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Hadroproduction of scalar P -wave quarkonia in the light-front k_T -factorization approach

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ABSTRACT: In this work, we present a thorough analysis of scalar P -wave χ_{Q0} , $Q = c, b$ quarkonia electromagnetic form factors for the $\gamma^*\gamma^* \rightarrow \chi_{Q0}$ couplings, as well as their hadroproduction observables in $k_\perp$ -factorisation using the light-front (LF) potential approach for the quarkonium wave function. The electromagnetic form factors are presented as functions of photon virtualities. We discuss the role of the Melosh spin-rotation and relativistic corrections estimated by comparing our results with those in the standard non-relativistic QCD (NRQCD) approach. Theoretical uncertainties of our approach are found by performing the analysis with two different unintegrated gluon densities and with five distinct models of the $Q\bar{Q}$ interaction potentials consistent with the meson spectra. The results for the rapidity and transverse momentum distributions of χ_{Q0} produced in high-energy pp collisions at $\sqrt{s} = 13$ TeV are shown.

KEYWORDS: QCD Phenomenology

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1 Introduction

The production of quarkonia in hadronic collisions continues to attract a lot of interest [1–4]. Different approaches were used in the past, for the case of χ_c, χ_b production, see e.g. [5–12]. Here, we continue our investigation [13, 14] of quarkonia production observables in the framework of the $k_\perp$ -factorisation approach [15–19] in the color-singlet formulation employing the light-front quarkonia wave functions. The previous works in refs. [5–8] have observed that in the $k_\perp$ -factorisation approach the color-singlet mechanism dominates in the production of P -wave quarkonia. At the next natural step we pursue in this direction, it is instructive to explore the role of relativistic corrections and the shape of the quarkonia wave function in the associated observables.

In our recent study [13, 14] we discussed inclusive production of pseudoscalar $\eta_c(1S, 2S)$ charmonia in proton-proton collisions. We explicitly calculated the corresponding electromagnetic $\gamma^* + \gamma^* \rightarrow \eta_c(1S, 2S)$ form factors as functions of photon virtualities. Similar form factors were calculated also for the $g^* + g^* \rightarrow \eta_c(1S, 2S)$ matrix element that was used in the k_T -factorization formula for differential hadroproduction cross section. We have shown there that inclusion of both form factors in such an approach is essential. In our calculations, we adopted the color-singlet model, which treats the quarkonium as a two-body bound state of heavy quark and antiquark.

Such a formalism was used previously for the production of χ_{cJ} ($J = 0, 1, 2$) quarkonia (see e.g. ref. [8] and references therein), and a relatively good agreement with data was

obtained in the k_T -factorization approach with an unintegrated gluon distribution (UGD), which effectively includes higher-order corrections. These previous calculations were done within the nonrelativistic QCD (NRQCD) approach.

In this work, we discuss production of the lowest P -wave scalar state for both $c\bar{c}$ and $b\bar{b}$ systems, χ_{Q0} , $Q = c, b$, in the same approach which we dubbed at the light-front $k_\perp$ -factorisation. As will be discussed in detail below, the situation with scalar χ_{c0} production is a bit more complicated than that for the pseudoscalar quarkonia. The leading-order partonic subprocess at high energies is the off-shell gluon-gluon fusion, $g^* + g^* \rightarrow \chi_{Q0}$. The main ingredient of such a subprocess is the off-shell matrix element written in terms of two independent form factors. Here we go beyond the NRQCD approach and present a similar analysis of χ_{Q0} form factors and hadroproduction as was done earlier for pseudoscalar (S -wave) charmonia. We will present the corresponding form factors, compare the new results to those obtained in NRQCD approaches and make predictions for the $pp \rightarrow \chi_{Q0}$ reactions at the LHC energies.

The paper is organised as follows. In section 2, we recapitulate the main details of the $k_\perp$ -factorisation formalism. In section 3 we derive the off-shell matrix element expressed in terms of two independent form factors written in terms of light-front quarkonia wave functions. These form factors can be related to the standard electromagnetic transition form factors, which encode the helicity dependence of the virtual-photon fusion amplitudes. We derive the relevant expressions in section 4. We then turn to numerical results, presenting the light front wave functions and radiative decay widths in section 5. In section 6 we present our results for the electromagnetic form factors. Our results for the transverse momentum and rapidity distributions of χ_{c0} and χ_{b0} mesons are shown in section 7. In section 8 we conclude.

2 k_T -factorization for inclusive hadroproduction

Let us briefly recapitulate the k_T -factorization formalism we use in this work. The leading process is the gluon-gluon fusion into the color-singlet heavy quark pair. Gluons carry transverse momenta, and their four momenta are written as ($\sqrt{s}$ is the pp center-of-mass energy, and we use the lightcone components of four-vectors $a = (a_+, a_-, \mathbf{a})$, $a^2 = 2a_+a_- - \mathbf{a}^2$):

$$q_1 = (q_{1+}, 0, \mathbf{q}_1), \quad q_2 = (0, q_{2-}, \mathbf{q}_2), \quad (2.1)$$

with

$$q_{1+} = x_1 \sqrt{\frac{s}{2}}, \quad q_{2-} = x_2 \sqrt{\frac{s}{2}}. \quad (2.2)$$

An important distinction from the collinear approach is that the gluon momenta are (space-like) off-shell, $q_i^2 = -\mathbf{q}_i^2$, $i = 1, 2$. Our starting point is the inclusive cross section for the $2 \rightarrow 1$ gluon-gluon fusion mode obtained from

$$d\sigma = \int \frac{dx_1}{x_1} \int \frac{d^2\mathbf{q}_1}{\pi\mathbf{q}_1^2} \mathcal{F}(x_1, \mathbf{q}_1^2, \mu_F^2) \int \frac{dx_2}{x_2} \int \frac{d^2\mathbf{q}_2}{\pi\mathbf{q}_2^2} \mathcal{F}(x_2, \mathbf{q}_2^2, \mu_F^2) \frac{1}{2x_1x_2s} |\overline{\mathcal{M}}|^2 d\Phi(2 \rightarrow 1). \quad (2.3)$$

Here the unintegrated gluon distributions are normalized such that the collinear glue is given by

$$xg(x, \mu_F^2) = \int^{\mu_F^2} \frac{d\mathbf{q}^2}{q^2} \mathcal{F}(x, \mathbf{q}^2, \mu_F^2), \quad (2.4)$$

where from now on we will no longer show the dependence on the factorization scale μ_F^2 explicitly. Let us denote the four-momentum of χ meson by p and parametrize it in light-cone coordinates as

$$p = (p_+, p_-, \mathbf{p}) = \left(\frac{m_T}{\sqrt{2}} e^y, \frac{m_T}{\sqrt{2}} e^{-y}, \mathbf{p} \right), \quad m_T = \sqrt{\mathbf{p}^2 + M_\chi^2}. \quad (2.5)$$

Here M_χ is the mass of the χ_{Q0} -meson, $\mathbf{p}$ is its transverse momentum, and y its rapidity in the pp c.m. frame. The phase-space element is

$$\begin{aligned} d\Phi(2 \rightarrow 1) &= (2\pi)^4 \delta^{(4)}(q_1 + q_2 - p) \frac{d^4 p}{(2\pi)^3} \delta(p^2 - M_\chi^2) \\ &= \frac{2\pi}{s} \delta\left(x_1 - \frac{m_T}{\sqrt{s}} e^y\right) \delta\left(x_2 - \frac{m_T}{\sqrt{s}} e^{-y}\right) \delta^{(2)}(\mathbf{q}_1 + \mathbf{q}_2 - \mathbf{p}) dy d^2 \mathbf{p}. \end{aligned} \quad (2.6)$$

We therefore obtain for the inclusive cross section

$$\frac{d\sigma}{dy d^2 \mathbf{p}} = \int \frac{d^2 \mathbf{q}_1}{\pi \mathbf{q}_1^2} \mathcal{F}(x_1, \mathbf{q}_1^2) \int \frac{d^2 \mathbf{q}_2}{\pi \mathbf{q}_2^2} \mathcal{F}(x_2, \mathbf{q}_2^2) \delta^{(2)}(\mathbf{q}_1 + \mathbf{q}_2 - \mathbf{p}) \frac{\pi}{(x_1 x_2 s)^2} |\overline{\mathcal{M}}|^2, \quad (2.7)$$

where the momentum fractions of gluons are fixed as $x_{1,2} = m_T \exp(\pm y)/\sqrt{s}$. The off-shell matrix element is written in terms of the Feynman amplitude as (we restore the color-indices):

$$\mathcal{M}^{ab} = \frac{q_{1\perp}^\mu q_{2\perp}^\nu}{|\mathbf{q}_1| |\mathbf{q}_2|} \mathcal{M}_{\mu\nu}^{ab} = \frac{q_{1+} q_{2-}}{|\mathbf{q}_1| |\mathbf{q}_2|} n_\mu^+ n_\nu^- \mathcal{M}_{\mu\nu}^{ab} = \frac{x_1 x_2 s}{2 |\mathbf{q}_1| |\mathbf{q}_2|} n_\mu^+ n_\nu^- \mathcal{M}_{\mu\nu}^{ab}. \quad (2.8)$$

