between muons and H. As in ref. [52], we shall consider the A2HDM case [97] (see ref. [98] for the recent global fit of A2HDM), in which the lepton Yukawa couplings with H are given by

$$-\mathcal{L}_Y = \sum_f y_f^H \frac{M_f}{v} \bar{f}_L f_R H + \text{H.c.}, \tag{5.2}$$

where M_f is the mass of the lepton flavor f and

$$y_f^H = (s_{\beta-\alpha}\zeta_f - c_{\beta-\alpha}) \,. \tag{5.3}$$

Here ζ_f is a parameter in the A2HDM, whose benchmark value is taken to be $\zeta_f = -100$ following ref. [52].

The contribution to the muon $(g-2)$ from the Barr-Zee diagrams is given by [99]

$$\Delta a_\mu = \sum_{\phi_i} \frac{\alpha m_\mu^2}{8\pi^3 m_H^2} \mathrm{Re}\Big(y_f^H\Big) \lambda_{H\phi_i\phi_i^*} \mathcal{F}\left(\frac{m_{\phi_i}^2}{m_H^2}\right), \tag{5.4}$$

where $\phi_i = \delta^\pm,\ \delta^{\pm\pm}$, the trilinear couplings $\lambda_{H\phi_i\phi_i^*}$ are given in eq. (D.1), and the loop function is given by

$$\mathcal{F}(\omega) = \frac{1}{2}\int_0^1 dx \frac{x(x-1)}{\omega - x(1-x)} \ln\left(\frac{\omega}{x(1-x)}\right). \tag{5.5}$$

Since the Barr-Zee diagrams in figure 1 dominate the anomalous muon $g-2$, we can ignore other one- or two-loop $(g-2)_\mu$ contributions in our following numerical calculations.

In order to search for the parameter space allowed by the perturbative unitarity, we scan over the quartic couplings λ_8, λ_9, λ_{11}, and λ_{12} in the range of $(-8\pi - 8\pi)$ and the doubly-charged scalar mass $m_{\delta^{\pm\pm}}$ in the range of (10–1000) GeV. For the 2HDM sector, we take $\lambda_{1,2,\ldots,5} = 0.2$, $t_\beta = 5$, and $m_H = 300\,\text{GeV}$ for conservative estimations. The other parameters in the model are set to zero. The relations among various parameters in the generic scalar basis and the Higgs basis are summarized in appendix B, while the masses of scalars are determined in appendix C.

We show the scan results in figure 2. From the upper two plots, we observe that the mass squared difference between the singly-charged and doubly-charged scalars in the triplet should be $|m_{\delta^\pm}^2 - m_{\delta^{\pm\pm}}^2|/v^2 \lesssim 6$, which is restricted by unitarity bound on the quartic coupling $\tilde{\lambda}_{11}$ as seen in eqs. (C.4) and (C.5). Furthermore, for $m_{\delta^{\pm\pm}} \lesssim 200\,\text{GeV}$ we have $m_{\delta^\pm}^2 - m_{\delta^{\pm\pm}}^2 > 0$. The colorbar of this figure represents the distribution of the predicted Δa_μ values. The lower two plots of figure 2 show that large values of $|\Delta a_\mu|$ prefer large values of trilinear scalar couplings, $|\lambda_{H\delta^\pm\delta^\pm}|$ and $|\lambda_{H\delta^{\pm\pm}\delta^{\pm\pm}}|$, as well as small values of $m_{\delta^{\pm\pm}}$. Note that if Δa_μ is positive as required by experiments, it picks the parameter

