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# Supersymmetric Rényi entropy and Weyl anomalies in six-dimensional (2,0) theories

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Abstract: We propose a closed formula of the universal part of supersymmetric Rényi entropy $S_q$ for $(2,0)$ superconformal theories in six-dimensions. We show that $S_q$ across a spherical entangling surface is a cubic polynomial of $\gamma := 1/q$, with all coefficients expressed in terms of the newly discovered Weyl anomalies $a$ and $c$. This is equivalent to a similar statement of the supersymmetric free energy on conic (or squashed) six-sphere. We first obtain the closed formula by promoting the free tensor multiplet result and then provide an independent derivation by assuming that $S_q$ can be written as a linear combination of 't Hooft anomaly coefficients. We discuss a possible lower bound $\frac{a}{c} \geq \frac{3}{7}$ implied by our result.

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## Contents



# 1 Introduction

Exact results in interacting quantum field theories are rare. Even less is known about the six-dimensional $(2,0)$ theories, although they are the local conformal field theories (CFTs) with maximal supersymmetry in the maximum number of dimensions [1, 2], which actually play important roles in understanding lower dimensional supersymmetric physics [3–7].

The main obstacle is that the proper formulation of the interacting theories is still lacking, for instance in the path integral formalism.[1] This also makes it challenging to study the theories in curved spaces. In particular it is unclear how to perform the supersymmetric localization [18–20] directly.

Recently alternative approaches to $6d$ $(2,0)$ theories, such as effective actions on the moduli space and the superconformal bootstrap, are advocated in [21, 22] and in [23, 24], respectively. In particular, the Weyl anomaly coefficients $a_{\mathfrak{g}}$ and $c_{\mathfrak{g}}$ have been determined for the $(2,0)$ superconformal field theory (SCFT) characterized by a Lie algebra $\mathfrak{g}$,[2]

$$\bar{a}_{\mathfrak{g}} := \frac{a_{\mathfrak{g}}}{a_{\mathfrak{u}(1)}} = \frac{16}{7} h^{\vee}_{\mathfrak{g}} d_{\mathfrak{g}} + r_{\mathfrak{g}}\,, \quad \bar{c}_{\mathfrak{g}} := \frac{c_{\mathfrak{g}}}{c_{\mathfrak{u}(1)}} = 4 h^{\vee}_{\mathfrak{g}} d_{\mathfrak{g}} + r_{\mathfrak{g}}\,, \tag{1.1}$$

where $r_{\mathfrak{g}}$, $d_{\mathfrak{g}}$ and $h^{\vee}_{\mathfrak{g}}$ are the rank, dimension and dual Coxeter number of the compact simply-laced Lie algebra $\mathfrak{g}$, respectively. $a$ and $c$ appear generally as coefficients of the anomalous trace of the stress tensor in a six-dimensional curved background [25, 26],

$$\langle T^{\mu}_{\mu} \rangle \sim a\, E_6 + \sum_{i=1}^{3} c_i\, I_i\,, \tag{1.2}$$

where $E_6$ is the Euler density while $I_i$ are Weyl invariants. In the presence of $(2,0)$ superconformal symmetry, $c_{i=1,2,3}$ are proportional to a single coefficient $c$. One interesting fact is that both $\bar{a}_{\mathfrak{g}}$ and $\bar{c}_{\mathfrak{g}}$ will be uniquely fixed once we assume that they are linear combinations of the 't Hooft anomaly coefficients, $h^{\vee}_{\mathfrak{g}} d_{\mathfrak{g}}$ and $r_{\mathfrak{g}}$. This can be done by fitting to the large $N$ values (from holography [27–30]) and the free tensor multiplet values [31, 32].

As robust observables, the 't Hooft anomalies of the continuous global symmetries in $6d$ $(2,0)$ theories have been worked out [33–39]. They are organized in an 8-form anomaly polynomial,

$$\mathcal{I}_8 = h^{\vee}_{\mathfrak{g}} d_{\mathfrak{g}}\, \frac{p_2(R)}{24} + r_{\mathfrak{g}}\, \mathcal{I}_{\mathfrak{u}(1)}\,, \tag{1.3}$$

where $p_2(R)$ is the second Pontryagin class of the field strength of the SO(5) R-symmetry background and $\mathcal{I}_{\mathfrak{u}(1)}$ is the anomaly polynomial of a free Abelian tensor multiplet.

As in other even dimensions, it is known that $a_{\mathfrak{g}}$ determines both the universal part[3] of the sphere partition function and the universal entanglement entropy associated with a spherical entangling surface (in flat space) [40]. On the other hand, it was pointed out that $c_{\mathfrak{g}}$ determines both the 2-point and the 3-point functions of the stress tensor in the vacuum in flat space [23, 24]. Due to the intrinsic relations between the flat space stress tensor correlators and the nearly-round sphere partition function, it is therefore attempting to ask whether one can fully determine the partition function on a branched ($q$-deformed) sphere,[4] which is directly related to the supersymmetric Rényi entropy $S_q$.

[1] For the attempts to write down a Lagrangian, see for instance [8–12] and for other field theoretical attempts, see [13–17].

[2] $\mathfrak{g} = \mathfrak{u}(1)$ corresponds to a free Abelian tensor multiplet.

[3] By "universal" we mean scheme-independent.

[4] A branched sphere is a sphere with a conical singularity with the deformation parameter $q - 1$.



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Supersymmetric Rényi entropy was first introduced in three-dimensions [41–43], and later studied in four-dimensions [44, 45, 47], five-dimensions [48, 49] and for free tensor multiplets in six-dimensions [50].[5] By turning on certain R-symmetry background fields (chemical potentials), one can calculate the partition function $Z_q$ on a $q$-branched sphere $\mathbb{S}^d_q$, and define the supersymmetric Rényi entropy as

$$S_q = \frac{1}{1-q}\left[\log Z_q(\mu(q)) - q\log Z_1(0)\right] , \tag{1.4}$$

which is a supersymmetric refinement of the ordinary Rényi entropy (which is non-supersymmetric because of the conical singularity).[6] The quantities defined in (1.4) are UV divergent in general but one can extract universal parts free of ambiguities. For instance, for $\mathcal{N}=4$ SYM in four-dimensions, the log coefficient of $S_q$ as a function of $q$ and three chemical potentials $\mu_1, \mu_2, \mu_3$ (corresponding to three independent R-symmetry Cartans) has been shown to be protected from the interactions [44]. It also receives a precise check from the holographic computation on the 5$d$ BPS STU topological black holes [44]. Furthermore, there are universal relations between the Weyl anomaly coefficients $a, c$ and the supersymmetric Rényi entropy in 4$d$ $\mathcal{N}=1,2$ SCFTs, which provides a new way to understand the Hofman-Maldacena bounds [45].[7] The above facts indicate that the supersymmetric Rényi entropy may be used as a new robust observable to understand SCFTs.

### 1.1 Summary of results

The main result in this paper is the exact supersymmetric Rényi entropy of 6$d$ (2, 0) SCFTs. We show that, for theories characterized by simply-laced Lie algebra $\mathfrak{g}$, it is given by a cubic polynomial of $\gamma := \frac{1}{q}$

$$S_\gamma[\mathfrak{g}] = \sum_{n=0}^{3} s_n(\gamma-1)^n , \tag{1.5}$$

with four coefficients

$$s_0 = \frac{7}{12}\,\bar{a}_\mathfrak{g} , \quad s_1 = \frac{1+2r_1r_2}{12}\,\bar{c}_\mathfrak{g} , \quad s_2 = \frac{r_1r_2}{12}\,\bar{c}_\mathfrak{g} , \quad s_3 = \frac{r_1^2r_2^2}{12}\,\frac{7\bar{a}_\mathfrak{g}-3\bar{c}_\mathfrak{g}}{4} , \tag{1.6}$$

where $\bar{a}_\mathfrak{g}, \bar{c}_\mathfrak{g}$ are given by (1.1) and $r_{1,2}$ are background parameters denoting the weights of the two U(1) chemical potentials associated to the two R-symmetry Cartans, satisfying the supersymmetry constraint $r_1 + r_2 = 1$.[8] The basic ingredients in our argument are the following:

(A) $S_\gamma$ has been computed for the free tensor multiplet by the author with collaborators [50]. The result takes the form (2.15)

$$S_\gamma[\mathfrak{u}(1)] = \frac{r_1^2r_2^2}{12}(\gamma-1)^3 + \frac{r_1r_2}{12}(\gamma-1)^2 + \frac{1+2r_1r_2}{12}(\gamma-1) + \frac{7}{12} . \tag{1.7}$$

This will be reviewed in section 2.

[5] The supersymmetric Rényi entropy was recently studied in two-dimensional (2, 2) SCFTs [51] in a slightly different way.

[6] For CFTs, the Rényi entropy (or supersymmetric one) associated with a spherical entangling surface in flat space can be mapped to that on a sphere. Throughout this work we take the "regularized cone" boundary conditions.

[7] Some of $a/c$ bounds by Hofman and Maldacena [46] coincide with Rényi entropy inequalities.

[8] We only consider non-negative weights of the chemical potentials, $r_1 \geq 0$ and $r_2 \geq 0$.



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(B) Based on (A) and (E)(F) below, a reasonable assumption is that the general form of supersymmetric Rényi entropy for $(2,0)$ theories is a cubic polynomial in $\gamma-1$. This assumption will be used in section 3. So far we do not have a sharp argument for this assumption.

