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Holographic DC conductivity for backreacted NLED in massive gravity

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ABSTRACT: In this work a holographic model with the charge current dual to a general non-linear electrodynamics (NLED) is discussed in the framework of massive gravity. Massive graviton can break the diffeomorphism invariance in the bulk and generates momentum dissipation in the dual boundary theory. The expression of DC conductivities in a finite magnetic field are obtained, with the backreaction of NLED field on the background geometry. General transport properties in various limits are presented, and then we turn to the three of specific NLED models: the conventional Maxwell electrodynamics, the Maxwell-Chern-Simons electrodynamics, and the Born-Infeld electrodynamics, to study the parameter-dependence of in-plane resistivities. Two mechanisms leading to the Mott-insulating behaviors and negative magneto-resistivities are revealed at zero temperature, and the role played by the massive gravity coupling parameters are discussed.

KEYWORDS: AdS-CFT Correspondence, Black Holes

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1 Introduction

The discovery of gauge/gravity duality makes it possible to deal with the strongly-coupled gauge theories on the boundary from the classical gravitational theories in the higher dimensional bulk [1–6]. And the well-known prediction on the ratio of the shear viscosity to the entropy density for $\mathcal{N} = 4$ super Yang-Mills (SYM) theory was found to be close to the experimental results of real quark-gluon plasma (QGP) [7–10], making it more convincing to physical community. In recent years the idea of gauge/gravity duality has been applied to hydrodynamics [11–14], quantum chromodynamics (QCD) [15–24], nuclear physics [25, 26], and strongly-coupled condensed matter systems [27–37], etc., and new insights are brought into these physical branches.

The establishment of quantum many-body theory [38, 39] in a fairly low energy scale compared with high energy physics is another magnificent and fascinating story apart from that of the standard model, which have profoundly extended and deepen our understanding of realistic matters. Many characterization methods, based on the linear response theory or quantum transport theory, have been developed in the scattering or transport experiments, to reveal the electronic or lattice structures, surface topography, defects and disorder, and study the transport properties of the material samples. In the framework of the energy band

theory the materials are roughly divided into three categories: metals, semiconductors, and insulators, which differs in their electrical conductivities. And it is known to all of us the classical electron motion in the presence of the magnetic field has been studied since 1879, which is the famous classical Hall effect [40]. Its quantum version, known as the quantum Hall effect, was first observed in 1980 by von Klitzing [41], has aroused a wide research enthusiasm in the last two decades and witnessed impressive theoretical and experimental breakthroughs [42, 43]. The new quantum phases of matter, termed topological insulators, exhibit another kind of bulk-boundary correspondence between the gapped insulated bulk and gapless metallic edge states on the boundary. Its exotic transport properties have also attracted lots of research interests, among which the conductivity behavior in the presence of magnetic field is of vital importance, for it provides a promising way on controlling the electrical properties with the help of external fields. For normal metals the resistivity always increases with the magnetic field strength [44], showing the positive magneto-resistivity. However, the experimental measurements [45–49] in topological materials, such as Dirac or Weyl semimetals, etc., demonstrated the presence of negative magneto-resistivity or crossover for positive to negative, which are mainly attributed to chiral anomaly [50–54]. In addition, the negative magneto-resistivity behaviors are also found in holographic chiral anomalous systems [55–58].

Mott insulators, which are believed to be parent materials of cuprate high T_c superconductors [59], is another example beyond the framework of energy band theory due to the strong Coulomb repulsive interaction. The strong electron-electron interaction would prevent the available charge carriers to efficiently transport charges as if suffers the electronic traffic jam. The holographic construction of Mott insulators has been a heated topic and much progress has been made [32, 60–67]. In ref. [68], the holographic model coupled with a particular NLED named iDBI was proposed to study the strong electron-electron interaction by introducing the self-interaction of NLED field, and the Mott-insulating behaviors appear for large enough self-interaction strength.

In this paper, we construct a holographic model coupled with a general NLED field to investigate the magneto-transport of the strongly-interacting system on the boundary from the perspective of gauge/gravity duality. The backreaction of NLED field on the bulk geometry is taken into consideration as in ref. [69]. The massive gravity, with massive potentials associated with the graviton mass, is adopted to break the diffeomorphism invariance in the bulk producing momentum relaxation in the dual boundary theory [70]. This model can serve as an effective description of strange metal phase and exhibiting the negative magneto-resistivities and Mott-insulating behaviors in certain circumstance.

This paper is organized as follows. In section 2 we establish our holographic model of massive gravity with NLED field. Then the DC conductivities with non-zero magnetic field are derived as the function of ρ , h , T , and related massive gravity coupling parameters in section 3, with various limit situation being discussed. Then in section 4 we present detailed investigation on the in-plane resistivities in the framework of conventional Maxwell electrodynamics, the CP-violating Maxwell-Chern-Simons electrodynamics, and Born-Infeld electrodynamics. Finally, we make our conclusion in section 5. We will use the unit $\hbar = G = k_B = l = 1$.

2 Holographic setup

The 4-dimensional massive gravity [70] with a negative cosmological constant Λ coupled to a nonlinear electromagnetic field A_μ we are considering is given by

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - 2\Lambda + m^2 \sum_{j=1}^4 c_j \mathcal{U}_j(g, f) + \mathcal{L}(s, p) \right], \quad (2.1)$$

where $\Lambda = -3/l^2$, m is the mass parameter, f is a fixed symmetric tensor called the reference metric, c_j are coupling constants,¹ and $\mathcal{U}_j(g, f)$ denotes the symmetric polynomials of the eigenvalue of the 4×4 matrix $\mathcal{K}_\nu^\mu = \sqrt{g^{\mu\lambda} f_{\lambda\nu}}$ given as

$$\begin{aligned} \mathcal{U}_1 &= [\mathcal{K}], \\ \mathcal{U}_2 &= [\mathcal{K}]^2 - [\mathcal{K}^2], \\ \mathcal{U}_3 &= [\mathcal{K}]^3 - 3[\mathcal{K}] [\mathcal{K}^2] + 2[\mathcal{K}^3], \\ \mathcal{U}_4 &= [\mathcal{K}]^4 - 6[\mathcal{K}]^2 [\mathcal{K}^2] + 8[\mathcal{K}] [\mathcal{K}^3] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]. \end{aligned} \quad (2.2)$$

The square root in $\mathcal{K}$ means $(\sqrt{\mathcal{K}})_\lambda^\mu (\sqrt{\mathcal{K}})_\nu^\lambda = \mathcal{K}_\nu^\mu$ and $[\mathcal{K}] \equiv \mathcal{K}_\mu^\mu$. The NLED Lagrangian $\mathcal{L}(s, p)$ in eq. (2.1) is constructed as the function of two independent nontrivial scalar using the field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$$s = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}, \quad (2.3a)$$

$$p = -\frac{1}{8} \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}. \quad (2.3b)$$

Here $\epsilon^{abcd} \equiv -[a b c d] / \sqrt{-g}$ is a totally antisymmetric Lorentz tensor, and $[a b c d]$ is the permutation symbol. In the weak field limit we assume the NLED reduces to the Maxwell-Chern-Simons Lagrangian $\mathcal{L}(s, p) \approx s + \theta p$ with θ defined as $\mathcal{L}^{(0,1)}(0, 0)$ for later convenience.

