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The dual formulation of M5-brane action

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Abstract: We construct a dual formulation, with respect to the conventional PST formalism, of the M5-brane action propagating in a generic 11d supergravity background. Constraint analysis is performed to further justify that our theory has the correct number of degrees of freedom. Comparison of this action with the existing M5-brane actions is carried out. We also show that a conventional D4-brane action is obtained upon double dimensional reduction.

Keywords: Duality in Gauge Field Theories, Gauge Symmetry, M-Theory, Space-Time Symmetries

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1 Introduction

In the literature, two main approaches are used to describe dynamics of M5-brane theory. The first approach is called the superembedding approach [1], in which a supersymmetric M5-brane worldvolume is embedded into an 11-dimensional supersymmetric target space. This approach was carried out in [2, 3]. Alternatively, the approach called the Green-Schwarz approach, in which a bosonic M5-brane worldvolume is embedded into an 11-dimensional supersymmetric target space can also be used. The first successful attempt on this approach is shown in [4–6], in which the action known as the PST action is constructed. Within the Green-Schwarz approach, it is also possible to construct alternative actions which are expected to serve some specific purposes better. In particular, with the hope to understand the connection between the five-brane proposal [7, 8] and the known complete M5 action, the alternative action [9, 10] is constructed. Although it is yet unclear whether this alternative action would eventually serve its original intended purpose, the possibility to have more than one action which fully describes a single supersymmetric M5-brane should already be a good motivation to seek further alternative actions. In this paper, we construct yet another alternative action. An attempt toward the 2+4 formulation of M5-brane action is put forward in [11]. However, it is still unclear whether the completion of [11] to an M5-brane is possible.

Making symmetries of underlying theory manifest is always beneficial, and it is no exception for duality symmetry. For example, the construction of the duality symmetric action of 11d supergravity allows its direct coupling to both the M2– and M5-branes [12]. For us, we are interested in the duality-symmetric worldvolume action for the M5-brane. In [13, 14], in order to reflect the duality property of M5-brane, the dual action to the quadratic PST action [15] for chiral 2-form is constructed. This action, however, still does not describe the M5-brane as the non-linearisation and the couple to the other fields have yet to be implemented. The main goal in our paper is to obtain the M5-brane action in the dual formalism.

In order to achieve this goal, we start from considering the quadratic action for the chiral 2-form in the dual formulation. In a sense, this is obtained from using the gauge freedom to fix the auxiliary field of the model in [13, 14]. We have checked that despite the fact that the Lorentz symmetry is not manifest due to a certain space-like direction is singled out, this action can be shown to have the modified version of Lorentz symmetry. We next couple this action with 6d gravity and show that it possesses a modified version of diffeomorphism symmetry. Next, we non-linearise the action by utilising the idea of [16–18], in which one starts from the known Hamiltonian for the gauge-fixed PST action, relax certain constraints, and then work out the action. Having obtained the linearised action, the extension to the M5-brane action is straightforward. The most non-trivial check is whether the action possesses the kappa symmetry. We have shown that this is the case.

In [4, 6], it was shown that the double dimensional reduction of the gauge-fixed PST M5-brane action gives rise to the dual D4-brane action [19]. With the dual nature of the dual 1+5 M5 action, it is anticipated that a standard D4 action [20, 21] written in terms of a worldvolume vector gauge field would be obtained upon double dimensional reduction. We have carried out the dimensional reduction and found that it is indeed the case.

The paper is organized as follows. In section 2, we first review the M5-brane action in the PST formulation, and then present the M5-brane action in the dual 1+5 formulation. Its derivation is shown in section 3. In section 4, the constraint analysis of the action is discussed. This presents further verifications on the action. In section 5, we show that the on-shell values of the dual 1+5 action equals to those of the 1+5 and 3+3 actions. It is shown in section 6 that the double dimensional reduction of dual 1+5 M5 action gives directly the standard D4 action. In Conclusion we summarise our results and discuss some open problems and possible future works.

2 The M5-brane actions

The action for a single supersymmetric M5-brane in the Green-Schwarz approach describes an M5-brane embedded into an 11 dimensional target superspace. Within this approach its first formulation, known as the PST formulation, is presented in [5, 6]. In the PST formulation, an auxiliary scalar field is introduced. There is a local gauge symmetry which reflects the auxiliary nature of this field. After a gauge fixing of this symmetry, the auxiliary field is identified as one of the coordinates of the 6d worldvolume. As a result, the 6d worldvoulme indices are separated into 1d and 5d ones. The 6d covariance is therefore no

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longer manifest. However, it can be shown that the resulting theory still has the full 6d diffeomorphism symmetry, which is modified. Furthermore, as in the case of the original PST formulation, the resulting action legitimately describes a single supersymmetric M5-brane in a generic 11d supergravity superbackground. Therefore due to the way the indices are separated, we call this action, which is a result of the gauge-fixing of the auxiliary field from the one in PST formulation, as being in the 1+5 formulation. Note that in this formulation, the singled out direction can either be space-like or time-like. In particular, we will call the 1+5 formulation in which the space-like direction is singled out as the PS1+5 formulation, whereas the one in which the time-like direction is singled out will be called the HT1+5 formulation.

More recently it is shown by construction in [10] that, also within the Green-Schwarz approach, there exists an alternative formulation of a single supersymmetric M5-brane. In this formulation, there are three auxiliary scalar fields. After the gauge-fixing of these scalar fields, the 6d worldvolume indices are separated into 3d and 3d ones. The resulting theory also legitimately describes a single supersymmetric M5-brane, and is said as being in the 3+3 formulation. Having an extra formulation at hand, it is natural to expect that this would eventually prove useful in order to understand more about the nature of M5-brane, and of course it would be natural to seek for other formulations. In [11], an attempt was made in order to construct the 2+4 formulation. However, it has not yet been clear whether such a construction would be possible.

In this paper, we construct and present yet another formulation, called the dual 1+5 formulation. To the best of our knowledge, the complete M5-brane action in the dual 1+5 formulation has not been presented nor discussed before in the literature. As for the case of the 1+5 formulation, one direction on the 6d worldvolume is singled out. However, the singled out direction can only be space-like. This feature is different from the 1+5 formulation, in which the singled out direction can either be space-like or time-like.

In this paper, the signature of the metric of the 11-dimensional target superspace is taken to be mostly plus. It is parametrized by $Z^{\mathcal{M}} = (X^M, \theta)$, in which X^M are eleven bosonic coordinates and θ are 32 real fermionic coordinates. The geometry of the 11d supergravity are described by tangent-space vector super-vielbeins $E^A(Z) = dZ^{\mathcal{M}} E_{\mathcal{M}}{}^A(Z)$ $(A = 0, 1, 2, \cdots, 10)$ and Majorana-spinor super-vielbeins $E^\alpha(Z) = dZ^{\mathcal{M}} E_{\mathcal{M}}{}^\alpha(Z)$ $(\alpha = 1, 2, \cdots, 32)$.

The vector super-vielbein satisfies the following essential torsion constraint, which is required for proving the kappa-symmetry of the $M5$-brane action,

$$T^A = DE^A = dE^A + E^B \Omega_B{}^A = -iE^\alpha \Gamma^A_{\alpha\beta} E^\beta \,, \tag{2.1}$$

where $\Omega_B{}^A(Z)$ is the 1-form spin connection in eleven dimension, $\Gamma^A_{\alpha\beta} = \Gamma^A_{\beta\alpha}$ are real symmetric gamma matrices and the exterior differential acts from the right.

The coordinates x^μ $(\mu = 0, 1, \cdots, 5)$ parametrize the worldvolume of the M5-brane which carries the chiral 2-form gauge field $B_2(x) = \frac{1}{2} dx^\mu dx^\nu B_{\nu\mu}(x)$. The induced metric on the $M5$-brane worldvolume is constructed with the pull-backs of the vector super-

vielbeins $E^A(Z)$

$$g_{\mu\nu}(x) = E^A_\mu E^B_\nu \eta_{AB}, \qquad E^A_\mu = \partial_\mu Z^{\mathcal{N}} E_{\mathcal{N}}{}^A(Z(x)). \tag{2.2}$$

The $M5$-brane couples to the 11d supergravity 3-form gauge superfield, $C_3(Z) = \frac{1}{3!} dZ^{\mathcal{M}_1} dZ^{\mathcal{M}_2} dZ^{\mathcal{M}_3} C_{\mathcal{M}_3\mathcal{M}_2\mathcal{M}_1}$, and its $C_6(Z)$ dual. Their field strengths are constrained as follows

$$\begin{aligned} dC_3 &= -\frac{i}{2} E^A E^B E^\alpha E^\beta (\Gamma_{BA})_{\alpha\beta} + \frac{1}{4!} E^A E^B E^C E^D F^{(4)}_{DCBA}(Z)\,, \\ dC_6 - C_3 dC_3 &= \frac{2i}{5!} E^{A_1}\cdots E^{A_5} E^\alpha E^\beta (\Gamma_{A_5\cdots A_1})_{\alpha\beta} + \frac{1}{7!} E^{A_1}\cdots E^{A_7} F^{(7)}_{A_7\cdots A_1}(Z) \\ F^{(7)\,A_1\cdots A_7} &= \frac{1}{4!}\epsilon^{A_1\cdots A_{11}} F^{(4)}_{A_8\cdots A_{11}}\,, \qquad \epsilon^{0\ldots 10} = -\epsilon_{0\ldots 10} = 1. \end{aligned} \tag{2.3}$$

The extended field strengths of $B_2(x)$ which appears in the M5-brane action is

$$H_3 = dB_2 + C_3\,, \tag{2.4}$$

where $C_3(Z(x))$ is the pullback of the 3-form gauge field on the M5-brane worldvolume .

Having discussed the background set-up, we next proceed by briefly reviewing the original form of the M5-brane action and then will present our main result, namely, the alternative worldvolume action for the M5-brane in a generic $D = 11$ supergravity background.

