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Black holes with spindles at the horizon

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Abstract: We construct $\mathrm{AdS}_4 \times \Sigma$ and $\mathrm{AdS}_2 \times \Sigma \times \Sigma_{\mathfrak{g}}$ solutions in F(4) gauged supergravity in six dimensions, where Σ is a two dimensional manifold of non-constant curvature with conical singularities at its two poles, called a spindle, and $\Sigma_{\mathfrak{g}}$ is a constant curvature Riemann surface of genus $\mathfrak{g}$. We find that the first solution realizes a "topologically topological twist", while the second class of solutions gives rise to an "anti twist". We compute the holographic free energy of the $\mathrm{AdS}_4 \times \Sigma$ solution and find that it matches the entropy computed by extremizing an entropy functional that is constructed by gluing gravitational blocks. For the $\mathrm{AdS}_2 \times \Sigma \times \Sigma_{\mathfrak{g}}$ solution, we find that the Bekenstein-Hawking entropy is reproduced by extremizing an appropriately defined entropy functional, which leads us to conjecture that this solution is dual to a three dimensional SCFT on a spindle. A class of the $\mathrm{AdS}_2 \times \Sigma \times \Sigma_{\mathfrak{g}}$ solutions can be embedded in four dimensional T^3 gauged supergravity, which is a subtruncation of the six dimensional theory.

Keywords: Black Holes in String Theory, AdS-CFT Correspondence

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1 Introduction and summary

An important tool to study the AdS/CFT correspondence has been to construct supersymmetric solutions by wrapping branes on supersymmetric cycles. Was originally done by following the idea of [1], where in order for the lower dimensional solution to preserve supersymmetry, the background R-symmetry gauge field cancels the spin connection on the compactification manifold — a mechanism dubbed "topological twisting". The holographic duals of these gravity theories are topologically twisted superconformal field theories (SCFTs) [2]. Such constructions with branes wrapped on a two dimensional constant curvature Riemann surface $\Sigma_{\mathfrak{g}}$ of genus $\mathfrak{g}$ have been extensively studied e.g., for M2 branes in [3], M5 branes in [4], D3 branes in [5], and D4 branes in [6].

Recently, new AdS/CFT constructions which do not rely on topological twisting have been studied, starting with [7] where D3 branes wrapped on a two dimensional surface known as a spindle were studied. The spindle is a weighted projective plane $\mathbb{WCP}^1_{[n_\pm]}$, with weights given by two positive coprime integers $n_\pm$. The metric on such a space is regular everywhere except at the poles where there are conical singularities parametrized by the integers $n_\pm$. This compactification preserves supersymmetry, not via a topological twist, but rather via an "anti twist" [7] or a "topologically topological twist" [8]. This has paved the way for a new class of constructions where branes are wrapped on a spindle instead of a constant curvature Riemann surface. M5 and M2 branes were studied in [8] and [9], a multicharge solution from D3 branes was studied in [10], and very recently, D4 branes were studied in [11]. A family of charged rotating solutions of the form $\mathrm{AdS}_2 \times \Sigma$ were constructed in 4d $\mathcal{N} = 4$ gauged supergravity in [12]. Compactifications on a topological disc, which preserve supersymmetry in a similar way, have been studied e.g. in [13, 14]. It was shown in [15] that D3 branes wrapped on a topological disc are a different global completion of the same local solution as in [16]. Supersymmetric solutions corresponding to D3 branes and M5 branes wrapped on a topological disc were constructed in [17], D4-D8 solutions were constructed in [18], and M2 branes in [19].

In this paper, we construct a new class of solutions of the form $\mathrm{AdS}_2 \times \Sigma \times \Sigma_\mathfrak{g}$, starting from six dimensional F(4) gauged supergravity. This arises as a consistent truncation of massive IIA (mIIA) supergravity compactified on a warped S^4. We find that these solutions preserve supersymmetry via an "anti twist". These solutions can be interpreted in the four dimensional theory obtained by compactifying the six dimensional theory on $\Sigma_\mathfrak{g}$. One such class of solutions exists for a specific choice of parameters and corresponds to the "gauged T^3 supergravity", while a second class corresponds to minimal gauged supergravity in four dimensions. These four dimensional theories can also be uplifted to eleven dimensional supergravity on $\mathrm{AdS}_4 \times S^7$, which is dual to three dimensional ABJM theory [20]. So the four dimensional gravity solutions can also be related to the ABJM theory. We compute the entropy by extremizing an off-shell entropy functional obtained by appropriately gluing "gravitational blocks" [21]. Remarkably we find that the result agrees with the computation in gravity. We also construct supersymmetric $\mathrm{AdS}_4 \times \Sigma$ solutions in the six dimensional F(4) gauged supergravity, where we instead find that supersymmetry is preserved via "a topologically topological twist". We expect this to be dual to a five dimensional SCFT on the spindle. We again compute the free energy on S^3 by extremizing the entropy functional constructed by gluing gravitational blocks and find agreement with the gravity result obtained from our solution.

The outline of this paper is as follows. We begin with a quick overview of the six dimensional F(4) gauged supergravity theory in section 2. We then construct the supersymmetric $\mathrm{AdS}_4 \times \Sigma$ solution in section 3, and the supersymmetric $\mathrm{AdS}_2 \times \Sigma \times \Sigma_\mathfrak{g}$ solutions in section 4. We conclude with some discussion in section 5.

Note. While writing up this paper, [11] appeared on `arXiv` whose results partially overlap with ours. They construct $\mathrm{AdS}_4 \times \Sigma$ solutions in 6d F(4) gauged supergravity and consistent with our observations, they also find that the solution is realized as "a topologically topological twist".

2 6d F(4) gauged supergravity

We begin by recalling some important aspects of six dimensional F(4) gauged supergravity. F(4) superalgebra is the minimal extension of the SO(2, 5) symmetry group of six dimen-

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sional AdS. It contains $\mathfrak{so}(2,5)\otimes\mathfrak{su}(2)$ as the maximal bosonic subalgebra, and is therefore the natural candidate for a six dimensional supergravity theory with 16 supercharges i.e., $\mathcal{N}=2$ in $d=6$. The minimal F(4) supergravity theory (containing only the gravitino multiplet) was constructed in [22], while the theory coupled to vector multiplets (which are the only possible massive long multiplets in $\mathcal{N}=2$) was constructed in [23, 24].

The bosonic fields contained in the gravitino multiplet are the metric $g_{\mu\nu}$, four gauge fields A^α_μ corresponding to the symmetry group $\mathrm{U}(1)\times\mathrm{SU}(2)_R$ (where $\alpha\in\{0,r\}$, with $r\in\{1,2,3\}$ being an index in the adjoint representation of $\mathrm{SU}(2)_R$), a two form $B_{\mu\nu}$ and the dilaton σ, where $\mu,\nu\in\{0,1\ldots,5\}$ are spacetime indices. The fermionic fields consist of two gravitini ψ^A_μ, and two spin-1/2 fermions χ^A, where $A\in\{1,2\}$, transforming in the fundamental representation of $\mathrm{SU}(2)_R$.

The gravity multiplet can be coupled to n_V vector multiplets labelled by an index $I\in\{1,\ldots,n_V\}$. Each vector multiplet contains a gauge field A_μ, four scalars ϕ_α, and a spin-1/2 fermion λ_A, where α and A are indices in the adjoint and fundamental representations of $\mathrm{SU}(2)_R$ respectively, as above. The $4n_V$ scalars span the coset manifold

$$\frac{\mathrm{SO}(4,n_V)}{\mathrm{SO}(4)\times\mathrm{SO}(n_V)}\,.$$

The scalars ϕ_α can be encoded in a coset representative $L^\Lambda{}_\Sigma\in\mathrm{SO}(4,n_V)$, where $\Lambda\in\{\alpha,I\}$ and I counts the number of vector multiplets $I\in\{1,\ldots,n_V\}$. The gauged six dimensional theory can be obtained by a compact gauging of $\mathcal{G}=\mathrm{SU}(2)_R\times G$, where G is a n_V dimensional compact subgroup of $\mathrm{SO}(n_V)$. The six dimensional bosonic Lagrangian, in the notation of [24] is:

$$\begin{aligned}\mathcal{L}=&-\frac{R}{4}-\frac{1}{8}e^{-2\sigma}\mathcal{N}_{\Lambda\Sigma}\hat{F}^\Lambda_{\mu\nu}\hat{F}^{\Sigma\mu\nu}+\frac{3}{64}e^{4\sigma}H_{\mu\nu\rho}H^{\mu\nu\rho}+\partial^\mu\sigma\partial_\mu\sigma-\frac{1}{4}P^{I\alpha\mu}P_{I\alpha\mu}\\&-\frac{1}{64}\epsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}\left(\eta_{\Lambda\Sigma}\hat{F}^\Lambda_{\rho\sigma}\hat{F}^\Sigma_{\lambda\tau}+mB_{\rho\sigma}\hat{F}^0_{\lambda\tau}+\frac{1}{3}m^2B_{\rho\sigma}B_{\lambda\tau}\right)-V\,,\end{aligned}\tag{2.1}$$

where

$$\begin{aligned}\mathcal{N}_{\Lambda\Sigma}&=L_\Lambda{}^\alpha\left(L^{-1}\right)_{\alpha\Sigma}-L_\Lambda{}^I\left(L^{-1}\right)_{I\Sigma}\,,\\P^I_\alpha&=\left(L^{-1}\right)^I{}_\Lambda\left(\mathrm{d}L^\Lambda{}_\alpha-f_\Gamma{}^\Lambda{}_\Pi A^\Gamma L^\Pi{}_\alpha\right)\,,\end{aligned}\tag{2.2}$$

with $f^\Lambda{}_{\Pi\Gamma}$ being the structure constants of the gauge group $\mathcal{G}$. g is the gauge coupling constant and m is the mass parameter associated to the two form. The minimal six dimensional F(4) gauged supergravity theory can be obtained as a consistent truncation of massive type IIA supergravity in ten dimensions on a warped S^4 [25]. This was dualized to a truncation of type IIB supergravity via a non-Abelian T-duality in [26], and was further generalized to a large class of geometries in [27]. Substantial evidence was provided in [28] that even the six dimensional theory coupled to a vector multiplet can be obtained as a consistent truncation of ten dimensional mIIA supergravity. It was also shown in [29] that the theory with one vector multiplet can be obtained from a consistent truncation of type IIB supergravity on a general class of manifolds, which includes the Abelian T-dual of the mIIA background considered here. The parameter m is then related to Romans' mass $m = F_{(0)}$.

