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One-loop free energy of tensionless type IIB string in $AdS_5 \times S^5$

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Abstract: Considering the zero 't Hooft coupling limit of $\mathcal{N} = 4$ super-Yang-Mills theory, the exact spectrum of all single-trace operators can be accessed in terms of the underlying $so(2,4)$ character. This makes it possible in turn to compute the one-loop free energy of the tensionless type IIB string theory in $AdS_5 \times S^5$ background, with help of the recently developed method of *character integral representation of zeta function (CIRZ)*. We calculate first the one-loop free energy of the string states in the $(p-1)$-th Regge trajectory and find the result to be p times the free energy of a single $\mathcal{N} = 4$ Maxwell multiplet. The full one-loop free energy is hence proportional to the divergent series $\sum_{p=2}^{\infty} p$. The divergence arises as a result of interrupting the regularization procedure in an intermediate stage. With a reorganization of states, we extract the finite part of free energy after summing over the Regge trajectories. This way gives us a finite result which is minus of the free energy of the $\mathcal{N} = 4$ multiplet. Hence, this bulk one-loop result matches the -1 term in the $N^2 - 1$ factor of the boundary result.

Keywords: 1/N Expansion, AdS-CFT Correspondence, Higher Spin Gravity, Higher Spin Symmetry

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Contents

1 Introduction

Ever since the proposal of the duality between four-dimensional $\mathcal{N}=4$ SU(N) super-Yang-Mills (SYM) and Type-IIB superstring theory on $AdS_5 \times S^5$ [1–3], there have been innumerous tests on this conjecture. Here, we revisit the match of free energies of the two theories. Considering the boundary geometry as S^4 or $S^1 \times S^3$, the free energy of the $\mathcal{N}=4$ SYM gives the conformal a-anomaly or Casimir energy on S^3, respectively. These quantities are protected from renormalization and given exactly by

$$\begin{aligned} F^{\mathrm{SU}(N)}_{\mathcal{N}=4}(S^4) &= (N^2-1)\,\log\Lambda\,, \\ F^{\mathrm{SU}(N)}_{\mathcal{N}=4}(S^1_{\beta}\times S^3) &= (N^2-1)\,\frac{3}{16}\,\beta + \mathcal{O}(\beta^0)\,, \end{aligned} \tag{1.1}$$

where Λ is the UV cut-off and β is the radius of the thermal cycle S^1. On the other hand, the bulk free energy admits the loop expansion,

$$\frac{1}{G}\,\Gamma_{\mathrm{IIB}} = \frac{1}{G}\,S_{\mathrm{IIB}} + \Gamma^{(1)}_{\mathrm{IIB}} + \mathcal{O}(G)\,. \tag{1.2}$$

With the dictionary $1/G = N^2$, the leading order S_{IIB} reproduces the N^2 term of the boundary free energy in the supergravity limit [4, 5]. Moving to the next order $\Gamma^{(1)}_{\mathrm{IIB}}$, one needs to consider one-loop effects in type-IIB string theory in the bulk [6–8].

The excitations of type-IIB string in $AdS_5 \times S^5$ can be organized into multiplets of the supergroup PSU$(2,2|4)$ [9–14]. The 10d massless modes correspond to BPS ones — the 5d $\mathcal{N}=8$ supergravity multiplet and infinite tower of massive Kaluza-Klein (KK) multiplets [15] — whereas the 10d massive modes correspond to various non-BPS multiplets whose masses are determined by the string tension. The contribution of the massive modes to the one-loop free energy is expected to vanish as the boundary result (1.1) does not

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depend on the 't Hooft coupling or string tension. In contrast, the contributions of the massless modes do not vanish (they are studied first in [6, 7] and revised in [8]):

$$\Gamma^{(1)}_{\mathrm{KK},p}(\mathrm{AdS}_5\times S^5) = p\,\log R\,,$$
$$\Gamma^{(1)}_{\mathrm{KK},p}(\mathrm{AdS}_5^{\beta}\times S^5) = p\,\frac{3}{16}\,\beta + \mathcal{O}(\beta^0)\,, \tag{1.3}$$

where $p-1$ is the KK level and R is the IR cut-off of the radial coordinate in AdS_5 which maps to the boundary UV cut-off Λ. AdS_5^{β} is the thermal AdS_5 with $S^1_{\beta}\times S^3$ boundary. Therefore, with the assumption that the contributions of 10d massive states vanish, the full one-loop free energy,

$$\Gamma^{(1)}_{\mathrm{IIB}}(\mathrm{AdS}_5\times S^5) = A\,\log R\,,$$
$$\Gamma^{(1)}_{\mathrm{IIB}}(\mathrm{AdS}_5^{\beta}\times S^5) = A\,\frac{3}{16}\,\beta + \mathcal{O}(\beta^0)\,, \tag{1.4}$$

is given with a divergent series $A=\sum_{p=2}^{\infty} p$. An appropriate regularization scheme is needed for a physically meaningful answer. The desirable result is $A=-1$ so that the semi-classical and one-loop results sum up to reproduce exactly the N^2-1 factor of the boundary free energy.