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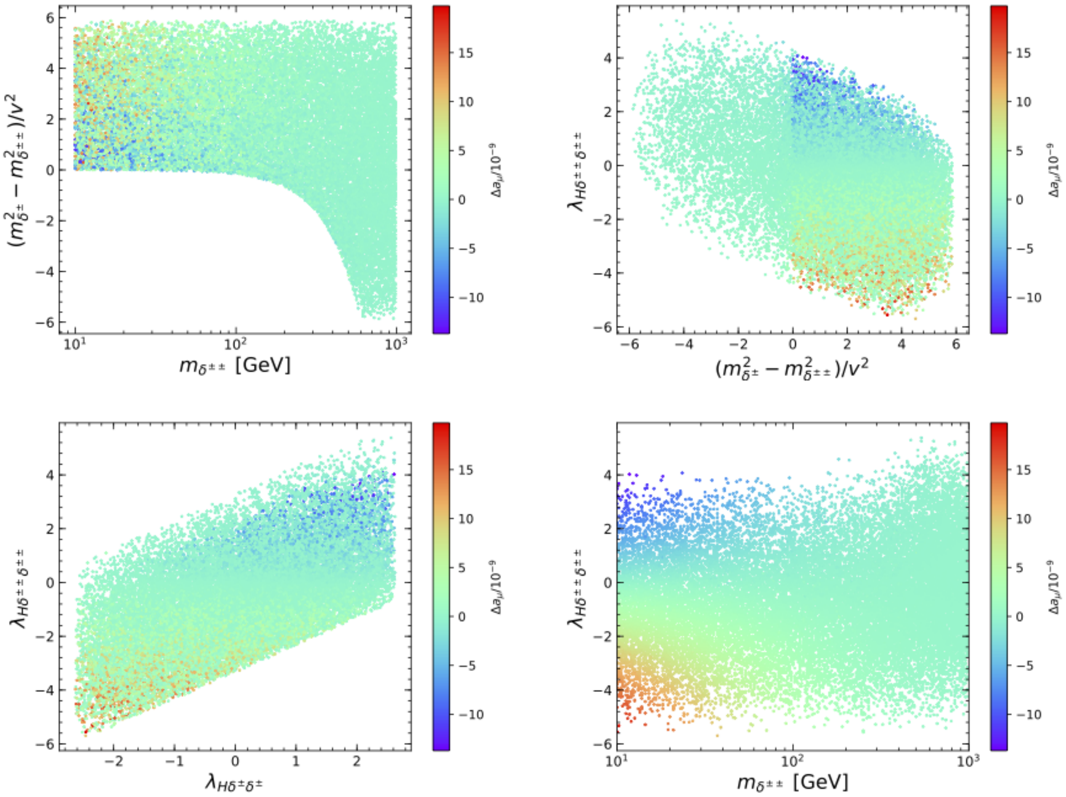

Figure 2. Unitarity bounds on the relevant parameter spaces of the complex triplet extension of the 2HDM. The colorbar represents the values of $\Delta a_\mu/10^{-9}$.

space with negative values of $\lambda_{H\delta^\pm\delta^\pm}$ and/or $\lambda_{H\delta^{\pm\pm}\delta^{\pm\pm}}$, which are well constrained by the unitarity consideration with $|\lambda_{H\delta^\pm\delta^\pm}| \lesssim 2.6$ and $|\lambda_{H\delta^{\pm\pm}\delta^{\pm\pm}}| \lesssim 5.0$. From the Barr-Zee Feynman diagrams and their expressions in eq. (5.4), the trilinear couplings are directly related to the dominant contribution to the muon $g-2$, and perturbative unitarity can thus put very useful constraints on this model. Finally, we note that the unitarity bounds given in this section are rather conservative since the doublet-triplet mixings are ignored due to the nearly vanishing triplet VEV. In the case with $v_\Delta \sim 1\,\mathrm{GeV}$, the mixings between Higgs doublet and triplet components can become significant, which would further enhance the unitarity bounds on the scalar masses.

We make several final remarks before closing this subsection. The difference between ref. [52] and ours is obvious. Ref. [52] aims to explain the muon $g-2$ in the context of the extension 2HDM with a complex triplet scalar. Our work focus on the derivations of the unitarity bounds for this specific extension of 2HDM. In this section, we have applied the unitarity bounds to constrain the trilinear scalar couplings $\lambda_{H\delta^\pm\delta^\pm}$ and $\lambda_{H\delta^{\pm\pm}\delta^{\pm\pm}}$, which has not been done in ref. [52]. Our new findings are depicted in figure 2, which indicates that the following parameter regions

$$|\lambda_{H\delta^\pm\delta^\pm}| \gtrsim 2.6, \quad |\lambda_{H\delta^{\pm\pm}\delta^{\pm\pm}}| \gtrsim 5.0, \quad \text{and} \quad |m_{\delta^\pm}^2 - m_{\delta^{\pm\pm}}^2| \gtrsim 6v^2 \tag{5.6}$$

