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Dark Matter in scalar $\mathrm{Sp}(\mathcal{N})$ gauge dynamics

Giacomo Landini^{a,b} and Jin-Wei Wang^{b,c,d}

^a*INFN — Sezione di Pisa,*

Largo Bruno Pontecorvo 3, I-56127 Pisa, Italy

^b*Dipartimento di Fisica “E. Fermi”, Università di Pisa,*

Largo Bruno Pontecorvo 3, I-56127 Pisa, Italy

^c*Key Laboratory of Particle Astrophysics, Institute of High Energy Physics,*

Chinese Academy of Sciences, 19B Yuquan Road, Beijing 100049, China

^d*School of Physical Sciences, University of Chinese Academy of Sciences,*

19A Yuquan Road, Beijing 100049, China

E-mail: giacomo.landini@phd.unipi.it, wangjinwei@ihep.ac.cn

ABSTRACT: We consider a model with Sp dark gauge group and a scalar field in the fundamental representation, which leads to two co-stable DM candidates at the perturbative level thanks to a global $\mathrm{U}(1)$ accidental symmetry. After gauge confinement at low energy scale, only one of the two candidates is still stable. We compute the DM relic abundance by solving the Boltzmann equations numerically. The presence of light dark glueballs gives extra cosmological effects and can affect Higgs physics. We study the DM phenomenology, providing the predictions for direct and indirect detection (including the Sommerfeld enhancement). We show that the model predicts a slightly suppressed indirect detection cross section in comparison to the usual WIMPs paradigm.

KEYWORDS: Beyond Standard Model, Gauge Symmetry, Cosmology of Theories beyond the SM

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1 Introduction

The possibility that Dark Matter (DM) originates from elementary scalar/gauge dynamics has been explored in [1] and [2]. In particular, [1] considered one new elementary scalar field $\mathcal{S}$ that fills the fundamental representation of a dark gauge group $\mathcal{G} = \{\text{SU}(\mathcal{N}), \text{SO}(\mathcal{N}), \text{Sp}(\mathcal{N}), G_2\}$. Every choice of $\mathcal{G}$ is characterized by an accidental symmetry, which leads to stable DM candidates with non-trivial features that are characteristics of each group. DM is always accompanied by light dark glueballs.

The dark group $\mathcal{G}$ can become strongly interacting (‘confined phase’) and/or get spontaneously broken by vacuum expectation values of $\mathcal{S}$ (‘Higgsed phase’ breaking $\mathcal{G}$ to a subgroup $\mathcal{H}$ that will also confine at low energy scale). A surprising equivalence between the confined and Higgsed phases for scalars $\mathcal{S}$ in the fundamental representation of the gauge groups $\mathcal{G}$ has been studied in [1, 3]. In these models $\mathcal{S}$ has a unique self-quartic, that leads to a unique symmetry breaking pattern where the only surviving scalar is a Higgs-like singlet, named scalon.

It is interesting to study the DM phenomenology of these models. Since the Higgs and confined phases, thanks to the equivalence between them, contain the same asymptotic particles, one can perform all computations in the Higgs phase for weak couplings. The phenomenology can be very different for each gauge group. In the SU case there turns out to be one single DM candidate and the related phenomenology has been studied in detail in [1]. The situation for the Sp dynamics is more involved: at the perturbative level there are two co-stable DM candidates arising from the spontaneous symmetry breaking (SSB) $\text{Sp}(\mathcal{N}+2) \rightarrow \text{Sp}(\mathcal{N})$. After the confinement of the subgroup $\text{Sp}(\mathcal{N})$, one of two candidates forms bound states (mesons) that can decay, leaving only one DM candidate. Therefore, we expect the DM phenomenology for Sp case to be more intriguing and worthy of more detailed calculations.

In this paper we compute the DM relic abundance of the Sp DM model: we consider a Sp dark gauge group and a scalar field that fills the fundamental representation and we solve the related Boltzmann equations numerically. We make some simplified assumptions to take into account gauge confinement, and we also consider the presence of light glueballs, which can dilute the DM relic abundance.

It is well known that light mediators can affect DM annihilations cross section through an enhancement factor (Sommerfeld enhancement) and bound state formation [4, 5]. The Sp DM model includes two kinds of light mediators: a scalar particle (the scalon) and massless gauge bosons, which arise from the spontaneous symmetry breaking of the gauge group. We also introduce how to compute Sommerfeld and bound state corrections and how to take them into account in the predictions for both the relic abundance and indirect detection signals.

We calculate the relevant cross sections for the direct and indirect detections and compare them with the current experimental limits as well as the future prospects, and we find that the Sp DM model is testable on the future indirect detection experiment like CTA [6]. We also investigate the phenomenology of the scalon and of light glueballs and their connection with Higgs physics.

This paper is outlined as follows: in section 2 we introduce the main features of our model, including the Lagrangian, the breaking pattern, the perturbative mass spectrum, the accidental symmetries and the formation of bound states after confinement. In section 3 we discuss the Sommerfeld enhancement factors for annihilation cross section and the formation of perturbative bound states. In section 4 we compute the DM relic abundance by solving the Boltzmann equations. In section 5 we discuss the predictions and the experimental limitations of this model, including scalon production, direct and indirect detections. In section 6 we briefly analyse the effects of the quadratic term in the scalar potential on the spectrum and the phenomenological results by abandoning the scale invariant hypothesis. Conclusions and the summary of results are given in section 7. Appendix A gives the explicit expressions of the Feynman rules of the Sp DM model in the Higgs phase.

2 Model detail

2.1 Lagrangian and mass spectrum

The group $\text{Sp}(\mathcal{N})$ is defined for even $\mathcal{N}$ as the transformations that leave invariant the tensor $\gamma_{\mathcal{N}} \equiv \mathbf{I}_{\mathcal{N}/2} \otimes \epsilon$, where ϵ_{ij} is the 2-dimensional anti-symmetric tensor. We consider a new $\text{Sp}(\mathcal{N} + 2)$ gauge sector, with vectors $\mathcal{G}_{\mu\nu}^a$ in the adjoint and one scalar field $\mathcal{S}$ in the fundamental representation, which is pseudo-real.¹ We thereby introduce a complex $(\mathcal{N} + 2)$ -dimensional scalar $\mathcal{S}$. The Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{\text{SM}} - \frac{1}{4} \mathcal{G}_{\mu\nu}^a \mathcal{G}^{a\mu\nu} - V_{\mathcal{S}} + |\mathcal{D}_{\mu} \mathcal{S}|^2, \quad (2.1)$$

with scalar potential

$$V_{\mathcal{S}} = -M_{\mathcal{S}}^2 |\mathcal{S}|^2 + \lambda_{\mathcal{S}} |\mathcal{S}|^4 - \lambda_{HS} |H|^2 |\mathcal{S}|^2, \quad (2.2)$$

where H is the SM Higgs field. The Lagrangian conserves an accidental global $\text{U}(1)$ symmetry (under which $\mathcal{S}$ has charge 1), by virtue of $\mathcal{S}^T \gamma \mathcal{S} = 0$. The RGEs at one-loop level are

$$(4\pi)^2 \frac{d\mathbf{g}}{d \ln \mu} = -\frac{11(\mathcal{N} + 2) + 21}{6} \mathbf{g}^3, \quad (2.3a)$$

$$(4\pi)^2 \frac{d\lambda_{\mathcal{S}}}{d \ln \mu} = \frac{3}{16} (\mathcal{N} + 6) \mathbf{g}^4 - 3(\mathcal{N} + 3) \mathbf{g}^2 \lambda_{\mathcal{S}} + 4(6 + \mathcal{N}) \lambda_{\mathcal{S}}^2, \quad (2.3b)$$

where $\mathbf{g}$ is the gauge coupling of the new sector. We normalize the generators in the fundamental representation as $\text{Tr}[T^a, T^b] = \frac{1}{2} \delta^{ab}$. It is interesting to consider dynamical symmetry breaking through the Coleman-Weinberg mechanism, obtained by setting $M_{\mathcal{S}} = 0$ (scale invariant hypothesis). In section 6 we briefly discuss the effects of the quadratic term $M_{\mathcal{S}}^2$ by abandoning the scale invariant hypothesis. Assuming that λ_{HS} is negligibly small, the RGEs give that $\mathcal{S}$ can radiatively acquire a vacuum expectation value $\mathcal{S}(x) = (0, \dots, 0, (w + \mathfrak{s}(x))/\sqrt{2})^T$, breaking the gauge group $\text{Sp}(\mathcal{N} + 2)$ to $\text{Sp}(\mathcal{N})$. At the same time, the accidental $\text{U}(1)$ symmetry gets rotated to an unbroken global $\text{U}(1)$ with generator

$$\text{diag}(1, \dots, 1, 1, 1) + \text{diag}(0, \dots, 0, 1, -1) = \text{diag}(1, \dots, 1, 2, 0), \quad (2.4)$$

obtained by combining the original $\text{U}(1)$ with the diagonal generator of the broken $\text{Sp}(2)$. We call this unbroken global $\text{U}(1)$ dark-baryon number. If the Higgs mass term is absten too, this model can also generate the weak scale $v \simeq 246 \text{ GeV}$ as [7]

$$v \simeq w \sqrt{\frac{\lambda_{HS}}{2\lambda_H}}, \quad (2.5)$$

¹We remind the reader that the $\text{Sp}(\mathcal{N})$ generators can be written as

$$\frac{T_{\text{asym}} \otimes \mathbb{K}_2}{\sqrt{2}}, \quad \frac{T_{\text{sym}} \otimes \sigma_k}{\sqrt{2}},$$

where σ_k ($k = 1, 2, 3$) are the Pauli matrices, $T_{\text{sym}} = \{T_{\text{real}}, \mathbb{K}_{\mathcal{N}/2}/\sqrt{\mathcal{N}}\}$ are the symmetric generators of $\text{U}(\mathcal{N}/2)$, and $T_{\text{asym}} = T_{\text{imag}}$ are the anti-symmetric ones [2].

assuming a small positive λ_{HS} (more complicated expressions hold if λ_{HS} is not negligibly small).

