On a gauge-invariant deformation of a classical gauge-invariant theory

I.L. Buchbinder and P.M. Lavrov

*Center of Theoretical Physics, Tomsk State Pedagogical University,
Kievskaya St. 60, 634061 Tomsk, Russia*

*National Research Tomsk State University,
Lenin Av. 36, 634050 Tomsk, Russia*

E-mail: joseph@tspu.edu.ru, lavrov@tspu.edu.ru

ABSTRACT: We consider a general gauge theory with independent generators and study the problem of gauge-invariant deformation of initial gauge-invariant classical action. The problem is formulated in terms of BV-formalism and is reduced to describing the general solution to the classical master equation. We show that such general solution is determined by two arbitrary generating functions of the initial fields. As a result, we construct in explicit form the deformed action and the deformed gauge generators in terms of above functions. We argue that the deformed theory must in general be non-local. The developed deformation procedure is applied to Abelian vector field theory and we show that it allows to derive non-Abelian Yang-Mills theory. This procedure is also applied to free massless integer higher spin field theory and leads to local cubic interaction vertex for such fields.

KEYWORDS: BRST Quantization, Gauge Symmetry

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1 Introduction

Gauge theories are an integral part of the Standard Model of Fundamental Interactions and the Standard Cosmological Model. Therefore, going beyond the standard models can be related to the construction and study of new gauge theories. In this paper, we propose an approach to generate new gauge theories beginning with some known and more or less simple gauge models. The approach is based on Batalin-Vilkovisky (BV) formalism [1–3] that allows to explore wide range of classical and quantum aspects of the gauge theories by unified and universal way (see the further development of the BV-formalism e.g. in [4–6] and the references therein).

The BV-formalism was initially developed to provide a generic universal method to quantize the general gauge theories. In this paper, we study the purely classical problem of constructing a deformation procedure of gauge theories with the preservation of gauge invariance. It is worth noting here a certain analogy with the use of the BRST-BFV method [7–9], constructed initially for the covariant canonical quantization of the gauge theories, in the classical higher-spin field theory (see e.g. [10, 11]).¹

The main object of the BV-formalism is the master equation which is formulated in terms of the antibrackets [1, 2]. The basic property of the antibracket is its invariance under the anticanonical transformations. Namely this property plays an important role in

¹There exists an extensive literature devoted mainly to quantum aspects of BV-formalism. Since we study the purely classical aspects, we are going to cite only the papers which can in principle be related to our work.

solving the different problems in classical and quantum descriptions of the gauge systems. In this paper we will use the anticanonical transformations to find out the general solution to classical master equation that allows the construction of an arbitrary gauge-invariant deformation of a given gauge theory.

The approach to solving the classical master equation was developed in the papers [12, 13], where it was proposed to look for solutions to this equation in form of expansion in some coupling parameters and reduce the classical master equation to an infinite system of cohomologies.² Applications of this approach to higher spin field theory were considered in the papers [16–18]. We develop a completely different approach that does not require expansions and does not use the cohomological analysis. The general solution of classical master equation is given in explicit form in terms of two independent generating functions responsible for deformation of initial action and initial gauge transformations.

The paper is organized as follows. In section 2 we briefly review the basic notations of the BV-formalism such as the antibracket, classical master equation, and anticanonical transformations. Section 3 is devoted to the solution of the classical master equation to construct the general deformation of the initial action in terms of a single generating function depending on initial fields. We prove that such deformation must in general be non-local, although in some special case the corresponding deformed action can have the local sector. In section 4 we describe a general deformation of gauge generators and derive the deformed gauge algebra. The deformed generators are defined by two functions, one of them is the same as for deformation of action and another function relates to the special deformation of the generators. It is interesting to point out that even if the initial theory is Abelian, the deformed theory will obligatorily be non-Abelian. In section 5 we show that the application of the deformation procedure under consideration to free Abelian vector field gauge theory leads to deformed theory containing the standard non-Abelian Yang-Mills field action among the other non-local terms in deformed action. Section 6 is devoted to the application of the above deformation theory to free massless integer higher spin field theory that allows the construction of the local cubic interaction vertex for such fields. In section 7 we summarize the results.

In the paper we systematically use the DeWitt’s condensed notations and employ the symbols $\varepsilon(A)$ for the Grassmann parity and $\text{gh}(A)$ for the ghost number respectively. The right and left functional derivatives are marked by special symbols “ $\leftarrow$ ” and “ $\rightarrow$ ” respectively. Arguments of any functional are enclosed in square brackets $[]$, and arguments of any function are enclosed in parentheses, $()$.

2 Antibracket and master equation

In this section, we briefly describe the basic notions of the BV-formalism which will be essentially used in the paper to describe a general gauge-invariant deformation of the classical gauge theory.

We consider a gauge theory of the fields $A = \{A^i\}$ with Grassmann parities $\varepsilon(A^i) = \varepsilon_i$ and ghost numbers $\text{gh}(A^i) = 0$. The theory is described by the initial action $S_0[A]$ and

²See also the recent papers [14, 15] and the references therein.

gauge generators $R^i_\alpha(A)$ ($\varepsilon(R^i_\alpha(A)) = \varepsilon_i + \varepsilon_\alpha$, $\text{gh}(R^i_\alpha(A)) = 0$). The action is invariant under the gauge transformations

$$\delta A^i = R^i_\alpha(A) \xi^\alpha, \quad (2.1)$$

where the gauge parameters ξ^α ($\varepsilon(\xi^\alpha) = \varepsilon_\alpha$) are the arbitrary functions of space-time coordinates. Condition of gauge invariance is written in the standard form

$$S_0[A] \overleftarrow{\partial}_{A^i} R^i_\alpha(A) = 0. \quad (2.2)$$

It is assumed that the fields $A = \{A^i\}$ are linear independent with respect to the index i however, in general, these generators may be linear dependent with respect to index α . Further, we restrict ourselves by the irreducible gauge transformations with a closed gauge algebra. In this case, the generators satisfy the following relation

$$R^i_{\alpha,j}(A) R^j_\beta(A) - (-1)^{\varepsilon_\alpha \varepsilon_\beta} R^i_{\beta,j}(A) R^j_\alpha(A) = -R^i_\gamma(A) F^\gamma_{\alpha\beta}(A), \quad R^i_{\alpha,j}(A) = R^i_\alpha(A) \overleftarrow{\partial}_{A^j}, \quad (2.3)$$

where $F^\gamma_{\alpha\beta}(A)$ ($\varepsilon(F^\gamma_{\alpha\beta}(A)) = \varepsilon_\alpha + \varepsilon_\beta + \varepsilon_\gamma$, $\text{gh}(F^\gamma_{\alpha\beta}(A)) = 0$) are the structure coefficients depending, in general, on the fields A^i with the following symmetry properties $F^\gamma_{\alpha\beta}(A) = -(-1)^{\varepsilon_\alpha \varepsilon_\beta} F^\gamma_{\beta\alpha}(A)$.

