The decay $h \rightarrow Z\gamma$ in the Standard-Model Effective Field Theory

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ABSTRACT: We calculate the S -matrix element for the Higgs boson decay to a Z -boson and a photon, $h \rightarrow Z\gamma$, at one-loop in the Standard-Model Effective Field Theory (SMEFT) framework and in linear R_ξ -gauges. Our SMEFT expansion includes all relevant operators up to dimension-6 considered in Warsaw basis without resorting to any flavour or CP-conservation assumptions. Within this approximation there are 23 dimension-6 operators affecting the amplitude, not including flavour and hermitian conjugation. The result for the on-shell $h \rightarrow Z\gamma$ amplitude is gauge invariant, renormalisation-scale invariant and gauge-fixing parameter independent. The calculated ratio of the SMEFT versus the SM expectation for the $h \rightarrow Z\gamma$ decay width is then written in a semi-numerical form which is useful for further comparisons with related processes. For example, the $h \rightarrow Z\gamma$ amplitude contains 16 operators in common with the $h \rightarrow \gamma\gamma$ amplitude and one can draw useful results about its feasibility at current and future LHC data.

KEYWORDS: Effective Field Theories, Higgs Physics, Beyond Standard Model

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1 Introduction

The Higgs boson decay processes $h \rightarrow \gamma\gamma$ and $h \rightarrow Z\gamma$ are extremely important probes for physics beyond the Standard Model (SM) and are under intensive research ever since the Higgs boson discovery at LHC [1, 2]. Experimental bounds for both $h \rightarrow \gamma\gamma$ and $h \rightarrow Z\gamma$ decays were set by the CMS and ATLAS collaborations at LHC [3–6]. Although the $h \rightarrow \gamma\gamma$ decay width has been observed to within 15% w.r.t. the SM prediction, the situation is not the same for $h \rightarrow Z\gamma$. An upper bound for $h \rightarrow Z\gamma$ given by ATLAS [6], with center-of-mass energy $\sqrt{s} = 13$ TeV proton-proton collisions, integrated luminosity 36.1 fb^{-1} , and Higgs boson mass $M_h = 125.09 \text{ GeV}$, finds that $\sigma(pp \rightarrow h) \times B(h \rightarrow Z\gamma)$ is 6.6 times the SM prediction with 95% confidence level. More specifically, it is

$$\mu_{h \rightarrow Z\gamma} = \frac{\sigma(pp \rightarrow h) \times \text{Br}(h \rightarrow Z\gamma)}{\sigma(pp \rightarrow h)_{\text{SM}} \times \text{Br}(h \rightarrow Z\gamma)_{\text{SM}}} \lesssim 6.6. \quad (1.1)$$

If physics beyond the SM does not affect the Higgs production,¹ which mainly goes via the gluon fusion process, $gg \rightarrow h$, then the bound of (1.1) is translated to a bound on a ratio

$$\mathcal{R}_{h \rightarrow Z\gamma} = \frac{\Gamma(\text{EXP}, h \rightarrow Z\gamma)}{\Gamma(\text{SM}, h \rightarrow Z\gamma)}. \quad (1.2)$$

The decay $h \rightarrow Z\gamma$ has been calculated for the first time in the SM in refs. [7–9]. To our knowledge, in the Standard-Model Effective Field Theory (SMEFT) this process has been studied using a partial list of $d = 6$ operators in refs. [10–12], while an analysis with a complete set of $d = 6$ operators has recently been performed in ref. [13]. Here, we advance the current status of the SMEFT one-loop calculation for $\mathcal{R}_{h \rightarrow Z\gamma}$ in eq. (1.2) by presenting

¹We shall comment upon this issue at the end of section 4.

- a clear and concise renormalisation framework in general R_ξ -gauges,
- a gauge invariant master formula for the amplitude which self-explains several issues even for the SM-amplitude,
- a semi-analytic formula for $\mathcal{R}_{h \rightarrow Z\gamma}$,
- correlations between the ratios $\mathcal{R}_{h \rightarrow Z\gamma}$ and $\mathcal{R}_{h \rightarrow \gamma\gamma}$.

Obviously there are many similarities in the calculation with the $h \rightarrow \gamma\gamma$ decay worked out at one loop in SMEFT in ref. [14]² and we follow faithfully the renormalisation framework and the results found in there. We shall only focus on technical aspects that arise strictly in calculating the $h \rightarrow Z\gamma$ amplitude. This involves some subtle issues regarding gauge invariance which we address in section 3. The operators relevant for $h \rightarrow Z\gamma$ are discussed in section 2 and their effects in $\mathcal{R}_{h \rightarrow Z\gamma}$ in section 4. We conclude in section 5.

2 Operators

Let the lightest of the heavy-particle masses be of order Λ . Following the decoupling theorem [20], their effects at low energies can be encoded in the renormalisation group running of the SM parameters in addition to the appearance of local non-renormalisable operators. The later are parameterised at low energies by a SMEFT Lagrangian, which takes the form

$$\mathcal{L} = \mathcal{L}_{\text{SM}}^{(4)} + \sum_X C^X Q_X^{(6)} + \sum_f C'^f Q_f^{(6)}. \quad (2.1)$$

Eq. (2.1) contains the, renormalisable, SM Lagrangian $\mathcal{L}_{\text{SM}}^{(4)}$, the dimension-6 operators $Q_X^{(6)}$ that do not involve fermion fields, and the dimension-6 operators $Q_f^{(6)}$ which are operators that contain fermion fields.³ All Wilson coefficients should be rescaled by Λ^2 , for instance $C^X \rightarrow C^X/\Lambda^2$. We shall restore $1/\Lambda^2$ explicitly in section 4 later on. The primed coefficients C'^f are written in the gauge invariant Warsaw basis of ref. [18], while the un-primed coefficients C^f in fermion mass basis are defined in ref. [19].

The operators contributing to the $h \rightarrow Z\gamma$ decay are collected in table 1. They are classified into 8 different classes according to the notation of ref. [18]. There are in total 23 relevant operators, not counting flavour structure and Hermitian conjugation. In unitary gauge, the coefficient C^φ associated with the operator $Q_\varphi = (\varphi^\dagger \varphi)^3$ does *not* appear in the calculation at $\mathcal{O}(\Lambda^{-2})$ and therefore does not contribute in the final amplitude.⁴ The four-fermion operator Q_{ll} enters indirectly into the calculation through the relation between the vacuum expectation value (VEV) and the Fermi coupling constant G_F . There are no contributions from CP-violating operators up to $1/\Lambda^2$ terms in the EFT expansion. This is based upon the fact that the SM amplitude is CP-invariant (symmetric in particle momenta interchange) and all interference terms with CP-violating coefficients (antisymmetric in particle momenta interchange) of $\mathcal{O}(1/\Lambda^2)$, vanish identically.

²For similar studies see also refs. [15–17].

³The single $d = 5$ lepton number violating operator does not affect $h \rightarrow Z\gamma$ at one-loop.

⁴On the contrary, in the R_ξ -gauges C^φ enters in individual diagrams, but it cancels out completely in the final sum. This adds to a list of several checks we performed in the final amplitude (cf. eq. (3.14)).