Tensionless limit of type IIB string theory. With the assumption that the massive 10d string modes do not contribute to the one-loop free energy, we avoid the problem of identifying such multiplets in the string spectrum. In the tensile case, this problem is highly non-trivial as it amounts to calculating exact anomalous dimensions for all single-trace operators. In the tensionless limit, the identification of all these single-trace operators — hence, the string excitation modes — becomes available in principle. In this paper, we calculate the one-loop free energy of the tensionless type IIB strings in a straightforward and thorough manner, that is, by properly collecting the contributions of all single-particle states in the string spectrum while making no *a priori* assumptions about the vanishing of the massive mode contribution. As we shall show, our result reproduces the expression (1.4) with $A=\sum_{p=2}^{\infty} p$, but now each p contribution comes from all the states residing in the $(p-1)$-th "Regge trajectory",[1] which contains in fact the $(p-1)$-th KK states.

Our work is motivated by the recent success of the dualities [16, 17] between higher spin gravity [18] and vector model conformal field theories, where the single-particle/trace states are simple enough to allow a direct evaluation of bulk one-loop free energy [19–21] (see [22–30] for more recent works). To extend the success of vector models to adjoint models, the authors of the current article have worked out two string-like bulk theories, one dual to free scalar [31] and the other dual to free Yang-Mills [32] with SU(N) adjoint symmetry. The key ingredient in our previous work is the use of Character Integral Representation of Zeta function (CIRZ) devised in [31], which allows us to calculate the free energy with the information encoded in the character only. Also in the current paper, the CIRZ method enables us to perform the computation in a straightforward manner.

[1] The term "Regge trajectory" refers to the collection of particles with the same mass versus spin ratio, and is useful to organize the string spectrum around flat spacetime. However, in *tensionless string* of $\mathrm{AdS}_5\times\mathrm{S}^5$, the spectrum does not have this pattern, and we use the term "Regge trajectory" for another way to organize the entire string spectrum. The precise definition will be given after the equation (2.2).

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2 Single-particle states/single-trace operators

In the tensionless limit ($\alpha' \to \infty$ while $L_{\rm AdS}$ finite) of type IIB string theory in the bulk [33–47], the 't Hooft coupling of the boundary $\mathcal{N}=4$ SYM goes to zero,

$$\lambda = N\, g_{\rm YM}^2 = \left(\frac{L_{\rm AdS}^2}{\alpha'}\right)^2 \to 0\,, \tag{2.1}$$

hence all the operators become free from any anomalous dimensions. In this case, the spectrum of single-trace operators can be exactly identified by group theoretical and combinatorial methods [48–51]. Via the AdS/CFT correspondence, we identify this with the spectrum of single-particle states in the bulk and consider the corresponding bulk theory, following [52], as our *working definition* for tensionless string theory. By exploiting this definition, one can collect and sum over the one-loop free energies of all single-particle states in the string spectrum [31, 32].

For the group theoretical identification of the operators made by p insertions of field in a single trace,

$$\mathrm{Tr}(\Phi_1 \cdots \Phi_p)\,, \qquad \Phi_i \,:\, \text{any field (or its derivative) of } \mathcal{N}=4 \text{ SYM}\,, \tag{2.2}$$

we need to consider the p-th tensor product of the $\mathcal{N}=4$ Maxwell multiplet and project it to the singlet under cyclic permutations. Each of the conformal primaries lying in this tensor product space is identified via the AdS/CFT correspondence to a field in AdS_5 as mentioned above. Henceforth, for notational convenience, we define the $(p-1)$-th Regge trajectory to be the set of all such fields.

The issue of tensor products and decompositions can be conveniently analyzed in terms of the $so(2,4)$ character. The cyclicity is required by the property of trace operation, and handling the cyclic projection is equivalent to the combinatorial problem of counting the number of different necklaces with p beads where each bead corresponds to a state — either primary or descendant — of $\mathcal{N}=4$ Maxwell multiplet. Polya's enumeration theorem solves this problem by making use of the cyclic index.

Partition function of $\mathcal{N}=4$ Maxwell multiplet. The cyclic index can be used to account the tensor products of $so(2,4)$ representation $\mathcal{H}$, if the index variables are replaced by $\mathrm{Tr}_{\mathcal{H}}(g^k)$ with $g \in \mathrm{SO}(2,4)$. In this way, the cyclic index of a bosonic system gives nothing but the $so(2,4)$ character. For more general systems with fermionic degrees of freedom, we should consider the weighted partition function [13, 53],

$$\mathcal{Z}_{\mathcal{H}}(q, x_1, x_2) = \mathrm{Tr}_{\mathcal{H}}\left((-1)^F\, q^{M_{05}}\, x_1^{M_{12}}\, x_2^{M_{34}}\right), \tag{2.3}$$

where M_{ab} are the generators of $so(2,4)$ and F is the fermion number. Even though we refer to this as partition function following previous references, it should be distinguished from the exponential of the one-loop free energy that we shall calculate in the next section. It can be equally viewed as a generalized Witten index. For an irreducible representation of $so(2,4)$, it is proportional to the character up to a sign factor:

$$\mathcal{Z}_{\mathcal{D}(\Delta,[j_+,j_-])} = (-1)^{2(j_++j_-)}\, \chi_{\mathcal{D}(\Delta,[j_+,j_-])}\,, \tag{2.4}$$

and for the $\mathcal{N}=4$ Maxwell multiplet, it is given by

$$\mathcal{Z}_{\mathcal{N}=4} = \chi_1 - 4\,\chi_{\frac{1}{2}} + 6\,\chi_0\,. \tag{2.5}$$