Varying the action with respect to $g^{\mu\nu}$ and A^μ we obtain the equations of motion

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{2} T_{\mu\nu}, \quad (2.4a)$$

$$\nabla_\mu G^{\mu\nu} = 0, \quad (2.4b)$$

where the energy-momentum tensor is

$$T_{\mu\nu} = g_{\mu\nu} \left(\mathcal{L}(s, p) - p \frac{\partial \mathcal{L}(s, p)}{\partial p} \right) + \frac{\partial \mathcal{L}(s, p)}{\partial s} F_\mu^\lambda F_{\nu\lambda} + m^2 \chi_{\mu\nu}, \quad (2.5)$$

with

$$\begin{aligned} \chi_{\mu\nu} &= c_1 (\mathcal{U}_1 g_{\mu\nu} - \mathcal{K}_{\mu\nu}) + c_2 (\mathcal{U}_2 g_{\mu\nu} - 2\mathcal{U}_1 \mathcal{K}_{\mu\nu} + 2\mathcal{K}_{\mu\nu}^2) \\ &\quad + c_3 (\mathcal{U}_3 g_{\mu\nu} - 3\mathcal{U}_2 \mathcal{K}_{\mu\nu} + 6\mathcal{U}_1 \mathcal{K}_{\mu\nu}^2 - 6\mathcal{K}_{\mu\nu}^3) \\ &\quad + c_4 (\mathcal{U}_4 g_{\mu\nu} - 4\mathcal{U}_3 \mathcal{K}_{\mu\nu} + 12\mathcal{U}_2 \mathcal{K}_{\mu\nu}^2 - 24\mathcal{U}_1 \mathcal{K}_{\mu\nu}^3 + 24\mathcal{K}_{\mu\nu}^4). \end{aligned} \quad (2.6)$$

¹To make the massive gravity theory self-consistent, all the c_j will be set to be negative.

And we introduce

$$G^{\mu\nu} = -\frac{\partial \mathcal{L}(s, p)}{\partial F_{\mu\nu}} = \frac{\partial \mathcal{L}(s, p)}{\partial s} F^{\mu\nu} + \frac{1}{2} \frac{\partial \mathcal{L}(s, p)}{\partial p} \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}. \quad (2.7)$$

We are looking for a black brane solution with asymptotic AdS spacetime by taking the following ansatz [70, 71] for the metric and the NLED field, and the reference metric:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2), \quad (2.8a)$$

$$A = A_t(r)dt + \frac{h}{2}(xdy - ydx), \quad (2.8b)$$

$$f_{\mu\nu} = \text{diag}(0, 0, \alpha^2, \alpha^2). \quad (2.8c)$$

where h is the magnetic field strength. The validity of eq. (2.8a) will be demonstrated in the appendix. From eqs. (2.8a) and (2.8b) we find the nontrivial scalars eq. (2.3) are

$$s = \frac{1}{2} \left(A_t'(r)^2 - \frac{h^2}{r^4} \right), \quad (2.9a)$$

$$p = -\frac{hA_t'(r)}{r^2}. \quad (2.9b)$$

Then the equations of motion are obtained as

$$rf'(r) + f(r) - 3r^2 = c_1\alpha m^2 r + c_2\alpha^2 m^2 + \frac{r^2}{2} \left(A_t'(r)G^{rt} + \mathcal{L}(s, p) \right), \quad (2.10a)$$

$$rf''(r) + 2f'(r) - 6r = c_1\alpha m^2 + r(\mathcal{L}(s, p) + hG^{xy}), \quad (2.10b)$$

$$\left[r^2 G^{rt} \right]' = 0, \quad (2.10c)$$

where the non-vanishing components $G^{\mu\nu}$ are

$$G^{rt} = \frac{\partial \mathcal{L}}{\partial p} \frac{h}{r^2} - \frac{\partial \mathcal{L}}{\partial s} A_t'(r), \quad (2.11a)$$

$$G^{xy} = \frac{\partial \mathcal{L}}{\partial s} \frac{h}{r^4} + \frac{\partial \mathcal{L}}{\partial p} \frac{A_t'(r)}{r^2}. \quad (2.11b)$$

eq. (2.10c) leads to $G^{tr} = \rho/r^2$ with ρ being a constant. The event horizon r_h is the root of $f(r)$, i.e., $f(r_h) = 0$, and the Hawking temperature of the black brane is given by

$$T = \frac{f'(r_h)}{4\pi}. \quad (2.12)$$

Then at $r = r_h$ eq. (2.10a) reduces to

$$4\pi T r_h - 3r_h^2 = c_1\alpha m^2 r_h + c_2\alpha^2 m^2 + \frac{r_h^2}{2} \left(A_t'(r_h)G_h^{rt} + \mathcal{L}(s_h, p_h) \right), \quad (2.13)$$

where

$$s_h = \frac{1}{2} \left(A_t'(r_h)^2 - \frac{h^2}{r_h^4} \right), \quad (2.14a)$$

$$p_h = -\frac{hA_t'(r_h)}{r_h^2}, \quad (2.14b)$$

$$G_h^{rt} = \mathcal{L}^{(0,1)}(s_h, p_h) \frac{h}{r_h^2} - \mathcal{L}^{(1,0)}(s_h, p_h) A_t'(r_h) \quad (2.14c)$$

3 DC conductivity

From the perspective of gauge/gravity duality, the black brane solution eqs. (2.8a) and (2.8b) in the bulk can describe an equilibrium state at finite temperature T given by eq. (2.13). And the conserved current $\mathcal{J}^\mu$ in the boundary theory is connected with the conjugate momentum of the NLED field in the bulk, which allows us to calculate the DC conductivity in the framework of linear response theory [72, 73].

