

Sequentially loop-generated quark and lepton mass hierarchies in an extended Inert Higgs Doublet model

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ABSTRACT: Extended scalar and fermion sectors offer new opportunities for generating the observed strong hierarchies in the fermion mass and mixing patterns of the Standard Model (SM). In this work, we elaborate on the prospects of a particular extension of the Inert Higgs doublet model where the SM hierarchies are generated sequentially by radiative virtual corrections in a fully renormalisable way, i.e. without adding any non-renormalisable Yukawa terms or soft-breaking operators to the scalar potential. Our model has a potential to explain the recently observed R_K and R_{K^*} anomalies, thanks to the non universal U_{1X} assignments of the fermionic fields that yield non universal Z' couplings to fermions. We explicitly demonstrate the power of this model for generating the realistic quark, lepton and neutrino mass spectra. In particular, we show that due to the presence of both continuous and discrete family symmetries in the considered framework, the top quark acquires a tree-level mass, lighter quarks and leptons get their masses at one- and two-loop order, while neutrino masses are generated at three-loop level. The minimal field content, particle spectra and scalar potential of this model are discussed in detail.

KEYWORDS: Beyond Standard Model, Discrete Symmetries, Neutrino Physics, Quark Masses and SM Parameters

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1 Introduction

The origin of various strong hierarchies in the fermion spectra of the Standard Model (SM) still remains a major unsolved problem of contemporary Particle Physics. A symmetry-based understanding of such hierarchies, in the framework of a single universal mechanism, consistent with the current phenomenological bounds on New Physics models, poses a challenging problem for the model-building community. In fact, a number of different potentially realizable mechanisms have been proposed so far, typically with certain limitations and deficiencies, and usually treating the quark, lepton and neutrino hierarchies on a separate footing. The most promising scenarios rely on the existence of horizontal (family) symmetries acting in the space of fermion generations and offering most times a more universal approach to the “fermion hierarchy problem” than other methods.

It seems natural and attractive to consider the viable prospect of a high-scale spontaneous discrete symmetry breaking, triggering the radiative generation of new mass operators in the corresponding low-energy effective field theory (EFT) [1–23]. This way, one arrives at the possible sequential generation (in general, due to a few sequential symmetry breakings at the high-energy scales) of the relevant mass (or SM Higgs Yukawa) terms in the SM, whose values are matched to zero or to a universal non-zero value in the high-scale

limit of a more symmetric, and hence more fundamental theory. The search for such ultraviolet (UV) completions, possessing a low-energy SM-like EFT, and explorations of their vast potential for explaining the origin of the SM structure have only begun recently [24, 25].

A model of radiatively generated fermion masses in the SM via sequential loop suppression has been proposed in ref. [24]. In this model, the top quark mass is generated at tree level, while the bottom, tau and muon lepton masses arise at one-loop level. Meanwhile, the smaller up, down and strange quark and electron masses are generated at two-loop level, while the light active neutrinos acquire their masses at four-loop level. However, such a natural sequential generation of fermion masses at various orders of the Perturbation Theory comes with a price. Namely, this model, based on the SM gauge symmetry, supplemented with the $S_3 \times Z_2$ discrete group, has a quite low cut-off, since it incorporates non-renormalizable Yukawa terms. In addition, it has another drawback in that the $S_3 \times Z_2$ discrete group is softly broken, yielding an unknown UV completion of the theory. This situation may indicate the need for horizontal continuous symmetries for a consistent description of hierarchies in the matter sectors in a fully renormalizable way.

As a follow up to this study, in the later work of ref. [25], we proposed such a renormalizable model based on the $SU_{3C} \times SU_{3L} \times U_{1X} \times U_{1L} \times Z_2^{(1)} \times Z_2^{(2)}$ symmetry, which generates the SM fermion mass hierarchies with an emergent sequential loop suppression mechanism. However, that model still retains some drawbacks of the previous formulation. Namely, the $Z_2^{(1)} \times Z_2^{(2)}$ symmetry is softly broken, and the scalar and fermion sectors are excessively large, making it very difficult to perform a reliable phenomenological analysis. In addition, that model does not explain the R_K and R_{K^*} anomalies, recently observed by the LHCb experiment [30–32], since it treats the first and second lepton families in the same footing. Furthermore, in the model of ref. [25], the masses for the light active neutrinos appear at two-loop level, as well as the masses for the electron and for the light up, down and strange quarks. Thus the smallness of the light active neutrino masses with respect to the electron mass does not receive a natural explanation in the model. As a natural step forward, it would be instructive to find a new analogous formulation that enables us to generate the SM charged fermion mass hierarchies via a sequential loop suppression mechanism and to generate three-loop level light active neutrino masses without the inclusion of soft breaking mass terms. It would also be relevant to further explore the potential of such formulations for explaining the LHCb anomalies.

In the current work, we propose a first renormalizable model, an extended variant of the Inert Higgs Doublet model (IDM) [26], that enables to generate strong fermion hierarchies via another sequential loop suppression pattern, not yet discussed in the literature, without introducing any soft family symmetry breaking mass terms. Similarly to the previous formulations, in the current model the top quark and exotic fermions do acquire tree level masses, whereas the masses of the remaining SM fermions are radiatively generated. Namely, the masses for the bottom, strange and charm quarks, tau and muon leptons are generated at one-loop level, whereas the masses for the up and down quarks, as well as the electron mass, arise at two-loop level. In variance to the previous version in ref. [24], the light active neutrinos acquire masses via radiative seesaw mechanisms at three-loop level, whereas in the former the light active neutrino masses were induced at two-loop

level. Finally, the minimal field content of the model is not as complicated as in the previous formulations, enabling us to explore several key phenomenological implications of this model, which is the main subject of a follow up study.

The current article is organized as follows. In section 2 we discuss generic conditions for a sequential loop suppression mechanism, providing a motivation for the proposed model. In section 3, we set up the formalism for the extended IDM, containing the basic details about the symmetries, particle content, Yukawa interactions and scalar potential crucial for the implementation of the sequential loop suppression mechanism. The scalar mass spectrum is discussed in detail in section 4. In section 5 we discuss the implications of our model for the radiative generation of the quark mass and mixing hierarchies. The charged lepton and neutrino mass spectra and the corresponding mixing patterns are discussed in section 6. Finally, our conclusions are briefly stated in section 7.

2 Sequential loop suppression mechanism

Before describing our model in detail, let us first explain the reasoning behind introducing the additional scalar and fermion degrees of freedom and the symmetries that are required for a consistent implementation of the sequential loop suppression mechanism for generating the SM fermion hierarchies.

2.1 Quark sector

First of all, it is worth noticing that the top quark mass can be generated at tree level by means of a renormalizable Yukawa operator, with the corresponding coupling of order one, i.e.

$$\bar{q}_{3L} \tilde{\phi}_1 u_{jR}, \quad j = 1, 2, 3, \quad (2.1)$$

where ϕ_1 is a SU_{2L} scalar doublet. To generate the charm quark mass at one loop level, it is necessary to forbid the operator:

$$\bar{q}_{nL} \tilde{\phi}_1 u_{jR}, \quad n = 1, 2, \quad j = 1, 2, 3, \quad (2.2)$$

at tree level and to allow other operators instead, which are described in what follows. Obviously, in this case the charm quark mass is not generated in the same way as the top quark mass, i.e., from a renormalizable Yukawa term, since this would imply setting the corresponding tree-level Yukawa coupling unnaturally small.

In order to generate a small charm quark mass at one-loop level, we will use the following operators:

$$\begin{aligned} \bar{q}_{nL} \tilde{\phi}_2 T_R, & \quad \bar{T}_L \eta^* u_{mR}, & n, m = 1, 2, \\ \bar{T}_L \sigma_1 T_R, & \quad \sigma_2^* \rho_2^* \sigma_1 \rho_3^*, & \eta \rho_2 \sigma_1, & \quad \left(\phi_1^\dagger \cdot \phi_2 \right) \sigma_2. \end{aligned} \quad (2.3)$$

For this to happen, we need to extend the SM gauge symmetry by adding the $U_{1X} \times Z_2^{(1)} \times Z_2^{(2)}$ symmetry, where the U_{1X} and $Z_2^{(1)}$ are spontaneously broken gauge and discrete symmetries, respectively, and $Z_2^{(2)}$ is a preserved (exact) discrete symmetry under which

the extra ϕ_2 scalar doublet, the σ_2, ρ_2, η electrically neutral scalar singlets, and the q_{jL}, u_{jR} ($j = 1, 2, 3$) quark fields are nontrivially charged. It is worth mentioning that the fields u_{nR} ($n = 1, 2$), ρ_2, ρ_3 and η are charged under the spontaneously broken $Z_2^{(1)}$ symmetry, whereas the remaining fields previously introduced are neutral under this symmetry. As we will be shown below, the σ_2, ρ_2, η electrically neutral scalar singlets will also mediate the one and two level radiative seesaw mechanisms that give masses to the second and third family of SM down type quarks and charged leptons as well as to the first family of SM charged fermions. Some of these scalars singlets will also participate in the three loop level radiative seesaw mechanism that produces the light active neutrino masses. Furthermore, the SU_{2L} singlet heavy quarks T_L, T_R with electric charges equal to $2/3$ have to be added to the fermion spectrum in order to implement the one-loop radiative seesaw mechanism that gives rise to the charm quark mass. Besides, an electrically neutral weak-singlet scalar σ_1 is needed in the scalar spectrum to provide a tree-level mass for the exotic T quark and to close the one-loop Feynman diagram that generates the charm quark mass. Let us note that σ_1 and ρ_3 are the only scalars neutral under the unbroken $Z_2^{(2)}$ symmetry, and thus they conveniently acquire vacuum expectation values (VEVs) that break the U_{1X} gauge symmetry and the $Z_2^{(1)}$ discrete symmetry, respectively.

In addition, we assume that the q_{nL} ($n = 1, 2$) fields have U_{1X} charges that are different from the charge of q_{3L} , while the SU_{2L} scalar doublets ϕ_1 and ϕ_2 have different U_{1X} charges as well. Thus, the third row of the up-type quark mass matrix is generated at tree level, whereas the first and second row emerge at one-loop level. Note that, since there is only one heavy exotic T quark mediating the one-loop radiative seesaw mechanism that generates the first and second row of the up-type quark mass matrix, the determinant of this matrix is equal to zero. Therefore, the up quark is massless at one-loop level, and in order to generate an up quark mass at two-loop level, the following operators are required:

$$\begin{aligned} \bar{q}_{nL}\phi_2 B_{3R}, \quad \bar{B}_{4L}\varphi_1^- u_{nR}, \quad \bar{B}_{3L}\sigma_2^* B_{4R}, \quad m_{B_k}\bar{B}_{kL}B_{kR}, \quad n = 1, 2, \\ \sigma_1\sigma_2\sigma_3^*\rho_3, \quad \varepsilon_{ab} \left(\phi_1^\dagger\right)^a \left(\phi_2^\dagger\right)^b \varphi_1^+ \sigma_3, \quad a, b = 1, 2, \quad k = 3, 4, \end{aligned} \tag{2.4}$$

where the extra SU_{2L} singlet heavy quarks $B_{3L}, B_{3R}, B_{4L}, B_{4R}$ have electric charges equal to $-1/3$, and σ_3 and φ_1^- are electrically neutral and charged weak-singlet scalars, respectively. The scalar fields σ_3 and φ_1^- are charged under the spontaneously broken $Z_2^{(1)}$ symmetry. The operators given above enable to generate the two-loop contributions to the first and second rows of the up-type quark mass matrix, and these contributions yield a nonvanishing determinant for the up-type quark mass matrix, giving rise to a suppressed two-loop up quark mass. It is worth mentioning that the nonvanishing parts of the SM up type quark mass matrix top quark are the $(3, 3)$ entry, which appears at tree level and the upper left 2×2 block which receive one and two loop level contributions. Let us note that in order that the two loop contribution to the upper left 2×2 block of the SM up type quark mass matrix gives a nonvanishing two loop level up quark mass, the one and two loop level contributions should not have common vertices. If they have a common vertex, the two loop level contribution can be treated as an effective 1-loop one having the same the left and (or) right handed fermionic field in the internal line as in the former one loop

level contribution. Thus, the net result of the sum of both contributions will correspond to one loop level diagram having in the fermionic line a seesaw mediator, which will be linear combination of the fermionic mediators in both contributions, thus leading to a vanishing up quark mass. Another reason for avoiding a common vertex in the one and two loop level contributions to the upper left 2×2 block of the SM up type quark mass matrix, is to prevent a proportionality between row and columns that will result in a vanishing eigenvalue. Because of the aforementioned reason, we have employed exotic down type quarks and electrically charged scalars in the two loop level contribution of the upper left 2×2 block of the SM up type quark mass matrix, thus giving rise to a up quark mass at two loop level. As it will be shown below, the two loop level contributions to the SM charged fermion mass matrices, will be generated by electrically charged scalars and exotic fermions running in the internal lines of the loop, thus yielding two loop level masses for the first generation of SM charged fermions.

