## The conformal brane-scan: an update

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Abstract: Generalizing the The Membrane at the End of the Universe, a 1987 paper Supersingletons by Blencowe and the author conjectured the existence of BPS p-brane configurations ( $p=2,3,4,5$ ) and corresponding CFTs on the boundary of anti-de Sitter space with symmetries appearing in Nahm's classification of superconformal algebras: $\operatorname{OSp}(N \mid 4) N=8,4,2,1 ; \operatorname{SU}(2,2 \mid N) N=4,2,1 ; F^{2}(4) ; \operatorname{OSp}\left(8^{*} \mid N\right), N=4,2$. This correctly predicted the $D 3$-brane with $\mathrm{SU}(2,2 \mid 4)$ on $A d S_{5} \times S^{5}$ and the $M 5$-brane with $\operatorname{OSp}\left(8^{*} \mid 4\right)$ on $A d S_{7} \times S^{4}$, in addition to the known $M 2$-brane with $\operatorname{OSp}(8 \mid 4)$ on $A d S_{4} \times S^{7}$. However, finding non-singular AdS solutions matching the other symmetries was less straightforward. Here we perform a literature search and confirm that all of the empty slots have now been filled, thanks to a number of extra ingredients including warped products and massive Type IIA. Orbifolds, orientifolds and S-folds also play a part providing examples not predicted: $\operatorname{SU}(2,2 \mid 3), \operatorname{OSp}(3 \mid 4), \operatorname{OSp}(5 \mid 4)$ and $\operatorname{OSp}(6 \mid 4)$ but not $\operatorname{OSp}(7 \mid 4)$. We also examine the status of $p=(0,1)$ configurations.

Keywords: Conformal Field Models in String Theory, M-Theory, P-Branes

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> Our mistake is not that we take our theories too seriously, but that we do not take them seriously enough.
> Steven Weinberg

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## 1 Supersingletons

The Membrane at the End of the Universe [1-10] was the name given to a supermembrane [11] (later called the M2-brane) on the $S^{1} \times S^{2}$ boundary of $A d S_{4} \times S^{7}$ described by a SCFT with symmetry

$$
\begin{equation*}
\mathrm{OSp}(8 \mid 4) \supset \mathrm{SO}(3,2) \times \mathrm{SO}(8) \tag{1.1}
\end{equation*}
$$

namely the $N=8$ singleton supermultiplet with 8 scalar and 8 spinors and $\mathrm{SO}(8) R$ symmetry. We recall that representations of $\mathrm{SO}(3,2)$ are denoted $D\left(E_{0}, s\right)$ where $E_{0}$ is the lowest energy eigenvalue which occurs and $s$ is the total angular momentum quantum number of the lowest energy state, analogous to the mass and spin of the Poincare group. However, Dirac's singletons $D(1 / 2,0)$ and $D(1,1 / 2)$ have no four-dimensional Poincare analogue [12] and are best interpreted a residing on the three-dimensional boundary $[2,13,14]$.

Accordingly, in 1987 Blencowe and the author [3] conjectured the existence of other BPS $p$-brane configurations with $p=(2,3,4,5)$ on the $S^{1} \times S^{p}$ boundary of $A d S_{(p+2)}$ and corresponding CFTs with other symmetries appearing in Nahm's classification of superconformal algebras [15], listed in table 1.

In each case the boundary CFT is described by the corresponding singleton (scalar), doubleton (scalar or vector) or tripleton (scalar or tensor) supermultiplet ${ }^{1}$ as shown in table 2. The number of dimensions transverse to the brane, $D-d$, equals the number of scalars in the supermultiplets. None of these BPS brane CFTs is self-interacting. (For non-BPS see $[18,19]$ ).

A plot of spacetime dimension $D$ vs worldvolume dimension $d=p+1$, known as the brane-scan, is shown in table 3 . This correctly predicted the $D 3$-brane [20-25] with $\mathrm{SU}(\mathbf{2}, \mathbf{2} \mid \mathbf{4})$ on $A d S_{5} \times S^{5}$ and the $M 5$-brane $[22,23,26]$ with $\operatorname{OSp}\left(\boldsymbol{8}^{*} \mid \mathbf{4}\right)$ on $A d S_{7} \times S^{4}$,