3 k_T -factorization element for final state $Q\bar{Q}$

Let us introduce the reduced amplitude

$$\mathcal{T}_{\mu\nu} \equiv \frac{1}{4\pi\alpha_{\text{em}} e_Q^2} \int \frac{dz d^2 \mathbf{k}}{z(1-z) 16\pi^3} \sum_{\lambda, \bar{\lambda}} \Psi_{\lambda\bar{\lambda}}^*(z, \mathbf{k}) \mathcal{M}_{\mu\nu}^{\lambda\bar{\lambda}}(\gamma^* \gamma^* \rightarrow Q_\lambda(zP_+, \mathbf{p}_Q) \bar{Q}_{\bar{\lambda}}((1-z)P_+, \mathbf{p}_{\bar{Q}})). \quad (3.1)$$

Here the transverse momenta of quark and antiquark are

$$\mathbf{p}_Q = \mathbf{k} + z\mathbf{P}, \quad \mathbf{p}_{\bar{Q}} = -\mathbf{k} + (1-z)\mathbf{P}. \quad (3.2)$$

The photon-photon fusion and gluon-gluon fusion amplitudes are proportional to the reduced amplitude as follows:

$$\begin{aligned} \mathcal{M}_{\mu\nu}(\gamma^* \gamma^* \rightarrow \chi_{Q0}(P_+, \mathbf{P})) &= 4\pi\alpha_{\text{em}} e_Q^2 \frac{\text{Tr}[\mathbb{1}]}{\sqrt{N_c}} \mathcal{T}_{\mu\nu} = 4\pi\alpha_{\text{em}} e_Q^2 \sqrt{N_c} \mathcal{T}_{\mu\nu}, \\ \mathcal{M}_{\mu\nu}^{ab}(g^* g^* \rightarrow \chi_{Q0}(P_+, \mathbf{P})) &= 4\pi\alpha_s \frac{\text{Tr}[t^a t^b]}{\sqrt{N_c}} \mathcal{T}_{\mu\nu} = \frac{2\pi\alpha_s}{\sqrt{N_c}} \delta^{ab} \mathcal{T}_{\mu\nu}. \end{aligned} \quad (3.3)$$

For the purpose of k_T -factorization it is most convenient to use the contraction of the gluon-fusion amplitude with light-cone vectors $n_\mu^\pm$. Using the standard covariant Feynman rules, we can obtain the reduced amplitude $\mathcal{T} = n^{+\mu} n^{-\nu} \mathcal{T}_{\mu\nu}$ as

$$\begin{aligned} \mathcal{T} = & (-2) \int \frac{dz d^2 \mathbf{k}}{\sqrt{z(1-z)} 16\pi^3} \\ & \times \left\{ -m_Q \left[\frac{1}{l_A^2 + \varepsilon^2} - \frac{1}{l_B^2 + \varepsilon^2} \right] \left(\sqrt{2} (e(-) \mathbf{q}_1) \Psi_{++}^*(z, \mathbf{k}) + \sqrt{2} (e(+) \mathbf{q}_1) \Psi_{--}^*(z, \mathbf{k}) \right) \right. \\ & + (2z(1-z) \mathbf{q}_1^2 + (1-2z)(\mathbf{k} \mathbf{q}_1)) \left[\frac{1}{l_A^2 + \varepsilon^2} - \frac{1}{l_B^2 + \varepsilon^2} \right] (\Psi_{+-}^*(z, \mathbf{k}) + \Psi_{-+}^*(z, \mathbf{k})) \\ & \left. - (1-2z)(\mathbf{q}_1 \mathbf{q}_2) \left[\frac{1-z}{l_A^2 + \varepsilon^2} + \frac{z}{l_B^2 + \varepsilon^2} \right] (\Psi_{+-}^*(z, \mathbf{k}) + \Psi_{-+}^*(z, \mathbf{k})) \right\}. \end{aligned} \quad (3.4)$$

Here we have introduced the shorthand notation

$$\varepsilon^2 = z(1-z) \mathbf{q}_1^2 + m_Q^2, \quad l_A = \mathbf{k} - (1-z) \mathbf{q}_2, \quad l_B = \mathbf{k} + z \mathbf{q}_2. \quad (3.5)$$

We have also made use of the fact, that for the scalar meson

$$\Psi_{+-}^*(z, \mathbf{k}) - \Psi_{-+}^*(z, \mathbf{k}) = 0. \quad (3.6)$$

We have collected the necessary information on the light-front wave function $\Psi_{\lambda\bar{\lambda}}(z, \mathbf{k})$ in appendix A. Inserting the identities which follow from eq. (A.18):

$$\begin{aligned} \sqrt{2} (e(-) \mathbf{q}_1) \Psi_{++}^*(z, \mathbf{k}) + \sqrt{2} (e(+) \mathbf{q}_1) \Psi_{--}^*(z, \mathbf{k}) &= \frac{2(\mathbf{q}_1 \mathbf{k})}{\sqrt{z(1-z)}} \psi(z, \mathbf{k}), \\ \Psi_{+-}^*(z, \mathbf{k}) + \Psi_{-+}^*(z, \mathbf{k}) &= \frac{2m_Q(1-2z)}{\sqrt{z(1-z)}} \psi(z, \mathbf{k}), \end{aligned} \quad (3.7)$$

we can simplify the amplitude further to read

$$\begin{aligned} \mathcal{T} = & (-4m_Q) \int \frac{dz d^2 \mathbf{k}}{z(1-z) 16\pi^3} \psi(z, \mathbf{k}) \left\{ 2z(1-z)(1-2z) \mathbf{q}_1^2 \left[\frac{1}{l_A^2 + \varepsilon^2} - \frac{1}{l_B^2 + \varepsilon^2} \right] \right. \\ & \left. - 4z(1-z)(\mathbf{k} \mathbf{q}_1) \left[\frac{1}{l_A^2 + \varepsilon^2} - \frac{1}{l_B^2 + \varepsilon^2} \right] - (1-2z)^2 (\mathbf{q}_1 \mathbf{q}_2) \left[\frac{1-z}{l_A^2 + \varepsilon^2} + \frac{z}{l_B^2 + \varepsilon^2} \right] \right\}. \end{aligned} \quad (3.8)$$

There now emerge two independent form factors $G_{1,2}(\mathbf{q}_1^2, \mathbf{q}_2^2)$:

$$\mathcal{T} = |\mathbf{q}_1| |\mathbf{q}_2| G_1(\mathbf{q}_1^2, \mathbf{q}_2^2) + (\mathbf{q}_1 \cdot \mathbf{q}_2) G_2(\mathbf{q}_1^2, \mathbf{q}_2^2). \quad (3.9)$$

The form factors $G_{1,2}(\mathbf{q}_1^2, \mathbf{q}_2^2)$ have the integral representations

$$\begin{aligned} G_1(\mathbf{q}_1^2, \mathbf{q}_2^2) &= |\mathbf{q}_1| |\mathbf{q}_2| \frac{4m_Q}{\mathbf{q}_2^2} \int \frac{dz d^2 \mathbf{k}}{z(1-z) 16\pi^3} \psi(z, \mathbf{k}) 2z(1-z)(2z-1) \left[\frac{1}{l_A^2 + \varepsilon^2} - \frac{1}{l_B^2 + \varepsilon^2} \right] \\ G_2(\mathbf{q}_1^2, \mathbf{q}_2^2) &= 4m_Q \int \frac{dz d^2 \mathbf{k}}{z(1-z) 16\pi^3} \psi(z, \mathbf{k}) \left[\frac{1-z}{l_A^2 + \varepsilon^2} + \frac{z}{l_B^2 + \varepsilon^2} \right] \\ &+ \frac{4m_Q}{\mathbf{q}_2^2} \int \frac{dz d^2 \mathbf{k}}{z(1-z) 16\pi^3} \psi(z, \mathbf{k}) 4z(1-z) \left[\frac{\mathbf{q}_2 \cdot l_A}{l_A^2 + \varepsilon^2} - \frac{\mathbf{q}_2 \cdot l_B}{l_B^2 + \varepsilon^2} \right]. \end{aligned} \quad (3.10)$$

We explain in appendix A how the light-front radial WF $\psi(z, \mathbf{k})$ can be obtained from the potential model WF using the Melosh-transform technique for the spin-orbit part [20, 21] augmented by the Terent'ev prescription [22].