Here, χ_s are the characters of massless spin s representations given in [54]

$$\begin{aligned}
\chi_1(\beta,\alpha_1,\alpha_2) &= \chi_{\mathcal{D}(2,[1,0])}(\beta,\alpha_1,\alpha_2) + \chi_{\mathcal{D}(2,[0,1])}(\beta,\alpha_1,\alpha_2) \\
&= \frac{e^{-\beta}\,(\cosh\beta - \cos\alpha_1 - \cos\alpha_2) + \cos\alpha_1\,\cos\alpha_2}{(\cos\alpha_1 - \cosh\beta)(\cos\alpha_2 - \cosh\beta)}\,, \qquad (2.6)\\
\chi_{\frac{1}{2}}(\beta,\alpha_1,\alpha_2) &= \chi_{\mathcal{D}\left(\frac{3}{2},[\frac{1}{2},0]\right)}(\beta,\alpha_1,\alpha_2) + \chi_{\mathcal{D}\left(\frac{3}{2},[0,\frac{1}{2}]\right)}(\beta,\alpha_1,\alpha_2) \\
&= \frac{2\,\cos\frac{\alpha_1}{2}\,\cos\frac{\alpha_2}{2}\,\sinh\frac{\beta}{2}}{(\cos\alpha_1 - \cosh\beta)(\cos\alpha_2 - \cosh\beta)}\,, \qquad (2.7)\\
\chi_0(\beta,\alpha_1,\alpha_2) &= \chi_{\mathcal{D}(1,[0,0])}(\beta,\alpha_1,\alpha_2) = \frac{\sinh\beta}{2(\cos\alpha_1 - \cosh\beta)(\cos\alpha_2 - \cosh\beta)}\,. \qquad (2.8)
\end{aligned}$$

We thus obtain

$$\begin{aligned}
&\mathcal{Z}_{\mathcal{N}=4}(\beta,\alpha_1,\alpha_2) \qquad (2.9)\\
&= \frac{e^{-\beta}(\cosh\beta - \cos\alpha_1 - \cos\alpha_2) + \cos\alpha_1\cos\alpha_2 - 8\,\cos\frac{\alpha_1}{2}\,\cos\frac{\alpha_2}{2}\,\sinh\frac{\beta}{2} + 3\,\sinh\beta}{(\cos\alpha_1 - \cosh\beta)(\cos\alpha_2 - \cosh\beta)}\,,
\end{aligned}$$

where the variables β,α_1,α_2 are related to the ones in (2.3) by $q = e^{-\beta}$, $x_1 = e^{i\,\alpha_1}$ and $x_2 = e^{i\,\alpha_2}$.

Partition function of the $(p-1)$-th Regge trajectory of tensionless type IIB string. As mentioned before, the partition function (2.3) takes the fermionic statistics properly into account in making cyclic projection [13, 53]. This quantity is computed over the cyclic tensor product of $\mathcal{N}=4$ Maxwell multiplet is given by

$$\begin{aligned}
\mathcal{Z}^{\mathrm{cyc}(p)}_{\mathcal{N}=4}(q,x_1,x_2) &= \frac{1}{p}\sum_{k|p}\varphi(k)\left(\mathcal{Z}_{\mathcal{N}=4}(q^k,x_1^k,x_2^k)\right)^{\frac{p}{k}} \\
&= \mathcal{Z}_{\mathrm{IIB}(p)}(q,x_1,x_2) = \sum_{\Delta,j_+,j_-} N^{\mathrm{IIB}(p)}_{\mathcal{D}(\Delta,[j_+,j_-])}\,\mathcal{Z}_{\mathcal{D}(\Delta,[j_+,j_-])}(q,x_1,x_2)\,. \qquad (2.10)
\end{aligned}$$

Here, $\varphi(k)$ is the Euler totient function, which counts the number of relative primes of k in $\{1,2,\ldots,k\}$, and $N^{\mathrm{IIB}(p)}_{\mathcal{D}(\Delta,[j_+,j_-])}$ is the number of the $\mathcal{D}(\Delta,[j_+,j_-])$ representation in the $(p-1)$-th Regge trajectory of the type IIB string theory in the tensionless limit.

3 One-loop free energy of tensionless type IIB string

The one-loop free energy of type IIB string in $\mathrm{AdS}_5\times S^5$ background is simply the sum of the individual free energies for all string states. Therefore, as a matter of principle, it is possible to obtain the full quantity in the tensionless limit since the information about the exact multiplicities $N^{\mathrm{IIB}(p)}_{\mathcal{D}(\Delta,[j_+,j_-])}$ is encoded in the partition function (2.10). However, in practice, it is *not* possible to extract analytic expressions for all these multiplicities, needless to mention about the eventual resummation.

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3.1 Character Integral Representation for Zeta function

To overcome this problem, a new method of computing one-loop free energy has been devised in [31]. This method, referred to as Character Integral Representation of Zeta function (CIRZ), makes it possible to obtain the spectral zeta function associated to the one-loop free energy as a certain integral of the $so(2,4)$ character. In this way, we can handle the contributions of all single-particle states without identifying their actual content. We refer [31] for the detailed derivation of this method. In our previous works [30–32], we have considered only bosonic systems, hence the method involved the $so(2,4)$ character. In order to generalize the method to incorporate fermionic states, it is sufficient to replace the $so(2,4)$ character by the partition function (2.3) since the one-loop free energies of bosons and fermions in odd-dimensional AdS are given in a completely analogous form but only with an overall minus sign factor. In below, we summarize the result of the CIRZ method.