3.1 Derivation of DC conductivity

The following perturbations on the metric and the NLED field are applied to derive the DC conductivity:

$$\delta g_{ti} = r^2 h_{ti}(r), \quad (3.1a)$$

$$\delta g_{ri} = r^2 h_{ri}(r), \quad (3.1b)$$

$$\delta A_i = -E_i t + a_i(r), \quad (3.1c)$$

where $i = x, y$. We first consider the t component. The absence of $A_t(r)$ in the eq. (2.9b) leads to the radially independent conjugate momentum $\partial_r \Pi^i = 0$, with

$$\Pi^t = \frac{\partial \mathcal{L}(s, p)}{\partial (A'_t(r))}. \quad (3.2)$$

Then the expectation value of $\mathcal{J}^t$ in the dual boundary field theory is given by

$$\langle \mathcal{J}^t \rangle = \Pi^t. \quad (3.3)$$

In the linear level we have $\langle \mathcal{J}^t \rangle = \rho$, which indicate that ρ can be interpreted as the charge density in the dual field theory. At the event horizon $r = r_h$ the charge density ρ is given by

$$\rho = \mathcal{L}^{(1,0)}(s_h, p_h) r_h^2 A'_t(r_h) - \mathcal{L}^{(0,1)}(s_h, p_h) h. \quad (3.4)$$

Then we consider the planar components. The NLED is explicitly independent of $a_i(r)$, making the conjugate momentum of the field $a_i(r)$

$$\Pi^i = \frac{\partial \mathcal{L}(s, p)}{\partial (a'_i(r))} = \frac{\partial \mathcal{L}(s, p)}{\partial (\partial_r A_i)} = \sqrt{-g} G^{ir}, \quad (3.5)$$

being radially independent as well. And the charge currents in the dual field theory are given by $\langle \mathcal{J}^i \rangle = \Pi^i$, and can be expressed with the perturbed metric and field components h_{ti} , h_{ri} , a'_i and E_i :

$$\langle \mathcal{J}^x \rangle = -\mathcal{L}^{(1,0)}(s, p) \left[f(r) a'_x(r) + h f(r) h_{ry}(r) + r^2 A'_t(r) h_{tx}(r) \right] - \mathcal{L}^{(0,1)}(s, p) E_y, \quad (3.6a)$$

$$\langle \mathcal{J}^y \rangle = -\mathcal{L}^{(1,0)}(s, p) \left[f(r) a'_y(r) - h f(r) h_{rx}(r) + r^2 A'_t(r) h_{ty}(r) \right] + \mathcal{L}^{(0,1)}(s, p) E_x. \quad (3.6b)$$

The perturbed metric components h_{ti} and h_{ri} are coupled to E_i in gravitational field equation and can be reduced. Thus we take the first order of the perturbed Einstein's equations of tx and ty components and we get:

$$\left(\frac{h^2}{r^2} - \frac{\alpha m^2 c_1 r}{\mathcal{L}^{(1,0)}(s, p)}\right) h_{tx}(r) - h A'_t(r) f(r) h_{ry}(r) = A'_t(r) f(r) a'_x(r) - \frac{E_y h}{r^2}, \quad (3.7a)$$

$$\left(\frac{h^2}{r^2} - \frac{\alpha m^2 c_1 r}{\mathcal{L}^{(1,0)}(s, p)}\right) h_{ty}(r) + h A'_t(r) f(r) h_{rx}(r) = A'_t(r) f(r) a'_y(r) + \frac{E_x h}{r^2}. \quad (3.7b)$$

Next by using the regularity constraints of the metric and field near the event horizon [73]:

$$\begin{aligned} f(r) &= 4\pi T(r - r_h) + \dots, \\ A_t(r) &= A'_t(r_h)(r - r_h) + \dots, \\ a_i(r) &= -\frac{E_i}{4\pi T} \ln(r - r_h) + \dots, \\ h_{ri}(r) &= \frac{h_{ti}(r)}{f(r)} + \dots, \end{aligned} \quad (3.8)$$

the eqs. (3.7a) and (3.7b) at the event horizon $r = r_h$ are

$$h A'_t(r_h) h_{ty}(r_h) - \left(\frac{h^2}{r_h^2} - \frac{\alpha m^2 c_1 r_h}{\mathcal{L}^{(1,0)}(s_h, p_h)}\right) h_{tx}(r_h) = A'_t(r_h) E_x + \frac{h}{r_h^2} E_y, \quad (3.9a)$$

$$\left(\frac{h^2}{r_h^2} - \frac{\alpha m^2 c_1 r_h}{\mathcal{L}^{(1,0)}(s_h, p_h)}\right) h_{ty}(r_h) + h A'_t(r_h) h_{tx}(r_h) = \frac{h}{r_h^2} E_x - A'_t(r_h) E_y. \quad (3.9b)$$

Solving eqs. (3.9a) and (3.9b) for $h_{ti}(r_h)$ in terms of E_i and inserting into eqs. (3.6a) and (3.6b), one can evaluate the current $\langle \mathcal{J}^i \rangle$ to the electric fields E_i at the event horizon $r = r_h$ via $\langle \mathcal{J}^i \rangle = \sigma_{ij} E_j$, where the DC conductivities are given by

$$\sigma_{xx} = \sigma_{yy} = \frac{\alpha m^2 c_1 r_h \left(\frac{\alpha m^2 c_1 r_h}{\mathcal{L}^{(1,0)}(s_h, p_h)} - \frac{h^2}{r_h^2} + r_h^2 A_t'^2(r_h) \right)}{\left(\frac{\alpha m^2 c_1 r_h}{\mathcal{L}^{(1,0)}(s_h, p_h)} - \frac{h^2}{r_h^2} \right)^2 + h^2 A_t'^2(r_h)}, \quad (3.10a)$$

$$\sigma_{xy} = -\sigma_{yx} = \frac{\mathcal{L}^{(1,0)}(s_h, p_h) r_h^2 A'_t(r_h)}{h} \left(1 - \frac{\left(\frac{\alpha m^2 c_1 r_h}{\mathcal{L}^{(1,0)}(s_h, p_h)} \right)^2}{\left(\frac{\alpha m^2 c_1 r_h}{\mathcal{L}^{(1,0)}(s_h, p_h)} - \frac{h^2}{r_h^2} \right)^2 + h^2 A_t'^2(r_h)} \right) - \mathcal{L}^{(0,1)}(s_h, p_h). \quad (3.10b)$$

