Electroweak axion string and superconductivity

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ABSTRACT: We study the axion strings with the electroweak gauge flux in the DFSZ axion model and show that these strings, called the electroweak axion strings, can exhibit superconductivity without fermionic zeromodes. We construct three types of electroweak axion string solutions. Among them, the string with W -flux can be lightest in some parameter space, which leads to a stable superconducting cosmic string. We also show that a large electric current can flow along the string due to the Peccei-Quinn scale much higher than the electroweak scale. This large current induces a net attractive force between the axion strings with the same topological charge, which opens a novel possibility that the axion strings form Y-junctions in the early universe.

KEYWORDS: Beyond Standard Model, Cosmology of Theories beyond the SM, Higgs Physics, Solitons Monopoles and Instantons

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1 Introduction

The strong CP problem is one of the unresolved mysteries in the Standard Model (SM) of particle physics. The problem can be naturally solved by the Peccei-Quinn mechanism, in which a global symmetry denoted by $U(1)_{\text{PQ}}$ is assumed to be spontaneously broken and provides a (pseudo) Nambu-Goldstone particle, the axion [1–4]. The axion is a promising candidate for a viable cold dark matter [5–7].

Among various models bringing the axion (for recent reviews, see, e.g., refs. [8–12]), the DFSZ model [13, 14] has been studied extensively, as well as the KSVZ model [15, 16]. In the DFSZ model, the scalar sector of the SM is extended to have two Higgs doublets and one SM-singlet complex scalar. The scalar fields and the SM fermions are assumed to be charged under the $U(1)_{\text{PQ}}$ symmetry, which is spontaneously broken by a vacuum expectation value (VEV) of the complex scalar. The axion is a linear combination of imaginary components of the doublets and the complex scalar.

The $U(1)_{\text{PQ}}$ symmetry in the DFSZ model is anomalous due to one-loop contributions from the SM fermions and is broken down to a discrete subgroup $\mathbb{Z}_3$ (or $\mathbb{Z}_6$), which produces

a domain wall at the QCD phase transition. The energy density of the domain walls dominate soon that of the universe, leading to the domain wall problem. One possible scenario to solve the problem is to assume that the $U(1)_{\text{PQ}}$ symmetry is broken during or before the cosmological inflation. There exists, however, a stringent constraint on isocurvature perturbation produced by the axion during the inflation [17]. Another simple way is introducing a tiny term breaking the symmetry explicitly, called the bias [18, 19], which resolves the vacuum degeneracy [20–22]. For other scenarios and their studies, see, e.g., refs. [23–28].

As well as most axion models enjoying the $U(1)_{\text{PQ}}$ symmetry, the DFSZ model predicts the axion string [29], which is a global cosmic string. The axion string is created by the Kibble-Zurek mechanism [30, 31] (see also ref. [32]) when the $U(1)_{\text{PQ}}$ symmetry is spontaneously broken. For the scenario that the domain wall problem is avoided by the inflation, the axion strings are diluted away and seem to play no role in cosmology. But for the other scenarios, they become interesting ingredients in the universe and have been studied in various contexts (see ref. [33]). We assume the latter scenarios in this paper. After the creation, the strings form a network whose energy density has a scaling property. To understand the evolution of the network, it is important to study the interaction between the axion strings. The interaction is thought to be dominated by exchange of the (massless) axion as a long-range force.

On the other hand, cosmic strings sometimes can be superconducting strings [34] when the electromagnetic gauge symmetry is spontaneously broken inside the strings. It is known that the axion strings necessarily become superconducting states [35–38] because they must have fermionic zero modes traveling on the string [39, 40]. The maximum amount of the supercurrent is determined by the (bulk) mass of the fermions. In the DFSZ model, however, the axion string cannot carry significant amount of the current because the model has no heavy fermion. Thus, the superconductivity seems to play no crucial role for cosmological properties.

In this paper, we show that the axion string in the DFSZ model becomes the electroweak string after the breaking of the electroweak symmetry. The electroweak string is a string containing flux tubes of the $SU(2)_W \times U(1)_Y$ gauge fields like the Abrikosov-Nielsen-Olesen vortex [41, 42] and has been studied in the SM [43–51] (see ref. [52] for review), and in two Higgs doublet models (2HDM) [53–62] (for recent comprehensive studies, see refs. [63, 64]). An essence is that the two Higgs doublets in the DFSZ model also acquire the VEVs after the electroweak phase transition and they must have winding in the $SU(2)_W \times U(1)_Y$ gauge orbits for the single-valuedness, as well as the winding for $U(1)_{\text{PQ}}$. We call such strings *the electroweak axion strings*. In particular, we show that there are at least three types of the electroweak axion string in the DFSZ model. Interestingly, some of them have very similar properties to those of the electroweak string in 2HDM.

Furthermore, we show that one of the electroweak axion strings (dubbed the type-C string) can be a superconducting string *without* fermionic fields. This is because the charged fields, the charged Higgs and W bosons, acquire non-zero values inside the string and the $U(1)_{\text{EM}}$ symmetry is spontaneously broken there. This is a similar situation to superconductivity of non-Abelian vortices [65, 66] and of the $U(1) \times \tilde{U}(1)$ model considered by Witten [34]. Remarkably, due to the coupling between the Higgs doublets and the

	H_1	H_2	S
$SU(2)_W$	2	2	1
$U(1)_Y$	1	1	0
$U(1)_{PQ}$	X_1	X_2	X_s

Table 1. The scalar field contents and their quantum charges.

complex scalar, the amount of the supercurrent can be of order of the $U(1)_{PQ}$ breaking scale resulting in large magnetic energy even in the DFSZ model. As a consequence, the strings feel a large magnetic interaction, which can overcome the one from the axion exchange. Therefore, superconductivity could drastically change the cosmological scenario of the axion strings after the electroweak phase transition in the DFSZ model.

The rest of this paper is organized as follows. In section 2, the DFSZ axion model is reviewed and our notation is introduced. For later use, we present a definition of the $U(1)_{EM}$ in general soliton backgrounds. In section 3, after a brief review of the conventional axion string, we discuss the electroweak axion strings. There are at least three types of the electroweak axion strings (type-A, B and C). We compare the tensions of the strings. In section 4, we show that the type-C string can be superconducting. A linearized equation of motion for massless zero modes traveling on the string is presented. In addition, we estimate the maximum amount of the supercurrent flowing on the string to be of order of the $U(1)_{PQ}$ breaking scale. Section 5 is devoted to the conclusion. In appendix A, we present the derivation of the linearized equation used in section 4.

2 The model

2.1 DFSZ axion model

The particle contents and the charge assignments under the SM gauge group and the $U(1)_{PQ}$ are shown in table 1. We introduce a SM-singlet complex scalar S and two $SU(2)_W$ doublets, H_1 and H_2 , both with the hypercharge $Y = 1$. The Lagrangian which describes the electroweak and scalar sectors is written as

$$\mathcal{L} = -\frac{1}{4}(Y_{\mu\nu})^2 - \frac{1}{4}(W_{\mu\nu}^a)^2 + \sum_{i=1,2} |D_\mu H_i|^2 + |\partial_\mu S|^2 - V(H_1, H_2, S). \quad (2.1)$$

Here, $Y_{\mu\nu}$ and $W_{\mu\nu}^a$ describe field strength tensors of the hypercharge and weak gauge interactions, respectively, with μ (ν) and a being Lorentz and weak iso-spin indices, respectively. D_μ represents the covariant derivative acting on the Higgs fields, and the index i runs $i = 1, 2$. The scalar potential $V(H_1, H_2, S)$ being invariant under the charge assignments of table 1 is

$$V(H_1, H_2, S) = V_H + V_S + V_{\text{mix}}, \quad (2.2)$$

where each part is given by

$$V_H = m_{11}^2 H_1^\dagger H_1 + m_{22}^2 H_2^\dagger H_2 + \frac{\beta_1}{2} (H_1^\dagger H_1)^2 + \frac{\beta_2}{2} (H_2^\dagger H_2)^2 \\ + \beta_3 (H_1^\dagger H_1) (H_2^\dagger H_2) + \beta_4 (H_1^\dagger H_2) (H_2^\dagger H_1), \quad (2.3)$$

$$V_S = -m_S^2 |S|^2 + \lambda_S |S|^4, \quad (2.4)$$

$$V_{\text{mix}} = \left(\kappa S^2 H_1^\dagger H_2 + \text{h.c.} \right) + \kappa_{1S} |S|^2 |H_1|^2 + \kappa_{2S} |S|^2 |H_2|^2, \quad (2.5)$$

with $m_S^2 > 0$ which admits S to acquire a non-zero VEV: $\langle S \rangle = v_s$. Without loss of generality, we can suppose that the Higgs fields develop VEVs as $\langle H_1 \rangle = (0, v_1)^T$, $\langle H_2 \rangle = (0, v_2)^T$ with $v_1, v_2 \in \mathbb{R}$.¹ Then the electroweak scale, $v_{\text{EW}} (\simeq 246 \text{ GeV})$, can be expressed by these VEVs as $v_{\text{EW}}^2 = 2(v_1^2 + v_2^2)$. We also define $\tan \beta \equiv v_2/v_1$. In order for V_{mix} to be invariant under the $U(1)_{\text{PQ}}$ symmetry, the $U(1)_{\text{PQ}}$ charges in table 1 should satisfy the relation $2X_s - X_1 + X_2 = 0$. If the first term in eq. (2.5) has a structure like $S H_1^\dagger H_2$ instead of $S^2 H_1^\dagger H_2$, the assignment of the $U(1)_{\text{PQ}}$ charges should change, but qualitative properties of the axion strings we discuss below are almost same. In particular, the string becomes superconducting also in such a case.

The Yukawa interaction terms are given by

$$\mathcal{L}_{\text{Yukawa}} = -y_U \bar{Q} (i\sigma_2 H_1^*) u_R - y_D \bar{Q} H_2 d_R - y_e \bar{L} H_2 e_R + \text{h.c.}, \quad (2.6)$$

and the SM fermions carry the $U(1)_{\text{PQ}}$ charge so that this Lagrangian is invariant under $U(1)_{\text{PQ}}$. The new singlet scalar S couples to the SM fermions via Higgs sector. In the following parts of this paper, we leave aside the Yukawa terms.