Writing the gauge bosons as

$$T^a \mathcal{G}_\mu^a = \left(\begin{array}{c|cc} \mathcal{A}_\mu & \mathcal{X}_\mu^\dagger/2 & \gamma_{\mathcal{N}} \mathcal{X}_\mu/2 \\ \hline \mathcal{X}_\mu/2 & \mathcal{Z}_\mu/2 & \mathcal{W}_\mu/\sqrt{2} \\ \gamma_{\mathcal{N}} \mathcal{X}_\mu^\dagger/2 & -\mathcal{W}_\mu^\dagger/\sqrt{2} & -\mathcal{Z}_\mu/2 \end{array} \right), \quad (2.6)$$

the perturbative mass spectrum in the broken phase is:

- the singlet scalon $\mathfrak{s}$ with one-loop suppressed mass $M_{\mathfrak{s}}^2 = \beta_{\lambda_S} w^2$;
- one real $\mathcal{Z}$ with mass $M_{\mathcal{Z}} = \mathfrak{g} w/2$ and zero dark-baryon number;
- one complex $\mathcal{W}$, with mass $M_{\mathcal{W}} = M_{\mathcal{Z}}$ and dark-baryon number 2.

For $\mathcal{N} = 0$ this is the $\text{Sp}(2) = \text{SU}(2)$ model of [3, 7, 8] where $\mathcal{W}$ and $\mathcal{Z}$ are co-stable DM candidates. For $\mathcal{N} \geq 2$ the spectrum contains extra particles:

- $\mathcal{N}$ complex massive vectors $\mathcal{X}$ in the fundamental representation of $\text{Sp}(\mathcal{N})$ with mass $M_{\mathcal{X}} = M_{\mathcal{W}}/\sqrt{2}$ and dark-baryon number 1;
- $\mathcal{N}(\mathcal{N} + 1)/2$ massless vectors $\mathcal{A}$ in the adjoint representation of $\text{Sp}(\mathcal{N})$.

The $\mathcal{Z}$ boson decays into $\mathcal{A}$'s through loops involving the $\mathcal{Z}\mathcal{W}\mathcal{W}^\dagger$ and $\mathcal{Z}\mathcal{X}\mathcal{X}^\dagger$ gauge couplings. At perturbative level the $\mathcal{W}$ and $\mathcal{X}$ are DM candidates, co-stable thanks to accidental baryon number conservation.

Condensation of $\text{Sp}(\mathcal{N})$. The theory becomes strongly coupled and the unbroken $\text{Sp}(\mathcal{N})$ confines at the energy scale

$$\Lambda_{\text{DC}} = M_{\mathcal{W}} \exp \left[-\frac{12\pi}{11(\mathcal{N} + 2)\alpha(M_{\mathcal{W}})} \right]. \quad (2.7)$$

with $\alpha(M_{\mathcal{W}}) = \mathfrak{g}^2(M_{\mathcal{W}})/4\pi$, where $\mathfrak{g}^2(M_{\mathcal{W}})$ is the value of the dark gauge coupling at the scale $M_{\mathcal{W}}$. After confinement we get the following spectrum of asymptotic states:

- The scalon s , the $\mathcal{Z}$ and $\mathcal{W}$ bosons, and dark glue-balls $\mathcal{A}\mathcal{A}$.
- Two kinds of dark mesons: the unstable $\mathcal{X}^\dagger \mathcal{X}$ and $\mathcal{X}^\dagger \mathcal{D}_\mu \mathcal{X}$, which have the same quantum numbers as s and $\mathcal{Z}$, and $\mathcal{M}_\mu = \mathcal{X}^T \gamma_{\mathcal{N}} \mathcal{D}_\mu \mathcal{X}$, with dark-baryon number 2 as $\mathcal{W}$. Only one linear combination of $\mathcal{M}$ and $\mathcal{W}$ appears among the stable asymptotic states, while the other corresponds to a resonance. A similar situation holds for s and $\mathcal{X}^\dagger \mathcal{X}$, and for $\mathcal{Z}$ and $\mathcal{X}^\dagger \mathcal{D} \mathcal{X}$.
- Dark baryons $\mathcal{B}$ (defined as states formed with one $\epsilon_{i_1 \dots i_{\mathcal{N}-2}}$ tensor) are not stable because the ϵ tensor can be decomposed as $\epsilon_{i_1 \dots i_{\mathcal{N}-2}} = \gamma_{i_1 i_2} \dots \gamma_{i_{\mathcal{N}-3} i_{\mathcal{N}-2}} + \text{permutations}$ [9]. This means that $\mathcal{B}$ splits into $\mathcal{N}/2 - 1$ mesons $\mathcal{M}$.

Both the $\mathcal{W}$ and the mesons $\mathcal{M}$ carry charge 2 under conserved $\text{U}(1)$ baryon number.

2.2 Lagrangian in Higgs phase: $\text{Sp}(\mathcal{N} + 2) \rightarrow \text{Sp}(\mathcal{N})$

After symmetry breaking the Lagrangian in eq. (2.1) can be further expanded as:

$$-\frac{\mathcal{G}_{\mu\nu}^a \mathcal{G}^{a\mu\nu}}{4} + |D_\mu \mathcal{S}|^2 = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\mathcal{WZ}} + \mathcal{L}_{\mathcal{WX}} + \mathcal{L}_{\mathcal{XZ}} + \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{other}}. \quad (2.8)$$

The kinetic terms are

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4} \mathcal{A}_{\mu\nu}^a \mathcal{A}^{a\mu\nu} - \frac{1}{2} \mathcal{W}_{\mu\nu}^\dagger \mathcal{W}^{\mu\nu} - \frac{1}{4} \mathcal{Z}_{\mu\nu} \mathcal{Z}^{\mu\nu} - \frac{1}{2} \mathcal{X}_{\mu\nu}^\dagger \mathcal{X}^{\mu\nu}. \quad (2.9)$$

The interaction terms between $\mathcal{WZ}$ and $\mathcal{WX}$ are

$$\begin{aligned} \mathcal{L}_{\mathcal{WZ}} = & + i\mathfrak{g} \left((\partial_{[\mu} \mathcal{W}_{\nu]}^\dagger) \mathcal{W}^\nu \mathcal{Z}^\mu - \mathcal{W}_\nu^\dagger (\partial^{[\mu} \mathcal{W}^{\nu]}) \mathcal{Z}_\mu - \mathcal{W}_{[\mu}^\dagger \mathcal{W}_{\nu]} \partial^\mu \mathcal{Z}^\nu \right) \\ & - \mathfrak{g}^2 \left(\mathcal{W}_\mu^\dagger \mathcal{W}^\mu \mathcal{Z}_\nu \mathcal{Z}^\nu - \mathcal{W}_\mu^\dagger \mathcal{W}_\nu \mathcal{Z}^\mu \mathcal{Z}^\nu \right), \end{aligned} \quad (2.10)$$

$$\begin{aligned} \mathcal{L}_{\mathcal{WX}} = & -\frac{\mathfrak{g}^2}{2} \left(\mathcal{W}_\mu^\dagger \mathcal{W}^\mu \mathcal{X}_\nu^\dagger \mathcal{X}^\nu + \mathcal{W}_\mu^\dagger \mathcal{W}_\nu \mathcal{X}^{\dagger\nu} \mathcal{X}^\mu - 2\mathcal{W}_\mu^\dagger \mathcal{W}_\nu \mathcal{X}^{\dagger\mu} \mathcal{X}^\nu \right) \\ & - \frac{i}{\sqrt{2}} \mathfrak{g} \left(\frac{1}{2} \gamma_{\mathcal{N}}^{ij} (\mathcal{X}_{[\nu}^i \mathcal{X}_{\mu]}^j) \partial^\mu \mathcal{W}^{\dagger\nu} + \gamma_{\mathcal{N}}^{ij} \mathcal{W}_{[\nu}^\dagger \mathcal{X}_{\mu]}^i \partial^\mu \mathcal{X}^{j\nu} \right). \end{aligned} \quad (2.11)$$

The interaction terms between $\mathcal{XZ}$ are

$$\begin{aligned} \mathcal{L}_{\mathcal{XZ}} = & \frac{i}{2} \mathfrak{g} \left((\partial_{[\mu} \mathcal{X}_{\nu]}^\dagger) \mathcal{X}^\nu \mathcal{Z}^\mu - \mathcal{X}_\nu^\dagger (\partial^{[\mu} \mathcal{X}^{\nu]}) \mathcal{Z}_\mu - \mathcal{X}_{[\mu}^\dagger \mathcal{X}_{\nu]} \partial^\mu \mathcal{Z}^\nu \right) \\ & - \frac{\mathfrak{g}^2}{4} \left(\mathcal{X}_\mu^\dagger \mathcal{X}^\mu \mathcal{Z}_\nu \mathcal{Z}^\nu - \mathcal{X}_\mu^\dagger \mathcal{X}_\nu \mathcal{Z}^\mu \mathcal{Z}^\nu \right). \end{aligned} \quad (2.12)$$

The kinetic and interaction terms for the scalar are

$$\begin{aligned} \mathcal{L}_{\text{scalar}} = & \frac{1}{2} \partial_\mu \mathfrak{s} \partial^\mu \mathfrak{s} + M_{\mathcal{W}}^2 \left(1 + \frac{\mathfrak{s}}{w} \right)^2 \mathcal{W}_\mu^\dagger \mathcal{W}^\mu + \frac{1}{2} M_{\mathcal{Z}}^2 \left(1 + \frac{\mathfrak{s}}{w} \right)^2 \mathcal{Z}_\mu \mathcal{Z}^\mu \\ & + M_{\mathcal{X}}^2 \left(1 + \frac{\mathfrak{s}}{w} \right)^2 \mathcal{X}_\mu^\dagger \mathcal{X}^\mu. \end{aligned} \quad (2.13)$$