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have been excluded by perturbative unitarity. We also note that ref. [52] applied the value $\lambda_{H\delta^{\pm\pm}\delta^{\pm\pm}} = 5$ for their estimations of muon $g-2$. This value just lies at the $\lambda_{H\delta^{\pm\pm}\delta^{\pm\pm}}$ upper limit allowed by the unitarity bounds in eq. (5.6). Therefore, we conclude that the main conclusion drawn by ref. [52] that the 2HDM with a complex triplet can explain the muon $g-2$ does not change dramatically even if the unitarity bounds are appropriately addressed.

6 Conclusions

The perturbative unitarity is one of the most significant theoretical constraints on the Higgs sector, beyond which the perturbation calculation in the theory breaks down. It has proven to be successful in predicting the upper limit on the Higgs boson mass in the minimal SM, and applying to constrain many new physics models such the 2HDM. In this work, we focus on deriving the perturbative unitarity bounds on the extension of the 2HDM with an additional real or complex Higgs scalar triplet. Since the total hypercharge and isospin are conserved in the high-energy limit of scatterings, we explicitly give the two-particle state basis according to their $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ charges by decomposing the direct product of two Higgs multiplets into direct sums of irreducible representations under electroweak gauge groups. The classification of the two-particle state basis is summarized in tables 2–7, in which the states are expressed in terms of component fields. With these two-particle bases, the $2 \to 2$ scattering amplitudes among scalars can be simplified into the block-diagonal forms, which are easily determined by expanding the quartic scalar terms in the potential. We then impose the unitarity bound on the eigenvalues of the scattering matrices. The associated analytical results are summarized in sections 3.2 and 4.2.

We then numerically apply our derived unitarity bounds to the extension of 2HDM with a complex Higgs triplet, which was recently shown to be of great phenomenological interest. We have shown that the unitarity can put strong upper limits on the trilinear scalar couplings and the mass differences of the charged triplet scalars. Since the contributions to the muon $g-2$ from the Barr-Zee diagrams with a charged scalar running in the loop are proportional to the trilinear scalar couplings, the unitarity bounds on these couplings can also constrain new solutions to the long-standing muon $g-2$ anomaly. In the near future, together with the experimental measurements of the Higgs trilinear coupling and the Higgs signal strengths of different channels at the LHC Run 3, we hope that the unitarity bounds would help us to understand the structure of Higgs sector more deeply.

Acknowledgments

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A Eigenvalues of the $(Y, I) = (0, 0)$ scattering matrix in the triplet extension of the 2HDM with a Z_2 symmetry

In this section we analytically solve the eigenvalues for the 5-dimensional scattering matrix, which appears in the $(Y, I) = (0, 0)$ sector of the extension of 2HDM with a softly-broken Z_2 symmetry. By imposing the Z_2 symmetry, the 5-dimensional scattering matrix in eq. (4.5) can be simplified into the following form

$$X = \begin{pmatrix} a_1 & a_2 & 0 & 0 & c_5 \\ a_3 & a_4 & 0 & 0 & c_6 \\ 0 & 0 & b_1 & b_2 & 0 \\ 0 & 0 & b_3 & b_4 & 0 \\ c_5 & c_6 & 0 & 0 & c_7 \end{pmatrix}, \tag{A.1}$$

which can be further decomposed into a 2×2 matrix and a 3×3 one as follows

$$X^{(2)} = \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}, \quad X^{(3)} = \begin{pmatrix} a_1 & a_2 & c_5 \\ a_3 & a_4 & c_6 \\ c_5 & c_6 & c_7 \end{pmatrix}. \tag{A.2}$$