(C) The first and second derivatives of $S_\gamma$ at $\gamma=1$ can be expressed in terms of integrated two- and three-point functions of operators in the stress tensor multiplet, and so are proportional to $c_{\mathfrak{g}}$. This will be demonstrated in appendix A. Moreover their dependence on $r_{1,2}$ is seen to be universal. Because of this, one has

$$\left.\frac{\partial_\gamma S_\gamma[\mathfrak{g}]}{\partial_\gamma S_\gamma[\mathfrak{u}(1)]}\right|_{\gamma=1}=\left.\frac{\partial_\gamma^2 S_\gamma[\mathfrak{g}]}{\partial_\gamma^2 S_\gamma[\mathfrak{u}(1)]}\right|_{\gamma=1}=\frac{c_{\mathfrak{g}}}{c_{\mathfrak{u}(1)}}\ . \tag{1.8}$$

This will be used in section 3.

(D) The value of $S_\gamma$ at $\gamma=1$ is the spherical entanglement entropy [41], which is proportional to $a_{\mathfrak{g}}$ [40]. As such, one has

$$\frac{S_{\gamma=1}[\mathfrak{g}]}{S_{\gamma=1}[\mathfrak{u}(1)]}=\frac{a_{\mathfrak{g}}}{a_{\mathfrak{u}(1)}}\ . \tag{1.9}$$

This will also be used in section 3.

(E) The large $\gamma$ behavior of the supersymmetric Rényi entropy is controlled by the "supersymmetric Casimir energy", which has been computed in [82].[9] This gives (4.26)

$$\lim_{\gamma\to\infty}\frac{S_\gamma[\mathfrak{g}]}{\gamma^3}=\frac{r_1^2r_2^2}{12}(r_{\mathfrak{g}}+h_{\mathfrak{g}}^\vee d_{\mathfrak{g}})\ . \tag{1.10}$$

This is our main result in section 4.

(F) $S_\gamma$ can be computed for the $A_{N-1}$ type $(2,0)$ theories at large $N$ using the AdS/CFT correspondence, with the result appearing in (5.19):

$$\frac{S_\gamma[A_{N\to\infty}]}{N^3}=\frac{r_1^2r_2^2}{12}(\gamma-1)^3+\frac{r_1r_2}{3}(\gamma-1)^2+\frac{1+2r_1r_2}{3}(\gamma-1)+\frac{4}{3}\ . \tag{1.11}$$

This is our main result in section 5.

From (A)(B)(C)(D)(E) listed above, one can uniquely determine the general expression of $S_\gamma[\mathfrak{g}]$ given in (1.5), (1.6), as we do in section 3. The precise agreement between (F) and the large $N$ limit of (1.5) for $A_{N-1}$ type theories can be considered as a nontrivial test of our result. Except for (A), (B)-(F) are new as far as we know.

One may notice that both our result (1.5) and $a,c$ anomalies (1.1) are linear combinations of $h_{\mathfrak{g}}^\vee d_{\mathfrak{g}}$ and $r_{\mathfrak{g}}$, which determine the anomaly polynomial (1.3). In fact, once such a relationship between the supersymmetric Rényi entropy and 't Hooft anomalies is assumed, (1.5) and (1.6) can be obtained by fitting to the known result of a free tensor multiplet [50] and the holographic result presented in section 5.

[9] Similar relation was first advertised in [45] in four-dimensions.

This paper is organized as follows. We begin with the review of the supersymmetric Rényi entropy of free tensor multiplets in section 2. Then we promote it to a general expression which works for general $(2,0)$ theories in section 3. In section 4 we demonstrate a relation between the $q \to 0$ behavior of supersymmetric Rényi entropy and supersymmetric Casimir energy, which is used to determine the remaining unfixed coefficient in the general formula in the previous section. We give a precise test of our result by comparing with the holographic results in section 5 and conjecture a lower bound for $\bar{a}/\bar{c}$ in section 6.

## 2 Review of abelian tensor multiplet

The six-dimensional $(2,0)$ superconformal algebra is $\mathfrak{osp}(8^*|4)$. While it is easy to identify a free Abelian tensor multiplet that realizes the $(2,0)$ superconformal symmetry, the existence of interacting $(2,0)$ theories was only inferred from decoupling limits of string constructions [62–64]. See for instance [65] for a review of various aspects of $6d$ $(2,0)$ theories.

Now we review the supersymmetric Rényi entropy of free tensor multiplets [50]. For free fields, the Rényi entropy associated with a spherical entangling surface in flat space can be computed by working on a hyperbolic space $\mathbb{S}^1_\beta \times \mathbb{H}^5$ and using heat kernel method.[10] A six-dimensional $(2,0)$ tensor multiplet includes 5 real scalars, 2 Weyl fermions and a 2-form field with self-dual strength. The 2-form field with self-dual strength can be considered as a chiral 2-form field with half of the degrees of freedom.

### 2.1 Heat kernel

The partition function of free fields on $\mathbb{S}^1_{\beta=2\pi q} \times \mathbb{H}^5$ can be obtained by heat kernel method,[11]

$$\log Z(\beta) = \frac{1}{2}\int_0^\infty \frac{dt}{t} K_{\mathbb{S}^1_\beta \times \mathbb{H}^5}(t)\,, \tag{2.1}$$

where $K_{\mathbb{S}^1_\beta \times \mathbb{H}^5}(t)$ is the heat kernel of the associated conformal Laplacian. The kernel can be factorized when the spacetime is a direct product,

$$K_{\mathbb{S}^1_\beta \times \mathbb{H}^5}(t) = K_{\mathbb{S}^1_\beta}(t)\, K_{\mathbb{H}^5}(t)\ . \tag{2.2}$$

The kernel on a circle $K_{\mathbb{S}^1_\beta}(t)$ is known to be[12]

$$K_{\mathbb{S}^1_\beta}(t) = \frac{\beta}{\sqrt{4\pi t}} \sum_{n\neq 0,\in\mathbb{Z}} e^{\frac{-\beta^2 n^2}{4t}}\ . \tag{2.3}$$

In the presence of a chemical potential $\mu$, it is twisted to be [55]

$$\widetilde{K}_{\mathbb{S}^1_\beta}(t) = \frac{\beta}{\sqrt{4\pi t}} \sum_{n\neq 0,\in\mathbb{Z}} e^{\frac{-\beta^2 n^2}{4t} + i2\pi n\mu + i\pi n f}\,, \tag{2.4}$$

[10] Six-dimensional $(2,0)$ theories have been studied in $AdS_5 \times S^1$ recently in the viewpoint of rigid holography [66].

[11] For Rényi entropy of free fields in other higher dimensions, see for instance [67–70].

[12] For fermions, the boundary conditions are anti-periodic.



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where $f = 0$ for scalars and $f = 1$ for fermions. Finally the kernels on the hyperbolic space $K_{\mathbb{H}^5}(t)$ can be written as follows because $\mathbb{H}^5$ is homogeneous,

$$K_{\mathbb{H}^5}(t) = \int d^5x\sqrt{g}\; K_{\mathbb{H}^5}(x,x,t) = V_5\, K_{\mathbb{H}^5}(0,t)\ . \tag{2.5}$$

The regularized volume $V_5 = \pi^2 \log(\ell/\epsilon)$. $\epsilon$ is the UV cutoff of the theory in the original space[13] and $\ell$ is the curvature radius of $\mathbb{H}^5$. Note that the kernels $K_{\mathbb{H}^5}(0,t)$ for free fields with different spins are known. See [50] and references there.

### 2.2 Rényi entropy

The total Rényi entropy of a tensor multiplet can be obtained by summing up the contributions of 5 real scalars, 2 Weyl fermions and a chiral 2-form,

$$S_q^{\text{free}} = 5 \times \frac{S_q^s}{2} + 2S_q^f + \frac{S_q^v}{2}\ , \tag{2.6}$$

where the Rényi entropy for fields with different spins can be computed by using the corresponding heat kernels. For the details of this computation we refer to [50]. We will instead list the results here. The Rényi entropy of a $6d$ real scalar is

$$S_q^s = \frac{(q+1)\left(3q^2+1\right)\left(3q^2+2\right)}{15120q^5}\frac{V_5}{\pi^2}\ , \tag{2.7}$$

and the Rényi entropy of a $6d$ Weyl fermion is

$$S_q^f = \frac{(q+1)\left(1221q^4+276q^2+31\right)}{120960q^5}\frac{V_5}{\pi^2}\ , \tag{2.8}$$

and that of a $6d$ 2-from field is

$$S_q^v = \frac{(q+1)\left(37q^2+2\right)+877q^4+4349q^5}{5040q^5}\frac{V_5}{\pi^2}\ . \tag{2.9}$$

It is worth to mention that, to get the correct Rényi entropy for the two form field, one has to take into account a $q$-independent constant shift due to the edge modes [50], like what should be done for the gauge field in $4d$ [71, 72]. Finally the Rényi entropy for a free $(2,0)$ tensor multiplet is

$$S_q^{\text{free}} = \frac{(q+1)(28q^2+3)+313q^4+1305q^5}{2880q^5}\frac{V_5}{\pi^2}\ . \tag{2.10}$$

It has been checked that $\partial^0_{q=1}$, $\partial^1_{q=1}$ and $\partial^2_{q=1}$ of $S_q^{\text{free}}$ are consistent [50] with the previous results about the tensor multiplet [31, 32, 73].

