

## Global analysis of leptophilic $Z'$ bosons

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**ABSTRACT:** New neutral heavy gauge bosons ( $Z'$ ) are predicted within many extensions of the Standard Model. While in case they couple to quarks the LHC bounds are very stringent, leptophilic  $Z'$  bosons (even with sizable couplings) can be much lighter and therefore lead to interesting quantum effects in precision observables (like  $(g-2)_\mu$ ) and generate flavour violating decays of charged leptons. In particular,  $\ell \rightarrow \ell' \nu \bar{\nu}$  decays, anomalous magnetic moments of charged leptons,  $\ell \rightarrow \ell' \gamma$  and  $\ell \rightarrow 3\ell'$  decays place stringent limits on leptophilic  $Z'$  bosons. Furthermore, in case of mixing  $Z'$  with the SM  $Z$ ,  $Z$  pole observables are affected. In light of these many observables we perform a global fit to leptophilic  $Z'$  models with the main goal of finding the bounds for the  $Z'$  couplings to leptons. To this end we consider a number of scenarios for these couplings. While in generic scenarios correlations are weak, this changes once additional constraints on the couplings are imposed. In particular, if one considers an  $L_\mu - L_\tau$  symmetry broken only by left-handed rotations, or considers the case of  $\tau - \mu$  couplings only. In the latter setup, one can explain the  $(g-2)_\mu$  anomaly and the hint for lepton flavour universality violation in  $\tau \rightarrow \mu \nu \bar{\nu} / \tau \rightarrow e \nu \bar{\nu}$  without violating bounds from electroweak precision observables.

**KEYWORDS:** Beyond Standard Model, Gauge Symmetry, Neutrino Physics

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## 1 Introduction

In 2012, the LHC confirmed the predictions of the Standard Model (SM) of particle physics by discovering the (Brout-Englert) Higgs boson [1, 2]. However, so far no particles beyond the ones of the SM have been observed in high energy searches. In particular, the bounds from di-jet [3, 4] and di-lepton [5–7] searches on particles that can be produced resonantly in the s-channel are very stringent. This also puts tight bounds on heavy neutral gauge bosons ( $Z'$ s), which are predicted by many new physics models (see [8–14]), in case they have sizable couplings to quarks.

However, in case the resonances are neutral and couple (to a good approximation) only to leptons, mostly LEP searches apply and the bounds are much weaker [15, 16], i.e. significantly below the TeV scale. Therefore, such leptophilic  $Z'$  bosons can have sizable couplings while at the same time being quite light. They can thus lead to relevant quantum corrections to leptonic precision observables and generate lepton flavour violating decays of leptons that are extremely suppressed in the SM since they vanish in the limit of massless neutrinos. The  $(g - 2)_\mu$  discrepancy [17, 18], recently reinforced by the  $g - 2$  experiment at Fermilab [19–22], with a tension of  $4.2\sigma$  compared to the SM prediction [23], can be explained with a  $Z'$  boson heavier than the electroweak (EW) scale if it couples flavour violatingly to the second and third lepton generation [24–43].

At first sight, new neutral gauge bosons coupling only to leptons and not to quarks might appear artificial. But as already in the SM gluons couple only to quarks and not to leptons, it is actually an interesting possibility that for a  $Z'$  boson the situation could be reversed. For example, gauged abelian flavour symmetries in the lepton sector, such as  $L_\mu - L_\tau$  [44–46], can naturally generate the observed pattern of the PMNS matrix [47–49] and lead by definition to leptophilic  $Z'$  bosons, which, after the breaking of the symmetry, can also induce charged LFV processes such as  $\tau \rightarrow 3\mu$  [30, 50, 51] and  $h \rightarrow \mu\tau$  [51–53].

Such scenarios are particularly interesting as within recent years several hints for the violation of lepton flavour universality (LFU) have been acquired. These include  $\tau \rightarrow \mu\nu\bar{\nu}/\tau \rightarrow e\nu\bar{\nu}$ ,  $\tau \rightarrow \mu\nu\bar{\nu}/\mu \rightarrow e\nu\bar{\nu}$  [54, 55] and the Cabibbo angle anomaly [56–59], which can also be interpreted as a sign of LFUV [60–71]. Furthermore, even though in this case also (small) couplings to bottom and strange quarks are necessary,  $Z'$  bosons are among the prime candidates for explaining the discrepancies between the SM predictions and data in  $b \rightarrow sl^+\ell^-$  transitions [12, 34, 51, 53, 67, 72–113]. Here, LHCb measurements [114, 115] indicate a deficit in muons with respect to electrons, i.e. LFUV with a combined significance of  $\approx 4\sigma$  [116–126]. This is consistent with many other measurements involving the same current, in particular with angular observables [127, 128], where data also shows a deficit in muonic channels [129, 130] such that the most up-to-date global analysis finds several NP scenarios to be preferred over the SM at the  $5 - 6\sigma$  level [123–125, 131–133]. In order to respect LHC bounds, it is again advantageous if the couplings to quarks are small, i.e. if the  $Z'$  is to a good approximation leptophilic, which can e.g. be achieved by generating the quark couplings effectively via heavy vector-like quarks [34, 51, 134].

Therefore, it is very interesting to explore the phenomenology of leptophilic  $Z'$  bosons. Even in the absence of quark couplings, such a  $Z'$  boson affects many observables with the most interesting being

- $\ell \rightarrow \ell'\gamma$  decays,
- Anomalous magnetic moments (AMMs) and electric dipole moments (EDMs) of charged leptons,
- $\ell \rightarrow 3\ell'$  decays (7 in total),
- $Z \rightarrow \ell\ell^{(\prime)}$  decays,

- LFU violation in  $\ell \rightarrow \ell' \nu \bar{\nu}$
- Neutrino trident production
- LEP searches for contact interactions

Therefore, in order to fully explore the allowed/preferred parameter space of such a new physics scenario, the aim of this article is to perform a global fit to all available data in a number of scenarios for  $Z'$  couplings to leptons. For this we will use the publicly available HEPfit code [135] which also allows us to perform a global fit taking into account many degrees of freedom at the same time.

The article is structured as follows: in the next section we will define our setup before we consider the relevant observables, and calculate the relevant  $Z'$  contributions to them in section 3. We then perform our phenomenological analysis in section 4 before concluding in section 5. In appendix A we give some details on the LEP-II bounds and in appendix B we list the contributions of QED penguins to LFV decays like  $\ell \rightarrow 3\ell'$  and to  $\mu \rightarrow e$  conversion in nuclei. In appendix C we present additional scenarios for LFV couplings beyond those presented in the main text.

## 2 Setup

We extend the SM by adding a heavy neutral gauge boson  $Z'_0$  (i.e. with a mass above the electroweak symmetry breaking scale). Following refs. [136, 137] we supplement the SM Lagrangian by a part containing the kinetic terms and the mass terms of the  $Z'_0$ -field,

$$\mathcal{L}_{Z'_0} = -\frac{1}{4} Z'_{0,\mu\nu} Z'^{\mu\nu}_0 + \frac{M_{Z'}^2}{2} Z'_{0\mu} Z'^{\mu}_0, \quad (2.1)$$

where  $Z'_{0,\mu\nu} \equiv \partial_\mu Z'_{0\nu} - \partial_\nu Z'_{0\mu}$  is the field strength tensor associated to the  $Z'_0$ -field, and a part describing the interactions of the  $Z'_0$ -field with the SM fields,

$$\begin{aligned} \mathcal{L}_{Z'_0}^{\text{int}} = & g_{Z'} Z'_{0\mu} Z'^{\mu}_0 \phi^\dagger \phi - i g_{Z'}^\phi Z'_{0\mu} \phi^\dagger \overleftrightarrow{D}_\mu \phi \\ & + \bar{\ell}_i \left( g_{ij}^L \gamma_\mu P_L + g_{ij}^R \gamma_\mu P_R \right) \ell_j Z'_0{}^\mu + \bar{\nu}_i g_{ij}^L \gamma_\mu P_L \nu_j Z'_0{}^\mu, \end{aligned} \quad (2.2)$$

where  $\overleftrightarrow{D}_\mu = \overrightarrow{D}_\mu - (\overleftarrow{D}_\mu)^\dagger$  and  $g_{Z'}^\phi$  is real. Note that here the subscript 0 refers to the fact that these are not mass, but rather interaction eigenstates.  $g_{ij}^{L/R}$  are hermitian and due to  $SU(2)_L$  invariance the coupling to neutrinos is the same as to left-handed charged leptons.<sup>1</sup>  $\phi$  is the SM Higgs  $SU(2)_L$  doublet and we use

$$D_\mu = \partial_\mu + i g_2 W_\mu^a T^a + i g_1 Y B_\mu, \quad (2.3)$$

as the definition of the covariant derivative.

The coupling  $g_{Z'}^\phi$  leads to mixing of the  $Z'_0$ -boson with the SM  $Z$ . The corresponding mass matrix in the interaction eigenbasis  $(Z_0, Z'_0)$  is then given by

$$\mathcal{M}^2 = \begin{pmatrix} m_{Z_0}^2 & -\frac{y}{c_W} \\ -\frac{y}{c_W} & M_{Z'_0}^2 \end{pmatrix}, \quad y \equiv \frac{v^2}{2} g_2 g_{Z'}^\phi \quad (2.4)$$

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<sup>1</sup>Here we neglected small active neutrino masses and therefore set the PMNS matrix to the unit matrix.

with  $m_{Z_0}^2 = \frac{v^2}{4} (g_1^2 + g_2^2)$  and  $\frac{v}{\sqrt{2}} \approx 174$  TeV. To order  $\frac{v^2}{m_{Z_0}^2}$  the eigenvalues are

$$m_Z^2 \simeq m_{Z_0}^2 - \frac{y^2}{c_W^2 M_{Z_0}^2} \equiv m_{Z_0}^2 (1 + \delta m_Z^2), \quad (2.5)$$

$$M_{Z'}^2 \simeq M_{Z_0}^2 + \frac{y^2}{c_W^2 M_{Z_0}^2}. \quad (2.6)$$

Hence the corrections to the mass of the SM  $Z_0$  can only be destructive. The mass eigenstates  $Z^{(\prime)}$  can then be expressed as

$$\begin{pmatrix} Z' \\ Z \end{pmatrix} = \begin{pmatrix} Z_0' \cos \xi - Z_0 \sin \xi \\ Z_0' \sin \xi + Z_0 \cos \xi \end{pmatrix} \quad (2.7)$$

where

$$\sin \xi \simeq \frac{y}{c_W M_{Z_0}^2} \quad (2.8)$$

describes the  $Z_0 - Z_0'$  mixing. Note that only the relative phase between  $\sin \xi$  and  $g_{ij}^{L,R}$  is physical. Therefore, one can assume one of the diagonal couplings  $g_{ii}^{L,R}$  to be positive without loss of generality. We write the interactions of the SM  $Z$  with fermions as

$$\begin{aligned} \mathcal{L}_{Zff} = & \bar{\ell}_i \gamma_\mu (\Delta_{ij}^{\ell L} P_L + \Delta_{ij}^{\ell R} P_R) \ell_j Z^\mu + \bar{\nu}_i \gamma_\mu \Delta_{ij}^{\nu L} P_L \nu_j Z^\mu \\ & + \bar{u}_k \gamma_\mu (g_{SM}^{uL} P_L + g_{SM}^{uR} P_R) u_k Z^\mu + \bar{d}_l \gamma_\mu (g_{SM}^{dL} P_L + g_{SM}^{dR} P_R) d_l Z^\mu, \end{aligned} \quad (2.9)$$

with  $i, j = e, \mu, \tau$ ,  $k = u, c, t$ ,  $l = d, s, b$  and

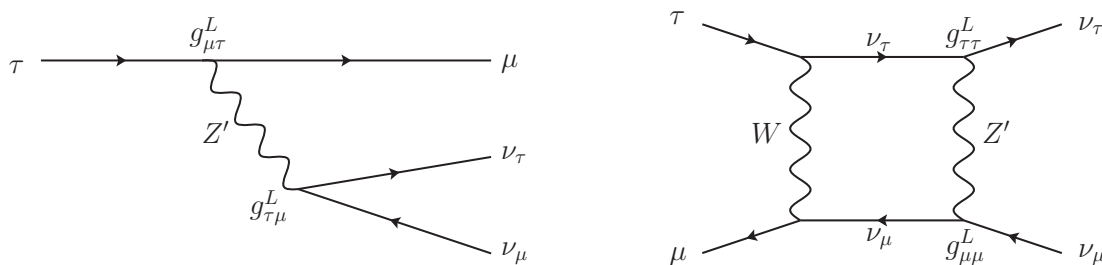
$$\Delta_{ij}^{\ell L,R} \simeq \sin \xi g_{ij}^{L,R} + g_{SM}^{\ell L,R} \delta_{ij}, \quad \Delta_{ij}^{\nu L} \simeq \sin \xi g_{ij}^L + g_{SM}^{\nu L} \delta_{ij}, \quad (2.10)$$

where  $g_{SM}^{\ell,\nu L,R}$  are the SM  $Z\bar{\ell}\ell$  and  $Z\bar{\nu}\nu$  couplings given by

$$\begin{aligned} g_{SM}^{\nu L} &= -\frac{e}{2s_W c_W}, \\ g_{SM}^{\ell L} &= -\frac{e}{2s_W c_W} (-1 + 2s_W^2), & g_{SM}^{\ell R} &= -\frac{e s_W}{c_W}, \\ g_{SM}^{uL} &= -\frac{e}{s_W c_W} \left( \frac{1}{2} - \frac{2}{3} s_W^2 \right), & g_{SM}^{uR} &= \frac{2}{3} \frac{e s_W}{c_W}, \\ g_{SM}^{dL} &= -\frac{e}{s_W c_W} \left( -\frac{1}{2} + \frac{1}{3} s_W^2 \right), & g_{SM}^{dR} &= -\frac{1}{3} \frac{e s_W}{c_W}, \end{aligned} \quad (2.11)$$

with  $e = g_1 g_2 / \sqrt{g_1^2 + g_2^2} = g_1 c_W = g_2 s_W$  being the electric charge.

$Z'$  scenarios are in general subject to gauge anomalies, which are often assumed to be canceled by additional heavy fields at a higher scale [9, 138, 139]. This is what we will do in the present paper. Gauge anomaly cancellation in  $Z'$  models was discussed recently in refs. [108, 110, 118, 140].



**Figure 1.** Feynman diagrams illustrating the leading  $Z'$ -contributions to the process  $\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu$ . In the presence of flavour off-diagonal  $Z'$ -couplings, the process arises already at tree-level (see left diagram), whereas in the case of flavour-diagonal couplings, corrections to  $\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu$  are induced by four boxes, one of which is shown on the right.

### 3 Basic formulae for observables

#### 3.1 Lepton flavor universality

With this setup we are in a position to calculate the effects in the relevant observables. Flavour off-diagonal couplings already generate at tree-level  $\ell \rightarrow \ell' \nu \bar{\nu}$  amplitudes which interfere with the SM one, while flavour diagonal couplings can only do this via box diagrams (see figure 1). Neglecting non-interfering contributions we thus have

$$R[\tau \rightarrow \mu] = \frac{\mathcal{A}(\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu)}{\mathcal{A}(\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu)_{\text{SM}}} = 1 + 2 \frac{|g_{\mu\tau}^L|^2 m_W^2}{g_2^2 M_{Z'}^2} - \frac{3}{8\pi^2} g_{\mu\mu}^L g_{\tau\tau}^L \frac{\ln\left(\frac{m_W^2}{M_{Z'}^2}\right)}{1 - \frac{M_{Z'}^2}{m_W^2}}, \quad (3.1)$$

where  $\mathcal{A}$  denotes the amplitude. Analogous expressions for  $\tau \rightarrow e \nu_\tau \bar{\nu}_e$  and  $\mu \rightarrow e \nu_\mu \bar{\nu}_e$  follow by a straightforward exchange of indices. This has to be compared to the experimental results [55]

$$\begin{aligned} \frac{\mathcal{A}[\tau \rightarrow \mu \nu \bar{\nu}]}{\mathcal{A}[\mu \rightarrow e \nu \bar{\nu}]_{\text{EXP}}} &= 1.0029 \pm 0.0014, \\ \frac{\mathcal{A}[\tau \rightarrow \mu \nu \bar{\nu}]}{\mathcal{A}[\tau \rightarrow e \nu \bar{\nu}]_{\text{EXP}}} &= 1.0018 \pm 0.0014, \\ \frac{\mathcal{A}[\tau \rightarrow e \nu \bar{\nu}]}{\mathcal{A}[\mu \rightarrow e \nu \bar{\nu}]_{\text{EXP}}} &= 1.0010 \pm 0.0014, \end{aligned} \quad (3.2)$$

with the correlation matrix [55]

$$\begin{pmatrix} 1.00 & 0.49 & 0.51 \\ 0.49 & 1.00 & -0.49^2 \\ 0.51 & -0.49 & 1.00 \end{pmatrix}. \quad (3.3)$$

<sup>2</sup>The HFLAV collaboration reports -0.50, however for the practical usage we choose -0.49 to have a positive semi-definite correlation matrix.

Furthermore, the effect in  $\mu \rightarrow e \nu_\mu \bar{\nu}_e$  is related to a modification of the Fermi constant which enters not only electroweak precision observables (to be discussed later) but also in the determination of  $V_{ud}$  from beta decays, in particular super-allowed beta decays, which allow for the most precise determination of  $V_{ud}$ . Here a tension with kaon, tau and  $D$  decays has been observed, whose significance depends strongly on the radiative corrections applied to  $\beta$  decays [59, 141–147], but also on the treatment of tensions between  $K_{\ell 2}$  and  $K_{\ell 3}$  decays [148] and on the bounds from  $\tau$  decays [55], see ref. [61] for more details. In the end, quoting a significance of  $3\sigma$  should provide a realistic representation of the current situation, and for definiteness we will thus use the estimate of the first-row CKM unitarity violation from ref. [149]

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(5). \tag{3.4}$$

In addition, note that there is also a deficit in the first-column CKM unitarity relation [149]

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 0.9970(18), \tag{3.5}$$

less significant than eq. (3.4), but suggesting that if the deficits were due to NP, they would likely be related to  $\beta$  decays. This unitarity deficit constitutes the so-called Cabibbo Angle Anomaly (CAA) and could be alleviated by our NP effect given by

$$R(\mu \rightarrow e) = 0.00075 \pm 0.00025, \tag{3.6}$$

with  $R(\mu \rightarrow e)$  defined in eq. (3.1) with properly changed flavour indices. Note that in these tests of LFU we can neglect the modifications of the  $W\ell\nu$  vertices since these would be loop-induced and thus suppressed by a factor  $m_\ell^2/M_{Z'}^2$ .

### 3.2 Lepton flavor violation in $\ell_j \rightarrow \ell_i \gamma$

Defining the effective Hamiltonian by

$$\mathcal{H}_{\text{NP}} = c_L^{ij} \bar{\ell}_i \sigma_{\mu\nu} P_L \ell_j F^{\mu\nu} + \text{h.c.}, \tag{3.7}$$

we find

$$c_L^{ij} = \frac{e}{48\pi^2 M_{Z'}^2} \sum_k \left( m_j g_{ik}^R g_{kj}^R - 3 m_k g_{ik}^R g_{kj}^L + m_i g_{ik}^L g_{kj}^L \right) \tag{3.8}$$

and  $c_R^{ij}$ , which can be obtained from  $c_L^{ij}$  by interchanging  $L$  and  $R$ . We find then the branching ratio<sup>3</sup>

$$\text{Br}[\ell_j \rightarrow \ell_i \gamma] = \frac{m_j^3}{4\pi\Gamma_j} \left( |c_L^{ij}|^2 + |c_R^{ij}|^2 \right). \tag{3.9}$$

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<sup>3</sup>The coefficients  $c_L^{ij}$  in the case of  $Z'$  contribution can be obtained from its contribution through the chromomagnetic penguin to  $b \rightarrow s \gamma$  decay calculated in ref. [150]. Using formulae in that paper and adjusting the couplings to the case at hand we obtain (at leading order in  $m_\ell/m_{Z'}$ ) consistent results with the generic formula of ref. [151]. However, our result for the branching ratio is a factor 1/2 smaller than the result of ref. [152].

The current experimental limits for lepton flavour violation processes at 90% C.L. are [153–155]:

$$\begin{aligned} \text{Br}[\mu \rightarrow e\gamma] &\leq 4.2 \times 10^{-13}, \\ \text{Br}[\tau \rightarrow \mu\gamma] &\leq 4.4 \times 10^{-8}, \\ \text{Br}[\tau \rightarrow e\gamma] &\leq 3.3 \times 10^{-8}. \end{aligned} \tag{3.10}$$

Improvements of approximately one order of magnitude for tau decays can be achieved at BELLE II [156] and MeG II will further increase the sensitivity for  $\mu \rightarrow e\gamma$  [157].