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2 \varphi^3$	
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger \varphi) \Box (\varphi^\dagger \varphi)$	$Q_{e\varphi}$	$(\varphi^\dagger \varphi) (\bar{l}'_p e'_r \varphi)$
		$Q_{\varphi D}$	$(\varphi^\dagger D^\mu \varphi)^* (\varphi^\dagger D_\mu \varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger \varphi) (\bar{q}'_p u'_r \tilde{\varphi})$
				$Q_{d\varphi}$	$(\varphi^\dagger \varphi) (\bar{q}'_p d'_r \varphi)$
$X^2 \varphi^2$		$\psi^2 X \varphi$		$\psi^2 \varphi^2 D$	
$Q_{\varphi W}$	$\varphi^\dagger \varphi W_{\mu\nu}^I W^{I\mu\nu}$	Q_{eW}	$(\bar{l}'_p \sigma^{\mu\nu} e'_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi) (\bar{l}'_p \gamma^\mu l'_r)$
$Q_{\varphi B}$	$\varphi^\dagger \varphi B_{\mu\nu} B^{\mu\nu}$	Q_{eB}	$(\bar{l}'_p \sigma^{\mu\nu} e'_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi) (\bar{l}'_p \tau^I \gamma^\mu l'_r)$
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi W_{\mu\nu}^I B^{\mu\nu}$	Q_{uW}	$(\bar{q}'_p \sigma^{\mu\nu} u'_r) \tau^I \tilde{\varphi} W_{\mu\nu}^I$	$Q_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi) (\bar{e}'_p \gamma^\mu e'_r)$
		Q_{uB}	$(\bar{q}'_p \sigma^{\mu\nu} u'_r) \tilde{\varphi} B_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi) (\bar{q}'_p \gamma^\mu q'_r)$
		Q_{dW}	$(\bar{q}'_p \sigma^{\mu\nu} d'_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi) (\bar{q}'_p \tau^I \gamma^\mu q'_r)$
		Q_{dB}	$(\bar{q}'_p \sigma^{\mu\nu} d'_r) \varphi B_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi) (\bar{u}'_p \gamma^\mu u'_r)$
				$Q_{\varphi d}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi) (\bar{d}'_p \gamma^\mu d'_r)$
		ψ^4			
		Q_{ll}	$(\bar{l}'_p \gamma_\mu l'_r) (\bar{l}'_s \gamma^\mu l'_t)$		

Table 1. Dimension-6 operators contributing to $h \rightarrow Z\gamma$ decay. For brevity we suppress fermion chiral indices L, R . We follow here the notation of refs. [18, 19]. The operator class $\psi^2 \phi^2 D$ does not enter the $h \rightarrow \gamma\gamma$ amplitude.

The 16 out of 23 operators affecting $h \rightarrow Z\gamma$ are identical with those affecting the $h \rightarrow \gamma\gamma$ amplitude.⁵ The 7 operators that appear only in $h \rightarrow Z\gamma$ (those belonging to category $\psi^2 \varphi^2 D$ of table 1) may provide assistance in disentangling models for new physics in case of a $h \rightarrow Z\gamma$ experimental discovery. This is interesting because, if perturbative decoupling of the UV theory is assumed, the operators in $\psi^2 \varphi^2 D$ category are potentially tree-level generated [21, 22]. If the two amplitudes, $h \rightarrow Z\gamma$ and $h \rightarrow \gamma\gamma$, are calculated in the same renormalisation input scheme, we can compare the relative strengths of the various contributions assuming dominance of one operator at a time. Within EFT we should be able to pose predictions on possible sensitivity at LHC and future colliders for the $h \rightarrow Z\gamma$ decay rate.

3 Renormalisation of the $h \rightarrow Z\gamma$ amplitude

Our renormalisation procedure follows an old but clear description invented by A. Sirlin [23]. This procedure has already been applied successfully in a SMEFT calculation for $h \rightarrow \gamma\gamma$ in ref. [14] and is quickly repeated here for completeness before applying it to the calculation of the $h \rightarrow Z\gamma$ on-shell matrix element.

⁵The operator $Q_{\varphi l}^{(3)}$ does in fact enters in the $h \rightarrow \gamma\gamma$ amplitude, as well as in $h \rightarrow Z\gamma$, but only through the Fermi coupling constant redefinition, and not directly to $h \rightarrow \gamma\gamma$ one-loop amplitude.

3.1 Counterterms

We first start with the part of SMEFT Lagrangian bilinear in gauge fields in gauge basis given in eq. (3.14) of ref. [19], and write all bare parameters as differences between renormalised parameters and corresponding counterterms, for example $g_0 = g - \delta g$. Then, mass diagonalisation for vector fields is performed by the matrix $\mathbb{X}$ given in eq. (3.19) of ref. [19]. As we are only interested in an S -matrix element, we keep all fields unrenormalised but multiplying the $h \rightarrow Z\gamma$ one-particle irreducible (1PI) amplitude by proper LSZ constants [24] for the external fields of h , γ and Z . In this way and after some algebra, counterterms are generated and connected to self-energy corrections for vector bosons. We work at one-loop in $\hbar$ -expansion, and at $1/\Lambda^2$ in EFT expansion according to our discussion below eq. (2.1).

The definition of 2- and 3-point 1PI correlation functions contains all information we need to calculate the amplitude. Our definitions and conventions follow directly those of refs. [14] and [23]. We introduce the unrenormalised, but regularised, self-energies, that is 1PI diagrams for scalars $s_{1,2}(=h)$, and vector bosons $V_{1,2}(=W^\pm, Z, \gamma)$,

$$\begin{array}{c} s_1 \\ \text{---} \\ \text{---} \end{array} \text{---} \text{---} \begin{array}{c} s_2 \\ \text{---} \\ \text{---} \end{array} = -i\Pi_{s_1 s_2}(q^2), \quad (3.1)$$

$$\begin{array}{c} V_1^\mu \\ \text{---}\text{---}\text{---} \\ \xrightarrow{q} \end{array} \bullet \begin{array}{c} V_2^\nu \\ \text{---}\text{---}\text{---} \end{array} = i\Pi_{V_1 V_2}^{\mu\nu}(q^2) = iA_{V_1 V_2}(q^2)g^{\mu\nu} + iB_{V_1 V_2}(q^2)q^\mu q^\nu, \quad (3.2)$$

$$\begin{array}{c} V_1^\mu \\ \text{---} \otimes \text{---} V_2^\nu \\ \text{---} \xrightarrow{q} \end{array} = ig^{\mu\nu} \delta m_{V_1 V_2}^2 + iq^\mu q^\nu \delta^{(q)} m_{V_1 V_2}^2. \quad (3.3)$$

We also include the definition (3.3) for the vector boson counterterms since these are needed in the final amplitude. Physical masses for vector bosons, M_W and M_Z , are defined to keep their tree-level form in SMEFT, i.e. eqs. (3.17) and (3.21) of ref. [19], by choosing the corresponding counterterms such that

$$\delta m_W^2 = \text{Re } A_{WW}(M_W^2), \quad \text{and} \quad \delta m_Z^2 = \text{Re } A_{ZZ}(M_Z^2). \quad (3.4)$$