Finally, the factorization formula for the hadroproduction process $pp \rightarrow \chi_{Q0} X$ reads as follows

$$\begin{aligned}
 \frac{d\sigma}{dy d^2\mathbf{p}} &= \int \frac{d^2\mathbf{q}_1}{\pi\mathbf{q}_1^4} \frac{d^2\mathbf{q}_2}{\pi\mathbf{q}_2^4} \delta^{(2)}(\mathbf{p} - \mathbf{q}_1 - \mathbf{q}_2) \mathcal{F}(x_1, \mathbf{q}_1) \mathcal{F}(x_2, \mathbf{q}_2) \frac{\pi}{4(N_c^2 - 1)^2} \sum_{a,b} |\mathcal{M}_{\mu\nu}^{ab} n_\mu^+ n_\nu^-|^2 \\
 &= \int \frac{d^2\mathbf{q}_1}{\pi\mathbf{q}_1^4} \frac{d^2\mathbf{q}_2}{\pi\mathbf{q}_2^4} \delta^{(2)}(\mathbf{p} - \mathbf{q}_1 - \mathbf{q}_2) \mathcal{F}(x_1, \mathbf{q}_1) \mathcal{F}(x_2, \mathbf{q}_2) \frac{\pi^3 \alpha_S^2 |\mathcal{T}|^2}{N_c(N_c^2 - 1)} \\
 &= \int \frac{d^2\mathbf{q}_1}{\pi\mathbf{q}_1^2} \frac{d^2\mathbf{q}_2}{\pi\mathbf{q}_2^2} \delta^{(2)}(\mathbf{p} - \mathbf{q}_1 - \mathbf{q}_2) \mathcal{F}(x_1, \mathbf{q}_1) \mathcal{F}(x_2, \mathbf{q}_2) \frac{\pi^3 \alpha_S^2}{N_c(N_c^2 - 1)} \\
 &\quad \times \left(G_1(\mathbf{q}_1^2, \mathbf{q}_2^2) + \cos(\phi_1 - \phi_2) G_2(\mathbf{q}_1^2, \mathbf{q}_2^2) \right)^2, \tag{3.11}
 \end{aligned}$$

with $\phi_{1,2}$ being the azimuthal angles of gluon transverse momenta $\mathbf{q}_{1,2}$. In our numerical calculations presented below, we set the factorization scale to $\mu_F^2 = m_T^2$, and the QCD coupling constant squared in the differential cross section above is taken in the following form:

$$\alpha_s^2 \rightarrow \alpha_s(\max\{m_T^2, \mathbf{q}_1^2\}) \alpha_s(\max\{m_T^2, \mathbf{q}_2^2\}). \tag{3.12}$$

3.1 NRQCD limit

Let us now take the nonrelativistic (NR) limit. In this case, one should Taylor-expand around $z = 1/2$ and $\mathbf{k} = 0$. Let us introduce

$$z = \frac{1}{2} + \xi, \quad 1 - z = \frac{1}{2} - \xi. \tag{3.13}$$

We can see, that in zeroth order the form factors G_1, G_2 vanish. To the first order in ξ and $\mathbf{k}$, we obtain

$$\frac{1}{\mathbf{l}_A^2 + \varepsilon^2} - \frac{1}{\mathbf{l}_B^2 + \varepsilon^2} = \frac{2}{\mu^4} \left(\xi \mathbf{q}_2^2 + (\mathbf{k} \mathbf{q}_2) \right), \tag{3.14}$$

with

$$\mu^2 = \frac{1}{4} \left(\mathbf{q}_1^2 + \mathbf{q}_2^2 + 4m_c^2 \right). \tag{3.15}$$

In the NR limit, we also replace $2m_Q \rightarrow M_\chi$, and obtain for the two form factors

$$\begin{aligned}
 G_1(\mathbf{q}_1^2, \mathbf{q}_2^2) &= \frac{|\mathbf{q}_1||\mathbf{q}_2|}{\mu^4} \frac{4}{M_\chi} \int \frac{dz d^2\mathbf{k}}{z(1-z)16\pi^3} \psi(z, \mathbf{k}) \xi^2 M_\chi^2, \\
 G_2(\mathbf{q}_1^2, \mathbf{q}_2^2) &= \frac{2}{\mu^4} \frac{1}{M_\chi} \int \frac{dz d^2\mathbf{k}}{z(1-z)16\pi^3} \psi(z, \mathbf{k}) \left(\mathbf{k}^2 M_\chi^2 + 4\xi^2 M_\chi^2 \mu^2 \right). \tag{3.16}
 \end{aligned}$$

Now, again in the NR limit, we can replace $\xi M_\chi = k_z$, and substitute

$$\frac{dz d^2\mathbf{k}}{z(1-z)16\pi^3} \psi(z, \mathbf{k}) \rightarrow \frac{1}{4\pi^2 \sqrt{M_\chi}} \frac{1}{2\sqrt{2}} d^3k \frac{u(k)}{k^2}. \tag{3.17}$$

Using the relation

$$\int_0^\infty dk k^2 u(k) = 3\sqrt{\frac{\pi}{2}} R'(0), \quad (3.18)$$

we see that both form factors are proportional to the first derivative of the radial wave function at the origin $R'(0)$:

$$G_1(\mathbf{q}_1^2, \mathbf{q}_2^2) = \frac{2|\mathbf{q}_1||\mathbf{q}_2|}{[M_\chi^2 + \mathbf{q}_1^2 + \mathbf{q}_2^2]^2} \lambda, \quad G_2(\mathbf{q}_1^2, \mathbf{q}_2^2) = \frac{3M_\chi^2 + \mathbf{q}_1^2 + \mathbf{q}_2^2}{[M_\chi^2 + \mathbf{q}_1^2 + \mathbf{q}_2^2]^2} \lambda. \quad (3.19)$$

Here we introduced the short-hand notation

$$\lambda = \frac{8}{\sqrt{\pi}} \frac{R'(0)}{M^{3/2}}. \quad (3.20)$$

The reduced amplitude $\mathcal{T}$ now becomes:

$$\mathcal{T} = \frac{\lambda}{[M_\chi^2 + \mathbf{q}_1^2 + \mathbf{q}_2^2]^2} \left(2\mathbf{q}_1^2 \mathbf{q}_2^2 + (\mathbf{q}_1 \mathbf{q}_2) (3M_\chi^2 + \mathbf{q}_1^2 + \mathbf{q}_2^2) \right). \quad (3.21)$$

This result is in full agreement with refs. [6, 24].

4 The $\gamma^* \gamma^* \rightarrow \chi_{Q0}$ amplitude and electromagnetic transition form factors

The photon-photon fusion amplitude can be expressed in terms of two form-factors F_{TT}, F_{LL} , referring to photons having transverse or longitudinal polarizations in the $\gamma^* \gamma^*$ c.m.s. with the quantization axis taken along the $\gamma^* \gamma^*$ collision axis. Using the standard techniques reviewed in [25, 26], it can be written covariantly as

$$\frac{1}{4\pi\alpha_{\text{em}}} \mathcal{M}_{\mu\nu}(\gamma^*(\mathbf{q}_1) \gamma^*(\mathbf{q}_2) \rightarrow \chi_{Q0}(\mathbf{P})) = -\delta_{\mu\nu}^\perp(q_1, q_2) F_{TT}(q_1^2, q_2^2) + e_\mu^L(q_1) e_\nu^L(q_2) F_{LL}(q_1^2, q_2^2). \quad (4.1)$$

Here we have introduced the projector on transverse polarization states

$$-\delta_{\mu\nu}^\perp(q_1, q_2) = -g_{\mu\nu} + \frac{1}{X} \left((q_1 \cdot q_2)(q_{1\mu} q_{2\nu} + q_{1\nu} q_{2\mu}) - q_1^2 q_{2\mu} q_{2\nu} - q_2^2 q_{1\mu} q_{1\nu} \right), \quad (4.2)$$

with $X = (q_1 \cdot q_2)^2 - q_1^2 q_2^2$. The longitudinal polarization states of virtual photons read:

$$e_\mu^L(q_1) = \sqrt{\frac{-q_1^2}{X}} \left(q_{2\mu} - \frac{q_1 \cdot q_2}{q_1^2} q_{1\mu} \right), \quad e_\nu^L(q_2) = \sqrt{\frac{-q_2^2}{X}} \left(q_{1\nu} - \frac{q_1 \cdot q_2}{q_2^2} q_{2\nu} \right). \quad (4.3)$$

Notice that here we always consider polarization in the $\gamma\gamma$ -c.m. frame. From here, we can obtain the decay width for $S \rightarrow \gamma\gamma$ as

$$\Gamma(\chi_{Q0} \rightarrow \gamma\gamma) = \frac{\pi\alpha_{\text{em}}^2}{M_\chi} |F_{TT}(0, 0)|^2, \quad (4.4)$$

where M_χ is the mass of the scalar meson.