### 3.3 Anomalous magnetic moments and electric dipole moments

Using the coefficients in eq. (3.8) for the flavour conserving case, we obtain for the NP contributions to anomalous magnetic moments  $\Delta a_i$  and the electric dipole moments  $d_i$ ,

$$\begin{aligned} \Delta a_i &= -\frac{4m_i}{e} \text{Re} [c_R^{ii}], \\ d_i &= -2 \text{Im} [c_R^{ii}]. \end{aligned} \tag{3.11}$$

These expressions have to be compared with experimental bounds

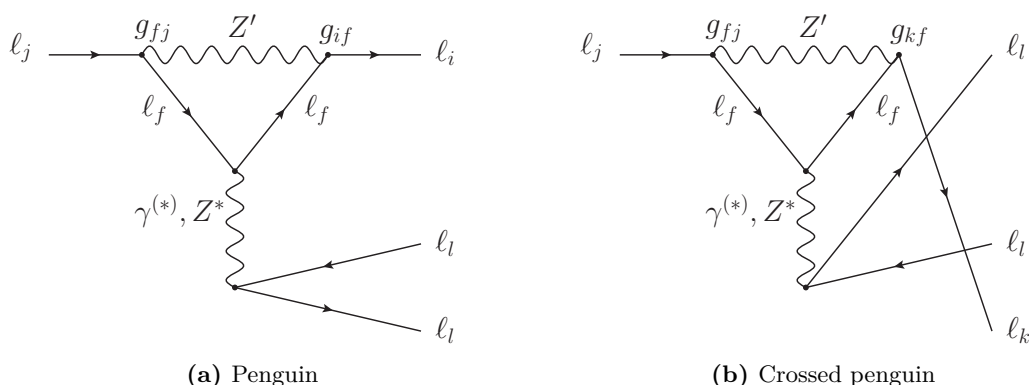
$$\begin{aligned} \Delta a_e^{\text{Cs}} &= a_e^{\text{exp}} - a_e^{\text{SM,Cs}} = -0.88(36) \times 10^{-12}, \\ \Delta a_e^{\text{Rb}} &= a_e^{\text{exp}} - a_e^{\text{SM,Rb}} = 0.48(30) \times 10^{-12}, \\ \Delta a_\mu &= a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 251(59) \times 10^{-11}, \\ d_e &< 1.1 \times 10^{-29} \text{ e cm}, \\ d_\mu &< -0.1(0.9) \times 10^{-19} \text{ e cm}, \\ -0.22 \times 10^{-16} \text{ e cm} &< \text{Re}(d_\tau) < 0.45 \times 10^{-16} \text{ e cm}, \\ -0.25 \times 10^{-16} \text{ e cm} &< \text{Im}(d_\tau) < 0.08 \times 10^{-16} \text{ e cm}. \end{aligned}$$

Here the value of  $\Delta a_e$  extracted from the corresponding measurement [158] and the SM prediction [159, 160], using the fine structure constant from  $^{133}\text{Cs}$  [161], is incompatible with the determination using  $\alpha$  from  $^{87}\text{Rb}$  [162]. Therefore, we quoted both values and we will also distinguish between these cases in our numerical analysis (where relevant).  $\Delta a_\mu$  is extracted from the measurement of refs. [17, 18] and from the recent results from the Fermilab Muon  $g - 2$  experiment [19–22]. The theory consensus is taken from ref. [23]<sup>4</sup> while the bound on  $d_\mu$  originates from ref. [189] and  $d_e$  from refs. [190, 191]. Note that while currently the bounds from tau leptons [192–194] and the muon EDM are not constraining, the latter could be significantly improved by a dedicated experiment proposed at PSI [195].

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<sup>4</sup>This result is based on refs. [163–182]. The recent lattice result of the Budapest-Marseilles-Wuppertal collaboration (BMWc) for the hadronic vacuum polarization (HVP) [183], on the other hand, is not included. This result would render the SM prediction of  $a_\mu$  compatible with experiment. However, the BMWc results are in tension with the HVP determined from  $e^+e^- \rightarrow \text{hadrons}$  data [167–172]. Furthermore, the HVP also enters the global EW fit [184], whose (indirect) determination is below the BMWc result [185]. Therefore, the BMWc determination of the HVP would increase tension in the EW fit [186, 187] and we opted for using the community consensus of ref. [23].





**Figure 2.** Feynman diagrams showing the  $Z$ - and the photon-penguin contributions to  $\ell_j \rightarrow \ell_i \bar{\ell}_l \ell_k$ -processes.

### 3.4 Lepton flavor violation in $\ell \rightarrow 3\ell'$

Three body decays to charged leptons receive the following contributions:

1. Tree-level  $Z'$  and  $Z$  exchanges (in the presence of  $Z - Z'$  mixing). The latter contributions to processes with just one flavour transition, such as  $\tau^- \rightarrow \mu^- e^+ e^-$  or  $\tau \rightarrow 3\mu$ , are proportional to  $\sin \xi$  for the  $Z$ - $Z'$  interference and proportional to  $\sin^2 \xi$  for the  $Z$  contributions alone. The processes with two flavour transitions, such as  $\tau^- \rightarrow \mu^- e^+ \mu^-$ , are suppressed by higher powers of  $\sin \xi$  and neglected in the following. However, for the  $Z'$ -mediated tree-level contributions we include the possibility that both vertices are flavour changing.
2. One-loop effects in dipole operators (as defined in eq. (3.8) and depicted in figure 2) entering via on-shell photon. There contributions again only affect decays with same flavour  $\ell^+ \ell^-$  pairs in the final states.
3. One-loop contributions generated through the mixing of tree-level induced 4-lepton operators into the operators

$$\begin{aligned}
 \mathcal{O}_{e\mu}^{LL} &= (\bar{e}\gamma_\mu P_L \mu)(\bar{\ell}\gamma^\mu P_L \ell), & \mathcal{O}_{e\tau}^{LL} &= (\bar{e}\gamma_\mu P_L \tau)(\bar{\ell}\gamma^\mu P_L \ell), \\
 \mathcal{O}_{\mu\tau}^{LL} &= (\bar{\mu}\gamma_\mu P_L \tau)(\bar{\ell}\gamma^\mu P_L \ell), & \mathcal{O}_{e\mu}^{RL} &= (\bar{e}\gamma_\mu P_R \mu)(\bar{\ell}\gamma^\mu P_L \ell), \\
 \mathcal{O}_{e\tau}^{RL} &= (\bar{e}\gamma_\mu P_R \tau)(\bar{\ell}\gamma^\mu P_L \ell), & \mathcal{O}_{\mu\tau}^{RL} &= (\bar{\mu}\gamma_\mu P_R \tau)(\bar{\ell}\gamma^\mu P_L \ell),
 \end{aligned} \tag{3.12}$$

and the corresponding ones with  $L$  and  $R$  interchanged. See figure 2 for an illustration of the penguin contributions. The details of this mixing through the QED penguin diagrams are discussed in appendix B. There we list the results for the Wilson coefficients of the operators in question which, due to the vectorial nature of the photon coupling, satisfy the following relations

$$C_{e\mu}^{LL} \Big|_{\text{QED}} = C_{e\mu}^{LR} \Big|_{\text{QED}}, \quad C_{e\mu}^{RL} \Big|_{\text{QED}} = C_{e\mu}^{RR} \Big|_{\text{QED}}, \tag{3.13}$$

with analogous relations for the remaining coefficients.

Note that the formulae for the coefficients  $C_{ij}^{AB}$  in appendix B carry no  $\ell\ell$  indices (contrary in  $C_{ij,\ell\ell}^{AB}$ ) and consequently apply to all operators. The difference will, however, be present in the parameters  $X_{ij,kl}^{AB}$  which are sensitive to the choice of  $kl$ . The latter are defined by

$$X_{ij,kl}^{AB} \equiv C_{ij}^{AB} \delta_{kl} + \frac{\Delta_{ij}^A \Delta_{kl}^B}{m_Z^2} \delta_{kl} + \frac{g_{ij}^A g_{kl}^B}{M_{Z'}^2}. \quad (3.14)$$

where  $A, B = L, R$ , which combines the contributions from tree-level  $Z'$  and  $Z$  exchanges, and from QED penguin contributions. The couplings  $\Delta_{ij}^A$  are defined in eq. (2.10). We then find the following branching ratios for flavour-violating  $\tau$  decays:

$$\text{Br} [\tau^\mp \rightarrow e^\mp \mu^\pm \mu^\mp] = -\frac{e^2 m_\tau^3}{48 \pi^3 \Gamma_\tau} \left( |c_L^{e\tau}|^2 + |c_R^{e\tau}|^2 \right) \left( \ln \left( \frac{m_\mu^2}{m_\tau^2} \right) + 3 \right) \quad (3.15)$$

$$+ \frac{m_\tau^5}{1536 \pi^3 \Gamma_\tau} \left( \left| X_{e\tau,\mu\mu}^{LL} + X_{\mu\tau,e\mu}^{LL} \right|^2 + \left| X_{e\tau,\mu\mu}^{LR} \right|^2 + \left| X_{\mu\tau,e\mu}^{LR} \right|^2 + L \leftrightarrow R \right) \\ + \frac{e m_\tau^4}{192 \pi^3 \Gamma_\tau} \left( \text{Re} \left[ c_R^{e\tau*} \left( X_{e\tau,\mu\mu}^{LL} + X_{\mu\tau,e\mu}^{LL} + X_{e\tau,\mu\mu}^{LR} + X_{\mu\tau,e\mu}^{LR} \right) \right] + L \leftrightarrow R \right),$$

$$\text{Br} [\tau \rightarrow 3\mu] = -\frac{e^2 m_\tau^3}{192 \pi^3 \Gamma_\tau} \left( |c_L^{\mu\tau}|^2 + |c_R^{\mu\tau}|^2 \right) \left( 4 \ln \left( \frac{m_\mu^2}{m_\tau^2} \right) + 11 \right)$$

$$+ \frac{m_\tau^5}{1536 \pi^3 \Gamma_\tau} \left( 2 \left| X_{\mu\tau,\mu\mu}^{LL} \right|^2 + \left| X_{\mu\tau,\mu\mu}^{LR} \right|^2 + L \leftrightarrow R \right) \quad (3.16)$$

$$+ \frac{e m_\tau^4}{192 \pi^3 \Gamma_\tau} \left( \text{Re} \left[ c_R^{\mu\tau*} \left( 2 X_{\mu\tau,\mu\mu}^{LL} + X_{\mu\tau,\mu\mu}^{LR} \right) \right] + L \leftrightarrow R \right),$$

$$\text{Br} [\tau^\mp \rightarrow \mu^\mp e^\pm \mu^\mp] = \frac{m_\tau^5}{1536 \pi^3 \Gamma_\tau} \left( 2 \left| X_{\mu\tau,\mu e}^{LL} \right|^2 + \left| X_{\mu\tau,\mu e}^{LR} \right|^2 + L \leftrightarrow R \right). \quad (3.17)$$

The expression for  $\tau \rightarrow 3e$  is obtained from  $\tau \rightarrow 3\mu$  by replacing  $\mu$  by  $e$ , the one for  $\mu \rightarrow 3e$  from  $\tau \rightarrow 3\mu$  by replacing index  $\tau$  by  $\mu$  and  $\mu$  by  $e$ . Finally the expression for  $\tau^\mp \rightarrow e^\mp \mu^\pm e^\mp$  is obtained from last formula by interchanging  $\mu$  and  $e$ .

The experimental bounds [55, 196–199] at 90% C.L. are:

$$\begin{aligned} \text{Br} [\mu^- \rightarrow e^- e^+ e^-] &\leq 1.0 \times 10^{-12}, \\ \text{Br} [\tau^- \rightarrow e^- e^+ e^-] &\leq 1.4 \times 10^{-8}, \\ \text{Br} [\tau^- \rightarrow e^- \mu^+ \mu] &\leq 1.6 \times 10^{-8}, \\ \text{Br} [\tau^- \rightarrow \mu^- e^+ \mu^-] &\leq 9.8 \times 10^{-9}, \\ \text{Br} [\tau^- \rightarrow \mu^- e^+ e^-] &\leq 1.1 \times 10^{-8}, \\ \text{Br} [\tau^- \rightarrow e^- \mu^+ e^-] &\leq 8.4 \times 10^{-9}, \\ \text{Br} [\tau^- \rightarrow \mu^- \mu^+ \mu^-] &\leq 1.1 \times 10^{-8}. \end{aligned} \quad (3.18)$$

Here we can expect future improvements in  $\tau$  decays by BELLE II [156] (and also LHCb [200]) and for  $\mu \rightarrow 3e$  by Mu3e [201, 202].

### 3.5 $\mu \rightarrow e$ conversion

We define

$$\mathcal{L}_{\text{eff}} = \sum_{q=u,d} \left( C_{e\mu,qq}^{LL} O_{e\mu,qq}^{LL} + C_{e\mu,qq}^{LR} O_{e\mu,qq}^{LR} \right) + (L \leftrightarrow R) + \text{h.c.}, \quad (3.19)$$

with

$$\begin{aligned} O_{e\mu,qq}^{LL} &= (\bar{e}\gamma^\mu P_L \mu)(\bar{q}\gamma_\mu P_L q), & O_{e\mu,qq}^{RL} &= (\bar{e}\gamma^\mu P_R \mu)(\bar{q}\gamma_\mu P_L q), \\ O_{e\mu,qq}^{LR} &= (\bar{e}\gamma^\mu P_L \mu)(\bar{q}\gamma_\mu P_R q), & O_{e\mu,qq}^{RR} &= (\bar{e}\gamma^\mu P_R \mu)(\bar{q}\gamma_\mu P_R q). \end{aligned} \quad (3.20)$$

In the presence of  $Z - Z'$  mixing, the flavour off-diagonal  $Z'$ -couplings lead to  $\mu \rightarrow e$  conversion already at tree-level

$$C_{e\mu,qq}^{AB} = \frac{\sin \xi}{m_Z^2} g_{\text{SM}}^{qB} \Big|_{Z-Z'}, \quad (3.21)$$

with the  $\sin \xi$  given in eq. (2.7) and  $A, B = L, R$ . In addition, for small  $Z_0 - Z'_0$  mixing, the mixing of four-lepton operators into  $O_{qq}^{V,LL}$  and  $O_{qq}^{V,LR}$  can be relevant and is obtained analogously to the off-shell photon effects in  $\ell \rightarrow 3\ell'$  decays (see appendix B). The standard renormalization group evolution is then performed from scale  $M_{Z'}$  down to  $m_\mu$ , taking into account that the  $\tau$ -lepton is integrated out at  $m_\tau$ . Taking into account that for operators with three electrons or three muons two different Wick contractions exist, which leads to a relative factor of 2 w.r.t. the  $\tau$ -leptons case, and only considering the contributions of the hidden operators,<sup>5</sup> we find

$$\begin{aligned} C_{e\mu,qq}^{LL} &= \frac{e^2 Q_q}{16\pi^2} \frac{2}{3M_{Z'}^2} \left( g_{e\mu}^L (2g_{\mu\mu}^L + g_{\mu\mu}^R) \ln \left( \frac{M_{Z'}^2}{m_\mu^2} \right) + (g_{e\tau}^L g_{\tau\mu}^L + g_{e\mu}^L (g_{\tau\tau}^L + g_{\tau\tau}^R)) \ln \left( \frac{M_{Z'}^2}{m_\tau^2} \right) \right) \Big|_{\text{QED}}, \\ C_{e\mu,qq}^{RL} &= \frac{e^2 Q_q}{16\pi^2} \frac{2}{3M_{Z'}^2} \left( g_{e\mu}^R (2g_{\mu\mu}^R + g_{\mu\mu}^L) \ln \left( \frac{M_{Z'}^2}{m_\mu^2} \right) + (g_{e\tau}^R g_{\tau\mu}^R + g_{e\mu}^R (g_{\tau\tau}^L + g_{\tau\tau}^R)) \ln \left( \frac{M_{Z'}^2}{m_\tau^2} \right) \right) \Big|_{\text{QED}}, \end{aligned} \quad (3.22)$$

and

$$C_{e\mu,qq}^{LL} \Big|_{\text{QED}} = C_{e\mu,qq}^{LR} \Big|_{\text{QED}}, \quad C_{e\mu,qq}^{RL} \Big|_{\text{QED}} = C_{e\mu,qq}^{RR} \Big|_{\text{QED}}, \quad (3.23)$$

where  $Q_q$  is the electric charge of the quarks ( $Q_u = +\frac{2}{3}$ ,  $Q_d = -\frac{1}{3}$ ).

The transition rate  $\Gamma_{\mu \rightarrow e}^N \equiv \Gamma(\mu N \rightarrow e N)$  is given by

$$\Gamma_{\mu \rightarrow e}^N = \frac{m_\mu^5}{4} \left| \frac{c_L^{e\mu}}{m_\mu} D_N + 4 \sum_{q=u,d} \left( C_{e\mu,qq}^{RL} + C_{e\mu,qq}^{RR} \right) \left( f_{V_p}^{(q)} V_N^p + f_{V_n}^{(q)} V_N^n \right) \right|^2 + (L \leftrightarrow R). \quad (3.24)$$

The quantities  $D_N$  and  $V_{p/n}^N$  are related to the overlap integrals between the lepton wave functions and the nucleon densities, and thus depend on the nature of the target  $N$ . We use the numerical values [203]

$$D_{\text{Au}} = 0.189, \quad V_{\text{Au}}^p = 0.0974, \quad V_{\text{Au}}^n = 0.146. \quad (3.25)$$

<sup>5</sup>For the contributions of the visible operators we refer to eq. (B.6) of appendix B.2 where this naming of operators is explained.

The nucleon vector form factors are the same as the ones measured in elastic electron-hadron scattering, i.e.

$$f_{V_p}^{(u)} = 2, \quad f_{V_n}^{(u)} = 1, \quad f_{V_p}^{(d)} = 1, \quad f_{V_n}^{(d)} = 2. \quad (3.26)$$

Finally, the branching ratio of  $\mu \rightarrow e$  conversion is defined as the transition rate divided by the  $\mu$  capture rate:

$$\text{Br}[\mu \rightarrow e] = \frac{\Gamma^{\text{conv}}}{\Gamma^{\text{capt}}}, \quad (3.27)$$

and for the latter we use [204]

$$\Gamma_{\text{Au}}^{\text{capt}} = 8.7 \times 10^{-18} \text{ GeV}. \quad (3.28)$$

The experimental limit on  $\mu \rightarrow e$  conversion from SINDRUM II is [153]

$$\frac{\Gamma_{\text{Au}}^{\text{conv}}}{\Gamma_{\text{Au}}^{\text{capt}}} < 7.0 \times 10^{-13}. \quad (3.29)$$

It is expected to be improved by three orders of magnitude by COMET and Mu2e collaborations in the coming years [205].

### 3.6 Electroweak precision observables

The EW sector of the SM has been tested with a very high precision at LEP and Tevatron [15, 206]. Since it can be parametrised by only three Lagrangian parameters, we choose the set with the smallest experimental error consisting of the Fermi constant ( $G_F = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$  [207]), the mass of the Z boson ( $m_Z = 91.1875(21)$  [206]) and the fine structure constant ( $\alpha_{em} = 7.2973525664(17) \times 10^{-3}$  [207]).

In our model, the Lagrangian values for  $G_F$  and  $m_Z$  are shifted with respect to their measurements. In particular, the effect in  $\mu \rightarrow e\nu\bar{\nu}$  leads to the following relation

$$\frac{G_F}{G_F^{\mathcal{L}}} = 1 + 2 \frac{|g_{e\mu}^L|^2}{g_2^2} \frac{m_W^2}{M_{Z'}^2} \equiv 1 + \delta G_F, \quad (3.30)$$

while the Z boson mass is modified via eq. (2.5). Moreover, taking into account the tree-level effects in eq. (2.10) and the loop effects in refs. [34, 208], we have the following modified Z couplings to leptons

$$\begin{aligned} \Delta_{ij}^{\ell L}(q^2 = m_Z^2) &= g_{\text{SM}}^{\ell L} \left( \delta_{ij} + \sin \xi \frac{g_{ij}^L}{g_{\text{SM}}^{\ell L}} + \sum_k \frac{g_{ik}^L g_{kj}^L}{(4\pi)^2} K_F \left( \frac{m_Z^2}{M_{Z'}^2} \right) \right), \\ \Delta_{ij}^{\nu L}(q^2 = m_Z^2) &= g_{\text{SM}}^{\nu L} \left( \delta_{ij} + \sin \xi \frac{g_{ij}^L}{g_{\text{SM}}^{\nu L}} + \sum_k \frac{g_{ik}^L g_{kj}^L}{(4\pi)^2} K_F \left( \frac{m_Z^2}{M_{Z'}^2} \right) \right), \\ \Delta_{ij}^{\ell R}(q^2 = m_Z^2) &= g_{\text{SM}}^{\ell R} \left( \delta_{ij} + \sin \xi \frac{g_{ij}^R}{g_{\text{SM}}^{\ell R}} + \sum_k \frac{g_{ik}^R g_{kj}^R}{(4\pi)^2} K_F \left( \frac{m_Z^2}{M_{Z'}^2} \right) \right), \end{aligned} \quad (3.31)$$

Observable	Experimental value
$m_W$ [GeV]	80.379(12)
$\Gamma_W$ [GeV]	2.085(42)
BR( $W \rightarrow$ had)	0.6741(27)
BR( $W \rightarrow$ lep)	0.1086(9)
$\sin^2 \alpha_{\text{eff}, e}^{\text{CDF}}$	0.23248(52)
$\sin^2 \alpha_{\text{eff}, e}^{\text{D0}}$	0.23146(47)
$\sin^2 \alpha_{\text{eff}, \mu}^{\text{CDF}}$	0.2315(10)
$\sin^2 \alpha_{\text{eff}, \mu}^{\text{CMS}}$	0.2287(32)
$\sin^2 \alpha_{\text{eff}, \mu}^{\text{LHCb}}$	0.2314(11)
$P_\tau^{\text{pol}}$	0.1465(33)
$A_e$	0.1516(21)
$A_\mu$	0.142(15)
$A_\tau$	0.136(15)
$\Gamma_Z$ [GeV]	2.4952(23)
$\sigma_h^0$ [nb]	41.541(37)
$R_e^0$	20.804(50)
$R_\mu^0$	20.785(33)
$R_\tau^0$	20.764(45)
$A_{\text{FB}}^{0,e}$	0.0145(25)
$A_{\text{FB}}^{0,\mu}$	0.0169(13)
$A_{\text{FB}}^{0,\tau}$	0.0188(17)
$R_b^0$	0.21629(66)
$R_c^0$	0.1721(30)
$A_{\text{FB}}^{0,b}$	0.0992(16)
$A_{\text{FB}}^{0,c}$	0.0707(35)
$A_b$	0.923(20)
$A_c$	0.670(27)

**Table 1.** Electroweak observables [206, 207] used in our fit which are calculated by HEPfit [135] using  $m_Z^L$ ,  $\alpha$  and  $G_F$  as input.

with

$$K_F(x) = -\frac{2(x+1)^2(\text{Li}_2(-x) + \ln(x)\ln(x+1))}{x^2} - \frac{7x+4}{2x} + \frac{(3x+2)\ln(x)}{x}. \quad (3.32)$$

For the numerical analysis, we implemented the EW observables shown in table 1 in HEPfit [135] taking into account the modifications induced by eq. (2.5), eq. (3.30) and eq. (3.31). In addition, the Higgs mass ( $M_H = 125.16 \pm 0.13$  GeV [209, 210]), the top mass ( $m_t =$

$\text{Br}[Z \rightarrow e^\pm \tau^\mp]$	$= (-0.1 \pm 3.5(\text{stat}) \pm 2.3(\text{syst})) \times 10^{-6}$	ATLAS: [214]
$\text{Br}[Z \rightarrow e^\pm \tau^\mp]$	$< 9.8 \times 10^{-6}$	LEP (OPAL): [149, 215]
$\text{Br}[Z \rightarrow \mu^\pm \tau^\mp]$	$= (4.3 \pm 2.8(\text{stat}) \pm 1.6(\text{syst})) \times 10^{-6}$	ATLAS: [214]
$\text{Br}[Z \rightarrow \mu^\pm \tau^\mp]$	$< 1.2 \times 10^{-5}$	LEP (DELPHI): [149, 216]
$\text{Br}[Z \rightarrow e^\pm \mu^\mp]$	$< 7.5 \times 10^{-7}$ (95% CL)	ATLAS: [149, 217]

**Table 2.** Experimental bounds on  $Z \rightarrow \ell\ell'$ .

172.80±0.40 GeV [211–213]), the strong coupling constant ( $\alpha_s(M_Z) = 0.1181 \pm 0.0011$  [207]) and the hadronic contribution to the running of  $\alpha_{em}$  ( $\Delta\alpha_{\text{had}} = 276.1(11) \times 10^{-4}$  [207]) have been used as input parameters, since they enter EW observables indirectly via loop effects.