The physical masses M_W and M_Z for the $W^\pm$ and Z vector bosons are inputs in our calculation. In general, tadpole and tadpole-counterterm diagrams also appear in the right side of (3.4). However, one can arrange a renormalisation condition where the tree-level VEV, v , is the exact one up to one-loop order or beyond. Such a condition implies that tadpole plus tadpole-counterterm diagrams vanish identically [25]. In addition, we define the weak mixing angle, θ_W , through

$$c^2 \equiv \cos^2 \theta_W = \frac{M_W^2}{M_Z^2}, \quad s^2 \equiv 1 - c^2, \quad t \equiv \frac{s}{c}. \quad (3.5)$$

3.2 The amplitude

The on-shell S -matrix element for the $h \rightarrow Z\gamma$ amplitude can be written as

$$\langle \gamma(\epsilon^\mu, p_1), Z(\epsilon^\nu, p_2) | S | h(q) \rangle = \sqrt{Z_h} \sqrt{Z_\gamma} \sqrt{Z_Z} [i\mathcal{A}^{\mu\nu}(h \rightarrow Z\gamma)] \epsilon_\mu^*(p_1) \epsilon_\nu^*(p_2), \quad (3.6)$$

with $q = p_1 + p_2$ the incoming Higgs boson momentum, and p_1 (p_2) the outgoing four-momentum of photon (Z -boson) along with the polarisation four-vectors, $\epsilon(p_1)$ [$\epsilon(p_2)$], respectively. Similar to the mass counterterms δm_V^2 of (3.4), the LSZ factors Z_h , Z_γ and Z_Z are calculated by the requirement for the full propagators to look like those of free particle states asymptotically. Diagrammatically, the amputated diagrams needed to sum up in eq. (3.6) are given in terms of 2- and 3-point 1PI Feynman diagrams calculated on the mass shell, $p_1^2 = 0$, $p_2^2 = M_Z^2$ and $p_1 \cdot p_2 = (M_h^2 - M_Z^2)/2$,

$$iA^{\mu\nu}(h \rightarrow Z\gamma)\epsilon_\mu^*(p_1) \epsilon_\nu^*(p_2) = -\frac{h}{Z} \text{[diagram]} +$$

$$\begin{aligned} &+ \text{[diagram]} + \text{[diagram]} + \text{[diagram]} \\ &+ \text{[diagram]} + \text{[diagram]} + \text{[diagram]}. \end{aligned} \quad (3.7)$$

A square (“■”) in a vertex stands for a vertex generated by *only* $d = 6$ operators. Shaded blobs in the second line denote, as before, 1PI 3-point $hZ\gamma$ -vertex and 2-point $Z\gamma$ - or γZ -mixing at one-loop, while diagrams with “ $\otimes$ ” symbol denote counterterms generated following the procedure described above.

Before deriving the master formula for the $h \rightarrow Z\gamma$ decay amplitude, it is worth noting a cancellation between some gauge non-invariant parts of the counterterms. For this reason, let us focus on the third line of the diagrams in (3.7) and collect the terms of the diagrams proportional to the gauge invariant quantity

$$\Delta^{\mu\nu}(p_1, p_2) = p_1^\nu p_2^\mu - (p_1 \cdot p_2) g^{\mu\nu}. \quad (3.8)$$

Then, the gauge non-invariant leftovers are proportional to $g^{\mu\nu}$ (“pure-metric” terms). For example, the $hZ\gamma$ -vertex counterterm expands diagrammatically as,

$$\begin{array}{c} \text{---} \otimes \text{---} \\ \text{---} \end{array} \equiv \begin{array}{c} \text{---} \otimes \text{---} \\ \text{---} \end{array} \Big|_{\Delta} + \begin{array}{c} \text{---} \otimes \text{---} \\ \text{---} \end{array} \Big|_g, \quad (3.9)$$

and similarly for the diagram containing the $Z\gamma$ -mixing counterterm. We can then prove that the sum of the “pure-metric” contributions from the first and the third diagram of (3.7)

vanishes:⁶

$$\left. \text{---} \bigcirc \begin{array}{l} \text{wavy} \\ \text{wavy} \end{array} \right|_g + \left. \text{---} \bigcirc \begin{array}{l} \text{wavy} \\ Z \end{array} \right|_g = 0. \quad (3.10)$$

As a result, only the gauge invariant parts of these two counterterm diagrams make it into the master formula for the amplitude below. Note that these counterterm contributions exist even in the pure SM amplitude but usually not discussed in the literature. One can of course exploit gauge invariance to start with, as it was done for example in the first $h \rightarrow Z\gamma$ complete calculation of ref. [8], but it is really a nice cross-check of the calculation to see how contributions turn out to be gauge-invariant, respecting the usual Ward identities. Finally, note that the second diagram in the third line of (3.7) is gauge invariant by itself.

We are now ready to present the on-shell reduced matrix element defined as

$$\langle \gamma(\epsilon^\mu, p_1), Z(\epsilon^\nu, p_2) | S | h(q) \rangle = (2\pi)^4 \delta^{(4)}(q - p_1 - p_2) [i\mathcal{M}^{\mu\nu}(h \rightarrow Z\gamma)] \epsilon_\mu^*(p_1) \epsilon_\nu^*(p_2). \quad (3.11)$$

Adding the diagrams in (3.7) together and by comparing eqs. (3.6) and (3.11) we obtain

$$\begin{aligned} i\mathcal{M}^{\mu\nu}(h \rightarrow Z\gamma) = & 4i\Delta^{\mu\nu}(p_1, p_2) \\ & \times \left\{ -scvC^{\varphi B} \left[1 + \mathcal{X}^{\varphi B} - \frac{1}{t} \frac{A_{Z\gamma}(M_Z^2) + \delta m_{Z\gamma}^2}{M_Z^2} + t \frac{A_{Z\gamma}(0) + \delta m_{Z\gamma}^2}{M_Z^2} \right] \right. \\ & + scvC^{\varphi W} \left[1 + \mathcal{X}^{\varphi W} + t \frac{A_{Z\gamma}(M_Z^2) + \delta m_{Z\gamma}^2}{M_Z^2} - \frac{1}{t} \frac{A_{Z\gamma}(0) + \delta m_{Z\gamma}^2}{M_Z^2} \right] \\ & + \frac{s^2 - c^2}{2} vC^{\varphi WB} \left[1 + \mathcal{X}^{\varphi WB} - \frac{2sc}{s^2 - c^2} \frac{A_{Z\gamma}(M_Z^2) + A_{Z\gamma}(0) + 2\delta m_{Z\gamma}^2}{M_Z^2} \right] \\ & + \frac{1}{M_W} \bar{\Gamma}^{\text{SM}} + \sum_{i \neq \varphi B, \varphi W, \varphi WB} vC^i \Gamma^i \left. \right\} \\ & - 4ig^{\mu\nu} \frac{1}{8} v(\bar{g}^2 + \bar{g}'^2) \left[1 + v^2 C^{\varphi \square} + \frac{3}{4} v^2 C^{\varphi D} + 2scv^2 C^{\varphi WB} \right] \frac{A_{Z\gamma}(0)}{M_Z^2} \\ & - 4i(p_1 \cdot p_2) g^{\mu\nu} \left[\frac{1}{M_W} \bar{\Gamma}_g^{\text{SM}} + \sum_i vC^i \Gamma_g^i \right]. \end{aligned} \quad (3.12)$$