We now wish to relate the helicity form factors F_{TT}, F_{LL} to the form factors of the (reggeized) gluon fusion amplitude, G_1, G_2 . We start by evaluating the contractions of off-shell “polarizations” $n_\mu^\pm$ with the projectors on transverse and longitudinal helicities:

$$-n^{+\mu}n^{-\nu}\delta_{\mu\nu}^\perp = -1 + \frac{(q_1 \cdot q_2)}{X}q_1^+q_2^- = \frac{1}{X}\left(q_1^2q_2^2 + (q_1 \cdot q_2)(q_1 \cdot q_2)\right), \quad (4.5)$$

and

$$e_\mu^L(q_1)e_\nu^L(q_2)n^{+\mu}n^{-\nu} = \frac{\sqrt{q_1^2q_2^2}}{X}q_1^+q_2^- = \frac{\sqrt{q_1^2q_2^2}}{X}\left((q_1 \cdot q_2) + (q_1 \cdot q_2)\right). \quad (4.6)$$

Inserting these projections into the general form of the amplitude and collecting coefficients in front of $q_1 \cdot q_2$ and $q_1^2q_2^2$, we obtain

$$\begin{aligned} \frac{n^{+\mu}n^{-\nu}}{4\pi\alpha_{\text{em}}}\mathcal{M}_{\mu\nu} = (q_1 \cdot q_2) & \left[\frac{(q_1 \cdot q_2)}{X}F_{TT}(q_1^2, q_2^2) + \frac{|q_1||q_2|}{X}F_{LL}(q_1^2, q_2^2) \right] \\ & + |q_1||q_2| \left[\frac{|q_1||q_2|}{X}F_{TT}(q_1^2, q_2^2) + \frac{(q_1 \cdot q_2)}{X}F_{LL}(q_1^2, q_2^2) \right]. \end{aligned} \quad (4.7)$$

Here we used, that $\sqrt{q_1^2q_2^2} = |q_1||q_2|$. Let us also note, that

$$(q_1 \cdot q_2) = \frac{1}{2}(M_\chi^2 + q_1^2 + q_2^2) \text{ and } X = \frac{M_\chi^4}{4} \left(1 + \frac{2(q_1^2 + q_2^2)}{M_\chi^2} + \frac{(q_1^2 - q_2^2)^2}{M_\chi^4} \right),$$

do not depend on the azimuthal angles of q_1 and q_2 . Thus we can now easily read off the relation between the two sets of form factors:

$$\begin{pmatrix} G_1 \\ G_2 \end{pmatrix} = \frac{1}{e_Q^2\sqrt{N_c}X} \begin{pmatrix} |q_1||q_2| & (q_1 \cdot q_2) \\ (q_1 \cdot q_2) & |q_1||q_2| \end{pmatrix} \begin{pmatrix} F_{TT} \\ F_{LL} \end{pmatrix}. \quad (4.8)$$

This set of linear equations can be easily inverted, so that helicity form factors can be calculated using the form factors $G_{1,2}$:

$$\begin{pmatrix} F_{TT} \\ F_{LL} \end{pmatrix} = e_Q^2\sqrt{N_c} \begin{pmatrix} -|q_1||q_2| & (q_1 \cdot q_2) \\ (q_1 \cdot q_2) & -|q_1||q_2| \end{pmatrix} \begin{pmatrix} G_1 \\ G_2 \end{pmatrix}. \quad (4.9)$$

At the on-shell point, $q_i^2 = 0$, we have

$$F_{TT}(0, 0) = \frac{M_\chi^2}{2} G_2(0, 0), \quad (4.10)$$

while $G_1(0, 0) = 0$. In the NRQCD limit, we obtain

$$F_{TT}(0, 0) = e_Q^2\sqrt{N_c} \frac{12}{\sqrt{\pi}} \frac{R'(0)}{M_\chi^{3/2}}, \quad (4.11)$$

which leads to the known relation of the radiative decay width to $R'(0)$:

$$\Gamma(\chi_{Q0} \rightarrow \gamma\gamma) = \frac{9 \cdot 2^4 \alpha_{\text{em}}^2 e_Q^4 N_c}{M_\chi^4} |R'(0)|^2. \quad (4.12)$$

We can also decompose form factors G_1, G_2 into TT and LL components. Indeed, the reduced amplitude $\mathcal{T}$ has the decomposition analogous to eq. (4.1):

$$\mathcal{T}_{\mu\nu} = -\delta_{\mu\nu}^\perp(q_1, q_2) \frac{F_{TT}(q_1^2, q_2^2)}{e_Q^2 \sqrt{N_c}} + e_\mu^L(q_1) e_\nu^L(q_2) \frac{F_{LL}(q_1^2, q_2^2)}{e_Q^2 \sqrt{N_c}}, \quad (4.13)$$

and hence

$$\mathcal{T} = n^{+\mu} n^{-\nu} \mathcal{T}_{\mu\nu} = \mathcal{T}_{TT} + \mathcal{T}_{LL}. \quad (4.14)$$

Evidently, each of the amplitudes $\mathcal{T}_{TT}, \mathcal{T}_{LL}$ has a decomposition like in eq. (3.9):

$$\begin{aligned} \mathcal{T}_{TT} &= |\mathbf{q}_1||\mathbf{q}_2| G_{1,TT}(\mathbf{q}_1^2, \mathbf{q}_2^2) + (\mathbf{q}_1 \cdot \mathbf{q}_2) G_{2,TT}(\mathbf{q}_1^2, \mathbf{q}_2^2), \\ \mathcal{T}_{LL} &= |\mathbf{q}_1||\mathbf{q}_2| G_{1,LL}(\mathbf{q}_1^2, \mathbf{q}_2^2) + (\mathbf{q}_1 \cdot \mathbf{q}_2) G_{2,LL}(\mathbf{q}_1^2, \mathbf{q}_2^2), \end{aligned} \quad (4.15)$$

with

$$\begin{aligned} G_{1,TT} &= \frac{|\mathbf{q}_1||\mathbf{q}_2|}{X} \left(\frac{1}{2} (M_\chi^2 + \mathbf{q}_1^2 + \mathbf{q}_2^2) G_2 - |\mathbf{q}_1||\mathbf{q}_2| G_1 \right), \\ G_{2,TT} &= \frac{M_\chi^2 + \mathbf{q}_1^2 + \mathbf{q}_2^2}{2X} \left(\frac{1}{2} (M_\chi^2 + \mathbf{q}_1^2 + \mathbf{q}_2^2) G_2 - |\mathbf{q}_1||\mathbf{q}_2| G_1 \right), \\ G_{1,LL} &= \frac{M_\chi^2 + \mathbf{q}_1^2 + \mathbf{q}_2^2}{2X} \left(\frac{1}{2} (M_\chi^2 + \mathbf{q}_1^2 + \mathbf{q}_2^2) G_1 - |\mathbf{q}_1||\mathbf{q}_2| G_2 \right), \\ G_{2,LL} &= \frac{|\mathbf{q}_1||\mathbf{q}_2|}{X} \left(\frac{1}{2} (M_\chi^2 + \mathbf{q}_1^2 + \mathbf{q}_2^2) G_1 - |\mathbf{q}_1||\mathbf{q}_2| G_2 \right). \end{aligned} \quad (4.16)$$

This decomposition puts into evidence that the fusion of two off-shell (reggeized) gluons contains transverse as well as longitudinally polarized gluons in the frame of the produced system. The longitudinal polarizations are absent in approaches based on on-shell gluons.

