Erratum: Dimension-six matrix elements for meson mixing and lifetimes from sum rules

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Eq. (5.15) in the original paper used a wrong numerical expression for the lifetime ratio $\tau(B_s^0)/\tau(B_d^0)$ from [16] and should read

$$\begin{aligned} \left. \frac{\tau(B_s^0)}{\tau(B^0)} \right|_{\text{exp}} &= 0.994 \pm 0.004, \\ \left. \frac{\tau(B_s^0)}{\tau(B^0)} \right|_{\overline{\text{MS}}} &= 1.0007 \pm 0.0025 \\ &= 1.0007 \pm 0.0014 (\text{had.}) \pm 0.0006 (\text{scale}) \pm 0.0020 (1/m_b^4), \end{aligned} \quad (5.15)$$

implying an agreement with experiment at the level of 1.4 instead of 1.1 σ as stated originally. Figure 9 is also affected and is updated below.

In the original paper we have set color factors in some places to their QCD values with $N_c = 3$ while keeping them general in others.¹ To restore generality one has to change the following eqs. to

$$\Delta\rho_{\tilde{Q}_i} \equiv \frac{N_c C_F}{4} \frac{\omega_1^2 \omega_2^2}{\pi^4} \frac{\alpha_s}{4\pi} r_{\tilde{Q}_i}(x, L_\omega), \quad (3.17)$$

$$w_{\tilde{Q}_i}(\omega_1, \omega_2) = \frac{\Delta\rho_{\tilde{Q}_i}^{\text{pert}}(\omega_1, \omega_2)}{\rho_{\Pi}^{\text{pert}}(\omega_1)\rho_{\Pi}^{\text{pert}}(\omega_2)} = \frac{C_F}{N_c} \frac{\alpha_s}{4\pi} r_{\tilde{Q}_i}(x, L_\omega), \quad (3.24)$$

$$\Delta B_{\tilde{Q}_i}^{\text{pert}}(\mu_\rho) = \frac{C_F}{N_c A_{\tilde{Q}_i}} \frac{\alpha_s(\mu_\rho)}{4\pi} r_{\tilde{Q}_i} \left(1, \log \frac{\mu_\rho^2}{4\Lambda^2} \right), \quad (3.25)$$

which coincide with their counterparts in the original paper in QCD with three colors.

¹We are grateful to A. Grozin and A. Pivovarov for pointing this out to us.

We noticed that the labels “This work (f_D^{lattice})” and “This work (f_D^{exp})” in figure 7 were interchanged. The figure with the corrected labels is shown below.

Our numerical code for ΔM_q and a_{sl}^q used a wrong input value for the top-quark mass.² The corresponding SM predictions in eqs. (4.4), (4.5), (4.7) and (4.8) change by a small amount to

$$\Delta M_s^{\text{SM}} = (16.6 \pm 1.7) \text{ ps}^{-1} = (16.6^{+1.2}_{-1.1} (\text{had.}) \pm 0.1 (\text{scale})^{+1.3}_{-1.2} (\text{param.})) \text{ ps}^{-1}, \quad (4.4)$$

$$\Delta M_d^{\text{SM}} = (0.56 \pm 0.08) \text{ ps}^{-1} = (0.56 \pm 0.04 (\text{had.}) \pm 0.00 (\text{scale}) \pm 0.07 (\text{param.})) \text{ ps}^{-1}, \quad (4.5)$$

$$a_{\text{sl}}^{s, \overline{\text{MS}}} = (2.3 \pm 0.3) \cdot 10^{-5} = (2.3 \pm 0.1 (\text{had.})^{+0.0}_{-0.1} (\text{scale}) \pm 0.3 (\text{param.})) \cdot 10^{-5},$$

$$a_{\text{sl}}^{s, \text{PS}} = (2.2 \pm 0.3) \cdot 10^{-5} = (2.2 \pm 0.1 (\text{had.})^{+0.0}_{-0.1} (\text{scale})^{+0.2}_{-0.3} (\text{param.})) \cdot 10^{-5},$$

$$a_{\text{sl}}^{s, 1\text{S}} = (2.1^{+0.2}_{-0.3}) \cdot 10^{-5} = (2.1 \pm 0.1 (\text{had.})^{+0.0}_{-0.1} (\text{scale})^{+0.2}_{-0.3} (\text{param.})) \cdot 10^{-5},$$

$$a_{\text{sl}}^{s, \text{kin}} = (2.2 \pm 0.3) \cdot 10^{-5} = (2.2 \pm 0.1 (\text{had.})^{+0.0}_{-0.1} (\text{scale})^{+0.2}_{-0.3} (\text{param.})) \cdot 10^{-5}, \quad (4.7)$$

$$a_{\text{sl}}^{d, \text{PS}} = (-4.4^{+0.6}_{-0.5}) \cdot 10^{-4} = (-4.4 \pm 0.1 (\text{had.})^{+0.2}_{-0.1} (\text{scale}) \pm 0.5 (\text{param.})) \cdot 10^{-4}, \quad (4.8)$$

which improves the agreement of ΔM_d^{SM} with the experimental value to better than one standard deviation. This also affects figure 8 as well as tables 6 and 7 which are corrected below. All our conclusions remain unchanged.