Turning now to a possible bottom quark mass generation at one-loop level, the following operators should be forbidden

$$\bar{q}_{3L}\phi_1 d_{jR}, \quad j = 1, 2, 3, \quad (2.5)$$

by means of, for example, the U_{1X} gauge symmetry. The fermion spectrum has to be extended by including the additional SU_{2L} -singlet heavy quarks B_{nL}, B_{nR} ($n = 1, 2$) with electric charges equal to $-1/3$, so that the mass for the bottom quark is generated with the help of the following operators:

$$\begin{aligned} \bar{q}_{3L}\phi_2 B_{nR}, & \quad \bar{B}_{nL}\eta d_{jR}, & \quad n = 1, 2, & \quad n, m = 1, 2, & \quad j = 1, 2, 3, \\ \bar{B}_{nL}\sigma_1^* B_{mR}, & \quad \sigma_2 \rho_2 (\sigma_1^*) \rho_3, & \quad \eta^* \rho_2^* \sigma_1^*, & \quad (\phi_1 \cdot \phi_2^\dagger) \sigma_2^*. \end{aligned} \quad (2.6)$$

Note that we have added two SU_{2L} -singlet heavy quarks B_n ($n = 1, 2$) instead of one, in order to fulfill the anomaly cancellation conditions discussed below.

In addition, in order to generate the first and second rows of the down-type quark mass matrix at one-loop level via a radiative seesaw mechanism, we need the following set of operators:

$$\begin{aligned} \bar{q}_{nL}\phi_2 B_{3R}, & \quad \bar{B}_{3L}\eta^* d_{jR}, & \quad n = 1, 2, & \quad j = 1, 2, 3, \\ m_{B_3}\bar{B}_{3L}B_{3R}, & \quad \eta (\phi_1 \cdot \phi_2^\dagger) \rho_3. \end{aligned} \quad (2.7)$$

Furthermore, in order to avoid tree-level masses for the down and strange quarks one has to forbid the terms

$$\bar{q}_{nL}\phi_1 d_{jR}, \quad n = 1, 2, \quad j = 1, 2, 3. \quad (2.8)$$

The latter can be excluded by assigning q_{nL} ($n = 1, 2$) to be even, while d_{jR} ($j = 1, 2, 3$) to be odd under the aforementioned spontaneously broken $Z_2^{(1)}$ symmetry.

Since there is only one fermionic seesaw mediator, i.e., the SU_{2L} singlet heavy quark T needed to generate the first and second rows of the down-type quark mass matrix at one-loop level, a nonvanishing one-loop strange quark mass emerges, whereas the down quark

remains massless at this point. Consequently, similarly to the up-type quark sector, the two-loop contributions to the first and second rows of the down-type quark mass matrix need to be generated in order to give rise to a down-type quark mass at two-loop level. For that purpose, the following operators are required:

$$\begin{aligned} \bar{q}_{nL} \tilde{\phi}_2 T_R, & \quad \bar{\tilde{T}}_L \varphi_2^+ d_{jR}, & \quad m_{\tilde{T}} \bar{\tilde{T}}_L \tilde{T}_R, & \quad n = 1, 2, & \quad j = 1, 2, 3, & \quad (2.9) \\ \bar{T}_L \sigma_1 T_R, & \quad \bar{T}_L \rho_2 \tilde{T}_R, & \quad \sigma_1 \sigma_2^* \rho_2 \rho_3, & \quad \varepsilon_{ab} (\phi_1)^a (\phi_2)^b \varphi_2^- \sigma_2, & \quad a, b = 1, 2, \end{aligned}$$

where an extra electrically charged weak-singlet scalar, φ_2^+ has been added to the scalar spectrum. Furthermore, the fermion spectrum has been extended by means of extra SU_{2L} -singlet heavy quarks \tilde{T}_L, \tilde{T}_R with electric charges equal to $2/3$. The two-loop contributions to the first and second rows of the down-type quark mass matrix provide its nonvanishing determinant, giving rise to a two-loop down quark mass.

2.2 Charged lepton sector

Now, consider the sequential loop suppression mechanism capable of explaining the observed hierarchy between the SM charged lepton masses. In what follows, let us discuss a possible way of generating the one-loop tau and muon masses as well as a two-loop electron mass. First of all, the following operators have to be forbidden

$$\bar{l}_{iL} \phi_1 l_{jR}, \quad i, j = 1, 2, 3.$$

by using the U_{1X} gauge symmetry, so that the SM charged lepton mass matrix is generated at one-loop level by means of the terms

$$\begin{aligned} \bar{l}_{kL} \phi_2 E_{3R}, & \quad \bar{E}_{3L} \rho_1 l_{nR}, & \quad (\phi_1 \cdot \phi_2^+) \sigma_2^*, & \quad \rho_1 \sigma_2 \sigma_1^*, & \quad n = 1, 3, \\ \bar{l}_{2L} \phi_2 E_{2R}, & \quad \bar{E}_{2L} \rho_1 l_{2R}, & \quad \bar{E}_{jL} \sigma_1^* E_{jR}, & \quad j = 2, 3, & \quad (2.10) \end{aligned}$$

where weak-singlet charged leptons E_{jL}, E_{jR} ($j = 2, 3$) have been included in the fermion spectrum. Let us note that these fields mediate the one-loop radiative seesaw mechanism that generates the $(1, 1), (2, 2), (3, 3), (1, 3)$ and $(3, 1)$ entries of the charged lepton mass matrix. Consequently, at this point the determinant of the charged lepton mass matrix is equal to zero and thus only the tau and muon leptons appear to be massive at one-loop level, whereas the electron remains massless. Two-loop corrections to the $(1, 1), (3, 3), (1, 3)$ and $(3, 1)$ entries are needed in order to induce a non-zero electron mass at two-loop level, and extra weak-singlet charged E_1 and neutral ν_{mR} ($m = 1, 3$), leptons Ψ_R as well as the electrically charged scalar singlets $\varphi_1^\pm, \varphi_k^\pm$ ($k = 3, 4, 5$) would be required for this purpose. Let us note that the neutral gauge singlet leptons ν_{mR} ($m = 1, 3$), leptons Ψ_R will also participate in the three loop level radiative seesaw mechanism that produces the small light active neutrino masses. To obtain the two-loop corrections of the SM charged lepton mass matrix that generate a two loop level electron mass, the following operators should be considered

$$\begin{aligned} \bar{l}_{kL} \tilde{\phi}_2 \nu_{nR}, & \quad \bar{\Psi}_R^C \varphi_3^+ l_{kR}, & \quad \varphi_4^- \varphi_5^+ \sigma_1, & \quad \varphi_1^+ \varphi_5^- \sigma_1^2, & \quad k, n = 1, 3, & \quad (2.11) \\ \bar{E}_{1L} \sigma_1^* E_{1R}, & \quad \bar{E}_{1L} \varphi_1^- \nu_{kR}, & \quad \bar{\Psi}_R^C \varphi_4^+ E_{1R}, & \quad \varepsilon_{ab} (\phi_1)^a (\phi_2)^b \varphi_3^-, & \quad a, b = 1, 2, \end{aligned}$$

As soon as the two-loop corrections generated by the above operators are included, the determinant of the charged lepton mass matrix becomes nonzero, thus giving rise to a two-loop electron mass, as expected.

Let us note that in this setup there is a mixing between the SM charged leptons of the first and third generations, but they do not mix with the second generation, which is a consequence of their U_{1X} assignments, as will be shown below. The second generation left-handed lepton SU_{2L} -doublet has a nonvanishing U_{1X} charge, whereas the first and third generations are not charged under U_{1X} . In addition, the first and third generations of SM right-handed charged leptons should have the same U_{1X} charge, which is different from the corresponding charge of the second generation.

2.3 Light active neutrino sector

Turning to the neutrino sector, in order to generate the SM light active neutrino masses at three-loop level as well as a realistic lepton mixing, a few operators have to be forbidden, namely,

$$\bar{l}_{iL}\phi_1\nu_{jR}, \quad (m_N)_{ij}\nu_{iR}\overline{\nu_{jR}^C}, \quad \nu_{iR}\sigma_1\overline{\nu_{jR}^C}, \quad \nu_{iR}\sigma_1^*\overline{\nu_{jR}^C} \quad m_\Omega\overline{\Omega_R^C}\Omega_R, \quad i, j = 1, 2, 3, \quad (2.12)$$

while the following operators are required

$$\begin{aligned} \bar{l}_{kL}\tilde{\phi}_2\nu_{nR}, & \quad \bar{l}_{2L}\tilde{\phi}_2\nu_{2R}, & \quad \overline{\Omega_{1R}^C}\eta^*\nu_{kR}, & \quad \overline{\Omega_{1R}^C}\sigma_3^*\nu_{2R}, & \quad k, n = 1, 3. \\ \overline{\Omega_{1R}^C}\eta\Psi_R, & \quad \overline{\Omega_{2R}^C}\eta^*\Psi_R, & \quad \overline{\Omega_{1R}^C}\sigma_2^*\Omega_{2R}, & \quad m_\Psi\overline{\Psi_R^C}\Psi_R, & \end{aligned} \quad (2.13)$$

where ν_{iR} ($i = 1, 2, 3$), Ω_R and Ψ_R are the SM-singlet right-handed Majorana neutrinos, and η , σ_3 and ρ_2 are the extra SM-singlet scalars. The latter fields have to be added in order to ensure three-loop level generation of the SM light active neutrino masses, as well as the lepton mixing parameters $\sin^2\theta_{23}$ and $\sin^2\theta_{12}$.

As was mentioned above, the charged lepton mass matrix has a mixing only in the (1,3)-plane such that the lepton mixing parameters $\sin^2\theta_{23}$ and $\sin^2\theta_{12}$ emerge from the neutrino sector. Let us note that the U_{1X} gauge symmetry prevents the light active neutrino masses to be generated at tree level, whereas the $Z_2^{(1)}$ symmetry, together with the U_{1X} gauge symmetry, forbid the mixing terms between the right-handed Majorana neutrinos ν_{kR} ($k = 1, 3$) and ν_{2R} . This enables us to avoid the appearance of light neutrino masses at one- and two-loop levels. Note also that the $Z_2^{(1)}$ symmetry, as well as the U_{1X} gauge symmetry, are crucial to forbid the terms $\nu_{2R}\sigma_1\overline{\nu_{kR}^C}$ and $\nu_{2R}\rho_3\overline{\nu_{kR}^C}$ ($k = 1, 3$) that could result in the appearance of SM neutrino masses at one loop.

3 The extended IDM model

In this section, we will summarize the main features of the first renormalizable model, an extended variant of the Inert Higgs Doublet model (IDM), that includes a sequential loop suppression mechanism for the generation of the SM fermion mass hierarchies, without the inclusion of soft breaking mass terms and, at the same time, allowing for an explanation of the R_K and R_{K^*} anomalies, thanks to the non-universal U_{1X} assignments of the fermionic

fields that yield non-universal Z' couplings to fermions. In a forthcoming work, we will show in detail how our model can fit the R_K and R_{K^*} anomalies. As previously stated in Introduction, we emphasize that our model, apart from having all the means for explaining the R_K and R_{K^*} anomalies not previously addressed in the model of ref. [24], has a more natural explanation for the smallness of the light active neutrino masses than the one provided in ref. [24], since in the former the masses for the light active neutrinos are generated at three-loop level, whereas in the latter they appear at two loops. Furthermore, unlike the model of ref. [24], our current model does not include soft breaking mass terms.

Let us summarize the structure of a minimal model where the sequential loop suppression mechanism capable of radiative generation of the mass and mixing hierarchies in the SM fermion sectors is realized. The reasons for choosing a particular field content and symmetries have been outlined in the previous section, and will be further substantiated below.