2.1 Original M5-brane action

In this case to ensure the $6d$ worldvolume covariance of the $M5$-brane action one uses an auxiliary scalar field $a(x)$, whose gradient $\partial_\mu a$ could be either time-like, i.e. in a certain gauge, $\partial_\mu a = \delta^0_\mu$ or space-like $\partial_\mu a = \delta^5_\mu$.

The $M5$-brane action in a generic $D = 11$ supergravity superbackground constructed in [4–6] has the following form:

$$\begin{aligned} S = &-\int_{\mathcal{M}_6} d^6x \left[\sqrt{-\det\left(g_{\mu\nu} + i\frac{\partial^\rho a}{\sqrt{(\partial a)^2}}\tilde{\tilde{H}}_{\rho\mu\nu}\right)} + \frac{\sqrt{-g}}{4(\partial a)^2}\partial_\lambda a \tilde{\tilde{H}}^{\lambda\mu\nu} H_{\mu\nu\rho}\partial^\rho a\right] \\ &+\frac{1}{2}\int_{\mathcal{M}_6}(C_6 + H_3 \wedge C_3)\,, \end{aligned} \tag{2.5}$$

with

$$\tilde{\tilde{H}}^{\rho\mu\nu} \equiv \frac{1}{6\sqrt{-g}}\,\epsilon^{\rho\mu\nu\lambda\sigma\tau} H_{\lambda\sigma\tau}, \quad g = \det g_{\mu\nu}\,, \tag{2.6}$$

where

$$\epsilon^{0\cdots 5} = -\epsilon_{0\cdots 5} = 1\,.$$

By utilising local gauge symmetries, it is possible to set $a = x^5$. The action then becomes

$$\begin{aligned} S = &-\int_{\mathcal{M}_6} d^6x \left[\sqrt{-\det(g_{\mu\nu} + i(\tilde{\tilde{H}}\cdot u)_{\mu\nu})} + \frac{\sqrt{-g}}{4}(\tilde{\tilde{H}}\cdot u)^{\mu\nu}(H\cdot u)_{\mu\nu}\right] \\ &+\frac{1}{2}\int_{\mathcal{M}_6}(C_6 + H_3 \wedge C_3)\,, \end{aligned} \tag{2.7}$$

with

$$(\tilde{\tilde{H}}\cdot u)_{\mu\nu}\equiv\tilde{\tilde{H}}_{\mu\nu\rho}u^{\rho},\qquad (H\cdot u)_{\mu\nu}\equiv H_{\mu\nu\rho}u^{\rho} \tag{2.8}$$

where

$$u_{\lambda}\equiv\frac{\delta^{5}_{\lambda}}{\sqrt{g^{55}}},\qquad u^{\lambda}\equiv\frac{g^{\lambda 5}}{\sqrt{g^{55}}}, \tag{2.9}$$

In the action (2.7), the 6d indices on the M5-brane worldvolume are separated into the 5d indices and the index 5. The 6d indices are represented by the Greek letters $\mu,\nu,\cdots=0,1,2,3,4,5$ while the 5d indices are represented by the underlined latin indices $\underline{a},\underline{b},\cdots=0,1,2,3,4$. Despite the explicit separation of the indices, the action still possess the diffeomorphism symmetries. See [22, 23] for example.

In addition to the conventional abelian gauge symmetry for the chiral 2-form, the action (2.7) has the following local gauge symmetry:

$$\delta B_{\underline{a}\underline{b}}=0,\qquad \delta B_{5\underline{a}}=\Phi_{\underline{a}}(x), \tag{2.10}$$

with $\Phi_{\underline{a}}(x)$ being arbitrary local functions on the woldvolume. The symmetry (2.10) ensures that the equation of motion of B_2 reduces to the non-linear self-duality condition

$$(H\cdot u)_{\mu\nu}=\mathcal{U}_{\mu\nu}(\tilde{\tilde{H}})\,, \tag{2.11}$$

where

$$\mathcal{U}^{\mu\nu}(\tilde{\tilde{H}})\equiv-2\,\frac{\delta\sqrt{\det(\delta^{\nu}_{\mu}+i(\tilde{\tilde{H}}\cdot u)_{\mu}{}^{\nu})}}{\delta(\tilde{\tilde{H}}\cdot u)_{\mu\nu}}. \tag{2.12}$$

The action (2.7) is also invariant under the local fermionic kappa-symmetry transformations with the parameter $\kappa^{\alpha}(x)$ which acts on the pullbacks of the target-space supervielbeins and the B_2 field strength as follows

$$\begin{aligned}
&i_{\kappa}E^{\alpha}\equiv\delta_{\kappa}Z^{\mathcal{M}}E^{\alpha}_{\mathcal{M}}=\frac{1}{2}(1+\bar{\Gamma})^{\alpha}{}_{\beta}\kappa^{\beta}, && i_{\kappa}E^{A}\equiv\delta_{\kappa}Z^{\mathcal{M}}E^{A}_{\mathcal{M}}=0. \\
&\delta g_{\mu\nu}=-4iE^{\alpha}_{(\mu}(\Gamma_{\nu)})_{\alpha\beta}\,i_{\kappa}E^{\beta}, && \delta H^{(3)}=i_{\kappa}dC^{(3)}, \qquad \delta_{\kappa}a(x)=0\,,
\end{aligned} \tag{2.13}$$

where $(1+\bar{\Gamma})/2$ is the projector of rank 16 with $\bar{\Gamma}$ having the following form

$$\begin{aligned}
\sqrt{\det(\delta^{\nu}_{\mu}+i(\tilde{\tilde{H}}\cdot u)_{\mu}{}^{\nu})}\,\bar{\Gamma}&=\gamma^{(6)}-\frac{1}{2}\Gamma^{\mu\nu\lambda}u_{\mu}(\tilde{\tilde{H}}\cdot u)_{\nu\lambda} \\
&\quad-\frac{1}{16\sqrt{-g}}\epsilon^{\mu_1\cdots\mu_6}(\tilde{\tilde{H}}\cdot u)_{\mu_1\mu_2}(\tilde{\tilde{H}}\cdot u)_{\mu_3\mu_4}\Gamma_{\mu_5\mu_6}, \\
\bar{\Gamma}^2&=1\,,\qquad \mathrm{tr}\bar{\Gamma}=0,
\end{aligned} \tag{2.14}$$

where

$$\Gamma_{\mu}=E_{\mu}{}^{A}\Gamma_{A}\,,\qquad \gamma^{(6)}=\frac{1}{6!\sqrt{-g}}\epsilon^{\mu_1\cdots\mu_6}\Gamma_{\mu_1\cdots\mu_6}\,. \tag{2.15}$$

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2.2 M5-brane action in the dual formulation

In this paper, we construct an M5-brane action in the dual formulation and show that it has all the required properties, that is it is self-interacting, diffeomorphism invariant and kappa symmetric. Let us first present the action along with basic discussions.

$$S = \int_{\mathcal{M}_6} d^6x \left[-\sqrt{-g}\sqrt{\det\left(\delta_\mu^\nu + (H\cdot v)_\mu{}^\nu\right)} + \frac{\sqrt{-g}}{4}(\tilde{\tilde{H}}\cdot v)^{\mu\nu}(H\cdot v)_{\mu\nu}\right] + \frac{1}{2}\int_{\mathcal{M}_6}(C_6 + H_3\wedge C_3)\,, \tag{2.16}$$

with

$$(\tilde{\tilde{H}}\cdot v)_{\mu\nu} \equiv \tilde{\tilde{H}}_{\mu\nu\rho}v^\rho, \qquad (H\cdot v)_{\mu\nu} \equiv H_{\mu\nu\rho}v^\rho, \tag{2.17}$$

where

$$v_\lambda \equiv \frac{g_{5\lambda}}{\sqrt{g_{55}}}, \qquad v^\lambda \equiv \frac{\delta_5^\lambda}{\sqrt{g_{55}}}. \tag{2.18}$$

This theory has the semi-local gauge symmetry

$$\delta B_{\underline{ab}} = \omega_{\underline{ab}}(x^{\underline{l}}), \qquad \delta B_{\underline{a}5} = 0, \tag{2.19}$$

where $\omega_{\underline{ij}} = \omega_{[\underline{ij}]}(x^{\underline{l}})$ are arbitrary functions of 5d coordinates $x^{\underline{l}}$. This semi-local gauge symmetry can be used to ensure that the equation of motion of B_2 reduces to the non-linear self-duality condition

$$-\frac{1}{3}\frac{1}{g_{55}}g_{5[5}\epsilon^{\underline{ablmn}}H_{\underline{lmn}]} + 3\frac{\delta V}{\delta H_{\underline{ab}5}} = 0, \tag{2.20}$$

where

$$V = V(g_{\mu\nu}, H_{5\underline{ab}}) = -\sqrt{-g}\sqrt{\det\left(\delta_{\underline{a}}^{\underline{b}} + \frac{1}{\sqrt{g_{55}}}H_{5\underline{a}}{}^{\underline{b}}\right)} \tag{2.21}$$

This alternative M5-brane action is also invariant under the kappa symmetry (2.13) with

$$\sqrt{\det(\delta_\mu^\nu + H_\mu{}^\nu)}\bar{\Gamma} = \gamma^{(6)} + \frac{1}{2}v_\alpha H_{\beta\gamma}\gamma^{(6)}\Gamma^{\alpha\beta\gamma} + \frac{1}{16\sqrt{-g}}\epsilon^{\mu_1\cdots\mu_6}H_{\mu_1\mu_2}H_{\mu_3\mu_4}\Gamma_{\mu_5\mu_6}, \tag{2.22}$$

which also satisfies

$$\bar{\Gamma}^2 = 1, \qquad \mathrm{tr}\bar{\Gamma} = 0. \tag{2.23}$$

Notice that the first line of the dual 1+5 action (2.16) may be obtained from the first line of the gauge-fixed PST action (2.7) by the replacement rule

$$i(\tilde{\tilde{H}}\cdot u)^{\mu\nu} \to (H\cdot v)^{\mu\nu}, \qquad i(H\cdot u)^{\mu\nu} \to (\tilde{\tilde{H}}\cdot v)^{\mu\nu}. \tag{2.24}$$

This formal relation above between actions is a typical characterisation of a formulation and its dual. For example, the standard Dp-brane actions in terms of their worldvolume vector fields are related to their electromagnetic dual counterparts, which is written in terms of the $(p-2)-$forms by a formal replacement rule similar to the above [4, 19]. However, being a self-dual gauge theory, the M5-brane action (2.7) is invariant under the

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worldvolume dualisation of the 2-form gauge field. Nevertheless, it is found in [13, 14] that the dualisation of linearised (2.5) with respect to the auxiliary field $a(x)$ gives the covariant form of the linearised (2.16). This is why we call (2.16) the dual formulation of the M5-brane action. Refs. [13, 14] suggest that (2.16) may be covariantized by an auxiliary 4-form. However, the covariantisation issue is quite complicated and we will not touch it upon throughout this paper.