Figure 7. Comparison of our results for the $\Delta C = 2$ matrix elements at the scale 3 GeV to the lattice values of ETM'14 [30], ETM'15 [31] and FNAL/MILC'17 [32]. The values for the matrix elements of the ETM collaboration are extracted from figure 16 of [32].

²We wish to thank G. Tetlalmatzi-Xolocotzi for pointing this out to us.

Figure 8. Comparison of our predictions for the mass and decay rate difference in the B_s (left) and B_d (right) system with the present experimental averages (error bars). We also show the results obtained with the lattice results of [5] for $f_{B_q}^2 \overline{B}_{Q_i}$ and the matrix element $\langle R_0 \rangle$ and the values given in appendix B for the other input parameters. The PS mass scheme for the bottom quarks has been used in both cases.

Figure 9. Comparison of our predictions for the lifetime ratios of heavy mesons with the present experimental averages.

	$\Delta M_s^{\text{SM}} [\text{ps}^{-1}]$	$\Delta \Gamma_s^{\text{PS}} [\text{ps}^{-1}]$	$a_{\text{sl}}^{s, \text{PS}} [10^{-5}]$
$\overline{B}_{Q_1}$	± 1.0	± 0.005	± 0.01
$\overline{B}_{Q_3}$	± 0.0	± 0.005	± 0.02
$\overline{B}_{R_0}$	± 0.0	± 0.003	± 0.00
$\overline{B}_{R_1}$	± 0.0	± 0.000	± 0.00
$\overline{B}_{R'_1}$	± 0.0	± 0.000	± 0.00
$\overline{B}_{R_2}$	± 0.0	± 0.016	± 0.00
$\overline{B}_{R_3}$	± 0.0	± 0.000	± 0.02
$\overline{B}_{R'_3}$	± 0.0	± 0.000	± 0.05
f_{B_s}	± 0.5	± 0.002	± 0.00
μ_1	± 0.0	$+0.007$ -0.018	$+0.04$ -0.09
μ_2	± 0.1	$+0.000$ -0.002	$+0.02$ -0.01
m_b	± 0.0	$+0.000$ -0.001	$+0.02$ -0.01
m_c	± 0.0	$+0.000$ -0.001	± 0.07
α_s	± 0.0	± 0.000	± 0.04
CKM	$+1.3$ -1.2	± 0.006	$+0.23$ -0.24

Table 6. Individual errors for the B_s mixing observables.

	$\Delta M_d^{\text{SM}} [\text{ps}^{-1}]$	$\Delta \Gamma_d^{\text{PS}} [10^{-3} \text{ps}^{-1}]$	$a_{\text{sl}}^{d, \text{PS}} [10^{-4}]$
$\overline{B}_{Q_1}$	± 0.03	± 0.16	± 0.02
$\overline{B}_{Q_3}$	± 0.00	$+0.17$ -0.16	± 0.03
$\overline{B}_{R_0}$	± 0.00	± 0.11	± 0.01
$\overline{B}_{R_1}$	± 0.00	± 0.01	± 0.00
$\overline{B}_{R'_1}$	± 0.00	± 0.01	± 0.00
$\overline{B}_{R_2}$	± 0.00	± 0.54	± 0.00
$\overline{B}_{R_3}$	± 0.00	± 0.00	± 0.04
$\overline{B}_{R'_3}$	± 0.00	± 0.01	± 0.10
f_B	± 0.02	± 0.11	± 0.00
μ_1	± 0.00	$+0.24$ -0.62	$+0.19$ -0.08
μ_2	± 0.00	$+0.00$ -0.08	$+0.01$ -0.03
m_b	± 0.00	$+0.01$ -0.03	$+0.01$ -0.03
m_c	± 0.00	± 0.02	± 0.14
α_s	± 0.00	± 0.01	$+0.09$ -0.08
CKM	± 0.07	$+0.36$ -0.37	± 0.49

Table 7. Individual errors for the B_d mixing observables.

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