3.1 Particle content and charges

With the necessary ingredients introduced above, in fact, we arrive at an extension of the inert 2HDM where the SM gauge symmetry is supplemented by the exact unbroken $Z_2^{(2)}$ discrete group and spontaneously broken $Z_2^{(1)}$ discrete and U_{1X} gauge symmetry groups. The unbroken $Z_2^{(2)}$ and continuous local U_{1X} (horizontal) family symmetries are crucial for the implementation of radiative seesaw mechanism of sequential loop suppression.

Besides the SM-like Higgs doublet ϕ_1 , the implementation of this mechanism requires an additional inert scalar SU_{2L} -doublet, ϕ_2 , seven electrically neutral, i.e., σ_j, ρ_j ($j = 1, 2, 3$), η , and five electrically charged φ_k^+ ($k = 1, 2, 3, 4, 5$), SU_{2L} -singlet scalars. The scalar sector of the considering model has the following $SU_{3c} \times SU_{2L} \times U_{1Y} \times U_{1X}$ charge assignments

$$\begin{aligned}
 \phi_1 &\sim \left(\mathbf{1}, \mathbf{2}, \frac{1}{2}, 1\right), & \phi_2 &\sim \left(\mathbf{1}, \mathbf{2}, \frac{1}{2}, 2\right), & \phi_3 &\sim \left(\mathbf{1}, \mathbf{2}, \frac{1}{2}, 2\right), & \sigma_1 &\sim (\mathbf{1}, \mathbf{1}, 0, -1), \\
 \sigma_2 &\sim (\mathbf{1}, \mathbf{1}, 0, -1), & \sigma_3 &\sim (\mathbf{1}, \mathbf{1}, 0, -2), & \rho_1 &\sim (\mathbf{1}, \mathbf{1}, 0, 0), & \rho_2 &\sim (\mathbf{1}, \mathbf{1}, 0, 0), \\
 \rho_3 &\sim (\mathbf{1}, \mathbf{1}, 0, 1), & \eta &\sim (\mathbf{1}, \mathbf{1}, 0, 1), & \varphi_1^+ &\sim (\mathbf{1}, \mathbf{1}, 1, 5), & \varphi_2^+ &\sim (\mathbf{1}, \mathbf{1}, 1, 2), \\
 \varphi_3^+ &\sim (\mathbf{1}, \mathbf{1}, 1, 3), & \varphi_4^+ &\sim (\mathbf{1}, \mathbf{1}, 1, 2), & \varphi_5^+ &\sim (\mathbf{1}, \mathbf{1}, 1, 3).
 \end{aligned} \tag{3.1}$$

As it will be shown in the next subsection, we provide a detailed analysis of the scalar potential showing that the only massless scalar fields are those ones corresponding to the SM Goldstone bosons and the Goldstone boson associated to the longitudinal component of the Z' gauge boson.

The corresponding $Z_2^{(1)} \times Z_2^{(2)}$ charges of the scalar fields are given by

$$\begin{aligned}
 \phi_1 &\sim (1, 1), & \phi_2 &\sim (1, -1), & \sigma_1 &\sim (1, 1), & \sigma_2 &\sim (1, -1), & \sigma_3 &\sim (-1, -1), \\
 \rho_1 &\sim (1, -1), & \rho_2 &\sim (-1, -1), & \rho_3 &\sim (-1, 1), & \eta &\sim (-1, -1), \\
 \varphi_1^+ &\sim (-1, 1), & \varphi_2^+ &\sim (1, 1), & \varphi_3^+ &\sim (1, -1), & \varphi_4^+ &\sim (-1, 1), & \varphi_5^+ &\sim (-1, 1).
 \end{aligned} \tag{3.2}$$

It is worth noticing that the model does not contain a weak-singlet scalar field with the following simultaneous three features: charged under the $Z_2^{(1)}$ discrete symmetry, neutral under the unbroken $Z_2^{(2)}$ symmetry with U_{1X} charge equal to ± 1 .

Let us note that the SM-type Higgs doublet, i.e., ϕ_1 , as well as the electroweak-singlet scalars σ_1 and ρ_3 are the only scalar fields neutral under the exact $Z_2^{(2)}$ discrete symmetry. Since the $Z_2^{(2)}$ symmetry is preserved, the Higgs doublet ϕ_1 and the singlets σ_1 and ρ_3 are the only scalar fields that acquire nonvanishing VEVs. The VEV in σ_1 is required to spontaneously break the U_{1X} local symmetry, whereas the ρ_3 VEV spontaneously breaks the $Z_2^{(1)}$ discrete symmetry, due to its nontrivial $Z_2^{(1)}$ charge.

Note that the exact $Z_2^{(2)}$ discrete symmetry guarantees the presence of several stable scalar dark matter candidates in our model. These are represented by the neutral components of the inert SU_{2L} scalar doublet ϕ_2 , as well as by the real and imaginary parts of the SM-singlet scalars σ_2 , σ_3 , ρ_1 , ρ_2 and η . Furthermore, the model can have a fermionic DM candidate, which is the only SM-singlet Majorana neutrino Ω_{1R} with a non-trivial $Z_2^{(2)}$ charge.

The set of SU_{2L} -singlet heavy quarks T_L, T_R, B_{iL}, B_{iR} ($i = 1, 2, 3$) represents the minimal amount of exotic quark degrees of freedom needed to implement the one-loop radiative seesaw mechanism that gives rise to the charm, bottom and strange quark masses. Furthermore, in order to ensure the radiative seesaw mechanism responsible for the generation of the up and down quark masses at two-loop level, the SU_{2L} singlet heavy quarks $\tilde{T}_L, \tilde{T}_R, B_{4L}, B_{4R}$, as well as the electrically neutral, σ_3, ρ_2 , and electrically charged, φ_1^+, φ_2^+ scalar SU_{2L} -singlets should also be present in the particle spectrum.

To summarize, the SM fermion sector of the considered model includes a total of six electrically charged weak-singlet leptons E_{jL} and E_{jR} ($j = 1, 2, 3$), four right-handed neutrinos ν_{jR} ($j = 1, 2, 3$), Ω_R , and twelve SU_{2L} -singlet heavy quarks $T_L, T_R, \tilde{T}_L, \tilde{T}_R, B_{kL}, B_{kR}$ ($k = 1, 2, 3, 4$). It is assumed that the heavy exotic T, \tilde{T} and B_k quarks have electric charges equal to $2/3$ and $-1/3$, respectively.

More specifically, the quark sector of the extended IDM under consideration has the following $SU_{3c} \times SU_{2L} \times U_{1Y} \times U_{1X}$ charges

$$\begin{aligned}
 q_{nL} &\sim \left(\mathbf{3}, \mathbf{2}, \frac{1}{6}, 0 \right), & q_{3L} &\sim \left(\mathbf{3}, \mathbf{2}, \frac{1}{6}, 1 \right), & n &= 1, 2, \\
 u_{jR} &\sim \left(\mathbf{3}, \mathbf{1}, \frac{2}{3}, 2 \right), & d_{jR} &\sim \left(\mathbf{3}, \mathbf{1}, -\frac{1}{3}, -1 \right), & j &= 1, 2, 3, \\
 T_L &\sim \left(\mathbf{3}, \mathbf{1}, \frac{2}{3}, 1 \right), & T_R &\sim \left(\mathbf{3}, \mathbf{1}, \frac{2}{3}, 2 \right), \\
 \tilde{T}_L &\sim \left(\mathbf{3}, \mathbf{1}, \frac{2}{3}, 1 \right), & \tilde{T}_R &\sim \left(\mathbf{3}, \mathbf{1}, \frac{2}{3}, 1 \right), \\
 B_{nL} &\sim \left(\mathbf{3}, \mathbf{1}, -\frac{1}{3}, 0 \right), & B_{nR} &\sim \left(\mathbf{3}, \mathbf{1}, -\frac{1}{3}, -1 \right), \\
 B_{3L} &\sim \left(\mathbf{3}, \mathbf{1}, -\frac{1}{3}, -2 \right), & B_{3R} &\sim \left(\mathbf{3}, \mathbf{1}, -\frac{1}{3}, -2 \right), \\
 B_{4L} &\sim \left(\mathbf{3}, \mathbf{1}, -\frac{1}{3}, -3 \right), & B_{4R} &\sim \left(\mathbf{3}, \mathbf{1}, -\frac{1}{3}, -3 \right)
 \end{aligned} \tag{3.3}$$

Field	ϕ_1	ϕ_2	σ_1	σ_2	σ_3	ρ_1	ρ_2	ρ_3	η	φ_1^+	φ_2^+	φ_3^+	φ_4^+	φ_5^+
SU_{3c}	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SU_{2L}	2	2	1	1	1	1	1	1	1	1	1	1	1	1
U_{1Y}	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	0	0	0	1	1	1	1	1
U_{1X}	1	2	-1	-1	-2	0	0	0	1	5	2	3	2	3
$Z_2^{(1)}$	1	1	1	1	-1	1	-1	-1	-1	-1	1	1	-1	-1
$Z_2^{(2)}$	1	-1	1	-1	-1	-1	-1	1	-1	1	1	-1	1	1

Table 1. Scalars assignments under the $SU_{3c} \times SU_{2L} \times U_{1Y} \times U_{1X} \times Z_2^{(1)} \times Z_2^{(2)}$ symmetry.

Field	q_{1L}	q_{2L}	q_{3L}	u_{1R}	u_{2R}	u_{3R}	d_{1R}	d_{2R}	d_{3R}	T_L	T_R	\tilde{T}_L	\tilde{T}_R	B_{1L}	B_{1R}	B_{2L}	B_{2R}	B_{3L}	B_{3R}	B_{4L}	B_{4R}	
SU_{3c}	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
SU_{2L}	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U_{1Y}	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
U_{1X}	0	0	1	2	2	2	-1	-1	-1	1	2	1	1	0	-1	0	-1	-2	-2	-3	-3	-3
$Z_2^{(1)}$	1	1	1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	1	1	1	1	1
$Z_2^{(2)}$	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	1	1	1	-1	-1

Table 2. Quark assignments under the $SU_{3c} \times SU_{2L} \times U_{1Y} \times U_{1X} \times Z_2^{(1)} \times Z_2^{(2)}$ symmetry.

Field	l_{1L}	l_{2L}	l_{3L}	l_{1R}	l_{2R}	l_{3R}	E_{1L}	E_{1R}	E_{2L}	E_{2R}	E_{3L}	E_{3R}	ν_{1R}	ν_{2R}	ν_{3R}	Ω_{1R}	Ω_{2R}	Ψ_R
SU_{3c}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SU_{2L}	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U_{1Y}	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0
U_{1X}	0	-3	0	-3	-6	-3	-3	-2	-6	-5	-3	-2	2	-1	2	-1	1	0
$Z_2^{(1)}$	1	-1	1	1	-1	1	-1	-1	-1	-1	1	1	1	-1	1	-1	-1	1
$Z_2^{(2)}$	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	-1	1	1

Table 3. Lepton charge assignments under the $SU_{3c} \times SU_{2L} \times U_{1Y} \times U_{1X} \times Z_2^{(1)} \times Z_2^{(2)}$ symmetry.

while their $Z_2^{(1)} \times Z_2^{(2)}$ charge assignments read

$$\begin{aligned}
 q_{nL} &\sim (1, -1), & q_{3L} &\sim (1, -1), & u_{nR} &\sim (-1, -1), & u_{3R} &\sim (1, -1), & d_{jR} &\sim (-1, -1), \\
 T_L &\sim (1, 1), & T_R &\sim (1, 1), & \tilde{T}_L &\sim (-1, -1), & \tilde{T}_R &\sim (-1, -1), & n &= 1, 2, j = 1, 2, 3, \\
 B_{jL} &\sim (1, 1), & B_{jR} &\sim (1, 1), & B_{4L} &\sim (1, -1), & B_{4R} &\sim (1, -1).
 \end{aligned} \tag{3.4}$$

A summary of all the field assignments with respect to the model symmetry is given in tables 1, 2, 3.

The radiative seesaw mechanism that generates the charged lepton mass hierarchy is similar to the one that produces the SM down-type quark mass hierarchy. The generation of one-loop tau and muon masses is mediated by the electrically charged weak-singlet leptons E_{rL} and E_{rR} ($r = 2, 3$), by the inert scalar SU_{2L} -doublet, ϕ_2 , and by the SU_{2L} -singlets σ_2 , ρ_1 . On the other hand, the radiative seesaw mechanism that give rises to a two loop level electron mass is mediated by electrically charged scalars as well as by the right-handed Majorana neutrinos Ψ , ν_{kR} ($k = 1, 3$) and the weak-singlet electrically charged leptons E_{1L} and E_{1R} .