The other interaction terms between $\mathcal{W}$, $\mathcal{Z}$, $\mathcal{X}$ and $\mathcal{A}$ are

$$\begin{aligned} \mathcal{L}_{\text{other}} = & -i\mathfrak{g} \left(\mathcal{X}_\mu^\dagger T_{\mathcal{N}}^a \mathcal{X}_\nu \right) \mathcal{A}^{a\mu\nu} \\ & - \frac{\mathfrak{g}^2}{2} \left(-(\mathcal{Z}_\mu \mathcal{A}_\nu^a + \mathcal{Z}_\nu \mathcal{A}_\mu^a) \left(\mathcal{X}^{\dagger\mu} T_{\mathcal{N}}^a \mathcal{X}^\nu \right) + 2\mathcal{Z}_\mu \mathcal{A}^{\mu a} \left(\mathcal{X}_\nu^\dagger T_{\mathcal{N}}^a \mathcal{X}^\nu \right) \right) \\ & - \frac{3\mathfrak{g}^2}{8\sqrt{2}} \gamma_{\mathcal{N}}^{ij} \mathcal{X}_{[\mu}^i \mathcal{X}_{\nu]}^j \left(\mathcal{W}^{\dagger\nu} \mathcal{Z}^\mu - \mathcal{W}^{\dagger\mu} \mathcal{Z}^\nu \right) \\ & + \frac{\mathfrak{g}^2}{\sqrt{2}} \gamma_{\mathcal{N}}^{il} T_{lk}^a \left(\mathcal{X}_\mu^i \mathcal{X}^{\mu k} \mathcal{W}_\nu^\dagger \mathcal{A}^{a\nu} - \frac{1}{2} \mathcal{X}_{\{\mu}^i \mathcal{X}_{\nu\}}^k \mathcal{W}_\mu^\dagger \mathcal{A}^{a\nu} \right) + \dots, \end{aligned} \quad (2.14)$$

where $\dots$ denotes $\mathcal{WW}^\dagger \mathcal{WW}^\dagger$ and $\mathcal{XX}^\dagger \mathcal{XX}^\dagger$ interactions and

$$\begin{aligned} \mathcal{W}_{\mu\nu} &= \partial_\mu \mathcal{W}_\nu - \partial_\nu \mathcal{W}_\mu, & \mathcal{Z}_{\mu\nu} &= \partial_\mu \mathcal{Z}_\nu - \partial_\nu \mathcal{Z}_\mu, & \mathcal{X}_{\mu\nu} &= \mathcal{D}_\mu \mathcal{X}_\nu - \mathcal{D}_\nu \mathcal{X}_\mu, \\ \mathcal{A}_{\mu\nu}^a &= \partial_\mu \mathcal{A}_\nu^a - \partial_\nu \mathcal{A}_\mu^a + \mathfrak{g} f_{\mathcal{N}}^{abc} \mathcal{A}_\mu^b \mathcal{A}_\nu^c, & \mathcal{D}_\mu &= \partial_\mu - i\mathfrak{g} T_{\mathcal{N}}^a \mathcal{A}_\mu^a; \end{aligned}$$

$f_{\mathcal{N}}^{abc}$ are the structure constants and $T_{\mathcal{N}}^a$ are the generators of the $\text{Sp}(\mathcal{N})$ group in the fundamental representation. Schematically, the interaction vertices can be summarized as:

- 3-gauge vertices: $\mathcal{A}\mathcal{A}\mathcal{A}$, $\mathcal{A}\mathcal{X}\mathcal{X}^\dagger$, $\mathcal{X}\mathcal{X}\mathcal{W}^\dagger$, $\mathcal{Z}\mathcal{W}\mathcal{W}^\dagger$, $\mathcal{Z}\mathcal{X}\mathcal{X}^\dagger$;
- 4-gauge vertices: $\mathcal{A}\mathcal{A}\mathcal{A}\mathcal{A}$, $\mathcal{A}^2\mathcal{X}\mathcal{X}^\dagger$, $\mathcal{A}\mathcal{Z}\mathcal{X}\mathcal{X}^\dagger$, $\mathcal{A}\mathcal{X}^2\mathcal{W}^\dagger$, $\mathcal{Z}\mathcal{X}^2\mathcal{W}^\dagger$, $\mathcal{Z}^2\mathcal{W}\mathcal{W}^\dagger$, $\mathcal{Z}^2\mathcal{X}\mathcal{X}^\dagger$, $\mathcal{X}\mathcal{X}^\dagger\mathcal{W}\mathcal{W}^\dagger$, $(\mathcal{W}\mathcal{W}^\dagger)^2$, $(\mathcal{X}\mathcal{X}^\dagger)^2$;
- Gauge-scalar vertices: $\mathfrak{s}\mathcal{Z}\mathcal{Z}$, $\mathfrak{s}\mathfrak{s}\mathcal{Z}\mathcal{Z}$, $\mathfrak{s}\mathcal{W}\mathcal{W}^\dagger$, $\mathfrak{s}\mathfrak{s}\mathcal{W}\mathcal{W}^\dagger$, $\mathfrak{s}\mathcal{X}\mathcal{X}^\dagger$, $\mathfrak{s}\mathfrak{s}\mathcal{X}\mathcal{X}^\dagger$.

By using the eqs. (2.9)–(2.14) we can get the desired Feynman rules for the interaction vertices, which are collected in appendix A.

3 Sommerfeld enhancement and bound states

The DM relic abundance as well as indirect detection signals are determined by the non-relativistic annihilation (and co-annihilation) cross sections among DM particles and with every particle that can interact with them. By using the Feynman rules listed in appendix A, we compute explicitly these cross sections in section 4.2. Besides, the DM relic abundance and the indirect detection predictions are calculated in section 4.3 and section 5.3, respectively.

The annihilation cross sections get enhanced if the particles in the initial states can exchange a light mediator, a contribution known in the literature as Sommerfeld enhancement [10–14]. This effect can be relevant for the computation of DM relic abundance as well as (and especially) for indirect detection. Generally speaking, it can be taken into consideration by multiplying the cross sections by an enhancement factor S .

In this section we summarize how to compute the enhancement factor for all the interesting cases that occur in our model. All the interaction processes that we are interested in can be divided into two categories:

- $\mathcal{X}\mathcal{X}^\dagger$ and $\mathcal{X}\mathcal{X}$ annihilations: they can exchange a massless dark gluon $\mathcal{A}$ generating a Coulomb potential. If the state $\mathcal{X}\mathcal{X}^\dagger(\mathcal{X}\mathcal{X})$ is in the representation J of $\text{Sp}(\mathcal{N})$, the cross section gets enhanced by [4]:

$$S_{\mathcal{A}} = \frac{2\pi\alpha_{\text{eff}}/v_{\text{rel}}}{1 - e^{-2\pi\alpha_{\text{eff}}/v_{\text{rel}}}} \quad (3.1)$$

with $\alpha_{\text{eff}} = \lambda_J\alpha$, where $\alpha = \mathfrak{g}^2/4\pi$ and λ_J is a group theory factor which is computed, as explained in [4], as $\lambda_J = C_R - C_J/2$, where C_J and C_R are the Casimir for the J and fundamental representation, respectively. v_{rel} is relative velocity among two particles $\mathcal{X}$.

- all other processes: the two particles in the initial state are $\{\mathcal{W}\mathcal{W}^\dagger, \mathcal{W}\mathcal{Z}, \mathcal{W}\mathcal{X}^\dagger, \mathcal{Z}\mathcal{Z}\}$. For every case, they can exchange a scalar (with mass M_s) generating an attractive potential. In the non-relativistic limit, the potential can be expressed as:

$$V(r) = -\frac{\mathfrak{g}^2}{16\pi} \frac{e^{-M_s r}}{r}, \quad (3.2)$$

so the cross sections get enhanced by [4]:

$$S_s(M) = \frac{2\pi\alpha_{\text{eff}}}{v_{\text{rel}}} \frac{\sinh(\pi M v_{\text{rel}}/\kappa M_s)}{\cosh(\pi M v_{\text{rel}}/\kappa M_s) - \cosh(\pi M v_{\text{rel}}\sqrt{1 - 4\alpha_{\text{eff}}\kappa M_s/M v_{\text{rel}}^2}/\kappa M_s)}, \quad (3.3)$$

where $\kappa \approx 1.74$, $\alpha_{\text{eff}} = \mathfrak{g}^2/16\pi$, v_{rel} is relative velocity among two particles of mass M . Under the thermal equilibrium assumption, v_{rel} can be estimated as $v_{\text{rel}} = \sqrt{2}v = \sqrt{16T/\pi M}$ (if the two particles have different masses (i.e. M_1 and M_2), one can use $v_{\text{rel}} = \sqrt{8T(M_1 + M_2)/\pi M_1 M_2}$).

There is a second phenomenon, which is related to Sommerfeld enhancement, can affect both the relic abundance and the indirect detection cross section: two scattering particles can form bound states through the exchange of a light mediator [4, 5, 15–21]. In particular, the interesting process is the formation of bound states via emission of the corresponding mediator (if the mediator is too heavy one can consider processes with emission of lighter particles, such as light glueballs).

The physics is very different for massive or massless mediators: if the mediator is massive, which is the case for the scalon exchange (with mass M_s) among $\mathcal{W}$, $\mathcal{X}$, and $\mathcal{Z}$ that we mentioned above, bound states can exist only if they satisfy the condition $\alpha_{\text{eff}}M > \kappa M_s$ (for the ground state; for excited states the condition is even stronger) [4, 5], where $\alpha_{\text{eff}} = \mathfrak{g}^2/16\pi$ and $M = \{M_{\mathcal{W}}, M_{\mathcal{X}}\}$ is the mass of the particles involved in the scattering. However, this condition is never satisfied in our model, so there will be no bound states generated by potential (16).

If instead the mediator is massless, which is the case for dark gluons exchange between $\mathcal{X}\mathcal{X}^\dagger(\mathcal{X}\mathcal{X})$, the corresponding potential is Coulombian, and therefore bound states are always allowed. In particular one gets the usual infinity of bound states with binding energies $E_n = \alpha_{\text{eff}}^2 M_{\mathcal{X}}/4n^2$, with α_{eff} as in eq. (3.1).

4 DM relic abundance

In this section we compute the cosmological relic abundance of DM by solving the related Boltzmann equations. As first step we consider perturbative annihilations and semi-annihilations of co-stable $\mathcal{W}$ and $\mathcal{X}$, which occur at high temperatures. Then we take into account the non-perturbative dynamics effects, i.e. the confinement of the $\text{Sp}(\mathcal{N})$ gauge group and the corresponding hadronization that become important at lower temperatures, in section 4.3.

We recall that for $\mathcal{N} = 0$ our model coincides with the $\text{Sp}(2) = \text{SU}(2)$ model, which has already been studied in [3, 7, 8]. Therefore, we will focus on the original cases for $\mathcal{N} \geq 2$.

4.1 Boltzmann equations

Notations. We use the following notations:

- we define a dimensionless parameter $z = M_{\mathcal{X}}/T$, where T is the temperature of the universe;
- $H = (T^2/M_{\text{P}})\sqrt{4\pi^3 g_{*s}/45}$ is the Hubble constant (M_{P} is the Planck mass);
- $s = 2\pi^2 g_{*s} T^3/45$ is the entropy density;
- g_{*s} is the number of relativistic d.o.f. If freeze-out of DM happens before EW phase transition, $g_{*s} = 106.75 + \mathcal{N}(\mathcal{N}+1) + 1$ where the extra d.o.f. comes from the massless dark gluons $\mathcal{A}$ and the presumably light scalon $\mathfrak{s}$;

- $Y_i = Y_i(z) = n_i/s$ is the number density of the i species divided by the entropy density;
- $Y_{i,\text{eq}} = Y_{i,\text{eq}}(z)$ is the equilibrium distribution of the i species. Before DM thermal freeze-out, all species are in thermal equilibrium. At freeze-out all species except DM ($\mathcal{W}$ and $\mathcal{X}$) keep staying in thermal equilibrium. DM is non-relativistic at freeze-out (which occurs roughly at $z \sim 25$). We remind that the equilibrium distribution for a non-relativistic species is given by $Y_{i,\text{eq}} \simeq 0.145 g_i (M_i/T)^{3/2} e^{-M_i/T} / g_{*s}$ where M_i is the mass of the particle and g_i its number of d.o.f. ($g_{\mathcal{W}} = 6$, $g_{\mathcal{X}} = 6\mathcal{N}$, $g_{\mathcal{Z}} = 3$);
- $\langle \sigma v \rangle(i \rightarrow j)$ is the thermal-averaged cross section for the $i \rightarrow j$ process in the non-relativistic limit; assuming s-wave annihilations we can take $\langle \sigma v \rangle \simeq \sigma_0$.