The field strength $\hat{F}^{\Lambda}_{\rho\sigma} = F^{\Lambda}_{\rho\sigma} - m\delta^{\Lambda 0}B_{\mu\nu}$ is dressed with this mass parameter, and we use the non-standard convention of [24] where $F = F_{\mu\nu}\mathrm{d}x^{\mu}\wedge\mathrm{d}x^{\nu}$, with $F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)/2$. Variation of the fermions (upto linear order in fermions) under an infinitesimal supersymmetry transformation are given by

$$\begin{aligned}
\delta\psi_{A\mu} &= \nabla_\mu\epsilon_A - \frac{i}{2}g\sigma^r_{AB}A_{r\mu}\epsilon^B + \frac{1}{16}e^{-\sigma}\left[\hat{T}_{[AB]\nu\lambda}\Gamma - T_{(AB)\nu\lambda}\right]\left(\Gamma_\mu{}^{\nu\lambda} - 6\delta^\nu_\mu\Gamma^\lambda\right)\epsilon^B \\
&\quad + \frac{i}{32}e^{2\sigma}H_{\nu\lambda\rho}\Gamma\left(\Gamma_\mu{}^{\nu\lambda\rho} - 3\delta^\nu_\mu\Gamma^{\lambda\rho}\right)\epsilon_A + S_{AB}\Gamma_\mu\epsilon^B\,, \\
\delta\chi_A &= \frac{i}{2}\Gamma^\mu\partial_\mu\sigma\epsilon_A + \frac{i}{16}e^{-\sigma}\left[\hat{T}_{[AB]\nu\lambda}\Gamma + T_{(AB)\nu\lambda}\right]\Gamma^{\nu\lambda}\epsilon^B + \frac{1}{32}e^{2\sigma}H_{\nu\lambda\rho}\Gamma\Gamma^{\nu\lambda\rho}\epsilon_A + N_{AB}\epsilon^B\,, \\
\delta\lambda^I_A &= iP^I_{r\mu}\sigma^r_{AB}\Gamma^\mu\epsilon^B - iP^I_{0\mu}\epsilon_{AB}\Gamma\Gamma^\mu\epsilon^B + \frac{i}{2}e^{-\sigma}T^I_{\mu\nu}\Gamma^{\mu\nu}\epsilon_A + M^I_{AB}\epsilon^B\,,
\end{aligned} \tag{2.3}$$

where Γ is the six dimensional chirality matrix $\Gamma = i\Gamma^0\ldots\Gamma^5$, and the dressed vector field strengths are defined as

$$\hat{T}_{[AB]\nu\lambda} = \epsilon_{AB}\left(L^{-1}\right)_{0\Lambda}\hat{F}^{\Lambda}_{\nu\lambda}\,,\quad T_{(AB)\nu\lambda} = \sigma^r_{AB}\left(L^{-1}\right)_{r\Lambda}F^{\Lambda}_{\nu\lambda}\,,\quad T_{I\nu\lambda} = \left(L^{-1}\right)_{I\Lambda}F^{\Lambda}_{\nu\lambda}\,, \tag{2.4}$$

and S_{AB}, N_{AB}, M_{AB} represent the extra contributions to the fermion variations due to gauging and the mass parameter. Greek indices are raised and lowered with the $\mathrm{SO}(4, n_V)$ invariant matrix $\eta_{\Lambda\Sigma} = \mathrm{diag}\,(1, 1, 1, 1, -1, \ldots, -1)$, and Roman indices with the $\mathrm{SU}(2)_\mathrm{R}$ tensor ϵ_{AB}

We further restrict ourselves to a theory which contains only one vector multiplet $n_V = 1$. We can consistently set all gauge fields to zero except $A^{r=3}_\mu$ and $A^{I=1}_\mu$ which will be necessary for the twisting, and for providing a magnetic charge for the black hole. Additionally, we require that the scalar fields in the vector multiplet are singlets under the gauge field $A^{r=3}_\mu$. Furthermore, requiring that the black holes are purely magnetic restricts the only non-zero component of the scalars to be ϕ_3. Following [28, 30], we choose a convenient parametrization of the scalar coset given by

$$L^{\Lambda}_{\Sigma} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \cosh\phi_3 & \sinh\phi_3 \\ 0 & 0 & 0 & \sinh\phi_3 & \cosh\phi_3 \end{pmatrix}. \tag{2.5}$$

With this parametrization, the kinetic matrix for the vector fields follows from equation (2.2)

$$N_{\Lambda\Sigma} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \cosh 2\phi_3 & -\sinh 2\phi_3 \\ 0 & 0 & 0 & -\sinh 2\phi_3 & \cosh 2\phi_3 \end{pmatrix}. \tag{2.6}$$

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In this parametrization, the shifts S_{AB}, N_{AB}, M_{AB} become

$$\begin{aligned} S_{AB} &= \frac{i}{4}\left(g e^{\sigma}\cosh\phi_3 + m e^{-3\sigma}\right)\epsilon_{AB}\,, \\ N_{AB} &= \frac{1}{4}\left(g e^{\sigma}\cosh\phi_3 - 3m e^{-3\sigma}\right)\epsilon_{AB}\,, \\ M_{AB} &= -2g e^{\sigma}\sinh\phi_3\,\sigma^3_{AB}\,. \end{aligned} \tag{2.7}$$

The 6d theory has AdS_6 as a vacuum solution if $g = 3m$. We are interested in near horizon solutions of higher dimensional objects whose full solution would represent a flow from AdS_6 to $\mathrm{AdS}_4 \times \Sigma$ or $\mathrm{AdS}_2 \times \Sigma \times \Sigma_{\mathfrak{g}}$ respectively. So we will choose $g = 3m$ in the rest of paper.

3 The supersymmetric $\mathrm{AdS}_4 \times \Sigma$ solution

3.1 Supersymmetry equations

We are interested in a solution of the form $\mathrm{AdS}_4 \times \Sigma$. To find this, we consider the following ansatz for the metric

$$\mathrm{d}s^2 = w(y)\left[\frac{4}{9}\mathrm{d}s^2_{\mathrm{AdS}_4} - \frac{\mathrm{d}y^2}{q(y)} - \frac{q(y)}{r(y)}\mathrm{d}z^2\right], \tag{3.1}$$

and assume that the two-form vanishes i.e., $B_{\mu\nu} = 0$. Let us pick the non-zero components of the two gauge fields to lie only along the spindle $A^{3,I} = A^{3,I}_z(y)\mathrm{d}z$, where the index I labels the gauge field from the vector multiplet. Maxwell's equations follow from the Lagrangian in equation (2.1), and in this case read

$$\partial_y\left(\sqrt{-g}\,e^{-2\sigma}\mathcal{N}_{\Lambda\Sigma}F^{\Sigma}_{yz}g^{yy}g^{zz}\right) = 0\,. \tag{3.2}$$

This determines the gauge field strengths along the spindle[1]

$$F^3 = -\frac{e^{2\sigma}}{w}\left(f_3\cosh 2\phi_3 + f_i\sinh 2\phi_3\right)\mathrm{vol}_\Sigma\,, \quad F^I = -\frac{e^{2\sigma}}{w}\left(f_3\sinh 2\phi_3 + f_i\cosh 2\phi_3\right)\mathrm{vol}_\Sigma\,, \tag{3.3}$$

where f_3, f_i are constants. We choose the following representation for the gamma matrices

$$\Gamma^a = \gamma^a \otimes \sigma_3, \quad \Gamma^4 = \mathbb{1}\otimes i\sigma_2, \quad \Gamma^5 = \mathbb{1}\otimes i\sigma_1\,, \tag{3.4}$$

where $a \in \{0,1,2,3\}$ are frame indices along the AdS_4 and indices $4,5$ are frame indices along the spindle. In six Lorenzian dimensions, spinors form a symplectic-Majorana pair which transform as $\mathcal{B}_6\epsilon_A = \varepsilon^{AB}\epsilon^*_B$ under the six dimensional matrix $\mathcal{B}_6$ defined by $\mathcal{B}_6\Gamma_m\mathcal{B}_6^{-1} = -\Gamma^*_m$, where $m \in \{0,\ldots,5\}$. We choose $\mathcal{B}_6 = \mathcal{B}_4 \otimes \mathcal{B}_2$, where $\mathcal{B}_4$ and $\mathcal{B}_2$ are matrices in 4d Lorenzian and 2d Euclidean space defined by $\mathcal{B}_4\gamma_a\mathcal{B}_4^{-1} = \gamma^*_a$, and

[1] For brevity, we will often not write the y dependence of the functions explicitly.

$\mathcal{B}_2\gamma_i\mathcal{B}_2^{-1} = -\gamma_i^*$ respectively. γ_a and $\gamma_i = (i\sigma_2, i\sigma_1)$, are the 4d Lorenzian and 2d Euclidean gamma matrices respectively. In particular, $\mathcal{B}_2$ is proportional to σ_1. With this choice, $\mathcal{B}_4\mathcal{B}_4^* = -1$, $\mathcal{B}_2\mathcal{B}_2^* = 1$ and $\mathcal{B}_6\mathcal{B}_6^* = -1$.

We make an ansatz for the 6d spinor to be of the form $\epsilon_1 = \beta_- \otimes \eta_1$, and $\epsilon_2 = \beta_+ \otimes \eta_2$, where $\beta_\pm$ satisfy $\nabla_a\beta_\pm = \pm(i/2)\gamma_a\beta_\pm$. For the choice of $\mathcal{B}_4$ above, $\mathcal{B}_4\beta_\pm = \mp(\beta_\mp)^*$. The action of $\mathcal{B}_2$ is given by $\mathcal{B}_2\eta_1 = \eta_2^*, \mathcal{B}_2\eta_2 = \eta_1^*$. In our conventions, $-\varepsilon_{AB}\epsilon^B = \epsilon_A$, $(\sigma^3)^A{}_B$ is the usual third Pauli Matrix and $\sigma^3_{AB} = -\varepsilon_{AC}\left(\sigma^3\right)^C{}_B$. Furthermore, we choose a gauge where the spinor is independent of the coordinate z.