Moreover, the three-loop radiative seesaw mechanism responsible for the generation of the light active neutrino masses is mediated by the right-handed neutrinos ν_{jR} ($j = 1, 2, 3$), Ω_R , as well as by the inert scalar SU_{2L} doublet ϕ_2 and the SU_{2L} -singlet σ_2 . To avoid tree-level mixing between the right-handed Majorana neutrinos ν_{kR} ($k = 1, 3$) and ν_{2R} triggered by Yukawa interactions with σ_1 , we need to impose a nontrivial $Z_2^{(1)}$ charge of ν_{2R} while keeping ν_{kR} ($k = 1, 3$) $Z_2^{(1)}$ -neutral.

In particular, the $SU_{3c} \times SU_{2L} \times U_{1Y} \times U_{1X}$ charges of the leptonic and neutrino fields of the model are defined as follows

$$\begin{aligned}
 l_{kL} &\sim \left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}, 0 \right), & l_{2L} &\sim \left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}, -3 \right), & k &= 1, 3, \\
 l_{kR} &\sim (\mathbf{1}, \mathbf{1}, -1, -3), & l_{2R} &\sim (\mathbf{1}, \mathbf{1}, -1, -6), \\
 E_{1L} &\sim (\mathbf{1}, \mathbf{1}, -1, -3), & E_{1R} &\sim (\mathbf{1}, \mathbf{1}, -1, -2), \\
 E_{2L} &\sim (\mathbf{1}, \mathbf{1}, -1, -6), & E_{2R} &\sim (\mathbf{1}, \mathbf{1}, -1, -5), \\
 E_{3L} &\sim (\mathbf{1}, \mathbf{1}, -1, -3), & E_{3R} &\sim (\mathbf{1}, \mathbf{1}, -1, -2), \\
 \nu_{kR} &\sim (\mathbf{1}, \mathbf{1}, 0, 2), & \nu_{2R} &\sim (\mathbf{1}, \mathbf{1}, 0, -1), \\
 \Omega_{1R} &\sim (\mathbf{1}, \mathbf{1}, 0, -1), & \Omega_{2R} &\sim (\mathbf{1}, \mathbf{1}, 0, 1), & \Psi_R &\sim (\mathbf{1}, \mathbf{1}, 0, 0), & (3.5)
 \end{aligned}$$

whereas the corresponding $Z_2^{(1)} \times Z_2^{(2)}$ charges are given by

$$\begin{aligned}
 l_{kL} &\sim (1, -1), & l_{2L} &\sim (-1, -1), & l_{kR} &\sim (1, -1), & l_{2R} &\sim (-1, -1), & k &= 1, 3, \\
 E_{1L} &\sim (-1, 1), & E_{1R} &\sim (-1, 1), & E_{2L} &\sim (-1, 1), & E_{2R} &\sim (-1, 1), & E_{3L} &\sim (1, 1), & E_{3R} &\sim (1, 1), \\
 \nu_{kR} &\sim (1, 1), & \nu_{2R} &\sim (-1, 1), & \Omega_{1R} &\sim (-1, -1), & \Omega_{2R} &\sim (-1, 1), & \Psi_R &\sim (1, 1), & k &= 1, 3. & (3.6)
 \end{aligned}$$

The $U_{1X} \times Z_2^{(1)} \times Z_2^{(2)}$ symmetry and the particular assignments listed above are crucial for avoiding the appearance of SM light active neutrino masses at one- and two-loop levels.

Let us note that the left-handed quark SU_{2L} doublets of the first and second generations are distinguished from the third generation by means of the U_{1X} charge assignments. In addition, the local U_{1X} family symmetry distinguishes the second generation left-handed lepton SU_{2L} doublet from the first and third generation ones. Such non-universal U_{1X} charge assignments in the fermion sector are crucial for implementing the sequential loop suppression mechanism and, hence, for the induced strong hierarchies in the SM fermion mass spectrum. Besides, as will be explicitly demonstrated in a forthcoming paper, such assignments are also relevant for explaining the R_K and R_{K^*} anomalies.

With the above assignments, we have numerically checked that the gauge anomaly cancellation conditions

$$\begin{aligned}
A_{[\text{SU}_{3c}]^2\text{U}_{1X}} &= \sum_Q X_{QL} - \sum_Q X_{QR}, & A_{[\text{SU}_{2L}]^2\text{U}_{1X}} &= \sum_L X_{LL} + 3 \sum_Q X_{QL}, \\
A_{[\text{U}_{1Y}]^2\text{U}_{1X}} &= \sum_{L,Q} (Y_{LL}^2 X_{LL} + 3Y_{QL}^2 X_{QL}) - \sum_{L,Q} (Y_{LR}^2 X_{LR} + 3Y_{QR}^2 X_{QR}), \\
A_{[\text{U}_{1X}]^2\text{U}_{1Y}} &= \sum_{L,Q} (Y_{LL} X_{LL}^2 + 3Y_{QL} X_{QL}^2) - \sum_{L,Q} (Y_{LR} X_{LR}^2 + 3Y_{QR} X_{QR}^2), \\
A_{[\text{U}_{1X}]^3} &= \sum_{L,Q} (X_{LL}^3 + 3X_{QL}^3) - \sum_{L,Q} (X_{LR}^3 + X_{NR}^3 + 3X_{QR}^3), \\
A_{[\text{Gravity}]^2\text{U}_{1X}} &= \sum_{L,Q} (X_{LL} + 3X_{QL}) - \sum_{L,Q,N} (X_{LR} + X_{NR} + 3X_{QR})
\end{aligned} \tag{3.7}$$

are satisfied in our model. Let us note that in the expression for $A_{[\text{SU}_{2L}]^2\text{U}_{1X}}$ the sum is performed only over the SU_{2L} doublets of left handed fermionic fields. On the other hand, in the expression for $A_{[\text{Gravity}]^2\text{U}_{1X}}$, the sum is performed over all left handed fermionic fields.

3.2 Yukawa interactions

With the above specified particle content and charge assignments, the most general renormalizable Lagrangian of Yukawa interactions and the exotic fermion mass terms, invariant under the $\text{SU}_{3c} \times \text{SU}_{2L} \times \text{U}_{1Y} \times \text{U}_{1X} \times Z_2^{(1)} \times Z_2^{(2)}$ symmetry, takes the following form

$$\begin{aligned}
\mathcal{L}_F &= y_{3j}^{(u)} \bar{q}_{3L} \tilde{\phi}_1 u_{3R} + \sum_{n=1}^2 x_n^{(u)} \bar{q}_{nL} \tilde{\phi}_2 T_R + \sum_{n=1}^2 z_j^{(u)} \bar{T}_L \eta^* u_{nR} + y_T \bar{T}_L \sigma_1 T_R + m_{\tilde{T}} \bar{\tilde{T}}_L \tilde{T}_R + x^{(T)} \bar{T}_L \rho_2 \tilde{T}_R \\
&+ \sum_{n=1}^2 x_n^{(d)} \bar{q}_{3L} \phi_2 B_{nR} + \sum_{n=1}^2 \sum_{j=1}^3 y_{nj}^{(d)} \bar{B}_{nL} \eta d_{jR} + \sum_{j=1}^3 z_j^{(d)} \bar{B}_{3L} \eta^* d_{jR} + \sum_{n=1}^2 w_n^{(u)} \bar{B}_{4L} \varphi_1^- u_{nR} \\
&+ \sum_{k=3}^4 m_{B_k} \bar{B}_{kL} B_{kR} + \sum_{n=1}^2 x_n^{(d)} \bar{q}_{nL} \phi_2 B_{3R} + \sum_{n=1}^2 \sum_{m=1}^2 y_{nm}^{(B)} \bar{B}_{nL} \sigma_1^* B_{mR} + z^{(B)} \bar{B}_{3L} \sigma_2^* B_{4R} \\
&+ \sum_{j=1}^3 w_j^{(d)} \bar{\tilde{T}}_L \varphi_2^+ d_{jR} + \sum_{k=1,3} x_{k3}^{(l)} \bar{l}_{kL} \phi_2 E_{3R} + \sum_{k=1,3} y_{3k}^{(l)} \bar{E}_{3L} \rho_1 l_{kR} + x_{22}^{(l)} \bar{l}_{2L} \phi_2 E_{2R} + y_{22}^{(l)} \bar{E}_{2L} \rho_1 l_{2R} \\
&+ \sum_{i=1}^3 y_i^{(E)} \bar{E}_{iL} \sigma_1^* E_{iR} + x_2^{(\nu)} \bar{l}_{2L} \tilde{\phi}_2 \nu_{2R} + \sum_{k=1,3} z_k^{(l)} \bar{\Psi}_R^C \varphi_3^+ l_{kR} + \sum_{k=1,3} z_k^{(\nu)} \bar{E}_{1L} \varphi_1^- \nu_{kR} + z^{(E)} \bar{\Psi}_R^C \varphi_4^+ E_{1R} \\
&+ \sum_{k=1,3} \sum_{n=1,3} x_{kn}^{(\nu)} \bar{l}_{kL} \tilde{\phi}_2 \nu_{nR} + \sum_{k=1,3} y_k^{(\Omega)} \bar{\Omega}_{1R}^C \eta^* \nu_{kR} + y^{(\Omega)} \bar{\Omega}_{1R}^C \sigma_3^* \nu_{2R} \\
&+ x_1^{(\Psi)} \bar{\Omega}_{1R}^C \eta \Psi_R + x_2^{(\Psi)} \bar{\Omega}_{2R}^C \eta^* \Psi_R + z_\Omega \bar{\Omega}_{1R}^C \sigma_2^* \Omega_{2R} + m_\Psi \bar{\Psi}_R^C \Psi_R + h.c.,
\end{aligned} \tag{3.8}$$

where the Yukawa couplings are $\mathcal{O}(1)$ parameters. From the quark Yukawa terms it follows that the top quark mass emerges due to an interaction involving the SM-like Higgs doublet ϕ_1 only. After spontaneous breaking of the electroweak symmetry, the observed hierarchies of SM fermion masses arise by means of a sequential loop suppression, according to the following pattern: tree-level top quark mass; one-loop bottom, strange, charm, tau and muon

masses; two-loop masses for the up, down quarks as well as for the electron. Furthermore, the SM light active neutrinos get their masses by means of a three-loop radiative seesaw mechanism.

A few comments on the phenomenological implications of the Lagrangian (3.8) are in order. Notice that the neutrino Yukawa coupling to, e.g. SM lepton and ϕ_2 doublets $x_{kn}^{(\nu)}$, is generally not suppressed. On the other hand it can be constrained by Z-boson decays. However, we do not expect this constraint to be very strong provided that ϕ_2 boson is heavy. Indeed, the corresponding one-loop amplitude of Z-boson decay would be suppressed by a large mass of ϕ_2 scalar in the propagators. Therefore, such an amplitude is, in any case, expected to be small.

It is also worth mentioning that the Yukawa interactions $\bar{E}_{2L}\rho_1 l_{2R}$ and $\bar{l}_{2L}\phi_2 E_{2R}$ presented in the 4th line of eq. (3.8) as well as the trilinear scalar interaction $\rho_2 (\phi_1 \cdot \phi_2^\dagger) \sigma_1^*$ generate a one loop level scalar contribution to the muon anomalous magnetic moment $g - 2$. The exchange with the heavy Z' gauge boson also yields a contribution to this observable. The possibility of explanation of the observed deviation of the $g - 2$ from the SM value will be studied in the forthcoming publication.

Let us also note that from the term $\bar{l}_{kL}\tilde{\phi}_2\nu_{nR}$ in eq. (3.8), it follows that the charged lepton flavor violating decay $\tau \rightarrow e\gamma$ is induced at one loop level by electrically charged scalar ϕ_2^+ (arising from the SU_{2L} inert doublet ϕ) and right handed Majorana neutrinos ν_{sR} ($s = 1, 3$) (whose masses are generated at two loop level) appearing in the internal lines of the loop. This decay also receives a one loop level contribution arising from the Z' exchange.