[13] This is the $q$-fold space with a conical singularity, which is used to compute Rényi entropy by replica trick.



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### 2.3 $S_q$ and $S_\gamma$

Before moving on, let us represent $S_q^{\text{free}}$ in terms of

$$S_\gamma := \frac{\pi^2}{V_5} S_q \,, \text{ with } \gamma := 1/q \,,$$

$$S_\gamma^{\text{free}} = \frac{1}{960}(\gamma-1)^5 + \frac{1}{160}(\gamma-1)^4 + \frac{7}{288}(\gamma-1)^3 + \frac{1}{18}(\gamma-1)^2 + \frac{\gamma-1}{6} + \frac{7}{12} \,. \tag{2.11}$$

The reason why $S_\gamma$ is convenient is that, the series expansion near $\gamma = 1$ has finite terms while the expansion of $S_q$ near $q = 1$ has infinite terms. We will use $S_\gamma$ instead of $S_q$ to express Rényi entropy and supersymmetric Rényi entropy from now on. It is worth to note the relations between the derivatives with respect to $q$ and the derivatives with respect to $\gamma$ at $q = 1/\gamma = 1$,

$$\partial_\gamma S_\gamma = -\partial_q S_q \Big|_{q=1/\gamma=1} \cdot \frac{\pi^2}{V_5} \,, \quad \partial_\gamma^2 S_\gamma = \left(2\partial_q S_q + \partial_q^2 S_q\right) \Big|_{q=1/\gamma=1} \cdot \frac{\pi^2}{V_5} \,. \tag{2.12}$$

### 2.4 Supersymmetric Rényi entropy

The supersymmetric Rényi entropy of a free tensor multiplet can be computed by the twisted kernel (2.4) on the supersymmetric background. The R-symmetry group of $6d$ $(2,0)$ theories is SO(5), which has two U(1) Cartans. Therefore one can turn on two independent R-symmetry background gauge fields (chemical potentials) to twist the boundary conditions for scalars and fermions along the replica circle $\mathbb{S}^1_\beta$. A general analysis of the Killing spinor equation on the conic space ($\mathbb{S}^6_q$ or $\mathbb{S}^1_{\beta=2\pi q} \times \mathbb{H}^5$) leads to the solution of the R-symmetry chemical potential [50][14]

$$\mu(q) := k_i A^i = \frac{q-1}{2} \,, \tag{2.13}$$

with $k_1$ and $k_2$ being the R-charges of the Killing spinor under the two U(1) Cartans, respectively. We choose $k_1 = k_2 = \frac{1}{2}$ and the two background fields can be expressed as

$$A^1 = (q-1)\, r_1 \,, \quad A^2 = (q-1)\, r_2 \,, \quad \text{with } \; r_1 + r_2 = 1 \,. \tag{2.14}$$

This is the most general background satisfying (2.13). For each component field in the tensor multiplet, one has to first figure out the Cartan charges $k_1$ and $k_2$ and then compute the chemical potential by $k_1 A^1 + k_2 A^2$. Then one can compute the free energy on $\mathbb{S}^1_\beta \times \mathbb{H}^5$ using the twisted heat kernel and get the supersymmetric Rényi entropy. For details, see [50].

After summing up all the component fields, the final supersymmetric Rényi entropy in terms of $\gamma$ can be expressed as,[15]

$$S_\gamma[\mathfrak{u}(1)] = \frac{1}{12} r_1^2 r_2^2 (\gamma-1)^3 + \frac{1}{12} r_1 r_2 (\gamma-1)^2 + \frac{1}{12}(1 + 2 r_1 r_2)(\gamma-1) + \frac{7}{12} \,. \tag{2.15}$$

[14] The Killing spinors on round sphere have been explored in [74].

[15] Although the form of this expression is a series expansion, the result itself is complete.

It is worth to note that, for a single U(1) background, $r_1 = 1, r_2 = 0$, the result becomes

$$S_\gamma = \frac{1}{12}(\gamma + 6)\,, \tag{2.16}$$

while for two U(1) backgrounds with equal values, $r_1 = r_2 = \frac{1}{2}$, we have

$$S_\gamma = \frac{1}{192}(\gamma-1)^3 + \frac{1}{48}(\gamma-1)^2 + \frac{1}{8}(\gamma-1) + \frac{7}{12}\,. \tag{2.17}$$

## 3 Interacting (2,0) theories

Given the supersymmetric Rényi entropy (2.15) for a free tensor multiplet, now we promote it to a general form which works for general $(2,0)$ SCFTs,

$$S_\gamma[\mathfrak{g}] = \frac{r_1^2 r_2^2}{12} \cdot A\,(\gamma-1)^3 + \frac{r_1 r_2}{12} \cdot B\,(\gamma-1)^2 + \frac{1+2r_1r_2}{12} \cdot C\,(\gamma-1) + \frac{7}{12}D\,, \tag{3.1}$$

where the coefficients $A\,, B\,, C\,, D$ will depend on the specific theory. As stated in the introduction, the assumption that $S_\gamma[\mathfrak{g}]$ is a cubic polynomial of $\gamma - 1$ is based on both the free multiplet result and the holographic result (as we will see in section 5).[16] Their dependence on $r_{1,2}$ is universal because $r_{1,2}$ originally come from the $\alpha_i$ ($\alpha_1 = r_1, \alpha_2 = r_2$) in (A.23), (A.27), which are background parameters independent of the specific theory. Later we will see that precisely the same $r_{1,2}$ dependence appears in the holographic supersymmetric Rényi entropy (5.19), which confirms this fact.

### 3.1 $S_{\gamma=1}$ and $a_{\mathfrak{g}}$

We would like to first determine the coefficient $D$ in (3.1). Recall that the entanglement entropy associated with a spherical entangling surface, $S_{\gamma=1}$, is proportional to the $a$-type Weyl anomaly. This is true for general CFTs in even dimensions as shown in [40]. Therefore

$$\frac{S_{\gamma=1}[\mathfrak{g}]}{S_{\gamma=1}[\mathfrak{u}(1)]} = \frac{a_{\mathfrak{g}}}{a_{\mathfrak{u}(1)}}\,. \tag{3.2}$$

This allows us to fix

$$D = \frac{a_{\mathfrak{g}}}{a_{\mathfrak{u}(1)}} = \frac{16}{7} h^\vee_{\mathfrak{g}} d_{\mathfrak{g}} + r_{\mathfrak{g}}\,, \tag{3.3}$$

where we have used the $a$-type Weyl anomaly result in $6d$ $(2,0)$ theories [22].

[16] Similar thing happens in $\mathcal{N} = 4$ SYM. Here we see an essential difference between the ordinary Rényi entropy and the supersymmetric one, because the type of $q$ scaling in the ordinary Rényi entropy is not protected [53, 75].

### 3.2 $\partial_\gamma S_{\gamma=1}$, $\partial_\gamma^2 S_{\gamma=1}$ and $c_{\mathfrak{g}}$

The coefficients $C$ and $B$ in (3.1) are determined by the first and the second $\gamma$-derivatives of $S_\gamma$ at $\gamma = 1$, respectively. $\gamma$-derivatives can be translated into $q$-derivatives. Taking $q$-derivatives is equal to taking derivatives with respect to background fields, therefore $\partial_\gamma S_{\gamma=1}$ and $\partial_\gamma^2 S_{\gamma=1}$ are intrinsically related to the corresponding correlators. This is illustrated in appendix A.

More explicitly, the first $\gamma$-derivative (which is minus the $q$-derivative at $q = 1/\gamma = 1$) is determined by a linear combination of the integrated stress tensor 2-point function and the integrated R-current 2-point function. The first $q$-derivative at $q = 1$ is given by the equation (A.23),

$$S'_{q=1} = -V_{d-1}\left(\frac{\pi^{\frac{d}{2}+1}\Gamma\left(\frac{d}{2}\right)(d-1)}{(d+1)!}C_T - \alpha^2\frac{\pi^{\frac{d+3}{2}}}{2^{d-3}(d-1)\Gamma\left(\frac{d-1}{2}\right)}C_v\right), \tag{3.4}$$

which works for general SCFTs with conserved R-symmetries in $d$-dimensions.

Similarly the second $\gamma$-derivative at $\gamma = 1$ is related to $q$-derivatives by (2.12). The second $q$-derivative at $q = 1$ is determined by a linear combination of the integrated stress tensor 3-point function, the integrated R-current 3-point function and some mixed 3-point functions. This is given explicitly by (A.27)

$$S''_{q=1} = \frac{1}{6}I'''_{q=1} = \frac{4\pi^3}{3}\left[\langle\hat{E}\hat{E}\hat{E}\rangle^c - \alpha^3\langle\hat{Q}\hat{Q}\hat{Q}\rangle^c - 3\alpha\langle\hat{E}\hat{E}\hat{Q}\rangle^c + 3\alpha^2\langle\hat{E}\hat{Q}\hat{Q}\rangle^c\right]_{\mathbb{S}^1_{q=1}\times\mathbb{H}^{d-1}}, \tag{3.5}$$

which also works for general SCFTs with conserved R-symmetries in $d$-dimensions.