We are looking for a supersymmetric solution. For simplicity, we pick a purely bosonic background by setting all the fermions to zero, and further demand that they remain zero under a supersymmetry transformation. This is imposed by demanding that the fermionic variations in equation (2.3) vanish. For the gravitino variation, this implies:

$$\begin{aligned}\delta\psi_{1a} &= \gamma_a\beta_- \otimes \left(-\eta_1 - A\eta_1 - B\sigma_3\eta_1 - \frac{iw'\sqrt{q}}{3w}\sigma_1\eta_1\right),\\ \delta\psi_{2a} &= \gamma_a\beta_+ \otimes \left(\eta_2 + A\eta_2 - B\sigma_3\eta_2 - \frac{iw'\sqrt{q}}{3w}\sigma_1\eta_2\right).\end{aligned} \tag{3.5}$$

where

$$A \coloneqq \frac{e^\sigma}{6w^{3/2}}\left(f_3\cosh\phi_3 + f_i\sinh\phi_3\right), \quad B \coloneqq \frac{\sqrt{w}}{3}\left(ge^\sigma\cosh\phi_3 + me^{-3\sigma}\right). \tag{3.6}$$

The equations for $\eta_{1,2}$ obtained above are invariant under the symplectic-Majorana condition, as expected. Since the two spinors η_1 and η_2 are related by $\eta_1 = \mathcal{B}_2\eta_2^*$, there is only one independent 2d spinor, and all the equations can be written in terms of it. Relabelling η_2 as ξ, we can now rewrite the set of equations in (2.3) as:

$$\begin{aligned}\delta\psi_{Aa} &: \left[1 + \frac{e^\sigma}{6w^{3/2}}\left(f_3\cosh\phi_3 + f_i\sinh\phi_3\right)\right]\xi - \frac{\sqrt{w}}{3}\left(ge^\sigma\cosh\phi_3 + me^{-3\sigma}\right)\sigma_3\xi\\ &\qquad - \frac{iw'\sqrt{q}}{3w}\sigma_1\xi \stackrel{!}{=} 0,\\ \delta\psi_{A4} &: -\frac{3ie^\sigma}{8w^2}\left(f_3\cosh\phi_3 + f_i\sinh\phi_3\right)\sigma_1\xi - \frac{1}{4}\left(ge^\sigma\cosh\phi_3 + me^{-3\sigma}\right)\sigma_2\xi + \sqrt{\frac{q}{w}}\xi' \stackrel{!}{=} 0,\\ \delta\psi_{A5} &: \frac{3ie^\sigma}{8w^2}\left(f_3\cosh\phi_3 + f_i\sinh\phi_3\right)\sigma_2\xi - \frac{1}{4}\left(ge^\sigma\cosh\phi_3 + me^{-3\sigma}\right)\sigma_1\xi - \frac{ig}{2}\sqrt{\frac{r}{wq}}A_z^3\xi\\ &\qquad + i\frac{\sqrt{r}}{2w}\left(\sqrt{\frac{wq}{r}}\right)'\sigma_3\xi \stackrel{!}{=} 0,\\ \delta\chi &: \frac{1}{2}\sqrt{\frac{q}{w}}\sigma'\sigma_2\xi - \frac{e^\sigma}{8w^2}\left(f_3\cosh\phi_3 + f_i\sinh\phi_3\right)\sigma_3\xi + \frac{1}{4}\left(ge^\sigma\cosh\phi_3 - 3me^{-3\sigma}\right)\xi \stackrel{!}{=} 0,\\ \delta\lambda &: \sqrt{\frac{q}{w}}\phi_3'\sigma_2\xi - \frac{e^\sigma}{w^2}\left(f_3\sinh\phi_3 + f_i\cosh\phi_3\right)\sigma_3\xi + 2g\sinh\phi_3 e^\sigma\xi \stackrel{!}{=} 0.\end{aligned} \tag{3.7}$$

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3.2 The solution

We solve the first equation by choosing $q(y) = q_1(y)q_2(y)$, and $\xi = n(y)\left(\sqrt{q_1(y)}, i\sqrt{q_2(y)}\right)$. This determines q_1, q_2 to be

$$q_{1,2} = \frac{3w}{w'}\left[\pm\left(1 + \frac{e^{\sigma}}{6w^{3/2}}\left(f_3\cosh\phi_3 + f_i\sinh\phi_3\right)\right) + \frac{\sqrt{w}}{3}\left(ge^{\sigma}\cosh\phi_3 + me^{-3\sigma}\right)\right]. \quad (3.8)$$

With this choice of the spinor ϵ, the four components of the last two equations immediately simplify to just two linearly independent equations which are solved by

$$e^{4\sigma} = \cosh\phi_3 + \frac{f_3}{f_i}\sinh\phi_3\,, \quad w = \frac{(2f_i\operatorname{csch}\phi_3)^{2/3}}{3^{4/3}}\left(\cosh\phi_3 + \frac{f_3}{f_i}\sinh\phi_3\right)^{1/6}. \quad (3.9)$$

Inserting these in the gravitino variation along Σ gives a full solution in terms of the scalar field ϕ_3. However, is convenient to choose a parametrization $\phi_3 = \operatorname{arccoth}(y)$ to further simplify the solution. The full solution including the gauge field, the scalars, and the normalization of the spinor, in this parametrization is as follows:

$$\begin{aligned}
w &= \frac{2^{2/3}}{3^{4/3}}\sqrt{f_i}\,(f_3 + f_i\,y)^{1/6}\left(y^2 - 1\right)^{1/4}, \qquad r = r_0\,(f_3 + f_i\,y)^{2/3}\left(y^2 - 1\right),\\
q_{1,2} &= \pm\left(\frac{9f_3}{2f_i} + \frac{9y}{2}\right) + 2\cdot 6^{1/3}m\,(f_3 + f_i\,y)^{1/3}\sqrt{y^2 - 1}\,,\\
A_z^3 &= \frac{6^{1/3}\left(3f_3\,y + 2f_i + f_i\,y^2\right)}{f_i\left(y^2 - 1\right)}\,, \qquad e^{4\sigma} = \frac{f_3 + f_i\,y}{f_i\sqrt{y^2 - 1}}\,,\\
n &= n_0\,(f_3 + f_i\,y)^{-1/8}\left(y^2 - 1\right)^{-3/16}.
\end{aligned} \quad (3.10)$$

3.3 Regularity of the solution

For the metric to have a definite signature, the functions q and r appearing in the metric must be positive throughout the interval on which it is defined. Taking $y > 0$, the metric function r is positive for $y > 1$, while the function $q = q_1q_2$ is a polynomial of degree 8. We want to find conditions for which the metric

$$\mathrm{d}s^2_\Sigma = \frac{\mathrm{d}y^2}{q} + \frac{q}{r}\mathrm{d}z^2\,, \quad (3.11)$$

is a smooth metric on the spindle i.e., a two dimensional weighted projective plane with weights represented by two positive coprime integers $n_\pm$: $\mathbb{WCP}^1_{[n_+,n_-]}$. For this, q must be positive in an interval $y \in [y_1, y_2]$ where $q(y_1) = q(y_2) = 0$, and $y_2 > y_1 > 0$.[2] Near the endpoints of this interval, the metric becomes (we denote both roots collectively as y_i)

$$\mathrm{d}s^2_\Sigma = \frac{1}{q'(y_i)}\left(\frac{\mathrm{d}y^2}{y - y_i} + \frac{\left(q'(y_i)\right)^2(y - y_i)}{r(y_i)}\mathrm{d}z^2\right) = \frac{1}{|q'(y_i)|}\left(dx^2 + x^2\frac{\left(q'(y_i)\right)^2}{4r(y_i)}\mathrm{d}z^2\right), \quad (3.12)$$

[2]In this article, we have chosen the roots to be positive. This is a choice and not a requirement. Allowing for negative roots could lead to more solutions, possibly with both types of twists, as recently found in [31, 32].

where in the second step we have changed coordinates to a "near the pole" coordinate x defined by $y - y_i = \pm x^2/4$. We demand that the z coordinate is periodic with period Δz. This requires the following conditions

$$\frac{q'(y_i)\Delta z}{2\sqrt{r(y_i)}} \overset{!}{=} \pm\frac{2\pi}{n_\pm}\,, \tag{3.13}$$

where the upper sign corresponds to $y = y_1$ where $q'(y_1) > 0$, and the lower sign to $y = y_2$ where $q'(y_2) < 0$. This corresponds to a metric on the spindle that is regular everywhere except at the endpoints of the interval $y \in [y_1, y_2]$ where there is a conical singularity with a deficit angle $\alpha = 2\pi(1 - n_\pm^{-1})$. Changing from y to $\tilde{y}$ defined by $y \mapsto (\tilde{y}^3 - f_3)/f_i$, we can solve equation (3.13) to find an implicit equation for the roots y_i

$$\tilde{y}_{1,2}^3 = f_3 + \tilde{y}_{1,2}\left(\frac{3^{7/3}}{2^{11/3}m^2} \pm \frac{3\pi}{16m^3 n_\pm \Delta z}\right). \tag{3.14}$$

We can now compute the Euler number for the metric in equation (3.11). This is given by the integral of the Ricci scalar over the manifold

$$\chi(\Sigma) = \frac{1}{4\pi}\int_\Sigma \mathrm{d}y\,\mathrm{d}z\sqrt{g}R_\Sigma = \frac{1}{4\pi}\int_\Sigma \mathrm{d}y\,\mathrm{d}z\left(\frac{qr' - q'r}{r^{3/2}}\right)' = \frac{\Delta z}{4\pi}\frac{qr' - q'r}{r^{3/2}}\bigg|_{y=y_2}^{y=y_3} = \left(\frac{1}{n_-} + \frac{1}{n_+}\right), \tag{3.15}$$

which is indeed the right result for the spindle. Let us now compute the flux of the R-charge gauge field on the spindle

$$\frac{g}{2\pi}\int_\Sigma F^3 = \left(\frac{1}{n_+} + \frac{1}{n_-}\right). \tag{3.16}$$

Remarkably, this is equal to the Euler number of the spindle. The present situation resembles the "topological twist" that happens when compactifying on a Riemann surface of constant curvature. However, the local curvature on a spindle is not constant, therefore, the twist is like a topological twist, but only topologically. Hence this was referred to in [8] as a topologically topological twist. In contrast, the situation that we will find in section 4.2 is usually called an "anti twist".