Numerical factors. We use the following convention in computing the cross sections and deriving the Boltzmann equations:

- a factor $1/2$ for identical particles in the final states is included in the cross sections;
- a factor $\kappa = 1/2$ for complex DM annihilations is included in the cross sections (more precisely $\kappa = 1/2$ if the initial states are both complex; $\kappa = 1$ if the initial states are both real or one is real and one is complex);
- we put a factor $1/2$ for identical particles in the initial state in the equations;
- we put a factor $\pm \Delta_i$ if in the process the number of DM particles of kind “ i ” increases (decreases) of Δ_i units.

With the above notations and numerical factors, we can write the general form of the Boltzmann equations as

$$\left(\frac{Hz}{s} \right) \frac{dY_i}{dz} = \sum_{\text{processes}} \Delta_i S \langle \sigma v \rangle (12 \rightarrow 34) \left[Y_1 Y_2 - \frac{Y_3 Y_4}{Y_3^{\text{eq}} Y_4^{\text{eq}}} Y_1^{\text{eq}} Y_2^{\text{eq}} \right], \quad (4.1)$$

where $S = 1/2$ if the initial particles are identical and $S = 1$ for other cases.

List of (semi-)annihilation processes. All the kinematics permitted annihilation and semi-annihilation processes that can change the DM number of 1 or 2 units are listed below (Note that some processes, e.g. $\mathcal{W} \rightarrow \mathcal{X}\mathcal{X}$, are forbidden by energy-momentum conservation):

- $\mathcal{W}^\dagger \mathcal{W} \rightarrow \mathcal{X}^\dagger \mathcal{X}, \mathfrak{s}\mathfrak{s}, \mathcal{Z}\mathfrak{s};$
- $\mathcal{X}^\dagger \mathcal{X} \rightarrow \mathcal{A}\mathcal{A}, \mathfrak{s}\mathfrak{s}, \mathcal{Z}\mathfrak{s}, \mathcal{A}\mathfrak{s}, \mathcal{Z}\mathcal{A};$
- $\mathcal{W}\mathcal{Z} \rightarrow \mathcal{X}\mathcal{X};$
- $\mathcal{X}^\dagger \mathcal{W} \rightarrow \mathcal{A}\mathcal{X}, \mathfrak{s}\mathcal{X};$
- $\mathcal{X}\mathcal{X} \rightarrow \mathcal{A}\mathcal{W}, \mathfrak{s}\mathcal{W};$
- $\mathcal{Z}\mathcal{Z} \rightarrow \mathcal{X}^\dagger \mathcal{X}.$

Equations. By using the eq. (4.1) and all (semi-)annihilation processes mentioned above, we can get the following coupled Boltzmann equations for the number densities of $\mathcal{W}$ and $\mathcal{X}$:

$$\begin{aligned}
 \frac{Hz}{s} \frac{dY_{\mathcal{W}}}{dz} = & -2\langle\sigma v\rangle(\mathcal{W}\mathcal{W}^\dagger \rightarrow \mathcal{X}\mathcal{X}^\dagger) \left(Y_{\mathcal{W}}^2 - \frac{Y_{\mathcal{X}}^2}{Y_{\mathcal{X},\text{eq}}^2} Y_{\mathcal{W},\text{eq}}^2 \right) \\
 & -2\langle\sigma v\rangle(\mathcal{W}\mathcal{W}^\dagger \rightarrow ss + \mathcal{Z}s) (Y_{\mathcal{W}}^2 - Y_{\mathcal{W},\text{eq}}^2) \\
 & -\langle\sigma v\rangle(\mathcal{W}\mathcal{Z} \rightarrow \mathcal{X}\mathcal{X}) \left(Y_{\mathcal{W}}Y_{\mathcal{Z},\text{eq}} - \frac{Y_{\mathcal{X}}^2}{Y_{\mathcal{X},\text{eq}}^2} Y_{\mathcal{W},\text{eq}}Y_{\mathcal{Z},\text{eq}} \right) \\
 & -\langle\sigma v\rangle(\mathcal{W}\mathcal{X}^\dagger \rightarrow \mathcal{A}\mathcal{X} + s\mathcal{X}) (Y_{\mathcal{W}}Y_{\mathcal{X}} - Y_{\mathcal{X}}Y_{\mathcal{W},\text{eq}}) \\
 & + \frac{1}{2}\langle\sigma v\rangle(\mathcal{X}\mathcal{X} \rightarrow \mathcal{A}\mathcal{W} + s\mathcal{W}) \left(Y_{\mathcal{X}}^2 - \frac{Y_{\mathcal{W}}}{Y_{\mathcal{W},\text{eq}}} Y_{\mathcal{X},\text{eq}}^2 \right) \quad (4.2)
 \end{aligned}$$

$$\begin{aligned}
 \frac{Hz}{s} \frac{dY_{\mathcal{X}}}{dz} = & 2\langle\sigma v\rangle(\mathcal{W}\mathcal{W}^\dagger \rightarrow \mathcal{X}\mathcal{X}^\dagger) \left(Y_{\mathcal{W}}^2 - \frac{Y_{\mathcal{X}}^2}{Y_{\mathcal{X},\text{eq}}^2} Y_{\mathcal{W},\text{eq}}^2 \right) \\
 & -2\langle\sigma v\rangle(\mathcal{X}\mathcal{X}^\dagger \rightarrow ss + \mathcal{A}s + \mathcal{A}\mathcal{A} + \mathcal{Z}\mathcal{A} + \mathcal{Z}s) (Y_{\mathcal{X}}^2 - Y_{\mathcal{X},\text{eq}}^2) \\
 & + 2\langle\sigma v\rangle(\mathcal{W}\mathcal{Z} \rightarrow \mathcal{X}\mathcal{X}) \left(Y_{\mathcal{W}}Y_{\mathcal{Z},\text{eq}} - \frac{Y_{\mathcal{X}}^2}{Y_{\mathcal{X},\text{eq}}^2} Y_{\mathcal{W},\text{eq}}Y_{\mathcal{Z},\text{eq}} \right) \\
 & -\langle\sigma v\rangle(\mathcal{X}\mathcal{X} \rightarrow s\mathcal{W} + \mathcal{A}\mathcal{W}) \left(Y_{\mathcal{X}}^2 - \frac{Y_{\mathcal{W}}}{Y_{\mathcal{W},\text{eq}}} Y_{\mathcal{X},\text{eq}}^2 \right) \\
 & + \langle\sigma v\rangle(\mathcal{Z}\mathcal{Z} \rightarrow \mathcal{X}\mathcal{X}^\dagger) \left(Y_{\mathcal{Z},\text{eq}}^2 - \frac{Y_{\mathcal{X}}^2}{Y_{\mathcal{X},\text{eq}}^2} Y_{\mathcal{Z},\text{eq}}^2 \right). \quad (4.3)
 \end{aligned}$$

4.2 Non-relativistic cross sections

We compute the tree-level non-relativistic cross section at the leading order in the relative velocity v_{rel} , i.e. taking into account only the s -wave contribution σ_0 (averaging over initial spin and gauge components, and multiplying by $\kappa = 1/2$ for 2 complex DM particles in the initial state or $\kappa = 1$ for other cases, as pointed out in the previous section). In this approximation, the thermal averaged cross section is simply $\langle\sigma v\rangle = \sigma_0$. For the group theory factors, we use the following notations:

- d_R is the dimension of the fundamental representation of $\text{Sp}(\mathcal{N})$, i.e. $d_R = \mathcal{N}$;
- d_G is the dimension of the adjoint representation of $\text{Sp}(\mathcal{N})$, i.e. $d_G = \mathcal{N}(\mathcal{N} + 1)/2$;
- C_R is the Casimir for the fundamental representation of $\text{Sp}(\mathcal{N})$, i.e. $C_R = (\mathcal{N} + 1)/4$;
- C_G is the Casimir for the adjoint representation of $\text{Sp}(\mathcal{N})$, i.e. $C_G = (\mathcal{N} + 2)/2$;
- The Dynkin Index of the fundamental representation of $\text{Sp}(\mathcal{N})$ is normalized as $1/2$ as explained in section 2.1.

The cross sections of all the relevant processes can be expressed as:

$$\langle\sigma v\rangle(\mathcal{W}\mathcal{W}^\dagger \rightarrow \mathfrak{s}\mathfrak{s}) = \frac{11M_{\mathcal{W}}^2}{288\pi w^4} = \frac{11\mathfrak{g}^4}{4608\pi M_{\mathcal{W}}^2}, \quad (4.4)$$

$$\langle\sigma v\rangle(\mathcal{X}^\dagger\mathcal{X} \rightarrow \mathfrak{s}\mathfrak{s}) = \frac{11M_{\mathcal{X}}^2}{288\pi d_R w^4} = \frac{11\mathfrak{g}^4}{4608\pi \mathcal{N} M_{\mathcal{X}}^2} = \frac{11\mathfrak{g}^4}{9216\pi \mathcal{N} M_{\mathcal{W}}^2}, \quad (4.5)$$

$$\langle\sigma v\rangle(\mathcal{W}\mathcal{W}^\dagger \rightarrow \mathfrak{s}\mathcal{Z}) = \frac{\mathfrak{g}^2(4M_{\mathcal{W}}^2 - M_{\mathcal{Z}}^2)^3}{576\pi M_{\mathcal{W}}^6 w^2} = \frac{3\mathfrak{g}^4}{256\pi M_{\mathcal{W}}^2}, \quad (4.6)$$

$$\langle\sigma v\rangle(\mathcal{X}^\dagger\mathcal{X} \rightarrow \mathfrak{s}\mathcal{Z}) = \frac{\mathfrak{g}^2(4M_{\mathcal{W}}^2 - M_{\mathcal{Z}}^2)^3}{2304\pi d_R M_{\mathcal{X}}^6 w^2} = \frac{\mathfrak{g}^4}{1152\pi \mathcal{N} M_{\mathcal{W}}^2}, \quad (4.7)$$