### 3.7 $Z \rightarrow \ell\ell'$

In the presence of  $Z - Z'$  mixing, we obtain a tree-level contribution to  $Z \rightarrow \ell_i \ell_j$ ,  $i \neq j$ , leading to the branching ratios

$$\begin{aligned} \text{Br}[Z \rightarrow \ell_i \bar{\ell}_j] &= \frac{1}{24\pi} \frac{m_Z}{\Gamma_Z} \left( |\Delta_{ij}^{\ell L}(q^2 = m_Z^2)|^2 + |\Delta_{ij}^{\ell R}(q^2 = m_Z^2)|^2 \right), \\ \text{Br}[Z \rightarrow \nu_i \bar{\nu}_j] &= \frac{1}{24\pi} \frac{m_Z}{\Gamma_Z} |\Delta_{ij}^{\nu L}(q^2 = m_Z^2)|^2, \end{aligned} \quad (3.33)$$

with  $\Gamma_Z = 2.4952 \pm 0.0023$  GeV [149]. We compare these results to the ATLAS and LEP measurements given in table 2.

### 3.8 Neutrino trident production

Neutrino trident production can be used to constrain couplings of muons to muon neutrinos [34, 36]. Generalizing the formula of ref. [36] to the case of chiral  $Z'$  couplings, we find

$$\frac{\sigma_{\text{SM+NP}}}{\sigma_{\text{SM}}} = 1 + 8 \frac{(1 + 4s_W^2) \frac{g_{22}^L (g_{22}^L + g_{22}^R) m_W^2}{g_2^2 M_{Z'}^2} - \frac{g_{22}^L (g_{22}^R - g_{22}^L)^2 m_W^2}{g_2^2 M_{Z'}^2}}{(1 + 4s_W^2)^2 + 1}. \quad (3.34)$$

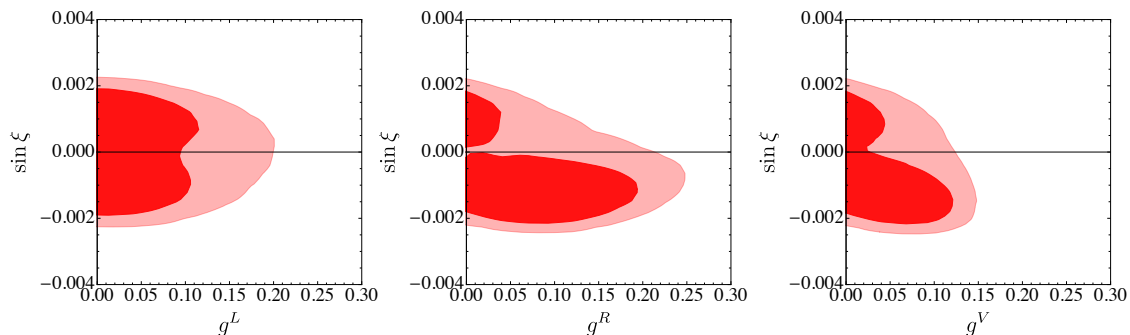
This ratio is bounded by the weighted average

$$\sigma_{\text{exp}}/\sigma_{\text{SM}} = 0.83 \pm 0.18, \quad (3.35)$$

obtained from averaging the CHARM-II [218], CCFR [219] and NuTeV results [220].

### 3.9 LEP-II bounds

LEP-II set stringent bounds on 4-lepton operators from  $e^+e^- \rightarrow \ell^+\ell^-$  (with  $\ell = e, \mu, \tau$ ) [15] for specific chiralities. Some of our more general scenarios cannot be matched to the 4-lepton operators as given in ref. [15]. For these cases we derive the constraints in appendix A and provide the formula which we implemented in HEPfit.



**Figure 3.** 68% and 95% CL regions for the three LFU cases  $g^L$ ,  $g^R$  and  $g^V$ . Note that due to the preference for a slightly lower  $W$  mass than predicted in the SM, the origin in the  $g^R$  scenario is not within the 68% CL region. The  $Z - Z'$  mixing angle is bounded to be  $\xi \lesssim 0.02$  and  $g^{L,R} \lesssim 0.2$ , while  $g^V \lesssim 0.1$  for  $M_{Z'} = 1$  TeV.

## 4 Phenomenological analysis

In our phenomenological analysis we perform a global fit taking into account all observables discussed in the last section. This includes EW precision observables, as implemented in the HEPfit distribution [135]. To include the other observables discussed previously, we added them to the HEPfit code such that we can perform a Bayesian statistical analysis using the Markov Chain Monte Carlo (MCMC) determination of posteriors of the Bayesian Analysis Toolkit (BAT) [221].

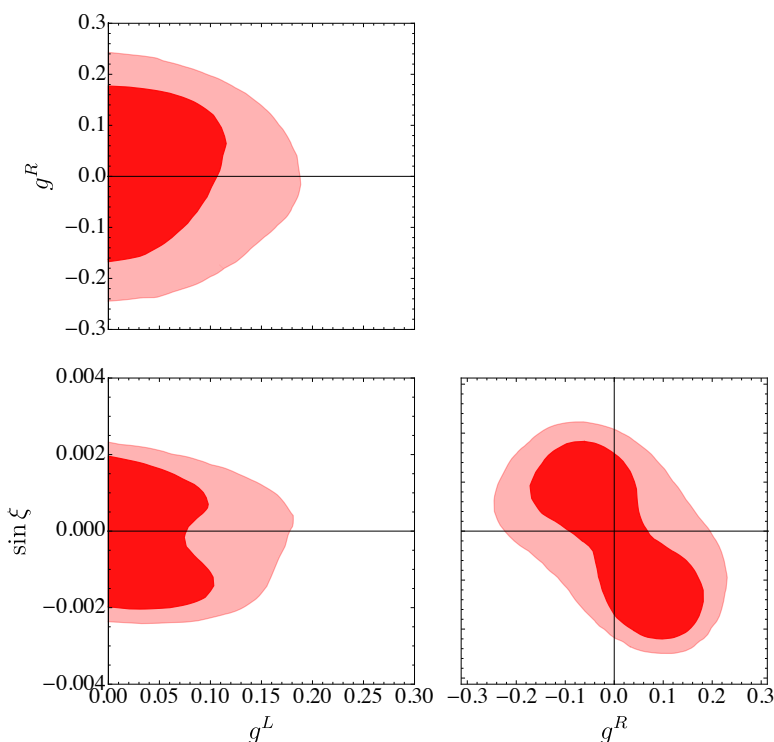
With this setup we can now consider several different scenarios. For our numerical analysis we fix  $M_{Z'} = 1$  TeV unless stated otherwise and assume real couplings. Note that despite small logarithmic corrections, the results we obtain scale like  $g^2/M_{Z'}^2$ . However, for the loop-induced modifications of  $Z\ell\ell$  couplings in the scenario which aims at an explanation of the data on the anomalous magnetic moment of the muon, these logarithmic corrections to the  $g^2/M_{Z'}^2$  scaling can indeed be relevant.

### 4.1 Lepton flavour universality

Here we consider four scenarios which respect lepton flavour universality:

1. Left-handed couplings:  $g_{ii}^L = g^L$
2. Right-handed couplings:  $g_{ii}^R = g^R$
3. Vectorial couplings:  $g_{ii}^L = g_{ii}^R = g^V$
4. Generic chiral couplings:  $g_{ii}^L = g^L, g_{ii}^R = g^R$

Each of the first three LFU scenarios is two dimensional, with the coupling and the  $Z-Z'$  mixing angle  $\xi$  being free parameters, and shown in figure 3. We can see that for a  $Z'$  mass of 1 TeV, the couplings should be smaller than  $\approx 0.2$  and the  $Z - Z'$  mixing is bounded to be less than  $\approx 0.002$  at 95% CL. In the case of  $g_L$  and  $g_R$  being independent of each other, shown in figure 4, there is a mild preference for a non-zero mixing angle at



**Figure 4.** 68% and 95% CL regions for the three-dimensional LFU fit, where  $g_L$ ,  $g_R$  and  $\xi$  are free parameters for  $M_{Z'} = 1$  TeV.

the 68% CL, which is due to the slight tension in the  $W$  mass prediction within the EW fit. However, neither the tension in  $\tau \rightarrow \mu\nu\bar{\nu}/\tau \rightarrow e\nu\bar{\nu}$  nor in the first row CKM unitarity or  $(g - 2)_\mu$  can be explained in these LFU setups.

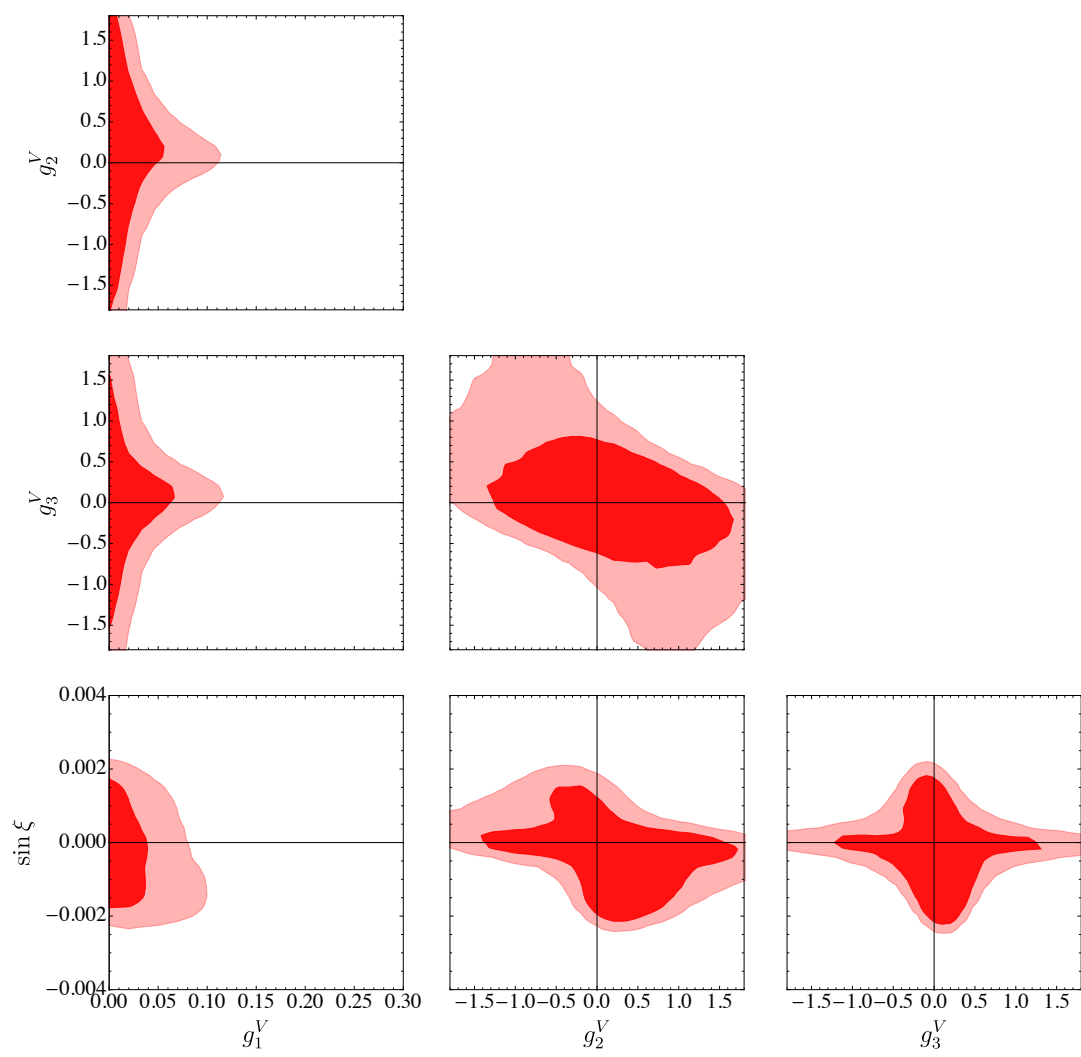
#### 4.2 Lepton flavour universality violation

Here we study the case in which the couplings that are diagonal in flavour space but not proportional to the unit matrix:

1. Vectorial couplings:  $g_{ii}^L = g_{ii}^R = g_i^V$
2. Left-handed couplings:  $g_{ii}^L = g_i^L$
3. Right-handed couplings:  $g_{ii}^R = g_i^R$

These scenarios are shown in figures 5–7. The couplings to electrons are very well constrained and can be at most of the order of 0.2 due to the LEP bounds on 4-electron contact interactions. The bounds on muon and tau couplings are less stringent and therefore can be as large as 2 for larger  $M_{Z'} = 1$  TeV. This is also due to the fact that  $\tau \rightarrow \mu\nu\bar{\nu}$  prefers larger couplings to muons and taus which enter via the box contributions. However, in this case effects in the EW fit are generated as well, such that no significant preference over the SM fit can be achieved.





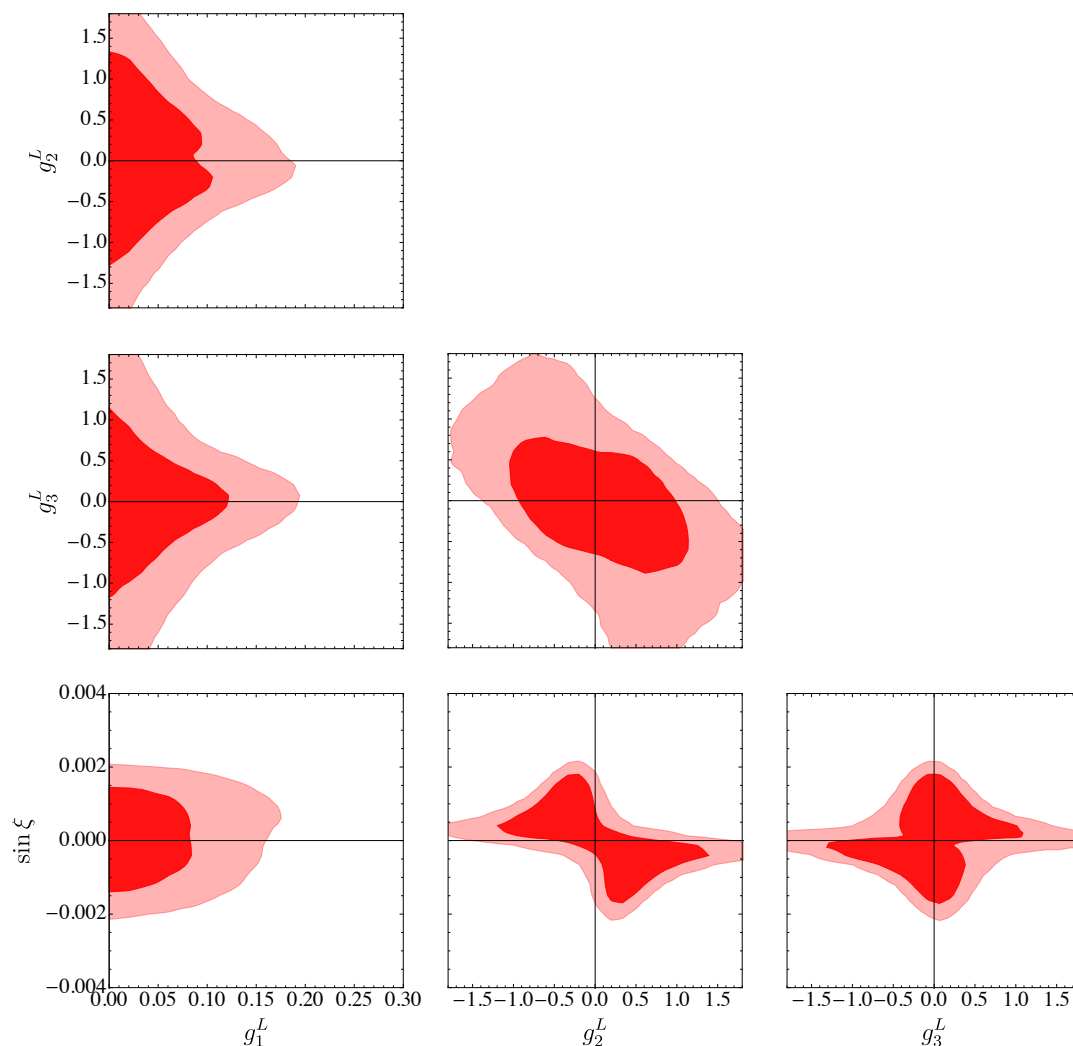
**Figure 5.** 68% and 95% CL regions for the LFUV case with vectorial couplings  $g_{ii}^L = g_{ii}^R = g_i^V$ .

### 4.3 Lepton flavour violation

Here we study the following scenarios for the couplings:

1. Flavour violating tau-muon couplings:  $g_{23}^R$  and  $g_{23}^L$
2.  $L_\mu - L_\tau$  symmetry with left-handed rotations
3. Vectorial couplings:  $g_{ij}^L = g_{ij}^R = g_{ij}^V$
4. Left-handed couplings:  $g_{ij}^L$
5. Right-handed couplings:  $g_{ij}^R$

Scenario 1, which assumes that only  $g_{23}^L$  and  $g_{23}^R$  are non-zero, allows us to find interesting correlations as shown in figure 8. Here one can see that it is possible to explain  $\tau \rightarrow \mu\nu\bar{\nu}$



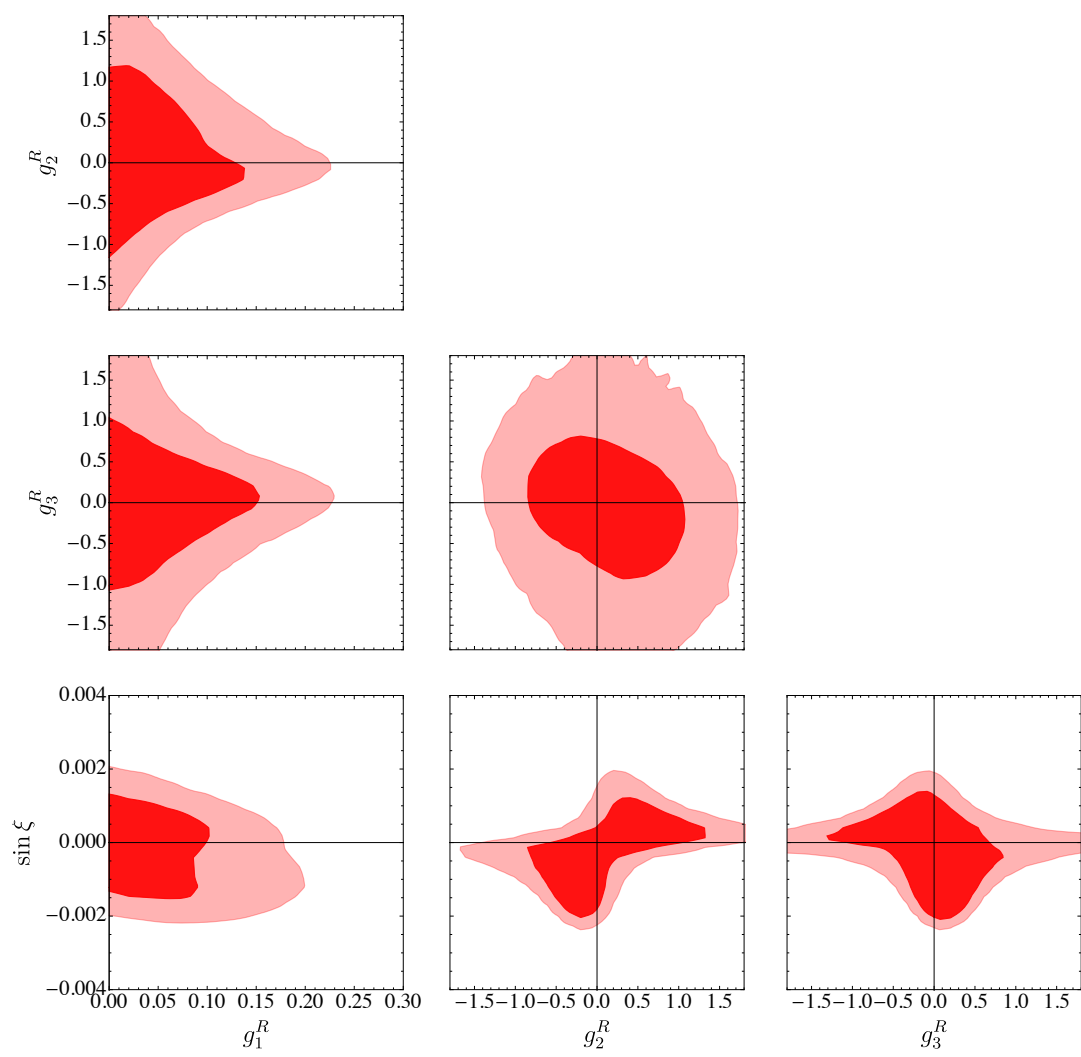
**Figure 6.** 68% and 95% CL regions for the LFUV case with left-handed couplings  $g_i^L = g_{ii}^L$ .

and  $(g - 2)_\mu$  simultaneously, predicting observable effects in  $Z \rightarrow \mu\bar{\mu}$  and  $Z \rightarrow \tau\bar{\tau}$  for  $M_{Z'} = 1$  TeV. However, due to the logarithmic corrections involved here, the effects in  $Z \rightarrow \mu\bar{\mu}$  and  $Z \rightarrow \tau\bar{\tau}$  become weaker for smaller masses. These effects will allow in the future to conquer the parameter space (assuming an explanation of  $(g - 2)_\mu$ ), as direct searches and EW precision constraints test complementary regions in parameter space.