The resistivity matrix is the inverse of the conductivity matrix:

$$R_{xx} = R_{yy} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} \quad \text{and} \quad R_{xy} = -R_{yx} = \frac{\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2}. \quad (3.11)$$

The event horizon radius r_h is the solution of eq. (2.13) and will be affected by the temperature T , the charge density ρ , the magnetic field h and the parameters $c_{1,2}$, m and α . For a given Lagrangian $\mathcal{L}(s, p)$ one first solve $A'_t(r_h)$ from eq. (3.4), and then plug it into eq. (2.13) to solve r_h , which will be brought into eqs. (3.10a) and (3.10b) to obtain the DC conductivity as a complicated function of the parameters mentioned above.

3.2 Various limits

The concrete examples of NLED will be discussed in section 4 and the properties of DC resistivity are discussed in detail. Before focus on the specific NLED model we first consider some general properties of the DC conductivity in various limits.

3.2.1 Massless and massive limits

The massless limit corresponds to $m \rightarrow 0$ and the system will restore the Lorentz invariance. It has been shown that in a Lorentz invariant theory that the DC conductivities in the presence of a magnetic field were $\sigma_{xx} = 0$ and $\sigma_{xy} = \rho/h$ [74]. For comparison, in the massless limit the DC conductivities in eqs. (3.10a) and (3.10b) become

$$\sigma_{xx} = \frac{\alpha m^2 |c_1| r_h^3}{h^2} + \mathcal{O}(m^4) \quad \text{and} \quad \sigma_{xy} = \frac{\rho}{h} + \mathcal{O}(m^4). \quad (3.12)$$

And in the massive limit $m \rightarrow +\infty$, the DC conductivities have the asymptotic behaviors

$$\sigma_{xx} = \mathcal{L}^{(1,0)}(s_h, p_h) + \mathcal{O}(m^{-2}) \quad \text{and} \quad \sigma_{xy} = -\mathcal{L}^{(0,1)}(s_h, p_h) + \mathcal{O}(m^{-2}), \quad (3.13)$$

which agree with the calculation by treating the NLED field as a probe one [75], for the geometry is dominated by the massive terms.

3.2.2 Zero field and charge density limits

In the zero field limit $h = 0$ case, the DC conductivities become

$$\sigma_{xx} = \mathcal{L}^{(1,0)}\left(\frac{A_t'^2(r_h)}{2}, 0\right) - \frac{\rho^2}{\alpha m^2 c_1 r_h^3} \quad \text{and} \quad \sigma_{xy} = -\mathcal{L}^{(0,1)}\left(\frac{A_t'^2(r_h)}{2}, 0\right), \quad (3.14)$$

where $A_t'(r_h)$ is obtained by solving

$$\rho = \mathcal{L}^{(1,0)}\left(\frac{A_t'^2(r_h)}{2}, 0\right) r_h^2 A_t'(r_h). \quad (3.15)$$

At zero charge density $\rho = 0$, the DC conductivities become

$$\sigma_{xx}^{-1} = \frac{1}{\mathcal{L}^{(1,0)}\left(-\frac{h^2}{2r_h^4}, 0\right)} - \frac{h^2}{\alpha m^2 c_1 r_h^3} \quad \text{and} \quad \sigma_{xy} = -\mathcal{L}^{(0,1)}\left(-\frac{h^2}{2r_h^4}, 0\right). \quad (3.16)$$

We find that the DC conductivities are in general non-zero and can be interpreted as incoherent contributions [76], known as the charge conjugation symmetric contribution σ_{ccs} . There is another contribution from explicit charge density relaxed by some momentum dissipation, σ_{diss} , depending on the charge density ρ . The results show that, for a general NLED model, the DC conductivities usually depend on σ_{diss} and σ_{ccs} in a nontrivial way.

Figure 1. Left panel: the metal-insulator phase diagram in the high temperature limit. The red region has positive temperature derivative of R_{xx} and thus describes a metal, while the blue region has negative temperature derivative, and hence an insulator. The black solid lines are the phase boundaries. Right panel: the Mott-insulating region (blue) and the negative magneto-resistivity (red) in the high temperature limit.

3.2.3 High temperature limit

Finally, we consider the high temperature limit $T \gg \max \left\{ \sqrt{h}, \sqrt{\rho}, |c_1| \alpha m^2, \sqrt{|c_2|} \alpha m \right\}$. In this limit, eq. (2.13) gives $T \approx \frac{3}{4\pi} r_h$. The longitudinal resistivity then reduces to

$$R_{xx} = \frac{1}{1 + \theta^2} \left\{ 1 + \frac{27}{64\pi^3 \alpha |c_1| T^3} \left[h^2 (1 + \theta^2) + 2h\theta\rho - \frac{1 - \theta^2}{1 + \theta^2} \rho^2 \right] \right\} + \mathcal{O}(T^{-6}). \quad (3.17)$$

The nonlinear effect will be suppressed by the temperature and we only keep the leading order. Usually the DC conductivities of metal and insulator have different temperature dependence. For the metallic materials the phonon scattering enlarges the resistivity, while the thermal excitation of carriers in insulating materials can promote the conductivity. And the metal-insulator transition can occur when the coefficient of $1/T^2$ changes the sign, which is shown in figure 1 in the $h/\rho - \theta$ parameter space. The θ term can break the $(\rho, h) \rightarrow (\rho, -h)$ or $(\rho, h) \rightarrow (-\rho, h)$ symmetries for σ_{ij} or R_{ij} , but σ_{ij} or R_{ij} are invariant under $(\rho, h) \rightarrow (-\rho, -h)$. And the phases are central symmetric in the parameter plane.