$$\langle\sigma v\rangle(\mathcal{X}^\dagger\mathcal{X} \rightarrow \mathcal{A}\mathcal{A}) = \frac{19C_R(4C_R - C_G)\mathfrak{g}^4}{576\pi d_R M_{\mathcal{X}}^2} = \frac{19\mathfrak{g}^4(\mathcal{N} + 1)}{2304\pi M_{\mathcal{W}}^2}, \quad (4.8)$$

$$\langle\sigma v\rangle(\mathcal{X}^\dagger\mathcal{X} \rightarrow \mathcal{A}\mathfrak{s}) = \frac{C_R\mathfrak{g}^2}{9\pi d_R w^2} = \frac{\mathfrak{g}^4(\mathcal{N} + 1)}{144\pi \mathcal{N} M_{\mathcal{W}}^2}, \quad (4.9)$$

$$\langle\sigma v\rangle(\mathcal{W}\mathcal{W}^\dagger \rightarrow \mathcal{X}^\dagger\mathcal{X}) = \frac{225\mathfrak{g}^4 d_R}{8192\sqrt{2}\pi M_{\mathcal{X}}^2} = \frac{225\mathfrak{g}^4 \mathcal{N}}{4096\sqrt{2}\pi M_{\mathcal{W}}^2}, \quad (4.10)$$

$$\langle\sigma v\rangle(\mathcal{W}\mathcal{Z} \rightarrow \mathcal{X}\mathcal{X}) = \frac{5\mathfrak{g}^4 d_R}{576\sqrt{2}\pi M_{\mathcal{X}}^2} = \frac{5\mathfrak{g}^4 \mathcal{N}}{288\sqrt{2}\pi M_{\mathcal{W}}^2}, \quad (4.11)$$

$$\langle\sigma v\rangle(\mathcal{X}\mathcal{X} \rightarrow \mathfrak{s}\mathcal{W}) = \frac{\mathfrak{g}^2(4M_{\mathcal{X}}^2 - M_{\mathcal{W}}^2)^3}{1152\pi d_R M_{\mathcal{X}}^6 w^2} = \frac{\mathfrak{g}^4}{576\pi \mathcal{N} M_{\mathcal{W}}^2}, \quad (4.12)$$

$$\langle\sigma v\rangle(\mathcal{W}\mathcal{X}^\dagger \rightarrow \mathfrak{s}\mathcal{X}) = \frac{(30 - 13\sqrt{2})\mathfrak{g}^4}{1152\pi M_{\mathcal{W}}^2}, \quad (4.13)$$

$$\langle\sigma v\rangle(\mathcal{Z}\mathcal{Z} \rightarrow \mathcal{X}^\dagger\mathcal{X}) = \frac{1705\mathfrak{g}^4 d_R}{18432\sqrt{2}\pi M_{\mathcal{Z}}^2} = \frac{1705\mathfrak{g}^4 \mathcal{N}}{18432\sqrt{2}\pi M_{\mathcal{W}}^2}, \quad (4.14)$$

$$\begin{aligned} \langle\sigma v\rangle(\mathcal{X}^\dagger\mathcal{X} \rightarrow \mathcal{Z}\mathcal{A}) &= \frac{g^4 d_G}{576\pi M_{\mathcal{X}}^2 d_R^2} \left(1 - \frac{M_{\mathcal{Z}}^2}{4M_{\mathcal{X}}^2}\right) \left(19 + \frac{M_{\mathcal{Z}}^2}{M_{\mathcal{X}}^2} + \frac{M_{\mathcal{Z}}^4}{4M_{\mathcal{X}}^4}\right) \\ &= \frac{11C_R\mathfrak{g}^2}{36\pi d_R w^2} = \frac{11\mathfrak{g}^4(\mathcal{N} + 1)}{576\pi \mathcal{N} M_{\mathcal{W}}^2}, \end{aligned} \quad (4.15)$$

$$\langle\sigma v\rangle(\mathcal{X}\mathcal{X} \rightarrow \mathcal{A}\mathcal{W}) = \frac{11C_R\mathfrak{g}^2}{18\pi d_R w^2} = \frac{11\mathfrak{g}^4(\mathcal{N} + 1)}{288\pi \mathcal{N} M_{\mathcal{W}}^2}, \quad (4.16)$$

$$\langle\sigma v\rangle(\mathcal{W}\mathcal{X}^\dagger \rightarrow \mathcal{A}\mathcal{X}) = \frac{(32 - 21\sqrt{2})C_R\mathfrak{g}^4}{72\pi M_{\mathcal{W}}^2} = \frac{(32 - 21\sqrt{2})\mathfrak{g}^4(\mathcal{N} + 1)}{144\pi M_{\mathcal{W}}^2}. \quad (4.17)$$

As a check of our computations, we have verified that all the cross sections scale as $1/s$ in the ultra-relativistic limit.²

4.3 Relic abundance

If the dynamics is purely perturbative, the DM abundance is given by the sum of $\mathcal{W}$ and $\mathcal{X}$ abundances, and one can use the following expressions (the subscript 0 means quantities

²For $\mathcal{N} = 0$, cross sections involving $\mathcal{A}$ and $\mathcal{X}$ vanish, $\mathcal{Z}$ becomes DM forming a degenerate triplet with $\mathcal{W}$, and the results in [8] is reproduced taking into account the extra cross sections $2\langle\sigma v\rangle(\mathcal{W}\mathcal{W}^\dagger \rightarrow \mathfrak{s}\mathfrak{s}) = \langle\sigma v\rangle(\mathcal{Z}\mathcal{Z} \rightarrow \mathfrak{s}\mathfrak{s})$ and $2\langle\sigma v\rangle(\mathcal{W}\mathcal{W}^\dagger \rightarrow \mathcal{Z}\mathfrak{s}) = \langle\sigma v\rangle(\mathcal{W}\mathcal{Z} \rightarrow \mathcal{W}\mathfrak{s}) = 3\mathfrak{g}^4/128\pi M_{\mathcal{W}}^2$.

evaluated at the present time):

$$\Omega_{\text{DM}} = \frac{\rho_{\text{DM}}}{\rho_{\text{cr}}} = \frac{s_0}{3H_0^2/8\pi G} (M_{\mathcal{W}}Y_{\mathcal{W}0} + M_{\mathcal{X}}Y_{\mathcal{X}0}) \quad (4.18)$$

or

$$\frac{\Omega_{\text{DM}}h^2}{0.110} = \frac{M_{\mathcal{W}}}{0.4\text{eV}} \left(Y_{\mathcal{W}0} + \frac{Y_{\mathcal{X}0}}{\sqrt{2}} \right). \quad (4.19)$$

Instead, in our case the $\text{Sp}(\mathcal{N})$ dynamics confines at the scale Λ_{DC} , given by eq. (2.7). After the confinement, $\mathcal{X}$ can form two kinds of dark mesons: (1) the meson $\mathcal{X}^\dagger\mathcal{X}$ (and $\mathcal{X}^\dagger\mathcal{D}_\mu\mathcal{X}$), which carry no dark-baryon number and decay into glueballs through the $\mathcal{X}\mathcal{X}^\dagger\mathcal{A}\mathcal{A}$ vertex; (2) the mesons $\mathcal{M} \sim \mathcal{X}^T\gamma_{\mathcal{N}}\mathcal{D}_\mu\mathcal{X}$ with dark-baryon number 2. The cubic vertex $\mathcal{X}\mathcal{X}\mathcal{W}^\dagger$ becomes a $\mathcal{M}\mathcal{W}$ mass mixing. In the limit of $\Lambda_{\text{DC}} \ll w$, their masses are $M_{\mathcal{M}} \simeq \sqrt{2}M_{\mathcal{W}}$ so that $\mathcal{M}$ will decay into $\mathcal{W}$ and glue-balls through the $\mathcal{A}\mathcal{X}\mathcal{X}\mathcal{W}^\dagger$ vertex (we assume that decays are faster than $\mathcal{M}^*\mathcal{M}$ annihilations), leaving $\mathcal{W}$ as the only DM candidate.

We take into account the confinement of $\text{Sp}(\mathcal{N})$ dynamics and the corresponding hadronization under the following simplified assumptions:

- confinement of $\text{Sp}(\mathcal{N})$ dynamics occurs after the perturbative freeze-out of $\mathcal{W}$ and $\mathcal{X}$, i.e. $\Lambda_{\text{DC}} \ll T_f \sim M_{\mathcal{W}}/25$ (this is roughly the correct order of magnitude for T_f , and can be verified through the numerical solutions of Boltzmann equations as shown in figure 1). This gives an upper limit on $\mathfrak{g}$ that is, using eq. (2.7):

$$\mathfrak{g}^2 \ll \frac{48\pi^2}{11(\mathcal{N}+2)\log 25}. \quad (4.20)$$

In the opposite regime confinement occurs before DM decoupling and invalidates our computations.

- about half $\mathcal{X}$ form charged mesons $\mathcal{M}$: whenever two $\mathcal{X}$ meet, they can form either the meson $\mathcal{X}^\dagger\mathcal{X}$ or the meson $\mathcal{M} \sim \mathcal{X}\mathcal{X}$, and each case has a 50% chance. However, only mesons $\mathcal{M}$ contribute to $\mathcal{W}$ abundance.
- all the mesons $\mathcal{M}$ decay to $\mathcal{W}$ (+ glueballs/scalon): these processes occurs through $\mathcal{X}\mathcal{X}\mathcal{A}\mathcal{W}$ and other vertices. As we have already pointed out, we assume that $\mathcal{M}$ decays always faster than $\mathcal{M}^*\mathcal{M}$ annihilations.

Under these assumptions, the DM relic abundance is given by

$$\frac{\Omega_{\text{DM}}h^2}{0.110} = \frac{M_{\mathcal{W}}}{0.4\text{eV}} \left(Y_{\mathcal{W}0} + \frac{Y_{\mathcal{X}0}}{4} \right). \quad (4.21)$$

Sommerfeld enhancement and bound states. In this subsection we discuss the effects of the Sommerfeld enhancement factors and (perturbative) bound states formation on the relic abundance. Following the analysis in section 3, we compute the Sommerfeld enhancement factors for the (semi-)annihilation cross sections in 4.2. Then we solve the Boltzmann equations to get the DM relic abundance as explained in section 4.3 and we compare the results obtained (i) with the enhancement factors and (ii) without the enhancement factors. We find that the difference between (i) and (ii) is negligible ($\lesssim 3\%$). So we conclude

that Sommerfeld enhancement gives only small corrections to the relic abundance and can be safely ignored (notice that the contribution of the Sommerfeld enhancement to indirect detection cross section is instead not negligible as we explain in section 5.3).