[^0]| d | G | H |  | Susy |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |
| 6 | $\mathrm{OSp}\left(8^{*} \mid N\right)$ | $\mathrm{SO}^{*}(8) \times \mathrm{USp}(N)$ | $N$ even | $8 N$ |
| 5 | $F^{2}(4)$ | $\mathrm{SO}(5,2) \times \mathrm{SU}(2)$ | $N \neq 4$ | $8 N$ |
| 4 | $\mathrm{SU}(2,2 \mid N)$ | $\mathrm{SU}(2,2) \times \mathrm{U}(N)$ |  | 32 |
|  | $\mathrm{SU}(2,2 \mid 4)$ | $\mathrm{SU}(2,2) \times \mathrm{SU}(4)$ |  | $4 N$ |
| 3 | $\mathrm{OSp}(N \mid 4)$ | $\mathrm{SO}(N) \times \mathrm{Sp}(4, \mathbb{R})$ |  |  |
| 2 | $G_{+} \times G_{-}$ |  |  |  |
| 1 | $G_{ \pm}=$ |  |  | $2 N$ |
|  | $\mathrm{OSp}(N \mid 2)$ | $\mathrm{O}(N) \times \mathrm{SU}(1,1)$ | $4 N$ |  |
|  | $\mathrm{SU}(N \mid 1,1)$ | $\mathrm{U}(N) \times \mathrm{SU}(1,1)$ | 8 |  |
|  | $\mathrm{SU}(2 \mid 1,1)$ | $\mathrm{SU}(2) \times \mathrm{SU}(1,1)$ | $8 N$ |  |
|  | $\mathrm{OSp}\left(4^{*} \mid 2 N\right)$ | $\mathrm{SU}(2) \times \mathrm{USp}(2 N) \times \mathrm{SU}(1,1)$ |  | 14 |
|  | $G(3)$ | $G_{2} \times \mathrm{SU}(1,1)$ | 16 |  |
|  | $F(4)$ | $\mathrm{Spin}(7) \times \mathrm{SU}(1,1)$ | 8 |  |

Table 1. Following [15, 16] we list the AdS supergroups in $d \leq 6$ and their bosonic subgroups in the notation of [17].
in addition to the known $M 2$-brane [11, 23] with $\operatorname{OSp}(\mathbf{8} \mid \mathbf{4})$ on $A d S_{4} \times S^{7}$. The purpose of the present paper is to report that all of the other slots have now been filled, thanks to a number of extra ingredients: warped products, massive Type IIA and Chern-Simons theories. Orbifolds, orientifolds and S-folds also play a part providing examples not predicted: $\operatorname{SU}(2,2 \mid 3), \operatorname{OSp}(3 \mid 4), \operatorname{OSp}(5 \mid 4)$ and $\operatorname{OSp}(6 \mid 4)$ but not $\operatorname{OSp}(7 \mid 4)$. We also examine the status of $p=(0,1)$ configurations.

## 2 The conformal brane-scan

Comments:

- The list in table 1 is complete if one assumes that the Killing superalgebras of AdS backgrounds are simple. However a more detailed investigation reveals that there may be some additional central generators in the Killing superlgebra for $A d S_{3}$ and $A d S_{5}$ backgrounds [27, 28]
- The supersingleton lagrangian and transformation rules were also spelled out explicitly in [3]. This conformal or (in later terminology) near-horizon brane-scan differs from the scan of Green-Schwarz type kappa-symmetric branes [29] which are not in general conformal and which, in any case, include only scalar supermultiplets. Further developments and elaborations on the brane-scan are summarized in Schreiber's n -lab and references therein.
- In early 1988, Nicolai, Sezgin and Tanii [5] independently put forward the same generalization of the Membrane at the End of the Universe idea, spelling out the doubleton