In the particular case of $6d$ $(2,0)$ SCFTs, the operators in the above two- and three-point functions stay in the same multiplet, the stress tensor multiplet. Therefore both the first and second derivative of $S_\gamma$ at $\gamma = 1$ are proportional to the central charge $c_{\mathfrak{g}}$ (1.1), as discussed in detail in appendix A.[17] Because of this, we have

$$\left.\frac{\partial_\gamma S_\gamma[\mathfrak{g}]}{\partial_\gamma S_\gamma[\mathfrak{u}(1)]}\right|_{\gamma=1} = \left.\frac{\partial^2_\gamma S_\gamma[\mathfrak{g}]}{\partial^2_\gamma S_\gamma[\mathfrak{u}(1)]}\right|_{\gamma=1} = \frac{c_{\mathfrak{g}}}{c_{\mathfrak{u}(1)}}. \tag{3.6}$$

This actually means we can fix

$$B = C = 4h^\vee_{\mathfrak{g}}d_{\mathfrak{g}} + r_{\mathfrak{g}}. \tag{3.7}$$

The remaining coefficient $A$ will be fixed by (4.26) in section 4

$$A = h^\vee_{\mathfrak{g}}d_{\mathfrak{g}} + r_{\mathfrak{g}} \tag{3.8}$$

by studying the asymptotic $q := 1/\gamma \to 0$ behavior of the supersymmetric Rényi entropy. Obviously, the leading contribution in the limit $\gamma \to \infty$ is controlled only by $A$.

[17] This actually explains the universal ratio $4N^3$ between the explicit results on $\langle TT\rangle, \langle TTT\rangle, \langle JJ\rangle, \langle JJJ\rangle$ in holography and those in free tensor multiplets [32, 76].

### 3.3 A closed formula

As a summary, we can uniquely determine a closed formula of supersymmetric Rényi entropy for $(2,0)$ SCFTs characterized by simply-laced Lie algebra $\mathfrak{g}$

$$\begin{aligned} S_\gamma[\mathfrak{g}] &= \frac{r_1^2 r_2^2}{12}(h_\mathfrak{g}^\vee d_\mathfrak{g} + r_\mathfrak{g})(\gamma-1)^3 + \frac{r_1 r_2}{12}(4h_\mathfrak{g}^\vee d_\mathfrak{g} + r_\mathfrak{g})(\gamma-1)^2 \\ &\quad + \frac{1+2r_1 r_2}{12}(4h_\mathfrak{g}^\vee d_\mathfrak{g} + r_\mathfrak{g})(\gamma-1) + \left(\frac{4h_\mathfrak{g}^\vee d_\mathfrak{g}}{3} + \frac{7r_\mathfrak{g}}{12}\right), \qquad (3.9) \\ &= \frac{r_1^2 r_2^2}{48}(7\bar{a}_\mathfrak{g} - 3\bar{c}_\mathfrak{g})(\gamma-1)^3 + \frac{r_1 r_2}{12}\bar{c}_\mathfrak{g}(\gamma-1)^2 + \frac{1+2r_1 r_2}{12}\bar{c}_\mathfrak{g}(\gamma-1) + \frac{7}{12}\bar{a}_\mathfrak{g}, \qquad (3.10) \end{aligned}$$

where in the last line we have used the normalized Weyl anomalies defined in (1.1).

For a single U(1) chemical potential,

$$r_1 = 1, \quad r_2 = 0, \tag{3.11}$$

the result is simplified to be

$$\begin{aligned} S_\gamma[\mathfrak{g}] &= \frac{1}{12}\bar{c}_\mathfrak{g}(\gamma-1) + \frac{7}{12}\bar{a}_\mathfrak{g}, \qquad (3.12) \\ &= h_\mathfrak{g}^\vee d_\mathfrak{g}\left(\frac{1}{3}\gamma + 1\right) + r_\mathfrak{g}\frac{(\gamma+6)}{12}. \qquad (3.13) \end{aligned}$$

As for two U(1) chemical potentials with equal values,

$$r_1 = r_2 = \frac{1}{2}, \tag{3.14}$$

the result is simplified to be

$$\begin{aligned} S_\gamma[\mathfrak{g}] &= \frac{1}{192\times 4}(7\bar{a}_\mathfrak{g} - 3\bar{c}_\mathfrak{g})(\gamma-1)^3 + \frac{1}{48}\bar{c}_\mathfrak{g}(\gamma-1)^2 + \frac{1}{8}\bar{c}_\mathfrak{g}(\gamma-1) + \frac{7}{12}\bar{a}_\mathfrak{g}, \qquad (3.15) \\ &= \frac{175+67\gamma+13\gamma^2+\gamma^3}{192}h_\mathfrak{g}^\vee d_\mathfrak{g} + \frac{91+19\gamma+\gamma^2+\gamma^3}{192}r_\mathfrak{g}. \qquad (3.16) \end{aligned}$$

## 4 $q \to 0$ asymptotics

In this section we discuss the relation between the $q \to 0$ limit ($\gamma \to \infty$) of supersymmetric Rényi entropy $S_q$ and supersymmetric Casimir energy. Recall the definition of $S_q$

$$S_q = \frac{qI_1 - I_q}{1-q}. \tag{4.1}$$

Assuming that in the limit $q \to 0$ the free energy behaves

$$I_q = I_{(0)} q^{-\alpha} + \cdots, \tag{4.2}$$

where $\alpha \geq 0$, one can easily get

$$S_{q\to 0} = -I_{q\to 0} \tag{4.3}$$

in the leading order. This relation does not depend on which geometric background we are working on.

The idea is that, $\mathbb{S}^d_q$ can be conformally mapped to $\mathbb{H}^1 \times \mathbb{S}^{d-1}_q$, therefore the Rényi entropy (or supersymmetric) is invariant [40]. In the case with supersymmetry, one has to make sure that in the limit $q \to 0$, the background field on $\mathbb{S}^d_q$ coincides with that on $\mathbb{H}^1 \times \mathbb{S}^{d-1}_q$. If that is the case, the asymptotic supersymmetric Rényi entropy $S_{q\to 0}$ on $\mathbb{S}^d_q$ will coincide with the minus free energy on $\mathbb{H}^1 \times \mathbb{S}^{d-1}_{q\to 0}$. The latter is determined by the supersymmetric Casimir energy [77]. We will illustrate the details in the following.

### 4.1 From $\mathbb{S}^d_q$ to $\mathbb{H}^{d-p} \times \mathbb{S}^p_q$

We start with the conformal transformation from conic sphere $\mathbb{S}^d_q$ to hyperbolic space $\mathbb{H}^{d-p} \times \mathbb{S}^p_q$. Of course $\mathbb{S}^d_q$ can be considered as the special case of $p = d$.

In the particular case $p = 1$, the transformation is nothing but the Weyl transformation discussed in [40], which offers a convenient way to compute Rényi entropy of CFTs. In this case, the branched $d$-sphere is described as[18]

$$\mathrm{d}s^2 = \sin^2\theta q^2 \mathrm{d}\tau^2 + \mathrm{d}\theta^2 + \cos^2\theta \mathrm{d}^2\Omega_{d-2}\,, \tag{4.4}$$

with domains of coordinates given by

$$\tau \in [0, 2\pi)\,, \quad \theta \in \left[0, \frac{\pi}{2}\right]\,, \tag{4.5}$$

and $\Omega_{d-2}$ is a standard $d$-2-dimensional round sphere. The metric (4.4) can be written as

$$\mathrm{d}s^2 = \sin^2\theta \left(q^2\mathrm{d}\tau^2 + \frac{1}{\sin^2\theta}\mathrm{d}\theta^2 + \cot^2\theta \mathrm{d}^2\Omega_{d-2}\right)\,, \tag{4.6}$$

which can be related to the following space by dropping an overall factor $\sin^2\theta$ and using a coordinate transformation $\cot\theta = \sinh\eta$

$$\mathrm{d}s^2 = q^2\mathrm{d}\tau^2 + \mathrm{d}\eta^2 + \sinh^2\eta \mathrm{d}^2\Omega_{d-2}\,, \tag{4.7}$$

where $\eta \in [0, +\infty)$. This is the space of $\mathbb{H}^{d-1} \times \mathbb{S}^1_q$, which indeed fits the case of $p = 1$.

Now we consider the general cases, $1 \le p < d$. The key observation is that, the branched sphere can be presented in different forms. For instance, we can represent $\mathbb{S}^d_q$ as

$$\mathrm{d}s^2 = \sin^2\theta(\mathrm{d}\chi^2 + \sin^2\chi q^2\mathrm{d}\tau^2) + \mathrm{d}\theta^2 + \cos^2\theta \mathrm{d}^2\Omega_{d-3}\,, \tag{4.8}$$

with domains

$$\chi \in [0, \pi]\,, \quad \tau \in [0, 2\pi)\,, \quad \theta \in \left[0, \frac{\pi}{2}\right]\,, \tag{4.9}$$

and $\Omega_{d-3}$ is a standard $d$-3-dimensional round sphere. Again by dropping an overall factor $\sin^2\theta$ and using a coordinate transformation $\cot\theta = \sinh\eta$ for the metric (4.8), one obtains

$$\mathrm{d}s^2 = \mathrm{d}\chi^2 + \sin^2\chi q^2\mathrm{d}\tau^2 + \mathrm{d}\eta^2 + \sinh^2\eta \mathrm{d}^2\Omega_{d-3}\,, \tag{4.10}$$

[18] We normalize the radius as unit.