Next, let us consider case 2 with a (broken)  $L_\mu - L_\tau$  symmetry. This means that the coupling matrix takes the form

$$g_{ij}^V = \begin{pmatrix} 0 & 0 & 0 \\ 0 & g & 0 \\ 0 & 0 & -g \end{pmatrix} \quad (4.1)$$

in the interaction basis. Now we assume that  $L_\mu - L_\tau$  is broken by the charged lepton



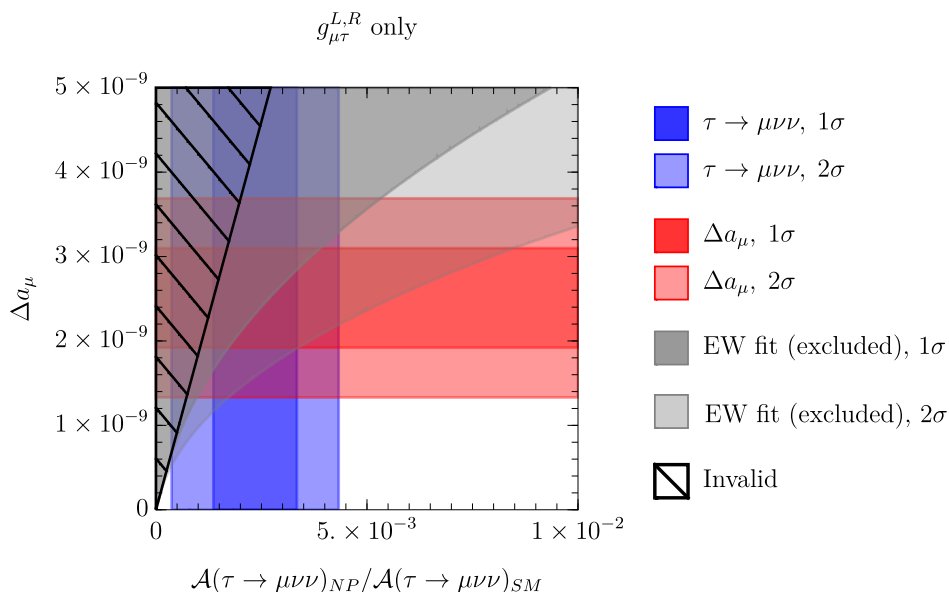
**Figure 7.** 68% and 95% CL regions for the LFUV case with right-handed couplings  $g_i^R = g_{ii}^R$ .

Yukawa couplings in the left-handed sector such that, after EW symmetry breaking,

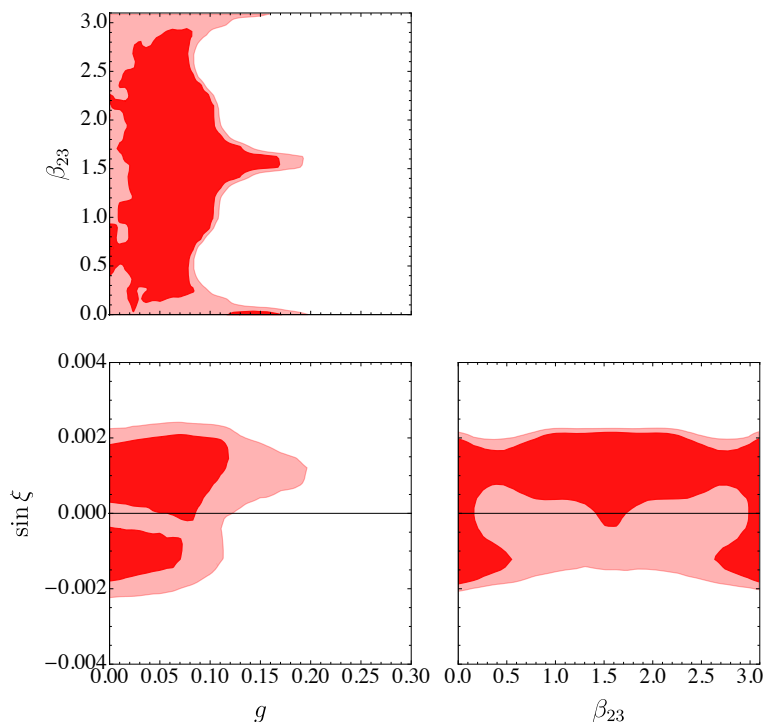
$$g_{ij}^L = \left( \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & \cos \beta_{23} & \sin \beta_{23} \\ 0 & -\sin \beta_{23} & \cos \beta_{23} \end{array} \right) \cdot \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & g & 0 \\ 0 & 0 & -g \end{array} \right) \cdot \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & \cos \beta_{23} & -\sin \beta_{23} \\ 0 & \sin \beta_{23} & \cos \beta_{23} \end{array} \right) \right)_{ij}, \quad (4.2)$$

for 2–3 rotations. The analogous formula for 1–2 rotations follows straightforwardly.

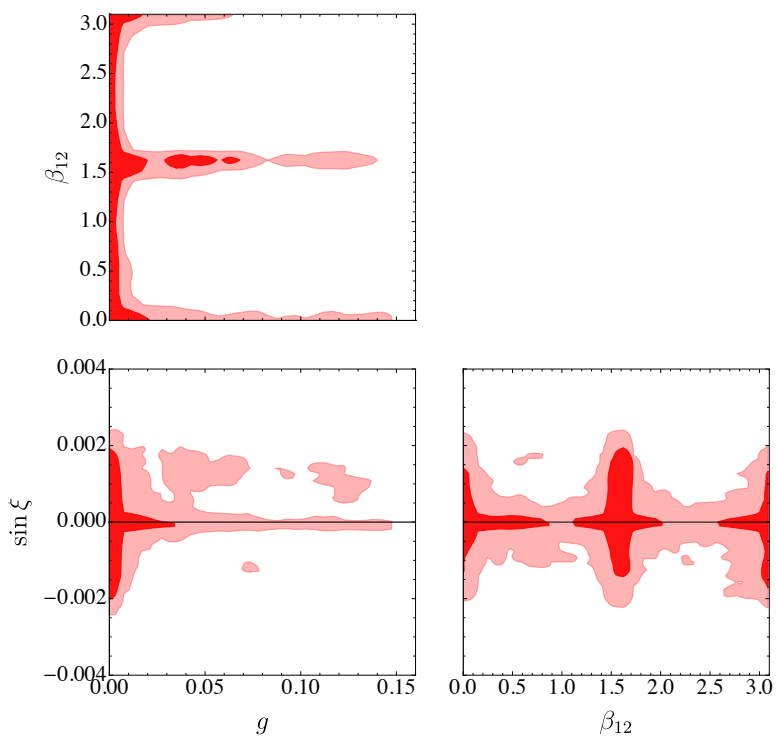
For this setup we show in figures 9 and 10 the 68% and 95% CL regions for the coupling  $g$  and the rotation angle  $\beta_{23}$  and  $\beta_{12}$ , respectively. In figure 11 we show the correlations between  $\text{Br}[\tau \rightarrow \mu\gamma]$  and  $\text{Br}[\tau \rightarrow \mu\mu\mu]$ , which display that in this scenario the present experimental upper bound on  $\text{Br}[\tau \rightarrow \mu\mu\mu]$  can easily be saturated, while  $\text{Br}[\tau \rightarrow \mu\gamma]$  is orders of magnitude below the present current bound. Therefore, finding in the coming years  $\text{Br}[\tau \rightarrow \mu\gamma]$  at the level of  $10^{-9}$  would rule out the  $L_\mu - L_\tau$  scenario. Similarly for



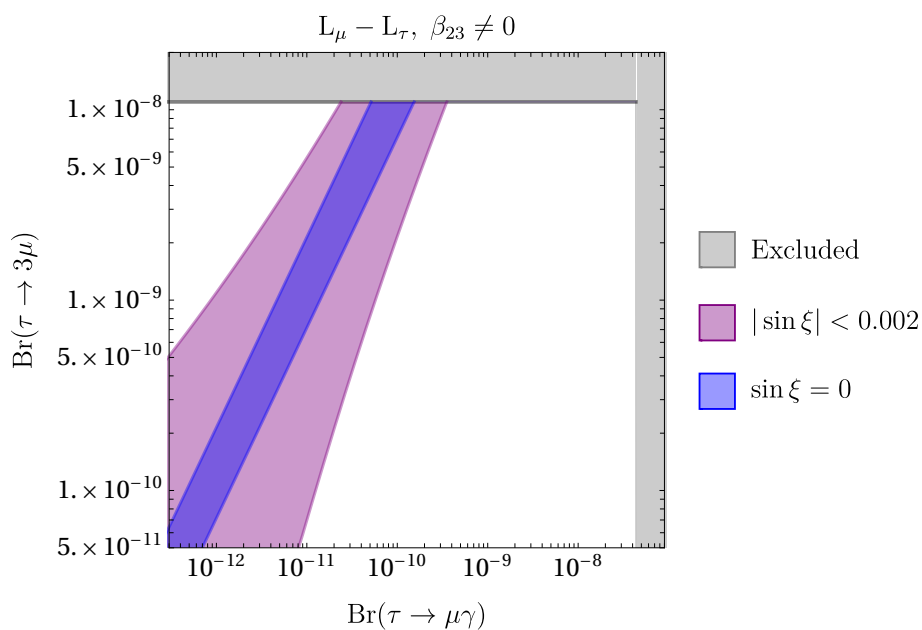
**Figure 8.** Correlations between  $(g - 2)_\mu$  and  $\mathcal{A}(\tau \rightarrow \mu\nu\bar{\nu})_{NP}/\mathcal{A}(\tau \rightarrow \mu\nu\bar{\nu})_{SM}$  in the scenario with zero  $Z - Z'$ -mixing ( $\sin \xi = 0$ ), where all  $Z'$  couplings to leptons are set to zero apart from  $g_{\mu\tau}^{L,R} \neq 0$ . The colored regions are preferred by the anomalous magnetic moment of the muon and by LFUV in tau decays, while the gray region is excluded by electroweak data for  $M_{Z'} = 1$  TeV. Note that for lighter  $Z'$  masses EW precision observables would be less constraining. The hatched region is excluded in this setup in the sense that points within it cannot be reached in this setup.



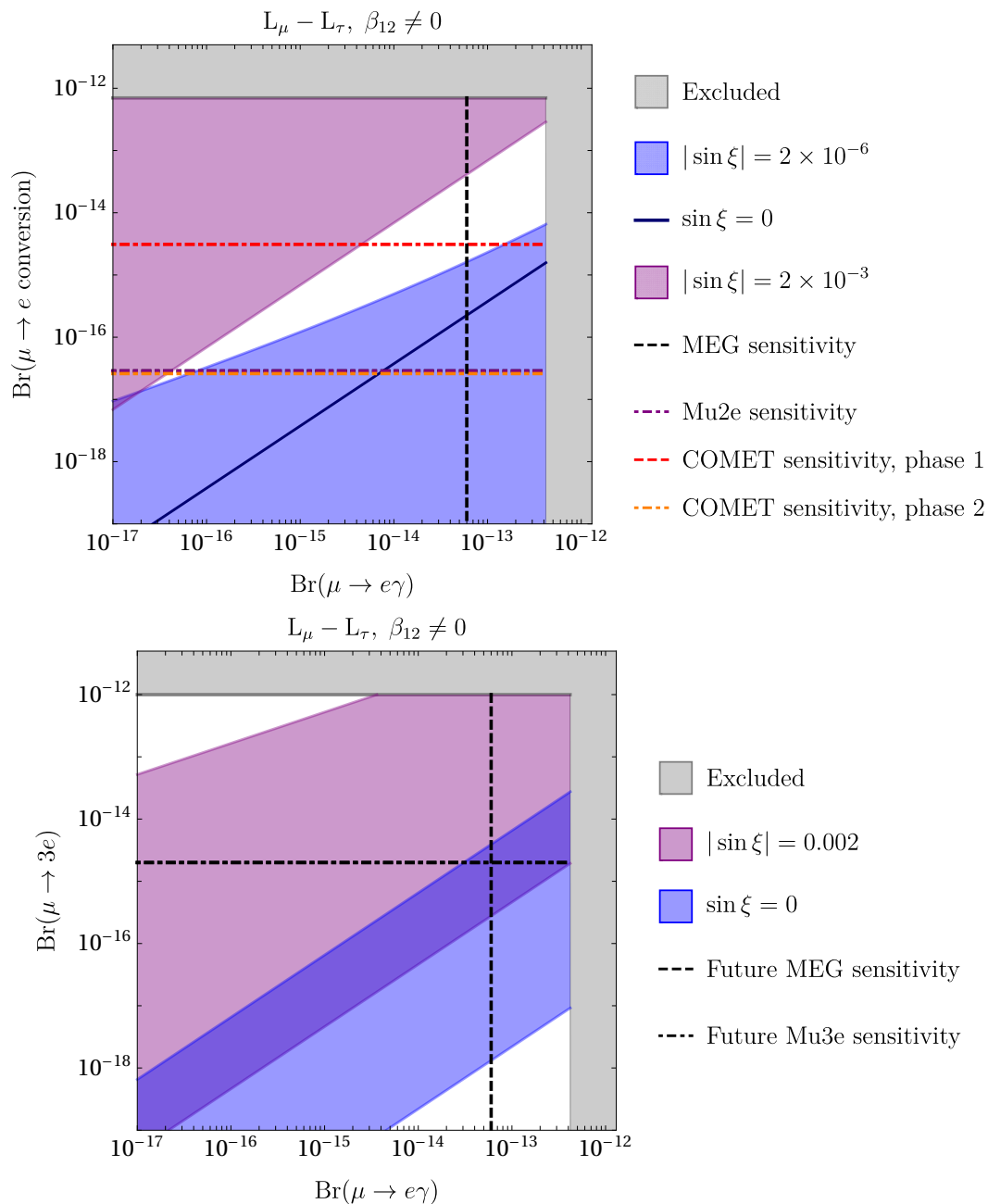
**Figure 9.** 68% and 95% C.L. regions for the  $L_\mu - L_\tau$  coupling  $g$ , the rotation angle  $\beta_{23}$  and the mixing angle  $\sin \xi$ .



**Figure 10.** 68% and 95% C.L. regions for the  $L_\mu - L_\tau$  coupling  $g$ , the rotation angle  $\beta_{12}$  and the mixing angle  $\sin \xi$ .



**Figure 11.** Correlations between  $\text{Br}(\tau \rightarrow \mu\mu\mu)$  and  $\text{Br}(\tau \rightarrow \mu\gamma)$  for different values of  $\sin \xi$  within the  $L_\mu - L_\tau$  scenario with left-handed 2-3 rotations ( $\beta_{23} \neq 0$ ). The colored regions are allowed within this setup.



**Figure 12.** Correlations between different  $\mu \rightarrow e$  transitions in the  $L_\mu - L_\tau$  scenario with left-handed 1-2 rotations ( $\beta_{12} \neq 0$ ).

$\mu \rightarrow e$  transitions: figure 12 demonstrates the importance of  $Z - Z'$  mixing and thereby the role of  $Z$  in the enhancement of  $\mu \rightarrow 3e$  and  $\mu \rightarrow e$  conversion. However, in this case, in contrast to  $\text{Br}[\tau \rightarrow \mu\gamma]$ , the branching ratio for  $\text{Br}[\mu \rightarrow e\gamma]$  can easily saturate the present experimental upper bound. MEG, Mu2e, Mu2e and COMET will constrain the allowed space in these plots in the coming years significantly.

We discuss the more general cases 3, 4 and 5, with seven free parameters each, in appendix C.

## 5 Conclusions

In this paper we have performed a global fit to leptophilic  $Z'$  models with the goal to obtain bounds on the  $Z'$  couplings to leptons in multi-dimensional scenarios. In our global analysis we took into account a large number of observables, including  $\ell \rightarrow \ell' \nu \bar{\nu}$  decays, anomalous magnetic moments of charged leptons,  $\ell \rightarrow \ell' \gamma$ ,  $\ell \rightarrow 3\ell'$  decays,  $\mu \rightarrow e$  conversion, electroweak precision observables, lepton flavour violating  $Z$  decays, neutrino trident production and LEP searches for four-lepton contact interactions.

Properly extending the HEPfit code [135] by implementing these observables, and performing a Bayesian statistical analysis, we obtained bounds on the  $Z'$  couplings in a number of generic scenarios, as listed in section 4. The results are presented in figures 5–17. The plots in these figures are self-explanatory but the main message is that the couplings involving electrons are much more strongly bounded than those involving muons or tau leptons. These results should turn out to be useful for building models where the patterns for the couplings are governed by flavour symmetries.

In more detail, we find that in the LFU scenario neither the tension in  $\tau \rightarrow \mu \nu \bar{\nu} / \tau \rightarrow e \nu \bar{\nu}$  nor in the first row CKM unitarity or in  $(g-2)_\mu$  can be explained. In the LFUV scenario the couplings to electrons are very well constrained and can be at most of the order of 0.2 due to the LEP bounds on 4-electron contact interactions. The bounds on muon and tau couplings are less stringent and therefore can be as large as 2 for larger  $M_{Z'} = 1$  TeV. We then studied more specific scenarios with constrained patterns for the couplings. Here we found that if only  $g_{23}^L$  and  $g_{23}^R$  are non-zero, the anomaly in the anomalous magnetic moment of the muon and the hint for LFUV in leptonic tau decays can be explained, with interesting predictions for EW precision observables. Furthermore, in scenarios with a  $L_\mu - L_\tau$  symmetry we were able to correlate  $\tau \rightarrow 3\mu$  to  $\tau \rightarrow \mu \gamma$  (and similarly for  $\mu \rightarrow e$  transitions). These correlations can be used to test this setup with future experiments.

## Acknowledgments

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## A LEP-II constraints for general coupling structure

LEP-II measured with high precision the total cross section  $\sigma_{\text{TOT}} = \sigma_F + \sigma_B$  and the Forward-Backward asymmetry  $A_{FB} = (\sigma_F - \sigma_B)/\sigma$  for the process  $e^+e^- \rightarrow \ell^+\ell^-$  (with  $\ell = e, \mu, \tau$ ) at various  $\sqrt{s}$  between 130 and 207 GeV (see tables 3.4 and 3.8–3.12 from ref. [15]). One can extract bounds on our model parameters by computing the BSM contribution to  $\sigma_F \pm \sigma_B$ . Here we work within an EFT approach with a Lagrangian defined

as  $\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{v^2} C_i O_i$  and we fierzed the operators giving an effect at the dimension six level into the basis:

$$\begin{aligned}
 [O_{\ell\ell}]_{11jj} &= (\ell_1 \gamma_\mu P_L \ell_1) (\ell_j \gamma^\mu P_L \ell_j), \\
 [O_{ee}]_{11jj} &= (\ell_1 \gamma_\mu P_R \ell_1) (\ell_j \gamma^\mu P_R \ell_j), \\
 [O_{\ell e}]_{11jj} &= (\ell_1 \gamma_\mu P_L \ell_1) (\ell_j \gamma^\mu P_R \ell_j), \\
 [O_{\ell e}]_{jj11} &= (\ell_j \gamma_\mu P_L \ell_j) (\ell_1 \gamma^\mu P_R \ell_1).
 \end{aligned}
 \tag{A.1}$$

Assuming that the  $Z - Z'$  mixing effects are sub-leading, and for vanishing lepton masses, the BSM contributions to  $\sigma_{\text{TOT}}$  and  $A_{FB}$  for  $\ell = \mu$  are given by:

$$\begin{aligned}
 \delta(\sigma_F + \sigma_B) &= \frac{1}{24\pi v^2} \left\{ e^2 ([c_{\ell\ell}]_{1122} + [c_{ee}]_{1122} + [c_{\ell e}]_{1122} + [c_{\ell e}]_{2211}) \right. \\
 &\quad + \frac{s(g_2^2 + g_1^2)}{s - m_Z^2} \left[ \left(-\frac{1}{2} + s_W^2\right)^2 [c_{\ell\ell}]_{1122} + s_W^4 [c_{ee}]_{1122} \right. \\
 &\quad \left. \left. + \left(-\frac{1}{2} + s_W^2\right) s_W^2 ([c_{\ell e}]_{1122} + [c_{\ell e}]_{2211}) \right] \right\},
 \end{aligned}
 \tag{A.2}$$

$$\begin{aligned}
 \delta(\sigma_F - \sigma_B) &= \frac{1}{32\pi v^2} \left\{ e^2 ([c_{\ell\ell}]_{1122} + [c_{ee}]_{1122} - [c_{\ell e}]_{1122} - [c_{\ell\ell}]_{2211}) \right. \\
 &\quad + \frac{s(g_2^2 + g_1^2)}{s - m_Z^2} \left[ \left(-\frac{1}{2} + s_W^2\right)^2 [c_{\ell\ell}]_{1122} + s_W^4 [c_{ee}]_{1122} \right. \\
 &\quad \left. \left. - \left(-\frac{1}{2} + s_W^2\right) s_W^2 ([c_{\ell e}]_{1122} + [c_{\ell e}]_{2211}) \right] \right\}.
 \end{aligned}
 \tag{A.3}$$

For the  $\tau^+\tau^-$  channel we have the same expressions with the index exchange  $2 \leftrightarrow 3$ .