AdS$_5$ with S^4 boundary. The one-loop free energy of a spectrum $\mathcal{H}$ in AdS$_5$ with S^4 boundary is

$$\Gamma_{\mathcal{H}}^{(1)\,\mathrm{ren}}(\mathrm{AdS}_5) = \log R \left(\gamma_{\mathcal{H}|2} + \gamma_{\mathcal{H}|1} + \gamma_{\mathcal{H}|0}\right). \tag{3.1}$$

Here, R is the IR cut-off for the radial coordinate in AdS$_5$ [55] and $\gamma_{\mathcal{H}|n}$ are given by

$$\int_0^\infty d\beta \, \frac{\left(\frac{\beta}{2}\right)^{2(z-1-n)}}{\Gamma(z-n)} \, f_{\mathcal{H}|n}(\beta) = -2\,\gamma_{\mathcal{H}|n} + \mathcal{O}(z)\,, \tag{3.2}$$

where $f_{\mathcal{H}|n}(\beta)$ are determined by the partition functions $\mathcal{Z}_{\mathcal{H}}$ as

$$\begin{aligned}
f_{\mathcal{H}|2}(\beta) &= \frac{\sinh^4\frac{\beta}{2}}{2}\,\mathcal{Z}_{\mathcal{H}}(\beta,0,0)\,,\\
f_{\mathcal{H}|1}(\beta) &= \sinh^2\frac{\beta}{2}\left[\frac{\sinh^2\frac{\beta}{2}}{3} - 1 - \sinh^2\frac{\beta}{2}\left(\partial_{\alpha_1}^2 + \partial_{\alpha_2}^2\right)\right]\mathcal{Z}_{\mathcal{H}}(\beta,\alpha_1,\alpha_2)\Big|_{\alpha_i=0}\,,\\
f_{\mathcal{H}|0}(\beta) &= \left[1 + \frac{\sinh^2\frac{\beta}{2}\left(3-\sinh^2\frac{\beta}{2}\right)}{3}\left(\partial_{\alpha_1}^2+\partial_{\alpha_2}^2\right)\right.\\
&\qquad \left. -\frac{\sinh^4\frac{\beta}{2}}{3}\left(\partial_{\alpha_1}^4 - 12\,\partial_{\alpha_1}^2\partial_{\alpha_2}^2 + \partial_{\alpha_2}^4\right)\right]\mathcal{Z}_{\mathcal{H}}(\beta,\alpha_1,\alpha_2)\Big|_{\alpha_i=0}\,.
\end{aligned} \tag{3.3}$$

The variables $\beta, \alpha_1, \alpha_2$ are related to the ones in (2.3) by $q = e^{-\beta}$, $x_1 = e^{i\,\alpha_1}$ and $x_2 = e^{i\,\alpha_2}$. When the functions $f_{\mathcal{H}|n}$ do not have any singularity apart from poles at origin, we can deform the integral (3.2) to a contour one and obtain

$$\gamma_{\mathcal{H}|n} = -\,(-4)^n\,n!\oint \frac{d\beta}{2\pi i}\,\frac{f_{\mathcal{H}|n}(\beta)}{\beta^{2(n+1)}}\,, \tag{3.4}$$

where the contour encircles the origin in the anticlockwise direction.

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Thermal AdS_5 with $S^1 \times S^3$ boundary. When the background is the thermal AdS_5^{β} with $S^1_{\beta} \times S^3$ boundary (β is the radius of S^1) the one-loop free energy reads

$$\Gamma_{\mathcal{H}}^{(1)\,\mathrm{ren}}(\mathrm{AdS}_5^{\beta}) = \beta\, \mathcal{E}_{\mathcal{H}} + \mathcal{O}(\beta^0)\,, \tag{3.5}$$

where $\mathcal{E}_{\mathcal{H}}$ is the Casimir energy given by

$$\int_0^{\infty} \frac{d\beta\, \beta^{z-1}}{\Gamma(z)}\, \mathcal{Z}_{\mathcal{H}}(\beta, 0, 0) = 2\, \mathcal{E}_{\mathcal{H}} + \mathcal{O}(z+1)\,. \tag{3.6}$$

If $\mathcal{Z}_{\mathcal{H}}(\beta, 0, 0)$ does not have any singularity apart from poles at the origin, we can deform the integral such that we obtain the Casimir energy as

$$\mathcal{E}_{\mathcal{H}} = -\frac{1}{2} \oint \frac{d\beta}{2\,\pi\, i\, \beta^2}\, \mathcal{Z}_{\mathcal{H}}(\beta, 0, 0)\,, \tag{3.7}$$

where the contour is again an anticlockwise circle around the origin.

We now compute the one-loop free energies (3.1) and (3.5) for the tensionless string. To begin with, we organize the string states according to the level of Regge trajectories and compute the one-loop free energy at each trajectory.

3.2 One-loop free energy of the $(p-1)$-th Regge trajectory

The partition function of the $(p-1)$-th Regge trajectory is

$$\mathcal{Z}_{\mathrm{IIB}(p)}(\beta, \alpha_1, \alpha_2) = \frac{1}{p} \sum_{k|p} \varphi(k)\, [\mathcal{Z}_{\mathcal{N}=4}(k\,\beta, k\,\alpha_1, k\,\alpha_2)]^{\frac{p}{k}}\,, \tag{3.8}$$

where the partition function of $\mathcal{N}=4$ Maxwell multiplet has the form of

$$\begin{aligned} &\mathcal{Z}_{\mathcal{N}=4}(\beta, \alpha_1, \alpha_2) = \\ &= \frac{e^{-\beta}(\cosh\beta - \cos\alpha_1 - \cos\alpha_2) + \cos\alpha_1 \cos\alpha_2 - 8\, \cos\frac{\alpha_1}{2}\, \cos\frac{\alpha_2}{2}\, \sinh\frac{\beta}{2} + 3\, \sinh\beta}{(\cos\alpha_1 - \cosh\beta)(\cos\alpha_2 - \cosh\beta)}\,. \end{aligned} \tag{3.9}$$