For later use, we rewrite the Higgs fields in a two-by-two matrix form [67], H , defined by

$$H = (i\sigma_2 H_1^*, H_2). \quad (2.7)$$

The matrix field H transforms under the electroweak $SU(2)_W \times U(1)_Y$ symmetry as

$$H \rightarrow \exp \left[\frac{i}{2} \theta_a(x) \sigma_a \right] H \exp \left[-\frac{i}{2} \theta_Y(x) \sigma_3 \right], \quad (2.8)$$

where the group element acting from the left belongs to $SU(2)_W$ and the other element acting from the right belongs to $U(1)_Y$. Therefore the covariant derivative on H can be expressed as

$$D_\mu H = \partial_\mu H - i \frac{g}{2} \sigma_a W_\mu^a H + i \frac{g'}{2} H \sigma_3 Y_\mu. \quad (2.9)$$

The VEV of H is expressed by a diagonal matrix $\langle H \rangle = \text{diag}(v_1, v_2)$, and the Higgs potential V_H can be written by using H as follows:

$$V_H = -m_1^2 \text{Tr} |H|^2 - m_2^2 \text{Tr} (|H|^2 \sigma_3) + \alpha_1 \text{Tr} |H|^4 \\ + \alpha_2 \left(\text{Tr} |H|^2 \right)^2 + \alpha_3 \text{Tr} (|H|^2 \sigma_3 |H|^2 \sigma_3) + \alpha_4 \text{Tr} (|H|^2 \sigma_3 |H|^2), \quad (2.10)$$

¹Note that we drop “ $1/\sqrt{2}$ ” in our notation for the VEVs.

where $|H|^2 \equiv H^\dagger H$ and the relations between the parameters in eq. (2.3) and in eq. (2.10) are given by

$$m_{11}^2 = -m_1^2 - m_2^2, \quad m_{22}^2 = -m_1^2 + m_2^2, \quad (2.11)$$

$$\beta_1 = 2(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4), \quad \beta_2 = 2(\alpha_1 + \alpha_2 + \alpha_3 - \alpha_4), \quad (2.12)$$

$$\beta_3 = 2(\alpha_1 + \alpha_2 - \alpha_3), \quad \beta_4 = 2(\alpha_3 - \alpha_1). \quad (2.13)$$

The mixing term V_{mix} is also rewritten by H as

$$V_{\text{mix}} = \left(\kappa S^2 \det H + \text{h.c.} \right) + \frac{1}{2} (\kappa_{1S} + \kappa_{2S}) |S|^2 \text{Tr} |H|^2 + \frac{1}{2} (\kappa_{1S} - \kappa_{2S}) |S|^2 \text{Tr} (|H|^2 \sigma_3). \quad (2.14)$$

The custodial transformation $\text{SU}(2)_C$ in the two Higgs doublet model [67, 68] is identified as the global unitary transformation of the Higgs matrix H as

$$H \rightarrow U^\dagger H U, \quad U \in \text{SU}(2)_C. \quad (2.15)$$

If $m_2^2 = \alpha_3 = \alpha_4 = 0$, and $\kappa_{1S} = \kappa_{2S}$, the scalar potential is invariant under the custodial transformation. This symmetry makes the two VEVs be equal, $\tan \beta = 1$.

2.2 Mass spectra and PQ transformation

The scalar fields develop the following VEVs

$$\langle H \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix}, \quad \langle S \rangle = v_s. \quad (2.16)$$

The stationary conditions are solved with the mass parameters m_1^2 , m_2^2 , m_S^2 as

$$m_1^2 = (\alpha_1 + 2\alpha_2 + \alpha_3 + \alpha_4) v_1^2 + (\alpha_1 + 2\alpha_2 + \alpha_3 - \alpha_4) v_2^2 + \frac{1}{2} \left[\kappa \left(\frac{v_1}{v_2} + \frac{v_2}{v_1} \right) + \kappa_{1S} + \kappa_{2S} \right] v_s^2, \quad (2.17)$$

$$m_2^2 = (\alpha_1 + \alpha_3 + \alpha_4) v_1^2 - (\alpha_1 + \alpha_3 - \alpha_4) v_2^2 + \frac{1}{2} \left[\kappa \left(\frac{v_2}{v_1} - \frac{v_1}{v_2} \right) + \kappa_{1S} - \kappa_{2S} \right] v_s^2, \quad (2.18)$$

$$m_S^2 = 2\lambda_S v_s^2 + \kappa_{1S} v_1^2 + \kappa_{2S} v_2^2 + 2\kappa v_1 v_2. \quad (2.19)$$

After the symmetry breaking, the gauge bosons and scalars become massive due to the above VEVs. The masses of the weak gauge bosons are given by

$$m_W = \frac{g v_{\text{EW}}}{2}, \quad m_Z = \frac{g v_{\text{EW}}}{2 \cos \theta_W}, \quad (2.20)$$

with the standard definitions of the weak mixing angle $\cos \theta_W = g / \sqrt{g^2 + g'^2}$, the Z boson $Z_\mu = W_\mu^3 \cos \theta_W - Y_\mu \sin \theta_W$, and the photon $A_\mu = W_\mu^3 \sin \theta_W + Y_\mu \cos \theta_W$.

In the scalar sector, we have three scalars, three pseudo scalars, and two charged scalars. Among these, one massless pseudo scalar and one massless charged scalar are eaten by the weak gauge bosons, and the other massless pseudo scalar becomes the axion, which obtains a mass from the non-perturbative QCD effect. There remain five physical

scalar bosons after the symmetry breaking. For example, the lightest real scalar has the mass eigenvalue

$$m_{h_1}^2 \simeq 4(\alpha_1 + \alpha_3) \frac{v_1^4 + v_2^4}{v_1^2 + v_2^2} + 4\alpha_2(v_1^2 + v_2^2) + 4\alpha_4(v_1^2 - v_2^2) - \frac{(\kappa_1 S v_1^2 + \kappa_2 S v_2^2 + 2\kappa v_1 v_2)^2}{\lambda_S(v_1^2 + v_2^2)}, \quad (2.21)$$

up to $\mathcal{O}(v_{\text{EW}}^2/v_s^2)$, and we identify it as the SM Higgs boson. The mass squared matrix for three pseudo scalars is expressed as

$$\kappa \begin{pmatrix} -\frac{v_2 v_s^2}{v_1} & v_s^2 & 2v_2 v_s \\ v_s^2 & -\frac{v_1 v_s^2}{v_2} & -2v_1 v_s \\ 2v_2 v_s & -2v_1 v_s & -4v_1 v_2 \end{pmatrix}. \quad (2.22)$$

This matrix has one massive and two exact zero modes. The non-vanishing mass eigenvalue is

$$m_{A_0}^2 = -\kappa \frac{4v_1^2 v_2^2 + v_1^2 v_s^2 + v_2^2 v_s^2}{v_1 v_2}. \quad (2.23)$$

In order to avoid the tachyonic mass, the portal coupling κ should be negative. One of the massless eigenvector is $(\cos \beta, \sin \beta, 0)$ which corresponds to the longitudinal mode of the Z boson. Another zero eigenvector is given by $(X_1 v_1, X_2 v_2, X_s v_s)$ as long as $2X_s - X_1 + X_2 = 0$ is satisfied. This flat direction corresponds to the axion. Imposing these two massless modes are orthogonal, we find

$$X_1 = 2 \sin^2 \beta, \quad X_2 = -2 \cos^2 \beta, \quad X_s = 1, \quad (2.24)$$

where X_s determines the normalization. Then the $U(1)_{\text{PQ}}$ transformation acts on the scalars as

$$H_1 \rightarrow e^{2i\alpha \sin^2 \beta} H_1, \quad H_2 \rightarrow e^{-2i\alpha \cos^2 \beta} H_2, \quad S \rightarrow e^{i\alpha} S. \quad (2.25)$$

The same result is obtained by defining the $U(1)_{\text{PQ}}$ current not to couple to the Z boson. For the matrix field H , the $U(1)_{\text{PQ}}$ transformation becomes

$$H \rightarrow e^{-i\alpha} H e^{i\alpha \sigma_3 \cos 2\beta}. \quad (2.26)$$

2.3 Definition of unbroken $U(1)_{\text{EM}}$ group

Unlike in the vacuum, in the presence of a soliton background, the definition of the unbroken $U(1)_{\text{EM}}$ generator is non-trivial. In this paper, it is defined as²

$$\hat{Q}H \equiv -n^a \frac{\sigma_a}{2} H - H \frac{\sigma_3}{2}, \quad (2.27)$$

where

$$n^a \equiv \frac{\sum_{i=1,2} |H_i|^2 n_i^a}{C}, \quad (2.28)$$

²The vector n^a corresponds to $\tilde{n}^a$ in ref. [69].

$$n_1^a \equiv \frac{H_1^\dagger \sigma^a H_1}{|H_1|^2}, \quad n_2^a \equiv \frac{H_2^\dagger \sigma^a H_2}{|H_2|^2}. \quad (2.29)$$

The positive normalization factor C is determined to satisfy $n^a n^a = 1$. Correspondingly, the $U(1)_Z$ subgroup in the $SU(2)_W \times U(1)_Y$ group is defined as

$$\hat{T}_Z H \equiv -n^a \frac{\sigma_a}{2} H - \sin^2 \theta_W \hat{Q} H. \quad (2.30)$$

Also, the $U(1)_Z$ and $U(1)_{\text{EM}}$ gauge fields are defined as

$$Z_\mu \equiv -n^a W_\mu^a \cos \theta_W - Y_\mu \sin \theta_W, \quad (2.31)$$

$$A_\mu \equiv -n^a W_\mu^a \sin \theta_W + Y_\mu \cos \theta_W. \quad (2.32)$$

In addition, the charged components of $SU(2)_W$ gauge group is defined as orthogonal components to $n^a \sigma_a$. In the vacuum, the Higgs field takes a constant VEV $\langle H \rangle = \text{diag}(v_1, v_2)$, and $n^a \sigma_a = -\sigma_3$. The above definitions reduce to the conventional ones. The VEV is invariant under $U(1)_{\text{EM}}$,

$$\hat{Q} \langle H \rangle = 0, \quad (2.33)$$

which means that the $U(1)_{\text{EM}}$ symmetry is not spontaneously broken in the vacuum.

It may be useful to rewrite the above expressions for the two doublets H_1 and H_2 ,

$$\hat{Q} H_i = \left(-n^a \frac{\sigma_a}{2} + \frac{1}{2} \mathbf{1} \right) H_i, \quad (2.34)$$

$$\hat{T}_Z H_i = \left(-n^a \frac{\sigma_a}{2} - \sin^2 \theta_W \hat{Q} \right) H_i, \quad (2.35)$$

for $i = 1, 2$.

3 Electroweak axion strings

Similarly to other axion models, the DFSZ axion model provides a vortex string solution known as the axion string corresponding to the breaking of $U(1)_{\text{PQ}}$. On the other hand, after the electroweak phase transition, the axion string can contain flux tubes of the $SU(2)_W \times U(1)_Y$ gauge fields like the Abrikosov-Nielsen-Olesen vortex [41, 42] since the two Higgs doublets also acquire the VEVs. We call such vortex strings the electroweak axion strings. In this section, we show that there are (at least) three types of the electroweak axion strings in the DFSZ model. Interestingly, some of them have similar properties to those of (non-Abelian) vortices in two Higgs doublet models, in which there is a global symmetry for a relative rotation of the two Higgs doublets. In particular, some part of our argument in this section refers to that in refs. [63, 64].

3.1 Axion string in DFSZ model

We first review the conventional axion string in this subsection. Let us consider a case that the $U(1)_{\text{PQ}}$ symmetry is spontaneously broken by $\langle S \rangle \neq 0$ but the electroweak symmetry remains, $\langle H_1 \rangle = \langle H_2 \rangle = (0, 0)^T$. This situation realized in the early universe when the

temperature T_{th} satisfies $v_{EW} \ll T_{th} \ll v_s$. In this case, as is well-known, a vortex-string configuration associated with the global $U(1)_{PQ}$ symmetry exists as a solution to the equation of motion (EOM), which is called the axion string in the literature. The configuration located on the z -axis is described by the following ansatz

$$S = v_s e^{i\theta} \phi(r), \quad H_1 = H_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad (3.1)$$

where r and θ are the distance from the z -axis and the rotational angle, respectively. Namely, $x + iy = r e^{i\theta}$. The profile function $\phi(r)$ satisfies the boundary conditions

$$\phi(0) = 0, \quad \phi(\infty) = 1. \quad (3.2)$$

The detailed form of $\phi(r)$ is determined by solving the EOM. This string has a winding number associated with the $U(1)_{PQ}$ symmetry, and hence is topologically stable. The $U(1)_{PQ}$ symmetry is restored on the string core because of $\phi(0) = 0$.