Due to the fact that electron is charged under U_{1X} the LEP measurements of $e^+e^- \rightarrow \mu^+\mu^-$ set a stringent limit [33] on the ratio

$$\frac{M_{Z'}}{g_X} > 12 \text{ TeV}. \tag{3.9}$$

Our model contains two electroweak doublet Higgs scalars $\phi_{1,2}$. As such we should take special care of Flavor Changing Neutral Currents (FCNCs). Our model automatically implements the alignment limit for the lightest 125 GeV Higgs boson, since all other scalar states appear to be decoupled in the mass spectrum and, hence, are very heavy by default. This means the SM-like Higgs boson state does not have tree-level FCNCs while such contributions from the heavier scalars are strongly suppressed by their large mass scale. While a detailed study of the FCNC constraints goes beyond the scope of the current work, we can make a generic statement about nonexistence of FCNCs in our model based upon the Glashow-Weinberg-Paschos theorem [34, 35]. This theorem states that there will be no tree-level FCNC coming from the scalar sector, if all right-handed fermions of a given electric charge couple to only one of the doublets. As seen from eq. (3.8) this condition is satisfied in our model. So, despite of an obvious mass suppression, any possible FCNC corrections would emerge at a loop level only, yielding the model safe with respect to the corresponding phenomenological constraints. Finally, any possible FCNC from the Z' mediation would, for sure, be very much suppressed by its large mass scale compared to the EW one, i.e. $m_{Z'} > 12 \text{ TeV}$ (for $g_X = 1$), according to the LEP constraint.

3.3 Scalar potential

The most general renormalizable scalar potential invariant under the gauge and discrete symmetries of our model is given by

$$\begin{aligned}
 V = & \sum_{i=1}^2 \left(\mu_{pi}^2 |\phi_i|^2 + \lambda_{pi} |\phi_i|^4 \right) + \sum_{j=1}^3 \left(\mu_{sj}^2 |\sigma_j|^2 + \lambda_{sj} |\sigma_j|^4 \right) + \sum_{j=1}^3 \left(\mu_{rj}^2 |\rho_j|^2 + \lambda_{rj} |\rho_j|^4 \right) + \mu_e^2 |\eta|^2 + \lambda_e |\eta|^4 \\
 & + \sum_{i=1}^5 \left(\mu_{fi}^2 \varphi_i^+ \varphi_i^- + \lambda_{fi} (\varphi_i^+ \varphi_i^-)^2 \right) + \sum_{i=1}^2 \sum_{j=1}^3 \alpha_{ij} |\phi_i|^2 |\sigma_j|^2 + \sum_{i=1}^2 \sum_{j=1}^3 \beta_{ij} |\phi_i|^2 |\rho_j|^2 + \sum_{i=1}^2 \kappa_{pi} |\phi_i|^2 |\eta|^2 \\
 & + \kappa_1 |\phi_1|^2 |\phi_2|^2 + \kappa_2 \left(\phi_1 \phi_2^\dagger \right) \left(\phi_2 \phi_1^\dagger \right) + \kappa_3 \left[\varepsilon_{ab} \varepsilon_{cd} (\phi_1)^a (\phi_2)^b (\phi_1^\dagger)^c (\phi_2^\dagger)^d + h.c. \right] \\
 & + \sum_{i=1}^3 \sum_{j=1}^3 \gamma_{ij} |\sigma_i|^2 |\rho_j|^2 + \sum_{i=1}^3 \alpha_{ei} |\eta|^2 |\sigma_i|^2 + \sum_{j=1}^3 \beta_{ej} |\eta|^2 |\rho_j|^2 + \sum_{i=1}^5 \sum_{j=1}^5 \kappa_{ij} (\varphi_i^+ \varphi_i^-) (\varphi_j^+ \varphi_j^-) \\
 & + \sum_{i=1}^5 \sum_{j=1}^2 \lambda_{ij} (\varphi_i^+ \varphi_i^-) |\phi_j|^2 + \sum_{i=1}^5 \sum_{j=1}^3 \varsigma_{ij} (\varphi_i^+ \varphi_i^-) |\sigma_j|^2 + \sum_{i=1}^5 \sum_{j=1}^3 \varrho_{ij} (\varphi_i^+ \varphi_i^-) |\rho_j|^2 + \sum_{i=1}^5 \varkappa_i (\varphi_i^+ \varphi_i^-) |\eta|^2 \\
 & + A_1 \left[\left(\phi_1^\dagger \cdot \phi_2 \right) \sigma_2 + h.c. \right] + A_2 \left[\varepsilon_{ab} (\phi_1)^a (\phi_2)^b \varphi_3^- + h.c. \right] + A_3 (\varphi_4^- \varphi_5^+ \sigma_1 + h.c.) + A_4 (\rho_1 \sigma_2 \sigma_1^* + h.c.) \\
 & + A_5 (\eta \sigma_2 \rho_3 + h.c.) + A_6 (\rho_1 \rho_2 \rho_3 + h.c.) + A_7 (\rho_2 \eta \sigma_1 + h.c.) + A_8 (\sigma_3 \sigma_1^* \eta + h.c.) + A_9 (\varphi_2^- \varphi_4^+ \rho_3 + h.c.) \\
 & + A_{10} (\varphi_1^- \varphi_3^+ \sigma_3^* + h.c.) + A_{11} (\varphi_2^- \varphi_3^+ \sigma_2 + h.c.) + A_{11} (\varphi_3^- \varphi_4^+ \eta + h.c.) + A_{12} (\varphi_3^- \varphi_5^+ \rho_2 + h.c.) \\
 & + \zeta_1 \left[\eta \left(\phi_1 \cdot \phi_2^\dagger \right) \rho_3 + h.c. \right] + \zeta_2 \left[\varepsilon_{ab} (\phi_1)^a (\phi_2)^b \varphi_2^- \sigma_2 + h.c. \right] + \zeta_3 \left[\varepsilon_{ab} (\phi_1)^a (\phi_2)^b \varphi_1^- \sigma_3^* + h.c. \right] \\
 & + \zeta_4 (\varphi_1^+ \varphi_5^- \sigma_1^2 + h.c.) + \zeta_5 (\sigma_1 \sigma_2 \sigma_3^* \rho_3 + h.c.) + \zeta_6 (\sigma_1 \sigma_2^* \rho_2 \rho_3 + h.c.) + \zeta_7 (\sigma_1^* \sigma_2 \rho_2 \rho_3 + h.c.) \\
 & + \zeta_8 \left[\sigma_1^2 (\sigma_2^*)^2 + h.c. \right] + \zeta_9 \left[\sigma_3 \rho_2 (\sigma_1^*)^2 + h.c. \right] + \zeta_{10} (\rho_1 \eta \rho_3 \sigma_1 + h.c.) + \zeta_{11} \left[\rho_1 \left(\phi_1 \cdot \phi_2^\dagger \right) \sigma_1^* + h.c. \right] \\
 & + \zeta_{12} \left[\rho_1^* \left(\phi_1 \cdot \phi_2^\dagger \right) \sigma_1^* + h.c. \right] + \zeta_{13} (\varphi_2^- \varphi_5^+ \sigma_1 \rho_3 + h.c.). \tag{3.10}
 \end{aligned}$$

From the minimization conditions for this potential, we find the following simple relations

$$\begin{aligned}
 \mu_{p1}^2 &= \frac{1}{2} \left(-2v^2 \lambda_{p1} - \beta_{13} v_\rho^2 - \alpha_{11} v_\sigma^2 \right), \\
 \mu_{s1}^2 &= \frac{1}{2} \left(-\gamma_{13} v_\rho^2 - 2v_\sigma^2 \lambda_{s1} - \alpha_{11} v^2 \right), \\
 \mu_{r5}^2 &= \frac{1}{2} \left(-2\lambda_{r3} v_\rho^2 - \gamma_{13} v_\sigma^2 - \beta_{13} v^2 \right), \tag{3.11}
 \end{aligned}$$

that will be used below in a discussion of the scalar mass spectrum of the model.

4 Scalar mass spectrum

Considering the scalar potential given above, we find that the squared mass matrices for the CP-even neutral scalar sector are have the following form

$$M_{\text{CPeven}} = \begin{pmatrix} M_{\text{CPeven}}^{(1)} & 0_{3 \times 8} \\ 0_{8 \times 3} & M_{\text{CPeven}}^{(2)} \end{pmatrix}, \tag{4.1}$$

where $M_{\text{CPeven}}^{(1)}$ and $M_{\text{CPeven}}^{(2)}$ are the squared mass matrices for the $Z_2^{(2)}$ -neutral and $Z_2^{(2)}$ -charged scalars, respectively. The matrix $M_{\text{CPeven}}^{(1)}$ in the basis $(\text{Re}(\phi_1^0), \text{Re}(\sigma_1), \rho_3)$ (re-

mind, ρ_3 is a real SM-singlet scalar), takes the form:

$$M_{\text{CPeven}}^{(1)} = \begin{pmatrix} v^2 \lambda_{p1} & \frac{1}{2} v v_\sigma \alpha_{11} & \frac{1}{2} v v_\rho \beta_{13} \\ \frac{1}{2} v v_\sigma \alpha_{11} & v_\sigma^2 \lambda_{s1} & \frac{1}{2} v_\rho v_\sigma \gamma_{13} \\ \frac{1}{2} v v_\rho \beta_{13} & \frac{1}{2} v_\rho v_\sigma \gamma_{13} & v_\rho^2 \lambda_{r3} \end{pmatrix}. \quad (4.2)$$

The second mass form $M_{\text{CPeven}}^{(2)}$ in the basis $(\text{Re}(\sigma_2), \text{Re}(\sigma_3), \text{Re}(\rho_1), \text{Re}(\rho_2), \text{Re}(\eta), \text{Re}(\phi_2^0))$ reads

$$M_{\text{CPeven}}^{(2)} = \begin{pmatrix} C_1 & C_2 \\ C_2^T & C_3 \end{pmatrix},$$

$$C_1 = \begin{pmatrix} \frac{1}{2} \left(\frac{\alpha_{12} v^2}{2} + \mu_{s2}^2 + \frac{1}{2} v_\rho^2 \gamma_{23} + v_\sigma^2 \zeta_8 \right) & 0 & \frac{A_4 v_\sigma}{2\sqrt{2}} \\ 0 & \frac{1}{4} (\alpha_{13} v^2 + 2\mu_{s3}^2 + v_\rho^2 \gamma_{33}) & 0 \\ \frac{A_4 v_\sigma}{2\sqrt{2}} & 0 & \frac{1}{4} (\beta_{12} v^2 + 2\mu_{r2}^2 + v_\sigma^2 \gamma_{11}) \end{pmatrix},$$

$$C_2 = \begin{pmatrix} \frac{1}{4} v_\rho v_\sigma (\zeta_6 + \zeta_7) & \frac{\sqrt{2}}{4} v_\rho A_5 & \frac{v A_1}{2\sqrt{2}} \\ \frac{1}{4} v_\sigma^2 \zeta_9 & \frac{A_8 v_\sigma}{2\sqrt{2}} & 0 \\ \frac{A_6 v_\rho}{2\sqrt{2}} & \frac{1}{4} v_\rho v_\sigma \zeta_{10} & \frac{1}{4} v v_\sigma (\zeta_{11} + \zeta_{12}) \end{pmatrix}, \quad (4.3)$$

$$C_3 = \begin{pmatrix} \frac{1}{4} (\beta_{14} v^2 + 2\mu_{r4}^2 + v_\sigma^2 \gamma_{12}) & \frac{A_7 v_\sigma}{2\sqrt{2}} & 0 \\ \frac{A_7 v_\sigma}{2\sqrt{2}} & \frac{1}{4} (\kappa_{p1} v^2 + 2\mu_e^2 + v_\sigma^2 \alpha_{e1} + v_\rho^2 \beta_{e3}) & 0 \\ 0 & 0 & \frac{1}{4} (\kappa_1 v^2 + \kappa_2 v^2 + 2\mu_{p2}^2 + v_\sigma^2 \alpha_{21} + v_\rho^2 \beta_{23}) \end{pmatrix}.$$