|  | Supergroup | Supermultiplet | $B^{-}$ | V | $\chi$ | $\phi$ | D |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A d S_{3}$ | $\operatorname{OSp}(n \mid 2) \times \operatorname{OSp}(8-n \mid 2)$ | $\left(n_{+}, n_{-}\right)=(n, 8-n), d=2$ singleton | 0 | 0 | 8 | 8 | 10 |
|  | $\operatorname{OSp}(n \mid 2) \times \operatorname{OSp}(4-n \mid 2)$ | $\left(n_{+}, n_{-}\right)=(n, 4-n), d=2$ singleton | 0 | 0 | 4 | 4 | 6 |
|  | $\operatorname{OSp}(n \mid 2) \times \operatorname{OSp}(2-n \mid 2)$ | $\left(n_{+}, n_{-}\right)=(n, 2-n), d=2$ singleton | 0 | 0 | 2 | 2 | 4 |
|  | $\operatorname{OSp}(n \mid 2) \times \operatorname{OSp}(1-n \mid 2)$ | $\left(n_{+}, n_{-}\right)=(n, 1-n), d=2$ singleton | 0 | 0 | 1 | 1 | 3 |
| $A d S_{4}$ | $\operatorname{OSp}(8 \mid 4)$ | $n=8, d=3$ singleton | 0 | 0 | 8 | 8 | 11 |
|  | $\operatorname{OSp}(4 \mid 4)$ | $n=4, d=3$ singleton | 0 | 0 | 4 | 4 | 7 |
|  | $\operatorname{OSp}(2 \mid 4)$ | $n=2, d=3$ singleton | 0 | 0 | 2 | 2 | 5 |
|  | $\operatorname{OSp}(1 \mid 4)$ | $n=1, d=3$ singleton | 0 | 0 | 1 | 1 | 4 |
| $A d S_{5}$ | $\mathrm{SU}(2,2 \mid 2)$ | $n=2, d=4$ doubleton | 0 | 0 | 2 | 4 | 8 |
|  | $\mathrm{SU}(2,2 \mid 1)$ | $n=1, d=4$ doubleton | 0 | 0 | 1 | 2 | 6 |
|  | $\mathrm{SU}(2,2 \mid 4)$ | $n=4, d=4$ doubleton | 0 | 1 | 4 | 6 | 10 |
|  | $\mathrm{SU}(2,2 \mid 2)$ | $n=2, d=4$ doubleton | 0 | 1 | 2 | 2 | 6 |
|  | $\mathrm{SU}(2,2 \mid 1)$ | $n=1, d=4$ doubleton | 0 | 1 | 1 | 0 | 4 |
| $A d S_{6}$ | $F^{2}(4)$ | $n=2, d=5$ doubleton | 0 | 0 | 2 | 4 | 9 |
| $A d S_{7}$ | $\operatorname{OSp}\left(8^{*} \mid 2\right)$ | $\left(n_{+}, n_{-}\right)=(1,0), d=6$ tripleton | 0 | 0 | 1 | 4 | 10 |
|  | $\operatorname{OSp}\left(8^{*} \mid 4\right)$ | $\left(n_{+}, n_{-}\right)=(2,0), d=6$ tripleton | 1 | 0 | 2 | 5 | 11 |
|  | $\operatorname{OSp}\left(8^{*} \mid 2\right)$ | $\left(n_{+}, n_{-}\right)=(1,0), d=6$ tripleton | 1 | 0 | 1 | 1 | 7 |

Table 2. Superconformal groups and their singleton, doubleton and tripleton representations. $B^{-}$, $V, \chi, \phi$ denote the number of chiral 2 -forms, vector, spinors and scalars in each multiplet. The spacetime dimension $D$ equals the worldvolume dimension $d$ plus the number of scalars.
and tripleton lagrangian and transformation rules, in addition to the singleton. However, by insisting on only scalar supermultiplets as in [29] their list excluded the vector or tensor brane-scans of table 3. In this case, as they point out, the spheres are just the parallelizable ones $S^{1}, S^{3}$ and $S^{7}$.

- The two factors appearing in the $p=1$ case, $G_{+} \times G_{-}$, are simply a reflection of the ability of strings to have left and right movers on the worldsheet [30]. In this case, there are many candidate supergroups as shown in table 1 , so for $p=0,1$ we did not attempt a complete list of which of these would eventually be realized. In [3], we focused on Type IIA, Type IIB and heterotic strings with $\operatorname{OSp}(n \mid 2)_{c} \times \operatorname{OSp}(8-n \mid 2)_{s}$, $\operatorname{OSp}(n \mid 2)_{c} \times \operatorname{OSp}(8-n \mid 2)_{c}$ and $\operatorname{OSp}(n \mid 2)_{c} \times \operatorname{Sp}(2, \mathbb{R})$, respectively, since the singleton CFTs (but not the supergravity $A d S_{3}$ solutions) had already been identified [30]. For concreteness the Type IIA case appears on the scan of table 3 .
- Even for $p \geq 2$ not all of the conformal algebras listed in table 1 appear in the scan. For example, since none of our CFTs is self-interacting, we restricted [3] $\mathrm{SU}(2,2 \mid N)$ to $N=1,2,4$ since perturbatively $N=3$ implies $N=4$. But we now know there
$\mathrm{D} \uparrow$