In principle, the formation of Coulombian bound states from $\mathcal{X}\mathcal{X}^\dagger(\mathcal{X}\mathcal{X})$ states through the exchange of dark gluons $\mathcal{A}$ (discussed in section 3) can also affect the relic abundance. However, the order of magnitude of this effect is roughly comparable to Sommerfeld corrections. Generally speaking, the bound states formation is relevant whenever Sommerfeld enhancement is significant. As we discussed above, this is not the case for our model. Therefore, we can reasonably neglect these Coulombian bound states in our computation.

As a final remark we notice that the effect of both the Sommerfeld enhancement factors and the perturbative bound state formation on the relic abundance is reasonably subleading if compared to the uncertainties due to the process of hadronization that occurs at $T \simeq \Lambda_{\text{DC}}$, when the theory confines, which we discussed in section 4.3.

Glueballs dilution. Dark glueballs (DG) can decay to SM particles through the Higgs portal. The one-loop effective interaction between the scalon $\mathfrak{s}$ and the dark gluons $\mathcal{A}$ has been computed in [1] as:

$$\mathcal{L}_{\text{eff}}^{\mathfrak{s}\mathcal{A}\mathcal{A}} = -\frac{7\alpha}{16\pi^2}(\mathcal{A}_{\mu\nu})^2 \left(\frac{\mathfrak{s}}{w} - \frac{\mathfrak{s}^2}{2w^2} + \dots \right) \quad (4.22)$$

with $\alpha = \mathfrak{g}^2/4\pi$. The lifetime of glueballs can be estimated as $\tau_{\text{DG}} = \Gamma_{\text{DG}}^{-1}$, which depends on the mass ordering. If the mass of the glueballs is larger than the weak scale, they decay into Higgs components as [1]:

$$\Gamma_{\text{DG}}(\text{DG} \rightarrow \mathfrak{s} \rightarrow H^\dagger H = hh + ZZ + WW) = \frac{49f_{\text{DG}}^2\alpha^2\lambda_{HS}^2}{2048\pi^3 M_{\text{DG}} M_{\mathfrak{s}}^4} \text{Re} \sqrt{1 - \frac{4M_{h,W,Z}^2}{M_{\text{DG}}^2}}. \quad (4.23)$$

The glueballs life-time can instead become cosmologically large if glueballs are enough lighter than the weak scale; in this case we get [1]:

$$\Gamma_{\text{DG}} = \left(\frac{7\alpha f_{\text{DG}}}{32\pi w} \right)^2 \sin^2 2\gamma \left(\frac{1}{M_{S_1}^2 - M_{\text{DG}}^2} - \frac{1}{M_{S_2}^2 - M_{\text{DG}}^2} \right)^2 \Gamma_{h_{\text{DG}}}, \quad (4.24)$$

where $M_{\text{DG}} \approx 7\Lambda_{\text{DC}}$ is the mass of the glueball [22], $f_{\text{DG}} \sim M_{\text{DG}}^3$ is a dark matrix element, $\Gamma_{h_{\text{DG}}}$ is the decay width of a SM Higgs with mass M_{DG} , γ is the mixing angle that diagonalises the Higgs-scalon mass matrix, M_{S_i} are the mass eigenvalues. In more detail we have:

$$\sin 2\gamma = \frac{v^2 \sqrt{8\lambda_H \lambda_{HS}}}{M_{S_2}^2 - M_{S_1}^2} \quad (4.25)$$

with $M_{S_2} \approx M_{\mathfrak{s}} = \sqrt{\beta\lambda_S}w$ and $M_{S_1} \approx M_h \simeq 125 \text{ GeV}$. If the lifetime is long enough ($\tau_{\text{DG}} \gg t_{\Lambda_{\text{DC}}}$, where $t_{\Lambda_{\text{DC}}}$ is the time at which confinement occurs, estimated by $H(T = \Lambda_{\text{DC}}) = t_{\Lambda_{\text{DC}}}^{-1}$), glueballs can dominate the energy density of the Universe while decaying into SM particles, and the reheating effect dilutes the DM density as $Y_{\text{diluted}} = YD$ [23].

The dilution factor is estimated as:

$$D = \frac{1}{1 + \frac{g_{\text{DG}}}{g_{\text{SM}}} \left(\frac{\Lambda_{\text{DG}}^2/M_P}{\Gamma_{\text{DG}}} \right)^{\frac{2}{3}}} \quad (4.26)$$

with $g_{\text{SM}} = 106.75$ and $g_{\text{DG}} = \mathcal{N}(\mathcal{N} + 1)$. This effect is taken into account into the computation by multiplying the r.h.s. of eq. (4.21) by a factor of D . However, it becomes irrelevant if the lifetime is very short.

We define the mixing angle ϵ between the Higgs and the dark glueball from eq. (4.24):

$$\epsilon = \frac{7\alpha f_{\text{DG}}}{32\pi w} \sin 2\gamma \left(\frac{1}{M_{S_1}^2 - M_{\text{DG}}^2} - \frac{1}{M_{S_2}^2 - M_{\text{DG}}^2} \right). \quad (4.27)$$

In figure 2 we show the predicted Higgs-glueball mixing angle ϵ^2 as a function of the glueball mass M_{DG} with an assumption that some extra new physics gives fast glueball decays through different channels. We also show the current limits at LHC (shaded regions) and the future sensitivity of SHiP experimental proposal [24] (dashed line).

The predicted DM relic density as a function of $\mathbf{g}^2$ and $M_{\mathcal{W}}$ is plotted in figure 1. In the red region the overall DM abundance turns out to be larger than the cosmological value ($\Omega_{\text{DM}} h^2 = 0.11$), while the green region indicates $\Omega_{\text{DM}} h^2 < 0.11$. The red-solid line is obtained by numerical integration of the Boltzmann equations and taking into account the dilution factor in eq. (4.26) due to long-lived glueballs. We also show, as the red dashed line, the result obtained by assuming that some extra new physics gives fast glueball decays (i.e. no dilution factor). Besides, the limitations of current (future) direct [25] and indirect [26, 27] detections are also displayed with black solid (dashed) lines, respectively. Future perspective on indirect detection (CTA) [6] can explore a significant region of the parameter space especially in the case in which glueballs decay very fast.

In figure 1 (bottom right corner) we show the solutions of the Boltzmann equations for values of the parameters such that the DM abundance is reproduced. We see some modifications with respect to the conventional WIMPs behaviour. In particular at freeze-out (roughly $z \sim 25$) the $\mathcal{W}$ abundance is initially highly suppressed with respect to $\mathcal{X}$ because of the mass gap between the two states, but it suddenly increases because of the conversion processes $\mathcal{X}\mathcal{X} \rightarrow \mathfrak{s}\mathcal{W}$ and $\mathcal{X}\mathcal{X} \rightarrow \mathcal{A}\mathcal{W}$.

5 Signals and bounds

5.1 Scalon production

The scalon $\mathfrak{s}$ behaves like an extra Higgs boson with couplings rescaled by $\sin \gamma$, where the mixing angle γ is given by eq. (4.25). We show in figure 2 (left) the predicted value of $\sin^2 \gamma$, which is equal to the production cross section for the scalon in SM Higgs cross section units, as a function of the scalon mass $M_{\mathfrak{s}}$. We assume that some extra new physics gives fast glueball decays (if glueballs decay slowly the production cross section is suppressed by many order of magnitude and not testable at current/future experiments). Various present and projected constraints from Higgs measurements and direct searches are also shown [28, 29].

Figure 1. The DM abundance is reproduced ($\Omega_{\text{DM}} h^2 = 0.11$) along the boundaries between the red/green regions: the red-solid line is obtained taking into account the dilution factor due to the long-lived glueballs, while the red-dashed line is computed with the assumption that some extra new physics gives fast glueball decays. The current (future) limits of direct and indirect detections are also shown in black solid (dashed) line, respectively. The left gray region is excluded by BBN because of too slow DG decays (of course it is not excluded if we assume extra DG decays). We show predictions for different values of $\mathcal{N}$. In the last plot we show the solutions of the Boltzmann equations for values of the parameters ($\mathcal{N} = 4$, $\mathbf{g} = 1.2$, $M_W = 10.5$ TeV) such that the cosmological DM abundance is reproduced. We find that the freeze-out temperature is roughly $T_f \sim M_W/25 \gg \Lambda_{\text{DC}}$, which is in agreement with our assumptions.

In particular, it can be seen that measuring Higgs couplings with a 10^{-3} precision, which can be attained at future lepton colliders, would allow to probe a significant region of the parameter space.

5.2 Direct detection

As pointed out in [1] the spin-independent cross section for direct detection is the same as in the SU(2) model, and it can be expressed as

$$\sigma_{\text{SI}} = \mathbf{g}^2 \sin^2 2\gamma \frac{m_N^4 f^2}{16\pi v^2} \left(\frac{1}{M_{\tilde{S}_1}^2} - \frac{1}{M_{\tilde{S}_2}^2} \right)^2, \quad (5.1)$$

Figure 2. On the left: predicted production cross section for the scalon, under the assumption that $\mathcal{W}$ reproduce the full DM abundance of the Universe and some extra new physics gives fast glueball decays. Signals are very suppressed if glueballs decay slowly. On the right: predicted Higgs-glueball mixing angle as a function of the glueball mass M_{DG} with the same assumptions.

where $f \simeq 0.3$ is a nuclear matrix element. In figure 3 (top left corner) we plot the DM direct detection cross section as a function of the DM mass under the assumptions that $\mathcal{W}$ reproduce the full DM abundance and some extra new physics gives fast glueball decays. We also show the constraints from Xenon1T [25] as well as neutrino floor. If glueballs decay very slowly, the direct detection cross section is suppressed and not observable at current and future experiments, as we can observe in figure 1.