5 Light-front wave functions and radiative decay widths

In figure 1 we show the nonrelativistic wave function $u(k)$ obtained with different $c\bar{c}$ potentials from the literature. We refer the reader to [13, 27] where more details can be found. The P-wave functions have extra zero at $k = 0$ compared to the S-wave function discussed in [13]. This will have important consequences for corresponding transition form factors.

The corresponding light-cone wave function is shown for example in figure 2 for the Buchmüller-Tye potential. Similar light-front wave functions were obtained from four other potentials from the literature. The wave functions are used then to calculate $\gamma^* \gamma^* \rightarrow \chi_{c0}$ and $\gamma^* \gamma^* \rightarrow \chi_{b0}$ form factors as discussed in the previous section. For both mesons the light-cone wave function has a similar shape but in the case of the χ_{b0} the range of k_T is broader.

Some basic information on χ_{c0} and χ_{b0} mesons is collected in table 1. The value of $|F_{TT}(0, 0)|$ and decay widths at the leading and next-to-leading order obtained from different potential models are collected in table 2 for χ_{c0} meson. We used the NLO expression

Figure 1. Quarkonium wave functions $u(k)$ for the χ_{c0} and the χ_{b0} for five potentials.

Figure 2. Light cone wave function $\psi(z, k_T)$ for the Buchmüller-Tye potential for χ_{c0} and χ_{b0} .

	$M_\chi(1P)[\text{MeV}]$	$\Gamma_{\text{tot}}[\text{MeV}]$	$\text{Br}(\chi_{Q0} \rightarrow \gamma\gamma)$
$\chi_{c0}(1P)$	3414.71 ± 0.30	10.8 ± 0.6	$(2.04 \pm 0.09) \cdot 10^{-4}$
$\chi_{b0}(1P)$	$9859.44 \pm 0.42 \pm 0.31$	—	—

Table 1. Values for meson mass, Γ_{tot} , and the branching ratio $\text{Br}(\chi_{Q0} \rightarrow \gamma\gamma) = \Gamma(\chi_{Q0} \rightarrow \gamma\gamma)/\Gamma_{\text{tot}}$ according to the Particle Data Group [29].

for the radiative decay width (see ref. [28]), inserting the LO relation between $\gamma\gamma$ decay width and $F(0, 0)$ from eq. (4.4):

$$\Gamma(\chi_{Q0} \rightarrow \gamma\gamma)_{\text{NLO}} = \Gamma(\chi_{Q0} \rightarrow \gamma\gamma)_{\text{LO}} \left[1 + \frac{\alpha_s}{\pi} \left(\frac{\pi^2}{3} - \frac{28}{9} \right) \right], \quad (5.1)$$

here we employed $\alpha_s = 0.26$ for χ_{c0} and $\alpha_s = 0.18$ for χ_{b0} (see ref. [28]).

Using eq. (5.1) and the experimental value for the radiative decay width we extracted $F_{TT}(0, 0)$ for both on-shell photons at leading order.

Similarly $|F_{TT}(0, 0)|$, but with the PDG value of m_c [29] in the form factor is presented in table 3. It is worth to notice, that the form factor is very sensitive to the quark mass. This feature of the form factor is manifested in tables 2, 3 and figure 7.

Potential type	$ F(0,0) [\text{GeV}]$	$\Gamma(\chi \rightarrow \gamma\gamma)_{\text{LO}}[\text{keV}]$	$\Gamma(\chi \rightarrow \gamma\gamma)_{\text{NLO}}[\text{keV}]$
Harmonic oscillator	0.18	1.56	1.58
Logarithmic	0.14	0.91	0.93
Powerlike	0.16	1.32	1.34
Cornell	0.10	0.44	0.46
Buchmüller-Tye	0.14	0.96	0.98
extracted from experiment [29]	0.21*		2.20 ± 0.16

Table 2. $F_{TT}(0,0)$ and $\Gamma(\chi \rightarrow \gamma\gamma)$ for χ_{c0} for five different potentials. The value marked by * is obtained from eq. (4.4), (5.1).

Potential type	$ F_{TT}(0,0) [\text{GeV}]$	$\Gamma(\chi_{c0} \rightarrow \gamma\gamma)_{\text{LO}}[\text{keV}]$	$\Gamma(\chi_{c0} \rightarrow \gamma\gamma)_{\text{NLO}}[\text{keV}]$
Harmonic oscillator	0.21	2.06	2.09
Logarithmic	0.18	1.54	1.56
Power-law	0.18	1.54	1.56
Cornell	0.17	1.41	1.43
Buchmüller-Tye	0.18	1.54	1.56
extracted from experiment [29]	0.21*		2.20 ± 0.16

Table 3. $F_{TT}(0,0)$ with $m_c = 1.27 \text{ GeV}$ and corresponding radiative widths.

Potential type	$m_b[\text{GeV}]$	$ F_{TT}(0,0) [\text{GeV}]$	$\Gamma(\chi_{b0} \rightarrow \gamma\gamma)_{\text{LO}}[\text{keV}]$	$\Gamma(\chi_{b0} \rightarrow \gamma\gamma)_{\text{NLO}}[\text{keV}]$
Harmonic oscillator	4.2	0.053	0.047	0.048
Logarithmic	5.0	0.032	0.017	0.017
Power-law	4.721	0.033	0.018	0.019
Cornell	5.17	0.028	0.014	0.014
Buchmüller-Tye	4.87	0.031	0.017	0.017

Table 4. $|F_{TT}(0,0)|$ for χ_{b0} with quark/antiquark mass corresponding to each potential. Here $\alpha_s = 0.18$ according to ref. [28].

Potential type	$ F_{TT}(0,0) [\text{GeV}]$	$\Gamma(\chi_{b0} \rightarrow \gamma\gamma)_{\text{LO}}[\text{keV}]$	$\Gamma(\chi_{b0} \rightarrow \gamma\gamma)_{\text{NLO}}[\text{keV}]$
Harmonic oscillator	0.053	0.048	0.049
Logarithmic	0.045	0.034	0.035
Power-law	0.042	0.030	0.030
Cornell	0.043	0.031	0.031
Buchmüller-Tye	0.042	0.030	0.030

Table 5. $|F_{TT}(0,0)|$ for χ_{b0} with quark/antiquark $m_b = 4.18 \text{ GeV}$ according to PDG [29].

We apply the same formalism also to the χ_{b0} meson. The corresponding results are summed up in tables 4 and 5, respectively for the potential model m_b value and for the PDG m_b value.

From eqs. (4.4), (4.12) and (5.1), using the radiative decay width for χ_{c0} , we extracted the first derivative of radial wave function at the origin $R'(0)$, which is contained in table 6 and compared to result obtained from eq. (3.18). In table 7 we listed $R'(0)$ values for χ_{b0} .

Potential type	m_c [GeV]	$R'(0)$ [GeV ^{5/2}]	$\Gamma(\chi_{c0} \rightarrow \gamma\gamma)_{\text{NLO}}$ [keV] Eq. (5.2a)	$\Gamma(\chi_{c0} \rightarrow \gamma\gamma)_{\text{NLO}}$ [keV] Eq. (5.2b)
Harmonic oscillator	1.4	0.27	2.42	5.54
Logarithmic	1.5	0.24	1.85	3.11
Powerlike	1.334	0.22	1.62	4.34
Cornell	1.84	0.32	2.51	3.38
Buchmüller-Tye	1.48	0.25	2.15	3.81
extracted from experiment [29]		0.25 ± 0.01	2.20 ± 0.16	

Table 6. $\Gamma(\chi_{c0} \rightarrow \gamma\gamma)_{\text{NLO}}$ according to eqs. (5.1), (5.2) with respect of quark mass for distinguished potential. $R'(0)$ is obtained from eq. (3.18).

Potential type	m_b [GeV]	$R'(0)$ [GeV ^{5/2}]	$\Gamma(\chi_{b0} \rightarrow \gamma\gamma)_{\text{NLO}}$ [keV] Eq. (5.2a)	$\Gamma(\chi_{b0} \rightarrow \gamma\gamma)_{\text{NLO}}$ [keV] Eq. (5.2b)
Harmonic oscillator	4.2	1.07	0.035	0.066
Logarithmic	5.0	1.22	0.045	0.043
Powerlike	4.721	0.98	0.029	0.035
Cornell	5.17	1.37	0.057	0.047
Buchmüller-Tye	4.87	1.13	0.038	0.041

Table 7. $\Gamma(\chi_{b0} \rightarrow \gamma\gamma)_{\text{NLO}}$ according to eqs. (5.1), (5.2) with respect of quark mass for distinguished potential. $R'(0)$ is obtained from eq. (3.18) and compared to $R'(0)$ extracted from experiment as explained in the text.