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which is the space $\mathbb{H}^{d-2} \times \mathbb{S}_q^2$ with $p = 2$. One can follow the same way to eventually figure out the Weyl transformations between $\mathbb{S}_q^d$ and $\mathbb{H}^{d-p} \times \mathbb{S}_q^p$ for any integer $1 \leq p < d$.

Since the Rényi entropy on $\mathbb{S}_q^d$ can not depend on which particular circle we choose to create the conical singularity, one eventually arrives at the conclusion by employing the same argument in [40]:[19]

*The universal part of CFT$_d$ Rényi entropy is invariant on $\mathbb{H}^{d-p} \times \mathbb{S}_q^p$ for different integer $p$, where $1 \leq p \leq d$.*

For later purpose, let us discuss the particular case $p = d-1$. In this case we describe the branched sphere $\mathbb{S}_q^d$ as

$$\mathrm{d}s^2 = \sin^2\theta(\mathrm{d}\chi^2 + \sin^2\chi q^2 \mathrm{d}\tau^2 + \cos^2\chi \mathrm{d}^2\Omega_{d-3}) + \mathrm{d}\theta^2 \,, \tag{4.11}$$

with domains

$$\chi \in \left[0, \frac{\pi}{2}\right] \,, \quad \tau \in [0, 2\pi) \,, \quad \theta \in [0, \pi] \ . \tag{4.12}$$

Again by dropping an overall factor $\sin^2\theta$ for the metric (4.11), one obtains

$$\mathrm{d}s^2 = \mathrm{d}\chi^2 + \sin^2\chi q^2 \mathrm{d}\tau^2 + \cos^2\chi \mathrm{d}^2\Omega_{d-3} + \mathrm{d}\eta^2 \,, \tag{4.13}$$

where $\cot\theta = \sinh\eta$ and $\eta \in (-\infty, +\infty)$. This is the space $\mathbb{S}_q^{d-1} \times \mathbb{H}^1$. Here we use $\mathbb{H}^1$ instead of $\mathbb{R}^1$ to emphasize that the volume of $\mathbb{H}^d$ may be regularized. For free fields, one can compute the CFT Rényi entropy on $\mathbb{S}_q^{d-1} \times \mathbb{H}^1$ and show explicitly that the result agrees with that computed from $\mathbb{S}_q^d$ or $\mathbb{S}_q^1 \times \mathbb{H}^{d-1}$. In consideration of supersymmetry, one has to add a background field $A_\tau$ along the replica $\tau$ circle inside $\mathbb{S}_q^{d-1}$, in order to find the agreement.

### 4.2 Coincidence of backgrounds

Our main concern is physical quantities for CFTs. For this purpose we can work on $\mathbb{S}_{\sqrt{q}}^{d-1} \times \mathbb{H}_{1/\sqrt{q}}^1$ instead of $\mathbb{S}_q^{d-1} \times \mathbb{H}^1$ because they are related by a scale transformation

$$\frac{1}{\sqrt{q}}[\mathbb{S}_q^{d-1} \times \mathbb{H}^1] \;=\; [\mathbb{S}_{\sqrt{q}}^{d-1} \times \mathbb{H}_{1/\sqrt{q}}^1] \ . \tag{4.14}$$

Furthermore, we focus on the limit $q \to 0$. For this purpose, one can instead consider $\mathbb{S}_{\sqrt{q}}^{d-1} \times \mathbb{S}_{1/\sqrt{q}}^1$ because it is equivalent to $\mathbb{S}_{\sqrt{q}}^{d-1} \times \mathbb{H}_{1/\sqrt{q}}^1$ in the limit $q \to 0$

$$\mathbb{S}_{\sqrt{q}}^{d-1} \times \mathbb{H}_{1/\sqrt{q}}^1\Big|_{q\to 0} = \mathbb{S}_{\sqrt{q}}^{d-1} \times \mathbb{S}_{1/\sqrt{q}}^1\Big|_{q\to 0} \ . \tag{4.15}$$

In consideration of supersymmetry, one can use the squashed sphere $\widetilde{\mathbb{S}}_{\sqrt{q}}^{d-1}$ to replace the conic sphere $\mathbb{S}_{\sqrt{q}}^{d-1}$ in the right hand side of (4.15), because supersymmetric partition functions do not depend on the resolving factor [42, 49, 78–81].[20] Eq. (4.15) is useful in the

[19] Again by the universal part of Rényi entropy we refer to the scheme independent part.

[20] For this reason, we will not distinguish $d$-1-dimensional squashed sphere and conic sphere in the following unless it is necessary.

sense that it offers a way to compute the asymptotic supersymmetric Rényi entropy for interacting SCFTs. To do this, one has to make sure that the background gauge field on $\mathbb{S}^{d-1}_{\sqrt{q}} \times \mathbb{S}^1_{1/\sqrt{q}}$ agrees with that on the original space $\mathbb{S}^d_q$. Fortunately we have more knowledge about supersymmetric partition functions on $\mathbb{S}^{d-1} \times \mathbb{S}^1$ or its generalized version $\mathbb{S}^{d-1}_b \times \mathbb{S}^1_\beta$, where $b$ is the squashing parameter.

### 4.3 Squashed Casimir energy

Now we make a connection between the asymptotic Rényi entropy and Casimir energy. It is known that the partition function $Z$ on $\mathbb{S}^{d-1}_b \times \mathbb{S}^1_\beta$ is determined by the Casimir energy on $\mathbb{S}^{d-1}_b$ in the limit $\beta \to \infty$

$$E_c := -\lim_{\beta\to\infty} \partial_\beta \log Z(\beta)\,, \tag{4.16}$$

which is equivalent to say

$$\lim_{\beta\to\infty} \log Z(\beta) = -\beta E_c\,. \tag{4.17}$$

In this work, we concern the case with supersymmetry. In the particular case of $6d$ $(2,0)$ theories, the supersymmetric Casimir energy has been studied in [82],[21] where the authors considered a general 5-sphere with squashing parameters $\vec{\omega} = (\omega_1, \omega_2, \omega_3)$. The squashing parameters are defined as parameters appearing in the Killing vector

$$K = \omega_1 \frac{\partial}{\partial\phi_1} + \omega_2 \frac{\partial}{\partial\phi_2} + \omega_3 \frac{\partial}{\partial\phi_3}\,, \tag{4.18}$$

where $\phi_1, \phi_2, \phi_3$ are three circles representing $\mathrm{U}(1)^3$ isometries of $\mathbb{S}^5$. The supersymmetric Casimir energy of an interacting $(2,0)$ theory is [82]

$$E_{\mathfrak{g}} = r_{\mathfrak{g}} E_{\mathfrak{u}(1)} - d_{\mathfrak{g}} h^\vee_{\mathfrak{g}} \frac{\sigma_1^2\sigma_2^2}{24\,\omega_1\omega_2\omega_3}\,, \tag{4.19}$$

where $\sigma_1$ and $\sigma_2$ are chemical potentials for the two Cartans of the $\mathrm{SO}(5)$ R-symmetry and $E_{\mathfrak{u}(1)}$ is given by

$$E_{\mathfrak{u}(1)} = -\frac{1}{48\omega_1\omega_2\omega_3}\left[\sigma_1^2\sigma_2^2 - \sum_{i<j}\omega_i^2\omega_j^2 + \frac{1}{4}\left(\sum_j \omega_j^2 - \sigma_1^2 - \sigma_2^2\right)^2\right]. \tag{4.20}$$

For the particular case of $\mathbb{S}^5_q \times \mathbb{S}^1$ (which is equivalent to $\mathbb{S}^5_{\sqrt{q}} \times \mathbb{S}^1_{\frac{1}{\sqrt{q}}}$ for CFTs), we should identify the shape parameters as

$$\omega_1 = \omega_2 = 1\,, \quad \omega_3 = \frac{1}{q}\,. \tag{4.21}$$

In the limit $q \to 0$, in order to match our chemical potentials (2.14), we set $\sigma_1$ and $\sigma_2$ as[22]

$$\sigma_1^2(q\to 0) = \frac{r_1^2}{q^2}\,, \quad \sigma_2^2(q\to 0) = \frac{r_2^2}{q^2}\,, \quad \text{with} \;\; r_1 + r_2 = 1\,. \tag{4.22}$$

[21] For the $6d$ $(2,0)$ superconformal index, see [83–85].

[22] The $q$ scalings in chemical potentials appear following the convention in [82].