The eigenvalues for $X^{(2)}$ and $X^{(3)}$ are the same as that directly obtained from the 5-dimensional matrix X. Note that, for a general matrix A, the eigenvalue f can be obtained by solving the equation

$$|fI - A| = 0\,. \tag{A.3}$$

For $X^{(2)}$ and $X^{(3)}$, the eigenvalue equation can be transformed into the following equations

$$(f-b_1)(f-b_4)-b_2b_3=0 \tag{A.4}$$
$$(f-a_1)(f-a_4)(f-c_7)-(f-a_1)c_6^2-a_2a_3(f-c_7)-a_2c_5c_6-a_3c_5c_6-(f-a_4)c_5^2=0\,, \tag{A.5}$$

respectively. The solutions for eq. (A.4) can be easily solved by

$$f_{1,2} = \frac{1}{2}\Big(b_1 + b_4 \pm \sqrt{(b_1 - b_4)^2 + 4b_2b_3}\Big). \tag{A.6}$$

For the case with $b_1 = b_4$ and $b_2 = b_3^*$, we have

$$f_{1,2} = b_1 \pm |b_2|. \tag{A.7}$$

Note that the scattering matrix should be Hermitian, which means $a_2 = a_3^*$ and $b_2 = b_3^*$. Furthermore, the eigenvalues for the Hermitian matrix are always real. Note that, for a general cubic equation $f^3+bf^2+cf+d = 0$, one representations of the three roots is given by

$$\begin{aligned} f_3 &= -\frac{b}{3} + 2\sqrt[3]{r}\cos\theta\,, \\ f_4 &= -\frac{b}{3} + 2\sqrt[3]{r}\cos\Big(\theta + \frac{2}{3}\pi\Big)\,, \\ f_5 &= -\frac{b}{3} + 2\sqrt[3]{r}\cos\Big(\theta + \frac{4}{3}\pi\Big)\,, \end{aligned} \tag{A.8}$$

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where

$$r=\sqrt{-\left(\frac{p}{3}\right)^3},\ \theta=\frac{1}{3}\arccos\left(-\frac{q}{2r}\right),\ \text{with}\ p=\frac{3c-b^2}{3},\ q=\frac{27d-9bc+2b^3}{27}. \tag{A.9}$$

By comparing eq. (A.5) with the general cubic equation, it is found that

$$\begin{aligned}b&=-(a_1+a_4+c_7)\,,\\ c&=a_1a_4+a_1c_7+a_4c_7-a_2a_3-c_5^2-c_6^2\,,\\ d&=a_1c_6^2+a_2a_3c_7+a_4c_5^2-a_2c_5c_6-a_3c_5c_6\,.\end{aligned} \tag{A.10}$$

In this way, we give the analytic solutions to the eigenvalues for the three-rank scattering matrix $X^{(3)}$.

B Parameters in the Higgs basis for the complex triplet extension of 2HDM

The electroweak gauge symmetry is spontaneously broken when the neutral components of the Higgs multiplets obtain VEVs. The VEVs can be complex and there may be a relative phase between them. Here we use ξ to denote the phase between the VEVs of doublets Φ_1 and Φ_2 in the triplet extension of the 2HDM. Concretely, one assumes real v_1 and complex $v_2e^{i\xi}$. Such a phase can be absorbed by the following phase redefinitions of the complex parameters:

$$\lambda_5\to e^{2i\xi}\lambda_5 \text{ and } m_{12}^2,\lambda_6,\lambda_7,\lambda_{10},\lambda_{13}\to e^{i\xi}\left\{m_{12}^2,\lambda_6,\lambda_7,\lambda_{10},\lambda_{13}\right\}, \tag{B.1}$$

so that the form of the potential keeps unchanged. Thus, we can start with real VEVs for scalars. We can rotate the generic scalar basis $\{\Phi_1,\Phi_2\}$ into the Higgs basis $\{H_1,H_2\}$ via the transformation

$$\begin{pmatrix}H_1\\ H_2\end{pmatrix}=\begin{pmatrix}\cos\beta & \sin\beta\\ -\sin\beta & \cos\beta\end{pmatrix}\begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix} \tag{B.2}$$

so that only H_1 has a non-vanishing VEV $v=\sqrt{v_1^2+v_2^2}\simeq 246\,\text{GeV}$. Here the quantity $\tan\beta$ is defined by the ratio of two Higgs field VEVs, i.e., $\tan\beta\equiv t_\beta=v_2/v_1$. In the following, we summarize the parameters of the potential in the Higgs basis as functions of those defined in the generic basis. For the parameters with mass dimensions, we have