The Mott-insulating and magneto-resistance behaviors are also presented in figure 1. The Mott-insulating behavior region is where $\partial_{|\rho|} R_{xx} > 0$ in the parameter plane. The strong electron-electron interaction prevents the charge carriers from transporting. From eq. (3.17) we see that as long as $\theta^2 > 1$ the Mott-insulating behavior emerges in the absence of the magnetic field. The magneto-resistance is defined as

$$MR_{xx} = \frac{R_{xx}(h) - R_{xx}(0)}{R_{xx}(0)}, \quad (3.18)$$

and we see that in the figure 1 the negative magneto-resistivity can only occur with non-zero θ . For $\theta = 0$, eq. (3.17) reduces to

$$R_{xx} = 1 + \frac{27(h^2 - \rho^2)}{64\pi^3\alpha|c_1|T^3} + \mathcal{O}(T^{-6}), \quad (3.19)$$

which gives metallic behavior for $|h/\rho| < 1$ and insulating behavior for $|h/\rho| > 1$. And one always has $\partial_{|\rho|}R_{xx} < 0$ and $\partial_{|h|}R_{xx} > 0$, indicating the absence of Mott-insulating behavior and negative magneto-resistivity.

4 Various NLED models

In this section, we will use eqs. (2.13), (3.4) and (3.10a) to study the dependence of the in-plane resistivity R_{xx} on the temperature T , the charge density ρ and the magnetic field h in some specific NLED models. For convenience we rescale the c_i 's as $c_1 \sim \alpha m^2 c_1$ and $c_2 \sim \alpha^2 m^2 c_2$. The conventional Maxwell electrodynamics is firstly presented, with a detail discussion on the R_{xx} and R_{xy} 's dependence on involved massive gravity coupling parameters c_1 and c_2 , the charge density ρ , the magnetic field h and the temperature T . Then the Chern-Simons θ term as an extension is introduced to investigate the CP-violating effect. Finally, we discuss the Born-Infeld electrodynamics and the influence of non-linear effect on the DC resistivity. The high temperature behaviors have been discussed in section 3.2.3, so we will mainly focus on the behavior of R_{xx} around $T = 0$ in this section.

4.1 Maxwell electrodynamics

We first consider the Maxwell electrodynamics, in which $\mathcal{L}(s, p) = s$. From eq. (3.4) we can solve out that

$$A'_t(r_h) = \frac{\rho}{r_h^2}, \quad (4.1)$$

and bring it into eq. (2.13) we get the equation

$$4\pi T r_h - 3r_h^2 - c_1 r_h - c_2 + \frac{\rho^2 + h^2}{4r_h^2} = 0. \quad (4.2)$$

It is notice that c_1 offers an effective temperature correction, and make the solution complicated even at $T = 0$. Although the c_2 is a constant playing a similar role as the momentum dissipation strength in ref. [77], the effect on the resistivity is quite different. The effect of c_1 and c_2 on R_{xx} and R_{xy} at zero temperature is presented in figure 2. For $c_1 = 0$ the $R_{xx} = 0$ and R_{xy} is constant, and is independent of c_2 . For non-zero c_1 , the R_{xx} increases and saturates, while the R_{xy} decreases to zero as c_1 becomes more negative. And a larger $|c_2|$ makes the surface steeper. In the following figures 3 to 5 we separately investigate the resistivities as functions of ρ and h under different T and c_2 . Because c_1 acts as an effective temperature modification, we set $c_1 = -1$.

First, we consider zero temperature $T = 0$, and $c_2 = 0$. No Mott-insulating behaviors and positive magneto-resistivities are observed in figure 3. In addition, non-zero charge density can suppress the resistivity as more charge carriers are introduced. In figure 5 the

Figure 2. The influence of c_1 and c_2 on R_{xx} and R_{xy} . We take $T = 0$ and $\rho = 0.3, h = 0.2$ as an example.

Figure 3. Upper left: R_{xx} as a function of ρ and h . Upper right: contour plot of R_{xx} . Lower left: R_{xx} vs ρ with $h = 0, 0.5, 1, 1.5, 2$. Lower right: MR_{xx} vs h with $\rho = 0, 0.1, 0.2, 0.3, 0.4, 0.5$. We set $T = 0$, $c_1 = -1$, and $c_2 = 0$.

Figure 4. Upper left: R_{xx} as a function of ρ and h . Upper right: contour plot of R_{xx} . Lower left: R_{xx} vs ρ with $h = 0, 0.5, 1, 1.5, 2$. Lower right: MR_{xx} vs h with $\rho = 0, 0.1, 0.2, 0.3, 0.4, 0.5$. We set $T = 0.4$, $c_1 = -1$, and $c_2 = 0$.

effect of c_2 at zero temperature is presented. The figures show similar feature as those in figure 4. The larger c_2 also suppress the resistivity, while the influence is not so obvious as the temperature. All the pictures presented here do not exhibit the Mott-insulating behaviors and negative magneto-resistivities. Then the finite temperature situation with $T = 0.4$ and $c_2 = 0$ is studied in figure 4, where the saddle surface is relatively flatter than that in figure 3. And the magneto-resistivity is greatly suppressed by the temperature and the differences induced by charge density ρ is smoothen out.

Then we present the dependence of R_{xx} on h/ρ and $T/\sqrt{\rho}$ for $c_1 = -1$. The effect of different c_1 and c_2 can be deduced by change the temperature correspondingly, based on the analysis above. For $h < \rho$, figure 6 shows that the temperature dependence of R_{xx} is monotonic, and corresponds to metallic behavior. For $h > \rho$, the R_{xx} increases first and then decreases monotonically after reaching a maximum. The insulating behavior appears at high temperatures in this case. Moreover, if we take larger c_i , the metallic behaviors at $h > \rho$ would disappear and the R_{xx} decreases monotonically with increasing temperature. So we come to the conclusion that increasing the magnetic field would induce

Figure 5. Upper left: R_{xx} as a function of ρ and h . Upper right: contour plot of R_{xx} . Lower left: R_{xx} vs ρ with $h = 0, 0.5, 1, 1.5, 2$. Lower right: MR_{xx} vs h with $\rho = 0, 0.1, 0.2, 0.3, 0.4, 0.5$. We set $T = 0$, $c_1 = -1$, and $c_2 = -4$.

a finite-temperature transition or crossover from metallic to insulating behavior. In the last sub-figure of figure 6 the influence of temperature on the magneto-resistivity is shown. As temperature increases, the magneto-resistivity is remarkably suppressed.

At the end of the discussion of Maxwell electrodynamics, the Hall angle θ_H , defined as

$$\theta_H = \arctan \frac{\sigma_{xy}}{\sigma_{xx}}, \quad (4.3)$$

is shown in figure 7. At zero temperature for large h or ρ the Hall angle get saturated, with the sign depend on those of h or ρ . The surface is anti-symmetric under the transformation $(\rho, h) \rightarrow (\rho, -h)$ and $(\rho, h) \rightarrow (-\rho, h)$. As we expect, the temperature would evidently suppress the Hall angle.