Evaluating (4.19) we get

$$E_{\mathfrak{g}}\Big|_{q\to 0} = -\frac{1}{24}\frac{r_1^2 r_2^2}{q^3}(r_{\mathfrak{g}} + d_{\mathfrak{g}} h_{\mathfrak{g}}^{\vee}) \ . \tag{4.23}$$

Therefore the free energy[23]

$$f[\mathbb{S}^5_{q\to 0}\times\mathbb{S}^1] = \frac{1}{\pi^3}\beta E_{\mathfrak{g}}\Big|_{q\to 0} = -\frac{1}{12\pi^2}\frac{r_1^2 r_2^2}{q^3}(r_{\mathfrak{g}} + d_{\mathfrak{g}} h_{\mathfrak{g}}^{\vee})\,, \tag{4.24}$$

where we have divided a $q$-independent volume factor Vol $[\mathbb{D}^4\times\mathbb{S}^1] = \pi^3$. Due to (4.15), we have

$$f[\mathbb{S}^5_{q\to 0}\times\mathbb{S}^1] = f[\mathbb{S}^1_{q\to 0}\times\mathbb{H}^5]\,, \tag{4.25}$$

from which we obtain the asymptotic supersymmetric Rényi entropy on $\mathbb{S}^1_q\times\mathbb{H}^5$

$$S_{q\to 0}[\mathfrak{g}] = -I_{q\to 0}[\mathfrak{g}] = \frac{1}{12}\frac{r_1^2 r_2^2}{q^3}(r_{\mathfrak{g}} + d_{\mathfrak{g}} h_{\mathfrak{g}}^{\vee}) \ . \tag{4.26}$$

This fixes the undetermined coefficient $A$ in (3.1) as

$$A = r_{\mathfrak{g}} + h_{\mathfrak{g}}^{\vee} d_{\mathfrak{g}} \ . \tag{4.27}$$

Notice that the fact that the free limit of (4.26) precisely agrees with the leading large $\gamma$ term of (2.15) by itself is nontrivial, which confirms the validity of (4.25) in the free case.

## 5 Large $N$ limit

In the large $N$ limit of the $(2,0)$ theory with $\mathfrak{g} = A_{N-1}$, the supersymmetric Rényi entropy (3.9) becomes

$$\begin{aligned}\frac{S_\gamma[A_{N\to\infty}]}{N^3} &= \frac{1}{12}r_1^2 r_2^2(\gamma-1)^3 + \frac{4}{12}r_1 r_2(\gamma-1)^2\\ &\quad+\frac{4}{12}(1+2r_1 r_2)(\gamma-1) + \frac{4}{3} \ .\end{aligned} \tag{5.1}$$

We will demonstrate in this section that the above large $N$ result precisely agrees with the holographic result from the seven-dimensional BPS topological black hole in gauged supergravity.

### 5.1 Gauged supergravity

The seven-dimensional gauged SO(5) supergravity can be obtained by Kaluza-Klein reduction of eleven-dimensional supergravity on $\mathbb{S}^4$. For our purpose, we consider a truncation where only the metic, two gauge fields associated to two Cartans of SO(5) and two scalars are retained. The seven-dimensional Lagrangian is given by [86]

$$\frac{1}{\sqrt{g}}\mathcal{L} = R - \frac{1}{2}(\partial\vec{\phi})^2 - \frac{4}{L^2}V - \frac{1}{4}\sum_{i=1}^{2}\frac{1}{X_i^2}\left(F^i_{(2)}\right)^2\,, \tag{5.2}$$

[23] $f := \frac{I}{V}$.

where $\vec{\phi} = (\phi_1, \phi_2)$ are two scalars and

$$X_i = e^{-\frac{1}{2}\vec{a}_i\cdot\vec{\phi}}\,, i = 1, 2\ . \quad \vec{a}_1 = \left(\sqrt{2}, \sqrt{\frac{2}{5}}\right)\,, \quad \vec{a}_2 = \left(-\sqrt{2}, \sqrt{\frac{2}{5}}\right)\ . \tag{5.3}$$

The potential is given by

$$V = -4X_1X_2 - 2X_0X_1 - 2X_0X_2 + \frac{1}{2}X_0^2\,, \quad X_0 = \frac{1}{X_1X_2}\ . \tag{5.4}$$

Note that for two equal scalars and two equal gauge strengths, the Lagrangian (5.2) can be further truncated. Turn to the CFT side, 6d (2, 0) theories have global SO(5) R-symmetry, which corresponds to the SO(5) gauge group in the bulk supergravity. Also there could be two U(1) background fields used to compensate the singularity on $\mathbb{S}^6_q$, which correspond to $A^1, A^2$ in the gauged supergravity.

### 5.2 Topological black hole

The 2-charge 7$d$ AdS black hole solution for (5.2) was found in [86]

$$\begin{aligned} \mathrm{d}s_7^2 &= -\frac{1}{[h_1h_2]^{\frac{4}{5}}}f(r)\mathrm{d}t^2 + [h_1h_2]^{\frac{1}{5}}\left(\frac{\mathrm{d}r^2}{f(r)} + r^2\mathrm{d}\Omega_{5,k}^2\right) \\ f(r) &= k - \frac{m}{r^4} + \frac{r^2}{L^2}h_1h_2\,, \qquad h_i = 1 + \frac{q_i}{r^4}\,, \end{aligned} \tag{5.5}$$

together with scalars and gauge fields

$$X_i = \frac{[h_1h_2]^{\frac{2}{5}}}{h_i}\,, \quad A^i = \left[\sqrt{k}\left(\frac{1}{h_i} - 1\right) + \mu_i\right]\mathrm{d}t\ . \tag{5.6}$$

$\mathrm{d}\Omega_{5,k}^2$ is the metric on a unit $\mathbb{S}^5$, $\mathbb{T}^5$ or $\mathbb{H}^5$ corresponding to $k = 1, 0, -1$, respectively. Since our concern is the 6$d$ SCFT on $\mathbb{S}^1 \times \mathbb{H}^5$, we are particularly interested in the extremal solution with hyperbolic foliation, where $m = 0$ and $k = -1$. We will first proceed in Lorentz signature and assume a well-defined Wick rotation.

The solution (5.5) is a BPS topological black hole with two charges. For convenience, define a rescaled charge

$$\kappa_i = \frac{q_i}{r_H^4}\,, \tag{5.7}$$

where the horizon $r_H$ is the largest root of the equation

$$f(r_H) = 0\ . \tag{5.8}$$

Then the horizon can be expressed in terms of $\kappa_i$

$$r_H = \frac{L}{\sqrt{(1+\kappa_1)(1+\kappa_2)}}\ . \tag{5.9}$$

The Hawking temperature of this black hole is

$$\begin{aligned} T &= \left.\frac{f'(r)}{4\pi\sqrt{h_1h_2}}\right|_{r=r_H} \\ &= \frac{1-\kappa_1-\kappa_2-3\kappa_1\kappa_2}{2\pi L(1+\kappa_1)(1+\kappa_2)}\ . \end{aligned} \tag{5.10}$$



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When all charges vanish, we get to the temperature of the uncharged black hole

$$T_0 = \frac{1}{2\pi L} \ . \tag{5.11}$$

The Bekenstein-Hawking entropy is given by the outer horizon area

$$S = \frac{V_5 L^5}{4G_7} \frac{1}{(1+\kappa_1)^2(1+\kappa_2)^2} \,, \tag{5.12}$$

where $G_7$ is the seven dimensional Newton constant and $V_5$ is the regularized volume of $\mathbb{H}^5$. The total charge $Q_i$ can be computed by Gauss law

$$\begin{aligned} Q_i &= \frac{1}{16\pi G_7} \int_{r\to\infty} -\sqrt{g} F^{rt} = \frac{V_5}{4\pi G_7} i q_i \\ &= \frac{V_5 L^4}{4\pi G_7} \frac{i\kappa_i}{(1+\kappa_1)^2(1+\kappa_2)^2} \ . \end{aligned} \tag{5.13}$$

The chemical potential is

$$\mu_i = \frac{i}{\kappa_i^{-1}+1} \ . \tag{5.14}$$

### 5.3 Precise check

To match the background gauge fields of the boundary CFT, we set

$$\mu_1 = i(1-\gamma)\frac{r_1}{2} \,, \quad \mu_2 = i(1-\gamma)\frac{r_2}{2} \,, \quad \text{with} \quad r_1 + r_2 = 1 \ . \tag{5.15}$$

By using these inputs, we can solve $\kappa_1$ and $\kappa_2$ by (5.14). Then all physical quantities $T, S, Q_i$ can be worked out explicitly. One can eventually compute the holographic supersymmetric Rényi entropy using the formula derived in [42]

$$S_q = \frac{q}{1-q} \int_q^1 \left( \frac{S(n)}{n^2} - \frac{Q_i(n)\mu_i'(n)}{T_0} \right) \mathrm{d}n \ . \tag{5.16}$$

Written in terms of $\gamma := 1/q$, the result is given by

$$S_\gamma = \frac{L^5\pi^2}{4G_7} \left[ \frac{r_1^2 r_2^2(\gamma-1)^3}{16} + \frac{(1+2r_1r_2)(\gamma-1)}{4} + \frac{(\gamma-1)^2 r_1 r_2}{4} + 1 \right] \ . \tag{5.17}$$

By identifying the bulk and boundary parameters,

$$\frac{L^5\pi^2}{4G_7} = \frac{4}{3} N^3 \,, \tag{5.18}$$

one can write the holographic result as

$$S_\gamma = N^3 \left( \frac{r_1^2 r_2^2(\gamma-1)^3}{12} + \frac{(1+2r_1r_2)(\gamma-1)}{3} + \frac{(\gamma-1)^2 r_1 r_2}{3} + \frac{4}{3} \right) \ . \tag{5.19}$$

This precisely agrees with the field theory result (5.1).