Eq. (3.12) is the master formula for the $h \rightarrow Z\gamma$ on-shell amplitude. The gauge-invariant quantity $\Delta^{\mu\nu}(p_1, p_2)$ has been defined in (3.8) while the self-energies and the counterterm $\delta m_{Z\gamma}^2$ in eqs. (3.2) and (3.3), respectively. Moreover, in (3.12) and for brevity, we defined the quantity

$$\mathcal{X}^i \equiv \Gamma^i - \frac{\delta C^i}{C^i} - \frac{\delta v}{v} + \frac{1}{2} \Pi'_{hh}(M_h^2) + \frac{1}{2} A'_{ZZ}(M_Z^2) + \frac{1}{2} A'_{\gamma\gamma}(0), \quad (3.13)$$

⁶We remark here that the counterterm for the $h\gamma\gamma$ -vertex is gauge-invariant by itself and, of course, zero in the SM.

where $i = \varphi B, \varphi W, \varphi WB$. In (3.13), Γ^i stands for 1PI contributions from the first diagram in the 2nd line of (3.7). $\bar{\Gamma}^{\text{SM}}$ is the SM contribution from triangle diagrams with W -bosons and fermions. In addition, δC^i and δv are counterterms for the Wilson coefficients with $i = \varphi B, \varphi W, \varphi WB$ and the VEV, respectively. The $\delta v/v$ counterterm is specified in eqs. (3.18)–(3.20) of ref. [14] after following the renormalisation scheme of refs. [23, 25]. The coefficients C^i (and in fact all Wilson coefficients in (2.1)) can be readily transformed in $\overline{\text{MS}}$ -scheme, $C - \delta C \rightarrow C(\mu) - \delta C$. As usual, in this scheme the counterterms δC^i subtract infinite parts proportional to $(\frac{2}{4-d} - \gamma + \log 4\pi)$ and can be read directly from eqs. (3.23)–(3.25) of ref. [14] as they have been adapted from refs. [26–28]. We confirm, even analytically, that these counterterms are capable of subtracting all infinities arising from the one-loop diagrams. The last three terms in the right-hand side arise from the product of the square roots of the LSZ factors in (3.6) where the prime denotes derivative with respect to q^2 , for example $\Pi'_{hh}(q^2) = d\Pi_{hh}(q^2)/dq^2$. Finally, note that the hadronic contributions from light quarks in $A'_{\gamma\gamma}(0)$, as given in eq. (4.21) of ref. [14], have been taken into account since they have an important contribution (one order of magnitude) in the non-logarithmic parts of the one-loop amplitude.

Note that eq. (3.12) is divided in two parts: the first part is proportional to the gauge-invariant quantity $\Delta^{\mu\nu}$, while the second part (last two lines of eq. (3.12)) is proportional to $g^{\mu\nu}$ and, therefore, is *not* gauge-invariant and violates the Ward-identity for charge conservation. We have proved that for every gauge-fixing choice these contributions vanish. To be more specific, we have checked explicitly that in unitary gauge $A_{Z\gamma}(0) = 0$ and that there are no leftover corrections proportional to $g^{\mu\nu}$, i.e. $\bar{\Gamma}_g^{\text{SM}} = \Gamma_g^i = 0$. What happens in R_ξ -gauges is discussed at the end of subsection 3.3.

We are now ready to write the $h \rightarrow Z\gamma$ amplitude at one-loop and at $1/\Lambda^2$ in EFT expansion. After removing the last two lines in (3.12) and checking that infinities cancel when applying the counterterms δC^i , we arrive at the matrix element

$$\begin{aligned}
 i\mathcal{M}^{\mu\nu}(h \rightarrow Z\gamma) = & 4i\Delta^{\mu\nu}(p_1, p_2) \\
 & \times \left\{ -scvC^{\varphi B} \left[1 + \mathcal{X}^{\varphi B} - \frac{1}{t} \frac{A_{Z\gamma}(M_Z^2) + \delta m_{Z\gamma}^2}{M_Z^2} + t \frac{A_{Z\gamma}(0) + \delta m_{Z\gamma}^2}{M_Z^2} \right] \right. \\
 & + scvC^{\varphi W} \left[1 + \mathcal{X}^{\varphi W} + t \frac{A_{Z\gamma}(M_Z^2) + \delta m_{Z\gamma}^2}{M_Z^2} - \frac{1}{t} \frac{A_{Z\gamma}(0) + \delta m_{Z\gamma}^2}{M_Z^2} \right] \\
 & + \frac{s^2 - c^2}{2} vC^{\varphi WB} \left[1 + \mathcal{X}^{\varphi WB} - \frac{2sc}{s^2 - c^2} \frac{A_{Z\gamma}(M_Z^2) + A_{Z\gamma}(0) + 2\delta m_{Z\gamma}^2}{M_Z^2} \right] \\
 & \left. + \frac{1}{M_W} \bar{\Gamma}^{\text{SM}} + \sum_{i \neq \varphi B, \varphi W, \varphi WB} vC^i \Gamma^i \right\}_{\text{finite}}, \quad (3.14)
 \end{aligned}$$

which is gauge invariant and renormalisation scale μ -independent, in a sense that $\mu d\mathcal{M}^{\mu\nu}/d\mu = 0$. The subscript “finite” means that infinities proportional to $(\frac{2}{4-d} - \gamma + \log 4\pi)$ have been removed from expressions such as A_{VV} , A'_{VV} , Γ^i , etc., with counterterms δC^i removed from the quantity $\mathcal{X}^i$ of (3.13) as well. All self-energies but $A_{Z\gamma}(M_Z^2)$ and $A'_{ZZ}(M_Z^2)$ appearing in (3.14) are given analytically in general R_ξ -gauges, in appendix A of ref. [14]

(see also [29] for formulae in $\xi = 1$). It is obvious from (3.14) that self-energies for the Higgs or vector bosons should be calculated *only* in the SM, not (necessarily) in SMEFT. The three-point vertex functions Γ^i are in general too lengthy and is not really illuminating to be given here.

Although we leave the expression (3.14) for the matrix element in a slightly involved form, it can be reduced further by noting the following. As in the case of the $h \rightarrow \gamma\gamma$ amplitude, there is a remarkable relation between factors multiplying the coefficients $C^{\varphi B}$ and $C^{\varphi W}$ when replacing

$$\tan \theta_W \rightarrow -\frac{1}{\tan \theta_W}, \quad (3.15)$$

while on the other hand, factors multiplying $C^{\varphi WB}$ in (3.14) remain invariant. In addition, elementary trigonometric relations may reduce eq. (3.14) further. For example, by using $\tan \theta_W - 1/\tan \theta_W = -2 \cot^2(2\theta_W)$ one may factor out $\delta m_{Z\gamma}^2/M_Z^2$ terms. We believe, however, that eq. (3.14) is more transparent and easily understood when read in conjunction with the list of diagrams and counterterms of eq. (3.7).