Following the BV-formalism, we introduce the minimal antisymplectic space of fields ϕ^A and antifields ϕ^*_A ,³

$$\phi^A = (A^i, C^\alpha), \quad \phi^*_A = (A^*_i, C^*_\alpha), \quad (2.4)$$

where C^α ($\varepsilon(C^\alpha) = \varepsilon_\alpha + 1$, $\text{gh}(C^\alpha) = 1$) are the ghost fields and antifields obeying the following properties

$$\varepsilon(\phi^*_A) = \varepsilon(\phi^A) + 1, \quad \text{gh}(\phi^*_A) = -1 - \text{gh}(\phi^A). \quad (2.5)$$

The basic object of the BV-formalism is the extended action $S = S[\phi, \phi^*]$ satisfying the classical master equation,

$$(S, S) = 0, \quad (2.6)$$

and the boundary condition,

$$S[\phi, \phi^*] \Big|_{\phi^*=0} = S_0[A]. \quad (2.7)$$

The master equation (2.6) is written in terms of antibracket which is defined for any functionals $F[\phi, \phi^*]$ and $H[\phi, \phi^*]$ in the form

$$(G, H) = G \left(\overleftarrow{\partial}_{\phi^A} \overrightarrow{\partial}_{\phi^*_A} - \overleftarrow{\partial}_{\phi^*_A} \overrightarrow{\partial}_{\phi^A} \right) H. \quad (2.8)$$

³We will only be interested in possible deformations of initial classical gauge systems consistent with basic properties of the BV-formalism. In this case, it is sufficient to consider the minimal antisymplectic space only.

The gauge invariance of the initial action $S_0[A]$ leads to the invariance of the action $S[\phi, \phi^*]$,

$$\delta_B S = 0 \quad (2.9)$$

under the global supersymmetry transformations (BRST transformations [19, 20])

$$\delta_B \phi^A = (\phi^A, S)\mu = \overrightarrow{\partial}_{\phi_A^*} S \mu, \quad \delta_B \phi_A^* = 0, \quad (2.10)$$

as a consequence that S satisfies the classical master equation. Here μ is a constant Grassmann parameter. In the case of Yang-Mills theories, this invariance in the sector of fields A^i is nothing but the gauge invariance of $S_0[A]$ under the standard gauge transformations with the special gauge parameters $\xi^\alpha = C^\alpha \mu$.

Taking into account the gauge invariance of the initial action (2.1) and the boundary condition (2.7), one can write the action $S = S[\phi, \phi^*]$ up to the terms linear in antifields in the form

$$S = S_0[A] + A_i^* R_\alpha^i(A) C^\alpha - \frac{1}{2} C_\gamma^* F_{\alpha\beta}^\gamma(A) C^\beta C^\alpha (-1)^{\varepsilon_\alpha} + O(\phi^{*2}). \quad (2.11)$$

We emphasize that the antibracket is an essential element of the compact description of the classical gauge theories within the BV-formalism. An important property of the antibracket (2.8), which we will use, is its invariance with respect to anticanonical transformations of fields and anti-fields [1, 2]. It leads to the statement that any of the two solutions of classical master equation (2.6), satisfying the same boundary condition (2.7), are related one to another by some anticanonical transformation [1, 2].⁴ Moreover, taking some solution of the classical master equation satisfying the given boundary condition and using an arbitrary anticanonical transformation in this solution, we will get again a solution of the classical master equation. Namely this fact will be used to construct a gauge-invariant deformation of the gauge theories.

In this paper, we will describe the general deformation of initial classical action and initial gauge symmetry on the base of the classical master equation. We will see that these deformations are completely formulated in terms of anticanonical transformations. It is known that there are two possibilities to present the anticanonical transformations, namely, in terms of generating functional or in terms of generators [4]. We suppose that for the problem under consideration, the description of anticanonical transformations with the help of generating functional seems to be more preferable.

3 Deformed action

In this section, we will describe the general structure of deformed action depending on the same set of fields A as the initial action $S_0[A]$.

⁴It is worth pointing out that the space of fields and antifields is analogous to phase space of classical mechanics and the antibracket is analogous to Poisson bracket. Then, the anticanonical transformations of the gauge theory are analogous to canonical transformations preserving the Poisson bracket.

We begin with the action (2.11) subjected to anticanonical transformations

$$\tilde{S}[\phi, \phi^*] = S[\Phi(\phi, \phi^*), \Phi^*(\phi, \phi^*)], \quad (3.1)$$

where $\Phi(\phi, \phi^*)$ and $\Phi^*(\phi, \phi^*)$ are the solutions to the equations

$$\phi_A^* = Y[\phi, \Phi^*] \overleftarrow{\partial}_{\phi^A}, \quad \Phi^A = \overrightarrow{\partial}_{\Phi_A^*} Y[\phi, \Phi^*], \quad (3.2)$$

and $Y = Y[\phi, \Phi^*]$ ($\varepsilon(Y) = 1$, $\text{gh}(Y) = -1$) is the generating functional of the anticanonical transformation. The action (3.1) satisfies the classical master equation,

$$(\tilde{S}, \tilde{S}) = 0 \quad (3.3)$$

and is invariant under the BRST transformations,

$$\delta_B \tilde{S} = 0, \quad \delta_B \phi^A = (\phi^A, \tilde{S})\mu = \overrightarrow{\partial}_{\phi_A^*} \tilde{S} \mu, \quad \delta_B \phi_A^* = 0. \quad (3.4)$$