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## 6 A possible $a/c$ bound

As what has been observed in $4d$ SCFTs [45], the Rényi entropy inequalities indicate $a/c$ bounds in field theories[24]

$$\partial_q H_q \leq 0\,, \tag{6.1}$$

$$\partial_q \left(\frac{q-1}{q} H_q\right) \geq 0\,, \tag{6.2}$$

$$\partial_q((q-1)H_q) \geq 0\,, \tag{6.3}$$

$$\partial_q^2((q-1)H_q) \leq 0\,, \tag{6.4}$$

where $H_q := S_q/S_1$. Imposing these conditions to our results (3.10)(3.12)(3.15), one obtains

$$0 < \frac{\bar{c}}{\bar{a}} \leq \frac{7}{3}\,, \tag{6.5}$$

or equivalently

$$\frac{\bar{a}}{\bar{c}} \geq \frac{3}{7}\,. \tag{6.6}$$

This lower bound can be derived alternatively by requiring a non-negative specific heat. In the limit $q \to 0$, the energy of the system can be read from (3.10)

$$E_{q\to 0} := \frac{1}{2\pi}\partial_q F_{q\to 0} = -\frac{1}{2\pi}\partial_q S_{q\to 0} = V_5 \frac{r_1^2 r_2^2}{32\pi^2}\frac{(7\bar{a}-3\bar{c})}{q^4} = V_5\pi\frac{r_1^2 r_2^2}{2}(7\bar{a}-3\bar{c})\,T^4\,, \tag{6.7}$$

where $T = 1/\beta = \frac{1}{2\pi q}$. It follows from the stability of the ensemble that the specific heat must be non-negative, $\frac{\partial E}{\partial T} \geq 0$, which gives (6.6).

Note that all the $a, c$ data of the currently known $6d$ $(2,0)$ SCFTs, listed in table 1 in appendix B, satisfy the inequality (6.5)(6.6). The lowest $\bar{a}/\bar{c}$ value in the current data, $4/7$, supported by the large $N$ limits, is greater than our bound $3/7$. Note that the expression of supersymmetric Rényi entropy in terms of $a, c$ anomalies could work for theories beyond the ADE type. It would be interesting to understand whether our bound implies new $(2,0)$ SCFTs. It would also be interesting to understand similar bounds in SCFTs with less supersymmetry. We leave these questions for future work.


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[24] The validity of these inequalities for supersymmetric Rényi entropy is expected although a proof is still in preparation.

## A Near $q = 1$ expansion

We study the perturbative expansion of supersymmetric Rényi entropy (associated with spherical entangling surface) around $q = 1$. This can be considered as an extension of the previous study of the ordinary Rényi entropy near $q = 1$. Although our main concern will be $6d$ $(2, 0)$ SCFTs, we keep the discussions in this section valid for any SCFT with conserved R-symmetries in $d$-dimensions.

Following the way in [52, 53],[25] we consider the supersymmetric partition function on $\mathbb{S}^1_q \times \mathbb{H}^{d-1}$ with background gauge fields (R-symmetry chemical potentials), which can be used to compute the supersymmetric Rényi entropy across a spherical entangling surface, see $\mathbb{S}^{d-2}$, in flat space. We work in grand canonical ensemble. The partition function on $\mathbb{S}^1_{\beta=2\pi q} \times \mathbb{H}^{d-1}$ can be written as

$$Z[\beta, \mu] = \mathrm{Tr}\left(e^{-\beta(\hat{E}-\mu\hat{Q})}\right) . \tag{A.1}$$

The state variables can be computed as follows

$$E = \left(\frac{\partial I}{\partial \beta}\right)_{\mu} - \frac{\mu}{\beta}\left(\frac{\partial I}{\partial \mu}\right)_{\beta} , \tag{A.2}$$

$$S = \beta\left(\frac{\partial I}{\partial \beta}\right)_{\mu} - I , \tag{A.3}$$

$$Q = -\frac{1}{\beta}\left(\frac{\partial I}{\partial \mu}\right)_{\beta} , \tag{A.4}$$

where $I := -\log Z$. Therefore we get energy expectation value by (A.2)

$$E = \frac{\mathrm{Tr}\left(e^{-\beta(\hat{E}-\mu\hat{Q})}\,\hat{E}\right)}{\mathrm{Tr}\left(e^{-\beta(\hat{E}-\mu\hat{Q})}\right)} , \tag{A.5}$$

and the charge expectation value by (A.4)

$$Q = \frac{\mathrm{Tr}\left(e^{-\beta(\hat{E}-\mu\hat{Q})}\,\hat{Q}\right)}{\mathrm{Tr}\left(e^{-\beta(\hat{E}-\mu\hat{Q})}\right)} . \tag{A.6}$$

In the presence of supersymmetry, both inverse temperature $\beta$ and chemical potential $\mu$ are functions of a single variable $q$ therefore $I$ is considered as

$$I_q := I[\beta(q), \mu(q)] . \tag{A.7}$$

The supersymmetric Rényi entropy is defined as

$$S_q = \frac{qI_1 - I_q}{1-q} . \tag{A.8}$$

Consider the Taylor expansion around $q = 1$, with $\delta q = q - 1$,

$$S_q = S_{\mathrm{EE}} + \sum_{n=2}^{\infty} \frac{1}{n!}\left(\frac{\partial^n I_q}{\partial q^n}\right)_{q=1} \delta q^{n-1} . \tag{A.9}$$

[25]See [54] from the viewpoint of twisted operator.

### A.1 $\partial_q I_q$

We will first consider $\partial_q I_q$. The first derivative with respect to $q$ can be written as

$$\frac{dI_q}{dq} = \left(\frac{\partial I}{\partial \beta}\right)_\mu \beta'(q) + \left(\frac{\partial I}{\partial \mu}\right)_\beta \mu'(q)\ . \tag{A.10}$$

Using (A.2) and (A.4), we can rewrite it as

$$\frac{dI_q}{dq} = (E - \mu Q)\,\beta'(q) - \beta Q\,\mu'(q)\ . \tag{A.11}$$

The $q$-dependence of the temperature and the chemical potential can be read off from the supersymmetric background (including metric and R-symmetry gauge field),

$$\beta(q) = 2\pi q\,, \quad \mu(q) = \alpha\frac{q-1}{q}\,, \tag{A.12}$$

where $\beta(q)$ is determined by the geometric fact and $\mu(q)$ is solved from the Killing spinor equation on the background. $\alpha$ is some number which may be different in various rigid supersymmetric backgrounds.[26] The first $q$-derivative of $I_q$ is simplified by using (A.12)

$$I'_q = 2\pi(E - \alpha Q)\ . \tag{A.13}$$

Notice that in general both $E$ and $Q$ are functions of $q$. Also $E$ and $Q$ here are expectation values rather than operators.

### A.2 $S'_{q=1}$ and $I''_{q=1}$

From (A.9) we see that

$$S'_{q=1} = \frac{1}{2}I''_{q=1}\ . \tag{A.14}$$

Let us take one more derivative above on the first derivative (A.13) and take use of (A.5) and (A.6)

$$I''_q = -4\pi^2\left(\frac{\mathrm{Tr}\left(e^{-\beta(\hat{E}-\mu\hat{Q})}\,(\hat{E}-\alpha\hat{Q})^2\right)}{\mathrm{Tr}\left(e^{-\beta(\hat{E}-\mu\hat{Q})}\right)} - \frac{\left[\mathrm{Tr}\left(e^{-\beta(\hat{E}-\mu\hat{Q})}\,(\hat{E}-\alpha\hat{Q})\right)\right]^2}{\left[\mathrm{Tr}\left(e^{-\beta(\hat{E}-\mu\hat{Q})}\right)\right]^2}\right)\,, \tag{A.15}$$

which can be simplified in the limit $q \to 1$ by using $\mu = 0$ at $q = 1$

$$S'_{q=1} = -2\pi^2\left(\frac{\mathrm{Tr}\left(e^{-\beta\hat{E}}\,(\hat{E}-\alpha\hat{Q})^2\right)}{\mathrm{Tr}\left(e^{-\beta\hat{E}}\right)} - \frac{\left[\mathrm{Tr}\left(e^{-\beta\hat{E}}\,(\hat{E}-\alpha\hat{Q})\right)\right]^2}{\left[\mathrm{Tr}\left(e^{-\beta\hat{E}}\right)\right]^2}\right)_{q=1}\ . \tag{A.16}$$

This can be rewritten as connected correlators

$$S'_{q=1} = -2\pi^2\left[\langle\hat{E}\hat{E}\rangle^c + \alpha^2\langle\hat{Q}\hat{Q}\rangle^c - 2\alpha\langle\hat{E}\hat{Q}\rangle^c\right]_{\mathbb{S}^1_{q=1}\times\mathbb{H}^{d-1}}\,, \tag{A.17}$$

[26] $\alpha$ characterizes the weight of the chemical potential. In the case of multiple chemical potentials, one should use $\alpha_{i=1,2\ldots R}$, where $R$ denotes the number of U(1) R-symmetry Cartans. $i$ should be summed over for $\alpha_i Q^i$.

where we have used $[\hat{E},\hat{Q}]=0$ to flip the order of $\hat{E}$ and $\hat{Q}$. Given that $\langle\hat{E}\hat{Q}\rangle^c=0$ and $\langle\hat{E}\hat{E}\rangle^c$ has been computed in [52], we get

$$S'_{q=1}=-V_{d-1}\frac{\pi^{d/2+1}\Gamma(d/2)(d-1)}{(d+1)!}C_T-2\pi^2\alpha^2\int_{\mathbb{H}^{d-1}}\int_{\mathbb{H}^{d-1}}\langle J_\tau(x)J_\tau(y)\rangle^c_{q=1}\,. \tag{A.18}$$