It is known that such strings are necessarily produced during the phase transition of the $U(1)_{PQ}$ symmetry breaking by the Kibble-Zurek mechanism. In the viewpoint of phenomenology, one of the important aspects of cosmic strings is the interaction between a pair of the cosmic strings having the same topological charge. For axion strings, the interaction is dominated by exchange of massless axion particles, resulting in the long-range repulsive force. The potential of the interaction $V_{st.}$ is approximately given as $V_{st.} \sim -v_s^2 \log R$ with R being the distance between the pair. Due to the repulsive interaction, a pair of the strings reconnects with probability of the order of unity when they collide to each other and does not form a bound state of the strings (such as the Y-junction [70–78]). As a result, the strings form a stationary network whose typical length scale remains to be the Hubble horizon scale (scaling regime). Such a scale-invariant evolution of the network prevents the energy density of the strings from dominating that of the universe, and thus axion models producing the axion strings are cosmologically viable as far as concerning the strings.

3.2 Vortex string with Z -flux (type-A string)

Next, we discuss the electroweak axion strings. Let us consider a string configuration after the electroweak phase transition $T_{th} \lesssim v_{EW}$. The two doublets also acquire the VEVs and their phases must also wind because they also have the $U(1)_{PQ}$ charges, otherwise divergent energy arises from V_{mix} .³ From the single-valuedness of the doublets, the string configuration has the form

$$\begin{cases} S = v_s e^{i\theta} \phi(r) \\ H_1 = v_1 e^{i\theta} \begin{pmatrix} 0 \\ f(r) \end{pmatrix} \\ H_2 = v_2 e^{-i\theta} \begin{pmatrix} 0 \\ h(r) \end{pmatrix} \end{cases}, \quad (3.3)$$

$$Z_i = \frac{2 \cos 2\beta}{g_Z} \frac{\epsilon_{ij} x_j}{r^2} (1 - z(r)), \quad (3.4)$$

³If the two doublets had no windings, the mixing term provides $\kappa v_s^2 v_1 v_2 \cos 2\theta$ at large distances, which means a divergent potential energy after the spatial integration.

and $W_i^\pm = A_i = 0$. g_Z is the coupling of Z -boson given by $g_Z = \sqrt{g^2 + g'^2}$. ϵ_{ij} is the anti-symmetric tensor satisfying $\epsilon_{12} = -\epsilon_{21} = 1$. We call this string configuration the type-A electroweak axion string.

The last two configurations in eq. (3.3) are equivalent to

$$H = e^{-i\theta} \begin{pmatrix} v_1 f(r) & 0 \\ 0 & v_2 h(r) \end{pmatrix}. \quad (3.5)$$

The profile functions $f(r), h(r)$ satisfy the same boundary conditions as that of $\phi(r)$, i.e.,

$$\phi(0) = f(0) = h(0) = 0, \quad \phi(\infty) = f(\infty) = h(\infty) = 1. \quad (3.6)$$

The profile function for the gauge field $z(r)$ should satisfy the boundary conditions

$$z(0) = 1, \quad z(\infty) = 0. \quad (3.7)$$

Eq. (3.6) indicates that the electroweak symmetry is restored inside the string as well as $U(1)_{PQ}$.

Noting that the two doublets have $U(1)_{PQ}$ charges as given in eq. (2.25), it is convenient to decompose the winding phases as

$$H_1 = v_1 e^{2i\theta s_\beta^2} e^{-i\theta \sigma_3 c_{2\beta}} \begin{pmatrix} 0 \\ f(r) \end{pmatrix}, \quad (3.8)$$

$$H_2 = v_2 e^{-2i\theta c_\beta^2} e^{-i\theta \sigma_3 c_{2\beta}} \begin{pmatrix} 0 \\ h(r) \end{pmatrix}, \quad (3.9)$$

with $c_X \equiv \cos(X)$ and $s_X \equiv \sin(X)$. Eqs. (3.8) and (3.9) mean that the configurations of the doublets have the winding number unity for the global $U(1)_{PQ}$ symmetry and the fractional winding number $-\cos 2\beta$ for the $U(1)_Z$ subgroup of the $SU(2)_W \times U(1)_Y$ gauge symmetry. Therefore, the gradient energy from the $U(1)_Z$ windings is canceled by the Z gauge field (3.4) with eq. (3.7) at large distances $r \rightarrow \infty$. It follows from eq. (3.4) that the string configuration has the Z -flux,

$$\Phi_Z = \oint_{r=\infty} dx_i Z_i = \frac{-4\pi \cos 2\beta}{g_Z}, \quad (3.10)$$

which is fractionally quantized because of the fractional winding number.

We discuss a qualitative property of the profile functions. The typical length scale for $\phi(r)$ is v_s^{-1} while those of $f(r), h(r)$ and $z(r)$ are given as $(v_{EW})^{-1}$. Thus the string is a “multi-scale solution”. Figure 1 shows a schematic picture of the energy density profile of the electroweak axion strings. The energy density has three structures. One is that from the radial component of the complex scalar, $\phi(r)$, whose typical scale is v_s^{-1} . This part looks as a thin object (Region I: $r \lesssim v_s^{-1}$). Another is from those of the two Higgs doublets and the gauge fields, whose typical scale is the EW-scale $(v_{EW})^{-1}$. This region, which we call Region II, is much fatter than Region I and is shown as the red region. The third part is the fattest part from the gradient energy of the axion. This is denoted by

Figure 1. Schematic picture of the energy density profile of the electroweak axion strings (type-A, B and C). The energy density consists of three parts. There is a one-dimensional thin object consisting of the radial component of the complex scalar $\phi(r)$ in Region I: $r \lesssim v_s^{-1}$. We call this region the core of the string. In Region II: $v_s^{-1} \lesssim r \lesssim (v_{EW})^{-1}$ (red region), the energy density is dominated by the Higgs fields and the electroweak gauge fields. This region is much fatter than Region I. In addition, there is the fattest part made from the gradient energy of the axion, leading to the logarithmically divergent tension. This is denoted by Region III: $r \gtrsim (v_{EW})^{-1}$ (blue cloud). Note that the type-A string with $\tan\beta = 1$ is a special case since it has only the global winding and its energy density does not have the Region II.

Region III: $r \gtrsim (v_{EW})^{-1}$ (blue cloud in the figure). When one calculates the tension of the string (energy per unit length) by integrating the energy density on the xy plane, the third part leads to the log-divergent tension $\sim 2\pi v_s^2 \log L$ where L is the IR-cutoff and is usually taken as the distance between neighbor two strings. The coefficient of the log divergence is the same as that of the conventional axion string shown in the last subsection because it depends only on the winding number of the global $U(1)_{PQ}$ symmetry.

Let us obtain the profile functions and calculate the string tension for the type-A string in a numerical way. For simplicity, we take $m_2^2 = \alpha_4 = 0$ and $\kappa_{1S} = \kappa_{2S}$, leading to $\tan\beta = 1$ (see the stationary condition (2.18)). The VEVs are denoted as $v_1 = v_2 \equiv v$. We should note that in this case, the Higgs doublets do not have winding number for the $U(1)_Z$ gauge subgroup, and thus the right hand side of eq. (3.4) vanishes. The Z -flux is constantly zero. After substituting the ansatz, the energy density is given by

$$\mathcal{E} \equiv |\partial_i S|^2 + \text{Tr}|D_i H|^2 + \frac{1}{4}(W_{ij}^a)^2 + \frac{1}{4}(Y_{ij})^2 + V(H, S) \quad (3.11)$$

$$\begin{aligned} &= \frac{v^2}{r^2} \left[r^2 \left(f'(r)^2 + h'(r)^2 \right) + f(r)^2 + h(r)^2 \right] \\ &\quad + v^2 \left[-m_1^2 (f(r)^2 + h(r)^2) + 2\alpha_2 v^2 f(r)^2 h(r)^2 + v^2 \alpha_{123} (f(r)^4 + h(r)^4) \right] \\ &\quad + v^2 v_s^2 \left[2\kappa f(r) h(r) \phi(r)^2 + \kappa_{1S} (f(r)^2 + h(r)^2) \phi(r)^2 \right] \\ &\quad + v_s^2 \left(-m_S^2 \phi(r)^2 + \lambda_S v_S^2 \phi(r)^4 \right) + \frac{v_s^2}{r^2} \left(r^2 \phi'(r)^2 + \phi(r)^2 \right), \end{aligned} \quad (3.12)$$

with $\alpha_{123} \equiv \alpha_1 + \alpha_2 + \alpha_3$ and $'$ denoting the derivative with respect to r .

The EOMs are obtained as

$$f''(r) + \frac{f'(r)}{r} - \frac{f(r)}{r^2} - \left(2\alpha_{123}v^2f(r)^2 + 2\alpha_2v^2h(r)^2 + \kappa_{1S}v_s^2\phi(r)^2 - m_1^2\right)f(r) - \kappa v_s^2h(r)\phi(r)^2 = 0, \quad (3.13)$$

$$h''(r) + \frac{h'(r)}{r} - \frac{h(r)}{r^2} - \left(2\alpha_{123}v^2h(r)^2 + 2\alpha_2v^2f(r)^2 + \kappa_{1S}v_s^2\phi(r)^2 - m_1^2\right)h(r) - \kappa v_s^2f(r)\phi(r)^2 = 0, \quad (3.14)$$

$$\phi''(r) + \frac{\phi'(r)}{r} - \frac{\phi(r)}{r^2} - \left(2\lambda_S v_s^2\phi(r)^2 + 2\kappa_{1S}v^2(f(r)^2 + h(r)^2) + 2\kappa v^2f(r)h(r) - m_S^2\right)\phi(r) = 0. \quad (3.15)$$

We adopt the so-called relaxation method to solve the EOMs. As a benchmark case, we take the parameters as

$$\alpha_1 = 1, \quad \alpha_2 = -0.3348, \quad \alpha_3 = 0, \quad \lambda_S = 1, \quad \kappa = -2\left(\frac{v}{v_s}\right)^2, \quad \kappa_{1S} = 0.4, \quad (3.16)$$

such that the lightest scalar mass $m_{h_1}^2$ reproduces the SM Higgs mass $(125 \text{ GeV})^2$. In addition, we set the VEV for S as $v_s = 10v$. Although this is too small and not viable in the phenomenological viewpoint, it does not matter because the qualitative picture of the type-A string does not change. If one takes them more hierarchical, huge numerical costs arise in the calculation. The obtained numerical solutions are shown in figure 2. In the left panel, $f(r)$ (blue line) is equal to $h(r)$ (dotted orange line) everywhere. $\phi(r)$ increases as $\phi \sim v_s r$ for $r \sim 0$ while $f = h \sim vr$. All of them approach to unity for $r \rightarrow \infty$. The right panel shows the energy density $\mathcal{E}$ (eq. (3.11)) divided by v_s^2/r^2 . The divided value approaches to unity, which means that the energy density has a polynomial tail like r^{-2} , instead of an exponential one. This leads to the logarithmically divergent tension.