3.4 Free energy on S^3

We have found a solution of the form $\mathrm{AdS}_4 \times \Sigma$ in 6d F(4) gauged supergravity. Since this is a consistent truncation of mIIA supergravity, the solution can be uplifted to 10d. The full 10d solution corresponding to this should be thought of as an interpolating solution between $\mathrm{AdS}_6 \times S^4$ and $\mathrm{AdS}_4 \times \Sigma \times S^4$. The $\mathrm{AdS}_6 \times S^4$ solution is dual to a 5d $\mathcal{N} = 1$ SCFT. So we expect the $\mathrm{AdS}_4 \times \Sigma \times S^4$ solution to be dual to the 3d SCFT obtained by compactifying the 5d SCFT on Σ. The free energy can be computed holographically to get

$$F_{S^3} = \frac{\pi L_{\mathrm{AdS}_4}^2}{2G_{4d}^{\mathrm{N}}} = \frac{2\pi L_{\mathrm{AdS}_4}^2}{9G_{6d}^{\mathrm{N}}}\int \mathrm{d}y\,\mathrm{d}z\,\frac{w^2}{\sqrt{r}} = \frac{2\pi L_{\mathrm{AdS}_4}^2}{9G_{6d}^{\mathrm{N}}}\left(\frac{\Delta z\,(\tilde{y}_2 - \tilde{y}_1)}{4\cdot 6^{1/3}m^2} - \frac{\pi\,(n_+\tilde{y}_2 + n_-\tilde{y}_1)}{12\cdot 6^{2/3}m^3 n_+ n_-}\right). \tag{3.17}$$

Since the function q is a polynomial of order 8, the explicit form of the roots $\tilde{y}_{1,2}$ is difficult to obtain. Instead, we expand the free energy as a perturbation series in the total magnetic charge on the spindle Q, which is defined as

$$Q = \frac{g}{2\pi}\int_\Sigma F^I = \frac{16\Delta z f_i m^3}{3\pi}\left(\frac{1}{\tilde{y}_1} - \frac{1}{\tilde{y}_2}\right). \tag{3.18}$$

Expanded around $Q = 0$, the free energy is[3]

$$\begin{aligned} F_{S^3} = &\frac{2\pi L^2_{\mathrm{AdS}_4}}{9G^{\mathrm{N}}_{6d}} \cdot \\ &\left[-\frac{\pi\left(n_+ + n_-\right)^3}{96m^4 n_+ n_-\left(n_+^2 - n_+ n_- + n_-^2\right)} + \frac{\pi n_+ n_-\left(n_+ + n_-\right)\left(n_+ - 2n_-\right)\left(2n_+ - n_-\right)Q^2}{192m^4\left(n_+^2 - n_+ n_- + n_-^2\right)^2}\right] \\ &+ \mathcal{O}\left(Q^4\right). \end{aligned} \tag{3.19}$$

We want to compare this to the free energy of the 3d SCFT dual to this solution. This can be obtained by computing the logarithm of the inverse of the partition function of the 5d SCFT placed on $S^3 \times \Sigma$, and then taking the large N limit. However, the same result can also be obtained holographically by using the technology of "gravitational blocks" introduced in [21]. This involves extremizing an entropy functional that is constructed by gluing gravitational blocks. The gravitational blocks are constructed from the prepotential which in this case is $\mathcal{F}(X_i) = (X_1 X_2)^{3/2}$, and is given by $\mathcal{B}(X_i) = \mathcal{F}(X_i)/\epsilon$. We then define the entropy functional to be

$$I = \frac{\pi^2 L^4_{\mathrm{AdS}_6}}{3G^{\mathrm{N}}_{6d}}\frac{8}{27}\left[\mathcal{B}\left(X_i^+\right) - \mathcal{B}\left(X_i^-\right) + \lambda\left(\Delta_1 + \Delta_2 - 2\right)\right], \tag{3.20}$$

where

$$X_1^\pm = \Delta_1 \mp \frac{\epsilon}{2n_\pm} \pm \frac{s\,\epsilon}{4}, \quad X_2^\pm = \Delta_2 \mp \frac{\epsilon}{2n_\pm} \mp \frac{s\,\epsilon}{4}. \tag{3.21}$$

Δ_i and ϵ are chemical potentials conjugated to the electric charges and the rotational symmetry of the spindle respectively.[4] λ is a Lagrange multiplier that enforces the constraint on the chemical potentials. The $\pm$ index on X_i corresponds to the values at the north and the south poles of the hemispheres which are glued together, and the relative minus sign between the blocks corresponds to the "A-gluing" in [21]. The coefficient involving L_{AdS_6} and G^{N}_{6d} in equation (3.20) is the free energy on S^5, and the factor of $8/27$ comes from the coefficient in front of $\mathcal{F}$, as well as our normalization of ϵ. This is an off-shell

[3]This perturbative expansion around an integer Q is just a trick that we have used due to the difficulty of finding the analytic form of the roots $\tilde{y}_{1,2}$, which are solutions to an order 8 polynomial equation. If one manages to find these analytical expressions, this trick can be avoided altogether and the computation can be done exactly. We have checked numerically that the free energy matches the extremized entropy functional for arbitrary Q, which justifies the trick in this case.

[4]Our entropy functional can related to that of [11] by taking their $r_i = 1$, $n_i = (1 \pm z)(n_- + n_+)/(n_- n_+)$, and redefining z in terms of s.

expression which needs to be extremized with respect to the chemical potentials. s is a continuous flavor charge and should correspond to Q in equation (3.18). We can perform the extremization

$$\partial_{\Delta_i} I = \partial_\epsilon I = 0, \tag{3.22}$$

perturbatively around $s = 0$. Upon extremization, we find that the entropy functional evaluated at the saddle point is

$$I_* = \frac{8\pi^2 L^4_{\mathrm{AdS}_6}}{81 G^{\mathrm{N}}_{6d}} \left[-\frac{3\,(n_+ + n_-)^3}{8 n_+ n_- \left(n_+^2 - n_+ n_- + n_-^2\right)} + \frac{3 n_+ n_- \,(n_+ + n_-)\,(n_+ - 2n_-)\,(2n_+ - n_-)\, s^2}{16\,\left(n_+^2 - n_+ n_- + n_-^2\right)^2} \right] + \mathcal{O}\left(s^4\right). \tag{3.23}$$

Comparing with equation (3.19), we see that they match with the identification $s = Q$.[5] The result can be checked to arbitrary order in the perturbation series. This shows that the free energy of the solution indeed matches the expectation from field theory and lends support to the duality that we suspected. We have performed further numerical checks for arbitrary charge Q and confirmed that the entropies are indeed equal.

4 Supersymmetric black hole solutions

4.1 Supersymmetry equations

We will now shift attention to solutions with a different topology, namely $\mathrm{AdS}_2 \times \Sigma \times \Sigma_{\mathfrak{g}}$, where $\Sigma_{\mathfrak{g}}$ is a smooth Riemann surface of genus $\mathfrak{g}$.[6] This can be thought of as the spacetime near the horizon of a black hole with the horizon geometry $\Sigma \times \Sigma_{\mathfrak{g}}$. Let us consider the following metric

$$\mathrm{d}s^2 = w(y) \left[\frac{4}{9} \mathrm{d}s^2_{\mathrm{AdS}_2} - \frac{\mathrm{d}y^2}{q(y)} - \frac{q(y)}{r(y)} \mathrm{d}z^2 \right] - w_1(y) \mathrm{d}s^2_{\Sigma_{\mathfrak{g}}}\,. \tag{4.1}$$

We assume that the scalar field $\phi_3(y)$ depends only on the coordinate y. This time, we turn on a non-zero two-form, and pick the non-zero components of the gauge fields (where again, the index i labels the gauge field from the vector multiplet) to be

$$A^3 = A^3_z(y)\mathrm{d}z + A^3_\phi(\theta)\mathrm{d}\phi\,, \quad A^i = A^i_z(y)\mathrm{d}z + A^i_\phi(\theta)\mathrm{d}\phi\,, \quad B = B_{tr}(y)\mathrm{d}t \wedge \mathrm{d}r\,,$$

where θ, ϕ are along the Riemann surface. It is consistent to choose a two-form with zero field strength i.e., $H_{\mu\nu\rho} = 0$, and we will make this simplifying assumption. With this, the only non-trivial equations of motion for the gauge field and the two-form are given by[7]

$$\begin{aligned} \partial_y \left(\sqrt{-g}\, e^{-2\sigma} \mathcal{N}_{\Lambda\Sigma} F^{\Sigma yz} \right) = \partial_\theta \left(\sqrt{-g}\, e^{-2\sigma} \mathcal{N}_{\Lambda\Sigma} F^{\Sigma\theta\phi} \right) = 0\,, \\ \frac{m^2}{4} \sqrt{-g}\, e^{-2\sigma} B_{tr} g^{tt} g^{rr} + \frac{1}{8} \eta_{\Lambda\Sigma} F^\Lambda_{yz} F^\Sigma_{\theta\phi} = 0\,. \end{aligned} \tag{4.2}$$

[5] Where we have used the identification $L^2_{\mathrm{AdS}_4} = 16 m^4 L^2_{\mathrm{AdS}_6}$.