$$\begin{aligned}\tilde{m}_{11}^2&=c_\beta^2m_{11}^2+s_\beta^2m_2^2-c_\beta s_\beta\left(m_{12}^2+m_{12}^{2*}\right),\\ \tilde{m}_{22}^2&=s_\beta^2m_{11}^2+c_\beta^2m_2^2+c_\beta s_\beta\left(m_{12}^2+m_{12}^{2*}\right),\\ \tilde{m}_{12}^2&=c_\beta s_\beta\left(m_{11}^2-m_{22}^2\right)+\left(c_\beta^2m_{12}^2-s_\beta^2m_{12}^{2*}\right),\\ \tilde{m}_\Delta^2&=m_\Delta^2,\\ \tilde{\mu}_1&=\mu_1c_\beta^2+\mu_3c_\beta s_\beta+\mu_2s_\beta^2,\\ \tilde{\mu}_2&=\mu_2c_\beta^2-\mu_3c_\beta s_\beta+\mu_1s_\beta^2,\\ \tilde{\mu}_3&=\mu_3\left(c_\beta^2-s_\beta^2\right)+2(\mu_2-\mu_1)c_\beta s_\beta.\end{aligned} \tag{B.3}$$

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The quartic couplings that relate only to the two Higgs doublets are given by

$$\begin{aligned}
\tilde{\lambda}_1 &= \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + 2(\lambda_3+\lambda_4+\mathrm{Re}\lambda_5)c_\beta^2 s_\beta^2 + 4\mathrm{Re}\lambda_6 c_\beta^3 s_\beta + 4\mathrm{Re}\lambda_7 c_\beta s_\beta^3,\\
\tilde{\lambda}_2 &= \lambda_1 s_\beta^4 + \lambda_2 c_\beta^4 + 2(\lambda_3+\lambda_4+\mathrm{Re}\lambda_5)c_\beta^2 s_\beta^2 - 4\mathrm{Re}\lambda_6 c_\beta s_\beta^3 - 4\mathrm{Re}\lambda_7 c_\beta^3 s_\beta,\\
\tilde{\lambda}_3 &= \frac{1}{4}s_{2\beta}^2[\lambda_1+\lambda_2-2(\lambda_3+\lambda_4+\mathrm{Re}\lambda_5)] + \lambda_3 - (\mathrm{Re}\lambda_6 - \mathrm{Re}\lambda_7)c_{2\beta}s_{2\beta},\\
\tilde{\lambda}_4 &= \frac{1}{4}s_{2\beta}^2[\lambda_1+\lambda_2-2(\lambda_3+\lambda_4+\mathrm{Re}\lambda_5)] + \lambda_4 - (\mathrm{Re}\lambda_6 - \mathrm{Re}\lambda_7)c_{2\beta}s_{2\beta},\\
\tilde{\lambda}_5 &= [\lambda_1+\lambda_2-2(\lambda_3+\lambda_4)]c_\beta^2 s_\beta^2 + \lambda_5 c_\beta^4 + \lambda_5^* s_\beta^4 - 2(\lambda_6-\lambda_7)c_\beta^3 s_\beta\\
&\quad + 2(\lambda_6^*-\lambda_7^*)c_\beta s_\beta^3,\\
\tilde{\lambda}_6 &= (-\lambda_1+\lambda_3+\lambda_4+\lambda_5^*)c_\beta^3 s_\beta + (\lambda_2-\lambda_3-\lambda_4-\lambda_5)c_\beta s_\beta^3 + \lambda_6^* c_\beta^4 - \lambda_7 s_\beta^4\\
&\quad - (\lambda_6^*-2\lambda_7^*+2\lambda_6-\lambda_7)c_\beta^2 s_\beta^2,\\
\tilde{\lambda}_7 &= (-\lambda_1+\lambda_3+\lambda_4+\lambda_5)c_\beta s_\beta^3 - (-\lambda_2+\lambda_3+\lambda_4+\lambda_5^*)c_\beta^3 s_\beta - \lambda_6 s_\beta^4 + \lambda_7^* c_\beta^4\\
&\quad + (2\lambda_6^*-\lambda_7^*+\lambda_6-2\lambda_7)c_\beta^2 s_\beta^2.
\end{aligned} \tag{B.4}$$