Finally, some words about calculating the diagrams appearing in the shaded blobs of (3.7). We used the Feynman Rules of ref. [19], given in general R_ξ -gauges, and passed them manually to the **Mathematica** package **FeynCalc** [30, 31]. The Feynman integrals are regulated with dimensional regularisation [32] with the Dirac algebra performed in d -dimensions. The result is reduced to basic Passarino-Veltman functions [33]. We then checked expressions for analytic functions, some of them presented in ref. [14], against the numerical library **LoopTools** [34, 35]. The most crucial (and time consuming) test is the gauge-fixing parameter independence of the amplitude (3.12).

3.3 Gauge-fixing parameters cancellation

Since the cancellation in the amplitude of the gauge-fixing parameters, collectively denoted as ξ , is a very involved and important cross-check of the validity of our calculation, let us give here some insight on this particular computational task. In general, there are two different ways of how ξ -dependent contributions arise in SMEFT. Let us call the result one finds by subtracting the unitary gauge result from the full result in R_ξ -gauges the ξ -dependent result. For an operator C^i there are *explicit* ξ -dependent contributions, coming from the ξ -dependent result which is proportional to the Wilson coefficient C^i . There are also *implicit* contributions, coming from the ξ -dependent SM-like result by Taylor-expanding the masses with C^i as an expansion parameter.

In the $h \rightarrow Z\gamma$ process there are two independent gauge-fixing parameters, ξ_W and ξ_Z . We therefore prove the ξ -cancellation in the amplitude for each of these parameters *independently*. Interchanging between gauge-fixing parameters is a great advantage of the Feynman rules written in general R_ξ -gauges in ref. [19]. We also checked gauge-invariance without any renormalisation scheme. In this case, one has to add a Higgs tadpole diagram in the “hhAZ” vertex. As explained in eq. (3.12), the last two lines do not appear in the unitary gauge at all. On the other hand, each of the terms in these lines contributes in the ξ -dependent part, so one has to prove that they add to zero. Note that there are explicit contribution from the SMEFT Γ s in the last line as well as from the vertex and the

$Z\gamma$ -mixing in the penultimate line, and also implicit contributions from the Z -boson mass and the $Z\gamma$ -mixing in the penultimate line and the ξ -dependent SM result in the last line.

It is important to stress here the analytic result of the $Z\gamma$ -mixing in SMEFT with $d = 6$ operators. One can prove that the result is simply given by

$$A_{Z\gamma}^{\text{SMEFT}}(0) = \left(1 + \frac{1}{2}v^2 C^{\varphi D}\right) A_{Z\gamma}^{\text{SM}}(0), \quad (3.16)$$

where $A_{Z\gamma}^{\text{SM}}(0)$ is the SM-like value at $q^2 = 0$, given analytically in eq. (A.6) of ref. [14]. Note that $A_{Z\gamma}^{\text{SM}}(0)$ is a function of the SMEFT couplings, the VEV and the W -boson mass. Therefore, the SM-like and the SM values coincide. We believe that (3.16) has interesting consequences in the general SMEFT renormalisation program.

Each coefficient has its own unique way of how the ξ -cancellation occurs. As an example, let us discuss here the $C^{\varphi WB}$ coefficient. Since $A_{Z\gamma}^{\text{SMEFT}}$ doesn't depend on this coefficient (either explicitly or implicitly) and the vertex contribution cancels that of the Z -boson mass, $C^{\varphi WB}$ cancels trivially in the penultimate line. Therefore, the implicit and explicit contributions from the last line should cancel among each other, which we have proved that this is exactly the case.

4 Results

4.1 $h \rightarrow Z\gamma$ in the Standard Model and the input parameters scheme

As it is well known, the $h \rightarrow Z\gamma$ SM contribution, $\bar{\Gamma}^{\text{SM}}/M_W$, in eq. (3.14) is a sum of one-loop diagrams with only $W^\pm$ bosons and charged fermions, f , circulating in the loop. In terms of the SMEFT parameters $\{\bar{g}, \bar{g}', v\}$, defined in ref. [19], we find that

$$\frac{\bar{\Gamma}^{\text{SM}}}{M_W} = \frac{\bar{g}\bar{g}'}{16\pi^2 v} \left[\sum_f N_{c,f} Q_f \left(T_f^3 - 2Q_f \frac{\bar{g}'^2}{\bar{g}^2 + \bar{g}'^2} \right) I_f + \frac{\bar{g}^2}{2(\bar{g}^2 + \bar{g}'^2)} I_W \right], \quad (4.1)$$

where the electromagnetic fermion charges and the third component of the weak isospin are given by

$$Q_f = \begin{cases} 0, & \text{for } f = \nu_e, \nu_\mu, \nu_\tau \\ -1, & \text{for } f = e, \mu, \tau \\ 2/3, & \text{for } f = u, c, t \\ -1/3, & \text{for } f = d, s, b \end{cases} \quad \text{and} \quad T_f^3 = \begin{cases} 1/2, & \text{for } f = \nu_e, \nu_\mu, \nu_\tau, u, c, t \\ -1/2, & \text{for } f = e, \mu, \tau, d, s, b \end{cases}. \quad (4.2)$$

and the colour factor $N_{c,f}$ is equal to 1 for leptons and 3 for quarks. In (4.1) I_f and I_W contain the contribution from the fermionic and the bosonic sector respectively. Explicitly,

these quantities are given in terms of PV functions as:⁷

$$I_f = \frac{m_f^2}{(M_h^2 - M_Z^2)^2} \left\{ 2M_Z^2 [B_0(M_h^2, m_f^2, m_f^2) - B_0(M_Z^2, m_f^2, m_f^2)] \right. \\ \left. - (M_h^2 - M_Z^2) [(M_h^2 - M_Z^2 - 4m_f^2)C_0(0, M_h^2, M_Z^2, m_f^2, m_f^2, m_f^2) - 2] \right\}, \quad (4.3)$$

and

$$I_W = \frac{1}{(M_h^2 - M_Z^2)^2} \left\{ \frac{M_Z^2}{M_W^2} [M_h^2(M_Z^2 - 2M_W^2) + 2M_W^2(M_Z^2 - 6M_W^2)] \right. \\ \times [B_0(M_h^2, M_W^2, M_W^2) - B_0(M_Z^2, M_W^2, M_W^2)] \\ + \frac{M_h^2 - M_Z^2}{M_W^2} [2M_W^2(M_h^2(6M_W^2 - M_Z^2) - 12M_W^4 - 6M_W^2 M_Z^2 + 2M_Z^4) \\ \left. \times C_0(0, M_h^2, M_Z^2, M_W^2, M_W^2, M_W^2) + M_h^2(M_Z^2 - 2M_W^2) + 2M_W^2(M_Z^2 - 6M_W^2)] \right\}. \quad (4.4)$$

We have proved explicitly that the SM matrix element is finite, gauge invariant and gauge-fixing parameter independent.