4.2 Maxwell-Chern-Simons electrodynamics

The Lorentz and gauge invariance do not forbid the appearance of CP-violating Chern-Simons θ term in the electrodynamics Lagrangian

$$\mathcal{L}(s, p) = s + \theta p, \quad (4.4)$$

Figure 6. Upper left: R_{xx} as a function of h/ρ and $T/\sqrt{\rho}$. Upper right: contour plot of R_{xx} . Lower left: R_{xx} vs $T/\sqrt{\rho}$ with $h/\rho = 0, 0.5, 1, 1.5, 2$. Lower right: MR_{xx} vs h with $T/\sqrt{\rho} = 0, 0.1, 0.2, 0.3, 0.4, 0.5$. We set $T = 0$, $c_1 = -1$, and $c_2 = 0$.

Figure 7. Hall angle at $T = 0$ (left panel) and $T = 0.4$ (right panel). $c_1 = -1$ and $c_2 = 0$.

and the Chern-Simons theory is of vital importance for the both integer and fractional quantum Hall effects in condensed matter physics [78–81]. And the value of θ can be related to the Hall conductivity in the unit of $e^2/\hbar$. After taking the Chern-Simons term into consideration, eq. (3.4) gives

$$A'_t(r_h) = \frac{\rho + \theta h}{r_h^2}, \quad (4.5)$$

and the eq. (2.13) becomes

$$4\pi T r_h - 3r_h^2 - c_1 r_h - c_2 + \frac{(1 + \theta^2) h^2 + 2\theta h \rho + \rho^2}{4r_h^2} = 0. \quad (4.6)$$

We then reinvestigate the dependence of DC resistivity R_{xx} on ρ and h . As one expects, the similar saddle surface to that of Maxwell electrodynamics is found, and the reflection asymmetry and central symmetry due to the θ term is shown in the upper panel of figure 8. The more surprising things are the appearance of Mott-insulating behavior $\partial_{|\rho|} R_{xx} > 0$ and negative magneto-resistivity $M R_{xx} < 0$. At zero field R_{xx} is the even function of ρ , and as $|\rho|$ increases, the value of R_{xx} increases and reaches a maximum, showing the feature of Mott-insulating behavior, and then decreases monotonically. While for finite magnetic field, the reflection symmetry for positive and negative ρ is broken. Along the positive ρ direction the behavior of R_{xx} is similar, however, for negative ρ it is more complicated. As $|\rho|$ increases, the value of R_{xx} decreases and reaches a minimum, and then shares the similar behaviors as the positive ρ case. We then further study how the value of θ affect the Mott-insulating behavior in figure 9. To clearly show the region $\partial_{|\rho|} R_{xx} > 0$ we set other region's value to be zero. Similar results have been obtained as in ref. [77]. At zero field there is no Mott-insulating behavior for $\theta = 1$. A larger θ makes it possible even with $h = 0$. On the other hand, the Mott-insulating region becomes larger but $\partial_{|\rho|} R_{xx}$ becomes smaller as θ increases. The negative magneto-resistivity revealed in figure 8 is also shown in the $\rho - h$ parameter plane in figure 10, from which we can see that the negative magneto-resistivity emerges in the finite interval of h/ρ of about $[-1.2, 0]$. The positive magneto-resistivity region's value is set to be zero as well. At larger magnetic field, the negative magneto-resistivity phenomenon disappears, and the magneto-resistivity increases almost linearly with h . For zero density the negative magneto-resistivity does not appear. Finally we show the behavior of Hall resistivity R_{xy} and Hall conductivity σ_{xy} in figure 11. The negative Hall resistivity comes from the negative transverse conductivity σ_{xy} depending on the direction of magnetic field h and the type of charge carriers. From the figure we see that at zero field the transverse conductivity σ_{xy} is exactly $-\theta$ in regardless of charge density ρ , which agrees with the scenario in quantum Hall physics [42, 80]. At finite magnetic field, for positive ρ the Hall conductivity will decreases to zero, and then changes its sign, while for negative ρ the Hall conductivity remains negative. And we can also study the impact of magnetic field on the Hall conductivity with fixed ρ . All the transverse conductivities for various ρ as strong field tend to be zero as the result of localization. For positive ρ , in the positive magnetic field, the Hall conductivity decreases to zero, then changes its sign and increases to a maximum, and finally decreases monotonically to zero from the positive

Figure 8. Upper left: R_{xx} as a function of h and ρ . Upper right: contour plot of R_{xx} . Lower left: R_{xx} vs ρ with $h = 0, 0.05, 0.1, 0.2, 0.3$. Lower right: MR_{xx} vs h with $\rho = -0.6, -0.3, 0, 0.3, 0.6$. We set $T = 0$, $c_1 = -1.5$, $c_2 = 0$ and $\theta = 2$.

Figure 9. Mott-insulating region for $\theta = 1, 2, 3$ in the parameter plane for Maxwell-Chern-Simons electrodynamics at zero temperature. We set $c_1 = -1.5$ and $c_2 = 0$.

Figure 10. Negative magneto-resistivity region for $\theta = 3$ in the parameter plane for Maxwell-Chern-Simons electrodynamics at zero temperature. We set $c_1 = -1.5$ and $c_2 = 0$.

side. And in the negative magnetic field, the Hall conductivity increases to a negative maximum, and then decreases to zero from the negative side. The case of negative ρ can be known by simply making transformation $(\rho, h) \rightarrow (-\rho, -h)$ in the discussion above.