In the NR limit there is an ambiguity whether calculate the decay width using the quark mass m_Q or the meson mass M_χ , as to the NR accuracy $M_\chi = 2m_Q$, i.e.:

$$\Gamma(\chi_{Q0} \rightarrow \gamma\gamma) = \frac{9 \cdot 2^4 \alpha_{\text{em}}^2 e_Q^4 N_c}{M_\chi^4} |R'(0)|^2, \quad \text{or} \quad (5.2a)$$

$$\Gamma(\chi_{Q0} \rightarrow \gamma\gamma) = \frac{9 \alpha_{\text{em}}^2 e_Q^4 N_c}{m_Q^4} |R'(0)|^2. \quad (5.2b)$$

We calculated $\Gamma(\chi_{Q0} \rightarrow \gamma\gamma)_{\text{NLO}}$ respectively for χ_{c0} and χ_{b0} (see table 6 and table 7) for five different potentials from the literature using first eq. (5.2a) (with meson mass as in table 1) and then eq. (5.2b) for comparison.

6 The $\gamma^* \gamma^* \rightarrow \chi_{c0}$ and $\gamma^* \gamma^* \rightarrow \chi_{b0}$ form factors: numerical results

Let us now focus for a while on the form factors calculated with the help of the light-cone wave functions (see eqs. (3.10), (4.9)).

The form factors F_{TT} and F_{LL} , which have a more straightforward interpretation than G_1, G_2 , are shown in figure 3 for χ_{c0} and figure 4 for χ_{b0} . These results were obtained for the Buchmüller-Tye (B-T) potential. There is a smooth dependence on virtualities with maxima at $Q_1^2 = 0$ and $Q_2^2 = 0$ for both mesons. One can see the required Bose symmetry under exchange of Q_1^2 and Q_2^2 . The two form factors have a quite different dependence on photon virtualities. This is demonstrated in the right panel of figure 3 where the F_{LL} -to- F_{TT} ratio is shown. We note, that $F_{LL} \rightarrow 0$ when either $Q_1^2 \rightarrow 0$ or $Q_2^2 \rightarrow 0$.

Figure 3. $|F_{TT}|$ (left) and F_{LL} (middle) form factors and their ratio (right) for the Buchmüller-Tye potential for the χ_{c0} wave function.

Figure 4. $|F_{TT}|$ (left) and F_{LL} (right) form factors for the Buchmüller-Tye potential for χ_{b0} .

Figure 5. $|F_{TT}|$ (left) and F_{LL} (right) form factors for the Buchmüller-Tye potential as a function of ω and $\bar{Q}^2$ for χ_{c0} .

In order to compare to the situation for $\gamma^*\gamma^* \rightarrow \eta_c(1S, 2S)$ discussed in ref. [13], in figure 5 we show the two form factors as a function of

$$\bar{Q}^2 = \frac{Q_1^2 + Q_2^2}{2} \quad \text{and} \quad \omega = \frac{Q_1^2 - Q_2^2}{Q_1^2 + Q_2^2}. \quad (6.1)$$

In contrast to the case of η_c production [13] we do not observe the scaling in ω , neither for F_{TT} , nor F_{LL} .

Figure 6. The ratio of light front to the NRQCD form factors, (F_{TT} — upper panels), (F_{LL} — lower panels) for χ_{c0} and χ_{b0} . Here, for illustration, the LF form factor is calculated from wave function according to the Buchmüller-Tye potential model, with $m_c = 1.48$ GeV, $m_b = 4.87$ GeV.

In order to understand the impact of relativistic corrections, we show in figure 6 the ratio of F_{TT} and F_{LL} obtained in the light-front approach to the ones using the NRQCD approximation for the quarkonia wave function. In order to highlight the impact on the shape of form factors, we adjusted $|R'(0)|^2$ for the NRQCD result, such that the value of $F_{TT}(0, 0)$ coincides for both the light-front and NRQCD approaches. We see, that the shapes of form factors are in general different. For the case of $\chi_{c0}(1P)$ we observe about 20 – 40% difference in the two approaches to the form factors, while for the $\chi_{b0}(1P)$ the difference is smaller $\sim 10 - 25\%$.

In the left panel of figure 7 we compare the normalized form factors $F_{TT}(Q^2, 0)/F_{TT}(0, 0)$ for $\eta_c(1S)$ with its counterpart for $\chi_{c0}(1P)$ and $\chi_{b0}(1P)$ in the virtuality interval $Q^2 \in (0, 50 \text{ GeV}^2)$. For the form factor of the χ_{c0} we used the mass of the c -quark $m_c = 1.27$ GeV, and for the χ_{b0} form factor, $m_b = 4.18$ GeV. Results for five different potential models are presented for $\chi_{c0}(1P)$ and $\chi_{b0}(1P)$, whereas for the case of $\eta_c(1S)$, we show the result from ref. [13] for the power-law potential model only. In the right panel we present the normalized form factors for five different potential models for χ_{c0} and χ_{b0} , but with corresponding quark masses specific to each potential model.

A first measurement in e^+e^- collisions of χ_{c0} production and single-tag Q^2 dependence of the cross section was done recently by the Belle collaboration [30] in the $K_S K_S$ final state. The statistics is, however, too low (about 10 events) to allow detailed studies.

Figure 7. Normalized to unity TT form factor for three different mesons: $\eta_c(1S)$, $\chi_{c0}(1P)$, $\chi_{b0}(1P)$.

7 Differential cross sections for the $pp \rightarrow \chi_{c0}$ and $pp \rightarrow \chi_{b0}$ reactions

In this subsection we shall present differential distributions in the χ_{c0} transverse momentum and rapidity for the $pp \rightarrow \chi_{c0}$ reaction as well as distributions in the same observables for $pp \rightarrow \chi_{b0}$. Here we shall set $\sqrt{s} = 13$ TeV. The grids for G_1 and G_2 form factors are used in an interpolation procedure to obtain values relevant for the cross section calculations.

In figure 8 we show transverse momentum distributions of χ_{c0} and χ_{b0} for the broad range of its rapidity $-6 < y < 6$ for different $c\bar{c}$ and $b\bar{b}$ potentials specified in the figure. There is a rather small dependence on the potential used, except for the region of $p_T \lesssim 5$ GeV. In the first row one can find the plots where we used in the form factor the quark mass corresponding to the respective $q\bar{q}$ potential model. The second row presents the plots with $m_c = 1.27$ GeV and $m_b = 4.18$ GeV. In this case, in the region of meson $p_T < 5$ GeV there is no strong dependence on the $q\bar{q}$ potential model, but in the region of $p_T > 5$ GeV for the χ_{c0} one can observe a spread of the curves.

In figure 9 we compare results obtained with two different UGDs. The UGDs, which we labeled in figures as “KMR” are based on the prescription [31] with updates [32, 33]. The KMR prescription requires as an input a collinear gluon distribution. For this purpose, we used the NLO collinear gluon distribution from [34]. We also employed the UGD “JH-2013-set2” constructed from HERA data on deep inelastic structure functions as a solution of the CCFM evolution equation [35–37], which has been obtained by Hautmann and Jung [38–40]. The corresponding results differ at small transverse momenta p_T , in the peak region of the distribution. A similar behaviour was observed recently for the production of η_c in [14].

In the present analysis we explicitly calculated the $\gamma^*\gamma^* \rightarrow \chi_{c0}$ as well as $\gamma^*\gamma^* \rightarrow \chi_{b0}$ form factors as presented in the previous section. Here we wish to compare our results of corresponding cross sections with the result obtained previously [6] within the NRQCD approximation. In figure 10 and figure 11 we compare our full result obtained with the Buchmüller-Tye potential with that of the NRQCD approach. Both results were obtained with the KMR UGDF.

Figure 8. Comparison of p_T distributions for five different potentials types with corresponding quark mass in the first row, and the second row is with identical (for all potentials) $m_c = 1.27$ GeV and $m_b = 4.18$ GeV used in calculating form factors. Here the KMR UGDF was used for $\chi_{c0}(1P)$ (left panel) and $\chi_{b0}(1P)$ (right panel).

Figure 9. Comparison of p_T distributions of χ_{c0} (left) and χ_{b0} (right) for two different UGDFs. Here the Buchmüller-Tye potential was used.