$C_T$ is defined in the flat space correlator

$$\langle T_{ab}(x)T_{cd}(0)\rangle=\frac{C_T}{x^{2d}}I_{ab,cd}(x)\,, \tag{A.19}$$

where

$$I_{ab,cd}(x)=\frac{1}{2}\left(I_{ac}(x)I_{bd}(x)+I_{ad}(x)I_{bc}(x)\right)-\frac{1}{d}\delta_{ab}\delta_{cd}\,,\quad I_{ab}(x)=\delta_{ab}-2\frac{x_a x_b}{x^2}\,. \tag{A.20}$$

Now the task is to compute the second term in (A.18). Following the way of computing $\langle TT\rangle$ on the hyperbolic space $\mathbb{S}^1_{q=1}\times\mathbb{H}^{d-1}$, one can take use of the flat space correlators in the CFT vacuum. The result is[27]

$$\langle\hat{Q}\hat{Q}\rangle^c=-\frac{\pi^{\frac{d-1}{2}}V_{d-1}}{2^{d-2}(d-1)\Gamma(\frac{d-1}{2})}C_v\,, \tag{A.21}$$

where $C_v$ is defined in the current correlator in flat space

$$\langle J_a(x)J_b(0)\rangle=\frac{C_v}{x^{2(d-1)}}I_{ab}(x)\,. \tag{A.22}$$

Then our final result of $S'_{q=1}$ becomes

$$S'_{q=1}=-V_{d-1}\left(\frac{\pi^{\frac{d}{2}+1}\Gamma(\frac{d}{2})(d-1)}{(d+1)!}C_T-\alpha^2\frac{\pi^{\frac{d+3}{2}}}{2^{d-3}(d-1)\Gamma(\frac{d-1}{2})}C_v\right)\,, \tag{A.23}$$

which tells us that the first $q$-derivative of supersymmetric Rényi entropy at $q=1$ is given by a linear combination of $C_T$ and $C_v$.[28] This is intuitively expected because in the presence of supersymmetry, taking the derivative with respect to $q$ is equivalent to taking the derivative with respect to $g_{\tau\tau}$ and $A_\tau$ in the same time.[29] $q$-deformation can be often equivalent to the squashing $b:=\sqrt{q}$, therefore this formula also shows the relation between $\partial^2_{b=1}$ of the free energy on squashed sphere and flat space correlators. It is clear from the above derivation that this formula works both for free theories and interacting SCFTs in general $d$-dimensions. In the particular case of $6d$ $(2,0)$ SCFTs, the 2-point function of the stress tensor is determined by the central charge $c_{\mathfrak{g}}$ in (1.1) [23, 24]. Therefore the integrated 2-point function is proportional to $c_{\mathfrak{g}}$. Furthermore, $S'_{q=1}$ is also proportional to $c_{\mathfrak{g}}$, because the stress tensor and the R-current in the right hand side of (A.23) live in the same multiplet.[30] The same thing happens in $\mathcal{N}=4$ SYM [44].

[27] $\langle J\hat{Q}\rangle$ was first computed in [55].
[28] In another word, a linear combination of the integrated stress tensor 2-point function and the integrated R-current 2-point function.
[29] This was first suggested in [44].
[30] For $(2,0)$ tensor multiplet, this supermultiplet was studied explicitly in [56].



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### A.3 $S''_{q=1}$ and $I'''_{q=1}$

From (A.9) we see that

$$S''_{q=1} = \frac{1}{6} I'''_{q=1}\,. \tag{A.24}$$

One may go straightforward to compute $I'''_q$ by taking one more derivative above on (A.15)

$$\begin{aligned}\frac{I'''_q}{8\pi^3} =& \frac{\mathrm{Tr}\Big(e^{-\beta(\hat{E}-\mu\hat{Q})}\,(\hat{E}-\alpha\hat{Q})^3\Big)}{\mathrm{Tr}\Big(e^{-\beta(\hat{E}-\mu\hat{Q})}\Big)} - 3\frac{\mathrm{Tr}\Big(e^{-\beta(\hat{E}-\mu\hat{Q})}(\hat{E}-\alpha\hat{Q})^2\Big)\,\mathrm{Tr}\Big(e^{-\beta(\hat{E}-\mu\hat{Q})}\,(\hat{E}-\alpha\hat{Q})\Big)}{\Big[\mathrm{Tr}\,\Big(e^{-\beta(\hat{E}-\mu\hat{Q})}\Big)\Big]^2}\\ &+2\frac{\Big[\mathrm{Tr}\,\Big(e^{-\beta(\hat{E}-\mu\hat{Q})}\,(\hat{E}-\alpha\hat{Q})\Big)\Big]^3}{\Big[\mathrm{Tr}\,\Big(e^{-\beta(\hat{E}-\mu\hat{Q})}\Big)\Big]^3}\,,\end{aligned} \tag{A.25}$$

which may be simplified at $q=1$ where $\mu=0$

$$\begin{aligned}\frac{I'''_{q=1}}{8\pi^3} =& \Bigg(\frac{\mathrm{Tr}\,\Big(e^{-\beta\hat{E}}\,(\hat{E}-\alpha\hat{Q})^3\Big)}{\mathrm{Tr}e^{-\beta\hat{E}}} - 3\frac{\mathrm{Tr}\,\Big(e^{-\beta\hat{E}}\,(\hat{E}-\alpha\hat{Q})^2\Big)\,\mathrm{Tr}\,\Big(e^{-\beta\hat{E}}\,(\hat{E}-\alpha\hat{Q})\Big)}{\Big[\mathrm{Tr}e^{-\beta\hat{E}}\Big]^2}\\ &+2\frac{\Big[\mathrm{Tr}(e^{-\beta\hat{E}}\,(\hat{E}-\alpha\hat{Q}))\Big]^3}{\Big[\mathrm{Tr}e^{-\beta\hat{E}}\Big]^3}\Bigg)_{q=1}\,.\end{aligned} \tag{A.26}$$

This can be further written in terms of connected correlation functions,

$$S''_{q=1} = \frac{1}{6}I'''_{q=1} = \frac{4\pi^3}{3}\Big[\langle\hat{E}\hat{E}\hat{E}\rangle^c - \alpha^3\langle\hat{Q}\hat{Q}\hat{Q}\rangle^c - 3\alpha\langle\hat{E}\hat{E}\hat{Q}\rangle^c + 3\alpha^2\langle\hat{E}\hat{Q}\hat{Q}\rangle^c\Big]_{\mathbb{S}^1_{q=1}\times\mathbb{H}^{d-1}}\,, \tag{A.27}$$

where we have used $[\hat{E},\hat{Q}]=0$ because $\hat{Q}$ is conserved charge. The integrated correlators in (A.27) can be computed by transforming the corresponding flat space correlators, $\langle TTT\rangle, \langle JJJ\rangle, \langle TTJ\rangle, \langle TJJ\rangle$ in the CFT vacuum.[31] These correlators in flat space can be determined up to some coefficients for general CFTs in $d$-dimensions by conformal Wald identities [57, 58]. In the presence of 6$d$ (2,0) superconformal symmetry, both the 2- and 3-point functions of the stress tensor multiplet are uniquely determined in terms of a single parameter, the central charge $c_{\mathfrak{g}}$ [23, 24]. And the right hand side of (A.27) should be proportional to $c_{\mathfrak{g}}$, because the stress tensor and the R-current belong to the same multiplet.[32] The same thing can be seen in $\mathcal{N}=4$ SYM [44].

[31] We leave the explicit computations of these correlators elsewhere.

[32] By representation theory, the stress tensor belongs to a half BPS multiplet. In superspace, the 2-, 3- and 4-point functions of all half BPS multiplets are known to admit a unique structure [59–61].

## B Data of simply-laced Lie algebra $\mathfrak{g}$

| $\mathfrak{g}$ | $r_\mathfrak{g}$ | $h^\vee_\mathfrak{g}$ | $d_\mathfrak{g}$ | $\bar{a}_\mathfrak{g}$ | $\bar{c}_\mathfrak{g}$ | $\bar{a}_\mathfrak{g}/\bar{c}_\mathfrak{g}$ |
|---|---|---|---|---|---|---|
| $A_{n-1}$ | $n-1$ | $n$ | $n^2-1$ | $\frac{16}{7}n^3-\frac{9}{7}n-1$ | $4n^3-3n-1$ | $\frac{3}{7(2n+1)^2}+\frac{4}{7}$ |
| $D_n$ | $n$ | $2n-2$ | $n(2n-1)$ | $\frac{64}{7}n^3-\frac{96}{7}n^2+\frac{39}{7}n$ | $16n^3-24n^2+9n$ | $\frac{3}{7(3-4n)^2}+\frac{4}{7}$ |
| $E_6$ | 6 | 12 | 78 | $\frac{15018}{7}$ | 3750 | $\sim 0.572114$ |
| $E_7$ | 7 | 18 | 133 | 5479 | 9583 | $\sim 0.571742$ |
| $E_8$ | 8 | 30 | 248 | $\frac{119096}{7}$ | 29768 | $\sim 0.571544 > \frac{4}{7}$ |

**Table 1**. The rank $r_\mathfrak{g}$, dual Coxeter number $h^\vee_\mathfrak{g}$, dimension $d_\mathfrak{g}$ of the simply-laced Lie algebras and the normalized $a, c$ anomalies for the associated $6d$ $(2,0)$ SCFTs [22].



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