We can express the SM-like result of eq. (4.1), or for that matter any other contribution in (3.14), in terms of well-measured quantities that will be taken as inputs in evaluating the $h \rightarrow Z\gamma$ amplitude. The set of well-measured quantities we have chosen, contains G_F , M_W and M_Z . Using the following expressions for $\bar{g}'$, $\bar{g}$ and v as functions of the Fermi coupling constant G_F , the physical W -boson mass, M_W , and the physical Z -boson mass, M_Z , we obtain:

$$\bar{g}' = 2^{5/4} \sqrt{M_Z^2 - M_W^2} \sqrt{G_F} \left[1 - \frac{1}{2\sqrt{2}G_F} \left(\frac{C_{11}^{\varphi l(3)}}{\Lambda^2} + \frac{C_{22}^{\varphi l(3)}}{\Lambda^2} - \frac{C_{1221}^{ll}}{\Lambda^2} \right) \right. \\ \left. - \frac{M_Z^2}{4\sqrt{2}G_F(M_Z^2 - M_W^2)} \left(\frac{C^{\varphi D}}{\Lambda^2} + 4\frac{M_W}{M_Z} \sqrt{1 - \frac{M_W^2}{M_Z^2}} \frac{C^{\varphi WB}}{\Lambda^2} \right) \right], \quad (4.5)$$

$$\bar{g} = 2^{5/4} M_W \sqrt{G_F} \left[1 - \frac{1}{2\sqrt{2}G_F} \left(\frac{C_{11}^{\varphi l(3)}}{\Lambda^2} + \frac{C_{22}^{\varphi l(3)}}{\Lambda^2} - \frac{C_{1221}^{ll}}{\Lambda^2} \right) \right], \quad (4.6)$$

$$v = \frac{1}{2^{1/4} \sqrt{G_F}} \left[1 + \frac{1}{2\sqrt{2}G_F} \left(\frac{C_{11}^{\varphi l(3)}}{\Lambda^2} + \frac{C_{22}^{\varphi l(3)}}{\Lambda^2} - \frac{C_{1221}^{ll}}{\Lambda^2} \right) \right]. \quad (4.7)$$

Finally we can express the parameters in eq. (4.1) as a function of the experimental quantities G_F , M_W and M_Z taken from PDG [36]. The reason for choosing the input scheme $\{G_F, M_W, M_Z\}$ is twofold: first, it has natural implementation⁸ into our renormalisation prescription discussed already in section 3 and especially into the simple definition

⁷Our notation for PV-functions is identical to those of `LoopTools` in ref. [34].

⁸At least more natural than the scheme with the input set $\{\alpha_{em}, G_F, M_Z\}$.

of the weak mixing angle in eq. (3.5)⁹ and second it is a scheme that is becoming increasingly popular after refs. [13, 17] with whom we would like to compare our results. Other advantages of this scheme have also been put forward by ref. [37].

After replacing $\bar{g}'$, $\bar{g}$ and v in eq. (4.1) with eqs. (4.5)–(4.7), it is rather more instructive to present also the numerical result here. This reads

$$\begin{aligned} \frac{\bar{\Gamma}^{\text{SM}}}{M_W} = & -(1.43 \times 10^{-5} - 1.11 \times 10^{-8}i) \\ & + (1.07 + 1.38 \times 10^{-4}i) \frac{C^{\varphi WB}}{\Lambda^2} + (0.64 + 8.28 \times 10^{-5}i) \frac{C^{\varphi D}}{\Lambda^2} \\ & + (1.30 - 1.00 \times 10^{-3}i) \left[\frac{C_{11}^{\varphi l(3)}}{\Lambda^2} + \frac{C_{22}^{\varphi l(3)}}{\Lambda^2} - \frac{C_{1221}^{\varphi l}}{\Lambda^2} \right]. \end{aligned} \quad (4.8)$$

As one can see, the imaginary part of the SM-like amplitude is more than three orders of magnitude smaller than the real part and can be safely ignored in the following. Our result agrees with ref. [38] and partially with refs. [8, 39].¹⁰ The pure SM contribution, $\bar{\Gamma}^{\text{SM}}/M_W$, can be factored out in the amplitude of (3.14) and after squaring and integrating over the phase space of the final state particles, γ and Z , one can easily find the decay rate for $h \rightarrow Z\gamma$ in the SM and in SMEFT. It is then useful to express our results in terms of the quantity

$$\mathcal{R}_{h \rightarrow Z\gamma} = \frac{\Gamma(\text{SMEFT}, h \rightarrow Z\gamma)}{\Gamma(\text{SM}, h \rightarrow Z\gamma)} \equiv 1 + \delta\mathcal{R}_{h \rightarrow Z\gamma}, \quad (4.9)$$

and compare with the experimental bound of eq. (1.2). In the next subsection we present corrections for $\delta\mathcal{R}_{h \rightarrow Z\gamma}$ from new physics in the form of running Wilson coefficients of the operators listed in table 1. In addition, we search for correlations with an analogous expression arising from the $h \rightarrow \gamma\gamma$ decay.

4.2 Semi-numerical expression for the ratio $\mathcal{R}_{h \rightarrow Z\gamma}$

In this section we finally present our results for $\delta\mathcal{R}_{h \rightarrow Z\gamma}$. As in ref. [14], we shall separate constant and renormalisation scale μ -dependent logarithmic parts which multiply RGE running Wilson coefficients, $C(\mu)$. In “Warsaw” mass-basis of ref. [19], by exploiting the input parameters scheme $\{G_F, M_W, M_Z\}$ with the new-physics scale Λ written in TeV-

⁹Note that the expression for $\bar{g}'$ in eq. (4.5) becomes much simpler upon substitution of the weak mixing angle definition of (3.5). Then the second line of (4.5) reads:

$$-\frac{1}{4\sqrt{2}s^2G_F} \left(\frac{C^{\varphi D}}{\Lambda^2} + 4sc \frac{C^{\varphi WB}}{\Lambda^2} \right).$$

¹⁰We have a minus sign difference in the term before the last parenthesis of eq. (4) of ref. [8]. Furthermore, our SM result agrees with ref. [39] only if the branches of the piecewise function $g(\tau)$ in eq. (2.56) are reversed.