On the other hand for  $e^+e^- \rightarrow e^+e^-$  ref. [15] gives also the angular differential cross section, measured at different  $\sqrt{s}$  and  $\cos\theta$  ranges (see tables 3.8–3.12 in ref. [15]). In this case the BSM contribution to the differential cross section is given by:

$$\begin{aligned}
 \delta \frac{d\sigma}{d\cos\theta} &= \frac{1}{8\pi s} \frac{1}{v^2} \left\{ 2u^2 \left[ e^2 ([c_{\ell\ell}]_{1111} + [c_{ee}]_{1111}) \left( \frac{1}{s} + \frac{1}{t} \right) \right. \right. \\
 &\quad \left. \left. + (g_2^2 + g_1^2) \left( \frac{1}{s - m_Z^2} + \frac{1}{t - m_Z^2} \right) \left( \left( \frac{1}{2} + s_W^2 \right)^2 [c_{\ell\ell}]_{1111} + s_W^4 [c_{ee}]_{1111} \right) \right] \right. \\
 &\quad \left. + t^2 \left[ [c_{\ell e}]_{1111} \frac{e^2}{s} + [c_{\ell e}]_{1111} \frac{(g_2^2 + g_1^2) \left( \frac{1}{2} + s_W^2 \right) s_W^2}{s - m_Z^2} \right] \right. \\
 &\quad \left. + s^2 \left[ [c_{\ell e}]_{1111} \frac{e^2}{t} + [c_{\ell e}]_{1111} \frac{(g_2^2 + g_1^2) \left( \frac{1}{2} + s_W^2 \right) s_W^2}{t - m_Z^2} \right] \right\},
 \end{aligned}
 \tag{A.4}$$

where  $t = -\frac{s}{2}(1 - \cos\theta)$  and  $u = -\frac{s}{2}(1 + \cos\theta)$ .



The relations between our model parameters and the SMEFT at dimension six level are as follows:

$$\begin{aligned}
 [c_{\ell\ell}]_{1122} &= -\frac{v^2}{M_{Z'}^2} \left( g_{ee}^L g_{\mu\mu}^L + g_{e\mu}^L g_{\mu e}^L \right), \\
 [c_{\ell e}]_{1122} &= -\frac{v^2}{M_{Z'}^2} \left( g_{ee}^L g_{\mu\mu}^R + g_{\mu e}^R g_{e\mu}^L \right), \\
 [c_{\ell e}]_{2211} &= -\frac{v^2}{M_{Z'}^2} \left( g_{ee}^R g_{\mu\mu}^L + g_{\mu e}^L g_{e\mu}^R \right), \\
 [c_{ee}]_{1122} &= -\frac{v^2}{M_{Z'}^2} \left( g_{ee}^R g_{\mu\mu}^R + g_{\mu e}^R g_{e\mu}^R \right), \\
 [c_{\ell\ell}]_{1111} &= -\frac{v^2}{2M_{Z'}^2} g_{ee}^L g_{ee}^L, \\
 [c_{ee}]_{1111} &= -\frac{v^2}{2M_{Z'}^2} g_{ee}^R g_{ee}^R, \\
 [c_{\ell e}]_{1111} &= -\frac{v^2}{M_{Z'}^2} g_{ee}^R g_{\mu\mu}^L.
 \end{aligned} \tag{A.5}$$

and similarly for  $2 \leftrightarrow 3$ .

## B QED penguin contributions

### B.1 Hidden operators

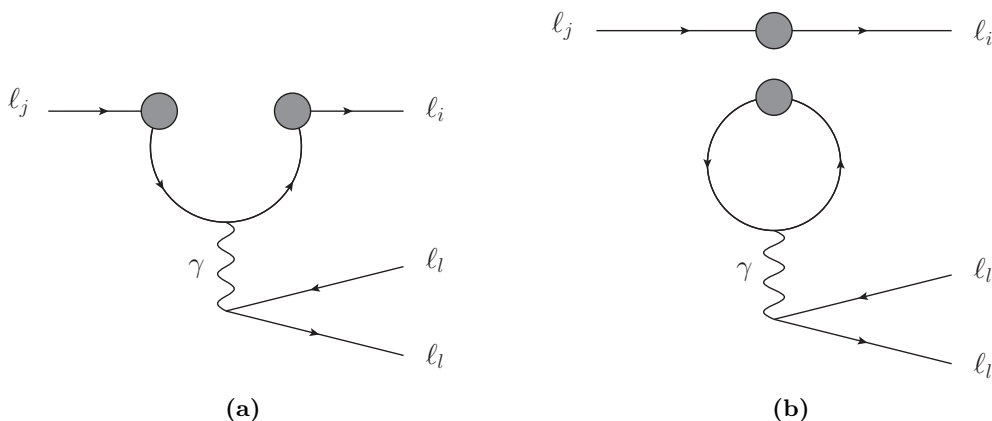
The following operators contained in eq. (3.12)

$$\mathcal{O}_{e\mu,\mu\mu}^{AB}, \quad \mathcal{O}_{e\mu,\tau\tau}^{AB}, \quad \mathcal{O}_{e\tau,\tau\mu}^{AB}, \quad \mathcal{O}_{e\tau,\tau\tau}^{AB}, \quad \mathcal{O}_{\mu\tau,\tau\tau}^{AB}, \tag{B.1}$$

with  $AB = LL, LR, RL, RR$ , do not contribute to the flavour changing processes considered by us at tree level and could thus be considered as *hidden operators*. However, they contribute to these processes through QED penguin diagrams (as depicted in figure 13) [222–224].

In the formal language, these diagrams generate mixing of the operators in eq. (B.1) into operators contributing already at tree-level to flavour changing processes. In this appendix we present the results for this additional effect.

This mixing can be found by means of standard methods [14]. That is by calculating the relevant one-loop anomalous dimensions obtained by inserting the operators in eq. (B.1) into one-loop off-shell photon penguin diagrams [222–224]. As shown in figure 13, there are two possible operator insertions which result in the same contributions to the Wilson coefficients, unless they vanish. In the cases at hand one should note that for the inserted  $LL$  and  $RR$  operators with three electrons or three muons both penguin topologies contribute, which brings in a factor of two for this mixing relative to the remaining operator insertions. However, in the case of  $RL$  and  $LR$  operators, only the insertions into diagram (b) in figure 13 contribute, and this implies that the operator  $\mathcal{O}_{e\tau,\tau\mu}^{V,RL}$  cannot contribute to this mixing.



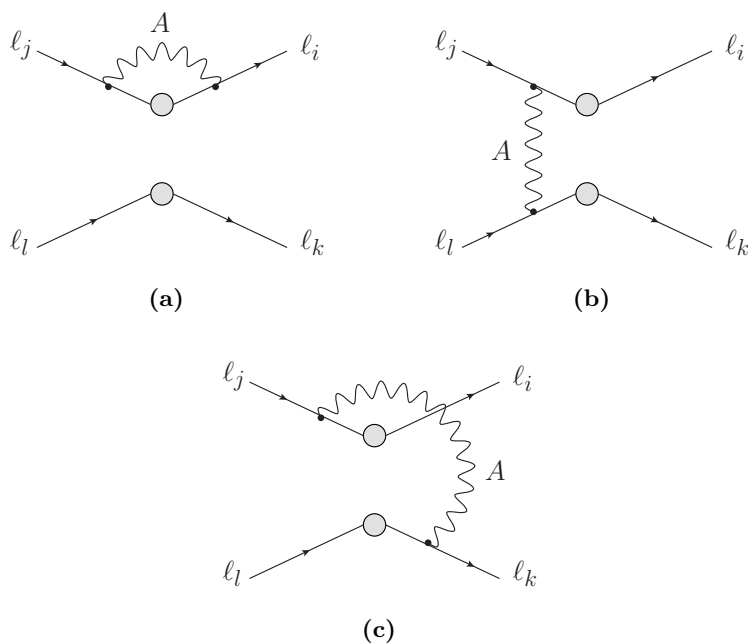
**Figure 13.** Feynman diagrams with two operator insertions corresponding to different Wick contractions in the EFT where the  $Z'$  is integrated out.

The standard renormalization group evolution is then performed from the scale  $M_{Z'}$  down to  $m_\tau$  or  $m_\mu$  for  $\tau$  decays and  $\mu$  decay, respectively. In the latter case one takes into account that the  $\tau$ -lepton is integrated out at  $m_\tau$ , which implies two different logarithms in the final results. Keeping only the leading logarithms, we find the following results for the additional contributions to the Wilson coefficients of the operators in eq. (3.12).

$$\begin{aligned}
 C_{e\mu}^{LL} &= \frac{-e^2}{16\pi^2 M_{Z'}^2} \frac{2}{3} \left( g_{e\mu}^L (2g_{\mu\mu}^L + g_{\mu\mu}^R) \ln \left( \frac{M_{Z'}^2}{m_\mu^2} \right) + (g_{e\tau}^L g_{\tau\mu}^L + g_{e\mu}^L (g_{\tau\tau}^L + g_{\tau\tau}^R)) \ln \left( \frac{M_{Z'}^2}{m_\tau^2} \right) \right), \\
 C_{e\mu}^{RL} &= \frac{-e^2}{16\pi^2 M_{Z'}^2} \frac{2}{3} \left( g_{e\mu}^R (2g_{\mu\mu}^R + g_{\mu\mu}^L) \ln \left( \frac{M_{Z'}^2}{m_\mu^2} \right) + (g_{e\tau}^R g_{\tau\mu}^R + g_{e\mu}^R (g_{\tau\tau}^R + g_{\tau\tau}^L)) \ln \left( \frac{M_{Z'}^2}{m_\tau^2} \right) \right), \\
 C_{e\tau}^{LL} &= \frac{-e^2}{16\pi^2 M_{Z'}^2} \frac{2}{3} g_{e\tau}^L (2g_{\tau\tau}^L + g_{\tau\tau}^R) \ln \left( \frac{M_{Z'}^2}{m_\tau^2} \right), \\
 C_{e\tau}^{RL} &= \frac{-e^2}{16\pi^2 M_{Z'}^2} \frac{2}{3} g_{e\tau}^R (g_{\tau\tau}^L + 2g_{\tau\tau}^R) \ln \left( \frac{M_{Z'}^2}{m_\tau^2} \right), \\
 C_{\mu\tau}^{LL} &= \frac{-e^2}{16\pi^2 M_{Z'}^2} \frac{2}{3} g_{\mu\tau}^L (2g_{\tau\tau}^L + g_{\tau\tau}^R) \ln \left( \frac{M_{Z'}^2}{m_\tau^2} \right), \\
 C_{\mu\tau}^{RL} &= \frac{-e^2}{16\pi^2 M_{Z'}^2} \frac{2}{3} g_{\mu\tau}^R (g_{\tau\tau}^L + 2g_{\tau\tau}^R) \ln \left( \frac{M_{Z'}^2}{m_\tau^2} \right). \tag{B.2}
 \end{aligned}$$

The vectorial nature of the photon implies  $C_{ij}^{LR} = C_{ij}^{LL}$  and  $C_{ij}^{RR} = C_{ij}^{RL}$ . It should be noted that these results apply universally to all operators with  $j = k$  independent of the lepton flavour, as seen in the first term on the r.h.s. in (3.14).

The Wilson coefficients of the operators contributing already at tree-level can also be affected by inserting them in penguin and one-loop current-current operators. But since these are already constrained by tree-level processes and loop-effects turn out to be subleading. Yet, for completeness we present expressions for these effects in the next appendix.



**Figure 14.** 1-loop QED corrections generating mixing among four-lepton operators.

## B.2 Running of visible operators

So far we have only included the contributions from hidden operators mixing into visible operators. But there are also contributions from the mixing of visible operators into visible operators, both through QED penguins and insertions in current-current topologies. The QED penguin diagrams are again given in figure 13, the current-current topologies in figure 14.

The QED penguin contributions imply shifts in the coefficients of the operators in (B.2). We find

$$\begin{aligned}
 \Delta C_{e\mu}^{LL} &= \frac{-e^2}{16\pi^2 M_{Z'}^2} \frac{2}{3} g_{e\mu}^L (2g_{ee}^L + g_{ee}^R) \ln\left(\frac{M_{Z'}^2}{m_\mu^2}\right), \\
 \Delta C_{e\mu}^{RL} &= \frac{-e^2}{16\pi^2 M_{Z'}^2} \frac{2}{3} g_{e\mu}^R (g_{ee}^L + 2g_{ee}^R) \ln\left(\frac{M_{Z'}^2}{m_\mu^2}\right), \\
 \Delta C_{e\tau}^{LL} &= \frac{-e^2}{16\pi^2 M_{Z'}^2} \frac{2}{3} \left( g_{e\tau}^L (2g_{ee}^L + g_{ee}^R) + (g_{e\mu}^L g_{\mu\tau}^L + g_{e\tau}^L (g_{\mu\mu}^L + g_{\mu\mu}^R)) \right) \ln\left(\frac{M_{Z'}^2}{m_\tau^2}\right), \\
 \Delta C_{e\tau}^{RL} &= \frac{-e^2}{16\pi^2 M_{Z'}^2} \frac{2}{3} \left( g_{e\tau}^R (2g_{ee}^R + g_{ee}^L) + (g_{e\mu}^R g_{\mu\tau}^R + g_{e\tau}^R (g_{\mu\mu}^R + g_{\mu\mu}^L)) \right) \ln\left(\frac{M_{Z'}^2}{m_\tau^2}\right), \\
 \Delta C_{\mu\tau}^{LL} &= \frac{-e^2}{16\pi^2 M_{Z'}^2} \frac{2}{3} \left( g_{\mu\tau}^L (2g_{\mu\mu}^L + g_{\mu\mu}^R) + (g_{\mu e}^L g_{e\tau}^L + g_{\mu\tau}^L (g_{ee}^L + g_{ee}^R)) \right) \ln\left(\frac{M_{Z'}^2}{m_\tau^2}\right), \\
 \Delta C_{\mu\tau}^{RL} &= \frac{-e^2}{16\pi^2 M_{Z'}^2} \frac{2}{3} \left( g_{\mu\tau}^R (2g_{\mu\mu}^R + g_{\mu\mu}^L) + (g_{\mu e}^R g_{e\tau}^R + g_{\mu\tau}^R (g_{ee}^R + g_{ee}^L)) \right) \ln\left(\frac{M_{Z'}^2}{m_\tau^2}\right),
 \end{aligned} \tag{B.3}$$

and for  $RR$  and  $LR$  with  $L$  and  $R$  interchanged.

On the other hand, the current-current contributions modify the coefficients  $X_{ij,kl}^{AB}$  in (3.14). We find

$$\Delta X_{ij,kl}^{LL} = \frac{-e^2}{16\pi^2 M_{Z'}^2} 6 g_{ij}^L g_{kl}^L \ln\left(\frac{M_{Z'}^2}{m_\tau^2}\right), \quad (\text{B.4})$$

$$\Delta X_{ij,kl}^{RL} = \frac{e^2}{16\pi^2 M_{Z'}^2} 6 g_{ij}^R g_{kl}^L \ln\left(\frac{M_{Z'}^2}{m_\tau^2}\right), \quad (\text{B.5})$$

where  $m_r = m_\tau$  and  $m_r = m_\mu$  for  $\tau$  and  $\mu$  decays, respectively. For  $RR$ - and  $LR$ -coefficients,  $L$  and  $R$  should be interchanged.

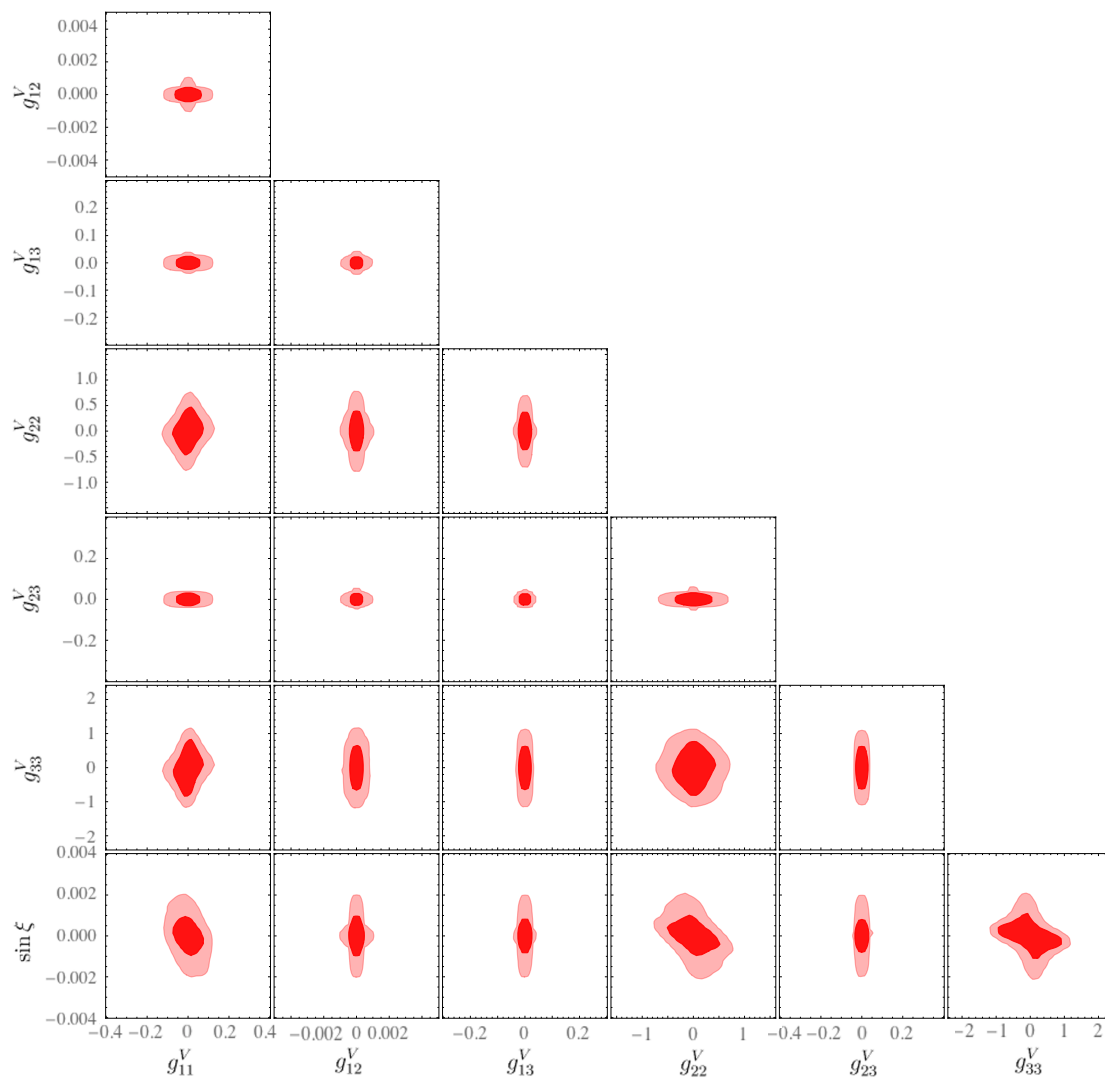
The visible operators also lead to contributions to  $\mu \rightarrow e$  conversion. The resulting shifts in the Wilson coefficients of eq. (3.22) are given by

$$\begin{aligned} \Delta C_{e\mu,qq}^{LL} &= \frac{e^2 Q_q}{16\pi^2} \frac{2}{3} \frac{1}{M_{Z'}^2} g_{e\mu}^L (2g_{ee}^L + g_{ee}^R) \log\left(\frac{M_{Z'}^2}{m_\mu^2}\right), \\ \Delta C_{e\mu,qq}^{RL} &= \frac{e^2 Q_q}{16\pi^2} \frac{2}{3} \frac{1}{M_{Z'}^2} g_{e\mu}^R (2g_{ee}^R + g_{ee}^L) \log\left(\frac{M_{Z'}^2}{m_\mu^2}\right). \end{aligned} \quad (\text{B.6})$$

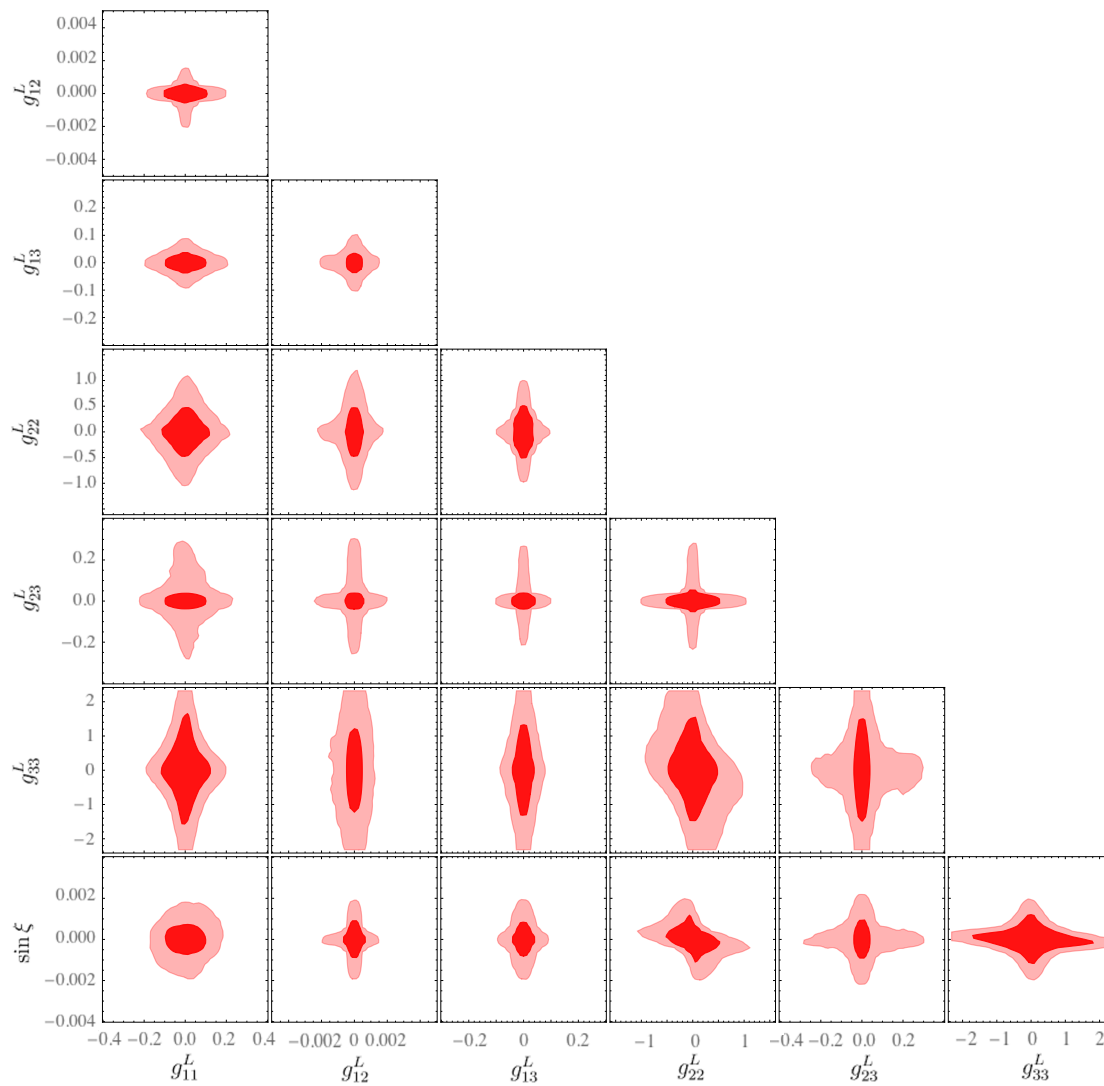
where, as in eq. (3.22),  $Q_q$  stands for the electric charge of the quarks.