We first note that both one-loop free energies (3.1) and (3.5) of the $(p-1)$-th Regge trajectory are given by a few series coefficients of $\mathcal{Z}_{\mathrm{IIB}(p)}(\beta, 0, 0)$ or $f_{\mathrm{IIB}(p)|n}(\beta)$ because the latter do not have any singularity near origin hence the residue theorem can be applied to the contour integrals (3.4) and (3.7). Since the latter is a bit more involved, let us begin with the former case, that is, the Casimir energy. We first consider the series expansion of the partition function $\mathcal{Z}_{\mathcal{N}=4}(\beta, 0, 0)$:

$$\mathcal{Z}_{\mathcal{N}=4}(\beta, 0, 0) = \frac{2 + 6\, e^{\frac{\beta}{2}}}{(1 + e^{\frac{\beta}{2}})^3} = 1 + a\,\beta + \mathcal{O}(\beta^3)\,, \qquad a = -\frac{3}{8}\,. \tag{3.10}$$

It is worth to remark the speciality of the above expression: the partition function of a generic multiplet consisting of n_1 spin 1, $n_{\frac{1}{2}}$ spin 1/2 and n_0 spin 0 has the series expansion,

$$\begin{aligned} &\mathcal{Z}_{\{n_1, n_{\frac{1}{2}}, n_0\}}(\beta, 0, 0) = \\ &= \frac{2\, n_0 - 4\, n_{\frac{1}{2}} + 4\, n_1}{\beta^3} + \frac{n_{\frac{1}{2}} - 4\, n_1}{2\,\beta} + n_1 - \frac{4\, n_0 + 17\, n_{\frac{1}{2}} + 88\, n_1}{480}\, \beta + \mathcal{O}(\beta^3)\,, \end{aligned} \tag{3.11}$$

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hence involves the β^{-3} and β^{-1} terms. Note that the β^{-3} term vanishes in any supersymmetric theory, but its physical meaning is not clear to the authors. When the SUSY is the maximal $\mathcal{N}=4$ one, not only the β^{-3} term but also the β^{-1} term (hence all negative powers) vanish. In this case, we find a drastic simplification in the expansion formula for $[\mathcal{Z}_{\mathcal{N}=4}(k\,\beta,0,0)]^{\frac{p}{k}}$:

$$[\mathcal{Z}_{\mathcal{N}=4}(k\,\beta,0,0)]^{\frac{p}{k}} = 1 - \frac{3}{8}\,p\,\beta + \mathcal{O}(\beta^2)\,, \tag{3.12}$$

which appear in the partition function of the cyclic tensor product (3.8). Moreover, since (3.12) is independent of k, we can perform the summation over k in (3.8) by using the identity,

$$\frac{1}{p}\sum_{k|p}\varphi(k) = 1\,. \tag{3.13}$$

Finally, we find that the series expansion of $\mathcal{Z}_{\text{IIB}(p)}(\beta,0,0)$ itself is given by the right hand side of (3.12). Hence, the thermal AdS_5 one-loop free energy is

$$\Gamma^{(1)\,\text{ren}}_{\text{IIB}(p)}(\text{AdS}_5^{\beta}\times S^5) = -\frac{1}{2}\,p\,a\,\beta + \mathcal{O}(\beta^0) = \frac{3\,p}{16}\,\beta + \mathcal{O}(\beta^0)\,, \tag{3.14}$$

for the tensionless string states in the $(p-1)$-th Regge trajectory. This result coincides with that of the $(p-1)$-th KK level (1.3).

Let us now move on to the one-loop free energy in AdS_5 with S^4 boundary. In this case, the partition function $\mathcal{Z}_{\text{IIB}(p)}(\beta,\alpha_1,\alpha_2)$ enters in the computation through the functions $f_{\text{IIB}(p)|n}(\beta)$ defined in (3.3). From the residue theorem, the $\gamma_{\text{IIB}(p)|n}$ coefficient (3.4) of the one-loop free energy (3.1) depends only on the β^{2n+1} coefficient of $f_{\text{IIB}(p)|n}(\beta)$. However, since the latter combines $\mathcal{Z}_{\text{IIB}(p)}(\beta,\alpha_1,\alpha_2)$ with hyperbolic functions, one may expect that the one-loop free energy depends on higher series coefficients of the partition function $\mathcal{Z}_{\text{IIB}(p)}(\beta,\alpha_1,\alpha_2)$. However, as we shall show now, this kind of complications do not happen due to the speciality of the $\mathcal{N}=4$ partition function. About the series expansion in α_1 and α_2, only the terms up to $\mathcal{O}(\alpha^6)$ are relevant, hence for the building block $\mathcal{Z}_{\mathcal{N}=4}(\beta,\alpha_1,\alpha_2)$, it is sufficient to consider the series expansion to the same order:

$$\begin{aligned}\mathcal{Z}_{\mathcal{N}=4}(\beta,\alpha_1,\alpha_2) = {}& \frac{2+6\,e^{\frac{\beta}{2}}}{(1+e^{\frac{\beta}{2}})^3} + \frac{\sinh^4\frac{\beta}{4}}{\sinh^5\frac{\beta}{2}}\left(\alpha_1^2+\alpha_2^2\right) - \frac{3+\cosh\beta}{2048\,\sinh^3\frac{\beta}{4}\,\cosh^7\frac{\beta}{4}}\,\alpha_1^2\,\alpha_2^2 \\ & + \frac{5-\cosh\frac{\beta}{2}}{3072\,\sinh\frac{\beta}{4}\,\cosh^7\frac{\beta}{4}}\left(\alpha_1^4+\alpha_2^4\right) + \mathcal{O}(\alpha^6)\,.\end{aligned} \tag{3.15}$$