To describe the possible deformations of the action, the generating functional Y should have the form

$$Y[\phi, \Phi^*] = \Phi_A^* \phi^A + X[\phi, \Phi^*], \quad (3.5)$$

so that

$$\Phi^A = \phi^A + \overrightarrow{\partial}_{\Phi_A^*} X[\phi, \Phi^*], \quad \phi_A^* = \Phi_A^* + X[\phi, \Phi^*] \overleftarrow{\partial}_{\phi^A}. \quad (3.6)$$

Now let us consider the Taylor expansion of the functional X in antifields,

$$X[\phi, \Phi^*] = \Phi_A^* H^A(\phi) + \frac{1}{2} \Phi_A^* \Phi_B^* H^{BA}(\phi) + O(\phi^{*3}). \quad (3.7)$$

Then we have

$$\Phi^A = \phi^A + H^A(\phi) + \Phi_B^* H^{BA}(\phi) + O(\phi^{*2}), \quad (3.8)$$

$$\phi_A^* = \Phi_A^* + \Phi_B^* H^B(\phi) \overleftarrow{\partial}_{\phi^A} + \frac{1}{2} \Phi_B^* \Phi_C^* H^{CB}(\phi) \overleftarrow{\partial}_{\phi^A} + O(\phi^{*3}). \quad (3.9)$$

First of all, we should express the Φ_A^* from (3.9) in the form

$$\Phi_A^* = \Phi_A^*(\phi, \phi^*) \quad (3.10)$$

and then to express Φ^A from (3.8) as a function of variables ϕ and ϕ^* ,

$$\Phi^A(\phi, \phi^*) = \phi^A + H^A(\phi) + \Phi_B^*(\phi, \phi^*) H^{BA}(\phi) + O(\phi^{*2}). \quad (3.11)$$

The solution (3.10) can be found perturbatively. The result reads

$$\Phi^A(\phi, \phi^*) = (\mathcal{A}^i, \mathcal{C}^\alpha) = \phi^A + H^A(\phi) + \phi_C^* (M^{-1}(\phi))^C_B H^{BA}(\phi) + O(\phi^{*2}), \quad (3.12)$$

$$\Phi_A^*(\phi, \phi^*) = (\mathcal{A}_i^*, \mathcal{C}_\alpha^*) = \phi_B^* (M^{-1}(\phi))^B_A + \frac{1}{2} \phi_B^* \phi_C^* H^{CB}(\phi) \overleftarrow{\partial}_{\phi^A} + O(\phi^{*3}), \quad (3.13)$$

or, in more detailed form,

$$\mathcal{A}^i = A^i + h^i(\phi) + \phi_A^*(M^{-1}(\phi))^A_B H^{Bi}(\phi) + O(\phi^{*2}), \quad (3.14)$$

$$\mathcal{C}^\alpha = C^\alpha + g^\alpha(\phi) + \phi_A^*(M^{-1}(\phi))^A_B H^{B\alpha}(\phi) + O(\phi^{*2}) \quad (3.15)$$

and

$$\mathcal{A}_i^* = A_j^*(M^{-1}(\phi))^j_i + C_\alpha^*(M^{-1}(\phi))^\alpha_i + \frac{1}{2}\phi_B^*\phi_C^*H^{CB}(\phi)\overleftarrow{\partial}_{A^i} + O(\phi^{*3}), \quad (3.16)$$

$$\mathcal{C}_\alpha^* = C_\beta^*(M^{-1}(\phi))^\beta_\alpha + A_j^*(M^{-1}(\phi))^j_\alpha + \frac{1}{2}\phi_B^*\phi_C^*H^{CB}(\phi)\overleftarrow{\partial}_{C^\alpha} + O(\phi^{*3}), \quad (3.17)$$

where $H^A(\phi) = (h^i(\phi), g^\alpha(\phi))$ and $(M^{-1}(\phi))^B_A$ is the inverse matrix for the matrix

$$M^B_A(\phi) = \delta^B_A + H^B(\phi)\overleftarrow{\partial}_{\phi^A}. \quad (3.18)$$

The matrices $M^B_A(\phi)$ and $(M^{-1}(\phi))^B_A$ will be used to describe the deformation of gauge symmetry.

From (3.13) it follows the important relation

$$\Phi_A^*(\phi, \phi^*)|_{\phi^*=0} = 0, \quad (3.19)$$

which will be employed to study the possible deformations of action

If to use the result (3.19) in relation (3.1), one obtains

$$\tilde{S}[\phi, \phi^*]|_{\phi^*=0} = S[\Phi(\phi, \phi^*=0), 0] = S[\phi + H(\phi), 0] = S_0[A + h(\phi)] \quad (3.20)$$

with functions $h^i(\phi)$ defined by the expansion (3.14). This a final result for deformation of the initial action $S[\phi, \phi^*]$ in the sector of initial fields A^i . The result is extremely simple. The arbitrary deformation of action is described by a simple shift of the field A^i in the initial action $S_0[A]$ by arbitrary function $h^i(\phi)$.

The result (3.20) looks so simple that it can be considered as trivial redefinition of the initial field A^i . Indeed, we can do the inverse redefinition, exclude the functions $h^i(\phi)$, and obtain a result that deformed theory is equivalent to initial theory. However, such inverse redefinition leads to equivalent theory only if the functions $h^i(\phi)$ are local. But when getting this result, nowhere it is assumed that these functions must be local. Hence the nontrivial deformed action is obtained only for non-local functions $h^i(\phi)$. Therefore we conclude that the deformed action, in general, must be non-local functional. Nevertheless, this does not mean that in some special cases a nontrivial deformed action can not be local. In principle, there can be such a situation that general non-local deformed action admits a closed local sector. It means that the deformed action has the following structure $S_0[A + h(A)] = S_1[A] + \text{non-local terms}$, where the action $S_1[A]$ is a local functional. If the deformed gauge transformations contain a local piece that leaves the action $S_1[A]$ invariant, we obtain the closed local sector of deformed theory. Let for example the initial Lagrangian has a form $\mathcal{L} \sim A \square A$ and let $h \sim \frac{1}{\square} F(A)$ with some local function $F(A)$. Then after some transformations, the deformed Lagrangian takes the form $\tilde{\mathcal{L}} \sim A \square A +$

$2F(A) + \text{non-local terms}$. If the deformed gauge transformations have a local piece leaving the Lagrangian $\tilde{\mathcal{L}}_1 \sim A \square A + 2F(A)$ invariant, we obtain the closed local sector of non-local theory. As we will see later, just such a situation is realized to derive non-Abelian Yang-Mills theory from free Abelian gauge theory. To conclude this section one notes that the most general deformed action is obtained from initial action $S_0[A]$ by the transformation $A^i \rightarrow A^i + h^i(A)$ with non-local functions $h^i(A)$.