4.3 Born-Infeld electrodynamics

The Born-Infeld electrodynamics is described by a square-root Lagrangian [82, 83]

$$\mathcal{L}(s, p) = \frac{1}{a} \left(1 - \sqrt{1 - 2as - a^2 p^2} \right), \quad (4.7)$$

where the coupling parameter $a = (2\pi\alpha')^2$ relates to the Regge slope α' . It is believed that such a NLED governs the dynamics of electromagnetic fields on D-branes. If we take the zero-slope limit $\alpha' \rightarrow 0$, the Maxwell Lagrangian is recovered

$$\mathcal{L}(s, p) = s + \mathcal{O}(a). \quad (4.8)$$

The Born-Infeld electrodynamics takes the advantage of eliminating the divergence of electrostatic self-energy and incorporating maximal electric field strength [84]. This can be seen from the solution of eq. (3.4)

$$A'_t(r) = \frac{\rho}{\sqrt{a(\rho^2 + h^2) + r^4}}, \quad (4.9)$$

which is finite when $r \rightarrow 0$. The r_h is solved form

$$4\pi r_h T - 3r_h^2 - c_1 r_h - c_2 + \frac{\rho^2}{2\sqrt{a(h^2 + \rho^2) + r_h^4}} + \frac{1}{2a} \left(\sqrt{\frac{(ah^2 + r_h^4)^2}{a(h^2 + \rho^2) + r_h^4}} - r_h^2 \right) = 0. \quad (4.10)$$

Figure 11. Upper left: R_{xy} as a function of h and ρ . Upper right: contour plot of R_{xy} . Lower left: σ_{xy} vs ρ with $h = 0, 0.05, 0.1, 0.2, 0.3$. Lower right: σ_{xy} vs h with $\rho = -6, -3, 0, 3, 6$. We set $T = 0$, $c_1 = -1.5$, $c_2 = 0$ and $\theta = 2$.

The DC resistivity is then studied in the framework of Born-Infeld electrodynamics and the results are shown in figure 13. For positive a the behavior of R_{xx} is similar to the Maxwell case [69, 77], and the negative a can bring in more interesting phenomenon. However, for negative a the eq. (4.9) suffers a singularity $r = r_s \equiv {}^{1/4}\sqrt{|a|(h^2 + \rho^2)}$, and the physical solution requires that $r_h > r_s$, setting an upper bound for $h^2 + \rho^2$ [77]. We then present two cases of $a = -0.4$ and $a = -1$, respectively. For $a = -0.4$, We see the saddle surface is similar to the previous results, and there is no Mott-insulating behaviors and negative magneto-resistivities. However, if one increases the absolute value of the negative a , the region admitting physical solution for R_{xx} shrinks and is truncated at the upper bound of $h^2 + \rho^2$. For zero and finite small magnetic field, the Mott-insulating behavior is absent, while for a larger field the Mott-insulating behavior emerges. And for a relatively small ρ , the negative magneto-resistivity is present, and increasing ρ can destroy it. To expose the role of a we expand the interaction between electrons to the first order [77]

$$F(a) = \rho A'_t(r) \sim \frac{\rho^2}{r^2} \left[1 - \frac{a}{2r^4} (h^2 + \rho^2) \right] + \mathcal{O}(a^2), \quad (4.11)$$

Figure 12. R_{xx} as a function of h and ρ and some intersection curves for fixed h and ρ for $a = -0.4$ (upper panel) and $a = -1$ (lower panel). The surface ends at some place where h and ρ touch the upper bound.

in which the leading order is the familiar Coulomb interaction, and the nonlinearity parameter serves as an effective modification. A positive a suppress the interaction and we do not expect different phenomenon compared with the Maxwell electrodynamics. However, for a negative a the interaction is enhanced at r_h

$$\frac{F(a)}{F(0)} = 1 + \frac{1}{2} \left(\frac{r_s}{r_h} \right)^4, \quad (4.12)$$

and we expect that the NLED model with negative a can grasp some features of strongly correlated systems. Besides, in the leading order expansion eq. (4.8) the Chern-Simons term does not appear, and can be deduced from eq. (3.19) that the system will not exhibit Mott-insulating behavior and negative magneto-resistivity. Thus we conclude that the Born-Infeld electrodynamics provides a new mechanism different from the Chern-Simons theory to give rise to the Mott-insulating behavior and negative magneto-resistivity, and the temperature can induce a transition at finite temperature.

One can also consider another kind of construction of square-root Lagrangian [84]

$$\mathcal{L}(s, p) = \frac{1}{a} \left(1 - \sqrt{1 - 2as} \right), \quad (4.13)$$

which has the same leading order expansion as eq. (4.8), and will re-derive the results of Born-Infeld electrodynamics in the presence of zero field. The solution of eq. (3.4) gives

$$A'_t(r) = \frac{\rho}{r^2} \sqrt{\frac{ah^2 + r^4}{a\rho^2 + r^4}}, \quad (4.14)$$

Figure 13. R_{xy} as a function of h and ρ and some intersection curves for fixed h and ρ for $a = -0.4$ (upper panel) and $a = -1$ (lower panel).

which can be deduced that the effective interaction is

$$F = \rho A'_t(r) \sim \frac{\rho^2}{r^2} \left[1 - \frac{a}{2r^4} (\rho^2 - h^2) \right] + \mathcal{O}(a^2). \quad (4.15)$$

The radius equation is

$$4\pi r_h T - 3r_h^2 - c_1 r_h - c_2 + \frac{h^2}{2r_h^2} + \frac{r_h^2}{2a} \left(\sqrt{\frac{a\rho^2 + r_h^4}{ah^2 + r_h^4}} - 1 \right) = 0. \quad (4.16)$$

We re-check the R_{xx} in the square-root electrodynamics for negative a . For $a = -0.4$ the saddle surface is similar to the previous results, and both the zero field and finite field no Mott insulating behaviors are shown. And for fixed ρ , it is surprising that for larger ρ , the transition from positive magneto-resistivity to negative magneto-resistivity is observed for about $\rho > h$. We then studied the case of larger negative a . To our surprise, the saddle surface changes its direction and looks as if it rotates 90° . We find that the Mott-insulating behavior appears and becomes significant at large ρ , and negative magneto-resistivity is observed for various ρ .