We can expand the above in β to get

$$\begin{aligned}\mathcal{Z}_{\mathcal{N}=4}(\beta,\alpha_1,\alpha_2) = {}& 1 + a\,\beta + \mathcal{O}(\beta^3) + \left(b\,\beta+\mathcal{O}(\beta^3)\right)\frac{\alpha_1^2+\alpha_2^2}{\beta^2} \\ & + \left(c\,\beta+\mathcal{O}(\beta^3)\right)\frac{\alpha_1^2\,\alpha_2^2}{\beta^4} + \mathcal{O}(\beta^3)\,\frac{\alpha_1^4+\alpha_2^4}{\beta^4} + \mathcal{O}(\alpha^6)\,,\end{aligned} \tag{3.16}$$

where a, b, c are constants given by

$$a = -\frac{3}{8}\,, \qquad b = \frac{1}{8}\,, \qquad c = -\frac{1}{8}\,. \tag{3.17}$$

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Again the absence of negative β powers in $\mathcal{Z}_{\mathcal{N}=4}(\beta,0,0)$ allows us to obtain a compact series expansion formula for $[\mathcal{Z}_{\mathcal{N}=4}(k\,\beta,k\,\alpha_1,k\,\alpha_2)]^{\frac{p}{k}}$:

$$[\mathcal{Z}_{\mathcal{N}=4}(k\,\beta,k\,\alpha_1,k\,\alpha_2)]^{\frac{p}{k}} = 1 + p\,a\,\beta + \mathcal{O}(\beta^2) + \big(p\,b\,\beta + \mathcal{O}(\beta^2)\big)\,\frac{\alpha_1^2+\alpha_2^2}{\beta^2} + \big(p\,c\,\beta + \mathcal{O}(\beta^2)\big)\,\frac{\alpha_1^2\,\alpha_2^2}{\beta^4} + \mathcal{O}(\beta^2)\,\frac{\alpha_1^4+\alpha_2^4}{\beta^4} + \mathcal{O}(\alpha^6)\,. \tag{3.18}$$

Moreover, the k dependence disappears again in the above expression and the summation over k in $\mathcal{Z}_{\text{IIB}(p)}(\beta,\alpha_1,\alpha_2)$ (3.8) can be performed with the identity (3.13). As a result, the partition function $\mathcal{Z}_{\text{IIB}(p)}(\beta,\alpha_1,\alpha_2)$ has exactly the same series expansion as the right hand side of (3.18). Even though $f_{\text{IIB}(p)|n}(\beta)$ involves a few hyperbolic functions, eventually the coefficients $\gamma_{\text{IIB}(p)|n}$ are all determined from the series coefficients of $\mathcal{Z}_{\text{IIB}(p)}$ that we have just identified. In the end, we obtain

$$\gamma_{\text{IIB}(p)|2} = -p\,a\,, \qquad \gamma_{\text{IIB}(p)|1} = -p\,(a+b)\,, \qquad \gamma_{\text{IIB}(p)|0} = -p\,(a+b+c)\,, \tag{3.19}$$

and combining these, the AdS_5 one-loop free energy is obtained as

$$\Gamma^{(1)\,\text{ren}}_{\text{IIB}(p)}(\text{AdS}_5\times S^5) = -p\,(3\,a+2\,b+c)\,\log R = p\,\log R\,, \tag{3.20}$$

for the tensionless string states in the $(p-1)$-th Regge trajectory. Again, this result coincides with that of the $(p-1)$-th KK level (1.3).

3.3 Full one-loop free energy

In the previous section, we have calculated the one-loop free energies $\Gamma^{(1)\,\text{ren}}_{\text{IIB}(p)}$ of the tensionless type IIB string in a given Regge trajectory. The full one-loop free energy is their sum over trajectories,

$$\Gamma^{(1)\,\text{ren}}_{\text{IIB}} = \sum_{p=2}^{\infty}\Gamma^{(1)\,\text{ren}}_{\text{IIB}(p)}\,. \tag{3.21}$$

With the results (3.20) and (3.14), the above gives (1.4) with the divergent series $A = \sum_{p=2}^{\infty} p$. In order to reproduce the physically desirable result, there should exist an appropriate regularization scheme giving rise to

$$\sum_{p=1}^{\infty} p = 0\,. \tag{3.22}$$

In [8], it has been shown that the standard zeta function regularization is compatible with the insertion of $e^{-\epsilon\,M_{05}}$ inside the trace. With the latter regularization, if we take the contribution of the entire KK towers of 10d supergravity keeping the regularization parameter ϵ finite and send ϵ to zero in the final stage, then we recover the result which is consistent with the scheme (3.22). Here, the key idea is that the extraction of the finite part from the regulator dependent quantity should be done after the summation over KK levels. Coming back to the standard zeta function regularization which the current paper is relying on, we can also attempt to extract the finite part of the result after performing