Finally, the quartic couplings involving the complex Higgs triplet are give by

$$\begin{aligned}
\tilde{\lambda}_8 &= \lambda_8 c_\beta^2 + \lambda_9 s_\beta^2 + 2\mathrm{Re}\lambda_{10}c_\beta s_\beta,\\
\tilde{\lambda}_9 &= \lambda_8 s_\beta^2 + \lambda_9 c_\beta^2 - 2\mathrm{Re}\lambda_{10}c_\beta s_\beta,\\
\tilde{\lambda}_{10} &= (-\lambda_8+\lambda_9)c_\beta s_\beta + \lambda_{10}c_\beta^2 - \lambda_{10}^* s_\beta^2,\\
\tilde{\lambda}_{11} &= \lambda_{11}c_\beta^2 + \lambda_{12}s_\beta^2 + 2\mathrm{Re}\lambda_{13}s_{2\beta},\\
\tilde{\lambda}_{12} &= \lambda_{11}s_\beta^2 + \lambda_{12}c_\beta^2 - 2\mathrm{Re}\lambda_{13}s_{2\beta},\\
\tilde{\lambda}_{13} &= (-\lambda_{11}+\lambda_{12})c_\beta s_\beta + \lambda_{13}c_\beta^2 - \lambda_{13}^* s_\beta^2,\\
\tilde{\lambda}_{\Delta 1} &= \lambda_{\Delta 1}, \quad \tilde{\lambda}_{\Delta 2} = \lambda_{\Delta 2}.
\end{aligned} \tag{B.5}$$

Note that under the U(1) transformation $H_1 \to e^{i\chi}H_1$ and $H_2 \to e^{-i\chi}H_2$, the scalar potential remains unchanged if the complex parameters of the scalar potential in the Higgs basis are transformed by the corresponding phase rotation [100]:

$$\tilde{\lambda}_5 \to e^{4i\chi}\tilde{\lambda}_5 \text{ and } \tilde{m}_{12}^2, \tilde{\lambda}_6, \tilde{\lambda}_7, \tilde{\lambda}_{10}, \tilde{\lambda}_{13} \to e^{2i\chi}\left\{\tilde{m}_{12}^2, \tilde{\lambda}_6, \tilde{\lambda}_7, \tilde{\lambda}_{10}, \tilde{\lambda}_{13}\right\}. \tag{B.6}$$

Therefore, beginning with the eqs. (B.3)–(B.5) in the phase $\{\xi = 0, \chi = 0\}$, we can obtain the relations between the two sets of parameters with arbitrary phases $\{\xi, \chi\}$ by applying the replacements (B.1) and (B.6) to the eqs. (B.3)–(B.5). Finally, The inversion of eqs. (B.3)–(B.5) can be obtained by making the replacements $\tilde{m}_{12}^2 \to m_{12}^2$, $\tilde{\lambda}_i \to \lambda_i$, and $\beta \to -\beta$.

C Mass matrices in the complex triplet extension of 2HDM

Here we provide some of the mass matrices elements for the extension of 2HDM with an additional complex Higgs triplet. In this appendix we express the neutral scalar in the triplet (4.2) as $\delta^0 = \frac{1}{\sqrt{2}}(d^0 + i\eta^0)$. The two doublet scalars in the Higgs basis can be expressed in components as follows,

$$H_1 = \begin{pmatrix} H_1^+ \\ \frac{1}{\sqrt{2}}(v + h_1^0 + iA_1) \end{pmatrix}, \quad \text{and} \quad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(h_2^0 + iA_2), \end{pmatrix} \tag{C.1}$$

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where h_1^0 and h_2^0 are CP-even neutral Higgs bosons, A_2 and H^+ are the physical neutral pseudoscalar and the charged scalar, respectively, while $H_1^\pm$and A_1 are the Goldstone bosons associated with the $W^\pm$ and Z gauge bosons.