4 Deformed gauge symmetry

In this section, we will describe the general structure of deformed gauge symmetry of the deformed action $S_0[A + h]$ as a direct consequence that the action $\tilde{S} = \tilde{S}[\phi, \phi^*]$ satisfies the classical master equation.

4.1 Consequences of master equation

To describe the deformation of gauge symmetry of initial classical action we take into account that the action $\tilde{S}$ satisfies the master equation. Therefore it can be written in the form analogous to (2.11)

$$\tilde{S} = S_0[\mathcal{A}] + \mathcal{A}_i^* R_\alpha^i(\mathcal{A}) C^\alpha - \frac{1}{2} \mathcal{C}_\gamma^* F_{\alpha\beta}^\gamma(\mathcal{A}) C^\beta C^\alpha (-1)^{\varepsilon_\alpha} + O(\phi^{*2}), \quad (4.1)$$

up to the second order in antifields ϕ_A^* . Then from the classical master equation for $\tilde{S}$ one gets the relation

$$\frac{1}{2} (S_0[\mathcal{A}], S_0[\mathcal{A}]) + (S_0[\mathcal{A}], \mathcal{A}_i^*) R_\alpha^i(\mathcal{A}) C^\alpha - \frac{1}{2} (S_0[\mathcal{A}], \mathcal{C}_\gamma^*) F_{\alpha\beta}^\gamma(\mathcal{A}) C^\beta C^\alpha (-1)^{\varepsilon_\alpha} + O(\phi^*) = 0. \quad (4.2)$$

The terms in the l.h.s. of (4.2) can be represented in the forms

$$\frac{1}{2} (S_0[\mathcal{A}], S_0[\mathcal{A}]) = S_0[\mathcal{A}] \overleftarrow{\partial}_{\phi^A} (M^{-1}(\phi))^A_B H^{Bj}(\phi) \overrightarrow{\partial}_{\mathcal{A}^j} S_0[\mathcal{A}] + O(\phi^*), \quad (4.3)$$

$$(S_0[\mathcal{A}], \mathcal{A}_i^*) R_\alpha^i(\mathcal{A}) C^\alpha = S_0[\tilde{A}] \overleftarrow{\partial}_{\phi^A} (M^{-1}(\phi))^A_i R_\alpha^i(\tilde{A}) \tilde{C}^\alpha + O(\phi^*), \quad (4.4)$$

$$(S_0[\mathcal{A}], \mathcal{C}_\gamma^*) F_{\alpha\beta}^\gamma(\mathcal{A}) C^\beta C^\alpha (-1)^{\varepsilon_\alpha} = S_0[\tilde{A}] \overleftarrow{\partial}_{\phi^A} (M^{-1}(\phi))^A_\alpha F_{\sigma\beta}^\alpha(\tilde{A}) \tilde{C}^\beta \tilde{C}^\sigma (-1)^{\varepsilon_\sigma} + O(\phi^*), \quad (4.5)$$

where the notations

$$\tilde{A}^i = A^i + h^i(\phi), \quad \tilde{C}^\alpha = C^\alpha + g^\alpha(\phi), \quad (4.6)$$

are used. Here the functions h^i and g^α are defined by the expansions (3.14) and (3.15) respectively.

The first term in the r.h.s. (4.3) contains the functions H^{Bj} which is defined by the expansion (3.8). To construct the deformed action in the sector of initial fields, we should switch off all antifields. Therefore the functions H^{Bj} do not enter the deformed action. Hence, it is sufficient to put

$$H^{AB}(\phi) = 0. \quad (4.7)$$

Moreover, this restriction leads to the correspondence

$$(S_0[A], S_0[A]) = 0 \quad \rightarrow \quad (S_0[\mathcal{A}], S_0[\mathcal{A}])|_{\phi^*=0} = 0. \quad (4.8)$$

Now taking in the relation (4.2) the limit $\phi^* \rightarrow 0$, one obtains

$$S_0[\tilde{A}] \overleftarrow{\partial}_{\phi^A} (M^{-1}(\phi))^A_i R_\alpha^i(\tilde{A}) \tilde{C}^\alpha - \frac{1}{2} S_0[\tilde{A}] \overleftarrow{\partial}_{\phi^A} (M^{-1}(\phi))^A_\alpha F_{\sigma\beta}^\alpha(\tilde{A}) \tilde{C}^\beta \tilde{C}^\sigma (-1)^{\varepsilon_\sigma} = 0. \quad (4.9)$$

Since the initial fields A^i obey the property $\text{gh}(A^i) = 0$, the generating functions h^i obey the analogous property $\text{gh}(h^i) = 0$. Therefore, the functions h^i do not depend on the ghost fields C^α ($\text{gh}(C^\alpha) = 1$),

$$h^i(\phi) = h^i(A). \quad (4.10)$$

It leads to the following relation

$$S_0[\tilde{A}] \overleftarrow{\partial}_{A^j} (M^{-1}(\phi))^j_i R_\alpha^i(\tilde{A}) \tilde{C}^\alpha - \frac{1}{2} S_0[\tilde{A}] \overleftarrow{\partial}_{A^j} (M^{-1}(\phi))^j_\alpha F_{\sigma\beta}^\alpha(\tilde{A}) \tilde{C}^\beta \tilde{C}^\sigma (-1)^{\varepsilon_\sigma} = 0. \quad (4.11)$$

The matrix $M_A^B(\phi)$ (3.18) has the triangular form

$$M^j_i(\phi) = \delta^j_i + h^j(A) \overleftarrow{\partial}_{A^i}, \quad M^j_\beta(\phi) = 0, \quad (4.12)$$

$$M^\alpha_i(\phi) = g^\alpha(\phi) \overleftarrow{\partial}_{A^i}, \quad M^\alpha_\beta(\phi) = \delta^\alpha_\beta + g^\alpha(\phi) \overleftarrow{\partial}_{C^\beta}. \quad (4.13)$$