units, we find:¹¹

$$\begin{aligned}
 \delta\mathcal{R}_{h\rightarrow Z\gamma} \simeq & 0.18 \frac{C_{1221}^{\ell\ell} - C_{11}^{\varphi\ell(3)} - C_{22}^{\varphi\ell(3)}}{\Lambda^2} + 0.12 \frac{C^{\varphi\Box} - C^{\varphi D}}{\Lambda^2} \\
 & - 0.01 \frac{C_{33}^{d\varphi} - C_{33}^{u\varphi}}{\Lambda^2} + 0.02 \frac{C_{33}^{\varphi u} + C_{33}^{\varphi q(1)} - C_{33}^{\varphi q(3)}}{\Lambda^2} \\
 & + \left[14.99 - 0.35 \log \frac{\mu^2}{M_W^2} \right] \frac{C^{\varphi B}}{\Lambda^2} - \left[14.88 - 0.15 \log \frac{\mu^2}{M_W^2} \right] \frac{C^{\varphi W}}{\Lambda^2} \\
 & + \left[9.44 - 0.26 \log \frac{\mu^2}{M_W^2} \right] \frac{C^{\varphi WB}}{\Lambda^2} + \left[0.10 - 0.20 \log \frac{\mu^2}{M_W^2} \right] \frac{C^W}{\Lambda^2} \\
 & - \left[0.11 - 0.04 \log \frac{\mu^2}{M_W^2} \right] \frac{C_{33}^{uB}}{\Lambda^2} + \left[0.71 - 0.28 \log \frac{\mu^2}{M_W^2} \right] \frac{C_{33}^{uW}}{\Lambda^2} \\
 & - \left[0.01 + 0.00 \log \frac{\mu^2}{M_W^2} \right] \frac{C_{22}^{uW}}{\Lambda^2} - \left[0.01 + 0.00 \log \frac{\mu^2}{M_W^2} \right] \frac{C_{33}^{dW}}{\Lambda^2} + \dots, \quad (4.10)
 \end{aligned}$$

where the ellipses denote contributions from operators that are less than $0.01 \times C/\Lambda^2$. Note that the VEV appearing at tree-level introduces one-loop corrections when exchanged for the Fermi constant through $\bar{G}_F = 1/[\sqrt{2}v^2(1 - \Delta r)]$. We follow here the same procedure as in ref. [14] below eq. (4.16). Formula (4.10) should be renormalisation scale (μ) independent at one-loop and up to terms with $1/\Lambda^2$ in EFT expansion. Assuming that Higgs boson production is not affected by the operators listed in table 1, the current experimental bound of (1.1) sets rather weak constraints on tree-level SMEFT Wilson coefficients. As an example, for $\mu = M_W$ we obtain

$$\frac{|C^{\varphi B}|}{\Lambda^2} \lesssim \frac{0.4}{(1 \text{ TeV})^2}, \quad \frac{|C^{\varphi W}|}{\Lambda^2} \lesssim \frac{0.4}{(1 \text{ TeV})^2}, \quad \frac{|C^{\varphi WB}|}{\Lambda^2} \lesssim \frac{0.7}{(1 \text{ TeV})^2}. \quad (4.11)$$

For loop-induced operators, the logarithmic part is of the same order of magnitude as of the constant part. Contributions in the first and second line of (4.10) arise from finite fermionic triangle diagrams that just rescale the SM result. Wilson coefficients $C_{33}^{\varphi u}$, $C_{33}^{\varphi q(1)}$, $C_{33}^{\varphi q(3)}$ are the new operators appearing now in $h \rightarrow Z\gamma$ decay relative to $h \rightarrow \gamma\gamma$ (see table 1). Interestingly, out of many operators only three made a contribution for more than 1% and in fact they are just barely pass that threshold!¹²

How to use eq. (4.10)? First, decouple heavy particles from a more fundamental theory. Match to Warsaw-basis operators relevant for $h \rightarrow Z\gamma$, listed in table 1. Set the coefficients, $C(\mu)$ at a scale $\mu = \Lambda$. Use RGEs to run the parameters down to the Higgs mass scale — one could use dedicated codes for this purpose like those in refs. [40, 41]. Plug in the results for $C(\mu = M_h)$ coefficients in eq. (4.10) and obtain $\delta\mathcal{R}_{h\rightarrow Z\gamma}$. As long as discussing the same physical process in the same input parameter scheme, the result should be unambiguous.

¹¹Our result is in agreement with the revised (arXiv v3) version of ref. [13].

¹²We consider 1% of corrections as an indicative limit that LHC can reach for $\delta\mathcal{R}_{h\rightarrow Z\gamma}$ at later stages of its run.

Keeping in mind the current experimental sensitivity for $h \rightarrow Z\gamma$, eq. (4.10) is not of much use. It is however, quite interesting to check for a $h \rightarrow Z\gamma$ projective reach by comparing $\delta\mathcal{R}_{h \rightarrow Z\gamma}$ of eq. (4.10) with $\delta\mathcal{R}_{h \rightarrow \gamma\gamma}$ taken from ref. [14] but translated into the $\{G_F, M_W, M_Z\}$ input scheme. We have,

$$\begin{aligned}
 \delta\mathcal{R}_{h \rightarrow \gamma\gamma} \simeq & 0.18 \frac{C_{1221}^{ll} - C_{11}^{\varphi l(3)} - C_{22}^{\varphi l(3)}}{\Lambda^2} + 0.12 \frac{C^{\varphi\Box} - 2C^{\varphi D}}{\Lambda^2} \\
 & - 0.01 \frac{C_{22}^{e\varphi} + 4C_{33}^{e\varphi} + 5C_{22}^{u\varphi} + 2C_{33}^{d\varphi} - 3C_{33}^{u\varphi}}{\Lambda^2} \\
 & - \left[48.04 - 1.07 \log \frac{\mu^2}{M_W^2} \right] \frac{C^{\varphi B}}{\Lambda^2} - \left[14.29 - 0.12 \log \frac{\mu^2}{M_W^2} \right] \frac{C^{\varphi W}}{\Lambda^2} \\
 & + \left[26.17 - 0.52 \log \frac{\mu^2}{M_W^2} \right] \frac{C^{\varphi WB}}{\Lambda^2} + \left[0.16 - 0.22 \log \frac{\mu^2}{M_W^2} \right] \frac{C^W}{\Lambda^2} \\
 & + \left[2.11 - 0.84 \log \frac{\mu^2}{M_W^2} \right] \frac{C_{33}^{uB}}{\Lambda^2} + \left[1.13 - 0.45 \log \frac{\mu^2}{M_W^2} \right] \frac{C_{33}^{uW}}{\Lambda^2} \\
 & - \left[0.03 + 0.01 \log \frac{\mu^2}{M_W^2} \right] \frac{C_{22}^{uB}}{\Lambda^2} - \left[0.01 + 0.00 \log \frac{\mu^2}{M_W^2} \right] \frac{C_{22}^{uW}}{\Lambda^2} \\
 & + \left[0.03 + 0.01 \log \frac{\mu^2}{M_W^2} \right] \frac{C_{33}^{dB}}{\Lambda^2} - \left[0.02 + 0.01 \log \frac{\mu^2}{M_W^2} \right] \frac{C_{33}^{dW}}{\Lambda^2} \\
 & + \left[0.02 + 0.00 \log \frac{\mu^2}{M_W^2} \right] \frac{C_{33}^{eB}}{\Lambda^2} - \left[0.01 + 0.00 \log \frac{\mu^2}{M_W^2} \right] \frac{C_{33}^{eW}}{\Lambda^2} + \dots \quad (4.12)
 \end{aligned}$$

One can draw interesting remarks by comparing eqs. (4.10) and (4.12). Wilson coefficients in the first line of both equations are dominated from input scheme dependencies.¹³ The only “real” difference is a factor of 2 enhancement in front of the coefficient $C^{\varphi D}$ in the case of $h \rightarrow \gamma\gamma$. Another issue is the surprisingly large loop enhancement of the C_{33}^{uB} coefficient (top-quark inside the loop) in $\delta\mathcal{R}_{h \rightarrow \gamma\gamma}$ as shown and discussed in ref. [14]. This enhancement has been reduced by a factor of 20 in $\delta\mathcal{R}_{h \rightarrow Z\gamma}$ in (4.10). The reason seems to be an accidental cancellation. In the $h \rightarrow \gamma\gamma$ case we have an overall factor $16sc^2 \approx 6$, while in the $h \rightarrow Z\gamma$ case we have an overall factor $3c^3 - 13cs^2 \approx -0.5$. It is this factor of (-10) that gives such a big difference in the relevant results. This suppression may be used to disentangle new-physics effects between the two observables. Interestingly, however, the coefficient C_{33}^{uW} does not suffer by similar accidental suppression.