Figure 10. Comparison of p_T distributions of χ_{c0} for our full model (solid line — with $m_c = 1.48$ GeV, dashed line — with $m_c = 1.27$ GeV) with the NRQCD result (see eq. (3.21)) (dash-dotted line). Here the B-T potential and the KMR UGDF were used. In the NRQCD matrix element we applied $R'(0) = 0.25 \text{ GeV}^{5/2}$ (the extracted value from the experimental radiative decay width, see table 6).

Figure 11. Comparison of p_T distributions for our full model (solid line — with $m_b = 4.87$ GeV, dashed line — with $m_b = 4.18$ GeV) with the NRQCD result (see eq. (3.21)) (dash-dotted line). Here the BT potential and the KMR UGDF are used.

In figure 12 we compare our results for χ_{c0} and χ_{b0} production with that for $\eta_c(1S)$ production from our recent analysis [14]. The cross section for χ_{c0} production is almost an order of magnitude smaller than that for $\eta_c(1S)$ production. While the distributions in rapidity have a similar shape, the distributions in transverse momentum are somewhat different. The different shape of transverse momentum distribution is a reflection of different matrix elements in both cases.

Since in addition, for the branching ratios to the $p\bar{p}$ channel, we have $Br(\chi_{c0} \rightarrow p\bar{p}) < Br(\eta_c \rightarrow p\bar{p})$, it may be very difficult to observe the χ_{c0} quarkonium in the proton-antiproton final state. Here the $\gamma\gamma$ final state could be a better option. The feasibility of

Figure 12. Differential cross section in terms of transverse momentum (l.h.s.) and rapidity (r.h.s.) of the meson (η_c , χ_{c0} , χ_{b0}).

Figure 13. On the l.h.s. differential cross section as a function of the transverse momentum of initial gluon (q_{1T} or q_{2T}) with rapidity cuts for the ATLAS and LHCb experiments. On the r.h.s. differential cross section as a function of the transverse momentum of χ_{c0} with rapidity cuts for the ATLAS and LHCb experiments.

such a measurement requires further Monte Carlo studies which go beyond the scope of the present paper.

Figures 13, 14 are devoted to specific experiments: ATLAS and LHCb. On the l.h.s. we show the q_{1T} or q_{2T} distributions. For the ATLAS experiment they are the same, but differ for the LHCb experiment (compare the red dashed and red dotted lines). On the r.h.s. we show the χ_{c0} transverse momentum distributions for the two experiments. A broader distribution is obtained for the ATLAS kinematics. The results presented on the r.h.s. could be directly confronted with experimental data provided such a measurement is feasible.

Let us now come to the discussion of some of the most striking results of our work, related to the polarizations of gluons. Indeed, eq. (4.13) to eq. (4.16) clearly show the

Figure 14. Differential cross section as a function of the transverse momentum of initial gluon (q_{1T} or q_{2T}) with rapidity cuts for the ATLAS and LHCb experiments. On the r.h.s. differential cross section as a function of the transverse momentum of χ_b with rapidity cuts for the ATLAS and LHCb experiments.

nontrivial helicity structure of the (reggeized) gluon-gluon fusion process. Differently from the collinear approach, gluons do not only carry transverse polarizations, but also longitudinal gluons enter. The χ -production cross section rather encodes the properties of a whole density matrix in the space of gluon polarizations.

In figure 15 and figure 16 we compare contributions from TT and LL amplitudes of the process $pp \rightarrow \chi_{c0}$ and $pp \rightarrow \chi_{b0}$ to their coherent sum. The LL component when separate is an order of magnitude smaller than the TT one. However, one can observe net negative interference effect — the cross section when including the LL component is smaller than that obtained for the TT component only. This is clearly visible for rapidity distributions for χ_{c0} . The details depend, however, on the meson transverse momentum. The interference effect changes sign at small p_T .

It is interesting to investigate how different the interplay between TT and LL components is in the full LF approach compared to the NRQCD approximation. To this end we show in figure 17 the p_T distribution for the TT and LL amplitudes as well as their coherent sum for the LF approach using the Buchmüller-Tye potential, as well as with both $F(0,0)$ adjusted to the same value. In the lower inlays of the plots, we show the ratios of the cross section of the LF and NRQCD approaches. It can be seen, that the difference in the full cross section is small, however there is a large difference in the — albeit small — longitudinal gluon contribution.

8 Conclusions

We have presented a formalism how to include $c\bar{c}$ wave functions for calculating production of χ_{c0} and χ_{b0} P-wave quarkonia in proton-proton collisions within the k_T -factorization approach. The corresponding radiative decay widths were discussed in detail. We have presented a discussion about the relevant quark mass dependence of the form factors at $Q_1^2 = 0$

Figure 15. The comparison of meson transverse momentum distributions with only the LL amplitude and only TT amplitude as well as for the full, coherent sum of TT and LL amplitudes. Here the B-T potential was used.

Figure 16. The comparison of meson rapidity distribution with only LL amplitude and only TT amplitude as well as for the full coherent sum of TT and LL amplitudes. Here the B-T potential was used.

and $Q_2^2 = 0$ and radiative decay widths. We have presented several tables demonstrating dependences on approximations used.

Different representations of the $\gamma^*\gamma^* \rightarrow \chi_{Q0}$ form factors were considered. We have presented and discussed the form factors as functions of (Q_1^2, Q_2^2) and $(\bar{Q}^2, \omega)$, where $\bar{Q}^2 = \frac{Q_1^2 + Q_2^2}{2}$ and $\omega = \frac{Q_1^2 - Q_2^2}{Q_1^2 + Q_2^2}$. In contrast to the η_c case we observe no scaling of the F_{TT} and F_{LL} form factors in ω for χ_{c0} . We have presented also the form factors as functions of single photon virtuality. Such form factors could be measured in future by Belle2 experiments.

Similar form factors were calculated next for $g^*g^* \rightarrow \chi_{c0}$. The form factors have then been used to calculate rapidity and transverse momentum distributions of χ_{c0} at $\sqrt{s} = 13$ TeV within the k_T -factorization approach. Two different UGDFs from the literature have been used.

Figure 17. The transverse momenta distributions of χ_{c0} and χ_{b0} with TT and LL components shown separately for the LF form factor as well as for the NRQCD form factor. The original masses from B-T potentials, $m_c = 1.48$ GeV, $m_b = 4.87$ GeV, are used to calculate the LF form factors. The derivative of radial wave functions at the origin were obtained from $F_{TT}(0,0)$ (tables 2, 4) and $R'(0) = 0.169$ GeV^{5/2}, $R'(0) = 0.736$ GeV^{5/2} for χ_{c0} , χ_{b0} , respectively.

We have compared the results of our full calculations with the form factors obtained with the light-cone $c\bar{c}$ wave function to the results of the NRQCD calculations as well as with those for $\eta_c(1S)$ production obtained recently [14]. The cross section for χ_{c0} production is somewhat smaller than that for the $\eta_c(1S)$ production which makes a measurement in the $p\bar{p}$ channel rather difficult. The $\gamma\gamma$ or $\gamma J/\psi$ final states might be a better option.

Finally we have performed a decomposition of the cross sections for $pp \rightarrow \chi_{c0}$ and $pp \rightarrow \chi_{b0}$ into TT, LL and their interference components. On average we have found a negative interference effect. For χ_{c0} production the inclusion of the LL components lowers the rapidity distributions — the result of the complete calculation is lower than that for the TT component alone. The interference effect depends, however, on χ_{c0} or χ_{b0} transverse momenta.

To compare our new light-cone WF results to the NRQCD limit, we have shown the ratio of corresponding form factors as a function of (Q_1^2, Q_2^2) , as well as the ratio of corresponding cross section as a function of meson transverse momentum. The ratios deviate from unity by 10 – 30%, when the transverse formfactor F_{TT} in both approaches is normalized to the same value at the on-shell point.