### C General LFV scenarios

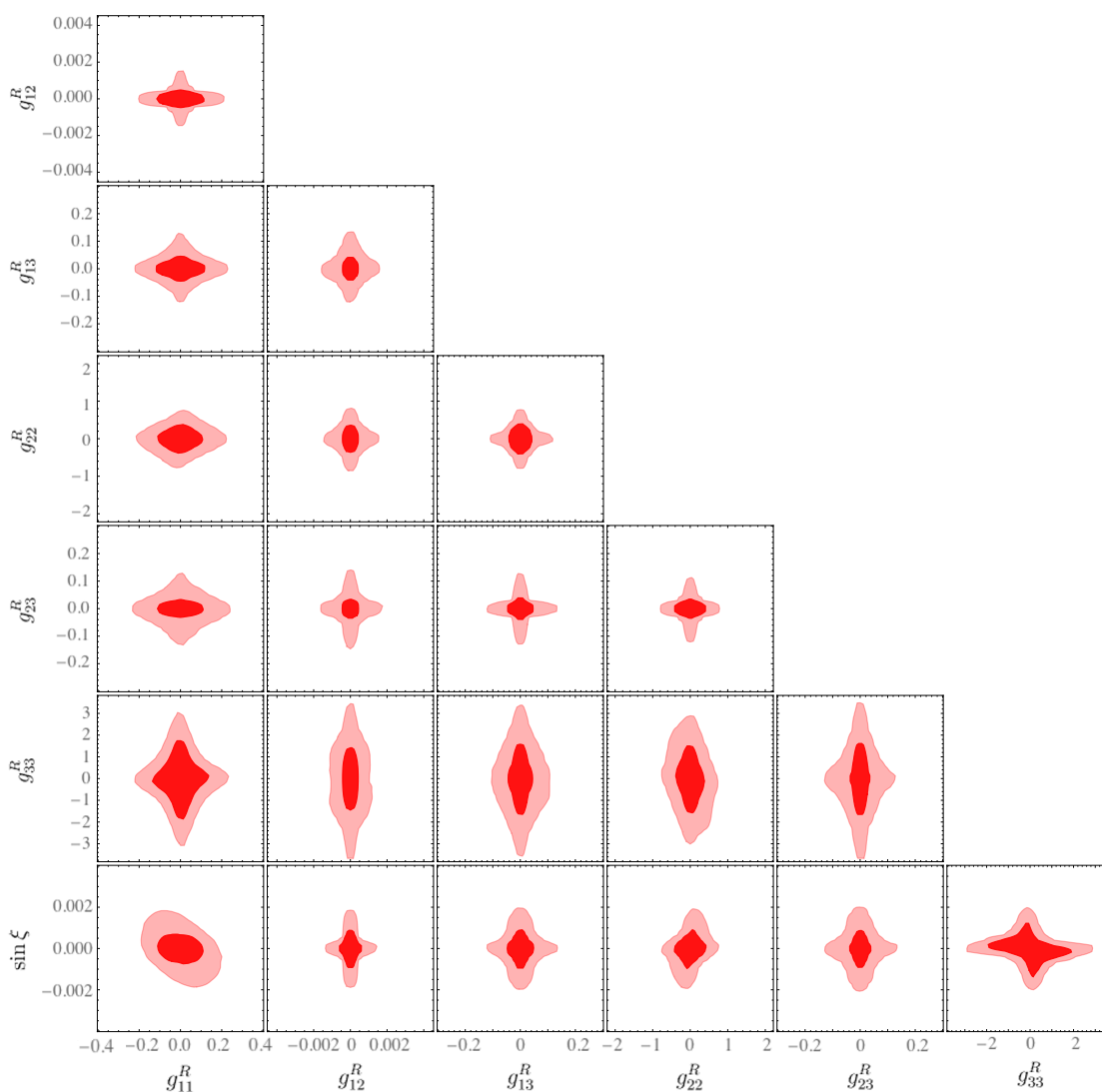
Here we study the general cases 4, 5 and 6 from section 4.3. Case 4, where  $g_{ij}^L = g_{ij}^R = g_{ij}^V$ , is shown in figure 15. Similarly to previous cases in sections 4.1 and 4.2, all couplings involving the first generation are strongly constrained. Also the flavour violating ones are strictly bounded while the flavour conserving ones involving muons and taus can be of order unity. Similarly, we present the results for the cases 5 and 6, where only  $g_{ij}^L$  and  $g_{ij}^R$  are non-zero, as shown in figure 16 and figure 17, respectively. Evidently, in these cases the couplings are allowed to be larger than for the case of vectorial couplings.



**Figure 15.** 68% and 95% CL regions for  $g_{ij}^V = g_{ij}^L = g_{ij}^R$  and  $\sin \xi$ .



**Figure 16.** 68% and 95% CL regions for the LFV case where only  $g_{ij}^L$  is non-zero.



**Figure 17.** 68% and 95% CL regions for the LFV case where only  $g_{ij}^R$  is non-zero.

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## References

- [1] ATLAS collaboration, *Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC*, *Phys. Lett. B* **716** (2012) 1 [[arXiv:1207.7214](https://arxiv.org/abs/1207.7214)] [[INSPIRE](https://inspirehep.net/literature/1207721)].
- [2] CMS collaboration, *Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC*, *Phys. Lett. B* **716** (2012) 30 [[arXiv:1207.7235](https://arxiv.org/abs/1207.7235)] [[INSPIRE](https://inspirehep.net/literature/1207735)].

- [3] ATLAS collaboration, *Search for new phenomena in dijet events using  $37 \text{ fb}^{-1}$  of pp collision data collected at  $\sqrt{s} = 13 \text{ TeV}$  with the ATLAS detector*, *Phys. Rev. D* **96** (2017) 052004 [[arXiv:1703.09127](#)] [[INSPIRE](#)].
- [4] CMS collaboration, *Search for narrow and broad dijet resonances in proton-proton collisions at  $\sqrt{s} = 13 \text{ TeV}$  and constraints on dark matter mediators and other new particles*, *JHEP* **08** (2018) 130 [[arXiv:1806.00843](#)] [[INSPIRE](#)].
- [5] ATLAS collaboration, *Search for high-mass dilepton resonances using  $139 \text{ fb}^{-1}$  of pp collision data collected at  $\sqrt{s} = 13 \text{ TeV}$  with the ATLAS detector*, *Phys. Lett. B* **796** (2019) 68 [[arXiv:1903.06248](#)] [[INSPIRE](#)].
- [6] CMS collaboration, *Search for a narrow resonance in high-mass dilepton final states in proton-proton collisions using  $140 \text{ fb}^{-1}$  of data at  $\sqrt{s} = 13 \text{ TeV}$* , *CMS-PAS-EXO-19-019* (2019).
- [7] CMS collaboration, *Search for resonant and nonresonant new phenomena in high-mass dilepton final states at  $\sqrt{s} = 13 \text{ TeV}$* , [arXiv:2103.02708](#) [[INSPIRE](#)].
- [8] P. Langacker and M. Plümacher, *Flavor changing effects in theories with a heavy  $Z'$  boson with family nonuniversal couplings*, *Phys. Rev. D* **62** (2000) 013006 [[hep-ph/0001204](#)] [[INSPIRE](#)].
- [9] P. Langacker, *The Physics of Heavy  $Z'$  Gauge Bosons*, *Rev. Mod. Phys.* **81** (2009) 1199 [[arXiv:0801.1345](#)] [[INSPIRE](#)].
- [10] A.J. Buras, F. De Fazio and J. Girrbach, *The Anatomy of  $Z'$  and  $Z$  with Flavour Changing Neutral Currents in the Flavour Precision Era*, *JHEP* **02** (2013) 116 [[arXiv:1211.1896](#)] [[INSPIRE](#)].
- [11] W. Altmannshofer and I. Yavin, *Predictions for lepton flavor universality violation in rare  $B$  decays in models with gauged  $L_\mu - L_\tau$* , *Phys. Rev. D* **92** (2015) 075022 [[arXiv:1508.07009](#)] [[INSPIRE](#)].
- [12] A. Crivellin, L. Hofer, J. Matias, U. Nierste, S. Pokorski and J. Rosiek, *Lepton-flavour violating  $B$  decays in generic  $Z'$  models*, *Phys. Rev. D* **92** (2015) 054013 [[arXiv:1504.07928](#)] [[INSPIRE](#)].
- [13] B.C. Allanach, J.M. Butterworth and T. Corbett, *Collider constraints on  $Z'$  models for neutral current  $B$ -anomalies*, *JHEP* **08** (2019) 106 [[arXiv:1904.10954](#)] [[INSPIRE](#)].
- [14] A. Buras, *Gauge Theory of Weak Decays*, Cambridge University Press, Cambridge U.K. (2020) [[INSPIRE](#)].
- [15] ALEPH, DELPHI, L3, OPAL and LEP ELECTROWEAK collaborations, *Electroweak Measurements in Electron-Positron Collisions at  $W$ -Boson-Pair Energies at LEP*, *Phys. Rept.* **532** (2013) 119 [[arXiv:1302.3415](#)] [[INSPIRE](#)].
- [16] F. del Aguila, M. Chala, J. Santiago and Y. Yamamoto, *Collider limits on leptophilic interactions*, *JHEP* **03** (2015) 059 [[arXiv:1411.7394](#)] [[INSPIRE](#)].
- [17] MUON G-2 collaboration, *Final Report of the Muon  $E821$  Anomalous Magnetic Moment Measurement at BNL*, *Phys. Rev. D* **73** (2006) 072003 [[hep-ex/0602035](#)] [[INSPIRE](#)].
- [18] P.J. Mohr, D.B. Newell and B.N. Taylor, *CODATA Recommended Values of the Fundamental Physical Constants: 2014*, *Rev. Mod. Phys.* **88** (2016) 035009 [[arXiv:1507.07956](#)] [[INSPIRE](#)].



- [19] MUON G-2 collaboration, *Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm*, *Phys. Rev. Lett.* **126** (2021) 141801 [[arXiv:2104.03281](#)] [[INSPIRE](#)].
- [20] MUON G-2 collaboration, *Magnetic-field measurement and analysis for the Muon  $g - 2$  Experiment at Fermilab*, *Phys. Rev. A* **103** (2021) 042208 [[arXiv:2104.03201](#)] [[INSPIRE](#)].
- [21] MUON G-2 collaboration, *Measurement of the anomalous precession frequency of the muon in the Fermilab Muon  $g - 2$  Experiment*, *Phys. Rev. D* **103** (2021) 072002 [[arXiv:2104.03247](#)] [[INSPIRE](#)].
- [22] MUON G-2 collaboration, *Beam dynamics corrections to the Run-1 measurement of the muon anomalous magnetic moment at Fermilab*, *Phys. Rev. Accel. Beams* **24** (2021) 044002 [[arXiv:2104.03240](#)] [[INSPIRE](#)].
- [23] T. Aoyama et al., *The anomalous magnetic moment of the muon in the Standard Model*, *Phys. Rept.* **887** (2020) 1 [[arXiv:2006.04822](#)] [[INSPIRE](#)].
- [24] R. Foot, X.G. He, H. Lew and R.R. Volkas, *Model for a light  $Z'$  boson*, *Phys. Rev. D* **50** (1994) 4571 [[hep-ph/9401250](#)] [[INSPIRE](#)].
- [25] S.N. Gninenko and N.V. Krasnikov, *The Muon anomalous magnetic moment and a new light gauge boson*, *Phys. Lett. B* **513** (2001) 119 [[hep-ph/0102222](#)] [[INSPIRE](#)].
- [26] B. Murakami, *The Impact of lepton flavor violating  $Z'$  bosons on muon  $g - 2$  and other muon observables*, *Phys. Rev. D* **65** (2002) 055003 [[hep-ph/0110095](#)] [[INSPIRE](#)].
- [27] S. Baek, N.G. Deshpande, X.G. He and P. Ko, *Muon anomalous  $g - 2$  and gauged  $L_\mu - L_\tau$  models*, *Phys. Rev. D* **64** (2001) 055006 [[hep-ph/0104141](#)] [[INSPIRE](#)].
- [28] E. Ma, D.P. Roy and S. Roy, *Gauged  $L_\mu - L_\tau$  with large muon anomalous magnetic moment and the bimaximal mixing of neutrinos*, *Phys. Lett. B* **525** (2002) 101 [[hep-ph/0110146](#)] [[INSPIRE](#)].
- [29] M. Pospelov, *Secluded U(1) below the weak scale*, *Phys. Rev. D* **80** (2009) 095002 [[arXiv:0811.1030](#)] [[INSPIRE](#)].
- [30] J. Heck and W. Rodejohann, *Gauged  $L_\mu - L_\tau$  Symmetry at the Electroweak Scale*, *Phys. Rev. D* **84** (2011) 075007 [[arXiv:1107.5238](#)] [[INSPIRE](#)].
- [31] H. Davoudiasl, H.-S. Lee and W.J. Marciano, *Dark Side of Higgs Diphoton Decays and Muon  $g-2$* , *Phys. Rev. D* **86** (2012) 095009 [[arXiv:1208.2973](#)] [[INSPIRE](#)].
- [32] C.D. Carone, *Flavor-Nonuniversal Dark Gauge Bosons and the Muon  $g-2$* , *Phys. Lett. B* **721** (2013) 118 [[arXiv:1301.2027](#)] [[INSPIRE](#)].
- [33] K. Harigaya, T. Igari, M.M. Nojiri, M. Takeuchi and K. Tobe, *Muon  $g - 2$  and LHC phenomenology in the  $L_\mu - L_\tau$  gauge symmetric model*, *JHEP* **03** (2014) 105 [[arXiv:1311.0870](#)] [[INSPIRE](#)].
- [34] W. Altmannshofer, S. Gori, M. Pospelov and I. Yavin, *Quark flavor transitions in  $L_\mu - L_\tau$  models*, *Phys. Rev. D* **89** (2014) 095033 [[arXiv:1403.1269](#)] [[INSPIRE](#)].
- [35] G. Tomar and S. Mohanty, *Muon anomalous magnetic moment and positron excess at AMS-02 in a gauged horizontal symmetric model*, *JHEP* **11** (2014) 133 [[arXiv:1403.6301](#)] [[INSPIRE](#)].
- [36] W. Altmannshofer, S. Gori, M. Pospelov and I. Yavin, *Neutrino Trident Production: A Powerful Probe of New Physics with Neutrino Beams*, *Phys. Rev. Lett.* **113** (2014) 091801 [[arXiv:1406.2332](#)] [[INSPIRE](#)].

- [37] H.-S. Lee, *Muon  $g - 2$  anomaly and dark leptonic gauge boson*, *Phys. Rev. D* **90** (2014) 091702 [[arXiv:1408.4256](#)] [[INSPIRE](#)].
- [38] B. Allanach, F.S. Queiroz, A. Strumia and S. Sun,  *$Z'$  models for the LHCb and  $g - 2$  muon anomalies*, *Phys. Rev. D* **93** (2016) 055045 [*Erratum ibid.* **95** (2017) 119902] [[arXiv:1511.07447](#)] [[INSPIRE](#)].
- [39] J. Heeck, *Lepton flavor violation with light vector bosons*, *Phys. Lett. B* **758** (2016) 101 [[arXiv:1602.03810](#)] [[INSPIRE](#)].
- [40] S. Patra, S. Rao, N. Sahoo and N. Sahu, *Gauged  $U(1)_{L_\mu - L_\tau}$  model in light of muon  $g - 2$  anomaly, neutrino mass and dark matter phenomenology*, *Nucl. Phys. B* **917** (2017) 317 [[arXiv:1607.04046](#)] [[INSPIRE](#)].
- [41] W. Altmannshofer, C.-Y. Chen, P.S. Bhupal Dev and A. Soni, *Lepton flavor violating  $Z'$  explanation of the muon anomalous magnetic moment*, *Phys. Lett. B* **762** (2016) 389 [[arXiv:1607.06832](#)] [[INSPIRE](#)].
- [42] S. Iguro, K.A. Mohan and C.P. Yuan, *Detecting a  $\mu\tau$ -philic  $Z'$  boson via photon initiated processes at the LHC*, *Phys. Rev. D* **101** (2020) 075011 [[arXiv:2001.09079](#)] [[INSPIRE](#)].
- [43] S. Iguro, Y. Omura and M. Takeuchi, *Probing  $\mu\tau$  flavor-violating solutions for the muon  $g - 2$  anomaly at Belle II*, *JHEP* **09** (2020) 144 [[arXiv:2002.12728](#)] [[INSPIRE](#)].
- [44] X.G. He, G.C. Joshi, H. Lew and R.R. Volkas, *New  $Z'$  phenomenology*, *Phys. Rev. D* **43** (1991) 22 [[INSPIRE](#)].
- [45] R. Foot, *New Physics From Electric Charge Quantization?*, *Mod. Phys. Lett. A* **6** (1991) 527 [[INSPIRE](#)].
- [46] X.-G. He, G.C. Joshi, H. Lew and R.R. Volkas, *Simplest  $Z'$  model*, *Phys. Rev. D* **44** (1991) 2118 [[INSPIRE](#)].
- [47] P. Binetruy, S. Lavignac, S.T. Petcov and P. Ramond, *Quasidegenerate neutrinos from an Abelian family symmetry*, *Nucl. Phys. B* **496** (1997) 3 [[hep-ph/9610481](#)] [[INSPIRE](#)].
- [48] N.F. Bell and R.R. Volkas, *Bottom up model for maximal  $\nu_\tau - \nu_\tau$  mixing*, *Phys. Rev. D* **63** (2001) 013006 [[hep-ph/0008177](#)] [[INSPIRE](#)].
- [49] S. Choubey and W. Rodejohann, *A Flavor symmetry for quasi-degenerate neutrinos:  $L_\mu - L_\tau$* , *Eur. Phys. J. C* **40** (2005) 259 [[hep-ph/0411190](#)] [[INSPIRE](#)].
- [50] G. Dutta, A.S. Joshipura and K.B. Vijaykumar, *Leptonic flavor violations in the presence of an extra  $Z$* , *Phys. Rev. D* **50** (1994) 2109 [[hep-ph/9405292](#)] [[INSPIRE](#)].
- [51] A. Crivellin, G. D'Ambrosio and J. Heeck, *Explaining  $h \rightarrow \mu^\pm \tau^\mp$ ,  $B \rightarrow K^* \mu^+ \mu^-$  and  $B \rightarrow K \mu^+ \mu^- / B \rightarrow K e^+ e^-$  in a two-Higgs-doublet model with gauged  $L_\mu - L_\tau$* , *Phys. Rev. Lett.* **114** (2015) 151801 [[arXiv:1501.00993](#)] [[INSPIRE](#)].
- [52] J. Heeck, M. Holthausen, W. Rodejohann and Y. Shimizu, *Higgs  $\rightarrow \mu\tau$  in Abelian and non-Abelian flavor symmetry models*, *Nucl. Phys. B* **896** (2015) 281 [[arXiv:1412.3671](#)] [[INSPIRE](#)].
- [53] W. Altmannshofer, M. Carena and A. Crivellin,  *$L_\mu - L_\tau$  theory of Higgs flavor violation and  $(g - 2)_\mu$* , *Phys. Rev. D* **94** (2016) 095026 [[arXiv:1604.08221](#)] [[INSPIRE](#)].
- [54] BABAR collaboration, *Measurements of Charged Current Lepton Universality and  $|V_{us}|$  using Tau Lepton Decays to  $e^- \bar{\nu}_e \nu_\tau$ ,  $\mu^- \bar{\nu}_\mu \nu_\tau$ ,  $\pi^- \nu_\tau$ , and  $K^- \nu_\tau$* , *Phys. Rev. Lett.* **105** (2010) 051602 [[arXiv:0912.0242](#)] [[INSPIRE](#)].