The derivation and discussions on the M5-brane action in the dual formulation (2.16) are presented in the subsequent sections.

3 Derivation

3.1 EOM from superembedding

The complete set of equations of motion of the action (2.5) has been shown [24, 25] to be equivalent to those obtained from the superembedding approach [2]. In particular, when constructing a single M5-brane action in the 3+3 formulation [10] and in the yet-incomplete 2+4 formulation [11], the chiral 2-form equations of motion obtained from the superembedding approach provide useful information on how the action which gives the required equations of motion should look like. As in the case of the other formulations, also in the dual 1+5 formulation, it is useful to discuss the equations of motion of the chiral 2-form obtained from the superembedding approach.

In the superembedding formulation of the M5-brane [2, 3] the field strength H_3 of the chiral field B_2 is expressed in terms of an auxiliary self-dual tensor $h_3 = *h_3$ as follows[1]

$$\frac{1}{4}H_{\mu\nu\rho} = m_\mu^{-1\lambda} h_{\lambda\nu\rho}\,, \qquad \frac{1}{4}\tilde{H}^{\mu_1\nu_1\rho_1} = \frac{1}{6}\epsilon^{\mu_1\nu_1\rho_1\mu\nu\rho} m_\mu^{-1\lambda} h_{\lambda\nu\rho} = Q^{-1} m^{\mu_1\lambda} h_\lambda{}^{\nu_1\rho_1} \tag{3.1}$$

where $m_\mu^{-1\lambda}$ is the inverse matrix of

$$m_\mu{}^\lambda = \delta_\mu{}^\lambda - 2k_\mu{}^\lambda\,, \qquad m_\mu^{-1\lambda} = Q^{-1}(2\delta_\mu{}^\lambda - m_\mu{}^\lambda), \qquad k_\mu{}^\lambda = h_{\mu\nu\rho}h^{\lambda\nu\rho} \tag{3.2}$$

and

$$Q = 1 - \frac{2}{3}\mathrm{tr}\, k^2\,, \qquad \tilde{H}^{\mu\nu\rho} = \frac{1}{3!}\epsilon^{\mu\nu\rho\alpha\beta\gamma}H_{\alpha\beta\gamma}. \tag{3.3}$$

As was shown in [24], by splitting the indices in eqs. (3.1) into 1+5 and expressing components of h_3 in terms of $H_{\mu\nu5}$, one gets the duality relation (in our convention)

$$\tilde{H} = \frac{\left(1 - \frac{1}{2}\mathrm{tr}(H^2)\right) H + H^3}{\sqrt{1 - \frac{1}{2}\mathrm{tr}(H^2) + \frac{1}{8}\mathrm{tr}(H^2)^2 - \frac{1}{4}\mathrm{tr}(H^4)}} \tag{3.4}$$

where H and $\tilde{H}$ are matrices with components

$$H^{\underline{a}}{}_{\underline{b}} \equiv H_5{}^{\underline{a}}{}_{\underline{b}}, \qquad \tilde{H}^{\underline{a}}{}_{\underline{b}} \equiv \tilde{H}_5{}^{\underline{a}}{}_{\underline{b}}. \tag{3.5}$$

[1] Our normalisation convention of the field strength differs from that in [24] by the factor of $\frac{1}{4}$ in front of H_3.

Inverting the equation (3.4) gives

$$H = \frac{\left(1 + \frac{1}{2}\mathrm{tr}(\tilde{H}^2)\right)\tilde{H} - \tilde{H}^3}{\sqrt{1 + \frac{1}{2}\mathrm{tr}(\tilde{H}^2) + \frac{1}{8}\mathrm{tr}(\tilde{H}^2)^2 - \frac{1}{4}\mathrm{tr}(\tilde{H}^4)}}. \tag{3.6}$$

Although both the equations (3.4) and (3.6) are essentially the same, only the latter one arises directly, as a consequence of Euler-Lagrange equation for the B_2 sector (with all background fields and other worldvolume fields turned off) of the action (2.7), which first presented by [4, 26]. On the other hand, to the best of our knowledge, the Lagrangian which directly gives rise to the equation (3.4) has not appeared before in the literature, let alone its extended version to describe the complete single M5-brane theory. Thus in this paper, we construct and present the complete single M5-brane theory in the form which serves this purpose. This action is given in the equation (2.16).

The construction of this action starts from constructing the flat space free theory and then its nonlinearisation. To achieve the latter, we appeal to the Hamiltonian analysis and apply the idea of [16–18].

3.2 Free theory in non-covariant form

Let us start by deriving the linearised version of (3.4) from an action principle.

We would like to derive the linear self-duality condition

$$H^{\mu\nu\rho} = \frac{1}{3!}\epsilon^{\mu\nu\rho\tau\sigma\lambda}H_{\tau\sigma\lambda} = \tilde{H}^{\mu\nu\rho} \tag{3.7}$$

on the 3-form field strength $H_3 = dB_2$ of a 2-form potential B_2 from a 6d Lagrangian. Consider the following 1+5 splitting of field strength,

$$H_{\mu\nu\rho} = (H_{\underline{lmn}}, H_{\underline{mn}5}), \qquad \underline{l}, \underline{m}, \underline{n} = 0, 1, 2, 3, 4. \tag{3.8}$$

The Levi-Civita symbol is split according to

$$\epsilon^{012345} = 1 = -\epsilon_{012345}, \quad \Rightarrow \quad \epsilon^{012345} = \epsilon^{01234}\epsilon^{5} = \epsilon^{01234} = -\epsilon_{01234}\epsilon_5 = -\epsilon_{01234}. \tag{3.9}$$

The Greek letters are 6d indices $\mu, \nu, \cdots = 0, 1, 2, 3, 4, 5$ while the underlined latin indices are 5d ones $\underline{l}, \underline{m}, \underline{n} = 0, 1, 2, 3, 4$. Therefore,

$$\tilde{H}^{\underline{lmn}} = \frac{1}{2!}\epsilon^{\underline{lmnpq}}H_{\underline{pq}5}, \quad \tilde{H}^{\underline{pq}5} = -\frac{1}{3!}\epsilon^{\underline{pqlmn}}H_{\underline{lmn}}. \tag{3.10}$$

The self-duality equation (3.7) could be derived from the following action:

$$S = -\frac{1}{4}\int d^6x \left(H_{\underline{mn}5}\left(H^{\underline{mn}}{}_5 - \tilde{H}^{\underline{mn}}{}_5\right)\right). \tag{3.11}$$

The action has the following semi-local gauge symmetry

$$\delta B_{\underline{mn}} = \Omega_{\underline{mn}}(x^{\underline{k}}), \qquad \delta B_{\underline{m}5} = 0, \tag{3.12}$$

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where $\Omega_{\underline{mn}}(x^{\underline{k}})$ are arbitrary functions of 5d coordinates $x^{\underline{k}}$. To be eligible as a gauge symmetry, the Noether charge associated with the semi-local symmetry must vanish at least on-shell. The conserved Noether current associated with (3.12) is

$$j^{\mu} = \frac{1}{2}\left(H^{\underline{mn}5} - \tilde{H}^{\underline{mn}5}\right)\Omega_{\underline{mn}}\delta^{\mu}_{5}. \tag{3.13}$$

It is clear that the Noether charge $Q = \int j^0 d^5x$ vanishes identically *off-shell*, as $j^0 = 0$. Had we aligned the temporal direction in the '1' of '1+5' splitting, this would not be the case. In other words, the special direction chosen in the dual 1+5 formulation must be a spatial one.

The equations of motion derived by varying (3.11) are

$$\begin{aligned}
\partial_5\left(\frac{1}{3!}\epsilon^{\underline{pqlmn}}(H_{\underline{lmn}} - \tilde{H}_{\underline{lmn}})\right) &= 0, \\
\partial_{\underline{p}}\left(\frac{1}{3!}\epsilon^{\underline{pqlmn}}(H_{\underline{lmn}} - \tilde{H}_{\underline{lmn}})\right) &= 0.
\end{aligned} \tag{3.14}$$

The general solution to the field equations (3.14) is

$$\frac{1}{3!}\epsilon^{\underline{pqlmn}}(H_{\underline{lmn}} - \tilde{H}_{\underline{lmn}}) = \frac{1}{2!}\epsilon^{\underline{pqlmn}}\partial_{\underline{l}}\omega_{\underline{mn}}(x^{\underline{k}}), \tag{3.15}$$

where $\omega_{\underline{mn}}(x^{\underline{k}})$ are arbitrary functions of 5d coordinates $x^{\underline{k}}$.