The mass matrix elements for the CP-even components of the neutral scalars in the model are given by

$$\begin{aligned}
m^2_{h_1^0 h_1^0} &= \frac{3\tilde{\lambda}_1 v^2}{2} + \frac{v_\Delta^2 \tilde{\lambda}_8}{2} + \frac{v_\Delta^2 \tilde{\lambda}_{11}}{2} - \sqrt{2} v_\Delta \tilde{\mu}_1 + \tilde{m}_1^2, \\
m^2_{h_2^0 h_2^0} &= \frac{\tilde{\lambda}_3 v^2}{2} + \frac{\tilde{\lambda}_4 v^2}{2} + \frac{1}{4} v^2 \mathrm{Re}\tilde{\lambda}_5 + \frac{v_\Delta^2 \tilde{\lambda}_9}{2} + \frac{v_\Delta^2 \tilde{\lambda}_{12}}{2} - \sqrt{2} v_\Delta \tilde{\mu}_2 + \tilde{m}_2^2, \\
m^2_{d^0 d^0} &= 3 v_\Delta^2 \tilde{\lambda}_{\Delta 1} + 3 v_\Delta^2 \tilde{\lambda}_{\Delta 2} + \tilde{m}_\Delta^2 + \frac{\tilde{\lambda}_8 v^2}{2} + \frac{\tilde{\lambda}_{11} v^2}{2}, \\
m^2_{h_1^0 h_2^0} &= \frac{3}{4} v^2 \mathrm{Re}\tilde{\lambda}_6 + \frac{1}{4} v_\Delta^2 \mathrm{Re}\tilde{\lambda}_{10} + \frac{1}{4} v_\Delta^2 \mathrm{Re}\tilde{\lambda}_{13} - \frac{v_\Delta \mathrm{Re}\tilde{\mu}_3}{2\sqrt{2}} - \frac{\mathrm{Re}\tilde{m}_{12}^2}{2}, \\
m^2_{h_1^0 d^0} &= v_\Delta \tilde{\lambda}_8 v + v_\Delta \tilde{\lambda}_{11} v - \sqrt{2} \tilde{\mu}_1 v, \\
m^2_{h_2^0 d^0} &= \frac{1}{2} v_\Delta v \mathrm{Re}\tilde{\lambda}_{10} + \frac{1}{2} v_\Delta v \mathrm{Re}\tilde{\lambda}_{13} - \frac{v \mathrm{Re}\tilde{\mu}_3}{2\sqrt{2}}\,,
\end{aligned} \tag{C.2}$$

while the mass matrix elements for the CP-odd components are shown as follows

$$\begin{aligned}
m^2_{A_1^0 A_1^0} &= \frac{\tilde{\lambda}_1 v^2}{2} + \frac{v_\Delta^2 \tilde{\lambda}_8}{2} + \frac{v_\Delta^2 \tilde{\lambda}_{11}}{2} + \sqrt{2} v_\Delta \tilde{\mu}_1 + \tilde{m}_1^2, \\
m^2_{A_2^0 A_2^0} &= \frac{\tilde{\lambda}_3 v^2}{2} + \frac{\tilde{\lambda}_4 v^2}{2} - \frac{1}{4} v^2 \mathrm{Re}\tilde{\lambda}_5 + \frac{v_\Delta^2 \tilde{\lambda}_9}{2} + \frac{v_\Delta^2 \tilde{\lambda}_{12}}{2} + \sqrt{2} v_\Delta \tilde{\mu}_2 + \tilde{m}_2^2, \\
m^2_{\eta^0 \eta^0} &= v_\Delta^2 \tilde{\lambda}_{\Delta 1} + v_\Delta^2 \tilde{\lambda}_{\Delta 2} + \tilde{m}_\Delta^2 + \frac{\tilde{\lambda}_8 v^2}{2} + \frac{\tilde{\lambda}_{11} v^2}{2}, \\
m^2_{A_1^0 A_2^0} &= \frac{v_\Delta^2 \mathrm{Re}\tilde{\lambda}_{10}}{4} + \frac{v_\Delta^2 \mathrm{Re}\tilde{\lambda}_{13}}{4} + \frac{v_\Delta \mathrm{Re}\tilde{\mu}_3}{2\sqrt{2}} - \frac{\mathrm{Re}\tilde{m}_{12}^2}{2} + \frac{\mathrm{Re}\tilde{\lambda}_6 v^2}{4}, \\
m^2_{A_1^0 \eta^0} &= -\sqrt{2} \tilde{\mu}_1 v, \\
m^2_{A_2^0 \eta^0} &= -\frac{\mathrm{Re}\tilde{\mu}_3 v}{2\sqrt{2}}.
\end{aligned} \tag{C.3}$$