3.3 Type-B string with Z-flux

We have shown that the phases of the two Higgs doublets must wind after the electroweak phase transition $T_{th} \lesssim v_{EW}$. Actually, there is another type of vortex string that is consistent with the single-valuedness and the potential minimum. This is obtained by giving an *additional winding* in the $U(1)_Z$ gauge orbit to the type-A string. We call this string the type-B electroweak axion string. The ansatz describing the string is given as

$$\begin{cases} S = v_s e^{i\theta} \phi(r) \\ H_1 = v_1 e^{2i\theta} \begin{pmatrix} 0 \\ f(r) \end{pmatrix} \\ H_2 = v_2 \begin{pmatrix} 0 \\ h(r) \end{pmatrix} \end{cases} \quad (3.17)$$

$$Z_i = \frac{4 \cos^2 \beta}{g_Z} \frac{\epsilon_{ij} x_j}{r^2} (1 - z(r)). \quad (3.18)$$

Figure 2. Numerical solution for the type-A string. We take benchmark parameters as eq. (3.16) and $v_s = 10v$. Also we adopt a length unit as $v_s^{-1} = 0.5$. (left): plots of profile functions. Note $f(r)$ (blue line) is equal to $h(r)$ (dotted orange line) everywhere. $\phi(r)$ increases as $\phi \sim v_s r$ for $r \sim 0$ while $f = h \sim vr$. All of them approach to unity for $r \rightarrow \infty$. (right): plot of energy density $\mathcal{E}$ (eq. (3.11)) divided by v_s^2/r^2 . Clearly, the energy density has a polynomial tail like r^{-2} , instead of an exponential one. This leads to the logarithmically divergent tension. The integrated value of the tension over $0 \leq r \leq 120 v_s^{-1}$ is 140.321.

The last two configurations in eq. (3.17) are equivalent to

$$H = e^{-i\theta} e^{-i\theta\sigma_3} \begin{pmatrix} v_1 f(r) & 0 \\ 0 & v_2 h(r) \end{pmatrix}. \quad (3.19)$$

The profile functions $f(r)$ and $\phi(r)$ satisfy the same boundary conditions,

$$\phi(0) = f(0) = 0, \quad \phi(\infty) = f(\infty) = 1, \quad (3.20)$$

but $h(r)$ should satisfy the following boundary conditions:

$$\partial_r h|_{r=0} = 0, \quad h(\infty) = 1. \quad (3.21)$$

The profile function for the gauge field $z(r)$ satisfies

$$z(0) = 1, \quad z(\infty) = 0. \quad (3.22)$$

Note that $h(r)$ is not fixed to zero on the center of the string because it does not have a winding phase. Therefore, the electroweak symmetry is *not* restored inside the string core (see figure 3.)

Again, we decompose the winding phases as

$$H_1 = v_1 e^{2i\theta s_\beta^2} e^{-2i\theta\sigma_3 c_\beta^2} \begin{pmatrix} 0 \\ f(r) \end{pmatrix}, \quad (3.23)$$

$$H_2 = v_2 e^{-2i\theta c_\beta^2} e^{-2i\theta\sigma_3 c_\beta^2} \begin{pmatrix} 0 \\ h(r) \end{pmatrix}, \quad (3.24)$$

which mean that the configurations of the doublets have the winding number unity for the global $U(1)_{\text{PQ}}$ symmetry and the fractional winding number $-2\cos^2\beta$ for the $U(1)_Z$

Figure 3. Numerical solution for the type-B string. We take the same parameters as ones in figure 2. Also we adopt a length unit as $v_s^{-1} = 0.5$. (left): plots of profile functions. $\phi(r)$ increases as $\phi \sim v_s r$ for $r \sim 0$ while $f(r)$ behaves as a quadratic function with respect to r . The three profile functions f, h and ϕ approach to unity for $r \rightarrow \infty$. The profile function of the Z field, $z(r)$, approaches to zero as $r \rightarrow \infty$ starting from unity at $r = 0$. (right): plots of energy density (3.26) divided by v_s^2/r^2 and the Z -flux density multiplied by -10 . The energy density has a polynomial tail like r^{-2} like the type-A string. The tension T integrated over $0 \leq r \leq 120 v_s^{-1}$ is 140.524. The Z -flux density decays exponentially for $r \rightarrow \infty$ like the usual Abrikosov-Nielsen-Olesen vortex. The total value of the Z -flux is calculated to -16.9539 , which is consistent with eq. (3.25).

subgroup. The difference between the winding number of the type-A and type-B strings is unity. Similarly to the previous case, the Z gauge field (3.18) cancels the gradient energy from the $U(1)_Z$ windings. It follows from eq. (3.4) that the string configuration has the Z -flux, which is calculated as

$$\Phi_Z = \oint_{r=\infty} dx_i Z_i = \frac{-8\pi \cos^2 \beta}{gZ}. \quad (3.25)$$

Interestingly, this string configuration is quite similar to the topologically stable Z -string (topological vortex with the Z -flux) in 2HDM [55, 56, 63, 64]. In addition to the winding number of $U(1)_Z$, the difference from the 2HDM is the existence of the singlet complex scalar carrying the $U(1)_{PQ}$ charge, which is replaced by the relative phase rotation of the two doublets in the 2HDM.

Let us obtain the profile functions and calculate the string tension for the type-B string in a numerical way. Again, we take $m_2^2 = \alpha_4 = 0$ and $\kappa_{1S} = \kappa_{2S}$, leading to $\tan \beta = 1$. The VEVs are denoted as $v_1 = v_2 \equiv v$. Unlike the type-A string, the Higgs doublets have winding numbers for the $U(1)_Z$ gauge subgroup even for $\tan \beta = 1$. Consequently, the Z -flux is non-zero and confined inside the string. The energy density is

$$\begin{aligned} \mathcal{E} = & \frac{v^2}{r^2} \left[r^2 \left(f'(r)^2 + h'(r)^2 \right) + f(r)^2 (1 + z(r))^2 + h(r)^2 (1 - z(r))^2 \right] \\ & + v^2 \left[-m_1^2 (f(r)^2 + h(r)^2) + 2\alpha_2 v^2 f(r)^2 h(r)^2 + v^2 \alpha_{123} (f(r)^4 + h(r)^4) \right] \\ & + v^2 v_s^2 \left[2\kappa f(r) h(r) \phi(r)^2 + \kappa_{1S} (f(r)^2 + h(r)^2) \phi(r)^2 \right] \\ & + v_s^2 \left(-m_S^2 \phi(r)^2 + \lambda_S v_S^2 \phi(r)^4 \right) + \frac{v_s^2}{r^2} \left(r^2 \phi'(r)^2 + \phi(r)^2 \right) + 2 \frac{z'(r)^2}{g_Z^2 r^2} \end{aligned} \quad (3.26)$$

Then, the EOMs are obtained as

$$f''(r) + \frac{f'(r)}{r} - \frac{(1+z(r))^2}{r^2} f(r) - \left(2\alpha_{123} v^2 f(r)^2 + 2\alpha_2 v^2 h(r)^2 + \kappa_{1S} v_s^2 \phi(r)^2 - m_1^2\right) f(r) - \kappa v_s^2 h(r) \phi(r)^2 = 0, \quad (3.27)$$

$$h''(r) + \frac{h'(r)}{r} - \frac{(-1+z(r))^2}{r^2} h(r) - \left(2\alpha_{123} v^2 h(r)^2 + 2\alpha_2 v^2 f(r)^2 + \kappa_{1S} v_s^2 \phi(r)^2 - m_1^2\right) h(r) - \kappa v_s^2 f(r) \phi(r)^2 = 0, \quad (3.28)$$

$$\phi''(r) + \frac{\phi'(r)}{r} - \frac{\phi(r)}{r^2} - \left(2\lambda_S v_s^2 \phi(r)^2 + \kappa_{1S} v^2 (f(r)^2 + h(r)^2) + 2\kappa v^2 f(r) h(r) - m_S^2\right) \phi(r) = 0, \quad (3.29)$$

$$z''(r) - \frac{z'(r)}{r} - \frac{g_Z^2 v^2}{2} f(r)^2 (1+z(r)) - \frac{g_Z^2 v^2}{2} h(r)^2 (-1+z(r)) = 0. \quad (3.30)$$

We take the same parameter choice as eq. (3.16). In addition, we set the VEV for S as $v_s = 10v$. The obtained numerical solutions are shown in figure 3. In the left panel, $f(r)$ ($\phi(r)$) behaves like $v_s r$ (vr^2) at the origin because the phases of $\phi(r)$ and $f(r)$ wind once and twice, respectively. $h(r)$ does not start from zero at the origin due to the Neumann condition at the origin. All of the scalar profile functions approach to unity for $r \rightarrow \infty$. The profile function for the gauge field $z(r)$ approaches to zero starting from unity. The right panel shows the energy density $\mathcal{E}$ (eq. (3.11)) divided by v_s^2/r^2 and the Z -flux density multiplied by -10 . Similarly to the previous case, the type-A string, the energy density has a polynomial tail like r^{-2} , instead of an exponential one. This leads to the logarithmically divergent tension. The schematic picture of the energy density profile is the same as figure 1. On the other hand, the Z -flux has an exponential tail.

In the above ansatz for the type-B string, only the one doublet H_1 has the winding number. There may be also an alternative string in which only H_2 has the winding phase. Roughly speaking, they are related by exchange of the two doublets H_1 and H_2 . Since the property is similar to the former, we do not study the latter one in this paper and also categorize the latter one as the type-B string.

3.4 Type-C string with W -flux

Finally, we consider the third type of the vortex string, called the type-C electroweak axion string. While the type-B string has been obtained by performing an additional rotation of $U(1)_Z$ on the type-A one, the type-C string has a winding in the $U(1)_{W^1}$ subgroup of the $SU(2)_W \times U(1)_Y$ symmetry. The ansatz for the scalar fields is given as

$$\begin{cases} S = v_s e^{i\theta} \phi(r) \\ H_1 = \frac{1}{2} v_1 e^{i\theta} \begin{pmatrix} f(r) e^{i\theta} - h(r) e^{-i\theta} \\ f(r) e^{i\theta} + h(r) e^{-i\theta} \end{pmatrix} \\ H_2 = \frac{1}{2} v_2 e^{-i\theta} \begin{pmatrix} h(r) e^{i\theta} - f(r) e^{-i\theta} \\ h(r) e^{i\theta} + f(r) e^{-i\theta} \end{pmatrix} \end{cases} \quad (3.31)$$

The profile functions $f(r), h(r), \phi(r)$ satisfy similar boundary conditions to those of the type-B string, (3.20) and (3.21), i.e.,

$$f(0) = \phi(0) = 0, \quad \partial_r h|_{r=0} = 0, \quad f(\infty) = h(\infty) = \phi(\infty) = 1. \quad (3.32)$$

Due to the non-zero value of $h(0)$, the electroweak symmetry is not restored inside the string as the type-B string.