It means that inverse matrix $(M^{-1}(\phi))^B_A$ has triangular structure as well,

$$(M^{-1}(\phi))^j_i, \quad (M^{-1}(\phi))^j_\beta = 0, \quad (4.14)$$

$$(M^{-1}(\phi))^\alpha_i, \quad (M^{-1}(\phi))^\alpha_\beta, \quad (4.15)$$

where $(M^{-1}(\phi))^j_i$ is inverse to $M^j_i(\phi)$,

$$(M^{-1}(\phi))^j_l M^l_i(\phi) = \delta^j_i, \quad (4.16)$$

and does not depend on fields C^α ,

$$(M^{-1}(\phi))^j_i = (M^{-1}(A))^j_i. \quad (4.17)$$

In its turn $(M^{-1}(\phi))^\alpha_\beta$ is inverse to $M^\alpha_\beta(\phi)$,

$$(M^{-1}(\phi))^\alpha_\gamma M^\gamma_\beta(\phi) = \delta^\alpha_\beta. \quad (4.18)$$

As a consequence, the relation (4.11) rewrites as

$$S_0[\tilde{A}] \overleftarrow{\partial}_{A^j} (M^{-1}(A))^j_i R_\alpha^i(\tilde{A}) \tilde{C}^\alpha = 0. \quad (4.19)$$

This relation is base to derive the general deformation of gauge generators.

4.2 Deformation of gauge generators

We proceed with the discussion of transformed gauge symmetry. Since $\text{gh}(g^\alpha(\phi)) = 1$ the generating functions $g^\alpha(\phi)$ is linear in the ghost fields C^α ,

$$g^\alpha(\phi) = g^\alpha_\beta(A)C^\beta \quad (4.20)$$

Therefore the relation (4.6) leads to

$$\tilde{C}^\alpha = M^\alpha_\beta(A)C^\beta. \quad (4.21)$$

Taking into account this relation, we rewrite the relation (4.19) in the form

$$S_0[\tilde{A}] \overleftarrow{\partial}_{A^j} (M^{-1}(A))^j_i R^i_\alpha(\tilde{A}) M^\alpha_\beta(A) = 0. \quad (4.22)$$

Denoting

$$\mathbf{R}^i_\alpha(A) = (M^{-1}(A))^j_i R^i_\alpha(\tilde{A}) M^\alpha_\beta(A) \quad (4.23)$$

and using the definition $S_0[\tilde{A}] = S[A]$ one gets

$$S[A] \overleftarrow{\partial}_{A^j} \mathbf{R}^j_\alpha(A) = 0. \quad (4.24)$$

This relation allows us to interpret the $\mathbf{R}^i_\alpha(A)$ (4.23) as the deformed gauge generators. Then the relation (4.24) means the condition of gauge invariance of the deformed action.

Let us now turn to the derivation of the gauge algebra for deformed generators. To do that we should calculate the quantity

$$(\mathbf{R}^i_\alpha(A) \overleftarrow{\partial}_{A^j}) \mathbf{R}^j_\beta(A) - (-1)^{\varepsilon_\alpha \varepsilon_\beta} (\mathbf{R}^i_\beta(A) \overleftarrow{\partial}_{A^j}) \mathbf{R}^j_\alpha(A) \quad (4.25)$$

The calculation is divided into few steps.

1. Let us write

$$\mathbf{R}^i_\alpha(A) = \tilde{R}^i_\beta(A) M^\beta_\alpha(A) \quad (4.26)$$

with

$$\tilde{R}^i_\beta(A) = (M^{-1}(A))^i_j R^j_\alpha(\tilde{A}). \quad (4.27)$$

Then the relation (2.2) can be represented in the form

$$S_0[\tilde{A}] \overleftarrow{\partial}_{\tilde{A}^i} R^i_\alpha(\tilde{A}) = 0, \quad (4.28)$$

or, equivalently, as

$$S[A] \overleftarrow{\partial}_{A^i} \tilde{R}^i_\alpha(A) = 0. \quad (4.29)$$

2. Using the relation (4.28) we rewrite the initial gauge identity (2.3) in the form

$$(R_\alpha^i(\tilde{A}) \overleftarrow{\partial}_{\tilde{A}^j}) R_\beta^j(\tilde{A}) - (-1)^{\varepsilon_\alpha \varepsilon_\beta} (R_\beta^i(\tilde{A}) \overleftarrow{\partial}_{\tilde{A}^j}) R_\alpha^j(\tilde{A}) = -R_\gamma^i(\tilde{A}) F_{\alpha\beta}^\gamma(\tilde{A}). \quad (4.30)$$

Then one takes into account the relation

$$R_\alpha^i(\tilde{A}) = M_j^i(A) \tilde{R}_\alpha^i(A) \quad (4.31)$$

and properties of the matrix $M_j^i(A)$ (3.18)

$$\begin{aligned} M_j^i(A) \overleftarrow{\partial}_{A^k} &= h^i(A) \overleftarrow{\partial}_{A^j} \overleftarrow{\partial}_{A^k}, \\ M_j^i(A) \overleftarrow{\partial}_{A^k} &= (-1)^{\varepsilon_j \varepsilon_k} M_k^i(A) \overleftarrow{\partial}_{A^j}. \end{aligned} \quad (4.32)$$

It allows us to rewrite the relation (4.30) in the form

$$(\tilde{R}_\alpha^i(A) \overleftarrow{\partial}_{A^j}) \tilde{R}_\beta^j(A) - (-1)^{\varepsilon_\alpha \varepsilon_\beta} (\tilde{R}_\beta^i(A) \overleftarrow{\partial}_{A^j}) \tilde{R}_\alpha^j(A) = -\tilde{R}_\gamma^i(A) \tilde{F}_{\alpha\beta}^\gamma(A), \quad (4.33)$$

where $\tilde{F}_{\alpha\beta}^\gamma(A) = F_{\alpha\beta}^\gamma(\tilde{A})$ and $F_{\alpha\beta}^\gamma(A)$ are the structure coefficients of initial gauge algebra.