Notice that the components of the field strength $H_{\underline{mn}5}$ are invariant under the transformation (3.12). Under (3.12), the left hand side of (3.15) transforms as

$$\delta\left(\frac{1}{3!}\epsilon^{\underline{pqlmn}}(H_{\underline{lmn}} - \tilde{H}_{\underline{lmn}})\right) = \frac{1}{2!}\epsilon^{\underline{pqlmn}}\partial_{\underline{l}}\Omega_{\underline{mn}}, \tag{3.16}$$

which is in exactly the same form as the right hand side of (3.15). Therefore, by the gauge-fixing $\Omega_{\underline{mn}} = \omega_{\underline{mn}}$, one obtains the self-duality equations

$$\frac{1}{3!}\epsilon^{\underline{pqlmn}}(H_{\underline{lmn}} - \tilde{H}_{\underline{lmn}}) = 0 \tag{3.17}$$

which is obviously equivalent to (3.7).

The action (3.11) is manifestly invariant under the $\mathrm{SO}(1,4)$ subgroup of the 6d Lorentz symmetry. However, although less obvious, it is also invariant under the following modified Lorentz transformation parametrized by $\Lambda_{\underline{m}5} \equiv \Lambda_{\underline{m}}$ mixing the x^5 and other directions $x^{\underline{m}}$:

$$\begin{aligned}
\delta B_{\underline{mn}} &= \left[(\Lambda\cdot x)\partial_5 - x^5(\Lambda\cdot\partial)\right]B_{\underline{mn}} - 2\Lambda_{[\underline{m}}B_{\underline{n}]5} + x_5\Lambda^{\underline{l}}\left(H_{\underline{mnl}} - \tilde{H}_{\underline{mnl}}\right) \\
\delta B_{\underline{m}5} &= \left[(\Lambda\cdot x)\partial_5 - x^5(\Lambda\cdot\partial)\right]B_{\underline{m}5} - \Lambda^{\underline{n}}B_{\underline{mn}},
\end{aligned} \tag{3.18}$$

where $(\Lambda\cdot x) = \Lambda_{\underline{m}5}x^{\underline{m}}$ and $(\Lambda\cdot\partial) = \Lambda^{\underline{m}}{}_5\partial_{\underline{m}}$. Therefore, the action (3.11) enjoys the full 6d Lorentz symmetry. The modified Lorentz symmetry reduces to the standard one when the field strength satisfies the self-duality equation.

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The free theory introduced above could be put on a curved 6d space. Consider the following action

$$S = -\frac{1}{4}\int \mathrm{d}^6x \left(\frac{\sqrt{-g}}{g_{55}} H_{5\underline{jk}} \left(H_5{}^{\underline{jk}} - \tilde{\tilde{H}}_5{}^{\underline{jk}} \right) \right), \tag{3.19}$$

where

$$\tilde{\tilde{H}}^{\mu\nu\rho} = \frac{1}{3!\sqrt{-g}} \epsilon^{\mu\nu\rho\tau\sigma\lambda} H_{\tau\sigma\lambda}. \tag{3.20}$$

Indices in (3.19) are pulled up and down by the 6d metric $g_{\mu\nu}$ with the mostly positive signature $(-+++++)$. The action is still invariant under the semi-local gauge symmetry (3.12). By varying the action (3.19), we obtain the field equations

$$\begin{aligned} \partial_5 \left(\epsilon^{\underline{lmnpq}} \frac{4}{3g_{55}} g_{5[5} (H - \tilde{\tilde{H}})_{\underline{lmn}]} \right) &= 0, \\ \partial_{\underline{q}} \left(\epsilon^{\underline{lmnpq}} \frac{4}{3g_{55}} g_{5[5} (H - \tilde{\tilde{H}})_{\underline{lmn}]} \right) &= 0. \end{aligned} \tag{3.21}$$

The general solution to (3.21) is

$$\epsilon^{\underline{lmnpq}} \frac{4}{3g_{55}} g_{5[5} (H - \tilde{\tilde{H}})_{\underline{lmn}]} = \epsilon^{\underline{pqijk}} \partial_{\underline{k}} \omega_{\underline{ij}}(x^{\underline{l}}), \tag{3.22}$$

where $\omega_{\underline{ij}} = \omega_{\underline{[ij]}}(x^{\underline{l}})$ are arbitrary functions of 5d coordinates $x^{\underline{l}}$.

One could obtain the self-duality equations

$$\epsilon^{\underline{lmnpq}} \frac{4}{3g_{55}} g_{5[5} (H - \tilde{\tilde{H}})_{\underline{lmn}]} = 0 \tag{3.23}$$

by an appropriate gauge-fixing of the semi-local gauge symmetry (3.12).

The action (3.19) enjoys the full 6d diffeomorphism. However, the diffeomorphism transformations of $\delta_\epsilon B_{\underline{mn}}$ are modified in the directions $\xi^{\underline{l}}$. Indeed, after a somewhat lengthy algebra, one shows that the action (3.19) is invariant (up to total derivative terms) under

$$\begin{aligned} \delta_\epsilon B_{\underline{ij}} &= \xi^\mu H_{\mu\underline{ij}} - 4\frac{\xi^{\underline{p}}}{g_{55}} g_{5[5} \left(H_{\underline{ijp}]} - \tilde{\tilde{H}}_{\underline{ijp}]} \right) \\ &= \xi^\mu H_{\mu\underline{ij}} + \frac{\sqrt{-g}}{2!} \epsilon_{\underline{ijpmn}} \frac{\xi^{\underline{p}}}{g_{55}} \left(H^{\underline{mn}}{}_5 - \tilde{\tilde{H}}^{\underline{mn}}{}_5 \right), \\ \delta_\epsilon B_{5\underline{m}} &= \xi^\mu H_{\mu 5\underline{m}}. \end{aligned} \tag{3.24}$$

In the next subsection, we will generalise (3.19) to a nonlinear theory following the idea of [16–18].

3.3 Dual 1+5 Lagrangian from Hamiltonian

The HT1+5 nonlinear theory, which is the chiral 2-form part of PST M5 action (2.5) with the gauge-fixing $a = x^0$, contains primary constraints $\tilde{H}^{0\hat{a}\hat{b}} + \pi^{\hat{a}\hat{b}} = 0$ $(\hat{a}, \hat{b} = 1, 2, 3, 4, 5)$, where $\pi^{\hat{a}\hat{b}}$ are conjugate momenta to $B_{\hat{a}\hat{b}}$. Refs. [16–18] showed that one could obtain the PS1+5 nonlinear theory, which is the chiral 2-form part of (2.7), if one replaces $\tilde{H}^{0\hat{a}\hat{b}}$ in the

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HT1+5 Hamiltonian by $(\tilde{H}^{0\hat{a}\hat{b}}-\pi^{\hat{a}\hat{b}})/2$ and then relax the primary constraints $\tilde{H}^{0\hat{a}\hat{b}}+\pi^{\hat{a}\hat{b}}=0$ to $\tilde{H}^{0i5}+\pi^{i5}=0$ $(i=1,2,3,4)$. The manifest SO(5) covariant form of HT1+5 formulation would be first decomposed to SO(4) by relaxing the primary constraints, and then the indices 0 and i would be recombined to get a PS1+5 Lagrangian with the manifest SO$(1,4)$ covariance. In this section, we will apply the similar technique by relaxing the primary constraints $\tilde{H}^{0\hat{a}\hat{b}}+\pi^{\hat{a}\hat{b}}=0$ to $\tilde{H}^{0ij}+\pi^{ij}=0$. The result is expected to be the nonlinear dual 1+5 Lagrangian written in terms of the components $H_{5\underline{mn}}$, where $\underline{m},\underline{n}=0,1,2,3,4$.

Let us start from HT1+5 Lagrangian

$$\mathcal{L}=-\frac{1}{4}\frac{\tilde{H}^{0\hat{a}\hat{b}}H^{0}{}_{\hat{a}\hat{b}}}{g^{00}}-\sqrt{-g}\sqrt{\det\left(\delta^{\hat{b}}_{\hat{a}}+\frac{1}{\sqrt{\gamma}}\tilde{H}^{0}{}_{\hat{a}}{}^{\hat{b}}\right)}, \tag{3.25}$$

which is manifestly SO(5) covariant and is already in a first-order form in the gauge field B_2. The $\tilde{H}^{0}{}_{\hat{a}}{}^{\hat{b}}$ is defined as $\tilde{H}^{0}{}_{\hat{a}}{}^{\hat{b}}\equiv\tilde{H}^{0\mu\hat{b}}g_{\mu\hat{a}}$. For the future convenience of this section, let us rescale the Lagrangian $\mathcal{L}\to\mathcal{L}'=4\mathcal{L}$ and then put it in the first-order form:

$$\begin{aligned}\mathcal{L}'=&\pi^{\hat{a}\hat{b}}H_{0\hat{a}\hat{b}}+\tilde{H}^{0\hat{a}\hat{b}}H_{\hat{c}\hat{a}\hat{b}}N^{\hat{c}}-4N\sqrt{\gamma}\sqrt{1-\frac{1}{2\gamma}\mathrm{tr}\tilde{H}^{2}-\frac{1}{4\gamma^{2}}\mathrm{tr}\tilde{H}^{4}+\frac{1}{8\gamma^{2}}(\mathrm{tr}\tilde{H}^{2})^{2}}\\&+\xi_{\hat{a}\hat{b}}\left(\pi^{\hat{a}\hat{b}}+\tilde{H}^{0\hat{a}\hat{b}}\right)\end{aligned} \tag{3.26}$$

where

$$\mathrm{tr}\tilde{H}^{2}\equiv\tilde{H}^{0}{}_{\hat{a}}{}^{\hat{b}}\tilde{H}^{0}{}_{\hat{b}}{}^{\hat{a}},\qquad \mathrm{tr}\tilde{H}^{4}\equiv\tilde{H}^{0}{}_{\hat{a}}{}^{\hat{b}}\tilde{H}^{0}{}_{\hat{b}}{}^{\hat{c}}\tilde{H}^{0}{}_{\hat{c}}{}^{\hat{d}}\tilde{H}^{0}{}_{\hat{d}}{}^{\hat{a}}, \tag{3.27}$$

and the metric is Arnowitt-Deser-Misner decomposed

$$g_{\mu\nu}=\begin{pmatrix}-N^{2}+\gamma_{\hat{a}\hat{b}}N^{\hat{a}}N^{\hat{b}} & \gamma_{\hat{b}\hat{c}}N^{\hat{c}}\\ \gamma_{\hat{a}\hat{c}}N^{\hat{c}} & \gamma_{\hat{a}\hat{b}}\end{pmatrix}. \tag{3.28}$$