Unlike the type-A and type-B strings, in the background of the type-C string, the $U(1)_{\text{EM}}$ and $U(1)_Z$ generators depend on the positions. As explained in section 2, the $U(1)_{\text{EM}}$ generator is defined by eq. (2.27) or eq. (2.34). Substituting the ansatz eq. (3.31), we obtain

$$n_1^a = \frac{H_1^\dagger \sigma^a H_1}{|H_1|^2} = \frac{2}{f^2 + h^2} \left((f^2 - h^2)/2, -fh \sin 2\theta, -fh \cos 2\theta \right), \quad (3.33)$$

$$n_2^a = \frac{H_2^\dagger \sigma^a H_2}{|H_2|^2} = \frac{2}{f^2 + h^2} \left((h^2 - f^2)/2, -fh \sin 2\theta, -fh \cos 2\theta \right), \quad (3.34)$$

and

$$n^a \frac{\sigma_a}{2} = -\frac{\sigma_2}{2} \sin 2\theta - \frac{\sigma_3}{2} \cos 2\theta. \quad (3.35)$$

Then the $U(1)_{\text{EM}}$ and $U(1)_Z$ generators are given by

$$\hat{Q}H_i = \left(\frac{\sigma_2}{2} \sin 2\theta + \frac{\sigma_3}{2} \cos 2\theta + \frac{1}{2} \mathbf{1} \right) H_i, \quad (3.36)$$

$$\hat{T}_Z H_i = \left(\sigma_2 \sin 2\theta + \sigma_3 \cos 2\theta - \sin^2 \theta_W \hat{Q} \right) H_i, \quad (3.37)$$

which depend on θ .

Let us see the asymptotic behaviors of the two doublets at large distances $r \rightarrow \infty$. From eq. (3.32), we obtain

$$H_1 \sim v_1 e^{i\theta} \begin{pmatrix} i \sin \theta \\ \cos \theta \end{pmatrix} = v_1 e^{2i\theta s_\beta^2} e^{-i\theta c_{2\beta} \sigma_Z} e^{i\theta \sigma_1} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (3.38)$$

$$H_2 \sim v_2 e^{-i\theta} \begin{pmatrix} i \sin \theta \\ \cos \theta \end{pmatrix} = v_2 e^{-2i\theta c_\beta^2} e^{-i\theta c_{2\beta} \sigma_Z} e^{i\theta \sigma_1} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (3.39)$$

where $\sigma_Z \equiv 2\hat{T}_Z$ (note $\hat{Q}H_i = 0$ for $r \rightarrow \infty$). It is clear that these configurations have a winding number unity for $U(1)_{\text{PQ}}$, $-\cos 2\beta$ for $U(1)_Z$ and unity for the $U(1)_{W^1}$ subgroup (σ_1 rotation) of the gauge symmetry. Therefore, to cancel the gradient energy from the windings for $U(1)_Z$ and $U(1)_{W^1}$, the ansatz for the gauge fields are given as

$$Z_i = \frac{2 \cos 2\beta}{g_Z} \frac{\epsilon_{ij} x_j}{r^2} (1 - z(r)), \quad (3.40)$$

$$W_i^1 = \frac{-2}{g} \frac{\epsilon_{ij} x_j}{r^2} (1 - w(r)), \quad (3.41)$$

and $A_i = 0$, where we have used the definitions of the gauge fields eqs. (2.31) and (2.32). The profile functions for the gauge fields $w(r)$ and $z(r)$ satisfy

$$w(0) = z(0) = 1, \quad w(\infty) = z(\infty) = 0. \quad (3.42)$$

Interestingly, the type-C string has both of the Z and W -fluxes for $\tan\beta \neq 1$. It follows from the ansatz (3.40) and (3.41) that

$$\Phi_Z = \oint_{r=\infty} dx_i Z_i = \frac{-4\pi \cos 2\beta}{g_Z}, \quad (3.43)$$

$$\Phi_{W^1} = \oint_{r=\infty} dx_i W_i^1 = \frac{4\pi}{g}, \quad (3.44)$$

where the latter flux is independent of $\tan\beta$.

The most important difference from the type-A and type-B strings is that the $U(1)_{\text{EM}}$ symmetry is broken inside the type-C string. This can be seen by using the concrete expression, eq. (3.31), as $\hat{Q}H_i \neq 0$ ($i = 1, 2$). Actually, that is generally inevitable for a configuration with a non-vanishing winding number for charged components, i.e., a configuration whose asymptotic form is $\exp[i\hat{T}(\theta)]H(\theta = 0)$ with a non-Abelian generator $\hat{T}(\theta)$ satisfying $[\hat{T}(\theta), \hat{Q}] \neq 0$ [65, 66]. ($\hat{T} = \sigma_1\theta$ in our case.) Due to the winding, the charged components cannot remain zero, and they acquire non-zero values inside the string, leading to the breaking of $U(1)_{\text{EM}}$. We should note that $U(1)_{\text{EM}}$ is restored at large distances from the string, $r \rightarrow \infty$.

Let us solve the EOMs with respect to the profile functions and calculate the string tension for the type-C string. Again, we take $m_2^2 = \alpha_4 = 0$ and $\kappa_{1S} = \kappa_{2S}$, leading to $\tan\beta = 1$. The VEVs are denoted as $v_1 = v_2 \equiv v$. The Higgs doublets have winding numbers for the $U(1)_{W^1}$ gauge subgroup but not for $U(1)_Z$. Consequently, only the W^1 -flux is non-zero and confined inside the string. Substituting the ansatz, the energy density is given by

$$\begin{aligned} \mathcal{E} = & \frac{v^2}{r^2} \left[r^2 \left(f'(r)^2 + h'(r)^2 \right) + f(r)^2(w(r) + 1)^2 + h(r)^2(w(r) - 1)^2 \right] \\ & - m_1^2 v^2 (f(r)^2 + h(r)^2) + v^4 \left[2(\alpha_2 + \alpha_3) f(r)^2 h(r)^2 + (\alpha_1 + \alpha_2) (f(r)^4 + h(r)^4) \right] \\ & + v^2 v_s^2 \left[2\kappa f(r) h(r) \phi(r)^2 + \kappa_{1S} (f(r)^2 + h(r)^2) \phi(r)^2 \right] \\ & + \frac{2w'(r)^2}{g^2 r^2} + v_s^2 \left(-m_S^2 \phi(r)^2 + \lambda_S v_s^2 \phi(r)^4 \right) + \frac{v_s^2}{r^2} \left(r^2 \phi'(r)^2 + \phi(r)^2 \right), \end{aligned} \quad (3.45)$$

and, the EOMs are given as follows:

$$f''(r) + \frac{f'(r)}{r} - \frac{(1 + w(r))^2}{r^2} f(r) \quad (3.46)$$

$$- \left(2(\alpha_1 + \alpha_2) v^2 f(r)^2 + 2(\alpha_2 + \alpha_3) v^2 h(r)^2 + \kappa_{1S} v_s^2 \phi(r)^2 - m_1^2 \right) f(r) - \kappa v_s^2 h(r) \phi(r)^2 = 0,$$

$$h''(r) + \frac{h'(r)}{r} - \frac{(-1 + w(r))^2}{r^2} h(r) \quad (3.47)$$

$$- \left(2(\alpha_1 + \alpha_2) v^2 h(r)^2 + 2(\alpha_2 + \alpha_3) v^2 f(r)^2 + \kappa_{1S} v_s^2 \phi(r)^2 - m_1^2 \right) h(r) - \kappa v_s^2 f(r) \phi(r)^2 = 0,$$

$$\phi''(r) + \frac{\phi'(r)}{r} - \frac{\phi(r)}{r^2} \quad (3.48)$$

$$- \left(2\lambda_S v_s^2 \phi(r)^2 + \kappa_{1S} v^2 (f(r)^2 + h(r)^2) + 2\kappa v^2 f(r) h(r) \right) \phi(r) = 0,$$

$$w''(r) - \frac{w'(r)}{r} - \frac{g^2 v^2}{2} f(r)^2 (1 + w(r)) - \frac{g^2 v^2}{2} h(r)^2 (-1 + w(r)) = 0. \quad (3.49)$$

Figure 4. Numerical solution for the type-C string. We take the same benchmark parameters as figure 2. Also we adopt a length unit as $v_s^{-1} = 0.5$. (left): plots of profile functions. The behavior of the scalar profile functions are the same as one in figure 3. The profile function of the W^1 field, $w(r)$, approaches to zero as $r \rightarrow \infty$ starting from unity at $r = 0$. (right): plots of energy density (3.45) divided by v_s^2/r^2 and the W^1 -flux density multiplied by 10. The tension integrated over $0 \leq r \leq 120 v_s^{-1}$ is 140.604. The total value of the W^1 -flux is calculated to 19.3208, which is consistent with eq. (3.44).

The obtained numerical solutions are shown in figure 4. We take the same parameter choice as eq. (3.16) and set the VEV for S as $v_s = 10 v$. The shapes of the profile functions, the energy density and the flux density are almost similar to those of the type-B string (figure 3). This can be understood by the argument on non-Abelian moduli in 2HDM in ref. [64]. That is, when $\tan \beta = 1$, the two ansatz for the Higgs fields (eqs. (3.17) and (3.31)) are related by the $SU(2)_C$ custodial transformation:

$$H \rightarrow U^\dagger H U \quad (3.50)$$

with $U = \exp[i\frac{\pi}{4}\sigma_2]$. This symmetry is respected in the potential $V(H, S)$ when $m_2^2 = \alpha_3 = \alpha_4 = 0$ and $\kappa_{1S} = \kappa_{2S}$, but is explicitly broken in the gauge sector because of $g' \neq 0$. Thus the shapes and hence the string tensions are slightly different between them.

While the above ansatz has the Z - and W^1 -fluxes in the string, there is also a string in which the W^2 -flux (and also mixtures of them in general) is confined. Due to the $U(1)_{\text{EM}}$ symmetry in the Lagrangian, they have the degenerated tension. Since the property is almost the same as the one we studied above, we do not consider them in this paper.

3.5 String tensions

The vortex strings we have considered above have the same winding number (unity) associated with the $U(1)_{\text{PQ}}$ symmetry. Because all possible configurations in the theory are classified into topological sectors characterized by the non-trivial first homotopy group $\pi_1(U(1)_{\text{PQ}}) = \mathbb{Z}$, the above fact means that they are in the same topological sector with the topological charge $1 \in \pi_1(U(1)_{\text{PQ}}) = \mathbb{Z}$ and that they can continuously deform to each others. Since their string tensions (energy per length unit) are generically not degenerated, heavier strings decay into the lightest one, which does not decay any further and is a stable solution to the EOMs. We here study the string tensions, i.e., stability of the strings.

For simplicity, we focus on a case with $\tan \beta = 1$. This is realized when $m_2^2 = \alpha_4 = 0$ in the Higgs potential (2.10) and $\kappa_{1S} = \kappa_{2S}$ in the mixing term (2.14). In this case, as stated above, the type-A string does not have the Z -flux and the type-B one has the winding number unity for $U(1)_Z$. In addition, the type-C string does not have the $U(1)_Z$ winding but does for $U(1)_{W^1}$. It may seem that the type-A one is lighter than type-B and type-C ones since the latter two have the Z and W -fluxes. However, this is not the case when the potential energy is more dominant than that of the gauge sector. Indeed, both the profile functions for the doublets in the type-A string vanish on the core, and thus that leads to a larger amount of the potential energy than those of the type-B and type-C.

On the other hand, the difference of the tensions between the type-B and type-C strings is controlled by the parameter α_3 . As explained in the last subsection, in the case $\tan \beta = 1$, the type-C string has only the W^1 -flux, and the two strings are related by the custodial $SU(2)_C$ transformation. If $\alpha_3 = 0$ (and $m_2^2 = \alpha_4 = 0$), the Higgs potential V_H respects this symmetry but the gauge sector does not due to the $U(1)_Y$ coupling constant $g' \neq 0$. This slightly lifts up the tension of the type-C because of $g_Z = \sqrt{g^2 + g'^2} > g$. The non-zero value of α_3 can change this relation. In refs. [62, 64], it is shown that, in 2HDM, the smaller (larger) value of α_3 tends to make the tension of the string with the W -flux heavier (lighter) than that of the string with the Z -flux. Thus, in our case, we expect that the type-C string is lighter than the type-A and type-B ones when α_3 is larger than a critical value depending on other parameters.