5.3 Indirect detection

After the confinement of the gauge subgroup $\text{Sp}(\mathcal{N})$ below the energy scale Λ_{DC} , $\mathcal{W}$ becomes the only DM candidate. When $\mathcal{W}$ and $\mathcal{W}^\dagger$ annihilate with each other, they can produce indirect detection signals, which can be expressed as

$$\sigma_{\mathcal{W}\mathcal{W}^\dagger v_{\text{rel}}} = \langle \sigma v \rangle (\mathcal{W}\mathcal{W}^\dagger \rightarrow \mathfrak{s}\mathfrak{s}, \mathfrak{s}\mathcal{Z}, \text{dark hadrons}), \quad (5.2)$$

where the dark hadrons are glueballs ($\mathcal{A}\mathcal{A}$ bound state) and the uncharged mesons ($\mathcal{X}^\dagger \mathcal{X}$ bound state). The energy of the scattering processes is roughly $M_{\mathcal{W}} \gg \Lambda_{DC}$, so we can use the perturbative results in section 4.2. In particular $\langle \sigma v \rangle (\mathcal{W}\mathcal{W}^\dagger \rightarrow \mathcal{A}\mathcal{A})$ is one-loop suppressed, so the relevant cross section into dark hadrons is given by $\langle \sigma v \rangle (\mathcal{W}\mathcal{W}^\dagger \rightarrow \mathcal{X}^\dagger \mathcal{X})$. Annihilations products will further decay into SM particles. The total annihilation cross section relevant for indirect detection is:

$$\sigma_{\mathcal{W}\mathcal{W}^\dagger v_{\text{rel}}} = \frac{\mathfrak{g}^4}{256\pi M_{\mathcal{W}}^2} \left(\frac{65}{18} + \frac{225}{16\sqrt{2}} \mathcal{N} \right). \quad (5.3)$$

We take into account the Sommerfeld enhancement considering the scalon exchange among $\mathcal{W}$ and $\mathcal{W}^\dagger$ through the $\mathfrak{s}\mathcal{W}\mathcal{W}^\dagger$ interactions. The cross section get enhanced as

$$(\sigma_{\mathcal{W}\mathcal{W}^\dagger v_{\text{rel}}})_{\text{enhanced}} = S \sigma_{\mathcal{W}\mathcal{W}^\dagger v_{\text{rel}}}. \quad (5.4)$$

Figure 3. The first plot: predicted cross section for DM direct detection under the assumption that $\mathcal{W}$ reproduce the full DM abundance of the Universe and some extra new physics gives fast glueball decays. Other plots: predicted annihilations cross sections for indirect detection under the assumption that $\mathcal{W}$ reproduce the full DM abundance of the Universe, for different values of $\mathcal{N}$. The solid lines are obtained taking into account the dilution due to long-lived glueballs; the dashed lines are obtained assuming that some extra new physics gives fast glueball decays. The red lines take into account the Sommerfeld enhancement in indirect detection cross section. The vertical dashed line is the boundary between the perturbative and condensed phase, which is estimated as the scale at which $M_s \approx \Lambda_{DC}$.

For $v_{\text{rel}} \ll M_s/M_{\mathcal{W}}$, which is the relevant regime for indirect detection, the eq. (3.3) can be further simplified as [4]

$$S = \frac{2\pi^2 \alpha_{\text{eff}} M_{\mathcal{W}}}{\kappa M_s} \left(1 - \cos 2\pi \sqrt{\frac{\alpha_{\text{eff}} M_{\mathcal{W}}}{\kappa M_s}} \right)^{-1} \quad (5.5)$$

with $\kappa \simeq 1.74$ and $\alpha_{\text{eff}} = g^2/16\pi$. The indirect detection cross section is not affected by formation of perturbative bound states: indeed, as we have pointed out at the end of

section 3, the potential (3.2) doesn't allow for the existence of $WW^\dagger$ bound states, for the values of the parameters of the model.

In figure 3 we plot the DM annihilation cross section as a function of the DM mass under the assumption that $\mathcal{W}$ reproduce the full DM abundance. The solid lines are obtained by taking into account the dilution due to long-lived glueballs, while the dashed lines assume that some extra new physics gives fast glueball decays. The blue lines have been obtained without taking into account the Sommerfeld enhancement, which is instead included in the red lines. The vertical dashed line is the boundary between the perturbative and condensed phase, which is estimated as the scale at which $M_s \approx \Lambda_{\text{DC}}$. We compare the annihilation cross section with the HESS and FERMI-LAT limits on gamma-ray signals [26, 27]. The resulting bounds are plotted in figures 1 and 3. We also show future prospects at CTA [6]. In the region excluded by indirect detection the predicted DM abundance is much smaller than the cosmological abundance. However, the CTA prospects can test a significant region of the parameter space, especially in the case in which glueball decays are very fast.

We notice that the indirect detection cross section turns out to be slightly suppressed with respect to the standard WIMPs scenario (in which one gets approximately the value $\langle\sigma v\rangle_{\text{cosmo}} \simeq 2.2 \times 10^{-26} \text{cm}^3/\text{sec}$). The suppression factor depends on $\mathcal{N}$: it is roughly a factor of 2 for small values of $\mathcal{N}$ (such as the ones plotted in figure 3), while it becomes negligible for larger values of $\mathcal{N}$. This is a characteristic prediction of our model, which is testable by future experiments at CTA. The suppression factor becomes smaller if one includes the Sommerfeld enhancement (red lines). We notice that if we take the glueballs dilution effects into consideration (solid lines), the indirect detection cross section is further suppressed by one or more order of magnitudes in a certain range of the parameters space.

6 A few comments on the scale invariant hypothesis

In the previous sections we have computed the mass spectrum and studied the phenomenology of the model within the scale invariant hypothesis $M_S^2 = 0$. Under this assumption the SSB is driven by the one-loop effective potential (Coleman-Weinberg mechanism). Furthermore, we have also dropped the Higgs mass term by demanding that the weak scale v is generated by the dynamical scale w (see eq. (2.5)). Now we would like to briefly discuss a more generic case by abandoning the scale invariant framework, and investigate its effects on the phenomenological results.

If M_S^2 is sufficiently small (positive or negative) the dynamics is spontaneously broken by Coleman-Weinberg mechanism as in the scale invariant case. Otherwise, if the quadratic term M_S^2 is not negligibly small the picture can change significantly. We distinguish two cases: (1) the quadratic term is negative $M_S^2 < 0$; (2) the quadratic term is positive $M_S^2 > 0$.

In the first case, if M_S^2 is not too small, the $\text{Sp}(\mathcal{N} + 2)$ dynamics is no longer spontaneously broken and confines at low energy (Λ_{DC}). The mass spectrum in the confined phase is composed by unstable glueballs and $\mathcal{S}$ bound states (some of which are stable) as discussed in [1]. As we have pointed out in section 1 there are strong indications of a duality between this phase and the Higgsed one.

In the second case the $\text{Sp}(\mathcal{N} + 2)$ dynamics is already spontaneously broken at tree level. The mass spectrum is the same as we discussed in section 2 with an important difference: the scalon mass is no longer one-loop suppressed but gets a tree-level value $M_{\mathcal{S}}^2 = 2\lambda_{\mathcal{S}}w^2$ (assuming a small $\lambda_{H\mathcal{S}}$). Therefore, if $\lambda_{\mathcal{S}}$ is not too small (e.g. $\lambda_{\mathcal{S}} > \mathfrak{g}^4/(4\pi)^2$) the scalon can be significantly heavier than in the scale invariant case, and this can affect the phenomenological results of the model.

Concerning the cosmological evolution, the presence of a heavier scalon can reduce the cross sections that contain the scalon in the final state due to the phase space suppression. However, the contributions of the processes that contain no scalon are dominant (see eqs. (4.4)–(4.17)), so the mass term has very small effects ($\sim \mathcal{O}(1\%)$) on the relic density.

Both the direct detection cross section σ_{SI} and the Higgs-glueballs mixing ϵ^2 get suppressed by a heavier scalon (see eqs. (4.27)–(5.1)). As for the indirect detection, the processes $\mathcal{W}\mathcal{W}^\dagger \rightarrow \mathfrak{s}\mathfrak{s}, \mathfrak{s}\mathcal{Z}$ are suppressed (even kinematically forbidden if the scalon is very heavy), while the process $\mathcal{W}\mathcal{W}^\dagger \rightarrow \mathcal{X}\mathcal{X}^\dagger$ (predominant) is unaffected, so the indirect detection cross section is roughly the same as in the scale invariant case.

Finally we notice that, if we drop the scale invariant hypothesis, the model contains 4 free parameters ($M_{\mathcal{S}}^2, \lambda_{H\mathcal{S}}, \mathfrak{g}, \lambda_{\mathcal{S}}$). On the contrary, for the scale invariant case there are only 2 independent parameters left, so the phenomenological analysis is less generic but more predictive.

7 Conclusions

Motivated by the previous work [1], we have studied the phenomenology of Sp DM model, which contains a new dark gauge group $\text{Sp}(\mathcal{N} + 2)$ that is spontaneously broken to $\text{Sp}(\mathcal{N})$ by a scalar $\mathcal{S}$ in the fundamental representation. We further restricted the parameter space assuming that:

- the cosmological DM abundance is reproduced thermally;
- the $\text{Sp}(\mathcal{N} + 2) \rightarrow \text{Sp}(\mathcal{N})$ symmetry breaking occurs dynamically à la Coleman-Weinberg;
- the $\mathcal{S}$ vacuum expectation value also induces the observed Higgs mass.

Thanks to these extra assumptions, DM phenomenology is described by only one free parameter, the DM mass $M_{\mathcal{W}}$ (or the dark gauge coupling $\mathfrak{g}$). We have taken into account the confinement of $\text{Sp}(\mathcal{N})$ dynamics and the corresponding formation of bound states. Of course, some of the above assumptions can be relaxed, giving more general phenomenology.

In section 2 we have summarized the main features of the model, including the Lagrangian before and after the symmetry breaking, the perturbative mass spectrum and the formation of bound states. The theory admits an accidental global dark-baryon number $\text{U}(1)$ symmetry, which renders two co-stable vector DM candidates at the perturbative level: $\mathcal{W}$ (with dark-baryon number 2) and $\mathcal{X}$ (with dark-baryon number 1) with masses $M_{\mathcal{X}} = M_{\mathcal{W}}/\sqrt{2}$. After confinement of $\text{Sp}(\mathcal{N})$ dynamics, two $\mathcal{X}$ can either form neutral mesons that decay to SM particles or charged mesons that decay into $\mathcal{W}$, leaving the $\mathcal{W}$ as the only DM candidate of the model.

In section 3 we discussed how to compute the corrections to DM annihilation cross sections due to the exchange of light mediators (namely the Sommerfeld enhancement and bound state formation), and we focused on the relevant processes which occur in both cosmological evolution and indirect detection signals.

In section 4 we computed the DM relic abundance by solving numerically the Boltzmann equations of $\mathcal{W}$ and $\mathcal{X}$ abundances at the perturbative level, and we also took into account the gauge confinement under some simplified assumptions. DM is accompanied by light, unstable glueballs, which can dilute the DM abundance and can be potentially probed through their coupling to the Higgs. We discussed the impact of the Sommerfeld enhancement and perturbative bound state formation on the relic abundance. All these results are summarized in figure 1. Besides, we also show the predictions of Higgs and glueballs phenomenology in the right panel of figure 2.