We define the inverse of $\gamma_{\hat{a}\hat{b}}$ and its determinant as $\gamma^{\hat{a}\hat{b}}$, and γ, respectively. The inverse of the metric is

$$g^{\mu\nu}=\begin{pmatrix}-N^{-2} & \frac{N^{\hat{b}}}{N^{2}}\\ \frac{N^{\hat{a}}}{N^{2}} & \gamma^{\hat{a}\hat{b}}-\frac{N^{\hat{a}}N^{\hat{b}}}{N^{2}}\end{pmatrix}. \tag{3.29}$$

In [16–18], the PS1+5 Hamiltonian can be obtained from the HT1+5 Hamiltonian by first modifying

$$\tilde{H}^{0\hat{a}\hat{b}}\to\frac{\tilde{H}^{0\hat{a}\hat{b}}-\pi^{\hat{a}\hat{b}}}{2}, \tag{3.30}$$

and then modifying the constraint $\pi^{\hat{a}\hat{b}}+\tilde{H}^{0\hat{a}\hat{b}}=0$ to

$$\pi^{i5}=-\tilde{H}^{0i5}. \tag{3.31}$$

Using the obtained form of the Hamiltonian, then the PS1+5 Lagrangian can be obtained.

For us, we wish to obtain the dual 1+5 Lagrangian by following the similar way. So, we start from the first-order Lagrangian given in the equation (3.26). Let us modify

$$\tilde{H}^{0\hat{a}\hat{b}}\to\tilde{\mathcal{F}}^{\hat{a}\hat{b}}\equiv\frac{\tilde{H}^{0\hat{a}\hat{b}}-\pi^{\hat{a}\hat{b}}}{2}. \tag{3.32}$$

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Then modify the constraint $\pi^{\hat{a}\hat{b}} + \tilde{H}^{0\hat{a}\hat{b}} = 0$ to

$$\pi^{ij} = -\tilde{H}^{0ij}. \tag{3.33}$$

This gives

$$\tilde{\mathcal{F}}^a{}_b = \frac{\gamma_{b5}}{\gamma_{55}}\tilde{\mathcal{F}}^a{}_5 + \left(\gamma_{bc} - \frac{\gamma_{c5}\gamma_{b5}}{\gamma_{55}}\right)\tilde{H}^{0ac} \tag{3.34}$$

$$\tilde{\mathcal{F}}^5{}_b = -\frac{\gamma_{bc}}{\gamma_{55}}\tilde{\mathcal{F}}^c{}_5 + \frac{\gamma_{bc}}{\gamma_{55}}\tilde{H}^{0cd}\gamma_{d5} \tag{3.35}$$

$$\tilde{\mathcal{F}}^5{}_5 = -\frac{\gamma_{c5}}{\gamma_{55}}\tilde{\mathcal{F}}^c{}_5 \tag{3.36}$$

$$\tilde{\mathcal{F}}^a{}_5 = \tilde{\mathcal{F}}^a{}_5, \tag{3.37}$$

where $\tilde{\mathcal{F}}^{\hat{a}}{}_{\hat{b}} = \tilde{\mathcal{F}}^{\hat{a}\hat{c}}\gamma_{\hat{c}\hat{b}}$ and we have intended to maintain the matrix form of $\tilde{\mathcal{F}}$ which has one index up and one index down. Define $\tilde{p}^{\hat{a}\hat{b}} = \tilde{p}^{[\hat{a}\hat{b}]}$ such that

$$\tilde{p}^{ab} = \tilde{p}^{55} = 0, \qquad \tilde{p}^{5a} = -\tilde{p}^{a5} = \frac{\tilde{\mathcal{F}}^a{}_5}{g_{55}} \tag{3.38}$$

and $\tilde{T}_{\hat{b}}{}^{\hat{a}}$ as

$$\tilde{T}_b{}^a = \left(\gamma_{bc} - \frac{\gamma_{c5}\gamma_{b5}}{\gamma_{55}}\right)\tilde{H}^{0ac}, \qquad \tilde{T}_b{}^5 = \frac{\gamma_{bc}}{\gamma_{55}}\tilde{H}^{0cd}\gamma_{d5}, \qquad \tilde{T}_5{}^{\hat{a}} = 0, \tag{3.39}$$

so that we have

$$\tilde{\mathcal{F}}^{\hat{a}}{}_{\hat{b}} = g_{\hat{b}\hat{c}}\tilde{p}^{\hat{c}\hat{a}} + \tilde{T}_{\hat{b}}{}^{\hat{a}} = \tilde{p}_{\hat{b}}{}^{\hat{a}} + \tilde{T}_{\hat{b}}{}^{\hat{a}}. \tag{3.40}$$

The matrix $\tilde{T}_{\hat{b}}{}^{\hat{a}}$ encodes the components H_{5ab}, while $\tilde{p}^{5a}$ contains the conjugate momenta π^{a5} which must be replaced with its equation of motion in order to get the Lagrangian of the theory.

It is natural to expect the resulting dual 1+5 theory is described by the SO(1, 4) covariant tensor $H_{5\underline{ab}}$ ($\underline{a}, \underline{b} = 0, 1, 2, 3, 4$). Therefore, following the similar spirit of decomposition, we do the SO(1, 4) → SO(4) decomposition by writing

$$H_5{}^a{}_b = \frac{g^{a0}}{g^{00}}H_5{}^0{}_b + \gamma^{ac}H_{5cb} \tag{3.41}$$

$$H_5{}^a{}_0 = -\frac{g^{ab}}{g^{00}}H_5{}^0{}_b - \gamma^{ab}N^cH_{5cb} \tag{3.42}$$

$$H_5{}^0{}_0 = -\frac{g^{0a}}{g^{00}}H_5{}^0{}_a \tag{3.43}$$

$$H_5{}^0{}_b = H_5{}^0{}_b. \tag{3.44}$$

Let us define $p_{\underline{ab}} = p_{[\underline{ab}]}$ such that

$$p_{ab} = p_{00} = 0, \qquad p_{0a} = -p_{a0} = \frac{H_5{}^0{}_a}{g^{00}} \tag{3.45}$$

and $T^{\underline{a}}{}_{\underline{b}}$ such that

$$T^{0}{}_{\underline{b}} = 0, \qquad T^{a}{}_{b} = \gamma^{ac} H_{5cb}, \qquad T^{a}{}_{0} = -\gamma^{ab} N^{c} H_{5cb}. \tag{3.46}$$

Therefore,

$$H_{5}{}^{\underline{a}}{}_{\underline{b}} = g^{\underline{ac}} p_{\underline{cb}} + T^{\underline{a}}{}_{\underline{b}}. \tag{3.47}$$

After performing our version of [16–18] procedure, the full nonlinear Lagrangian is then

$$\begin{aligned} \mathcal{L}' = & -\tilde{H}^{0ab} H_{0ab} + 2\tilde{H}^{0a5} H_{0a5} - 4\tilde{p}^{a5} p_{a0} + 4\frac{\tilde{H}^{0ac}\gamma_{c5} H^{0}{}_{a5}}{\gamma_{55} g^{00}} + \frac{1}{2}\tilde{H}^{0ab}\epsilon_{abmn}\tilde{H}^{0mn} N^{5} \\ & - 4N\sqrt{\gamma}\sqrt{\mathcal{A} + \mathcal{C}_{ab}\tilde{p}^{a5}\tilde{p}^{b5}}, \end{aligned} \tag{3.48}$$

where

$$\mathcal{A} = 1 - \frac{1}{\gamma}\tilde{T}_{a}{}^{[b}\tilde{T}_{b}{}^{a]} - \frac{1}{\gamma^{2}}\tilde{T}_{a}{}^{[b}\tilde{T}_{b}{}^{c}\tilde{T}_{c}{}^{d}\tilde{T}_{d}{}^{a]} \tag{3.49}$$

$$\mathcal{C}_{ab} = (-g_{55}g_{mb} + g_{m5}g_{b5})\left(-\frac{1}{\gamma}\delta^{m}_{a} - \frac{1}{\gamma^{2}}\left(\tilde{T}_{a}{}^{n}\tilde{T}_{n}{}^{m} - \frac{1}{2}\delta^{m}_{a}\tilde{T}_{i}{}^{j}\tilde{T}_{j}{}^{i}\right)\right). \tag{3.50}$$

To get the final form of the Lagrangian, we need to replace $\tilde{p}^{a5}$ (which contains the conjugate momenta π^{a5}) with its equation of motion. The equation of motion of $\tilde{p}^{a5}$ is

$$0 = \frac{\delta\mathcal{L}}{\delta\tilde{p}^{a5}} = -4p_{a0} - 4\frac{N\sqrt{\gamma}}{\sqrt{\mathcal{A} + \mathcal{C}_{ab}\tilde{p}^{a5}\tilde{p}^{b5}}}(\mathcal{C}_{ab}\tilde{p}^{b5}), \tag{3.51}$$

or

$$0 = p_{a0} + \frac{N\sqrt{\gamma}}{\sqrt{\mathcal{A} + \mathcal{C}_{ab}\tilde{p}^{a5}\tilde{p}^{b5}}}(\mathcal{C}_{ab}\tilde{p}^{b5}). \tag{3.52}$$