The mass matrix elements for the singly-charged scalars are

$$\begin{aligned}
m^2_{H_1^+ H_1^-} &= \frac{\tilde{\lambda}_1 v^2}{2} + \frac{v_\Delta^2 \tilde{\lambda}_8}{2} + \tilde{m}_1^2, \\
m^2_{H_2^+ H_2^-} &= \frac{\tilde{\lambda}_3 v^2}{2} + \frac{v_\Delta^2 \tilde{\lambda}_9}{2} + \tilde{m}_2^2, \\
m^2_{\delta^+ \delta^-} &= \frac{\tilde{\lambda}_8 v^2}{2} + \frac{\tilde{\lambda}_{11} v^2}{4} + v_\Delta^2 \tilde{\lambda}_{\Delta 1} + v_\Delta^2 \tilde{\lambda}_{\Delta 2} + \tilde{m}_\Delta^2, \\
m^2_{H_1^+ H_2^-} &= \frac{1}{2} v^2 \tilde{\lambda}_6^* + \frac{1}{2} v_\Delta^2 \tilde{\lambda}_{10}^* - \tilde{m}_{12}^{2*} = m^{2*}_{H_1^- H_2^+}, \\
m^2_{H_1^+ \delta^-} &= \frac{v_\Delta \tilde{\lambda}_{11} v}{2\sqrt{2}} - \tilde{\mu}_1 v = m^{2*}_{H_1^- \delta^+}, \\
m^2_{H_2^+ \delta^-} &= \frac{v_\Delta \tilde{\lambda}_{13} v}{2\sqrt{2}} - \frac{\tilde{\mu}_3 v}{2} = m^{2*}_{H_2^- \delta^+}.
\end{aligned} \tag{C.4}$$

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Since there is only one doubly-charged scalar in the model, so there is not any mixing and its mass is simply given by

$$m^2_{\delta^{++}\delta^{--}} = v_\Delta^2 \tilde{\lambda}_{\Delta 1} + \tilde{m}_\Delta^2 + \frac{\tilde{\lambda}_8 v^2}{2} \tag{C.5}$$

The parameters with tilde denote those in the Higgs basis, with the transformation relation to the parameters in the generic basis given in appendix B. Due to CP-violating effects, there may exist mixings between the CP-even and CP-odd components in the neutral scalars. Since we do not use them in our work, we do not provide their explicit formulae here.

D Trilinear couplings in the complex triplet extension of 2HDM

The trilinear couplings between a neutral scalar and a pair of charged scalars in the triplet extension of the 2HDM are summarized as follows:

$$\begin{aligned}
\lambda_{h_1^0 H_2^+ H_2^-} &= \tilde{\lambda}_3, & \lambda_{h_2^0 H_2^+ H_2^-} &= \mathrm{Re}\tilde{\lambda}_7, \\
\lambda_{h_1^0 \delta^+ \delta^-} &= \tilde{\lambda}_8 + \frac{1}{2}\tilde{\lambda}_{11}, & \lambda_{h_2^0 \delta^+ \delta^-} &= \mathrm{Re}\tilde{\lambda}_{10} + \frac{1}{2}\mathrm{Re}\tilde{\lambda}_{13}, \\
\lambda_{h_1^0 \delta^{++} \delta^{--}} &= \tilde{\lambda}_8, & \lambda_{h_2^0 \delta^{++} \delta^{--}} &= \mathrm{Re}\tilde{\lambda}_{10}.
\end{aligned} \tag{D.1}$$

Note that in the aligned and decoupling limits, we have $h_1^0 \equiv h$ and $h_2^0 \equiv H$.

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