Keeping $\kappa_{1S} = \kappa_{2S}$, we take the benchmark parameters as

$$\alpha_1 = 1, \quad \lambda_S = 1, \quad \kappa = -2 \left(\frac{v}{v_s} \right)^2, \quad v_s = 10 v, \quad m_2^2 = \alpha_4 = 0, \quad (3.51)$$

and take α_2 such that the lightest scalar mass $m_{h_1}^2$ is equal to the SM Higgs mass $(125 \text{ GeV})^2$. We use a length unit $v_s^{-1} = 0.5$. The remaining two parameters α_3 and κ_{1S} are taken as free parameters. The tensions T are calculated over $0 \leq r \leq L$ with the IR cutoff $L = 120 v_s^{-1}$. Note that, although each tension depends on the IR cutoff as $\sim \log L$, the differences do not, so that we can compare them as far as L is fixed.

Figure 5 shows the relation of the string tensions between the three strings. In the left panel, we fix $\alpha_3 = 0$ and scan κ_{1S} in the range $-1 \leq \kappa_{1S} \leq 1$. The tension of the type-A string increases as κ_{1S} becomes larger, but those of the type-B and type-C ones change to decrease for $\kappa_{1S} \gtrsim 0.5$. It can be seen that the difference of the tensions between type-B and type-C is independent of κ_{1S} , which is reasonable because it is controlled only by the $SU(2)_C$ breaking parameters g' and α_3 as stated above.

In the right panel, we fix $\kappa_{1S} = 1.0$ and scan α_3 in the range $-1 \leq \alpha_3 \leq 1$. As is expected, the tension of the type-C (type-B) strings increases (decreases) as α_3 increases. In this parameter choice, that of the type-A string is almost constant, but this tendency depends on the parameters in general. The type-C string becomes the lightest one for $\alpha_3 \gtrsim 0$.

Before closing this section, we stress that the type-C string can be the lightest and stable string for rather wide parameter space. Although we have concentrated on the case of $\tan \beta = 1$, it would not be crucial. Therefore, the axion string produced by the breaking of $U(1)_{PQ}$ in the early universe necessarily becomes the type-C electroweak axion string

Figure 5. String tensions of the type-A, type-B and type-C strings. The parameters are taken as eq. (3.51) and the tension is calculated by integration over $0 \leq r \leq 120 v_s^{-1}$. (left): α_3 is fixed as 0 and κ_{1S} is taken as a free parameter. The tension of the type-A string increases as κ_{1S} becomes larger, but those of the type-B and type-C ones change to decrease for $\kappa_{1S} \gtrsim 0.5$. (right): κ_{1S} is fixed as 1.0 and α_3 is taken as a free parameter. As is expected, the tension of the type-B (type-C) strings decreases (increases) as α_3 becomes larger. The type-C string becomes the lightest one for $\alpha_3 \gtrsim 0$.

after the electroweak phase transition, depending on the parameters in the DFSZ model. In the string, the $U(1)_{\text{EM}}$ symmetry is spontaneously broken. This property causes an interesting phenomenon on the string, superconductivity of vortex strings, as we see in the next section.

4 Superconducting DFSZ string

It is known [34] that cosmic strings can be superconductors, i.e., the electric current can flow along a string without resistance, when the electromagnetic gauge symmetry is broken inside the string core. Superconducting strings are often realized by using scalar fields that develop non-zero VEVs only inside the strings or fermionic fields whose gapless modes are confined on the string. Further, non-Abelian vortex strings, in which charged particles such as charged vector bosons are condensed, can also support superconductivity [65, 66]. In this section, we show that the type-C string discussed in section 3 can be a superconducting string.

In the type-C string, the W^1 -flux is confined, and the $U(1)_{\text{EM}}$ symmetry is broken by the charged Higgs components and the W^1 gauge field. Corresponding to the breaking of $U(1)_{\text{EM}}$, the string has a $U(1)_{\text{EM}}$ moduli parameter, which is a flat direction around the string configuration. The existence of the moduli ensures that a (z, t) -dependent fluctuation in the direction of the moduli is a zero mode (massless excitation) in the string background and can travel on the string with the speed of light. It can carry an electric current without resistance, resulting in a supercurrent. We can rephrase this explanation into a more concrete one. The breaking of $U(1)_{\text{EM}}$ means that the charged components of the Higgs field gets the non-zero VEV inside the string, $\hat{Q}H \neq 0$, and that they can play a similar role to the charged scalar field in ref. [34].

In addition, the string can carry large electric current which induces a large magnetic interaction between the strings. That may affect the cosmological evolution of the string in the DFSZ axion model. In the following analysis, we assume $\tan \beta = 1$ for simplicity, but it is irrelevant to the argument on superconductivity.

4.1 Zero modes along the string

Let $\tilde{S}$, $\tilde{H}$, $\tilde{W}_\mu$ and $\tilde{Y}_\mu$ be the background configuration for the type-C string given by eqs. (3.31) and (3.41). Note that $\tilde{Z}_\mu = 0$ due to $\tan \beta = 1$. To find a zero mode excitation, we consider the (z, t) -modulated “gauge transformation” around the type-C string:

$$S = \tilde{S}, \quad (4.1)$$

$$H = \exp[i\eta(z, t)\chi(r, \theta)] \tilde{H} \exp\left[i\eta(z, t)\xi(r, \theta)\frac{\sigma_3}{2}\right], \quad (4.2)$$

$$W_\mu = \exp[i\eta(z, t)\chi(r, \theta)] \left(\tilde{W}_\mu - \frac{i}{g} \delta_\mu^j \partial_j \right) \exp[-i\eta(z, t)\chi(r, \theta)], \quad (4.3)$$

$$Y_\mu = \tilde{Y}_\mu + \frac{1}{g'} \delta_\mu^j \eta(z, t) \partial_j \xi(r, \theta) \quad (4.4)$$

where $\chi = \chi^a \sigma_a / 2$ and $j = r, \theta$. This is not a mere gauge transformation unless η is independent of z and t , but is a (z, t) -dependent physical excitation described by $\eta(z, t)$, $\xi(r, \theta)$ and $\chi^a(r, \theta)$.

Instead of the above expressions, for later use, we analyze an alternative ansatz that is obtained by performing the $SU(2)_W \times U(1)_Y$ gauge transformation with the gauge parameters $(\eta\chi, \eta\xi)$. The transformed ansatz is given by

$$S = \tilde{S}, \quad (4.5)$$

$$H = \tilde{H}, \quad (4.6)$$

$$W_\mu = \tilde{W}_\mu + \delta W_\mu, \quad (4.7)$$

$$Y_\mu = \tilde{Y}_\mu + \delta Y_\mu \quad (4.8)$$

with

$$\delta W_\mu = \frac{1}{g} \delta_\mu^\alpha \chi \partial_\alpha \eta, \quad (4.9)$$

$$\delta Y_\mu = -\frac{1}{g'} \delta_\mu^\alpha \xi \partial_\alpha \eta, \quad (4.10)$$

where $\alpha = t, z$. For these ansatz (4.5)–(4.8), the field strength tensors are given by

$$W_{\mu\nu} = \tilde{W}_{\mu\nu} + \tilde{D}_\mu \delta W_\nu - \tilde{D}_\nu \delta W_\mu, \quad (4.11)$$

$$Y_{\mu\nu} = \tilde{Y}_{\mu\nu} + \partial_\mu \delta Y_\nu - \partial_\nu \delta Y_\mu, \quad (4.12)$$

where $\tilde{W}_{\mu\nu}$, $\tilde{Y}_{\mu\nu}$ and $\tilde{D}_\mu$ are the field strengths and the covariant derivative consisting of the background gauge configurations $\tilde{W}_\mu$ and $\tilde{Y}_\mu$.

The linearized EOMs for the excitation η , χ^a and ξ are obtained by substituting the ansatz into the full EOMs,

$$(D_\nu W^{\nu\mu})^a = -j_W^{\mu,a}, \quad (4.13)$$

$$\partial_\nu Y^{\nu\mu} = -j_Y^\mu, \quad (4.14)$$

$$D_\mu D^\mu H = -\frac{\delta V(H, S)}{\delta H^\dagger}, \quad (4.15)$$

where $j_W^{\mu,a}$ and j_Y^μ are the $SU(2)_W$ and $U(1)_Y$ currents:

$$j_W^{\mu,a} = \frac{i}{2}g \operatorname{Tr} \left[H^\dagger \sigma^a D^\mu H - (D^\mu H)^\dagger \sigma^a H \right], \quad (4.16)$$

$$j_Y^\mu = -\frac{i}{2}g' \operatorname{Tr} \left[\sigma_3 H^\dagger D^\mu H - (D^\mu H)^\dagger H \sigma_3 \right]. \quad (4.17)$$

Then we obtain the following equations from eqs. (4.13) and (4.14) (see appendix A for the derivation),

$$\partial^\alpha \eta \left(\tilde{D}_j \tilde{D}^j \chi \right)^a = \frac{-g^2}{2} \partial^\alpha \eta \left(\chi^a \operatorname{Tr} |\tilde{H}|^2 + \xi \operatorname{Tr} \left[\tilde{H}^\dagger \sigma_a \tilde{H} \sigma_3 \right] \right), \quad (4.18)$$

$$\partial^\alpha \eta \partial_j \partial^j \xi = \frac{-g'^2}{2} \partial^\alpha \eta \left(\xi \operatorname{Tr} |\tilde{H}|^2 + 2 \operatorname{Tr} \left[\tilde{H}^\dagger \chi \tilde{H} \sigma_3 \right] \right), \quad (4.19)$$

$$\tilde{D}^j \chi \partial^\alpha \partial_\alpha \eta = 0, \quad (4.20)$$

$$\partial^j \xi \partial^\alpha \partial_\alpha \eta = 0, \quad (4.21)$$

and from the EOM for H (4.15)

$$\partial^\alpha \partial_\alpha \eta \left(2\chi \tilde{H} + \xi \tilde{H} \sigma^3 \right) = 0. \quad (4.22)$$

We have used the fact that the background configurations $\tilde{H}$, $\tilde{W}_\mu$ and $\tilde{Y}_\mu$ solve the EOMs.

The above equations (4.20), (4.21) and (4.22) are satisfied with

$$\partial^\alpha \partial_\alpha \eta = (\partial_t^2 - \partial_z^2) \eta = 0, \quad (4.23)$$

which is a (1+1)-dimensional wave equation. This has the zero mode solutions $\eta = \eta^+(z+t)$ and $\eta = \eta^-(z-t)$ with some functions $\eta^\pm$, and the general solution can be written by a linear combination of these modes. When one is particularly interested in the static case, the t -independent solution is given by

$$\eta(z) = \omega z, \quad (\omega : \text{const.}) \quad (4.24)$$

implying the constant current along the z direction. On the other hand, the radial and angular dependence of the excitations is determined by eqs. (4.18) and (4.19). We can see that there are four independent zero modes corresponding to the solutions χ^a ($a = 1, 2, 3$) and ξ . This is understood from the fact that the whole symmetry of $SU(2)_W \times U(1)_Y$ (even $U(1)_{\text{EM}}$) is broken inside the string. In other words, four zero modes can induce the $SU(2)_W$ and $U(1)_Y$ currents on the string.