In comparing eqs. (4.10) and (4.12), even the dominant contributions from the operators $C^{\varphi B}$ and $C^{\varphi WB}$ are smaller in $h \rightarrow Z\gamma$ by factor of 3 and only the coefficient of $C^{\varphi W}$ is similar in both $\delta\mathcal{R}_{h \rightarrow Z\gamma}$ and $\delta\mathcal{R}_{h \rightarrow \gamma\gamma}$. This is very interesting for disentangling among the three operators in case new physics enters through those. For example, one may envisage a new-physics scenario, like the one of ref. [42], with a heavy hypercharged $SU(2)_L$ -singlet scalar, which is decoupled from the theory at the TeV scale. Since this will only make $C^{\varphi B}$ non-zero, and say positive in $h \rightarrow \gamma\gamma$, it will only make a suppressed reduction in case of

¹³The large scheme dependence can be understood by comparing (4.12) with the first line of eq. (5.1) of ref. [14].

$h \rightarrow Z\gamma$. However, there are Wilson coefficients like the prefactor of C^W that are similar in both cases.

The real power, however, of EFT, is when using experimental data to constrain Wilson coefficients of various operators and therefore making estimates for projective reach of observables. For example, bounds have been set in some of the coefficients appearing in $\delta\mathcal{R}_{h \rightarrow \gamma\gamma}$ in refs. [14, 17]. We easily see that, if we consider *one coupling at a time*, bounds from $h \rightarrow \gamma\gamma$ on these coefficients kill any possible excess arising from these operators in the $h \rightarrow Z\gamma$ process. In addition to the already mentioned cancellation in top-quark loop for $h \rightarrow Z\gamma$, the relevant operators bounded from $h \rightarrow \gamma\gamma$ are now numerically completely irrelevant for $h \rightarrow Z\gamma$.

That is quite a lot one can infer by just comparing only two observables! One may use best fit values to EW observables to check upon other coefficients, such as C^W , C_{1221}^u , $C^{\varphi l(3)}$, $C^{\varphi u}$, $C^{\varphi q(1,3)}$, $C^{\varphi D}$ that enter similarly in $\delta\mathcal{R}_{h \rightarrow Z\gamma}$ and $\delta\mathcal{R}_{h \rightarrow \gamma\gamma}$ of eqs. (4.10) and (4.12), respectively. By taking, for instance, the best fit values from the 4th column of table 6 in ref. [37]¹⁴ we obtain that it is unlikely to discover any possible new-physics effect through $h \rightarrow Z\gamma$ decay in current LHC data before seeing a $h \rightarrow \gamma\gamma$ anomaly. Of course this statement weakens if one allows for more operators to be present at the same time.

As we already mentioned in Introduction, below eq. (1.1), in deriving bounds from $\delta\mathcal{R}_{h \rightarrow Z\gamma}$ we implicitly assumed that at least the dominant Higgs-boson production mechanism (gluon fusion) is not affected by the operators involved in the $h \rightarrow Z\gamma$ decay. Indeed, for the same reason we explained in section 2, only CP-invariant operators contribute to $gg \rightarrow h$ process. The main, i.e. tree level in SMEFT, gluon-higgs operator, $Q_{\varphi G}$, as well as the ones affecting the one-loop diagrams, Q_{uG} , Q_{dG} , do not interfere with the list of operators in table 1 relevant to $h \rightarrow Z\gamma$. However, operators $Q_{u\varphi}$ and $Q_{d\varphi}$ enter in both $h \rightarrow Z\gamma$ and $h \rightarrow gg$ but their associated Wilson coefficients are multiplied by small numbers in $\delta\mathcal{R}_{h \rightarrow Z\gamma}$ of eq. (4.10). Finally, the combination $(C^{\varphi\Box} + 1/4 C^{\varphi D})$ enters only multiplicatively in all three observables, $h \rightarrow gg$, $h \rightarrow Z\gamma$ and $h \rightarrow \gamma\gamma$ which is just a rescale effect. From these two coefficients, $C^{\varphi D}$ is a custodial violating parameter and therefore highly suppressed. Of course the safest is to calculate $h \rightarrow gg$ at one-loop in SMEFT. The reader is referred to refs. [11, 45, 46]

What about future $h \rightarrow Z\gamma$ sensitivity? Only at later stages of high luminosity of 3000 fb^{-1} at LHC, ATLAS will have enough significance ($\sim 5\sigma$) for the $h \rightarrow Z\gamma$ mode [47]. Assuming SM Higgs production and decay, the signal strength is expected to be measured with $\delta\mathcal{R}_{h \rightarrow Z\gamma} \approx \pm 0.24$ uncertainty. On the other hand for $h \rightarrow \gamma\gamma$, and the same projective reach, ATLAS expects $\delta\mathcal{R}_{h \rightarrow \gamma\gamma} \approx \pm 0.04$ for a SM Higgs boson produced from gluon fusion process and decaying dominantly to $b\bar{b}$. By comparing, our EFT calculations for $\delta\mathcal{R}_{h \rightarrow Z\gamma}$ and $\delta\mathcal{R}_{h \rightarrow \gamma\gamma}$ in eqs. (4.10) and (4.12), we obtain that any new-physics signal for $h \rightarrow Z\gamma$ is unlikely to be seen at near future LHC upgrades without seeing new physics first at $h \rightarrow \gamma\gamma$ data.

¹⁴Similar results one can draw from other tree-level studies, see refs. [43, 44].

5 Epilogue

We have performed a one-loop calculation for the Higgs-boson decay to a Z -boson and a photon, $h \rightarrow Z\gamma$, in SMEFT with $d = 6$ operators written in Warsaw basis. We find a general formula for the amplitude (3.14) which is finite, it respects the Ward-identities, and is gauge-fixing parameter independent. We present our result in terms of the ratio $\delta\mathcal{R}_{h \rightarrow Z\gamma}$ in eq. (4.10) and compare this with the previously calculated ratio $\delta\mathcal{R}_{h \rightarrow \gamma\gamma}$. We find that, for most Wilson-coefficients, $\delta\mathcal{R}_{h \rightarrow Z\gamma}$ is less sensitive to new physics than $\delta\mathcal{R}_{h \rightarrow \gamma\gamma}$. Some of the operators entering in $h \rightarrow Z\gamma$, but *not* in $h \rightarrow \gamma\gamma$, can modify $\delta\mathcal{R}_{h \rightarrow Z\gamma}$ at a rate hardly noticeable, currently or in the near future, at the LHC.

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