[6] Normalizing the metric such that $R_{mn} = \kappa g_{mn}$, volume of the Riemann surface is $\mathrm{vol}_{\Sigma_{\mathfrak{g}}} = 4\pi|\mathfrak{g} - 1|$ for $\mathfrak{g} \neq 1$, and $\mathrm{vol}_{\Sigma_{\mathfrak{g}}} = 2\pi$ for $\mathfrak{g} = 1$. For $\kappa = -1$, the metric is locally H^2, and can be quotiented to obtain a constant curvature Riemann surface with $\mathfrak{g} > 1$.

[7] As before, for brevity of presentation, we will not write the y dependence of the fields explicitly.

For the U(1) gauge fields, this implies

$$\begin{aligned} F^3 &= \frac{e^{2\sigma}}{w_1}\left(f_3\cosh 2\phi_3 + f_i \sinh 2\phi_3\right)\mathrm{vol}_\Sigma + \tilde{f}_3 \mathrm{vol}_{\Sigma_\mathfrak{g}}\,, \\ F^I &= \frac{e^{2\sigma}}{w_1}\left(f_3\sinh 2\phi_3 + f_i \cosh 2\phi_3\right)\mathrm{vol}_\Sigma + \tilde{f}_i \mathrm{vol}_{\Sigma_\mathfrak{g}}\,, \end{aligned} \tag{4.3}$$

where $f_3, f_i, \tilde{f}_3, \tilde{f}_i$ are constants. The equation of motion for the two-form is purely algebraic and can be solved to give

$$B_{tr} = \frac{2R^4}{9m^2}\,, \tag{4.4}$$

where R is a constant. Additionally, $H_{\mu\nu\rho} = 0$ fixes the dilaton σ in terms of the scalar field ϕ_3, and the warp factor on the Riemann surface w_1

$$e^{-4\sigma} w_1^2 R^4 = \left(f_3\tilde{f}_3 - f_i\tilde{f}_i\right)\cosh 2\phi_3 + \left(f_i\tilde{f}_3 - f_3\tilde{f}_i\right)\sinh 2\phi_3 \;\coloneqq\; f{\cdot}\tilde{f}\,. \tag{4.5}$$

We choose the following representation for the 8×8 gamma matrices[8]

$$\Gamma^{0,1} = \gamma^{0,1}\otimes\sigma_3\otimes\sigma_3,\; \Gamma^2 = \mathbb{1}\otimes i\sigma_2\otimes\sigma_3,\; \Gamma^3 = \mathbb{1}\otimes i\sigma_1\otimes\sigma_3,\; \Gamma^4 = \mathbb{1}\otimes\mathbb{1}\otimes i\sigma_2,\; \Gamma^5 = \mathbb{1}\otimes\mathbb{1}\otimes i\sigma_1\,. \tag{4.6}$$

In this section, we will consider compactifications on a negatively curved Riemann surface i.e., $\kappa = -1$. For the 6d spinor, we choose $\Gamma^{45}\epsilon_A = i\epsilon_A$, and for the remaining 4d part of the spinor, we proceed similar to section 3. In addition, we choose to work in a gauge where the spinor is independent of the coordinate z as well as the coordinates on the Riemann surface. The supersymmetric solution can now be obtained by imposing that the fermionic variations in equation (2.3) vanish. Similar to section 3, we define an appropriate combination of the components of the spinor on the spindle, which we call ξ. In terms of this, we obtain the following set of BPS equations

$$\begin{aligned} \delta\chi &: \left(\frac{9me^{-\sigma}}{32w}B_{tr} - \frac{e^\sigma a_1}{8ww_1}\right)\sigma_3\xi - \left(\frac{e^{-\sigma}a_2}{8w_1} + \frac{a_5}{4}\right)\xi - \frac{1}{2}\sqrt{\frac{q}{w}}\sigma'\sigma_2\xi \overset{!}{=} 0\,, \\ \delta\lambda &: \frac{e^\sigma a_4}{ww_1}\sigma_3\xi + \left(\frac{e^{-\sigma}a_3}{w_1} + 2ge^\sigma\sinh\phi_3\right)\xi + \sqrt{\frac{q}{w}}\phi_3'\sigma_2\xi \overset{!}{=} 0\,, \\ \delta\psi_{Aa} &: \left(1 - \frac{e^\sigma a_1}{6\sqrt{w}w_1} - \frac{9me^{-\sigma}}{8\sqrt{w}}B_{tr}\right)\xi - \left(\frac{e^{-\sigma}a_2}{6w_1} + \frac{a_6}{3}\right)\sqrt{w}\sigma_3\xi - \frac{iw'\sqrt{q}}{3w}\sigma_1\xi \overset{!}{=} 0\,, \\ \delta\psi_{A2} &: i\left(\frac{3e^\sigma a_1}{8ww_1} + \frac{9me^{-\sigma}}{32w}B_{tr}\right)\sigma_1\xi - \left(\frac{e^{-\sigma}a_2}{8w_1} + \frac{a_6}{4}\right)\sigma_2\xi + \sqrt{\frac{q}{w}}\xi' \overset{!}{=} 0\,, \\ \delta\psi_{A3} &: -i\left(\frac{3e^\sigma a_1}{8ww_1} + \frac{9me^{-\sigma}}{32w}B_{tr}\right)\sigma_2\xi - \left(\frac{e^{-\sigma}a_2}{8w_1} + \frac{a_6}{4}\right)\sigma_1\xi - \frac{ig\sqrt{r}}{2\sqrt{wq}}A_z^3\xi \\ &\qquad + \frac{i\sqrt{r}}{2w}\left(\sqrt{\frac{wq}{r}}\right)'\sigma_3\xi \overset{!}{=} 0\,, \\ \delta\psi_{A4} &: \left(-\frac{e^\sigma a_1}{8ww_1} + \frac{9me^{-\sigma}}{32w}B_{tr}\right)\sigma_3\xi + \left(\frac{3e^{-\sigma}a_2}{8w_1} - \frac{a_6}{4}\right)\xi + \frac{w_1'\sqrt{q}}{4w_1\sqrt{w}}\sigma_2\xi \overset{!}{=} 0\,, \end{aligned} \tag{4.7}$$

[8] As before, numerical indices are frame indices. $0,1$ lie along AdS_2, $2,3$ lie along the spindle Σ, and $4,5$ lie along the Riemann surface $\Sigma_\mathfrak{g}$.

where we have defined the following combinations

$$\begin{aligned} a_1 &= (f_3 \cosh\phi_3 + f_i \sinh\phi_3) \,, & a_2 &= \left(\tilde{f}_3 \cosh\phi_3 - \tilde{f}_i \sinh\phi_3\right) \,, \\ a_3 &= \left(\tilde{f}_i \cosh\phi_3 - \tilde{f}_3 \sinh\phi_3\right) \,, & a_4 &= (f_3 \sinh\phi_3 + f_i \cosh\phi_3) \\ a_5 &= \left(g\cosh\phi_3 e^{\sigma} - 3m e^{-3\sigma}\right) \,, & a_6 &= \left(g\cosh\phi_3 e^{\sigma} + m e^{-3\sigma}\right) \,. \end{aligned} \tag{4.8}$$

The remaining variation $\delta\psi_{A5} \stackrel{!}{=} 0$ gives the same condition as $\delta\psi_{A4} \stackrel{!}{=} 0$, along with the condition that the R-symmetry gauge field along $\Sigma_{\mathfrak{g}}$ cancels the spin connection. This is the usual topological twisting condition when compactifying on a Riemann surface[9]

$$\tilde{f}_3 + \frac{\kappa}{2g} = 0 \Rightarrow \tilde{f}_3 = \frac{1}{6m}. \tag{4.9}$$

Note that all of the BPS equations above should be supplemented with the constraint in equation (4.5) and the value of the two-form in equation (4.4). For the sake of brevity, we don't write them explicitly.

4.2 Solution with a constant scalar

We will now solve the BPS equations. To simplify the system of equations, we further restrict to a family of solutions in which the fluxes on the Riemann surface are identified: $\tilde{f}_3 = \tilde{f}_i$. With this choice, equation (4.5) becomes

$$e^{-4\sigma} w_1^2 R^4 = \tilde{f}_i \left(f_3 - f_i\right) e^{-2\phi_3} \,. \tag{4.10}$$

Motivated by this, we pick a particular value of R that simplifies the equations significantly:

$$R^4 = 144 m^4 \tilde{f}_i \left(f_3 - f_i\right) \,. \tag{4.11}$$

This fixes the two-form B_{tr}, which we take to be real and therefore restrict ourselves to the family of fluxes which have $f_3 > f_i$. As a further simplification, we assume $\phi_3' = 0$. We will drop this assumption in the next subsection and construct a solution for arbitrary ϕ_3. With these simplifications, we can solve the BPS equations to find a simple solution

$$\begin{aligned} q_{1,2} &= \frac{1}{w'}\left[\pm\left(3w - 24\cdot 3^{1/8} m^2 f_i \sqrt{w}\right) + 4\cdot 3^{3/8} m w^{3/2}\right] \,, \\ w_1 &= \frac{1}{4\cdot 3^{3/4} m^2} \,, \qquad e^{2\phi_3} = \frac{1}{3} \,, \qquad r = r_0 \frac{w^3}{(w')^2} \,, \\ A_z^3 &= \frac{2\cdot 3^{3/8}}{\sqrt{w r_0}}\left(\sqrt{w} - 16\cdot 3^{1/8} m^2 f_i\right) \,, \qquad n = n_0 \sqrt{\frac{w'}{w}} \,, \end{aligned} \tag{4.12}$$

with $f_3 = 2 f_i$. r_0 is again an unphysical parameter that can be absorbed in a coordinate redefinition of z. It does not appear in any physical quantity and so we will not bother to specify it here.

[9] Recall that we have chosen $\kappa = -1$ and $g = 3m$.