Using the above equation of motion to rewrite $-4\tilde{p}^{a5}p_{a0}$ in the Lagrangian. This gives

$$\begin{aligned} \mathcal{L}' = & -\tilde{H}^{0ab} H_{0ab} + 2\tilde{H}^{0a5} H_{0a5} + \frac{1}{2}\tilde{H}^{0ab}\epsilon_{abmn}\tilde{H}^{0mn} N^{5} - 4\frac{1}{g^{00}} H^{0}{}_{d5}\tilde{H}^{0nd}\frac{\gamma_{n5}}{\gamma_{55}} \\ & - 4\frac{\mathcal{A}N\sqrt{\gamma}}{\sqrt{\mathcal{A} + \mathcal{C}_{ab}\tilde{p}^{a5}\tilde{p}^{b5}}} \\ = & \frac{\sqrt{-g}}{g_{55}} H_{5\underline{ab}}\tilde{\tilde{H}}_{5}{}^{\underline{ab}} - 4\frac{\mathcal{A}N\sqrt{\gamma}}{\sqrt{\mathcal{A} + \mathcal{C}_{ab}\tilde{p}^{a5}\tilde{p}^{b5}}} \end{aligned} \tag{3.53}$$

By noting

$$T^{a}{}_{m}T^{m}{}_{n}\gamma^{nb} = \frac{1}{2}\frac{\gamma_{55}}{\gamma}\tilde{T}_{m}{}^{n}\tilde{T}_{n}{}^{m}\gamma^{ab} - \frac{\gamma_{55}}{\gamma}\gamma^{am}\tilde{T}_{m}{}^{n}\tilde{T}_{n}{}^{b}, \tag{3.54}$$

and using (3.52) we can obtain

$$\mathcal{L}' = \frac{\sqrt{-g}}{g_{55}} H_{5\underline{ab}}\tilde{\tilde{H}}_{5}{}^{\underline{ab}} - 4N\sqrt{\gamma}\sqrt{X + Z^{ab}p_{a0}p_{b0}}, \tag{3.55}$$

where

$$X = 1 - \frac{1}{g_{55}} T^a{}_{[b} T^b{}_{a]} - \frac{1}{g_{55}^2} T^a{}_{[b} T^b{}_c T^c{}_d T^d{}_{a]}, \tag{3.56}$$

$$Z^{ab} = -\frac{1}{g_{55}} \frac{\gamma^{ab}}{N^2} - \frac{1}{g_{55}^2} \left(\frac{1}{N^2} T^a{}_m T^m{}_n \gamma^{nb} - \frac{1}{2} \frac{1}{N^2} \gamma^{ab} T^m{}_i T^i{}_m \right) \tag{3.57}$$

Note that the manifestly SO(4) covariant expression inside the square root can be assembled back to a nice SO(1, 4) covariant form

$$X + Z^{ab} p_{a0} p_{b0} = 1 - \frac{1}{g_{55}} H_5{}^{\underline{a}}{}_{[\underline{b}} H_{|5|}{}^{\underline{b}}{}_{\underline{a}]} - \frac{1}{g_{55}^2} H_5{}^{\underline{a}}{}_{[\underline{b}} H_{|5|}{}^{\underline{b}}{}_{\underline{c}} H_{|5|}{}^{\underline{c}}{}_{\underline{d}} H_{|5|}{}^{\underline{d}}{}_{\underline{a}]}. \tag{3.58}$$

The nonlinear dual 1+5 action is then

$$S' = \int \mathrm{d}^6 x \left(\frac{\sqrt{-g}}{g_{55}} H_{5\underline{ab}} \tilde{\tilde{H}}_5{}^{\underline{ab}} - 4\sqrt{-g} \sqrt{\det \left(\delta_{\underline{a}}^{\underline{b}} + \frac{1}{\sqrt{g_{55}}} H_{5\underline{a}}{}^{\underline{b}} \right)} \right), \tag{3.59}$$

or

$$S = \int \mathrm{d}^6 x \left(\frac{1}{4} \frac{\sqrt{-g}}{g_{55}} H_{5\underline{ab}} \tilde{\tilde{H}}_5{}^{\underline{ab}} - \sqrt{-g} \sqrt{\det \left(\delta_{\underline{a}}^{\underline{b}} + \frac{1}{\sqrt{g_{55}}} H_{5\underline{a}}{}^{\underline{b}} \right)} \right). \tag{3.60}$$

As the nonlinear part of the action depends on the field strength through $H_{5\underline{ab}}$, the action still enjoys the semi-local gauge symmetry (3.12). As a result, the procedures of gauge-fixing to get the self-duality equations follows exactly the steps presented for the free theory. The upshot is that field equations of the above action is equivalent to the nonlinear self-duality equations (2.20).

Note that the action (3.60) enjoys the full 6d diffeomorphism invariance. However, the diffeomorphism transformations of $\delta_\epsilon B_{\underline{mn}}$ are modified in the directions $\xi^{\underline{l}}$. Indeed, after a somewhat lengthy algebra, one shows that the action (3.60) is invariant (up to total derivative terms) under

$$\begin{aligned} \delta_\epsilon B_{\underline{ij}} &= \xi^\mu H_{\mu\underline{ij}} - \xi^{\underline{p}} \left(4 \frac{1}{g_{55}} g_{5[5} H_{\underline{ijp}]} + 3 \epsilon_{\underline{abijp}} \frac{\delta V}{\delta H_{\underline{ab}5}} \right), \\ \delta_\epsilon B_{5\underline{m}} &= \xi^\mu H_{\mu 5\underline{m}}. \end{aligned} \tag{3.61}$$

The above transformations reduce to the standard diffeomorphism rules if the self-duality equations are satisfied.

In this subsection, we have obtained the dual 1+5 Lagrangian from the Hamiltonian. However, in order for this derivation to be justified, one of the requirements is to show that the theory has the correct number of degrees of freedom. In the next section, we will show that this is indeed the case.

4 Constraint analysis

Recall that from section 3.3, we have used the first-order Lagrangian for dual 1+5 theory

$$\begin{aligned} \mathcal{L}' = {} & \pi^{\hat{a}\hat{b}} H_{0\hat{a}\hat{b}} + \frac{1}{2} \tilde{\mathcal{F}}^{\hat{a}\hat{b}} \tilde{\mathcal{F}}^{\hat{x}\hat{y}} \epsilon_{\hat{a}\hat{b}\hat{c}\hat{x}\hat{y}} N^{\hat{c}} \\ & - 4N\sqrt{\gamma} \sqrt{1 - \frac{1}{2\gamma} \mathrm{tr}\tilde{\mathcal{F}}^2 - \frac{1}{4\gamma^2} \mathrm{tr}\tilde{\mathcal{F}}^4 + \frac{1}{8\gamma^2} (\mathrm{tr}\tilde{\mathcal{F}}^2)^2} \\ & + \xi_{ij} (\pi^{ij} + \tilde{H}^{0ij}) \end{aligned} \tag{4.1}$$

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to arrive at the non-linear dual 1+5 action (3.59). By reading off from the first-order Lagrangian (4.1), the Hamiltonian and momentum densities are then given by The Hamiltonian and momentum densities are then given by

$$\mathcal{H}_0 = 4\sqrt{\gamma}\sqrt{1 - \frac{1}{2\gamma}\mathrm{tr}\tilde{\mathcal{F}}^2 - \frac{1}{4\gamma^2}\mathrm{tr}\tilde{\mathcal{F}}^4 + \frac{1}{8\gamma^2}(\mathrm{tr}\tilde{\mathcal{F}}^2)^2}, \tag{4.2}$$

$$\mathcal{H}_{\hat{c}} = -\frac{1}{2}\tilde{\mathcal{F}}^{\hat{a}\hat{b}}\tilde{\mathcal{F}}^{\hat{x}\hat{y}}\epsilon_{\hat{a}\hat{b}\hat{c}\hat{x}\hat{y}}. \tag{4.3}$$

After combining the Lagrangian (4.1) with that of 6d gravity, one then follows for example the Dirac constraint analysis [27, 28]. Finally, One obtains the first-class constraints:

$$\Pi_\mu \approx 0\ (6), \qquad \pi^{0\hat{a}} \approx 0\ (5), \qquad \mathcal{H}_\mu \approx 0\ (6), \qquad \partial_{\hat{a}}\pi^{\hat{a}\hat{b}} \approx 0\ (4), \tag{4.4}$$

where Π_μ is the conjugate momenta of N^μ, and the second-class constraints:

$$\tilde{H}^{0ab} + \pi^{ab} \approx 0\ (6). \tag{4.5}$$

In total, the nonlinear dual 1+5 chiral 2-form theory has 21 first-class constraints and 6 second-class constraints. As there are 72 phase space variables (30 from $B_{\mu\nu}$ and $\pi^{\mu\nu}$ and 42 from $g_{\mu\nu}$ and $\Pi^\mu, \zeta^{\hat{m}\hat{n}}$), the number of degrees of freedom is then

$$\frac{72 - 2\times 21 - 6}{2} = 12 = 9 + 3. \tag{4.6}$$

We thus have 9 propagating degrees of freedom for graviton and 3 for chiral 2-form.