Since the 126 GeV SM-like Higgs boson is found in the squared mass matrix $M_{\text{CPeven}}^{(1)}$, and considering the fact that the scalar potential has a very large number of parameters, in this first study it is sufficient to diagonalize only $M_{\text{CPeven}}^{(1)}$ in the scalar sector. In addition, since this matrix cannot be diagonalized in analytically closed form, and for the sake of simplicity, here we focus on a particular scenario with $v_\sigma = v_\rho$. In this scenario, the matrix $M_{\text{CPeven}}^{(1)}$ can be diagonalized as follows

$$\left(R_{\text{CPeven}}^{(1)} \right)^T M_{\text{CPeven}}^{(1)} R_{\text{CPeven}}^{(1)} \simeq \begin{pmatrix} \frac{8}{11} \lambda v^2 & 0 & 0 \\ 0 & \frac{1}{2} (4 - \sqrt{5}) \lambda v_\sigma^2 & 0 \\ 0 & 0 & \frac{1}{2} (4 + \sqrt{5}) \lambda v_\sigma^2 \end{pmatrix}, \quad (4.4)$$

$$R_{\text{CPeven}}^{(1)} \simeq \begin{pmatrix} -1 + \frac{13}{121} x^2 & -\frac{1}{11} \sqrt{13 + \frac{19}{\sqrt{5}} x} & \frac{1}{11} \sqrt{13 - \frac{19}{\sqrt{5}} x} \\ \frac{5x}{11} & -\sqrt{\frac{1}{2} + \frac{1}{\sqrt{5}}} & \sqrt{\frac{1}{2} - \frac{1}{\sqrt{5}}} \\ \frac{x}{11} & \frac{1}{\sqrt{10 + 4\sqrt{5}}} & \sqrt{\frac{1}{2} + \frac{1}{\sqrt{5}}} \end{pmatrix}, \quad x = \frac{v}{v_\sigma}.$$

Consequently, the physical scalar states contained in the matrix $M_{\text{CPeven}}^{(1)}$ are given by:

$$\begin{pmatrix} h \\ \chi_1 \\ \chi_2 \end{pmatrix} \simeq \begin{pmatrix} -1 + \frac{13}{121} x^2 & -\frac{1}{11} x \sqrt{\frac{19}{5} \sqrt{5} + 13} & \frac{1}{11} x \\ -\frac{1}{11} x \sqrt{\frac{19}{5} \sqrt{5} + 13} & -\sqrt{\frac{1}{5} \sqrt{5} + \frac{1}{2}} & \frac{1}{2} \frac{\sqrt{2}}{\sqrt{2\sqrt{5} + 5}} \\ \frac{1}{11} x \sqrt{13 - \frac{19}{5} \sqrt{5}} & \sqrt{\frac{1}{2} - \frac{1}{5} \sqrt{5}} & \sqrt{\frac{1}{5} \sqrt{5} + \frac{1}{2}} \end{pmatrix} \begin{pmatrix} \phi_{1R}^0 \\ \sigma_{1R} \\ \rho_3 \end{pmatrix},$$

$$\phi_{1R}^0 = \text{Re}(\phi_1^0), \quad \sigma_{1R} = \text{Re}(\sigma_1), \quad (4.5)$$

where h is the 126 GeV SM-like Higgs boson, whereas χ_1 and χ_2 are the physical heavy scalar fields, which acquire masses at the scale of U_{1X} breaking. The squared masses of these fields are given by

$$m_h^2 \simeq \frac{8}{11} \lambda v^2, \quad m_{\chi_1}^2 \simeq \frac{1}{2} (4 - \sqrt{5}) \lambda v_\sigma^2, \quad m_{\chi_2}^2 \simeq \frac{1}{2} (4 + \sqrt{5}) \lambda v_\sigma^2. \quad (4.6)$$

Furthermore, we find that the SM-like Higgs boson h has the couplings that are very close to the SM expectation, with small deviations of the order of $\sim v^2/v_\sigma^2$.

Considering the CP-odd neutral scalar sector, we find that the squared mass matrices for the electrically neutral CP-odd scalars in the basis, are $(Im(\phi_1^0), Im(\sigma_1), Im(\sigma_2), Im(\sigma_3), Im(\rho_2), Im(\rho_4), Im(\eta), Im(\phi_2^0), Im(\rho_1), Im(\rho_3))$ are given by

$$\begin{aligned} M_{\text{CPodd}} &= \begin{pmatrix} M_{\text{CPodd}}^{(1)} & 0_{2 \times 8} \\ 0_{8 \times 2} & M_{\text{CPodd}}^{(2)} \end{pmatrix}, & M_{\text{CPodd}}^{(1)} &= 0_{2 \times 2}, & M_{\text{CPodd}}^{(2)} &= \begin{pmatrix} D_1 & D_2 \\ D_2^T & D_3 \end{pmatrix}, \\ D_1 &= \begin{pmatrix} \frac{1}{2} \left(\frac{\alpha_{12} v^2}{2} + \mu_{s2}^2 + \frac{1}{2} v_\rho^2 \gamma_{23} - v_\sigma^2 \zeta_8 \right) & 0 & -\frac{A_4 v_\sigma}{2\sqrt{2}} \\ 0 & \frac{1}{4} (\alpha_{13} v^2 + 2\mu_{s3}^2 + v_\rho^2 \gamma_{33}) & 0 \\ -\frac{A_4 v_\sigma}{2\sqrt{2}} & 0 & \frac{1}{4} (\beta_{12} v^2 + 2\mu_{r2}^2 + v_\sigma^2 \gamma_{11}) \end{pmatrix}, \\ D_2 &= \begin{pmatrix} \frac{1}{4} v_\rho v_\sigma (\zeta_6 - \zeta_7) & -\frac{\sqrt{2}}{4} v_\rho A_5 & -\frac{v A_1}{2\sqrt{2}} \\ -\frac{1}{4} v_\sigma^2 \zeta_9 & -\frac{A_8 v_\sigma}{2\sqrt{2}} & 0 \\ -\frac{A_6 v_\rho}{2\sqrt{2}} & -\frac{1}{4} v_\rho v_\sigma \zeta_{10} & \frac{1}{4} v v_\sigma (\zeta_{11} - \zeta_{12}) \end{pmatrix}, \\ D_3 &= \begin{pmatrix} \frac{1}{4} (\beta_{14} v^2 + 2\mu_{r4}^2 + v_\sigma^2 \gamma_{12}) & -\frac{A_7 v_\sigma}{2\sqrt{2}} & 0 \\ -\frac{A_7 v_\sigma}{2\sqrt{2}} & \frac{1}{4} (\kappa_{p1} v^2 + 2\mu_e^2 + v_\sigma^2 \alpha_{e1} + v_\rho^2 \beta_{e3}) & 0 \\ 0 & 0 & \frac{1}{4} (\kappa_1 v^2 + \kappa_2 v^2 + 2\mu_{p2}^2 + v_\sigma^2 \alpha_{21} + v_\rho^2 \beta_{23}) \end{pmatrix}, \end{aligned} \quad (4.7)$$

where $M_{\text{CPodd}}^{(1)}$ and $M_{\text{CPodd}}^{(2)}$ are the squared mass matrices for the CP-odd scalars, neutral and charged under Z_4 , respectively. Note that the squared mass matrix $M_{\text{CPodd}}^{(1)}$ (which is written in the basis $(Im(\phi_1^0), Im(\sigma_1))$) is exactly zero, since $Im(\phi_1^0)$ and $Im(\sigma_1)$ are the Goldstone bosons associated with the longitudinal components of the Z and Z' gauge bosons, respectively.

Finally, the squared mass matrix for the charged scalar fields in the basis $(\phi_1^+, \phi_2^+, \varphi_3^+, \varphi_4^+, \varphi_5^+, \varphi_4^+, \varphi_5^+)$ reads

$$\begin{aligned} M_C &= \begin{pmatrix} 0 & 0_{1 \times 2} & 0_{1 \times 4} \\ 0_{2 \times 1} & M_C^{(1)} & 0_{2 \times 4} \\ 0_{4 \times 1} & 0_{4 \times 2} & M_C^{(2)} \end{pmatrix}, & M_C^{(2)} &= \begin{pmatrix} M_C^{(2a)} & M_C^{(2b)} \\ (M_C^{(2b)})^T & M_C^{(2c)} \end{pmatrix}, \\ M_C^{(1)} &= \begin{pmatrix} \frac{1}{2} (\kappa_1 v^2 + 2\kappa_3 v^2 + 2\mu_{p2}^2 + v_\sigma^2 \alpha_{21} + v_\rho^2 \beta_{23}) & \frac{v A_2}{\sqrt{2}} \\ & \frac{v A_2}{\sqrt{2}} \\ & \frac{1}{2} (\lambda_{31} v^2 + 2\mu_{f3}^2 + \varrho_{33} v_\rho^2 + \varsigma_{31} v_\sigma^2) \end{pmatrix}, \\ M_C^{(2a)} &= \begin{pmatrix} \frac{1}{2} (\lambda_{11} v^2 + 2\mu_{f1}^2 + \varrho_{13} v_\rho^2 + \varsigma_{11} v_\sigma^2) & 0 \\ & 0 \\ & \frac{1}{2} (\lambda_{21} v^2 + 2\mu_{f2}^2 + \varrho_{23} v_\rho^2 + \varsigma_{21} v_\sigma^2) \end{pmatrix}, \\ M_C^{(2b)} &= \begin{pmatrix} 0 & \frac{\zeta_4 v_\sigma^2}{2} \\ \frac{A_9 v_\rho}{\sqrt{2}} & \frac{\zeta_{13} v_\sigma v_\rho}{2} \end{pmatrix}, & M_C^{(2c)} &= \begin{pmatrix} \frac{1}{2} (\lambda_{41} v^2 + 2\mu_{f4}^2 + \varrho_{43} v_\rho^2 + \varsigma_{41} v_\sigma^2) & \frac{A_3 v_\sigma}{\sqrt{2}} \\ \frac{A_3 v_\sigma}{\sqrt{2}} & \frac{1}{2} (\lambda_{51} v^2 + 2\mu_{f5}^2 + \varrho_{53} v_\rho^2 + \varsigma_{51} v_\sigma^2) \end{pmatrix}, \end{aligned} \quad (4.8)$$

such that ϕ_1^\pm are the electrically charged massless scalar states corresponding to the Goldstone bosons associated with the longitudinal components of the W^\pm gauge bosons.

5 Radiatively generated quark mass and mixing hierarchies

The quark Yukawa interactions determined by eq. (3.8) give rise to the following up and down mass matrices for the SM quarks, respectively,

$$M_U = \begin{pmatrix} \varepsilon_{11}^{(u)} + \tilde{\varepsilon}_{11}^{(u)} & \varepsilon_{12}^{(u)} + \tilde{\varepsilon}_{12}^{(u)} & 0 \\ \varepsilon_{21}^{(u)} + \tilde{\varepsilon}_{21}^{(u)} & \varepsilon_{22}^{(u)} + \tilde{\varepsilon}_{22}^{(u)} & 0 \\ 0 & 0 & y_{33}^{(u)} \end{pmatrix} \frac{v}{\sqrt{2}}, \quad M_D = \begin{pmatrix} \varepsilon_{11}^{(d)} + \tilde{\varepsilon}_{11}^{(d)} & \varepsilon_{12}^{(d)} + \tilde{\varepsilon}_{12}^{(d)} & \varepsilon_{13}^{(d)} + \tilde{\varepsilon}_{13}^{(d)} \\ \varepsilon_{21}^{(d)} + \tilde{\varepsilon}_{21}^{(d)} & \varepsilon_{22}^{(d)} + \tilde{\varepsilon}_{22}^{(d)} & \varepsilon_{23}^{(d)} + \tilde{\varepsilon}_{23}^{(d)} \\ \varepsilon_{31}^{(d)} & \varepsilon_{32}^{(d)} & \varepsilon_{33}^{(d)} \end{pmatrix} \frac{v}{\sqrt{2}}, \quad (5.1)$$

where the dimensionless parameters $\varepsilon_{nm}^{(u)}$ ($n, m = 1, 2$) and $\varepsilon_{ij}^{(d)}$ ($i, j = 1, 2, 3$) are generated at one-loop level, whereas $\tilde{\varepsilon}_{nj}^{(d)}$ and $\tilde{\varepsilon}_{ij}^{(d)}$ arise at two-loop level. The characteristic Feynman loop diagrams contributing to the entries of the SM quark mass matrices are shown in figure 1.