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the summation over the entire Regge trajectories. In our method of CIRZ, this would correspond to consider the full partition function,

$$\mathcal{Z}_{\text{IIB}} = \sum_{p=2}^{\infty} \mathcal{Z}_{\text{IIB}(p)} \,, \tag{3.23}$$

before performing various integrals. In fact, by using the re-summation,

$$\sum_{p=1}^{\infty} \sum_{k|p} \Big(\cdots \Big) = \sum_{k=1}^{\infty} \sum_{n=1}^{\infty} \Big(\cdots \Big) \qquad [p = k\, n] \,, \tag{3.24}$$

we can consider the partition function $\mathcal{Z}_{\text{IIB}}$ as a different series [52],

$$\mathcal{Z}_{\text{IIB}}(\beta, \alpha_1, \alpha_2) = -\mathcal{Z}_{\mathcal{N}=4}(\beta, \alpha_1, \alpha_2) - \sum_{k=1}^{\infty} \frac{\varphi(k)}{k} \log[1 - \mathcal{Z}_{\mathcal{N}=4}(k\,\beta, k\,\alpha_1, k\,\alpha_2)] \,, \tag{3.25}$$

as a result of the summation over n, which can be viewed as an effective sum over the Regge trajectories.

Now let us apply the CIRZ method to the formula (3.25). We know already the contribution of $-\mathcal{Z}_{\mathcal{N}=4}$ to $\Gamma^{(1)\,\text{ren}}_{\text{IIB}}$ hence move to the series part. Dealing with the summation over k is technically prohibitive, but since the summand with fixed k contains the contributions from the entire Regge trajectories, this alternative organization of spectrum (3.25) may have a better chance to provide a convergent series for one-loop free energy.[2] With this idea in mind, we proceed to the computation for each summand with help of CIRZ method. Again, all the relevant information is again encoded in the series expansion of the summand,

$$\begin{aligned} \log[1-\mathcal{Z}_{\mathcal{N}=4}(k\beta, k\alpha_1, k\alpha_2)] = \log\frac{3}{8} + \log(k\beta) + \mathcal{O}(\beta^2) + \left(-\frac{1}{3} + \mathcal{O}(\beta^2)\right) \frac{\alpha_1^2 + \alpha_2^2}{\beta^2} \\ + \left(\frac{2}{9} + \mathcal{O}(\beta^2)\right) \frac{\alpha_1^2\,\alpha_2^2}{\beta^4} + \left(-\frac{1}{18} + \mathcal{O}(\beta^2)\right) \frac{\alpha_1^4 + \alpha_2^4}{\beta^4} + \mathcal{O}(\alpha^4) \,. \end{aligned} \tag{3.26}$$

Following the contour integral prescription and taking the β^{2n+1} coefficient from the function $f_{\mathcal{H}|n}$, we can check that the above, that is the summand in (3.25) with fixed k, does not give any contribution to the one-loop free energy. Therefore in this prescription, the full one-loop free energy is given by minus times that of $\mathcal{N} = 4$ Maxwell multiplet:

$$\Gamma^{(1)\,\text{ren}}_{\text{IIB}} = -\Gamma^{(1)\,\text{ren}}_{\mathcal{N}=4} = -F^{\text{U}(1)}_{\mathcal{N}=4} \,, \tag{3.27}$$

which reproduces perfectly the -1 of the factor $N^2 - 1$ in (1.1).

[2] Note however the reorganization (3.24) does not admit a precise physical interpretation. In fact, the integrand with fixed k cannot be considered as the partition function over a certain vector space. Hence, the role of symmetries (conformal symmetry or supersymmetry) on a fixed k is not clear.

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Branch cut contribution. One may wonder why we could simply neglect the $\log(k\,\beta)$ term even though its branch cut prevents the use of the contour integral description in the CIRZ method. In the following, we shall prove that the $\log(k\,\beta)$ term is actually irrelevant. To show that, we first subtract

$$\mathcal{Z}_{\text{mod},k}(\beta) = \log[\tanh(k\,\beta)]\,, \tag{3.28}$$

from $\log[1-\mathcal{Z}_{\mathcal{N}=4}(k\,\beta, k\,\alpha_1, k\,\alpha_2)]$ in order to remove the $\log(k\,\beta)$ term in the expansion (3.26). To compensate what is subtracted, we have to put back the contribution of $\mathcal{Z}_{\text{mod},k}(\beta)$. Since the latter is a relatively simple function, one can explicitly check that it does not contribute to the full one-loop free energy.

Let us consider first the one-loop free energy in thermal AdS_5 background with $S^1_\beta \times S^3$ boundary. Due to the branch cut of the logarithm in (3.28), we need to consider the real line integral representation (3.6),

$$\tilde{\chi}_{\text{mod},k}(z) = \int_0^\infty \frac{d\beta\,\beta^{z-1}}{\Gamma(z)}\,\log[\tanh(k\,\beta)]\,, \tag{3.29}$$

instead of the contour one. The one-loop free energy gets the contribution of $\frac{1}{2}\,\tilde{\chi}_{\text{mod},k}(-1)\,\beta$ but since

$$\tilde{\chi}_{\text{mod},k}(z) = -(4\,k)^{-z}\,\zeta\left(z+1,\frac{1}{2}\right) = \mathcal{O}(z+1)\,, \tag{3.30}$$

this does not affect the result of the one-loop free energy.