By using the identities

$$(\tilde{\mathcal{F}}^3)_{\hat{a}\hat{b}}\epsilon^{\hat{p}\hat{a}\hat{b}\hat{x}\hat{y}}\tilde{\mathcal{F}}_{\hat{x}\hat{y}} = \frac{1}{4}\mathrm{tr}(\tilde{\mathcal{F}}^2)\epsilon^{\hat{p}\hat{x}\hat{y}\hat{m}\hat{n}}\tilde{\mathcal{F}}_{\hat{m}\hat{n}}\tilde{\mathcal{F}}_{\hat{x}\hat{y}}, \tag{4.7}$$

$$(\tilde{\mathcal{F}}^3)_{\hat{a}\hat{b}}\epsilon^{\hat{p}\hat{a}\hat{b}\hat{x}\hat{y}}(\tilde{\mathcal{F}}^3)_{\hat{x}\hat{y}} = -\left(\frac{1}{4}\mathrm{tr}(\tilde{\mathcal{F}}^4) - \frac{1}{8}\mathrm{tr}(\tilde{\mathcal{F}}^2)^2\right)\epsilon^{\hat{p}\hat{m}\hat{n}\hat{x}\hat{y}}\tilde{\mathcal{F}}_{\hat{m}\hat{n}}\tilde{\mathcal{F}}_{\hat{x}\hat{y}}, \tag{4.8}$$

it can be shown that the hypersurface deformation algebra [29]

$$[\mathcal{H}_0^{\mathrm{full}}(x), \mathcal{H}_0^{\mathrm{full}}(x')] = (\gamma^{\hat{a}\hat{b}}(x)\mathcal{H}_{\hat{a}}^{\mathrm{full}}(x) + \gamma^{\hat{a}\hat{b}}(x')\mathcal{H}_{\hat{a}}^{\mathrm{full}}(x'))\partial_{\hat{b}}\delta^{(5)}(x, x'), \tag{4.9}$$

$$[\mathcal{H}_{\hat{a}}^{\mathrm{full}}(x), \mathcal{H}_0^{\mathrm{full}}(x')] = \mathcal{H}_0^{\mathrm{full}}(x)\partial_{\hat{a}}\delta^{(5)}(x, x') + \partial_{\hat{m}}\pi^{\hat{m}\hat{n}}(x)\frac{\delta\mathcal{H}_0}{\delta\tilde{\mathcal{F}}^{\hat{n}\hat{a}}}(x)\delta^{(5)}(x, x'), \tag{4.10}$$

$$\begin{aligned}[\mathcal{H}_{\hat{a}}^{\mathrm{full}}(x), \mathcal{H}_{\hat{b}}^{\mathrm{full}}(x')] = {} & \mathcal{H}_{\hat{a}}^{\mathrm{full}}(x')\partial_{\hat{b}}\delta^{(5)}(x, x') + \mathcal{H}_{\hat{b}}^{\mathrm{full}}(x)\partial_{\hat{a}}\delta^{(5)}(x, x') \\ & + \partial_{\hat{m}}\pi^{\hat{m}\hat{n}}(x)\epsilon_{\hat{b}\hat{j}\hat{k}\hat{a}\hat{n}}(x)\tilde{\mathcal{F}}^{\hat{j}\hat{k}}(x)\delta^{(5)}(x, x'),\end{aligned} \tag{4.11}$$

where $\mathcal{H}_\mu^{\mathrm{full}} = \mathcal{H}_\mu^{(g)} + \mathcal{H}_\mu$, is satisfied. The pure gravity energy and momentum densities are given by

$$\mathcal{H}_0^{(g)} = -\sqrt{\gamma}R + \frac{1}{\sqrt{\gamma}}\left(\zeta^{\hat{i}\hat{j}}\zeta^{\hat{m}\hat{n}}\gamma_{\hat{i}\hat{m}}\gamma_{\hat{j}\hat{n}} - \frac{1}{2}(\zeta^{\hat{m}\hat{n}}\gamma_{\hat{m}\hat{n}})^2\right), \qquad \mathcal{H}_{\hat{m}}^{(g)} = -2\gamma_{\hat{m}\hat{n}}\nabla_{\hat{p}}\zeta^{\hat{n}\hat{p}}, \tag{4.12}$$

where $\gamma_{\hat{m}\hat{n}}$ is spatial 5d metric, $\zeta^{\hat{m}\hat{n}}$ is the conjugate momenta to $\gamma_{\hat{m}\hat{n}}$, R is 5d Ricci scalar and $\nabla_{\hat{m}}$ is γ-compatible covariant derivative.

By following for example the procedures outlined in [28], it can be shown that the Hamiltonian density $\mathcal{H}_0^{\mathrm{full}}$ and momentum densities $\mathcal{H}_{\hat{a}}^{\mathrm{full}}$ generate the modified diffeomorphism transformation presented in equation (3.61).

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5 Comparison of on-shell actions

Although the duality-symmetric actions corresponding to formulations with different splittings of space-time are different off-shell, they should agree with each other on-shell. For free chiral 2-form theories, it was found that the free theory actions with different splittings all vanish on-shell in [9]. In [10], it was shown that the chiral 2-form part of both 1+5 and 3+3 M5-brane action agree with each other and is given by

$$S^{(\text{on-shell})} = -\int d^6x\sqrt{-g}\frac{2}{Q} + \frac{1}{2}\int_{\mathcal{M}_6}(C_6 + H_3 \wedge C_3) \tag{5.1}$$

on-shell.[2] Physically, the on-shell value of the chiral 2-form part of the M5 action determines the tension of the string soliton [26]. This on-shell property of duality-symmetric action was used in [11] to obtain the nonlinearisation of the 2+4 action.

To put the dual 1+5 M5-brane action on-shell, the superembedding equations

$$\begin{aligned}\tilde{H}_{\underline{ab}5} &= 4Q^{-1}\left((1-2\text{tr}f^2)f_{\underline{ab}} + 8(f^3)_{\underline{ab}}\right),\\ H_{\underline{ab}5} &= 4Q^{-1}\left((1+2\text{tr}f^2)f_{\underline{ab}} - 8(f^3)_{\underline{ab}}\right),\end{aligned} \tag{5.2}$$

where $f_{\underline{ab}} = h_{\underline{ab}5}$, are substituted into the M5-brane action. We found that the on-shell dual 1+5 M5-brane action is also given by (5.1), and hence it agrees with the on-shell actions for the 1+5 and 3+3 cases, despite the fact that all of them have different off-shell actions from one another.

The off-shell differences are of interests because their understanding may shed some light on the issue of quantising self-dual fields. However, this is still an open problem.

6 Double dimensional reduction

It is known that M-theory on a circle is dual to type IIA string theory [30, 31]. Indeed, if one wraps the M5-brane on the compact direction, one expects D4-brane be obtained. This is called double dimensional reduction because both dimensions of the worldvolume of the M5-brane as well as the target space are reduced. It was shown in [4, 6] that the gauge-fixed PST M5 action gives rise to the dual D4-brane action [19] upon double dimensional reduction. In this section, we will show that the dual 1+5 M5 action reduces to the standard D4-brane action written in terms of the worldvolume vector gauge field directly.

Let X^{10} be the compact direction that x^5 wraps on. After the dimensional reduction, only the zero Fourier modes are kept. For simplicity, let us consistently neglect the vector and scalar fields that arise from the reduction of the metric tensor. In particular, the various objects reduce according to

$$\begin{aligned} H_{\mu\nu\rho} &\to (H_{\underline{mnp}}, F_{\underline{mn}}),\\ g_{\mu\nu} &\to g_{\underline{mn}},\\ C_{\mu\nu\rho} &\to (C_{\underline{mnp}}, C_{\underline{mn}}),\\ C_{\mu_1\cdots\mu_6} &\to C_{\underline{m}_1\cdots\underline{m}_5},\\ v_\lambda \to \delta^5_\lambda, &\qquad v^\lambda \to \delta^\lambda_5, \end{aligned} \tag{6.1}$$

[2] The action in [10] is twice as our (2.5) in the convention of overall numerical normalisation.

where we have defined

$$H_{\underline{mn}5} \equiv F_{\underline{mn}}, \quad C_{\underline{mn}5} \equiv C_{\underline{mn}}, \quad C_{\underline{m}_1\cdots\underline{m}_5 5} \equiv C_{\underline{m}_1\cdots\underline{m}_5}. \tag{6.2}$$

The $C_{\underline{mnp}}$ is the Ramond-Ramond 3-form while $C_{\underline{mn}}$ serves as Kalb-Ramond field in string theory.

A straightforward computation leads to

$$S_5 = -\int d^5x \sqrt{-\det(g_{\underline{mn}} + F_{\underline{mn}})} - \int_{\mathcal{M}_5} \left(e^{F_2} \wedge (C_3 + C_5')\right)_5 , \tag{6.3}$$

where

$$C_5' = \frac{1}{2}C_5 - \frac{1}{2}C_2 \wedge C_3, \tag{6.4}$$

$$F_{\underline{mn}} = \partial_{\underline{m}} A_{\underline{n}} - \partial_{\underline{n}} A_{\underline{m}} + C_{\underline{mn}} \tag{6.5}$$

is the extended field strength with $A_{\underline{a}} \equiv B_{\underline{a}5}$. The $H_{\underline{mnp}}$ components appear in total derivative terms after reduction and hence are discarded. As a result, only $B_{\underline{a}5}$ components of the chiral 2-form survives and serves as the vector gauge field $A_{\underline{a}}$ in the D4 worldvolume. The Wess-Zumino term is written in a formal manner that only 5-forms out of the wedge product of $(C_3 + C_5')$ with the formal expansion of $\exp(F_2)$ are integrated.

The D4-brane action (6.3) obtained by double dimensional reduction of dual 1+5 M5 action is in a standard form [20, 21]. It is obtained by trivial computation without the need to further dualise any resulting worldvolume gauge field.

7 Conclusion

We have constructed a dual 1+5 formulation with respect to the conventional PST formalism for the single M5-brane action propagating in a generic 11d supergravity background. The dual 1+5 M5 action has both the required local gauge symmetries on the worldvolume, i.e. general coordinate diffeomorphism invariance and kappa symmetry, although the action is in a non-manifestly covariant form. To equalise the field equations and the self-duality conditions, a semi-local gauge symmetry is utilised. In order for this semi-local symmetry to be eligible as a gauge symmetry, the special direction singled out from 6d must be spatial. Similar restrictions on the choices of temporal direction from the subspaces of splittings of worldvolume space was also observed in 2+4 formulation [11]. The use of semi-local gauge symmetry is also necessary for theories of chiral forms in topologically nontrivial space-time [32–34]. The dual 1+5 M5-brane formulation will be even more useful if we could validate its usage on topologically nontrivial worldvolume in the future.