3. Now one introduces the deformed generators (4.23) and use the relations (4.33) and explicit form of the matrix $M_\beta^\alpha(A)$

$$M_\beta^\alpha(A) = \delta_\beta^\alpha + g_\beta^\alpha(A). \quad (4.34)$$

It allows us to transform the relation (4.33) into the following relation

$$(\mathbf{R}_\alpha^i(A) \overleftarrow{\partial}_{A^j}) \mathbf{R}_\beta^j(A) - (-1)^{\varepsilon_\alpha \varepsilon_\beta} (\mathbf{R}_\beta^i(A) \overleftarrow{\partial}_{A^j}) \mathbf{R}_\alpha^j(A) = -\mathbf{R}_\gamma^i(A) \mathbf{F}_{\alpha\beta}^\gamma(A). \quad (4.35)$$

Here

$$\begin{aligned} \mathbf{F}_{\alpha\beta}^\gamma(A) &= -(M^{-1})_\lambda^\gamma(A) \tilde{F}_{\rho\sigma}^\lambda(A) M_\alpha^\sigma(A) M_\beta^\rho(A) (-1)^{\varepsilon_\alpha \varepsilon_\rho} - \\ &\quad - ((M^{-1})_\mu^\gamma(A) \overleftarrow{\partial}_{A^j}) \mathbf{R}_\alpha^j(A) M_\beta^\mu(A) (-1)^{\varepsilon_\alpha \varepsilon_\mu} + \\ &\quad + ((M^{-1})_\mu^\gamma(A) \overleftarrow{\partial}_{A^j}) \mathbf{R}_\beta^j(A) M_\alpha^\mu(A) (-1)^{\varepsilon_\alpha (\varepsilon_\beta + \varepsilon_\mu)}. \end{aligned} \quad (4.36)$$

The relation (4.35) is the final form of the gauge algebra for deformed generators (4.23). This algebra is irreducible and closed and includes the deformed structure coefficients (4.36). It is interesting to point out that even if the initial gauge theory is Abelian, the deformed gauge theory is non-Abelian.

As a result, we have completely built a deformed gauge theory that is given by deformed action and deformed generators. The deformed generators (4.23) includes the matrices $M(A)^i_j = \delta^i_j(A) + h^i_j(A)$ and $M_\beta^\alpha(A) = \delta_\beta^\alpha + g_\beta^\alpha(A)$. As we saw, the arbitrary functions $h^i_j(A)$ define the deformation of the initial action. Apart from the function $h^i_j(A)$, the deformed generators are also defined by the arbitrary functions $g_\beta^\alpha(A)$. Therefore we can conclude that the arbitrary deformation in the initial gauge theory is defined by two (in general, non-local) arbitrary generating functions of the initial fields A^i .

In some cases, one can expect that only one of these arbitrary functions will be independent. As such an example, we consider the form of the deformed generators in the first order in functions h and g . The general relation (4.23) leads to

$$\mathbf{R}_\alpha^i(A) = R_\alpha^i(A) - (h^i(A) \overleftarrow{\partial}_{A^j}) R_\alpha^j(A) + (R_\alpha^i(A) \overleftarrow{\partial}_{A^j}) h^j(A) + R_\beta^i(A) g_\alpha^\beta(A) + \cdots, \quad (4.37)$$

where $\cdots$ means higher terms in generating functions h and g . Let us assume that the following equation is fulfilled

$$R_\beta^i(A) g_\alpha^\beta(A) = (h^i(A) \overleftarrow{\partial}_{A^j}) R_\alpha^j(A). \quad (4.38)$$

Then we have a relation between the functions g and h and only one of these functions becomes independent.

5 The Yang-Mills theory as the deformation of the Abelian gauge theory

In this section, we will demonstrate how the general theory under consideration allows to obtain the non-Abelian Yang-Mills theory as deformation of the Abelian gauge theory. To be more precise, we will show that such a non-local deformed theory possesses the closed local sector.

We begin with a free vector field model with the action

$$S_0[A] = -\frac{1}{4} F_{0\mu\nu}^a(A) F_0^{a\mu\nu}(A), \quad (5.1)$$

where

$$F_{0\mu\nu}^a(A) = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a, \quad (5.2)$$

and a is the index of some semi-simple Lie algebra with the structure constants f^{abc} . The action $S_0[A]$ is invariant,

$$\delta_\xi S_0[A] = 0, \quad (5.3)$$

under the Abelian gauge transformations,

$$\delta_\xi A_\mu^a = \partial_\mu \xi^a, \quad R_\mu^{ab} = \delta^{ab} \partial_\mu. \quad (5.4)$$

Here ξ^a are arbitrary fields of space-time coordinates x and R_μ^{ab} are generators of initial gauge symmetry.

According to the general deformation procedure described in section 3, the deformed action is given by the relation

$$\begin{aligned} S[A] &= -\frac{1}{4} F_{0\mu\nu}^a(A + h(A)) F_0^{a\mu\nu}(A + h(A)) = \\ &= -\frac{1}{4} F_{0\mu\nu}^a(A) F_0^{a\mu\nu}(A) - \frac{1}{2} F_{0\mu\nu}^a(A) F_0^{a\mu\nu}(h(A)) - \\ &\quad - \frac{1}{4} F_{0\mu\nu}^a(h(A)) F_0^{a\mu\nu}(h(A)), \end{aligned} \quad (5.5)$$

where $h(A) = \{h_\mu^a(A)\}$ is non-local function responsible for the deformation of initial action (5.1). Using integration by parts we obtain the following expressions for the two last terms in the r.h.s. of (5.5),

$$\frac{1}{2}F_{0\mu\nu}^a(A)F_0^{a\mu\nu}(h(A)) = -A_\mu^a\Box h^{a\mu}(A) + A_\mu^a\partial_\nu\partial^\mu h^{a\nu}(A), \quad (5.6)$$

$$\frac{1}{4}F_{0\mu\nu}^a(h(A))F_0^{a\mu\nu}(h(A)) = -\frac{1}{2}h_\mu^a(A)\Box h^{a\mu}(A) + \frac{1}{2}h_\mu^a(A)\partial_\nu\partial^\mu h^{a\nu}(A) \quad (5.7)$$