In section 5 we studied the signatures of this model. We calculated the scalon production cross section and the results are shown in the left panel of figure 2. Besides, we also computed cross sections for direct and indirect detection and compared them with present experimental data and future prospects. We predicted a non standard indirect detection cross section, which turns out to be slightly suppressed in comparison to the usual WIMPs paradigm, and this is a characteristic and testable prediction of our model. The region in which the DM abundance is allowed by experimental bounds can be further explored by the future direct and indirect detection experiments, in particular the CTA. The phenomenology for direct and indirect detection is summarized in figure 3.

In section 6 we gave a brief investigation of the effects of the quadratic parameter M_S^2 on the mass spectrum as well as the phenomenological results. Under non scale invariant conditions, we found that the relic density and indirect detection are roughly the same as in the scale invariant case, while the direct detection cross section σ_{SI} and the Higgs-glueballs mixing ϵ^2 get suppressed. Besides, the theory becomes less predictive due to more free parameters.

As we sometimes relied on approximations, various aspects of the model can be more precisely computed. For example, if $\Lambda_{DC} \gtrsim T_f$ (i.e. confinement occurs before DM thermal freeze-out), the computation of the relic abundance will be qualitatively different. Therefore, it can be interesting for a new work to study this case with more detailed computation.

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A Feynman rules: $\text{Sp}(\mathcal{N} + 2) \rightarrow \text{Sp}(\mathcal{N})$

The Feynman vertices with all momenta p_i incoming are:

$$\mathcal{A}(1)\mathcal{A}(2)\mathcal{A}(3) : \\ \mathfrak{g}f_{\mathcal{N}}^{a_1 a_2 a_3} [g_{\mu_1 \mu_2} (p_1^{\mu_3} - p_2^{\mu_3}) - g_{\mu_1 \mu_3} (p_1^{\mu_2} - p_3^{\mu_2}) + g_{\mu_2 \mu_3} (p_2^{\mu_1} - p_3^{\mu_1})]$$

$$\mathcal{A}(1)\mathcal{A}(2)\mathcal{A}(3)\mathcal{A}(4) :$$

$$\begin{aligned} & i\mathfrak{g}^2 [f_{\mathcal{N}}^{a_1 a_3 c} f_{\mathcal{N}}^{a_2 a_4 c} (g_{\mu_1 \mu_4} g_{\mu_2 \mu_3} - g_{\mu_1 \mu_2} g_{\mu_3 \mu_4}) \\ & + f_{\mathcal{N}}^{a_1 a_2 c} f_{\mathcal{N}}^{a_3 a_4 c} (g_{\mu_1 \mu_4} g_{\mu_2 \mu_3} - g_{\mu_1 \mu_3} g_{\mu_2 \mu_4}) \\ & + f_{\mathcal{N}}^{a_1 a_4 c} f_{\mathcal{N}}^{a_2 a_3 c} (g_{\mu_1 \mu_3} g_{\mu_2 \mu_4} - g_{\mu_1 \mu_2} g_{\mu_3 \mu_4})] \end{aligned}$$

$$\mathcal{A}(1)\mathcal{X}(2)\mathcal{X}^\dagger(3) :$$

$$i\mathfrak{g}(T_{\mathcal{N}}^{a_1})_{m_3 m_2} [g_{\mu_1 \mu_2} (p_2^{\mu_3} - p_1^{\mu_3}) - g_{\mu_1 \mu_3} (p_3^{\mu_2} - p_1^{\mu_2}) + g_{\mu_2 \mu_3} (p_3^{\mu_1} - p_2^{\mu_1})]$$

$$\mathcal{A}(1)\mathcal{A}(2)\mathcal{X}(3)\mathcal{X}^\dagger(4) :$$

$$\begin{aligned} & i\mathfrak{g}^2 [(T_{\mathcal{N}}^{a_1} T_{\mathcal{N}}^{a_2})_{m_4 m_3} (g_{\mu_1 \mu_3} g_{\mu_2 \mu_4} - g_{\mu_1 \mu_2} g_{\mu_3 \mu_4}) \\ & + (T_{\mathcal{N}}^{a_2} T_{\mathcal{N}}^{a_1})_{m_4 m_3} (g_{\mu_1 \mu_4} g_{\mu_2 \mu_3} - g_{\mu_1 \mu_2} g_{\mu_3 \mu_4}) \\ & - i f_{\mathcal{N}}^{a_1 a_2 c} (T_{\mathcal{N}}^c)_{m_4 m_3} (g_{\mu_1 \mu_4} g_{\mu_2 \mu_3} - g_{\mu_1 \mu_3} g_{\mu_2 \mu_4})] \end{aligned}$$

$$\mathcal{W}(1)\mathcal{W}^\dagger(2)\mathcal{Z}(3) :$$

$$-i\mathfrak{g} [g_{\mu_1 \mu_2} (p_1^{\mu_3} - p_2^{\mu_3}) - g_{\mu_1 \mu_3} (p_1^{\mu_2} - p_3^{\mu_2}) + g_{\mu_2 \mu_3} (p_2^{\mu_1} - p_3^{\mu_1})]$$

$$\mathcal{X}(1)\mathcal{X}^\dagger(2)\mathcal{Z}(3) :$$

$$-i\frac{\mathfrak{g}}{2}(\delta_{\mathcal{N}})_{m_1 m_2} [g_{\mu_1 \mu_2} (p_1^{\mu_3} - p_2^{\mu_3}) - g_{\mu_1 \mu_3} (p_1^{\mu_2} - p_3^{\mu_2}) + g_{\mu_2 \mu_3} (p_2^{\mu_1} - p_3^{\mu_1})]$$

$$\mathcal{W}(1)\mathcal{W}^\dagger(2)\mathcal{Z}(3)\mathcal{Z}(4) :$$

$$-i\mathfrak{g}^2 (2g_{\mu_1 \mu_2} g_{\mu_3 \mu_4} - g_{\mu_1 \mu_4} g_{\mu_2 \mu_3} - g_{\mu_1 \mu_3} g_{\mu_2 \mu_4})$$

$$\mathcal{X}(1)\mathcal{X}^\dagger(2)\mathcal{Z}(3)\mathcal{Z}(4) :$$

$$-i\frac{\mathfrak{g}^2}{4}(\delta_{\mathcal{N}})_{m_1 m_2} (2g_{\mu_1 \mu_2} g_{\mu_3 \mu_4} - g_{\mu_1 \mu_4} g_{\mu_2 \mu_3} - g_{\mu_1 \mu_3} g_{\mu_2 \mu_4})$$

$$\mathcal{X}(1)\mathcal{X}^\dagger(2)\mathcal{Z}(3)\mathcal{A}(4) :$$

$$-i\frac{\mathfrak{g}^2}{2}(T_{\mathcal{N}}^{a_4})_{m_2 m_1} (2g_{\mu_1 \mu_2} g_{\mu_3 \mu_4} - g_{\mu_1 \mu_4} g_{\mu_2 \mu_3} - g_{\mu_1 \mu_3} g_{\mu_2 \mu_4})$$

$$\mathfrak{s}(1)\mathcal{X}(2)\mathcal{X}^\dagger(3) :$$

$$2i\frac{M_{\mathcal{X}}^2}{w}(\delta_{\mathcal{N}})_{m_2 m_3} g_{\mu_2 \mu_3}$$

$$\mathfrak{s}(1)\mathfrak{s}(2)\mathcal{X}(3)\mathcal{X}^\dagger(4) :$$

$$2i\frac{M_{\mathcal{X}}^2}{w^2}(\delta_{\mathcal{N}})_{m_3 m_4} g_{\mu_3 \mu_4}$$

$$\mathfrak{s}(1)\mathcal{W}(2)\mathcal{W}^\dagger(3) :$$

$$2i\frac{M_{\mathcal{W}}^2}{w} g_{\mu_2 \mu_3}$$

$$\mathfrak{s}(1)\mathfrak{s}(2)\mathcal{W}(3)\mathcal{W}^\dagger(4) :$$

$$2i\frac{M_{\mathcal{W}}^2}{w^2}g_{\mu_3\mu_4}$$

$$\mathfrak{s}(1)\mathcal{Z}(2)\mathcal{Z}(3) :$$

$$2i\frac{M_{\mathcal{Z}}^2}{w}g_{\mu_2\mu_3}$$

$$\mathfrak{s}(1)\mathfrak{s}(2)\mathcal{Z}(3)\mathcal{Z}(4) :$$

$$2i\frac{M_{\mathcal{Z}}^2}{w^2}g_{\mu_3\mu_4}$$

$$\mathcal{W}^\dagger(1)\mathcal{X}(2)\mathcal{X}(3)\mathcal{Z}(4) :$$

$$-\frac{3ig^2}{2\sqrt{2}}\gamma_{\mathcal{N}}^{m_2m_3}(g_{\mu_1\mu_3}g_{\mu_2\mu_4} - g_{\mu_1\mu_2}g_{\mu_3\mu_4})$$

$$\mathcal{W}(1)\mathcal{W}^\dagger(2)\mathcal{X}(3)\mathcal{X}^\dagger(4) :$$

$$\frac{ig^2}{2}(\delta_{\mathcal{N}})_{m_3m_4}(2g_{\mu_1\mu_3}g_{\mu_2\mu_4} - g_{\mu_1\mu_4}g_{\mu_2\mu_3} - g_{\mu_1\mu_2}g_{\mu_3\mu_4})$$

$$\mathcal{W}^\dagger(1)\mathcal{X}(2)\mathcal{X}(3) :$$

$$-\frac{ig}{\sqrt{2}}\gamma_{\mathcal{N}}^{m_2m_3}[g_{\mu_1\mu_2}(p_1^{\mu_3} - p_2^{\mu_3}) - g_{\mu_3\mu_1}(p_1^{\mu_2} - p_3^{\mu_2}) + g_{\mu_2\mu_3}(p_2^{\mu_1} - p_3^{\mu_1})]$$

$$\mathcal{A}(1)\mathcal{W}^\dagger(2)\mathcal{X}(3)\mathcal{X}(4) :$$

$$\frac{ig^2}{\sqrt{2}}(\gamma_{\mathcal{N}}T_{\mathcal{N}}^{a_1})_{m_3m_4}[2g_{\mu_1\mu_2}g_{\mu_3\mu_4} - g_{\mu_1\mu_4}g_{\mu_2\mu_3} - g_{\mu_1\mu_3}g_{\mu_2\mu_4}]$$

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