However, only one of them induces the $U(1)_{\text{EM}}$ current, which generates two-dimensional Coulomb (magnetic) potential far from the string. To illustrate this, let us consider the asymptotic behavior of χ and ξ at $r \rightarrow \infty$. At infinity, the $U(1)_{\text{EM}}$ is restored, in which the background configuration $\tilde{H}$ satisfies [69]

$$\tilde{H}\sigma^3 + n^a\sigma^a\tilde{H} = 0, \quad n^a = -\frac{\text{Tr}(\sigma^3\tilde{H}^\dagger\sigma^a\tilde{H})}{\text{Tr}|\tilde{H}|^2}, \quad (4.25)$$

and $(\tilde{D}_\mu n)^a = 0$ and $\text{Tr}|\tilde{H}|^2 = 2v^2$. Using these conditions, the Poisson-like equations (4.18) and (4.19) become

$$(\tilde{D}_j\tilde{D}^j\chi)^a = -g^2v^2(\chi^a - n^a\xi), \quad (4.26)$$

$$\partial_j\partial^j\xi = -g'^2v^2(\xi - \chi^an^a), \quad (4.27)$$

which describe the long-range behavior on the xy plane only when $\chi^a - \xi n^a = 0$. Thus, we have the two-dimensional Laplace equation:

$$\frac{1}{r}\partial_r(r\partial_r\xi) = 0 \quad (4.28)$$

with $\chi^a = \xi n^a$ for ξ being rotationally invariant ($\partial_\theta\xi = 0$). The asymptotic solution of eq. (4.28) behaves as $\xi \sim \log r$. Substituting this into the expressions of gauge fields gives

$$\delta W_z^a \sim \frac{\omega}{g}n^a\log r, \quad \delta Y_z \sim -\frac{\omega}{g'}\log r, \quad (4.29)$$

where we have taken the normalization of ξ such that $\xi \rightarrow \log r$ for $r \rightarrow \infty$. We find the form of the $U(1)_{\text{EM}}$ field strength

$$F_{rz}^{\text{EM}} = -\sin\theta_W n^a W_{rz}^a + \cos\theta_W Y_{rz}, \quad (4.30)$$

$$= -\sin\theta_W n^a \partial_r \delta W_z^a + \cos\theta_W \partial_r \delta Y_z, \quad (4.31)$$

$$\sim -\frac{\omega}{er} \quad (r \rightarrow \infty). \quad (4.32)$$

This is nothing but the magnetic long-range force on the two-dimensional xy plane. Correspondingly, the total amount of $U(1)_{\text{EM}}$ current J_{EM} along the string is estimated from eq. (4.32) as

$$J_{\text{EM}} \equiv -2\pi r F_{rz}^{\text{EM}} \sim \frac{2\pi\omega}{e}. \quad (4.33)$$

4.2 Current quenching and string interaction

In the above argument, it may seem that the magnitude of the current is given by the parameter ω and can be taken arbitrarily large. However this is not the case since we have ignored the backreaction from the zero modes to the background fields $\tilde{S}$, $\tilde{H}$, $\tilde{W}_\mu$ and $\tilde{Y}_\mu$ by linearizing the EOMs. To examine the backreaction, we look at the following term in the Lagrangian:

$$\mathcal{L} \supset -\text{Tr}|-ig\delta W_z\tilde{H} + i\frac{g'}{2}\tilde{H}\sigma_3\delta Y_z|^2 \quad (4.34)$$

$$= -\omega^2\text{Tr}|\chi\tilde{H} + \xi\tilde{H}\frac{\sigma_3}{2}|^2, \quad (4.35)$$

which is obtained by substituting the string ansatz and the solution for $\eta(z)$. This term induces a positive squared mass for the $SU(2)_W \times U(1)_Y$ charged components of $\tilde{H}$. For $\omega \rightarrow \infty$, the charged components vanish, $\chi^a \tilde{H} + \xi \tilde{H} \frac{\sigma_3}{2} \rightarrow 0$, which decreases the $SU(2)_W$ and $U(1)_Y$ currents from the Higgs field, $j_W^{z,a}$ and j_Y^z ((4.16) and (4.17)). Correspondingly, the right hand sides of eqs. (4.18) and (4.19) vanish everywhere, and they have only a trivial solution $\xi = \chi^a = 0$.⁴ Thus, the backreaction from an extremely large ω reduces the amount of the current. Such behavior is known as the current quenching [34].

Now, let us estimate the maximum value of the current. For the $U(1)_{EM}$ zero mode, $\chi^a = \xi n^a$, the mass term (4.35) reads

$$-(\omega\xi)^2 \text{Tr}|\hat{Q}\tilde{H}|^2 = -(\omega\xi)^2 \frac{v^2}{2}(f-h)^2, \quad (4.36)$$

where $\hat{Q}$ is the $U(1)_{EM}$ generator defined in eq. (2.27), and we have used the concrete expression of $\tilde{H}$ for the type-C string, eq. (3.31). On the other hand, using $\kappa_{1S} = \kappa_{2S}$ for $v_1 = v_2 = v$, the mass terms for f and h that are originally present in the Lagrangian are

$$-\mathcal{L} \supset \frac{v^2}{r^2} \left[(1+w)^2 f^2 + (1-w)^2 h^2 \right] + (-m_1^2 + \kappa_{1S} v_s^2 \phi^2)(f^2 + h^2). \quad (4.37)$$

Due to the backreaction term eq. (4.36), the two-by-two mass matrix M^2 for (f, h) is changed and not diagonal. It is sufficient to consider the signs of the eigenvalues of M^2 . If they are positive inside the string, then f and h tend to vanish, and hence $(f-h)^2 \propto \text{Tr}|\hat{Q}\tilde{H}|^2 = 0$, which means that the $U(1)_{EM}$ symmetry is restored even inside the string. On the other hand, if one of them is negative, the quenching is not significant and the $U(1)_{EM}$ symmetry is still broken. Then the current can be increased with $|\omega|$.

Inside the string, $r \lesssim v^{-1}$, the determinant of the matrix M^2 is calculated as

$$\det M^2 \sim m_1^4 + \frac{4}{r^2} \left(\omega^2 \xi^2 - m_1^2 + \kappa_{1S} \phi^2 \right) + \left(-m_1^2 + \kappa_{1S} v_s^2 \phi^2 \right) \omega^2 \xi^2, \quad (4.38)$$

where we have used $w(r) \sim 1$ there. Note that the mass matrix and hence the determinant vary with the radius r in and out the string core as

$$\det M^2 \sim \begin{cases} v_s^4 + \frac{4}{r^2} (\omega^2 \xi^2 - v^2) - v^2 \omega^2 \xi^2 & \text{for } r \gtrsim v_s^{-1}, \\ v_s^4 + \frac{4}{r^2} (\omega^2 \xi^2 - v_s^2) - v_s^2 \omega^2 \xi^2 & \text{for } r \lesssim v_s^{-1}. \end{cases} \quad (4.39)$$

Clearly, there is a critical value of $|\omega\xi|$, for which the sign of the term proportional to r^{-2} changes from negative to positive. For a value smaller than the critical one, there exists a region where the determinant is negative, avoiding the current quenching. The critical value of $|\omega\xi|$ is $\sim v_s$ around the core. Then the current magnitude becomes maximum for this value. Note that ξ is of order unity inside the core to be connected with the asymptotic form $\xi \rightarrow \log r$. Thus, $|\omega| \sim v_s$ in this case. Note that the definition of the $U(1)_{EM}$ moduli may be different a bit from $\chi^a = \xi n^a$ inside the string [69]. The difference is however subleading and negligible for the maximum amount of the supercurrent.

⁴A solution behaving like $\sim \log r$ for all r is singular at $r = 0$.

Finally, we consider the tension of the string (energy per unit length). When $\omega = 0$, the string reduces to the type-C string and the tension T is dominated by the gradient energy from the axion:

$$T(\omega = 0) \sim 2\pi \int^L r dr |\partial_i S|^2 \sim 2\pi v_s^2 \log L, \quad (4.40)$$

with L being an IR cutoff. The logarithmic divergence is natural because $U(1)_{\text{PQ}}$ is a global symmetry. On the other hand, for $\omega \neq 0$, there should be an additional contribution to the tension from the magnetic field induced by the $U(1)_{\text{EM}}$ current:

$$T(\omega) = T(0) + T_{\text{EM}} \quad (4.41)$$

where

$$T_{\text{EM}} \sim 2\pi \int^L r dr \left(F_{rz}^{\text{EM}}\right)^2 \sim \frac{2\pi\omega^2}{e^2} \log L. \quad (4.42)$$

Thus the current also induces a logarithmically divergent energy, which is comparable to that from the gradient term, for the maximal current $|\omega| \sim v_s$ discussed above.

This result provides an interesting sight for the interaction between the strings. Let us consider two superconducting strings that are well separated in the xy plane. They are assumed to have the same winding for $U(1)_{\text{PQ}}$ and $U(1)_{W1}$, and contain the superconducting currents with the same sign. As is well-known, the gradient energy of S gives a repulsive force $\sim v_s^2/R_0$ with R_0 being the distance between them. However, the magnetic interaction induced from the supercurrent provides an attractive force,

$$F = -\frac{(J_{\text{EM}})^2}{2\pi R_0} \sim -\frac{2\pi\omega^2}{e^2 R_0}. \quad (4.43)$$

Therefore, the superconducting strings can receive the attractive force overcoming the repulsive one with $|\omega| \sim v_s$.

4.3 Y-junction formation

In usual cases, a pair of axion strings reconnects with a probability of the order of unity when they collide. Thanks to this reconnection process, a network of strings in the early universe produces small string loops, which soon shrink and disappear by emitting radiations, and sufficiently loses the energy if the network is dense. The time evolution of the energy density approaches the so-called scaling behavior, and the total energy density of the universe is not dominated by the string network. However, the attractive interaction discussed above, which is induced by superconducting current, can change this picture drastically. In particular, it is known that an attractive interaction between strings could form the Y-junction [70–78], which is a bound state of two strings (see figure 6). If Y-junctions are formed frequently, that reduces the effective reconnection probability and makes non-trivial whether the string network evolves to the scaling solution. As the first step to study the Y-junctions of the axion strings, we present a rough estimation of the formation probability of Y-junctions in this section.

Figure 6. Reconnection process and formation of a Y-junction.

Firstly, let us assume the presence of a primordial magnetic field (PMF) after the electroweak phase transition. PMF is well motivated as an origin of intergalactic magnetic fields in order to explain the galactic magnetic fields observed today via the dinamo mechanism [79]. (For a review, see, e.g., ref. [80].) There are the observational lower and upper bounds on the present strength of intergalactic magnetic fields for a coherent length $\lambda \gtrsim 0.1 \text{ Mpc}$ as $3 \times 10^{-16} \text{ G} \lesssim B_0 \lesssim 10^{-11} \text{ G}$, where the lower one is set by the non-observation of secondary photons from the emission of highly collimated gamma rays by blazars [81] and the upper one is from the CMB observations [82]. For simplicity, we further assume that the power spectrum of PMF is scale invariant and almost coherent over the entire Hubble horizon, which could be realized for the inflationary magnetogenesis [80].