The functions $h_\mu^a(A)$ have the same indices as the fields A_μ^a . Having in mind this fact, one considers the following function $h_\mu^a(A)$

$$h_\mu^a(A) = \frac{1}{\Box} [c_1\partial^\nu(f^{abc}A_\nu^b A_\mu^c) + c_2 f^{abc}f^{cmn}A^{b\nu}A_\nu^m A_\mu^n], \quad (5.8)$$

where c_1, c_2 are some constants. Then, after some transformations, we obtain

$$\begin{aligned} \frac{1}{2}F_{0\mu\nu}^a(A)F_0^{a\mu\nu}(h(A)) &= \frac{1}{2}c_1 F_{0\mu\nu}^a(A)f^{abc}A^{b\mu}A^{c\nu} + c_2 f^{abc}A_\mu^b A_\nu^c f^{amn}A^{m\mu}A^{n\nu} + \\ &+ \text{non-local terms containing the } \frac{1}{\Box}. \end{aligned} \quad (5.9)$$

Taking the constants in the form $c_1 = 1, c_2 = \frac{1}{4}$ we get

$$S[A] = S_{YM}[A] + S_1[A], \quad (5.10)$$

where

$$S_{YM}[A] = -\frac{1}{4}F_{\mu\nu}^a(A)F^{a\mu\nu}(A), \quad F_{\mu\nu}^a(A) = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc}A_\mu^b A_\nu^c \quad (5.11)$$

is the standard local Yang-Mills action and $S_1[A]$ is some correction responsible for non-local deformed action. As a result, we see that the deformation procedure, described in section 3, allows us to derive the action of non-Abelian Yang-Mills theory beginning with free Abelian gauge theory.

The next step is to get the Yang-Mills gauge transformations. Taking into account the explicit expression for generating functions $h_\mu^a(A)$ (5.8) with fixed constants and using the relation (4.38) one obtains

$$\mathbf{R}_\mu^{ab}(A) = D_\mu^{ab}(A) + R_{1\mu}^{ab}(A), \quad (5.12)$$

where $D_\mu^{ab}(A)$ is a covariant derivative with respect to A_μ^a ,

$$D_\mu^{ab}(A) = \delta^{ab}\partial_\mu + f^{acb}A_\mu^c. \quad (5.13)$$

Here $D_\mu^{ab}(A)$ are the standard Yang-Mills generators and $R_{1\mu}^{ab}(A)$ corresponds to the part of gauge generators responsible for gauge invariance of the complete non-local deformed action. Thus, in the case under consideration, the non-local deformed action contains closed local sector which is nothing more than the Yang-Mills theory.⁵ It is interesting to point out that in the given case, the relation (4.38) allows to eliminate all the terms in deformed gauge transformations which contain the space-time derivatives of the gauge parameters besides $\partial_\mu \xi^a$.

⁵Note that the use of the gauge generators (5.13) makes it possible to automatically fix the coefficients c_1, c_2 in the function h_μ^a (5.8) as $c_1 = 1, c_2 = \frac{1}{4}$.

6 Cubic interaction vertex for massless integer higher spin fields as the deformation of the free massless higher spin theory

In this section, we will show how the approach to construction of the gauge-invariant deformation allows to derive the cubic interaction vertex in higher spin field theory.

We begin with free massless integer higher spin s field theory which is described by the Fronsdal action [21]

$$\begin{aligned}
 S^{(2)}[\phi] = \int d^4x \Big\{ & -\varphi_{\mu_1 \dots \mu_s} \square \varphi^{\mu_1 \dots \mu_s} - \frac{s}{2} \partial_\alpha \varphi^{\alpha \mu_2 \dots \mu_s} \partial^\beta \varphi_{\beta \mu_1 \dots \mu_s} \\
 & - \frac{s(s-1)}{2} \varphi^\rho{}_{\rho \mu_3 \dots \mu_s} \partial_\alpha \partial_\beta \varphi^{\alpha \beta \mu_3 \dots \mu_s} - \frac{s(s-1)}{4} \partial_\alpha \varphi^\rho{}_{\rho \mu_3 \dots \mu_s} \partial^\alpha \varphi_\sigma{}^{\sigma \mu_3 \dots \mu_s} \\
 & - \frac{s(s-1)(s-2)}{8} \partial_\alpha \varphi^\rho{}_{\rho \alpha \mu_4 \dots \mu_s} \partial^\beta \varphi^\sigma{}_{\sigma \beta \mu_4 \dots \mu_s} \Big\}. \quad (6.1)
 \end{aligned}$$

Here $\varphi_{\mu_1 \dots \mu_s}$ is a totally symmetric double traceless field with standard bosonic field dimension. The theory is invariant under the Abelian gauge transformations with the traceless totally symmetric parameters $\xi_{\mu_1 \dots \mu_{s-1}}$. We are going to apply our procedure of deformation to the free theory with action (6.1). However, a point needs to be made here. The deformation theory under consideration assumes that the fields and the gauge parameters are completely unconstrained. But the higher spin fields in action (6.1) and the corresponding gauge parameters obey the traceless constraints and our approach can not be applied in literal form. To simplify the situation we will follow the reasoning accepted in the higher spin field theory when constructing the interaction vertices (see e.g. [26] and the references therein). Namely, let us assume that there are no traceless restrictions on the fields and parameters from the very beginning and then impose needed restrictions on the fields in the vertices afterwards.⁶

According to the general procedure, described in section 3, to obtain the deformed action we should replace the field $\varphi_{\mu_1 \dots \mu_s}$ in the action (6.1) by the field $\varphi_{\mu_1 \dots \mu_s} + h_{\mu_1 \dots \mu_s}$ with the arbitrary totally symmetric non-local function $h_{\mu_1 \dots \mu_s}(\varphi)$. We will show that this function can be taken in such a way that the deformed theory contains at least cubic interaction local vertex.⁷

Let us consider the function $h_{\mu_1 \dots \mu_s}(\varphi)$ in the form

$$h^{\mu_1 \dots \mu_s} = c_k g^{s+2k} \frac{1}{\square} \partial^{\{\mu_1} \dots \partial^{\mu_k} \varphi^{\mu_{k+1} \dots \mu_s\} \nu_1 \dots \nu_k} \partial_{\nu_1} \dots \partial_{\nu_k} \partial^{\lambda_1} \dots \partial^{\lambda_s} \varphi_{\lambda_1 \dots \lambda_s}. \quad (6.2)$$

Here the parameter k takes the values $k = 0, 1, \dots, s$, c_k are the arbitrary real constants and g is a coupling constant of the dimension $\dim(g) = -1$. The used symbol $\{\mu_1 \dots \mu_k \mu_{k+1} \dots \mu_s\}$ means symmetrization with respect to indexes included. The function (6.2) is the only admissible function generating cubic vertex with help of transformation

⁶In principle there is a formulation of free higher spin field theory where the fields in action are completely unconstrained and the true constraints appear only on equations of motion [22, 23].