## SCALAR

11
10
9
8
7
6
5
4
3
2
1
0

## VECTOR

$\operatorname{OSp}(n \mid 2) \times \operatorname{OSp}(4-n \mid 2)$
$\operatorname{OSp}(4 \mid 4)$
$\operatorname{OSp}(2 \mid 4)$
$\operatorname{OSp}(1 \mid 4)$
$\operatorname{OSp}(n \mid 2) \times \operatorname{OSp}(8-n \mid 2) \quad \operatorname{OSp}\left(8^{*} \mid 2\right)$
$F^{2}(4)$
$\mathrm{SU}(2,2 \mid 2)$
$\mathrm{SU}(2,2 \mid 1)$
$\operatorname{OSp}(4 \mid 4)$
$\operatorname{OSp}(2 \mid 4)$
$\operatorname{OSp}(1 \mid 4)$
$\operatorname{OSp}(n \mid 2) \times \operatorname{OSp}(2-n \mid 2) \quad \operatorname{OSp}(1 \mid 4)$
$\operatorname{OSp}(n \mid 2) \times \operatorname{OSp}(1-n \mid 2)$
$\operatorname{OSp}(\mathbf{8} \mid \mathbf{4})$
$\mathrm{SU}(\mathbf{2}, \mathbf{2} \mid \mathbf{4})$
$\mathrm{SU}(2,2 \mid 2)$
$\mathrm{SU}(2,2 \mid 1)$

TENSOR
$\operatorname{OSp}\left(\mathbf{8}^{*} \mid \mathbf{4}\right)$
$\operatorname{OSp}\left(8^{*} \mid 2\right)$

Table 3. The brane-scans of superconformal groups: scalar supermultiplets: singletons $(p=1,2)$, doubletons $(p=3,4)$ and tripletons $(p=5)$; vector supermultiplets: doubletons $(p=3)$; tensor supermultiplets: tripletons $(p=5)$. The M2-, D3- and M5-branes are in boldface.
are nonperturbative interacting CFTs with just $N=3$ [31-35]. We also focussed on $N=1,2,4,8$ in $\operatorname{OSp}(N \mid 4)$ since they corresponded to the division algebra $\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$ interpretation of the four diagonal lines in the scalar branescan of table 3. The $N=3,5,6,7$ cases are discussed in section 4 .

## 3 Significance of the brane-scan

The significance of the $M 2, D 3$ and $M 5$ and indeed the other configurations on the branescan became clearer thanks to four major developments:

- Branes as solitons

The realization that string theory admits p-branes as solitons [20, 21, 23, 36-41]

- M-theory

The realization that the Type IIA superstring in $D=10$ could be interpreted [42] as a wrapped supermembrane in $D=11$ [11]. The membrane is a $1 / 2$ BPS solution of $D=11$ supergravity [43], whose spacetime approaches Minkowski space far away from the brane but $A d S_{4} \times S^{7}$ close to the brane, jumping to the full $\operatorname{OSp}(8 \mid 4)$ in the limit [44]. Regarded as an extremal black-brane, this limit was also called the near-horizon limit. Moreover multi-brane solutions could be obtained by stacking $N$ branes on top of one another [43], yielding quantized 4 -form flux. So $A d S_{4} \times S^{7}$ could equally well be regarded as the large $N$ limit. A similar story applied to its magnetic dual fivebrane [26] as a solution of $D=11$ supergravity. Moreover, the five string theories were merely different corners of an overarching M-theory [45-47] with $D=11$ supergravity as its low-energy limit. The membrane and fivebrane were accordingly renamed M2 and M5.

- D-branes

The realization that p -branes carrying RR charge, with a closed-string interpretation as solitons, admitted an alternative open string interpretation as Dirichlet-branes, surfaces of dimension $p$ on which open strings can end [25]. In particular the selfdual 3-brane, a solution of Type IIB supergravity with $A d S_{5} \times S^{5}$ and $\mathrm{SU}(2,2 \mid 4)$ in the large N limit, was reinterpreted as a D 3 -brane and renamed accordingly.

- AdS/CFT

The AdS/CFT conjecture [48-50] proposes that large N limits of certain conformal field theories in d dimensions can be described in terms of supergravity (and string theory) on the product of d+1-dimensional AdS space with a compact manifold. Another vital ingredient, missing in the early days, was the non-abelian nature of the symmetries that appear when we stack N branes on top of one another [51]. Examples include $N=4$ Yang-Mills in $D=4$ from $A d S_{5} \times S^{5}$ and ABJM theory [52] from $A d S_{4} \times S^{7} / Z_{n}$.