- [55] HFLAV collaboration, *Averages of  $b$ -hadron,  $c$ -hadron, and  $\tau$ -lepton properties as of 2018*, *Eur. Phys. J. C* **81** (2021) 226 [[arXiv:1909.12524](#)] [[INSPIRE](#)].
- [56] B. Belfatto, R. Beradze and Z. Berezhiani, *The CKM unitarity problem: A trace of new physics at the TeV scale?*, *Eur. Phys. J. C* **80** (2020) 149 [[arXiv:1906.02714](#)] [[INSPIRE](#)].
- [57] Y. Grossman, E. Passemar and S. Schacht, *On the Statistical Treatment of the Cabibbo Angle Anomaly*, *JHEP* **07** (2020) 068 [[arXiv:1911.07821](#)] [[INSPIRE](#)].
- [58] K. Shiells, P.G. Blunden and W. Melnitchouk, *Electroweak axial structure functions and improved extraction of the  $V_{ud}$  CKM matrix element*, [arXiv:2012.01580](#) [[INSPIRE](#)].
- [59] C.-Y. Seng, X. Feng, M. Gorchtein and L.-C. Jin, *Joint lattice QCD–dispersion theory analysis confirms the quark-mixing top-row unitarity deficit*, *Phys. Rev. D* **101** (2020) 111301 [[arXiv:2003.11264](#)] [[INSPIRE](#)].
- [60] A.M. Coutinho, A. Crivellin and C.A. Manzari, *Global Fit to Modified Neutrino Couplings and the Cabibbo-Angle Anomaly*, *Phys. Rev. Lett.* **125** (2020) 071802 [[arXiv:1912.08823](#)] [[INSPIRE](#)].
- [61] A. Crivellin and M. Hoferichter,  *$\beta$  Decays as Sensitive Probes of Lepton Flavor Universality*, *Phys. Rev. Lett.* **125** (2020) 111801 [[arXiv:2002.07184](#)] [[INSPIRE](#)].
- [62] C.A. Manzari, A.M. Coutinho and A. Crivellin, *Modified lepton couplings and the Cabibbo-angle anomaly*, *PoS(LHCP2020)242* (2021) [[arXiv:2009.03877](#)] [[INSPIRE](#)].
- [63] B. Capdevila, A. Crivellin, C.A. Manzari and M. Montull, *Explaining  $b \rightarrow s\ell^+\ell^-$  and the Cabibbo angle anomaly with a vector triplet*, *Phys. Rev. D* **103** (2021) 015032 [[arXiv:2005.13542](#)] [[INSPIRE](#)].
- [64] A. Crivellin, F. Kirk, C.A. Manzari and M. Montull, *Global Electroweak Fit and Vector-Like Leptons in Light of the Cabibbo Angle Anomaly*, *JHEP* **12** (2020) 166 [[arXiv:2008.01113](#)] [[INSPIRE](#)].
- [65] M. Kirk, *Cabibbo anomaly versus electroweak precision tests: An exploration of extensions of the Standard Model*, *Phys. Rev. D* **103** (2021) 035004 [[arXiv:2008.03261](#)] [[INSPIRE](#)].
- [66] A.K. Alok, A. Dighe, S. Gangal and J. Kumar, *The role of non-universal  $Z$  couplings in explaining the  $V_{us}$  anomaly*, [arXiv:2010.12009](#) [[INSPIRE](#)].
- [67] A. Crivellin, C.A. Manzari, M. Alguero and J. Matias, *Combined Explanation of the  $Z \rightarrow b\bar{b}$  Forward-Backward Asymmetry, the Cabibbo Angle Anomaly,  $\tau \rightarrow \mu\nu$  and  $b \rightarrow s\ell^+\ell^-$  Data*, [arXiv:2010.14504](#) [[INSPIRE](#)].
- [68] A. Crivellin, F. Kirk, C.A. Manzari and L. Panizzi, *Searching for lepton flavor universality violation and collider signals from a singly charged scalar singlet*, *Phys. Rev. D* **103** (2021) 073002 [[arXiv:2012.09845](#)] [[INSPIRE](#)].
- [69] A. Crivellin and L. Schnell, *Combined Constraints on First Generation Leptoquarks*, [arXiv:2104.06417](#) [[INSPIRE](#)].
- [70] A. Crivellin, M. Hoferichter and C.A. Manzari, *The Fermi constant from muon decay versus electroweak fits and CKM unitarity*, [arXiv:2102.02825](#) [[INSPIRE](#)].
- [71] A. Crivellin, C.A. Manzari and M. Montull, *Correlating Non-Resonant Di-Electron Searches at the LHC to the Cabibbo-Angle Anomaly and Lepton Flavour Universality Violation*, [arXiv:2103.12003](#) [[INSPIRE](#)].

- [72] A.J. Buras and J. Girrbach, *Left-handed  $Z'$  and  $Z$  FCNC quark couplings facing new  $b \rightarrow s\mu^+\mu^-$  data*, *JHEP* **12** (2013) 009 [[arXiv:1309.2466](#)] [[INSPIRE](#)].
- [73] R. Gauld, F. Goertz and U. Haisch, *On minimal  $Z'$  explanations of the  $B \rightarrow K^*\mu^+\mu^-$  anomaly*, *Phys. Rev. D* **89** (2014) 015005 [[arXiv:1308.1959](#)] [[INSPIRE](#)].
- [74] R. Gauld, F. Goertz and U. Haisch, *An explicit  $Z'$ -boson explanation of the  $B \rightarrow K^*\mu^+\mu^-$  anomaly*, *JHEP* **01** (2014) 069 [[arXiv:1310.1082](#)] [[INSPIRE](#)].
- [75] A. Crivellin, G. D'Ambrosio and J. Heeck, *Addressing the LHC flavor anomalies with horizontal gauge symmetries*, *Phys. Rev. D* **91** (2015) 075006 [[arXiv:1503.03477](#)] [[INSPIRE](#)].
- [76] C. Niehoff, P. Stangl and D.M. Straub, *Violation of lepton flavour universality in composite Higgs models*, *Phys. Lett. B* **747** (2015) 182 [[arXiv:1503.03865](#)] [[INSPIRE](#)].
- [77] D. Aristizabal Sierra, F. Staub and A. Vicente, *Shedding light on the  $b \rightarrow s$  anomalies with a dark sector*, *Phys. Rev. D* **92** (2015) 015001 [[arXiv:1503.06077](#)] [[INSPIRE](#)].
- [78] A. Carmona and F. Goertz, *Lepton Flavor and Nonuniversality from Minimal Composite Higgs Setups*, *Phys. Rev. Lett.* **116** (2016) 251801 [[arXiv:1510.07658](#)] [[INSPIRE](#)].
- [79] A. Falkowski, M. Nardecchia and R. Ziegler, *Lepton Flavor Non-Universality in B-meson Decays from a  $U(2)$  Flavor Model*, *JHEP* **11** (2015) 173 [[arXiv:1509.01249](#)] [[INSPIRE](#)].
- [80] A. Celis, W.-Z. Feng and D. Lüüst, *Stringy explanation of  $b \rightarrow s\ell^+\ell^-$  anomalies*, *JHEP* **02** (2016) 007 [[arXiv:1512.02218](#)] [[INSPIRE](#)].
- [81] A. Celis, J. Fuentes-Martin, M. Jung and H. Serodio, *Family nonuniversal  $Z'$  models with protected flavor-changing interactions*, *Phys. Rev. D* **92** (2015) 015007 [[arXiv:1505.03079](#)] [[INSPIRE](#)].
- [82] S.M. Boucenna, A. Celis, J. Fuentes-Martin, A. Vicente and J. Virto, *Non-abelian gauge extensions for B-decay anomalies*, *Phys. Lett. B* **760** (2016) 214 [[arXiv:1604.03088](#)] [[INSPIRE](#)].
- [83] S.M. Boucenna, A. Celis, J. Fuentes-Martin, A. Vicente and J. Virto, *Phenomenology of an  $SU(2) \times SU(2) \times U(1)$  model with lepton-flavour non-universality*, *JHEP* **12** (2016) 059 [[arXiv:1608.01349](#)] [[INSPIRE](#)].
- [84] A. Crivellin, J. Fuentes-Martin, A. Greljo and G. Isidori, *Lepton Flavor Non-Universality in B decays from Dynamical Yukawas*, *Phys. Lett. B* **766** (2017) 77 [[arXiv:1611.02703](#)] [[INSPIRE](#)].
- [85] I. Garcia Garcia, *LHCb anomalies from a natural perspective*, *JHEP* **03** (2017) 040 [[arXiv:1611.03507](#)] [[INSPIRE](#)].
- [86] G. Faisel and J. Tandean, *Connecting  $b \rightarrow s\ell\bar{\ell}$  anomalies to enhanced rare nonleptonic  $\bar{B}_s^0$  decays in  $Z'$  model*, *JHEP* **02** (2018) 074 [[arXiv:1710.11102](#)] [[INSPIRE](#)].
- [87] S.F. King, *Flavourful  $Z'$  models for  $R_{K^{(*)}}$* , *JHEP* **08** (2017) 019 [[arXiv:1706.06100](#)] [[INSPIRE](#)].
- [88] C.-W. Chiang, X.-G. He, J. Tandean and X.-B. Yuan,  *$R_{K^{(*)}}$  and related  $b \rightarrow s\ell\bar{\ell}$  anomalies in minimal flavor violation framework with  $Z'$  boson*, *Phys. Rev. D* **96** (2017) 115022 [[arXiv:1706.02696](#)] [[INSPIRE](#)].
- [89] S. Di Chiara et al., *Minimal flavor-changing  $Z'$  models and muon  $g - 2$  after the  $R_{K^*}$  measurement*, *Nucl. Phys. B* **923** (2017) 245 [[arXiv:1704.06200](#)] [[INSPIRE](#)].

- [90] P. Ko, Y. Omura, Y. Shigekami and C. Yu, *LHCb anomaly and B physics in flavored  $Z'$  models with flavored Higgs doublets*, *Phys. Rev. D* **95** (2017) 115040 [[arXiv:1702.08666](#)] [[INSPIRE](#)].
- [91] F. Sannino, P. Stangl, D.M. Straub and A.E. Thomsen, *Flavor Physics and Flavor Anomalies in Minimal Fundamental Partial Compositeness*, *Phys. Rev. D* **97** (2018) 115046 [[arXiv:1712.07646](#)] [[INSPIRE](#)].
- [92] S. Raby and A. Trautner, *Vectorlike chiral fourth family to explain muon anomalies*, *Phys. Rev. D* **97** (2018) 095006 [[arXiv:1712.09360](#)] [[INSPIRE](#)].
- [93] A. Falkowski, S.F. King, E. Perdomo and M. Pierre, *Flavourful  $Z'$  portal for vector-like neutrino Dark Matter and  $R_{K^{(*)}}$* , *JHEP* **08** (2018) 061 [[arXiv:1803.04430](#)] [[INSPIRE](#)].
- [94] R.H. Benavides, L. Muñoz, W.A. Ponce, O. Rodríguez and E. Rojas, *Minimal  $Z'$  models for flavor anomalies*, *J. Phys. G* **47** (2020) 075003 [[arXiv:1812.05077](#)] [[INSPIRE](#)].
- [95] P. Maji, P. Nayek and S. Sahoo, *Implication of family non-universal  $Z'$  model to rare exclusive  $b \rightarrow s(\bar{l}l, \nu\bar{\nu})$  transitions*, *PTEP* **2019** (2019) 033B06 [[arXiv:1811.03869](#)] [[INSPIRE](#)].
- [96] S. Singirala, S. Sahoo and R. Mohanta, *Exploring dark matter, neutrino mass and  $R_{K^{(*)},\phi}$  anomalies in  $L_\mu - L_\tau$  model*, *Phys. Rev. D* **99** (2019) 035042 [[arXiv:1809.03213](#)] [[INSPIRE](#)].
- [97] D. Guadagnoli, M. Reboud and O. Sumensari, *A gauged horizontal SU(2) symmetry and  $R_{K^{(*)}}$* , *JHEP* **11** (2018) 163 [[arXiv:1807.03285](#)] [[INSPIRE](#)].
- [98] B.C. Allanach and J. Davighi, *Third family hypercharge model for  $R_{K^{(*)}}$  and aspects of the fermion mass problem*, *JHEP* **12** (2018) 075 [[arXiv:1809.01158](#)] [[INSPIRE](#)].
- [99] M. Kohda, T. Modak and A. Soffer, *Identifying a  $Z'$  behind  $b \rightarrow s\bar{l}l$  anomalies at the LHC*, *Phys. Rev. D* **97** (2018) 115019 [[arXiv:1803.07492](#)] [[INSPIRE](#)].
- [100] S.F. King,  *$R_{K^{(*)}}$  and the origin of Yukawa couplings*, *JHEP* **09** (2018) 069 [[arXiv:1806.06780](#)] [[INSPIRE](#)].
- [101] G.H. Duan, X. Fan, M. Frank, C. Han and J.M. Yang, *A minimal U(1)' extension of MSSM in light of the B decay anomaly*, *Phys. Lett. B* **789** (2019) 54 [[arXiv:1808.04116](#)] [[INSPIRE](#)].
- [102] P. Rocha-Moran and A. Vicente, *Lepton flavor violation in a  $Z'$  model for the  $b \rightarrow s$  anomalies*, *Phys. Rev. D* **99** (2019) 035016 [[arXiv:1810.02135](#)] [[INSPIRE](#)].
- [103] S. Dwivedi, D. Kumar Ghosh, A. Falkowski and N. Ghosh, *Associated  $Z'$  production in the flavourful U(1) scenario for  $R_{K^{(*)}}$* , *Eur. Phys. J. C* **80** (2020) 263 [[arXiv:1908.03031](#)] [[INSPIRE](#)].
- [104] P. Foldenauer, *Phenomenology of Extra Abelian Gauge Symmetries*, Ph.D. Thesis, Heidelberg University, Heidelberg Germany (2019).
- [105] P. Ko, T. Nomura and C. Yu,  *$b \rightarrow s\mu^+\mu^-$  anomalies and related phenomenology in  $U(1)_{B_3-x_\mu L_\mu-x_\tau L_\tau}$  flavor gauge models*, *JHEP* **04** (2019) 102 [[arXiv:1902.06107](#)] [[INSPIRE](#)].
- [106] B.C. Allanach and J. Davighi, *Naturalising the third family hypercharge model for neutral current B-anomalies*, *Eur. Phys. J. C* **79** (2019) 908 [[arXiv:1905.10327](#)] [[INSPIRE](#)].
- [107] J. Kawamura, S. Raby and A. Trautner, *Complete vectorlike fourth family and new U(1)' for muon anomalies*, *Phys. Rev. D* **100** (2019) 055030 [[arXiv:1906.11297](#)] [[INSPIRE](#)].



- [108] W. Altmannshofer, J. Davighi and M. Nardecchia, *Gauging the accidental symmetries of the standard model, and implications for the flavor anomalies*, *Phys. Rev. D* **101** (2020) 015004 [[arXiv:1909.02021](#)] [[INSPIRE](#)].
- [109] L. Calibbi, A. Crivellin, F. Kirk, C.A. Manzari and L. Vernazza, *Z' models with less-minimal flavour violation*, *Phys. Rev. D* **101** (2020) 095003 [[arXiv:1910.00014](#)] [[INSPIRE](#)].
- [110] J. Aebischer, A.J. Buras, M. Cerdà-Sevilla and F. De Fazio, *Quark-lepton connections in Z' mediated FCNC processes: gauge anomaly cancellations at work*, *JHEP* **02** (2020) 183 [[arXiv:1912.09308](#)] [[INSPIRE](#)].
- [111] J. Kawamura, S. Raby and A. Trautner, *Complete vectorlike fourth family with U(1)′: A global analysis*, *Phys. Rev. D* **101** (2020) 035026 [[arXiv:1911.11075](#)] [[INSPIRE](#)].
- [112] B.C. Allanach, *U(1)<sub>B<sub>3</sub>-L<sub>2</sub></sub> explanation of the neutral current B-anomalies*, *Eur. Phys. J. C* **81** (2021) 56 [Erratum *ibid.* **81** (2021) 321] [[arXiv:2009.02197](#)] [[INSPIRE](#)].
- [113] A. Greljo, P. Stangl and A.E. Thomsen, *A Model of Muon Anomalies*, [arXiv:2103.13991](#) [[INSPIRE](#)].
- [114] LHCb collaboration, *Test of lepton universality with B<sup>0</sup> → K<sup>\*0</sup>ℓ<sup>+</sup>ℓ<sup>-</sup> decays*, *JHEP* **08** (2017) 055 [[arXiv:1705.05802](#)] [[INSPIRE](#)].
- [115] LHCb collaboration, *Search for lepton-universality violation in B<sup>+</sup> → K<sup>+</sup>ℓ<sup>+</sup>ℓ<sup>-</sup> decays*, *Phys. Rev. Lett.* **122** (2019) 191801 [[arXiv:1903.09252](#)] [[INSPIRE](#)].
- [116] B. Capdevila, A. Crivellin, S. Descotes-Genon, J. Matias and J. Virto, *Patterns of New Physics in b → sℓ<sup>+</sup>ℓ<sup>-</sup> transitions in the light of recent data*, *JHEP* **01** (2018) 093 [[arXiv:1704.05340](#)] [[INSPIRE](#)].
- [117] W. Altmannshofer, P. Stangl and D.M. Straub, *Interpreting Hints for Lepton Flavor Universality Violation*, *Phys. Rev. D* **96** (2017) 055008 [[arXiv:1704.05435](#)] [[INSPIRE](#)].
- [118] G. D’Amico et al., *Flavour anomalies after the R<sub>K</sub><sup>\*</sup> measurement*, *JHEP* **09** (2017) 010 [[arXiv:1704.05438](#)] [[INSPIRE](#)].
- [119] M. Ciuchini et al., *On Flavourful Easter eggs for New Physics hunger and Lepton Flavour Universality violation*, *Eur. Phys. J. C* **77** (2017) 688 [[arXiv:1704.05447](#)] [[INSPIRE](#)].
- [120] G. Hiller and I. Nisandzic, *R<sub>K</sub> and R<sub>K</sub><sup>\*</sup> beyond the standard model*, *Phys. Rev. D* **96** (2017) 035003 [[arXiv:1704.05444](#)] [[INSPIRE](#)].
- [121] L.-S. Geng, B. Grinstein, S. Jäger, J. Martin Camalich, X.-L. Ren and R.-X. Shi, *Towards the discovery of new physics with lepton-universality ratios of b → sℓℓ decays*, *Phys. Rev. D* **96** (2017) 093006 [[arXiv:1704.05446](#)] [[INSPIRE](#)].
- [122] T. Hurth, F. Mahmoudi, D. Martinez Santos and S. Neshatpour, *Lepton nonuniversality in exclusive b → sℓℓ decays*, *Phys. Rev. D* **96** (2017) 095034 [[arXiv:1705.06274](#)] [[INSPIRE](#)].
- [123] M. Algueró et al., *Emerging patterns of New Physics with and without Lepton Flavour Universal contributions*, *Eur. Phys. J. C* **79** (2019) 714 [Addendum *ibid.* **80** (2020) 511] [[arXiv:1903.09578](#)] [[INSPIRE](#)].
- [124] J. Aebischer, W. Altmannshofer, D. Guadagnoli, M. Reboud, P. Stangl and D.M. Straub, *B-decay discrepancies after Moriond 2019*, *Eur. Phys. J. C* **80** (2020) 252 [[arXiv:1903.10434](#)] [[INSPIRE](#)].

- [125] M. Ciuchini et al., *New Physics in  $b \rightarrow s\ell^+\ell^-$  confronts new data on Lepton Universality*, *Eur. Phys. J. C* **79** (2019) 719 [[arXiv:1903.09632](#)] [[INSPIRE](#)].
- [126] A. Arbey, T. Hurth, F. Mahmoudi, D.M. Santos and S. Neshatpour, *Update on the  $b \rightarrow s$  anomalies*, *Phys. Rev. D* **100** (2019) 015045 [[arXiv:1904.08399](#)] [[INSPIRE](#)].
- [127] J. Matias, F. Mescia, M. Ramon and J. Virto, *Complete Anatomy of  $\bar{B}_d \rightarrow \bar{K}^{*0}(\rightarrow K\pi)l^+l^-$  and its angular distribution*, *JHEP* **04** (2012) 104 [[arXiv:1202.4266](#)] [[INSPIRE](#)].
- [128] S. Descotes-Genon, T. Hurth, J. Matias and J. Virto, *Optimizing the basis of  $B \rightarrow K^*ll$  observables in the full kinematic range*, *JHEP* **05** (2013) 137 [[arXiv:1303.5794](#)] [[INSPIRE](#)].
- [129] LHCb collaboration, *Angular analysis of the  $B^0 \rightarrow K^{*0}\mu^+\mu^-$  decay using  $3\text{fb}^{-1}$  of integrated luminosity*, *JHEP* **02** (2016) 104 [[arXiv:1512.04442](#)] [[INSPIRE](#)].
- [130] LHCb collaboration, *Measurement of CP-Averaged Observables in the  $B^0 \rightarrow K^{*0}\mu^+\mu^-$  Decay*, *Phys. Rev. Lett.* **125** (2020) 011802 [[arXiv:2003.04831](#)] [[INSPIRE](#)].
- [131] M. Ciuchini, M. Fedele, E. Franco, A. Paul, L. Silvestrini and M. Valli, *Lessons from the  $B^{0,+} \rightarrow K^{*0,+}\mu^+\mu^-$  angular analyses*, *Phys. Rev. D* **103** (2021) 015030 [[arXiv:2011.01212](#)] [[INSPIRE](#)].
- [132] A. Carvunis, F. Dettori, S. Gangal, D. Guadagnoli and C. Normand, *On the effective lifetime of  $B_s \rightarrow \mu\mu\gamma$* , [arXiv:2102.13390](#) [[INSPIRE](#)].
- [133] W. Altmannshofer and P. Stangl, *New Physics in Rare B Decays after Moriond 2021*, [arXiv:2103.13370](#) [[INSPIRE](#)].
- [134] C. Bobeth, A.J. Buras, A. Celis and M. Jung, *Patterns of Flavour Violation in Models with Vector-Like Quarks*, *JHEP* **04** (2017) 079 [[arXiv:1609.04783](#)] [[INSPIRE](#)].
- [135] J. De Blas et al., *HEPfit: a code for the combination of indirect and direct constraints on high energy physics models*, *Eur. Phys. J. C* **80** (2020) 456 [[arXiv:1910.14012](#)] [[INSPIRE](#)].
- [136] F. del Aguila, J. de Blas and M. Pérez-Victoria, *Electroweak Limits on General New Vector Bosons*, *JHEP* **09** (2010) 033 [[arXiv:1005.3998](#)] [[INSPIRE](#)].
- [137] J. de Blas, J.M. Lizana and M. Pérez-Victoria, *Combining searches of  $Z'$  and  $W'$  bosons*, *JHEP* **01** (2013) 166 [[arXiv:1211.2229](#)] [[INSPIRE](#)].
- [138] R. Alonso, A. Carmona, B.M. Dillon, J.F. Kamenik, J. Martin Camalich and J. Zupan, *A clockwork solution to the flavor puzzle*, *JHEP* **10** (2018) 099 [[arXiv:1807.09792](#)] [[INSPIRE](#)].
- [139] A. Smolkovič, M. Tamaro and J. Zupan, *Anomaly free Froggatt-Nielsen models of flavor*, *JHEP* **10** (2019) 188 [[arXiv:1907.10063](#)] [[INSPIRE](#)].
- [140] J. Ellis, M. Fairbairn and P. Tunney, *Anomaly-Free Models for Flavour Anomalies*, *Eur. Phys. J. C* **78** (2018) 238 [[arXiv:1705.03447](#)] [[INSPIRE](#)].
- [141] W.J. Marciano and A. Sirlin, *Improved calculation of electroweak radiative corrections and the value of  $V(ud)$* , *Phys. Rev. Lett.* **96** (2006) 032002 [[hep-ph/0510099](#)] [[INSPIRE](#)].
- [142] C.-Y. Seng, M. Gorchtein, H.H. Patel and M.J. Ramsey-Musolf, *Reduced Hadronic Uncertainty in the Determination of  $V_{ud}$* , *Phys. Rev. Lett.* **121** (2018) 241804 [[arXiv:1807.10197](#)] [[INSPIRE](#)].
- [143] C.Y. Seng, M. Gorchtein and M.J. Ramsey-Musolf, *Dispersive evaluation of the inner radiative correction in neutron and nuclear  $\beta$  decay*, *Phys. Rev. D* **100** (2019) 013001 [[arXiv:1812.03352](#)] [[INSPIRE](#)].