We then show that PMF can induce large superconducting currents on the strings. Such large currents are sufficient to form the Y-junctions. The scale invariant PMF evolves as

$$B(t) \propto a(t)^{-2}, \quad (4.44)$$

where $B(t)$ and $a(t)$ are the magnetic field strength and the scale factor at the cosmological time t , respectively. It is convenient to define a ratio of the energy density of PMF (ρ_B) to that of photon (ρ_γ),

$$\epsilon \equiv \frac{\rho_B}{\rho_\gamma} \sim B(t)^2 G_N t^2, \quad (4.45)$$

with G_N being the Newton constant. The ratio ϵ is a constant as the universe expands. If we take the upper limit on B_0 , we have $\epsilon \sim 10^{-11}$.

In the early universe, the superconducting strings (Type-C EW axion strings) move with velocity v_{str} ($\sim \mathcal{O}(1)$) in the presence of PMF and hence feel the electric fields $E \sim B(t)v_{\text{str}}$, which induce the superconducting current,

$$J_{\text{PMF}} \sim e^2 B(t) \xi v_{\text{str}}, \quad (4.46)$$

Figure 7. Two strings (red and blue lines) collide with the crossing angle α . The black arrows indicate the directions of the topological winding number of $U(1)_{PQ}$. The red and blue arrows indicate the directions of the flowing electric currents, which are parallel for $\alpha = 0$. The green and purple planes are orthogonal and include the collision point.

with ξ being a string typical length. Since there are no superconducting strings before the electroweak phase transition, we can assume that the scaling property $\xi \sim t$ holds at least just after the phase transition, and obtain

$$J_{PMF} \sim 10^{12} \left(\frac{\epsilon}{10^{-11}} \right)^{1/2} \text{ GeV} \sim 10^{12} \left(\frac{B_0}{10^{-11} \text{ G}} \right) \text{ GeV}. \quad (4.47)$$

Note that this is independent of t . For the upper limit of B_0 , yielding $\epsilon \sim 10^{-11}$, the induced current (4.47) is larger than the maximum current estimated in the last subsection, $J_{EM} \sim 2\pi v_s/e$, with $v_s \simeq 10^{9-12} \text{ GeV}$. Therefore, the current is saturated to the maximum value by PMF. In the following argument, we use this maximum current as those the strings carry in the early universe.

Let us discuss the formation of Y-junctions. They can be formed by collisions of two superconducting strings when they feel an attractive force and are trapped in the potential. We suppose that the two strings collide with the crossing angle α and that they are identical and parallel for $\alpha = 0$. The superconducting currents are assumed to flow in the same direction for $\alpha = 0$ (the opposite case will be considered later). The three-dimensional dynamics of the collision event is rather complicated and difficult to analyze. However a qualitative picture can be understood by reducing it on two orthogonal planes including the colliding point [83–85] (see figure 7). On one of them (purple plane in figure 7), the situation is regarded as a collision (scattering) event of two point-like vortices in two dimensions while, on the other plane (green plane in figure 7), as a vortex-antivortex collision resulting in the annihilation in two dimensions. Note that this picture reproduces the reconnection process shown in the upper-right picture in figure 6 when the strings have no superconducting current; two scattered vortices on the purple plane and nothing on the green one.

We concentrate on the two-dimensional analysis of the superconducting vortex-vortex collision since the current does not change the annihilation. Unlike the non-superconducting case leading to the 90° scattering, the long-range magnetic interaction plays a crucial role

in our case. On the reduced plane, the vortex-vortex pair feel a net interaction potential,

$$V \sim \frac{v_s^2}{e^2} \cos \frac{\alpha}{2} \log r, \quad (4.48)$$

where r is the distance between the two vortices and we have ignored the axion-mediated repulsive interaction, which should be subleading by a factor e^2 compared to eq. (4.48). If the vortices do not have sufficient kinetic energy to escape to infinity, they become trapped by the potential. Thus we obtain a condition to form a bound state,

$$\frac{v_s^2}{e^2} \cos \frac{\alpha}{2} \log \frac{L}{\delta} \geq (\gamma - 1)\mu, \quad (4.49)$$

where γ and μ are the Lorentz factor and the tension of the colliding vortices. L and δ are the IR and UV cutoff for the potential, which we take as the Hubble radius and the width of the strings, respectively. As considering just after the electroweak phase transition, the logarithmic factor gives $\log(10^{17} \times v_s/v_{\text{EW}}) \sim 50$. In addition, we take a mildly relativistic velocity ~ 0.6 for the vortices, yielding $\gamma \sim 1.25$, and thus eq. (4.49) gives

$$\cos \frac{\alpha}{2} \geq 10^{-3}, \quad (4.50)$$

which means that they *almost always* form the bound state except for $\alpha \simeq \pi$.

On the other hand, they cannot form such a bound state in the case that the currents flow in the opposite directions (anti-parallel) on the vortex-vortex plane because they always feel a repulsive force. Therefore, we obtain a rough estimation for the formation probability of the bound state,

$$\begin{cases} \text{Y-junction formed} & (\text{current in the same directions}) \\ \text{not formed} & (\text{current in the opposite directions}) \end{cases} \Rightarrow \text{probability} \simeq \frac{1}{2}. \quad (4.51)$$

Note that this is based on the two-dimensional analysis focusing on the collision point. This picture may break down for the case that the dynamics after colliding is dominated by other parts of the strings than the collision point. Furthermore, the formed Y-junction could be peeled off, depending on the velocities and the crossing angle of the strings. For a more detailed study including these effects, it is necessary to perform a three-dimensional simulation, which is beyond the scope of the present paper.

Finally, we discuss a consequence of the formation of Y-junctions. When a superconducting string intersects with itself, it forms the Y-junction with probability $1/2$, produces a small loop with probability $p/2$, or passes through with probability $(1 - p)/2$ (see figure 8), where p is the reconnection probability without the current ($p \sim 1$). If such a Y-junction-connected loop (upper one in figure 8) is produced, it is not obvious whether the strings sufficiently lose their energy, and the scaling behavior of the energy density of strings is not ensured. Once the energy density of the universe is dominated by such superconducting strings, it causes large impacts on cosmology, and hence some parameter region in the DFSZ axion model may be constrained.

Figure 8. Self-intersection of a superconducting string. The Y-junction (green doubled line) is formed with probability $\frac{1}{2}$. Otherwise, it reconnects producing a small loop (blue line) or passes through. The reconnection probability is $p \sim 1$.

5 Conclusion

We have studied the axion strings with the electroweak gauge flux, and their superconductivity in the DFSZ model. We constructed three types of the electroweak axion string solutions, which have similar properties to those of (non-Abelian) vortices in two Higgs doublet model. We also showed that in some parameter space, the string with W -flux, we called the type-C string, can be lighter than those with Z -flux. The type-C string exhibits superconductivity and a large electric current can flow along the string. This large current may realize a net attractive force between the axion strings, which could form the so-called Y-junctions in the early universe. By considering the string collision reduced onto two-dimensional planes, the probability of the formation of the Y-junctions is estimated to be $\sim 1/2$.

Once such Y-junctions are formed, they can affect the evolution of the string network and make non-trivial whether the network evolves to the scaling solution. If no obeying the scaling behavior, the string network could dominate the energy density of the universe and the model is severely constrained. To conclude whether this is true or not, we need detailed numerical simulations on the time evolution of the network taking into account both the axionic and magnetic interactions. In addition, the Y-junction provides characteristic signals in astrophysical observations. It is worthwhile to study the Y-junction dynamics of the DFSZ axion strings⁵ within a phenomenologically allowed parameter space.

In this paper, we focus on the $U(1)_{\text{EM}}$ symmetry breaking through the W -flux and the charged Higgs components. On the other hand, the type-A and type-B strings also could be superconducting. The Z -flux give a negative mass contribution to the W -boson as

$$ig \cos \theta_W \tilde{Z}_{\mu\nu} W^{-,\mu} W^{+,\nu}, \quad (5.1)$$

⁵Y-junctions could also be formed in the KSVZ model when heavy extra fermions are $U(1)_{\text{EM}}$ charged and the strings are superconducting.

which leads the instability called the W -condensate [86–88]. This can be another possible mechanism realizing superconductivity. If the magnitude of the background flux is sufficiently large, the type-A and type-B strings also can be superconducting.

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A Derivation of the linearized EOMs

By substituting the string ansatz into the EOM (4.13) for $\mu = \alpha$, we have

$$\left(\tilde{D}_j \tilde{D}^j \delta W^\alpha\right)^a = \frac{-i}{2} g \operatorname{Tr} \left[\tilde{H}^\dagger \sigma^a \left(-ig \delta W^\alpha \tilde{H} + i \frac{g'}{2} \delta Y^\alpha \tilde{H} \sigma_3 \right) - \text{h.c.} \right], \quad (\text{A.1})$$

where we have used the fact that the background configurations $\tilde{H}$, $\tilde{W}_\mu$ and $\tilde{Y}_\mu$ solve the EOMs. The equation is explicitly written by the functions η , χ and ξ :

$$(\text{l.h.s.}) = \frac{1}{g} \partial^\alpha \eta \left(\tilde{D}_j \tilde{D}^j \chi \right)^a, \quad (\text{A.2})$$

$$(\text{r.h.s.}) = \frac{-g}{2} \partial^\alpha \eta \operatorname{Tr} \left[\tilde{H}^\dagger \{ \sigma^a, \chi \} \tilde{H} + \left(\tilde{H}^\dagger \sigma_a \tilde{H} \frac{\sigma_3}{2} + \frac{\sigma_3}{2} \tilde{H}^\dagger \sigma_a \tilde{H} \right) \xi \right] \quad (\text{A.3})$$

$$= \frac{-g}{2} \partial^\alpha \eta \left(\chi^a \operatorname{Tr} |\tilde{H}|^2 + \xi \operatorname{Tr} \left[\tilde{H}^\dagger \sigma^a \tilde{H} \sigma_3 \right] \right), \quad (\text{A.4})$$

which implies the linearized EOM (4.18). A similar procedure for the EOM of Y_μ (4.14) leads to the linearized one (4.19). On the other hand, for the EOM (4.13) for $\mu = j$, we have

$$\left(D_\nu W^{\nu j} \right)^a = -\frac{i}{2} g \operatorname{Tr} \left[\tilde{H}^\dagger \sigma^a \tilde{D}^j \tilde{H} - (\tilde{D}^j \tilde{H})^\dagger \sigma^a \tilde{H} \right]. \quad (\text{A.5})$$

The l.h.s. is divided into two pieces

$$\left(D_\nu W^{\nu j} \right)^a = \left(\tilde{D}_k W^{kj} \right)^a + \left(D_\alpha W^{\alpha j} \right)^a \quad (\text{A.6})$$

with $k = r, \theta$. The first term in the r.h.s. of eq. (A.6) is found to be equal to the r.h.s. of eq. (A.5), and they are cancelled out from the EOM. The remaining second term is given at the leading order of δW as

$$D_\alpha W^{\alpha j} = -\partial_\alpha \tilde{D}^j \delta W^\alpha + ig \left[\delta W_\alpha, \tilde{D}^j \delta W^\alpha \right] \simeq \frac{-1}{g} \tilde{D}^j \chi \partial^\alpha \partial_\alpha \eta, \quad (\text{A.7})$$

which implies the linearized EOM (4.20). The similar derivation holds for the gauge field Y_μ .

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