In what follows, we demonstrate that the mass matrices for SM quarks given above incorporate the observed hierarchies in the SM quark mass spectrum and the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. To this end, we proceed with a parametrization of the SM quark mass matrices in the following form

$$M_U = \begin{pmatrix} \left(a_{11}^{(u)}\right)^2 l & \left(a_{12}^{(u)}\right)^2 l & 0 \\ \left(a_{21}^{(u)}\right)^2 l & \left(a_{22}^{(u)}\right)^2 l & 0 \\ 0 & 0 & y_{33}^{(u)} \end{pmatrix} \frac{v}{\sqrt{2}}, \quad M_D = \begin{pmatrix} \left(a_{11}^{(d)}\right)^2 l & \left(a_{12}^{(d)}\right)^2 l & \left(a_{13}^{(d)}\right)^2 l \\ \left(a_{21}^{(d)}\right)^2 l & \left(a_{22}^{(d)}\right)^2 l & \left(a_{23}^{(d)}\right)^2 l \\ \left(a_{31}^{(d)}\right)^2 l & \left(a_{32}^{(d)}\right)^2 l & \left(a_{33}^{(d)}\right)^2 l \end{pmatrix} \frac{v}{\sqrt{2}}, \quad (5.2)$$

where $l \approx (1/4\pi)^2 \approx 2.0 \times \lambda^4$ is the loop suppression factor, and $\lambda = 0.225$ is the Wolfenstein parameter. As a consequence, we expect that $a_{nm}^{(u)}$, $a_{ij}^{(d)}$ ($n, m = 1, 2$ and $i, j = 1, 2, 3$) be $\mathcal{O}(1)$ parameters.

We remark that the Feynman diagrams contributing to the entries of the SM fermion mass matrices contain a large number of uncorrelated parameters that belong to the fermion and scalar sectors of our model. Nevertheless, these parameters can be absorbed into a limited number of effective parameters $\varepsilon_{nm}^{(u)}$, $\tilde{\varepsilon}_{nj}^{(d)}$, $\varepsilon_{ij}^{(d)}$, $\tilde{\varepsilon}_{ij}^{(d)}$ ($n, m = 1, 2$ and $i, j = 1, 2, 3$), which can be used to reproduce the experimental values of the physical observables in the quark sector¹

$$\begin{aligned} m_u(\text{MeV}) &= 1.45_{-0.45}^{+0.56}, & m_d(\text{MeV}) &= 2.9_{-0.4}^{+0.5}, & m_s(\text{MeV}) &= 57.7_{-15.7}^{+16.8}, \\ m_c(\text{MeV}) &= 635 \pm 86, & m_t(\text{GeV}) &= 172.1 \pm 0.6 \pm 0.9, & m_b(\text{GeV}) &= 2.82_{-0.04}^{+0.09}, \\ \sin \theta_{12} &= 0.2254, & \sin \theta_{23} &= 0.0414, & \sin \theta_{13} &= 0.00355, \\ J &= 2.96_{-0.16}^{+0.20} \times 10^{-5}. \end{aligned} \quad (5.3)$$

Here, $m_{t,u,c,d,s,b}$ are the SM quark masses, θ_{12} , θ_{23} , θ_{13} are the mixing angles, and J is the Jarlskog parameter.

¹We use the experimental values of the quark masses at the M_Z scale known from ref. [27], which are similar to those in ref. [28]. The experimental values of the CKM parameters are taken from ref. [29].

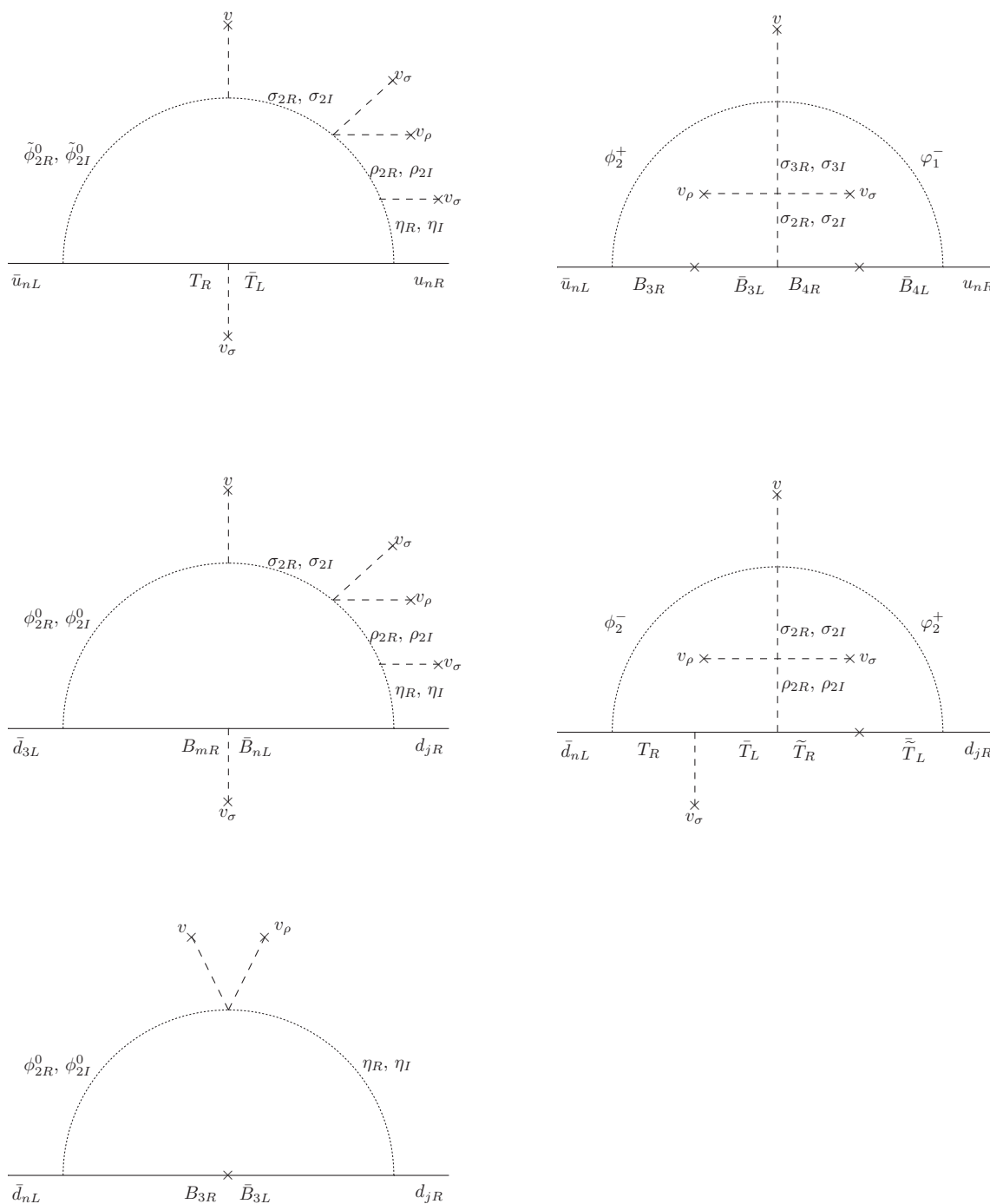


Figure 1. One- and two-loop Feynman diagrams contributing to the entries of the SM quark mass matrices. Here, $n, m = 1, 2$ and $j = 1, 2, 3$.

While our model does not predict the exact values of the physical observables, it offers a natural explanation of the observed (strong) hierarchies. As was previously mentioned, it only pretends to reproduce the existing pattern of quark masses and mixing caused by a sequential loop suppression predicted by the model. To this end, for the SM quark mass matrices given above, we look for the eigenvalue problem solutions reproducing the experimental values of the quark masses and the CKM parameters given by eq. (5.3), requiring that $a^{(u,d)}, b^{(u,d)}$ are all of the same order of one. The standard procedure renders the following solution

$$\begin{aligned}
 a_{11}^{(u)} &\simeq 0.708, & a_{12}^{(u)} = a_{21}^{(u)} &\simeq 0.567, & a_{22}^{(u)} &\simeq 0.456, & y_{33}^{(u)} &= 0.989, \\
 a_{11}^{(d)} &\simeq 0.191, & a_{12}^{(d)} = a_{21}^{(d)} &\simeq 0.182, & a_{13}^{(d)} = a_{31}^{(d)} &\simeq 0.325 + 0.009i, \\
 a_{23}^{(d)} = a_{32}^{(d)} &\simeq 0.269 - 0.016i, & a_{22}^{(d)} &\simeq 0.190, & a_{33}^{(d)} &\simeq 1.771.
 \end{aligned}
 \tag{5.4}$$

The above $\mathcal{O}(1)$ values exactly reproduce the measured central values of the SM quark masses and CKM parameters given in eq. (5.3). Hence, our model is consistent with and successfully reproduces the existing pattern of SM quark masses caused by the sequential loop suppression mechanism, with different quark flavors getting mass at different orders in Perturbation Theory as discussed above.

6 Radiatively generated lepton masses and mixings

The lepton and neutrino Yukawa interactions and mass terms given in eq. (3.8) give rise to the characteristic Feynman loop diagrams illustrated in figures 2 and 3 that necessarily generate the following SM charged lepton and light active neutrino mass forms:

$$M_l = \begin{pmatrix} \varepsilon_{11}^{(l)} + \tilde{\varepsilon}_{11}^{(l)} & 0 & \varepsilon_{13}^{(l)} + \tilde{\varepsilon}_{13}^{(l)} \\ 0 & \varepsilon_{22}^{(l)} & 0 \\ \varepsilon_{31}^{(l)} + \tilde{\varepsilon}_{31}^{(l)} & 0 & \varepsilon_{33}^{(l)} + \tilde{\varepsilon}_{33}^{(l)} \end{pmatrix} \frac{v}{\sqrt{2}}, \quad M_\nu = \begin{pmatrix} a_{11}^{(\nu)} & a_{12}^{(\nu)} & a_{13}^{(\nu)} \\ a_{21}^{(\nu)} & a_{22}^{(\nu)} & a_{23}^{(\nu)} \\ a_{31}^{(\nu)} & a_{32}^{(\nu)} & a_{33}^{(\nu)} \end{pmatrix}, \tag{6.1}$$

where $\varepsilon_{ii}^{(l)}, \varepsilon_{13}^{(l)}, \varepsilon_{31}^{(l)}$ are the dimensionless parameters generated at one-loop level, whereas the parameters $\tilde{\varepsilon}_{ii}^{(d)}, \tilde{\varepsilon}_{13}^{(d)}$ and $\tilde{\varepsilon}_{31}^{(d)}$ appear at two-loop level. In what follows, we show that the lepton mass matrices given above can accommodate the experimental data on the SM lepton masses and mixing. For this purpose, following the same strategy as for the quark mass forms discussed in the previous section, we parametrize the SM charged lepton mass matrix as follows:

$$M_l = \begin{pmatrix} \left(a_{11}^{(l)}\right)^2 l & 0 & \left(a_{13}^{(l)}\right)^2 l \\ 0 & \left(a_{22}^{(l)}\right)^2 l & 0 \\ \left(a_{31}^{(l)}\right)^2 l & 0 & \left(a_{33}^{(l)}\right)^2 l \end{pmatrix} \frac{v}{\sqrt{2}}. \tag{6.2}$$