4.2.1 Regularity of the solution

We have obtained a solution of the form $\mathrm{AdS}_2 \times \Sigma \times \Sigma_{\mathfrak{g}}$ where the scalar field ϕ_3 as well as the size of $\Sigma_{\mathfrak{g}}$ is constant, while the metric factors q and r depend on the warp factor w. Let us now examine the function q to ensure that the metric on Σ is smooth everywhere except at the poles. We choose a parametrization of the function w as $w(y) = y^2$. With this explicit choice, the metric coefficient r is positive, and q is a reduced quartic polynomial

$$q = q_1 q_2 = 4 \cdot 3^{3/4} m^2 y^4 - \frac{9y^2}{4} + 36 \cdot 3^{1/8} m^2 f_i \, y - 144 \cdot 3^{1/4} m^4 f_i^2 \,. \tag{4.13}$$

For $f_i > 0$, it has four real roots[10] given by

$$y_{1,2} = \frac{3^{1/8}}{8m} \left(-\sqrt{3} \mp \sqrt{3 + 128\sqrt{3} m^3 f_i} \right), \quad y_{3,4} = \frac{3^{1/8}}{8m} \left(\sqrt{3} \mp \sqrt{3 - 128\sqrt{3} m^3 f_i} \right), \tag{4.14}$$

where the order of the indices correspond to the upper and lower signs respectively. The function q is positive when y lies in the closed interval $y_2 \leq y \leq y_3$, and both roots are positive when $0 < 128 m^3 f_i < \sqrt{3}$. In fact, y_2 comes from $q_1 = 0$, while y_3 comes from $q_2 = 0$. Equation (3.13) can now be solved to find the period of z and to determine the flux

$$f_i = \frac{\sqrt{3}}{128 m^3} \left(\frac{n_-^2 - n_+^2}{n_-^2 + n_+^2} \right), \qquad \Delta z = \frac{\pi r_0}{3\sqrt{2} 3^{3/8} m} \sqrt{\frac{1}{n_-^2} + \frac{1}{n_+^2}} \,. \tag{4.15}$$

As a consistency check, we can again compute the Euler number for the metric in equation (3.11). This is given by the integral of the Ricci scalar

$$\chi(\Sigma) = \frac{1}{4\pi} \int_\Sigma \mathrm{d}y \, \mathrm{d}z \sqrt{g} R = \frac{\Delta z}{4\pi} \frac{q r' - q' r}{r^{3/2}} \bigg|_{y=y_2}^{y=y_3} = \left(\frac{1}{n_-} + \frac{1}{n_+} \right), \tag{4.16}$$

which is indeed the right result for the spindle. Let us now evaluate the total R-charge on the spindle. This integral receives contributions only from the endpoints of the interval to give

$$\frac{g}{2\pi} \int_\Sigma F^3 = \frac{g}{2\pi} \Delta z \left[A_z^3(y_3) - A_z^3(y_2) \right] = \left(\frac{1}{n_+} - \frac{1}{n_-} \right). \tag{4.17}$$

As alluded to in the previous section, this is not equal to the Euler number, but rather corresponds to an "anti twist" Finally, we can compute the Bekenstein-Hawking entropy of this black hole

$$S_{\mathrm{BH}} = \frac{\mathrm{Area}_{\Sigma \times \Sigma_{\mathfrak{g}}}}{4 G_{6d}^{\mathrm{N}}} = \frac{\mathrm{vol}_{\Sigma_{\mathfrak{g}}}}{4 G_{6d}^{\mathrm{N}}} \int \mathrm{d}y \mathrm{d}z \frac{w w_1}{\sqrt{r}} = - \frac{\pi \left(n_- + n_+ - \sqrt{2} \sqrt{n_-^2 + n_+^2} \right)}{48 \sqrt{3} m^4 n_- n_+} \frac{1}{4 G_{4d}^{\mathrm{N}}}, \tag{4.18}$$

where in the last step we have used $G_{4d}^{\mathrm{N}} = G_{6d}^{\mathrm{N}} / \mathrm{vol}_{\Sigma_{\mathfrak{g}}}$. Note that the expression within the parenthesis is always negative, and so the area has the correct sign.

[10] We have restricted ourselves to positive roots as discussed in footnote 2.

4.3 Solution with non-constant scalar

Let us now drop the assumption that $\phi_3' = 0$ and look for a solution with a non-constant scalar. We will still restrict ourselves to the family of fluxes where

$$\tilde{f}_3 = \tilde{f}_i, \quad R^4 = 144 m^4 \tilde{f}_i \left(f_3 - f_i\right) . \tag{4.19}$$

The BPS equations (4.7) now have the following solution

$$\begin{aligned}
q_{1,2} &= \frac{e^{-2\phi_3}}{8\left(f_3 - 2f_i\right)\phi_3'} \Big[\pm \Big(3\left(f_3 - f_i\right) - 6e^{2\phi_3}\left(2f_3 - f_i\right) + 9e^{4\phi_3}\left(f_3 + f_i\right) \Big) \\
&\quad -128 e^{3\phi_3} m^3 \left(f_3 - 2f_i\right)^2 \Big] , \\
w_1 &= \frac{e^{-\phi_3/2}}{12m^2} , \quad w = 256 e^{7\phi_3/2} m^4 \left(\frac{f_3 - 2f_i}{3e^{2\phi_3} - 1} \right)^2 , \quad r = r_0^2 \frac{e^{2\phi_3}}{(\phi_3')^2} , \\
A_z^3 &= \frac{2m^2 e^{-2\phi_3}}{r_0} \left[\left(3e^{4\phi_3} + 4e^{2\phi_3} - 3 \right) f_3 + \left(3e^{4\phi_3} - 2e^{2\phi_3} + 3 \right) f_i \right] , \\
n &= n_0 e^{3\phi_3/8} \sqrt{\frac{\phi_3'}{3e^{2\phi_3} - 1}} .
\end{aligned} \tag{4.20}$$

We have now obtained a solution in which the scalar field is not constant. The dilaton as well as the metric factors are determined in terms of the arbitrary scalar ϕ_3. The solution has two free parameters f_3 and f_i corresponding to the fluxes, in contrast to the solution with constant scalars which had only one free parameter f_i. So we expect this solution to reduce to the one in the previous subsection under a specific choice of fluxes. To see this, let us trade ϕ_3 for a new function $A(y)$ which we define by the following relation

$$e^{2\phi_3} = \frac{f_3 - 2f_i + A}{3A} , \tag{4.21}$$

and rescale the arbitrary constant r_0 to $r_0 = \tilde{r}_0 \left(f_3 - 2f_i\right)$. The solution in equation (4.20) can be rewritten in terms of A as follows

$$\begin{aligned}
w_1 &= \frac{1}{4 \cdot 3^{3/4} m^2} \left(\frac{A}{f_3 - 2f_i + A} \right)^{1/4} , \qquad w = \frac{256 m^4 A^{1/4}}{3 \cdot 3^{3/4}} \left(f_3 - 2f_i + A\right)^{7/4} , \\
r &= \frac{4A\tilde{r}_0^2}{3\left(A'\right)^2} \left(f_3 - 2f_i + A\right)^3 , \qquad n = \frac{n_0 \sqrt{A'}}{\sqrt{2} \left[27A^3 \left(f_3 - 2f_i + A\right)^5 \right]^{1/16}} , \\
q_{1,2} &= \frac{\mp 9\left(f_3 + f_i - 2A\right) + 128\sqrt{3} m^3 \sqrt{A} \left(f_3 - 2f_i + A\right)^{3/2}}{12A'} .
\end{aligned} \tag{4.22}$$

As before, the constraint in equation (4.10) determines σ. We have checked explicitly that this solution in terms of A is a solution to the BPS equations. It is now easy to see that imposing $f_3 = 2f_i$ takes us back to the solution with a constant scalar in section 4.2.

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4.3.1 Regularity of the solution

We have obtained a solution with the scalars and the metric factors depending on a single arbitrary function A. To analyse the structure of the metric, we will pick this function to be $A(y) = y$. We will further define fluxes g_1, g_2 as the following linear combinations of f_3, f_i

$$g_1 = f_3 + f_i\,, \quad g_2 = f_3 - 2f_i\,. \tag{4.23}$$

In terms of these fluxes, setting $g_2 = 0$ takes us back to the solution in section 4.2. With this choice,

$$q = \frac{1024}{3}m^6 y\,(y+g_2)^3 - \frac{9y^2}{4} + \frac{9g_1 y}{4} - \frac{9g_1^2}{16}\,, \tag{4.24}$$

which is again a quartic equation (but now including a y^3 term as well) with a positive coefficient for the leading term. This has four roots, with constraints on g_1, g_2 for all the roots to be real. In particular, we are interested in the cases where the middle two roots (we will call them y_2, y_3 like before) are positive. The interval $0 < y_2 \leq y \leq y_3$ then corresponds to a positive q. The metric coefficient r on the other hand is positive for $f_3 \geq 2f_i$. It is much more difficult to find the roots analytically in our present solution. Therefore, we have repeated the regularity analysis of section 4.2.1 by picking numerical values of the fluxes, and checked that it correctly reproduces the Euler character of the spindle, as well the total flux in equation (4.17).