5 Conclusion

In this work the black brane solution of four-dimensional massive gravity with backreacted NLED is obtained, and with the dictionary of gauge/gravity duality, the transport properties of the strongly correlated systems in the presence of finite magnetic field in 2+1-dimensional boundary is studied. In our holographic setup the bulk geometry and NLED

	Lagrangian	Parameter	Mott-insulating behavior	Negative magneto-resistivity
Maxwell	s		No	No
Maxwell-Chern-Simons	$s + \theta p$	θ	See figure 9	See figure 10
Born-Infeld	$\frac{1}{a} \left(1 - \sqrt{1 - 2as - a^2 p^2}\right)$	$a > 0$	No	No
		$a = -0.4$	No	No
		$a = -1$	Finite h and small ρ . See figure 12	Finite h and small ρ . See figure 12
Square	$\frac{1}{a} (1 - \sqrt{1 - 2as})$	$a > 0$	No	No
		$a = -0.4$	No	Larger ρ . See figure 13
		$a = -1$	Larger ρ . See figure 13	Yes

Table 1. Conclusion of Mott-insulating behavior and negative magneto-resistivity for various NLED models at zero temperature $T = 0$.

field are perturbed, and the DC conductivities are obtained in the linear response regime. Then some general properties are obtained in various limits, which agrees well with the previous work. To make it concrete, we present the study of the conventional Maxwell electrodynamics, the topological non-trivial Maxwell-Chern-Simons electrodynamics, and the Born-Infeld electrodynamics with string-theoretical correction taken into consideration. We concentrate on two interesting phenomena, i.e., the Mott-insulating behavior and negative magneto-resistivity, and results at zero temperature are summarized in table 1.

We briefly revisit the various limits for Born-Infeld electrodynamics. In the massless limit, the σ_{xx} has a lower bound controlled by r_s , which is affected by the NLED parameter a . In the massive limit, For the Maxwell-Chern-Simons electrodynamics, the DC conductivities is $\sigma_{xx} = 1$ and $\sigma_{xy} = -\theta$. While for the Born-Infeld electrodynamics, we have

$$\begin{aligned}\sigma_{xx} &= \frac{r_h^2 \sqrt{r_h^4 + a(h^2 + \rho^2)}}{r_h^4 + ah^2}, \\ \sigma_{xy} &= \frac{ah\rho}{r_h^4 + ah^2}.\end{aligned}\tag{5.1}$$

The σ_{xx} is suppressed for negative a and will vanishes at r_s . Besides, the negative a implies the Mott-insulating behavior. And the Hall conductivity does not vanish as a result of non-zero a . For the zero field or charge density limit, the Hall conductivity vanishes in both cases. And the longitudinal conductivity has the form

$$\begin{aligned}\sigma_{xx}(h = 0) &= \left(1 - \frac{a\rho^2}{a\rho^2 + r_h^4}\right)^{-1/2} - \frac{\rho^2}{\alpha m^2 c_1 r_h^3}, \\ \sigma_{xx}^{-1}(\rho = 0) &= \left(1 + \frac{ah^2}{r_h^4}\right)^{-1/2} - \frac{h^2}{\alpha m^2 c_1 r_h^3}.\end{aligned}\tag{5.2}$$

We see that when reaching r_s , the square root part is removed from the formula, leaving the rest part with a quadratic dependence on h or ρ .

The massive gravity coupling parameters' influence on the in-plane resistivity is compared with the temperature, where we find the c_1 behaves as the effective temperature correction, and non-zero one can lead to the DC conductivity. While the c_2 has less significant effect on R_{xx} than c_1 . Moreover, the dependence on ρ and h is shown and we find the field can induce the metal-insulator transition or Mott-insulating behavior and negative magneto-resistivity. Two different mechanism, the Chern-Simons term, and the negative nonlinearity parameter are proved that can give rise to Mott-insulating behavior and negative magneto-resistivity. The fact that the nonlinearity construction can result in the negative magneto-resistivity and Mott insulating behavior provides a different mechanism without CP-violation construction in Chern-Simons term. Besides, the effective interaction extracted from the NLED Lagrangian can qualitatively account for the presence of Mott insulating behavior. However, there are also some shortcomings. To avoid the appearance of naked singularity, we have to make some restriction on the radius of event horizon, which sets an upper bound for the magnetic field and charge density for the black hole, and the physical solution space shrinks. In addition, how the nonlinearities brings in the negative magneto-resistivity remains to be explored. We hope our work could explain some experimental phenomenon in strongly correlated systems.

A Consistency of metric ansatz

The reparametrisation freedom for the radial coordinate is not enough to fix both $g_{tt} = -1/g_{rr}$ and $g_{xx} = g_{yy} = r^2$ at the same time. To test the validity of eq. (2.8a), we write the metric as [71]:

$$ds^2 = -N(r)^2 f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 (dx^2 + dy^2), \quad (\text{A.1a})$$

$$A = A_t(r) dt + \frac{h}{2} (x dy - y dx), \quad (\text{A.1b})$$

$$f_{\mu\nu} = \text{diag}(0, 0, \alpha^2, \alpha^2), \quad (\text{A.1c})$$

where $N(r)$ is a scale function of r and will be proved to be a constant, and thus can be absorbed into the coordinate t . With the reference metric $f_{\mu\nu}$ we have

$$\mathcal{U}_1 = 2\alpha/r, \mathcal{U}_2 = 2\alpha^2/r^2, \mathcal{U}_3 = \mathcal{U}_4 = 0, \quad (\text{A.2})$$

and the non-vanishing $G^{\mu\nu}$ in eq. (2.7) in the main text are

$$G^{rt} = \frac{\partial \mathcal{L}}{\partial p} \frac{h}{N(r)r^2} - \frac{\partial \mathcal{L}}{\partial s} \frac{A'_t(r)}{N(r)^2}, \quad (\text{A.3a})$$

$$G^{xy} = \frac{\partial \mathcal{L}}{\partial s} \frac{h}{r^4} + \frac{\partial \mathcal{L}}{\partial p} \frac{A'_t(r)}{N(r)r^2}. \quad (\text{A.3b})$$

We now consider the equations of motion for t and r component:

$$rf'(r) + f(r) - 3r^2 = c_1\alpha m^2 r + c_2\alpha^2 m^2 + \frac{r^2}{2} \left(A'_t(r)G^{rt} + \mathcal{L}(s, p) \right), \quad (\text{A.4a})$$

$$rf'(r) + f(r) - 3r^2 = c_1\alpha m^2 r + c_2\alpha^2 m^2 + \frac{r^2}{2} \left(A'_t(r)G^{rt} + \mathcal{L}(s, p) \right) + 4rf(r)N(r)N'(r). \quad (\text{A.4b})$$

From the equations of motion above we see that eq. (A.4a) is the same as eq. (2.10a) in the main text for the t component, and eq. (A.4b) has an extra term $4rf(r)N(r)N'(r)$. We conclude that $4rf(r)N(r)N'(r) = 0$, and the physical solution is that $N'(r) = 0$. Therefore, $N(r)$ is a constant and can be eliminated by rescaling the t coordinate, and we come to the metric in eq. (2.8a) in the math text.

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