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A Light-cone wave function for the scalar meson $^{2S+1}L_J = {}^3P_0$, $J^{PC} = 0^{++}$

To construct the light-front wave function of the $Q\bar{Q}$ bound state of good angular momentum quantum numbers we use the procedure based on the Melosh transformation [20, 21] and Terentev substitution [22] starting from the quark-model rest-frame wave function (WF). For the scalar meson, which is a $J = 0$, $L = 1$, $S = 1$ state, the rest-frame WF has the form

$$\begin{aligned}\Psi_{\tau\bar{\tau}}(\vec{k}) &= \sum_{L_z, S_z} Y_{1L_z}(\hat{k}) \langle \frac{1}{2} \frac{1}{2} \tau\bar{\tau} | 1S_z \rangle \langle 11L_z S_z | 00 \rangle \frac{u(k)}{k} \\ &= \frac{1}{\sqrt{2}} \xi_Q^{\tau\dagger} \frac{(\vec{\sigma} \cdot \vec{k})}{k} i\sigma_2 \xi_{\bar{Q}}^{\bar{\tau}*} \frac{u(k)}{k} \frac{1}{\sqrt{4\pi}}.\end{aligned}\quad (\text{A.1})$$

Here, following the notation of [23], $\xi_Q^\tau, \xi_{\bar{Q}}^{\bar{\tau}*}$ are the canonical two-spinors of quark and antiquark respectively. The wave function is normalized as

$$\int d^3\vec{k} \sum_{\tau\bar{\tau}} |\Psi_{\tau\bar{\tau}}(\vec{k})|^2 = 1, \quad (\text{A.2})$$

which implies for the radial WF $u_1(k)$, that

$$\int_0^\infty dk u^2(k) = 1. \quad (\text{A.3})$$

We now wish to express the spin-orbit structure in terms of light front spinors $\chi_Q^\lambda, \chi_{\bar{Q}}^{\bar{\lambda}*}$, which are related to the canonical spinors via the Melosh-transform:

$$\begin{aligned}\xi_Q &= R(z, \mathbf{k}) \chi_Q \\ \xi_{\bar{Q}}^* &= R^*(1-z, -\mathbf{k}) \chi_{\bar{Q}}^*,\end{aligned}\quad (\text{A.4})$$

with the unitary matrix $R(z, \mathbf{k})$

$$R(z, \mathbf{k}) = \frac{m_Q + zM - i\vec{\sigma} \cdot (\vec{n} \times \mathbf{k})}{\sqrt{(m_Q + zM)^2 + \mathbf{k}^2}} = \frac{m_Q + zM - i\vec{\sigma} \cdot (\vec{n} \times \mathbf{k})}{\sqrt{zM(M + 2m_Q)}}. \quad (\text{A.5})$$

Here z and $1-z$ are the fractions of the meson’s lightfront plus-momentum carried by the quark and antiquark respectively, while their transverse momenta are denoted by $\pm\mathbf{k}$. We also introduced $\vec{n} = (0, 0, 1)$ and the vector product $\vec{n} \times \mathbf{k} = (-k_y, k_x, 0)$. The three-momentum $\vec{k}$ in the $Q\bar{Q}$ rest frame is parametrized as

$$\vec{k} = (\mathbf{k}, k_z) = \left(\mathbf{k}, \frac{1}{2}(2z-1)M \right), \quad (\text{A.6})$$

where M is the invariant mass of the $Q\bar{Q}$ system obtained from

$$M^2 = \frac{\mathbf{k}^2 + m_Q^2}{z(1-z)}. \quad (\text{A.7})$$

Notice also, that

$$k = |\vec{k}| = \frac{1}{2} \sqrt{M^2 - 4m_Q^2}. \quad (\text{A.8})$$

The light-front wave function depending on light-front helicities $\lambda\bar{\lambda}$ now has the form

$$\Psi_{\lambda\bar{\lambda}}(z, \mathbf{k}) = \chi_Q^{\lambda\dagger} \mathcal{O}' i\sigma_2 \chi_{\bar{Q}}^{\bar{\lambda}*} \phi(z, \mathbf{k}). \quad (\text{A.9})$$

Here the radial light-front wave function is expressed as

$$\phi(z, \mathbf{k}) = \frac{u(k)}{k} \frac{\sqrt{\mathcal{J}}}{\sqrt{4\pi}}, \quad (\text{A.10})$$

where $\mathcal{J}$ is a jacobian for the transition from integration over $\vec{k}$ to the relativistic two-body phase space parametrized through $z, \mathbf{k}$:

$$d^3\vec{p} = \mathcal{J} \frac{dz d^2\mathbf{k}}{z(1-z)16\pi^3}, \quad (\text{A.11})$$

which gives $\sqrt{\mathcal{J}} = 2\sqrt{M\pi^3}$, so that

$$\phi(z, \mathbf{k}) = \pi\sqrt{M} \frac{u(k)}{k}. \quad (\text{A.12})$$

Let us turn now to the spin-orbit factor. Denoting $\mathcal{O} = \vec{\sigma} \cdot \vec{k}/(\sqrt{2}k)$, we can write the Melosh-transformed spin-orbit vertex as

$$\mathcal{O}' = R^\dagger(z, \mathbf{k}) \mathcal{O} i\sigma_2 R^*(1-z, -\mathbf{k}) (i\sigma_2)^{-1} = R^\dagger(z, \mathbf{k}) \mathcal{O} R(1-z, -\mathbf{k}). \quad (\text{A.13})$$

In the last equality, we used the property of Pauli-matrices

$$i\sigma_2 \vec{\sigma}^* (i\sigma_2)^{-1} = -\vec{\sigma}. \quad (\text{A.14})$$

Inserting the explicit form of matrices R , we obtain

$$\begin{aligned} \mathcal{O}' = \frac{1}{\sqrt{z(1-z)}} \frac{1}{M(M+2m_Q)} \Big\{ & (m_Q^2 + m_Q M + z(1-z)M^2) \mathcal{O} \\ & - \vec{\sigma} \cdot (\vec{n} \times \mathbf{k}) \mathcal{O} \vec{\sigma} \cdot (\vec{n} \times \mathbf{k}) \\ & + (M+2m_Q) \frac{i}{2} \left(\mathcal{O} \vec{\sigma} \cdot (\vec{n} \times \mathbf{k}) + \vec{\sigma} \cdot (\vec{n} \times \mathbf{k}) \mathcal{O} \right) \\ & + (2z-1)M \frac{i}{2} \left(\mathcal{O} \vec{\sigma} \cdot (\vec{n} \times \mathbf{k}) - \vec{\sigma} \cdot (\vec{n} \times \mathbf{k}) \mathcal{O} \right) \Big\}. \end{aligned} \quad (\text{A.15})$$

For the case of interest, using the well known properties of Pauli matrices, we arrive at

$$\mathcal{O}' = \frac{1}{\sqrt{z(1-z)}} \frac{1}{2\sqrt{2}k} \left\{ \vec{\sigma} \cdot \mathbf{k} + (2z-1)m_Q \vec{\sigma} \cdot \vec{n} \right\}. \quad (\text{A.16})$$

We note in passing, that the effect of the Melosh rotation is simply a substitution of

$$\vec{k} \rightarrow \vec{k}' = \left(\frac{\mathbf{k}}{2\sqrt{z(1-z)}}, \frac{(2z-1)m_Q}{2\sqrt{z(1-z)}} \right), \quad (\text{A.17})$$

in the spin-orbit part of eq. (A.1).

Inserting this into eq. (A.9) gives us the following components of the light-front wave function:

$$\begin{aligned} \Psi_{\lambda\bar{\lambda}}(z, \mathbf{k}) &= \begin{pmatrix} \Psi_{++}(z, \mathbf{k}) & \Psi_{+-}(z, \mathbf{k}) \\ \Psi_{-+}(z, \mathbf{k}) & \Psi_{--}(z, \mathbf{k}) \end{pmatrix} \\ &= \frac{-1}{\sqrt{z(1-z)}} \begin{pmatrix} k_x - ik_y & m_Q(1-2z) \\ m_Q(1-2z) & -k_x - ik_y \end{pmatrix} \frac{\phi(z, \mathbf{k})}{\sqrt{2}\sqrt{M^2 - 4m_Q^2}}. \end{aligned} \quad (\text{A.18})$$

To lighten up the notation in the rest of the paper, we further absorb a factor $1/(2\sqrt{2}k)$ into the wave function, and introduce

$$\psi(z, \mathbf{k}) \equiv \frac{\phi(z, \mathbf{k})}{\sqrt{2}\sqrt{M^2 - 4m_Q^2}}. \quad (\text{A.19})$$

The normalization condition reads

$$\begin{aligned} 1 &= \int \frac{dz d^2\mathbf{k}}{z(1-z)16\pi^3} \sum_{\lambda, \bar{\lambda}} |\Psi_{\lambda\bar{\lambda}}(z, \mathbf{k})|^2 = \int \frac{dz d^2\mathbf{k}}{z(1-z)16\pi^3} |\phi(z, \mathbf{k})|^2 \\ &= \int \frac{dz d^2\mathbf{k}}{z(1-z)16\pi^3} 2(M^2 - 4m_Q^2) |\psi(z, \mathbf{k})|^2. \end{aligned} \quad (\text{A.20})$$

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