Furthermore, to get an analytic handle on the entropy, we have performed a perturbative expansion as a series in g_2 around $g_2 = 0$ and checked against numerical results. We will briefly present this here. The strategy is the following: the two equations (3.13) describing the deficit angles determine the flux g_1 as well as the periodicity Δz in terms of g_2, y_2, y_3, and positive coprime integers $n_\pm$. This leaves g_2 undetermined. Further, $q\,(y_2) = q\,(y_3) = 0$ determines $y_{2,3}$ in terms of $g_2, n_\pm$. Since $g_2 = 0$ corresponds to the constant scalar solution of section 4.2, it is natural to expand all quantities as a perturbation series in g_2. The roots $y_{2,3}$ expanded in g_2 read

$$\begin{aligned} y_2 &= -\frac{3\sqrt{3}}{128m^3}\left(1 - \frac{\sqrt{2}n_-}{\sqrt{n_+^2+n_-^2}}\right) - \frac{3g_2}{4} + \frac{4\sqrt{2}m^3 n_- g_2^2}{\sqrt{n_+^2+n_-^2}} + \mathcal{O}\left(g_2^3\right)\,, \\ y_3 &= \frac{3\sqrt{3}}{128m^3}\left(1 - \frac{\sqrt{2}n_+}{\sqrt{n_+^2+n_-^2}}\right) - \frac{3g_2}{4} - \frac{4\sqrt{2}m^3 n_+ g_2^2}{\sqrt{n_+^2+n_-^2}} + \mathcal{O}\left(g_2^3\right)\,. \end{aligned} \tag{4.25}$$

We can now also compute the area as an expansion in g_2. The relevant physical quantity corresponding to g_2 is the magnetic charge on the spindle

$$Q \coloneqq \frac{g}{2\pi}\int_\Sigma F^I\,. \tag{4.26}$$

In the absence of g_2, the flux on the spindle is $Q_0 = (n_+ - n_-)\,/\,(4n_-n_+)$. Subtracting this constant flux from Q, we define $\tilde{Q} \coloneqq Q - Q_0$, and rewrite g_2 as an expansion in this parameter to get

$$g_2 = \frac{\sqrt{6}n_+n_-\tilde{Q}}{16m^3\sqrt{n_+^2+n_-^2}} + \frac{n_+^2n_-^2\left(n_++n_-+\sqrt{2}\sqrt{n_+^2+n_-^2}\right)\tilde{Q}^2}{2\sqrt{6}m^3\left(n_+^2-n_-^2\right)\sqrt{n_+^2+n_-^2}} + \mathcal{O}\left(\tilde{Q}^3\right) . \tag{4.27}$$

We can now compute the area as an expansion in $\tilde{Q}$,

$$\begin{aligned} S_{\rm BH} &= \frac{\mathrm{Area}_{\Sigma\times\Sigma_{\mathfrak{g}}}}{4G^{\rm N}_{6d}} = \frac{\mathrm{vol}_{\Sigma_{\mathfrak{g}}}}{4G^{\rm N}_{6d}} \int \mathrm{d}y\, \mathrm{d}z \frac{w w_1}{\sqrt{r}} \\ &= \frac{1}{4G^{\rm N}_{4d}}\left[-\frac{\pi\left(n_-+n_+-\sqrt{2}\sqrt{n_-^2+n_+^2}\right)}{48\sqrt{3}m^4 n_- n_+} - \frac{\pi n_+ n_- \tilde{Q}^2}{72\sqrt{6}m^4\sqrt{n_-^2+n_+^2}}\right] + \mathcal{O}\left(\tilde{Q}^3\right) , \end{aligned} \tag{4.28}$$

where in the last line we have used $G^{\rm N}_{4d} = G^{\rm N}_{6d}/\mathrm{vol}_{\Sigma_{\mathfrak{g}}}$. As expected, this indeed reduces to equation (4.18) for $\tilde{Q} = 0$.

4.4 Subtruncation to 4d gauged T^3 supergravity

Let us now interpret the $\mathrm{AdS}_2\times\Sigma\times\Sigma_{\mathfrak{g}}$ solutions that we have found in sections 4.2 and 4.3, in terms of solutions to the 6d theory compactified on the Riemann surface $\Sigma_{\mathfrak{g}}$. A general compactification of this form gives a four dimensional $\mathcal{N}=2$ gauged supergravity [33], a particular subtruncation of which is the “gauged T^3 model”. This theory consists of a single vector multiplet whose scalars parametrize the coset manifold $\mathrm{SU}(1,1)/\mathrm{U}(1)$. The same 4d theory can also be obtained as a consistent truncation of the “gauged STU model”, which is the maximal four dimensional $\mathcal{N}=8$ supergravity obtained from a reduction of eleven dimensional supergravity on S^7 [34, 35].

The truncation from 6d F(4) gauged supergravity to the 4d “gauged T^3 model” was performed in [33], and the 6d solutions that we have found turn out to correspond to this subtruncation. To see this, we can compare properties of our solution to those presented in [33]. The 4d fields (χ_1,χ_2,σ) can be identified with combinations of the 6d fields as follows

$$e^{4\chi_1} = \frac{e^{4\sigma}}{w_1^2} , \quad \chi_2 = \phi_3 , \quad e^{2\phi} = \frac{e^{-2\sigma}}{w_1} . \tag{4.29}$$

In the T^3 model, these scalars should have $e^{2\phi} = e^{2\chi_1-\chi_2} = 12m^2$. Using equation (4.20) or (4.22), we see that indeed our solution reproduces this. Additionally, if we rewrite the fluxes through $\Sigma_{\mathfrak{g}}$ in terms of s_1 and s_2

$$2\int_{\Sigma_{\mathfrak{g}}} F^3 = s_1 + s_2 , \quad 2\int_{\Sigma_{\mathfrak{g}}} F^I = s_1 - s_2 , \tag{4.30}$$

then the T^3 model has $s_1 = 1/\left(3m\right), s_2 = 0$. Recalling from equation (4.3) that $\int_{\Sigma_{\mathfrak{g}}} F^3 = \int_{\Sigma_{\mathfrak{g}}} F^I = 1/(6m)$, we see that indeed the solutions that we have found correspond to the 4d gauged T^3 subtruncation of the 6d theory.

Analogous to the discussion in section 3, we expect the full solution to interpolate between $\mathrm{AdS}_2\times\Sigma\times\Sigma_{\mathfrak{g}}\times S^4$ and $\mathrm{AdS}_4\times\Sigma_{\mathfrak{g}}\times S^4$. The $\mathrm{AdS}_4\times\Sigma_{\mathfrak{g}}\times S^4$ solution is dual

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to a 3d SCFT [4]. So it is natural to expect that our solution is dual to this 3d theory on a spindle. To find support for this duality, we compute the Bekenstein-Hawking entropy and compare it with a holographic computation where we minimize an entropy functional obtained by gluing gravitational blocks. To construct this off-shell entropy functional, we start from the on-shell free energy of the $\mathrm{AdS}_4 \times \Sigma_{\mathfrak{g}}$ solution, which is the free energy on S^3 and in the large N limit is given by $F_{S^3} = \pi L^2_{\mathrm{AdS}_4}/G^N_4$. We then promote this to an off-shell quantity by using the prepotential of the four dimensional magnetic STU model (and identifying three of the four fields, corresponding to the T^3 model) $\mathcal{F} = \sqrt{X_1 X_2^3}$. Using gravitational blocks defined by $\mathcal{B}(X_i) = \mathcal{F}(X_i)/\epsilon$, we construct the following entropy functional [21]

$$I = \frac{\pi L^2_{\mathrm{AdS}_4}}{2G^{\mathrm{N}}_{4d}} \left[\mathcal{B}\left(X_i^+\right) + \mathcal{B}\left(X_i^-\right) + \lambda\left(\Delta_1 + 3\Delta_2 - 2\right)\right], \tag{4.31}$$

where

$$X_1^{\pm} = \left(\Delta_1 - \frac{\epsilon}{2n_{\pm}} \mp \frac{s\,\epsilon}{2}\right), \quad X_2^{\pm} = \left(\Delta_2 - \frac{\epsilon}{2n_{\pm}} \pm \frac{s\,\epsilon}{6}\right). \tag{4.32}$$

As before Δ_i and ϵ are chemical potentials conjugated to the electric charges and the rotational symmetry of the spindle respectively, and λ is a Lagrange multiplier that enforces the constraint $\Delta_1 + 3\Delta_2 = 2$.[11] The $\pm$ index on X_i refers to the north and the south pole across which the gravitational blocks are glued. The plus sign between the blocks in equation (4.31) corresponds to the identity gluing in [21]. To find the entropy, we now need to extremize this functional with respect to Δ_i and ϵ. We do this perturbatively in the parameter s that corresponds to the flavor charge, to find the following extremized value

$$I_* = \frac{\pi L^2_{\mathrm{AdS}_4}}{2G^{\mathrm{N}}_{4d}} \left[-\frac{\left(n_- + n_+ - \sqrt{2}\sqrt{n_-^2 + n_+^2}\right)}{2n_- n_+} - \frac{n_+ n_- s^2}{3\sqrt{2}\sqrt{n_-^2 + n_+^2}}\right] + \mathcal{O}\left(s^3\right). \tag{4.33}$$

The four dimensional AdS length is determined from the scalar potential of the four dimensional theory to give $1/L^2_{\mathrm{AdS}_4} = 48\sqrt{3}m^4$ [33]. Remarkably this reproduces the Bekenstein-Hawking entropy in equation (4.28) with the identification $s = \tilde{Q}$. The agreement can be checked to arbitrary orders in the perturbation series. We have also repeated the exact computation numerically and we see that the entropies indeed match exactly. This lends support to our expectation about the holographic duality.

4.5 Solution without equal fluxes

So far we have solved the BPS equations on the locus given by $\tilde{f}_3 = \tilde{f}_i$. We will now drop this assumption, as well as keep κ arbitrary, and look for new solutions. However, we restrict ourselves only to solutions with a constant scalar $\phi_3' = 0$. Solutions to the BPS

[11] Our entropy functional can be related to [11] by taking their $r_i = 1/2$, $n_1 = (1+z)(n_- - n_+)/(4n_- n_+)$, $n_2 = (1 - z/3)(n_- - n_+)/(4n_- n_+)$ so that $n_1 + 3n_2 = (n_- - n_+)/(n_- n_+)$, and redefining z in terms of s.

equations are obtained in a way analogous to that outlined in the previous sections. The functions appearing in the metric are

$$\begin{aligned} w_1 &= -\frac{\kappa\sqrt{f_3}\left(f_3^2-f_i^2\right)^{3/4}}{6\sqrt{6}m^2\left(f_3^2-2f_i^2\right)}\,, \quad r=\frac{r_0 w^3}{(w')^2}\,, \\ q_{1,2} &= \pm\left(\kappa\frac{12\cdot 2^{3/4}3^{1/4}m^2\sqrt{w}\left(f_3^2-2f_i^2\right)}{f_3^{3/4}\left(f_3^2-f_i^2\right)^{1/8}w'}+\frac{3w}{w'}\right)+\frac{2^{5/4}3^{3/4}m f_3^{3/4}w^{3/2}}{\left(f_3^2-f_i^2\right)^{3/8}w'}\,. \end{aligned} \tag{4.34}$$

The scalar ϕ_3 and the normalization of the spinor n are

$$e^{2\phi_3}=\frac{f_3-f_i}{f_3+f_i}\,, \quad n=n_0\sqrt{\frac{w'}{w}}\,. \tag{4.35}$$