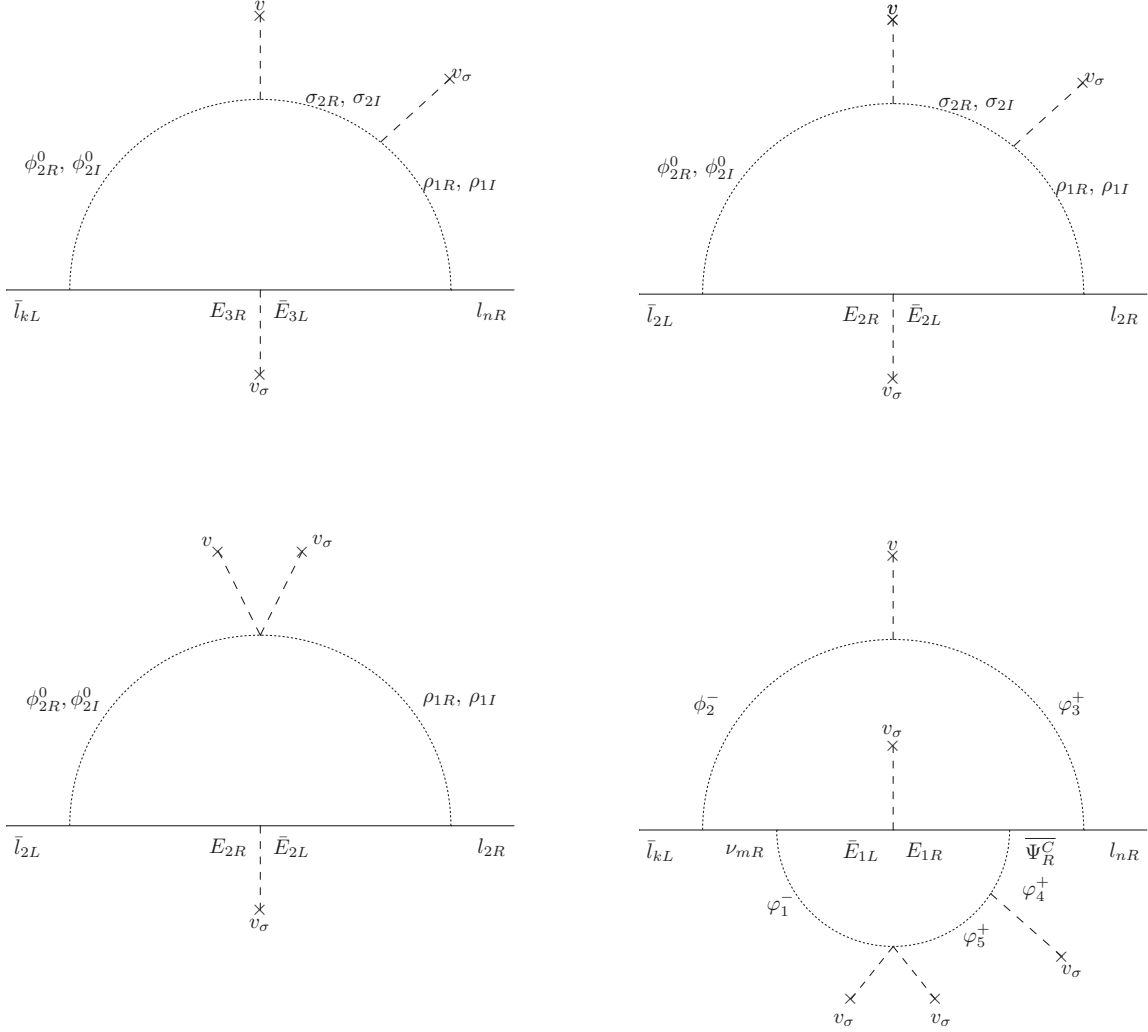


Figure 2. One- and two-loop Feynman diagrams contributing to the entries of the SM charged lepton mass matrix. Here, $k, m, n = 1, 3$.

In order to fit the measured values of the charged lepton masses, as well as the neutrino mass squared differences and lepton mixing parameters [37], we proceed by solving the eigenvalue problem for the SM lepton and light neutrino mass matrices. The following solution has been found:

$$a_{11}^{(l)} = a_{33}^{(l)} \simeq 0.491, \quad a_{13}^{(l)} = a_{31}^{(l)} \simeq 0.4905, \quad a_{22}^{(l)} \simeq 0.340, \quad (6.3)$$

$$M_\nu = \begin{cases} \begin{pmatrix} 0.0473664 - 0.00675494i & 0.00904226 - 0.0045673i & 0.00544117 + 0.000841261i \\ 0.00904226 - 0.0045673i & 0.0528446 + 0.000500202i & 0.0066006 + 0.00515561i \\ 0.00544117 + 0.000841261i & 0.0066006 + 0.00515561i & 0.04306 + 0.00575822i \end{pmatrix} \text{ eV for NH,} \\ \begin{pmatrix} 0.0573349 - 0.0100651i & -0.00894313 - 0.00802622i & -0.00466139 - 0.000301301i \\ -0.00894313 - 0.00802622i & 0.0495729 - 0.000243999i & -0.00936421 + 0.00765498i \\ -0.00466139 - 0.000301301i & -0.00936421 + 0.00765498i & 0.0560537 + 0.0101163i \end{pmatrix} \text{ eV for IH.} \end{cases} \quad (6.4)$$

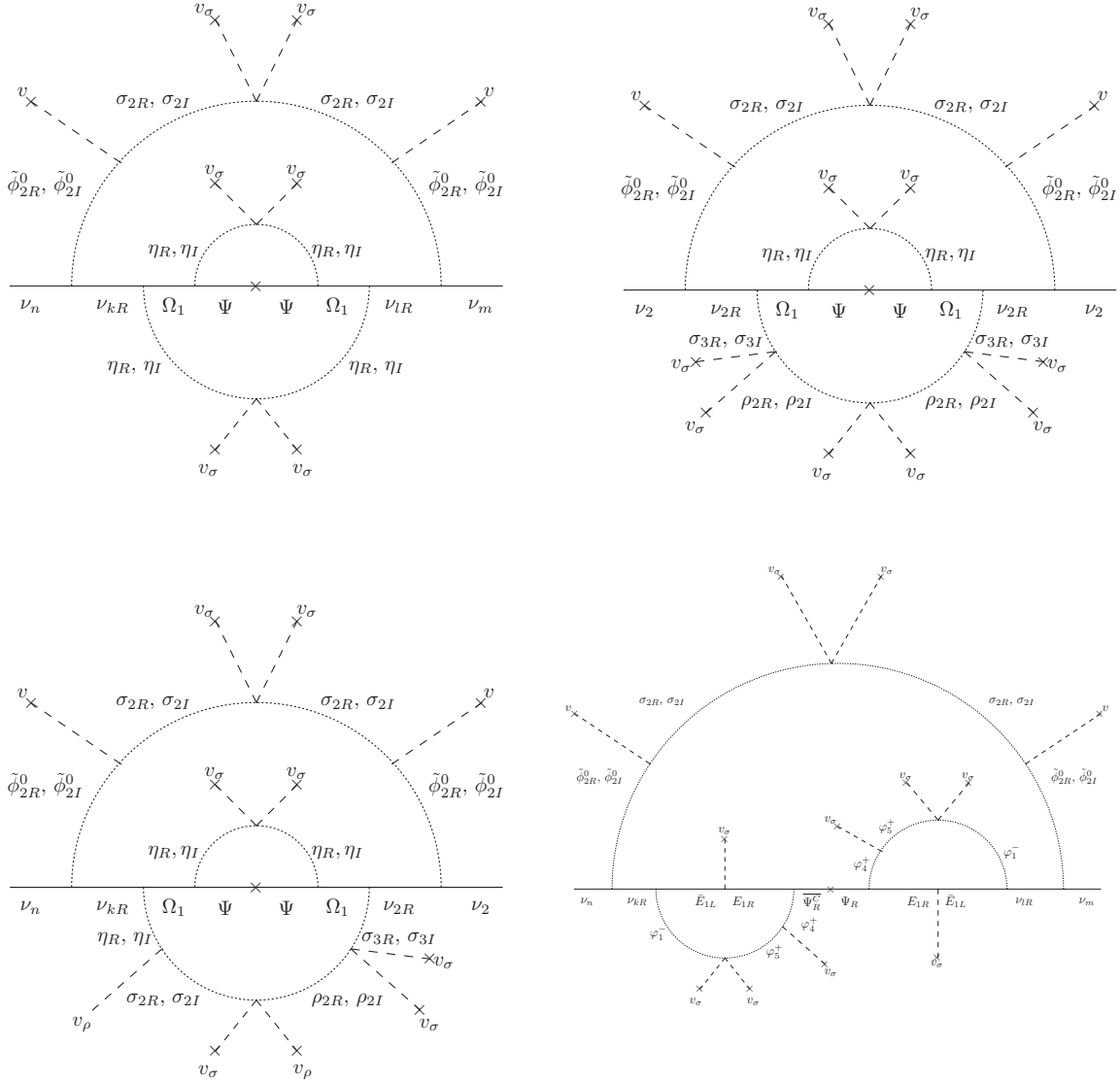


Figure 3. Three-loop Feynman diagrams contributing to the entries of the neutrino mass matrix. Here, $k, l, n, m = 1, 3$.

Thus we find that with $\mathcal{O}(1)$ values for the $a_{nm}^{(l)}$ ($n, m = 1, 3$), $a_{22}^{(l)}$ parameters and the above specified entries of the neutrino mass matrix satisfying $\mathcal{O}(10^{-3}) \text{ eV} \lesssim |(M_\nu)_{ij}| \lesssim \mathcal{O}(10^{-2}) \text{ eV}$ ($i, j = 1, 2, 3$), the experimental values for the physical observables of the lepton sector, i.e., the three charged lepton masses, the two neutrino mass squared splittings, the three leptonic mixing parameters and the leptonic Dirac CP violating phase can be successfully reproduced. Consequently, our model is consistent with and successfully reproduces the existing pattern of SM charged lepton masses generated by the sequential loop suppression mechanism.

7 Discussions and conclusions

We have constructed a first renormalizable extension of the Inert Doublet Model that enables an implementation of a sequential loop suppression mechanism, capable of explaining the observed SM fermion mass hierarchy without invoking soft breaking mass terms. In our model, the SM gauge symmetry is supplemented by the $U_{1X} \times Z_2^{(1)} \times Z_2^{(2)}$ family symmetry, where the gauge U_{1X} and discrete $Z_2^{(1)}$ symmetries are spontaneously broken, whereas the $Z_2^{(2)}$ symmetry is preserved.

Our model is consistent with the observed SM fermion mass spectrum and fermionic mixing parameters and allows for an explanation of the recently observed R_K and R_{K^*} anomalies, thanks to the non-universal Z' couplings to fermions. Let us point out that in the case of the studied sequential loop mechanism these anomalies cannot be explained by the Yukawa couplings. Indeed, as follows from eq. (3.8), there are no terms of the form $\bar{f}_{iL} S f_{jR}$ ($i, j=1, 2, 3$), where S is a scalar and f_i denote the SM fermions. Consequently, at tree level there is no scalar exchange contribution to the R_{K, K^*} anomalies. This contribution appears only at two loop level given that interactions of the form $\bar{b}_L \sigma \bar{s}_R$, $\bar{e}_L \sigma \bar{e}_R$ and $\bar{\mu}_L \sigma \bar{\mu}_R$ are generated at one loop level as seen from the one loop diagrams for the down type quark and the charged leptons masses shown in figures 1 and 2. Thus, the Yukawa contributions to the R_{K, K^*} anomalies are very much suppressed by the loop factors and the heavy particles in the loops.

We focused on an extension of the Inert Higgs Doublet model (IDM) that allows the implementation of the sequential loop suppression mechanism for the generation of SM fermion masses instead of an extension of the inert 3-3-1 model (model based on the $SU_{3C} \times SU_{3L} \times U_{1X}$ gauge symmetry). As previously mentioned, the extension of the inert 3-3-1 model of ref. [25] does not explain the R_K and R_{K^*} anomalies and the light active neutrino masses appear at two-loop level like the masses of the light SM charged fermions. Addressing the R_K and R_{K^*} anomalies in the framework of a 3-3-1 model would require to consider five families of $SU(3)_L$ leptonic triplets as done in ref. [36], in order to have different $U(1)_X$ charge assignments for the first and second lepton families, without spoiling the anomaly cancellation conditions. Thus, modifying the inert 3-3-1 model of ref. [25] to account for the R_K and R_{K^*} anomalies, and to generate the hierarchy of SM fermion masses by sequential loop suppression mechanism, with the light active neutrino masses appearing at three-loop level, will require a much larger particle content than the one adopted in the framework of an extended IDM.

In our model only the top quark and exotic fermions acquire tree-level masses, whereas the masses of the remaining SM fermions emerge from a radiative seesaw-like mechanism: the masses for the bottom, strange and charm quarks, tau and muon leptons are generated at one-loop level, whereas the masses for the up and down quarks as well as the electron mass appear at two-loop level. Furthermore, light active neutrino acquire masses by means of a radiative seesaw mechanism at three-loop level.

Due to an unbroken $Z_2^{(2)}$ discrete symmetry, our model has several stable scalar dark matter candidates, which can be the neutral components of the inert SU_{2L} scalar doublet ϕ_2 as well as the real and imaginary parts of the SM scalar singlets σ_2 , σ_3 , ρ_1 , ρ_2

and η . Furthermore, the model can have a fermionic dark matter candidate which is the only SM-singlet Majorana neutrino Ω_{1R} with a non-trivial $Z_2^{(2)}$ charge. A study of the phenomenological implications of our model goes beyond the scope of the present paper and will be performed in a forthcoming work.

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