Moving now to the case of the AdS_5 background with S^4 boundary, we first find that $\mathcal{Z}_{\text{mod},k}$ does not depend on α_1, α_2, hence the functions $f_{\text{mod},k|n}$ have simple form,

$$\begin{aligned} f_{\text{mod},k|2}(\beta) &= \frac{\sinh^4\frac{\beta}{2}}{2}\,\log[\tanh(k\,\beta)]\,,\\ f_{\text{mod},k|1}(\beta) &= \sinh^2\frac{\beta}{2}\left(\frac{\sinh^2\frac{\beta}{2}}{3}-1\right)\log[\tanh(k\,\beta)]\,,\\ f_{\text{mod},k|0}(\beta) &= \log[\tanh(k\,\beta)]\,. \end{aligned} \tag{3.31}$$

In order to extract the coefficients $\gamma_{\text{mod},k|n}$ from the above, we need to consider the real line integral (3.2) because the branch cut of the above functions prevent us to use the contour integral (3.4). They can be recast in terms of

$$I_k(z,a) = \int_0^\infty d\beta\,\frac{\left(\frac{\beta}{2}\right)^{2(z-1)}}{\Gamma(z)}\,e^{-a\,\beta}\,\log[\tanh(k\,\beta)]\,, \tag{3.32}$$

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as follows:

$$\int_0^\infty d\beta\, \frac{\left(\frac{\beta}{2}\right)^{2(z-1)}}{\Gamma(z)}\, f_{\mathrm{mod},k|0}(\beta) = I_k(z,0)\,, \tag{3.33}$$

$$\int_0^\infty d\beta\, \frac{\left(\frac{\beta}{2}\right)^{2(z-2)}}{\Gamma(z-1)}\, f_{\mathrm{mod},k|1}(\beta) =$$
$$= \frac{5}{8}\, I_k(z-1,0) - \frac{I_k(z-1,1)+I_k(z-1,-1)}{3} + \frac{I_k(z-1,2)+I_k(z-1,-2)}{48}\,, \tag{3.34}$$

$$\int_0^\infty d\beta\, \frac{\left(\frac{\beta}{2}\right)^{2(z-3)}}{\Gamma(z-2)}\, f_{\mathrm{mod},k|2}(\beta) =$$
$$= \frac{3}{16}\, I_k(z-2,0) - \frac{I_k(z-2,1)+I_k(z-2,-1)}{8} + \frac{I_k(z-2,2)+I_k(z-2,-2)}{32}\,. \tag{3.35}$$

The integral $I_k(z,a)$ is divergent for $a<-2k$ in the large β region. This can be interpreted as IR divergence and can be regularized by analytic continuation in a. We can evaluate $I_k(z,a)$ by series expanding the $\log[\tanh(k\,\beta)]$ term in $e^{-2k\,\beta}$ as

$$I_k(z,a) = -\frac{4^{2-3z}\,k^{1-2z}}{\Gamma(z)} \sum_{m=0}^{\infty} \frac{\Gamma(2z+m-1)\,\zeta(2z+m,\frac{1}{2})}{m!} \left(-\frac{a}{4\,k}\right)^m\,. \tag{3.36}$$

Series expanding the above around $z=0,-1,-2$, we find

$$I_k(z,a) = \left(\frac{1}{z}+\gamma-2\,\log(2\,k)\right) a + \mathcal{O}(z)\,, \tag{3.37}$$

$$I_k(z-1,a) = -\frac{8\,k^2}{3}\,a - \frac{2}{3}\left(\frac{1}{z}+\gamma-2\,\log(2\,k)-1\right) a^3 + \mathcal{O}(z)\,, \tag{3.38}$$

$$I_k(z-2,a) = \frac{224\,k^4}{45}\,a + \frac{32\,k^2}{9}\,a^3 + \frac{4}{15}\left(\frac{1}{z}+\gamma-2\,\log(2\,k)-\frac{3}{2}\right) a^5 + \mathcal{O}(z)\,, \tag{3.39}$$

where γ is the Euler-Mascheroni constant. Since the above are all odd functions in a, the integrals (3.33), (3.34) and (3.35) all vanish up to $\mathcal{O}(z)$ terms. Therefore, we find that the modification part $\mathcal{Z}_{\mathrm{mod},k}(\beta)$ (3.28) does not give any contribution to the one-loop free energy in AdS_5 with S^4 boundary and the result (3.27) still stands.

4 Conclusion

In this paper, we have calculated the one-loop free energy of tensionless type-IIB string theory in $AdS_5\times S^5$ background by making use of the CIRZ method and the partition function of the boundary $\mathcal{N}=4$ theory whose 't Hooft coupling vanishes. For a fixed but arbitrary Regge trajectory, we could analytically calculate the one-loop free energy thanks to the special form of the partition function of the $\mathcal{N}=4$ Maxwell multiplet. At the first place, we showed that the result is proportional to the trajectory number, hence the full quantity leads to a linearly divergent series. For a proper treatment of this divergence, we

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changed the organization of string states such that we can first perform the effective trajectory sum before sending the regularization parameter to zero. In this way, we obtained a finite result which is consistent with the prediction of the holographic conjecture.

We finally note that the simplifications observed in this paper for free energy computations for the AdS_5 superstring are somewhat in contrast to the generic situation in higher-spin theory where supersymmetry is not often useful. Indeed even in resolving the tensionless limit of string theory, supersymmetry has been of limited use. Nonetheless, the computations here are analytically possible only when the CFT spectrum coincides with the $\mathcal{N} = 4$ Maxwell multiplet, thanks to the cancellation of the negative β powers in (3.11). It would be interesting to understand this occurrence better as it might shed new light on the interplay between higher-spin symmetry and supersymmetry.

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