The R-charge gauge field along the spindle is

$$A_z^3=\frac{48m^2\kappa\left(f_3^2-2f_i^2\right)+2^{1/4}3^{3/4}f_3^{3/4}\left(f_3^2-f_i^2\right)^{1/8}\sqrt{w}}{\sqrt{\left(f_3^2-f_i^2\right)r_0 w}}\,, \tag{4.36}$$

and the fluxes $\tilde{f}_i$ are related to $f_{3,i}$ by

$$\tilde{f}_i=-\frac{\kappa f_3 f_i}{2g\left(f_3^2-2f_i^2\right)}\,. \tag{4.37}$$

This solution also turns out to have an interpretation in terms of the 6d F(4) gauged supergravity compactified on a Riemann surface. The particular subtruncation this corresponds to, is minimal supergravity in 4d. Its properties were discussed in [33], and we can check that the explicit solution obtained here has the correct properties. Parametrizing the fluxes in terms of s_1, s_2 using equation (4.30) as before, we find

$$s_1+s_2=-\frac{\kappa}{g}\,, \quad s_1-s_2=-\frac{\kappa f_3 f_i}{g\left(f_3^2-2f_i^2\right)}\,. \tag{4.38}$$

We can then compute the 4d scalars using the identification in equation (4.29) to find

$$\begin{aligned} \frac{e^{-2\sigma}}{w_1} &= \frac{24m^2}{-\kappa+m\sqrt{9\left(s_1-s_2\right)^2+4s_1s_2}}\,, \\ \frac{e^{4\sigma}}{w_1^2} &= \frac{96m^3}{m\left(9\left(s_1-s_2\right)^2+12s_1s_2\right)-\kappa\sqrt{9\left(s_1-s_2\right)^2+4s_1s_2}}\,, \\ e^{2\phi_3} &= \frac{2s_1}{\sqrt{9\left(s_1-s_2\right)^2+4s_1s_2}+3\left(s_1-s_2\right)}\,, \end{aligned} \tag{4.39}$$

which exactly matches the result for the 4d truncation in [33].

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4.5.1 Regularity of the metric

This solution is valid for a Riemann surface with arbitrary κ. However, to study the structure of the metric near the poles, we choose $\kappa = -1$, and pick the arbitrary function w to be $w = y^2$. The function q is a reduced quartic polynomial with a positive leading coefficient

$$q = \frac{3\sqrt{6} f_3^{3/2} m^2 y^4}{(f_3^2 - f_i^2)^{3/4}} - \frac{9y^2}{4} + \frac{18 \cdot 2^{3/4} \cdot 3^{1/4} (f_3^2 - 2f_i^2) m^2 y}{f_3^{3/4} (f_3^2 - f_i^2)^{1/8}} - \frac{72\sqrt{6} (f_3^2 - 2f_i^2)^2 m^4}{f_3^{3/2} (f_3^2 - f_i^2)^{1/4}} . \tag{4.40}$$

This has four roots

$$\begin{aligned} y_{1,2} &= \frac{3^{1/4} f_1^{1/8} \left(-f_1^{1/4} \pm \sqrt{f_1^{1/2} + 64 f_2 m^3} \right)}{4 \cdot 2^{1/4} m (2f_1 - f_2)^{3/8}} , \\ y_{3,4} &= \frac{3^{1/4} f_1^{1/8} \left(f_1^{1/4} \pm \sqrt{f_1^{1/2} - 64 f_2 m^3} \right)}{4 \cdot 2^{1/4} m (2f_1 - f_2)^{3/8}} , \end{aligned} \tag{4.41}$$

where we have defined the combination of fluxes $f_1 = f_3^2 - f_i^2$, and $f_2 = f_3^2 - 2f_i^2$. For the metric to be a smooth metric on the spindle, the middle two roots must be positive, when then specify the interval for y i.e., $y \in [y_2, y_3]$. The function r in the metric is always positive for $r_0 > 0$. There are conical singularities at the ends of this interval. Parametrizing the deficit angles by coprime integers $n_\pm$, we have

$$\frac{3^{7/4} 2^{1/4} (2f_1 - f_2)^{3/8} m \sqrt{f_1^{1/2} \pm 64 f_2 m^3}}{\sqrt{r_0} f_1^{5/8}} = \pm \frac{2\pi}{n_\pm \Delta z} . \tag{4.42}$$

This determines one of the fluxes and the periodicity in z in terms of the other flux and integers $n_\pm$. We can then compute the R-symmetry flux through the spindle, which turns out to be

$$\frac{g}{2\pi} \int_\Sigma F^3 = \left(\frac{1}{n_+} - \frac{1}{n_-} \right) . \tag{4.43}$$

Similar to the solutions in sections 4.2 and 4.3, this is of the "anti twist" type. The Euler character of the spindle can be computed similarly and indeed gives the correct result

$$\chi(\Sigma) = \left(\frac{1}{n_+} + \frac{1}{n_-} \right) . \tag{4.44}$$

Finally, let us compute the area of the horizon of this black hole, which is given straightforwardly in terms of the single free flux parameter. It is, however, more useful to use a different parametrization. Let us rewrite the R-symmetry flux and the magnetic flux through the Riemann surface in terms of a parameter ζ corresponding to the topological twist on $\Sigma_\mathfrak{g}$ which parametrizes the difference between the fluxes as follows

$$s_1 = -\frac{\kappa}{6m} \left(1 + \frac{\zeta}{\kappa} \right) , \quad s_2 = -\frac{\kappa}{6m} \left(1 - \frac{\zeta}{\kappa} \right) . \tag{4.45}$$

Restoring an arbitrary κ, the Bekenstein-Hawking entropy is

$$\begin{aligned} S_{\text{BH}} &= \frac{\text{Area}_{\Sigma\times\Sigma_{\mathfrak{g}}}}{4G^{\text{N}}_{6d}} \\ &= \frac{1}{4G^{\text{N}}_{4d}}\left[-\frac{\left(n_+ + n_- - \sqrt{2}\sqrt{n_+^2+n_-^2}\right)\pi}{n_+n_-}\cdot\frac{\left(\sqrt{\kappa^2+8\zeta^2}-3\kappa\right)^2}{864m^4\sqrt{2\kappa\left(\kappa-\sqrt{\kappa^2+8\zeta^2}\right)+4\zeta^2}}\right]. \end{aligned} \tag{4.46}$$

where in the last line, we have used $G^{\text{N}}_{4d} = G^{\text{N}}_{6d}/\text{vol}_{\Sigma_{\mathfrak{g}}}$. The second factor in the entropy is precisely $L^2_{\text{AdS}_4}$ for the four dimensional minimal supergravity obtained as a subtruncation of the six dimensional theory as obtained in [6, 33]. The first factor matches the entropy for $\text{AdS}_2\times\Sigma$ solution in 4d $\mathcal{N}=4$ gauged supergravity found in [9], in the absence of rotation. As a check, taking $\kappa=-1$ and $s_2=0$ (which corresponds to $\zeta=-1$) in the above, gives $1/L^2_{\text{AdS}_4} = 48\sqrt{3}m^4$, and correctly reproduces the entropy in equation (4.18).

We can again compute the entropy holographically using gravitational blocks. The prepotential is simply given by $\mathcal{F}(X) = X^2$, which defines the gravitational block $\mathcal{B}(X) = \mathcal{F}(X)/\epsilon$. The constraint now fixes $\Delta = 1/2$ to give

$$I = \frac{\pi L^2_{\text{AdS}_4}}{2G^{\text{N}}_{4d}}\left[\mathcal{B}\left(X^+\right)+\mathcal{B}\left(X^-\right)+\lambda\left(4\Delta-2\right)\right], \tag{4.47}$$

where

$$X^{\pm} = \left(\Delta - \frac{\epsilon}{2n_\pm}\right). \tag{4.48}$$

Extremizing this with respect to ϵ gives

$$I_* = \frac{\pi L^2_{\text{AdS}_4}}{2G^{\text{N}}_{4d}}\left[-\frac{\left(n_+ + n_- - \sqrt{2}\sqrt{n_+^2+n_-^2}\right)}{2n_+n_-}\right], \tag{4.49}$$

which exactly matches the entropy in equation (4.46) with $L^2_{\text{AdS}_4}$ identified as above.

5 Discussion

By solving the BPS equations in six dimensional F(4) gauged supergravity, we have found two classes of solutions: $\text{AdS}_4\times\Sigma$ and $\text{AdS}_2\times\Sigma\times\Sigma_{\mathfrak{g}}$. We conjectured that the $\text{AdS}_4\times\Sigma$ solution is dual to a five dimensional $\mathcal{N}=1$ SCFT on a spindle Σ, while the $\text{AdS}_2\times\Sigma\times\Sigma_{\mathfrak{g}}$ is dual to a three dimensional SCFT on Σ. We computed the entropy holographically by extremizing the entropy functional constructed from gravitational blocks and found that it agrees with the entropy computed from gravity. One class of our $\text{AdS}_2\times\Sigma\times\Sigma_{\mathfrak{g}}$ solutions corresponds to the gauged T^3 supergravity, while the other corresponds to minimal supergravity theory in four dimensions.

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Our solutions are obtained in a six dimensional truncation of mIIA supergravity, and an uplifted solution in ten dimensions can be constructed. The four dimensional T^3 sub-truncation of the six dimensional theory further admits an uplift in eleven dimensional supergravity. The solutions that we have found in this paper should then be expected to represent near horizon geometries of wrapped branes in ten or eleven dimensions. It would be very interesting to construct these uplifted solutions and understand the objects that they correspond to.

The solutions presented in this paper should be seen as fixed points of a flow from the supersymmetric AdS_6 solution. Constructing the full flow is often a challenging task. While there are a few examples of full analytic flows e.g., a rotating black hole in AdS_4 in [9, 12, 36], it can often only be done numerically. Nevertheless, it would be interesting to construct the full flow for the present solutions to better understand the objects they describe.

Lastly, we have constructed entropy functionals by appropriately gluing gravitational blocks and we have seen that they reproduce the entropy of the gravitational solutions. Finding an explanation of these entropy functionals from field theory would be very useful.

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