- [144] M. Gorchtein,  $\gamma W$  Box Inside Out: Nuclear Polarizabilities Distort the Beta Decay Spectrum, *Phys. Rev. Lett.* **123** (2019) 042503 [[arXiv:1812.04229](#)] [[INSPIRE](#)].
- [145] A. Czarnecki, W.J. Marciano and A. Sirlin, Radiative Corrections to Neutron and Nuclear Beta Decays Revisited, *Phys. Rev. D* **100** (2019) 073008 [[arXiv:1907.06737](#)] [[INSPIRE](#)].
- [146] L. Hayen, Standard Model  $\mathcal{O}(\alpha)$  renormalization of  $g_A$  and its impact on new physics searches, *Phys. Rev. D* **103** (2021) 113001 [[arXiv:2010.07262](#)] [[INSPIRE](#)].
- [147] J.C. Hardy and I.S. Towner, Superallowed  $0^+ \rightarrow 0^+$  nuclear  $\beta$  decays: 2020 critical survey, with implications for  $V_{ud}$  and CKM unitarity, *Phys. Rev. C* **102** (2020) 045501 [[INSPIRE](#)].
- [148] M. Moulson, Experimental determination of  $V_{us}$  from kaon decays, *PoS(CKM2016)033* (2017) [[arXiv:1704.04104](#)] [[INSPIRE](#)].
- [149] PARTICLE DATA GROUP collaboration, Review of Particle Physics, *PTEP* **2020** (2020) 083C01 [[INSPIRE](#)].
- [150] A.J. Buras, L. Merlo and E. Stamou, The Impact of Flavour Changing Neutral Gauge Bosons on  $\bar{B} \rightarrow X_s \gamma$ , *JHEP* **08** (2011) 124 [[arXiv:1105.5146](#)] [[INSPIRE](#)].
- [151] A. Crivellin, M. Hoferichter and P. Schmidt-Wellenburg, Combined explanations of  $(g-2)_{\mu,e}$  and implications for a large muon EDM, *Phys. Rev. D* **98** (2018) 113002 [[arXiv:1807.11484](#)] [[INSPIRE](#)].
- [152] M. Lindner, M. Platscher and F.S. Queiroz, A Call for New Physics: The Muon Anomalous Magnetic Moment and Lepton Flavor Violation, *Phys. Rept.* **731** (2018) 1 [[arXiv:1610.06587](#)] [[INSPIRE](#)].
- [153] SINDRUM II collaboration, A Search for muon to electron conversion in muonic gold, *Eur. Phys. J. C* **47** (2006) 337 [[INSPIRE](#)].
- [154] BABAR collaboration, Searches for Lepton Flavor Violation in the Decays  $\tau^\pm \rightarrow e^\pm \gamma$  and  $\tau^\pm \mu^\pm \gamma$ , *Phys. Rev. Lett.* **104** (2010) 021802 [[arXiv:0908.2381](#)] [[INSPIRE](#)].
- [155] MEG collaboration, Search for the lepton flavour violating decay  $\mu^+ \rightarrow e^+ \gamma$  with the full dataset of the MEG experiment, *Eur. Phys. J. C* **76** (2016) 434 [[arXiv:1605.05081](#)] [[INSPIRE](#)].
- [156] BELLE-II collaboration, The Belle II Physics Book, *PTEP* **2019** (2019) 123C01 [Erratum *ibid.* **2020** (2020) 029201] [[arXiv:1808.10567](#)] [[INSPIRE](#)].
- [157] MEG II collaboration, The design of the MEG II experiment, *Eur. Phys. J. C* **78** (2018) 380 [[arXiv:1801.04688](#)] [[INSPIRE](#)].
- [158] D. Hanneke, S. Fogwell and G. Gabrielse, New Measurement of the Electron Magnetic Moment and the Fine Structure Constant, *Phys. Rev. Lett.* **100** (2008) 120801 [[arXiv:0801.1134](#)] [[INSPIRE](#)].
- [159] T. Aoyama, T. Kinoshita and M. Nio, Revised and Improved Value of the QED Tenth-Order Electron Anomalous Magnetic Moment, *Phys. Rev. D* **97** (2018) 036001 [[arXiv:1712.06060](#)] [[INSPIRE](#)].
- [160] S. Laporta, High-precision calculation of the 4-loop contribution to the electron  $g-2$  in QED, *Phys. Lett. B* **772** (2017) 232 [[arXiv:1704.06996](#)] [[INSPIRE](#)].
- [161] R.H. Parker, C. Yu, W. Zhong, B. Estey and H. Müller, Measurement of the fine-structure constant as a test of the Standard Model, *Science* **360** (2018) 191 [[arXiv:1812.04130](#)] [[INSPIRE](#)].



- [162] L. Morel, Z. Yao, P. Cladé and S. Guellati-Khélifa, *Determination of the fine-structure constant with an accuracy of 81 parts per trillion*, *Nature* **588** (2020) 61 [INSPIRE].
- [163] T. Aoyama, M. Hayakawa, T. Kinoshita and M. Nio, *Complete Tenth-Order QED Contribution to the Muon  $g - 2$* , *Phys. Rev. Lett.* **109** (2012) 111808 [arXiv:1205.5370] [INSPIRE].
- [164] T. Aoyama, T. Kinoshita and M. Nio, *Theory of the Anomalous Magnetic Moment of the Electron*, *Atoms* **7** (2019) 28 [INSPIRE].
- [165] A. Czarnecki, W.J. Marciano and A. Vainshtein, *Refinements in electroweak contributions to the muon anomalous magnetic moment*, *Phys. Rev. D* **67** (2003) 073006 [Erratum *ibid.* **73** (2006) 119901] [hep-ph/0212229] [INSPIRE].
- [166] C. Gnendiger, D. Stöckinger and H. Stöckinger-Kim, *The electroweak contributions to  $(g - 2)_\mu$  after the Higgs boson mass measurement*, *Phys. Rev. D* **88** (2013) 053005 [arXiv:1306.5546] [INSPIRE].
- [167] M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, *Reevaluation of the hadronic vacuum polarisation contributions to the Standard Model predictions of the muon  $g - 2$  and  $\alpha(m_Z^2)$  using newest hadronic cross-section data*, *Eur. Phys. J. C* **77** (2017) 827 [arXiv:1706.09436] [INSPIRE].
- [168] A. Keshavarzi, D. Nomura and T. Teubner, *Muon  $g - 2$  and  $\alpha(M_Z^2)$ : a new data-based analysis*, *Phys. Rev. D* **97** (2018) 114025 [arXiv:1802.02995] [INSPIRE].
- [169] G. Colangelo, M. Hoferichter and P. Stoffer, *Two-pion contribution to hadronic vacuum polarization*, *JHEP* **02** (2019) 006 [arXiv:1810.00007] [INSPIRE].
- [170] M. Hoferichter, B.-L. Hoid and B. Kubis, *Three-pion contribution to hadronic vacuum polarization*, *JHEP* **08** (2019) 137 [arXiv:1907.01556] [INSPIRE].
- [171] M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, *A new evaluation of the hadronic vacuum polarisation contributions to the muon anomalous magnetic moment and to  $\alpha(m_Z^2)$* , *Eur. Phys. J. C* **80** (2020) 241 [Erratum *ibid.* **80** (2020) 410] [arXiv:1908.00921] [INSPIRE].
- [172] A. Keshavarzi, D. Nomura and T. Teubner,  *$g - 2$  of charged leptons,  $\alpha(M_Z^2)$ , and the hyperfine splitting of muonium*, *Phys. Rev. D* **101** (2020) 014029 [arXiv:1911.00367] [INSPIRE].
- [173] A. Kurz, T. Liu, P. Marquard and M. Steinhauser, *Hadronic contribution to the muon anomalous magnetic moment to next-to-next-to-leading order*, *Phys. Lett. B* **734** (2014) 144 [arXiv:1403.6400] [INSPIRE].
- [174] K. Melnikov and A. Vainshtein, *Hadronic light-by-light scattering contribution to the muon anomalous magnetic moment revisited*, *Phys. Rev. D* **70** (2004) 113006 [hep-ph/0312226] [INSPIRE].
- [175] P. Masjuan and P. Sanchez-Puertas, *Pseudoscalar-pole contribution to the  $(g_\mu - 2)$ : a rational approach*, *Phys. Rev. D* **95** (2017) 054026 [arXiv:1701.05829] [INSPIRE].
- [176] G. Colangelo, M. Hoferichter, M. Procura and P. Stoffer, *Dispersion relation for hadronic light-by-light scattering: two-pion contributions*, *JHEP* **04** (2017) 161 [arXiv:1702.07347] [INSPIRE].

- [177] M. Hoferichter, B.-L. Hoid, B. Kubis, S. Leupold and S.P. Schneider, *Dispersion relation for hadronic light-by-light scattering: pion pole*, *JHEP* **10** (2018) 141 [[arXiv:1808.04823](#)] [[INSPIRE](#)].
- [178] A. Gérardin, H.B. Meyer and A. Nyffeler, *Lattice calculation of the pion transition form factor with  $N_f = 2 + 1$  Wilson quarks*, *Phys. Rev. D* **100** (2019) 034520 [[arXiv:1903.09471](#)] [[INSPIRE](#)].
- [179] J. Bijnens, N. Hermansson-Truedsson and A. Rodríguez-Sánchez, *Short-distance constraints for the HLbL contribution to the muon anomalous magnetic moment*, *Phys. Lett. B* **798** (2019) 134994 [[arXiv:1908.03331](#)] [[INSPIRE](#)].
- [180] G. Colangelo, F. Hagelstein, M. Hoferichter, L. Laub and P. Stoffer, *Longitudinal short-distance constraints for the hadronic light-by-light contribution to  $(g - 2)_\mu$  with large- $N_c$  Regge models*, *JHEP* **03** (2020) 101 [[arXiv:1910.13432](#)] [[INSPIRE](#)].
- [181] T. Blum et al., *Hadronic Light-by-Light Scattering Contribution to the Muon Anomalous Magnetic Moment from Lattice QCD*, *Phys. Rev. Lett.* **124** (2020) 132002 [[arXiv:1911.08123](#)] [[INSPIRE](#)].
- [182] G. Colangelo, M. Hoferichter, A. Nyffeler, M. Passera and P. Stoffer, *Remarks on higher-order hadronic corrections to the muon  $g-2$* , *Phys. Lett. B* **735** (2014) 90 [[arXiv:1403.7512](#)] [[INSPIRE](#)].
- [183] S. Borsányi et al., *Leading hadronic contribution to the muon magnetic moment from lattice QCD*, *Nature* **593** (2021) 51 [[arXiv:2002.12347](#)] [[INSPIRE](#)].
- [184] M. Passera, W.J. Marciano and A. Sirlin, *The Muon  $g-2$  and the bounds on the Higgs boson mass*, *Phys. Rev. D* **78** (2008) 013009 [[arXiv:0804.1142](#)] [[INSPIRE](#)].
- [185] J. Haller, A. Hoecker, R. Kogler, K. Mönig, T. Peiffer and J. Stelzer, *Update of the global electroweak fit and constraints on two-Higgs-doublet models*, *Eur. Phys. J. C* **78** (2018) 675 [[arXiv:1803.01853](#)] [[INSPIRE](#)].
- [186] A. Crivellin, M. Hoferichter, C.A. Manzari and M. Montull, *Hadronic Vacuum Polarization:  $(g - 2)_\mu$  versus Global Electroweak Fits*, *Phys. Rev. Lett.* **125** (2020) 091801 [[arXiv:2003.04886](#)] [[INSPIRE](#)].
- [187] A. Keshavarzi, W.J. Marciano, M. Passera and A. Sirlin, *Muon  $g - 2$  and  $\Delta\alpha$  connection*, *Phys. Rev. D* **102** (2020) 033002 [[arXiv:2006.12666](#)] [[INSPIRE](#)].
- [188] B. Ananthanarayan, I. Caprini and D. Das, *Pion electromagnetic form factor at high precision with implications to  $a_\mu^{\pi\pi}$  and the onset of perturbative QCD*, *Phys. Rev. D* **98** (2018) 114015 [[arXiv:1810.09265](#)] [[INSPIRE](#)].
- [189] MUON (G-2) collaboration, *An Improved Limit on the Muon Electric Dipole Moment*, *Phys. Rev. D* **80** (2009) 052008 [[arXiv:0811.1207](#)] [[INSPIRE](#)].
- [190] ACME collaboration, *Order of Magnitude Smaller Limit on the Electric Dipole Moment of the Electron*, *Science* **343** (2014) 269 [[arXiv:1310.7534](#)] [[INSPIRE](#)].
- [191] ACME collaboration, *Improved limit on the electric dipole moment of the electron*, *Nature* **562** (2018) 355 [[INSPIRE](#)].
- [192] BELLE collaboration, *Search for the electric dipole moment of the tau lepton*, *Phys. Lett. B* **551** (2003) 16 [[hep-ex/0210066](#)] [[INSPIRE](#)].

- [193] DELPHI collaboration, *Study of tau-pair production in photon-photon collisions at LEP and limits on the anomalous electromagnetic moments of the tau lepton*, *Eur. Phys. J. C* **35** (2004) 159 [[hep-ex/0406010](#)] [[INSPIRE](#)].
- [194] S. Eidelman and M. Passera, *Theory of the tau lepton anomalous magnetic moment*, *Mod. Phys. Lett. A* **22** (2007) 159 [[hep-ph/0701260](#)] [[INSPIRE](#)].
- [195] A. Adelmann et al., *Search for a muon EDM using the frozen-spin technique*, [arXiv:2102.08838](#) [[INSPIRE](#)].
- [196] SINDRUM collaboration, *Search for the Decay  $\mu^+ \rightarrow e^+e^+e^-$* , *Nucl. Phys. B* **299** (1988) 1 [[INSPIRE](#)].
- [197] K. Hayasaka et al., *Search for Lepton Flavor Violating Tau Decays into Three Leptons with 719 Million Produced  $\tau^+\tau^-$  Pairs*, *Phys. Lett. B* **687** (2010) 139 [[arXiv:1001.3221](#)] [[INSPIRE](#)].
- [198] BABAR collaboration, *Limits on tau Lepton-Flavor Violating Decays in three charged leptons*, *Phys. Rev. D* **81** (2010) 111101 [[arXiv:1002.4550](#)] [[INSPIRE](#)].
- [199] LHCb collaboration, *Search for the lepton flavour violating decay  $\tau^- \rightarrow \mu^- \mu^+ \mu^-$* , *JHEP* **02** (2015) 121 [[arXiv:1409.8548](#)] [[INSPIRE](#)].
- [200] A. Cerri et al., *Report from Working Group 4: Opportunities in Flavour Physics at the HL-LHC and HE-LHC*, *CERN Yellow Rep. Monogr.* **7** (2019) 867 [[arXiv:1812.07638](#)] [[INSPIRE](#)].
- [201] A. Blondel et al., *Research Proposal for an Experiment to Search for the Decay  $\mu \rightarrow eee$* , [arXiv:1301.6113](#) [[INSPIRE](#)].
- [202] MU3E collaboration, *Status of the Mu3e Experiment at PSI*, *EPJ Web Conf.* **118** (2016) 01028 [[arXiv:1605.02906](#)] [[INSPIRE](#)].
- [203] R. Kitano, M. Koike and Y. Okada, *Detailed calculation of lepton flavor violating muon electron conversion rate for various nuclei*, *Phys. Rev. D* **66** (2002) 096002 [Erratum *ibid.* **76** (2007) 059902] [[hep-ph/0203110](#)] [[INSPIRE](#)].
- [204] T. Suzuki, D.F. Measday and J.P. Roalsvig, *Total Nuclear Capture Rates for Negative Muons*, *Phys. Rev. C* **35** (1987) 2212 [[INSPIRE](#)].
- [205] A. Baldini et al., *A submission to the 2020 update of the European Strategy for Particle Physics on behalf of the COMET, MEG, Mu2e and Mu3e collaborations*, [arXiv:1812.06540](#) [[INSPIRE](#)].
- [206] ALEPH, DELPHI, L3, OPAL, SLD, LEP ELECTROWEAK WORKING GROUP, SLD ELECTROWEAK GROUP and SLD HEAVY FLAVOUR GROUP collaborations, *Precision electroweak measurements on the Z resonance*, *Phys. Rept.* **427** (2006) 257 [[hep-ex/0509008](#)] [[INSPIRE](#)].
- [207] PARTICLE DATA GROUP collaboration, *Review of Particle Physics*, *Phys. Rev. D* **98** (2018) 030001 [[INSPIRE](#)].
- [208] U. Haisch and S. Westhoff, *Massive Color-Octet Bosons: Bounds on Effects in Top-Quark Pair Production*, *JHEP* **08** (2011) 088 [[arXiv:1106.0529](#)] [[INSPIRE](#)].
- [209] ATLAS collaboration, *Measurement of the Higgs boson mass in the  $H \rightarrow ZZ^* \rightarrow 4\ell$  and  $H \rightarrow \gamma\gamma$  channels with  $\sqrt{s} = 13$  TeV pp collisions using the ATLAS detector*, *Phys. Lett. B* **784** (2018) 345 [[arXiv:1806.00242](#)] [[INSPIRE](#)].

- [210] CMS collaboration, *A measurement of the Higgs boson mass in the diphoton decay channel*, *Phys. Lett. B* **805** (2020) 135425 [[arXiv:2002.06398](#)] [[INSPIRE](#)].
- [211] CDF and D0 collaborations, *Combination of CDF and D0 results on the mass of the top quark using up  $9.7 \text{ fb}^{-1}$  at the Tevatron*, [arXiv:1608.01881](#) [[INSPIRE](#)].
- [212] ATLAS collaboration, *Measurement of the top quark mass in the  $t\bar{t} \rightarrow \text{lepton+jets}$  channel from  $\sqrt{s} = 8 \text{ TeV}$  ATLAS data and combination with previous results*, *Eur. Phys. J. C* **79** (2019) 290 [[arXiv:1810.01772](#)] [[INSPIRE](#)].
- [213] CMS collaboration, *Measurement of the top quark mass in the all-jets final state at  $\sqrt{s} = 13 \text{ TeV}$  and combination with the lepton+jets channel*, *Eur. Phys. J. C* **79** (2019) 313 [[arXiv:1812.10534](#)] [[INSPIRE](#)].
- [214] ATLAS collaboration, *Charged-lepton-flavour violation at the LHC: a search for  $Z \rightarrow e\tau/\mu\tau$  decays with the ATLAS detector*, [arXiv:2010.02566](#) [[INSPIRE](#)].
- [215] OPAL collaboration, *A Search for lepton flavor violating  $Z0$  decays*, *Z. Phys. C* **67** (1995) 555 [[INSPIRE](#)].
- [216] DELPHI collaboration, *Search for lepton flavor number violating  $Z0$  decays*, *Z. Phys. C* **73** (1997) 243 [[INSPIRE](#)].
- [217] ATLAS collaboration, *Search for the lepton flavor violating decay  $Z \rightarrow e\mu$  in  $pp$  collisions at  $\sqrt{s} \text{ TeV}$  with the ATLAS detector*, *Phys. Rev. D* **90** (2014) 072010 [[arXiv:1408.5774](#)] [[INSPIRE](#)].
- [218] CHARM-II collaboration, *First observation of neutrino trident production*, *Phys. Lett. B* **245** (1990) 271 [[INSPIRE](#)].
- [219] CCFR collaboration, *Neutrino tridents and  $W Z$  interference*, *Phys. Rev. Lett.* **66** (1991) 3117 [[INSPIRE](#)].
- [220] NUTEV collaboration, *Neutrino trident production from NuTeV*, in *29th International Conference on High-Energy Physics*, 7, 1998 [[hep-ex/9811012](#)] [[INSPIRE](#)].
- [221] A. Caldwell, D. Kollar and K. Kroninger, *BAT: The Bayesian Analysis Toolkit*, *Comput. Phys. Commun.* **180** (2009) 2197 [[arXiv:0808.2552](#)] [[INSPIRE](#)].
- [222] A. Crivellin, S. Davidson, G.M. Pruna and A. Signer, *Renormalisation-group improved analysis of  $\mu \rightarrow e$  processes in a systematic effective-field-theory approach*, *JHEP* **05** (2017) 117 [[arXiv:1702.03020](#)] [[INSPIRE](#)].
- [223] G.M. Pruna and A. Signer, *The  $\mu \rightarrow e\gamma$  decay in a systematic effective field theory approach with dimension 6 operators*, *JHEP* **10** (2014) 014 [[arXiv:1408.3565](#)] [[INSPIRE](#)].
- [224] J. Aebischer, M. Fael, C. Greub and J. Virto, *B physics Beyond the Standard Model at One Loop: Complete Renormalization Group Evolution below the Electroweak Scale*, *JHEP* **09** (2017) 158 [[arXiv:1704.06639](#)] [[INSPIRE](#)].