## 4 The missing ingredients $p \geq 2$

Notwithstanding the success with $M 2, D 3$ and $M 5$, for quite some time the status of the other slots on the brane-scans remained obscure. ${ }^{2}$ Here we perform a literature search and confirm that all of the empty slots have now been filled, largely thanks to warped products, massive Type IIA, and Chern Simons theories as shown below

- d=6 $\operatorname{OSp}\left(8^{*} \mid N\right) N=4,2 ;[54-60]$
- $\mathrm{d}=5 \mathrm{~F}^{2}(4)[59,61-68]$
- d=4 SU(2,2|N) $N=4,3,2,1 ;[20,31-35,59,69-72]$.
- $\mathrm{d}=3 \operatorname{OSp}(N \mid 4) N=8,6,5,4,3,2,1[43,52,59,73-80]$.


## Comments

- We have included $N=3$ in the $d=4$ case and $N=3,5,6$ in the $d=3$ case, which, as previously noted, were not predicted in [3]. $N=6$ appears in ABJM [52]. and its $\operatorname{OSp}(6 \mid 4)$ symmetry in [80]. A useful reference on the absence of $N=7$ is [59].
- There are no $A d S_{7}$ solutions in Types IIA and IIB. In M all are locally isometric to $A d S_{7} \times S^{4}$.
- There are no maximally supersymmetric $A d S_{6}$ backgrounds in M, IIA or IIB. There are no half BPS ( 16 supersymmetries) $A d S_{6}$ backgrounds in M and IIA with compact internal space.
- There are no such $A d S_{5}$ solutions that preserve $>16$ supersymmetries in IIA and $\mathrm{D}=11 \mathrm{In}$ IIB, all supersymmetric solutions are locally isometric to $\operatorname{AdS} S_{5} \times S^{5}$. This means that all backgrounds preserving 24 supersymmetries in IIB are locally $A d S_{5} \times S^{5}$.
- There are no $>16 A d S_{4}$ supersymmetric solutions in IIA and IIB. In $\mathrm{D}=11$ all $>16$ supersymmetric solutions are locally isometric to $A d S_{4} \times S^{7}$. This means that all solutions with $20,24,28$ are locally $A d S_{4} \times S^{7}$.


## $5 \quad p=0,1$

- $\mathrm{d}=2[55,78,81-94]$
- $\mathrm{d}=1$ [95-106]


## Comment

- Not all of the algebras in Nahm's list correspond to known solutions and indeed there may be some for which no solutions exist. A thorough and up-to-date summary maybe found in [94].

[^1]
## 6 Conclusion

Thus not only the M2, D3 and M5 but all of the $p$-brane configurations on the $S^{1} \times S^{p}$ boundary of $\operatorname{AdS} S_{(p+1)}$ with $p=(5,4,3,2,1)$ mentioned explicitly in the 1987 paper as shown in table 3 have now been discovered: $\operatorname{OSp}(N \mid 4) N=8,4,2,1 ; \operatorname{SU}(\mathbf{2}, \mathbf{2} \mid \mathbf{N}) N=$ $4,2,1 ; F^{2}(4) ; \operatorname{OSp}\left(8^{*} \mid N\right), N=4,2$, as have most of the ( $p=0,1$ ) in Nahm's list not mentioned explicitly. Orbifolds, orientifolds and S-folds also play a part providing examples not predicted: $\operatorname{SU}(2,2 \mid 3), \operatorname{OSp}(3 \mid 4), \operatorname{OSp}(5 \mid 4)$ and $\operatorname{OSp}(6 \mid 4)$ but not $\operatorname{OSp}(7 \mid 4)$. To be fair, if our colleagues did not take our vector and tensor brane-scans seriously in 1987, it may be because, in the Weinberg sense, we did not take them seriously enough ourselves.

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[^0]:    ${ }^{1}$ Our nomenclature, based on the rank of $A d S_{p+2}$, is singleton $p=2$, doubleton $p=(2,3)$, tripleton $p=5$ and differs from that of Günaydin and Minic [17].

[^1]:    ${ }^{2}$ In [53] we entertained the idea that they might arise from classical branes whose symmetry is enhanced when $\alpha^{\prime}$ corrections are taken into account, but this did not pan out.