⁷We emphasize that at arbitrary choice of the generating function h the deformed action can not include the closed local sector. This can only be with a very special choice of the generating function, if even possible at all.

$\varphi \rightarrow \varphi + h$ in the action (6.1) and preserving the symmetry and dimension of φ . It is clear that in this case, we need a dimensional coupling constant. Transformation $\varphi \rightarrow \varphi + h$ in the first term of action (6.1) yields the family of the local cubic vertices

$$S_{\text{local}}^{(3)}[\varphi] = -2c_k g^{s+2k} \int d^4x \varphi^{\mu_1 \dots \mu_s} \partial_{\{\mu_1} \dots \partial_{\mu_k} \varphi_{\mu_{k+1} \dots \mu_s\} \nu_1 \dots \nu_k} \partial^{\nu_1} \dots \partial^{\nu_k} \partial_{\lambda_1} \dots \partial_{\lambda_s} \varphi^{\lambda_1 \dots \lambda_s} \quad (6.3)$$

with $k = 0, 1, \dots, s$. Deformation of the other terms in free action leads to non-local contributions. The obtained cubic vertices (6.3) correspond to the cubic vertices that were constructed in the papers on higher spin field theory by different methods (see e.g. the recent papers [24–27] and the references therein). In our approach, these vertices are a simple consequence of the general procedure, described in section 3.

The next question is finding the true local gauge transformations. Just as in deriving gauge transformations in Yang-Mills theory, we start with gauge transformations of free theory and apply the general relations (4.37). As a result, we obtain the non-local gauge transformations, corresponding to general non-local deformed theory. However, it is easy to see that these transformations contain a local piece, leaving invariant the action $S^{(2)} + S_{\text{local}}^{(3)}$ up to fourth order terms in φ . We see that the theory under consideration admits a closed local sector. Thus, the general deformation procedure allows to comparatively simply to describe the cubic vertices in the massless higher integer spin theory.

7 Conclusion

Let us summarize the results. We have described a general procedure of gauge-invariant deformation of classical gauge theories. The procedure is based on the use of the BV-formalism where the central objects are antibracket and the master equation. The arbitrary gauge deformation is formulated in terms of anticanonical transformation leaving invariant the antibracket. We have proved that the arbitrary gauge deformation of a given gauge-invariant theory is described by two arbitrary generating functions, where one is responsible for the deformation of the initial action and the other for the deformation of the gauge generators. The deformations are realized in the explicit form and given by the relations (3.20) and (4.23).

The deformation of initial action has extremely simple form and means a replacement in the initial action the gauge field ϕ by the field $\phi + h(\phi)$ with arbitrary non-local generating function $h(\phi)$. The deformation of gauge generators also has a simple enough form and is described by the same function $h(\phi)$ and another arbitrary function $g(\phi)$. We have calculated the algebra of deformed generators in the form (4.35) with deformed structure coefficients (4.36). We emphasize that even if the initial theory is Abelian, the corresponding deformed theory is non-Abelian.

The essential feature of the obtained deformed theory is that it is non-local in general. However, in special cases, there can be a situation when such a non-local theory contains a closed local sector. It means that the deformed action can involve the local piece which is invariant under the local piece of deformed gauge transformations. As a result, we can obtain some new local gauge theory.

The first important test for the theory under consideration is a possibility to derive Yang-Mills theory. We have shown that if to start with Abelian gauge theory and apply the transformation $A \rightarrow A + h(A)$, where the function $h_\mu^a(A)$ is given by (5.8) we obtain some non-local vector field theory with a local piece in action which just is the Yang-Mills action. The corresponding piece in the deformed gauge transformations is the Yang-Mills gauge transformation.

Also, we have considered a derivation of the massless integer higher spin cubic vertex in the framework of the general deformation theory. We again started with free theory and constructed in explicit form the non-local field deformation which generates the cubic interaction vertex consistent with ones in the literature. The generating function $h(A)$ should be nonlocal to reproduce nontrivial deformation of the free action. In the case under consideration, the non-locality is due to the presence of the operator $\frac{1}{\square}$ in the function h . To get the higher orders vertices, we have to consider the non-local contributions to this function containing e.g. the operators $\frac{1}{(\square)^n}$, $n = 1, 2, 3 \dots$. Therefore, one can expect that apparently the interaction vertices of the quartic and higher orders will obligatorily be non-local.⁸ This basically corresponds to results of the works [28–33]. We guess that the aspects of the locality of the vertices in the higher spin field theory deserve a comprehensive study (see e.g. [34] and the references therein).

Let us note some areas of further development and application of our deformation theory. First, we can generalize the approach for the theories with dependent generators. Second, it would be useful to study the relations of the quantum effective actions for the classical theories obtained one from another by non-local gauge-invariant deformation. Third, it would also be interesting to explore which nonlocal theories of gravity can be constructed by the deformation of the free massless symmetric second rank tensor field theory and whether there exists in deformed gravity theories a closed local sector corresponding to Einstein's gravity.

We believe that the most interesting applications of the developed deformation theory relate to the higher spin field theory. Since the true cubic interaction vertex for massless integer higher spin fields was derived on the base of deformation theory, one can expect that the other vertices for massless and massive higher spin fields can also be derived in the framework of such an approach. We plan to study all these issues in the forthcoming works.

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⁸Non-local contributions to fourth order vertex arise already at substituting the $\varphi \rightarrow \varphi + h$ with h given by (6.2) to (6.1).

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