The construction of the dual 1+5 M5 action starts from the free theory. The detailed analysis and gauge-fixing in Lagrangian formalism is presented. To nonlinearise the free theory, we followed the idea outlined in [16–18] by switching to Hamiltonian of HT1+5 and then relaxing certain constraints and finally completing the Legendre transformation. After obtaining the nonlinear dual 1+5 theory in curved 6d space in one go, we coupled the theory to background supergravity and found the kappa symmetry completion. Extending

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the idea of [16–18], it will be remarkable if one could find other possible formulations of self-interacting chiral 2-forms in a curved space by relaxing the primary constraints of HT1+5 in a different manner.

We computed the on-shell value of the dual 1+5 M5 action and found that it is written in terms of the superembedding scalar variable as in the case of its counterpart formulations, albeit the M5 actions in different splittings disagree with each other off-shell. On the other hand, we performed the double dimensional reduction on a circle and showed that dual 1+5 M5 action reduces directly to the standard conventional D4-brane action. This is in contrast to the conventional PST M5 action for which the double dimensional reduction results in a dual D4 action.

The dual 1+5 action presented here is in a non-manifestly covariant form. Although the splitting of 6d worldvolume by picking up a special direction is similar to the case of conventional 1+5 formalism, the PST covariantisation procedure [15, 35–37] with a scalar field seems to be not doable. Instead, an auxiliary 4-form is suggested by [13, 14] to covariantise the theory. However, we found this issue to be more nontrivial than we thought at this stage, and we will leave it as a possible future work. Similar obstacles in the PST covariantisation was also found in the 2+4 formulation [11]. We hope to report progress on these issues in the near future.

In [38], the PST M5-brane action in the background of $AdS_7 \times S^4$ is regarded as the *exact* effective action (called highly effective action there) of the (2,0) superconformal field theory in the Coulomb branch. It will be interesting to verify whether the dual 1+5 M5 action as well as the 3+3 M5 action [10] satisfy all the requirements [38] to be a highly effective action. Moreover, it will be interesting as well to see how these off-shell different actions capture the same quantum nature of the (2,0) superconformal field theory.

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References

[1] D.P. Sorokin, *Superbranes and superembeddings*, *Phys. Rept.* **329** (2000) 1 [hep-th/9906142] [INSPIRE].

JHEP06(2016)022

[2] P.S. Howe and E. Sezgin, $D = 11$, $p = 5$, *Phys. Lett.* **B 394** (1997) 62 [hep-th/9611008] [INSPIRE].

[3] P.S. Howe, E. Sezgin and P.C. West, *Covariant field equations of the M-theory five-brane*, *Phys. Lett.* **B 399** (1997) 49 [hep-th/9702008] [INSPIRE].

[4] P. Pasti, D.P. Sorokin and M. Tonin, *Covariant action for a $D = 11$ five-brane with the chiral field*, *Phys. Lett.* **B 398** (1997) 41 [hep-th/9701037] [INSPIRE].

[5] I.A. Bandos et al., *Covariant action for the superfive-brane of M-theory*, *Phys. Rev. Lett.* **78** (1997) 4332 [hep-th/9701149] [INSPIRE].

[6] M. Aganagic, J. Park, C. Popescu and J.H. Schwarz, *World volume action of the M-theory five-brane*, *Nucl. Phys.* **B 496** (1997) 191 [hep-th/9701166] [INSPIRE].

[7] P.-M. Ho and Y. Matsuo, *$M5$ from $M2$*, *JHEP* **06** (2008) 105 [arXiv:0804.3629] [INSPIRE].

[8] P.-M. Ho, Y. Imamura, Y. Matsuo and S. Shiba, *$M5$-brane in three-form flux and multiple $M2$-branes*, *JHEP* **08** (2008) 014 [arXiv:0805.2898] [INSPIRE].

[9] P. Pasti, I. Samsonov, D. Sorokin and M. Tonin, *BLG-motivated Lagrangian formulation for the chiral two-form gauge field in $D = 6$ and $M5$-branes*, *Phys. Rev.* **D 80** (2009) 086008 [arXiv:0907.4596] [INSPIRE].

[10] S.-L. Ko, D. Sorokin and P. Vanichchapongjaroen, *The $M5$-brane action revisited*, *JHEP* **11** (2013) 072 [arXiv:1308.2231] [INSPIRE].

[11] S.-L. Ko and P. Vanichchapongjaroen, *Towards $2 + 4$ formulation of $M5$-brane*, *JHEP* **12** (2015) 170 [arXiv:1511.05395] [INSPIRE].

[12] I.A. Bandos, N. Berkovits and D.P. Sorokin, *Duality symmetric eleven-dimensional supergravity and its coupling to M-branes*, *Nucl. Phys.* **B 522** (1998) 214 [hep-th/9711055] [INSPIRE].

[13] A. Maznytsia, C.R. Preitschopf and D.P. Sorokin, *Duality of selfdual actions*, *Nucl. Phys.* **B 539** (1999) 438 [hep-th/9805110] [INSPIRE].

[14] A. Maznytsia, C.R. Preitschopf and D.P. Sorokin, *Dual actions for chiral bosons*, in *International workshop supersymmetries and integrable systems*, June 22–26, Dubna, Russia (1998), hep-th/9808049 [INSPIRE].

[15] P. Pasti, D.P. Sorokin and M. Tonin, *On Lorentz invariant actions for chiral p forms*, *Phys. Rev.* **D 55** (1997) 6292 [hep-th/9611100] [INSPIRE].

[16] A. De Castro and A. Restuccia, *Superfive-brane Hamiltonian and the chiral degrees of freedom*, *Phys. Rev.* **D 66** (2002) 024037 [hep-th/0204052] [INSPIRE].

[17] A. De Castro and A. Restuccia, *Master canonical action and BRST charge of the M-theory bosonic five-brane*, *Nucl. Phys.* **B 617** (2001) 215 [hep-th/0103123] [INSPIRE].

[18] A. De Castro and A. Restuccia, *On the interacting chiral gauge field theory in $D = 6$ and the off-shell equivalence of dual Born-Infeld-like actions*, *Class. Quant. Grav.* **16** (1999) 2729 [hep-th/9809135] [INSPIRE].

[19] M. Aganagic, J. Park, C. Popescu and J.H. Schwarz, *Dual D-brane actions*, *Nucl. Phys.* **B 496** (1997) 215 [hep-th/9702133] [INSPIRE].

[20] M.R. Douglas, *Branes within branes*, in the proceedings of *Strings, branes and dualities*, May 26–June 14, NATO Advanced Study Institute, Cargese, France (1997), hep-th/9512077 [INSPIRE].

JHEP06(2016)022

[21] M.B. Green, C.M. Hull and P.K. Townsend, *D-brane Wess-Zumino actions, t duality and the cosmological constant*, *Phys. Lett.* **B 382** (1996) 65 [hep-th/9604119] [INSPIRE].

[22] M. Henneaux and C. Teitelboim, *Dynamics of chiral (selfdual) P forms*, *Phys. Lett.* **B 206** (1988) 650 [INSPIRE].

[23] J.H. Schwarz, *Coupling a selfdual tensor to gravity in six-dimensions*, *Phys. Lett.* **B 395** (1997) 191 [hep-th/9701008] [INSPIRE].

[24] P.S. Howe, E. Sezgin and P.C. West, *The six-dimensional selfdual tensor*, *Phys. Lett.* **B 400** (1997) 255 [hep-th/9702111] [INSPIRE].

[25] I.A. Bandos, K. Lechner, A. Nurmagambetov, P. Pasti, D.P. Sorokin and M. Tonin, *On the equivalence of different formulations of the M-theory five-brane*, *Phys. Lett.* **B 408** (1997) 135 [hep-th/9703127] [INSPIRE].

[26] M. Perry and J.H. Schwarz, *Interacting chiral gauge fields in six-dimensions and Born-Infeld theory*, *Nucl. Phys.* **B 489** (1997) 47 [hep-th/9611065] [INSPIRE].

[27] P.A.M. Dirac, *Generalized Hamiltonian dynamics*, *Can. J. Math.* **2** (1950) 129 [INSPIRE].

[28] . Henneaux and C. Teitelboim, *Quantization of gauge systems*, Princeton University Press, Princeton U.S.A. (1992).

[29] S.A. Hojman, K. Kuchar and C. Teitelboim, *Geometrodynamics regained*, *Annals Phys.* **96** (1976) 88 [INSPIRE].

[30] P.K. Townsend, *The eleven-dimensional supermembrane revisited*, *Phys. Lett.* **B 350** (1995) 184 [hep-th/9501068] [INSPIRE].

[31] E. Witten, *String theory dynamics in various dimensions*, *Nucl. Phys.* **B 443** (1995) 85 [hep-th/9503124] [INSPIRE].

[32] X. Bekaert and M. Henneaux, *Comments on chiral p forms*, *Int. J. Theor. Phys.* **38** (1999) 1161 [hep-th/9806062] [INSPIRE].

[33] I. Bandos, *On Lagrangian approach to self-dual gauge fields in spacetime of nontrivial topology*, *JHEP* **08** (2014) 048 [arXiv:1406.5185] [INSPIRE].

[34] H. Isono, *Note on the self-duality of gauge fields in topologically nontrivial spacetime*, *PTEP* **2014** (2014) 093B05 [arXiv:1406.6023] [INSPIRE].

[35] P. Pasti, D.P. Sorokin and M. Tonin, *Duality symmetric actions with manifest space-time symmetries*, *Phys. Rev.* **D 52** (1995) 4277 [hep-th/9506109] [INSPIRE].

[36] P. Pasti, D.P. Sorokin and M. Tonin, *Note on manifest Lorentz and general coordinate invariance in duality symmetric models*, *Phys. Lett.* **B 352** (1995) 59 [hep-th/9503182] [INSPIRE].

[37] P. Pasti, D.P. Sorokin and M. Tonin, *Comment on 'Covariant duality symmetric actions'*, *Phys. Rev.* **D 56** (1997) 2473 [hep-th/9607171] [INSPIRE].

[38] J.H. Schwarz, *Highly effective actions*, *JHEP* **01** (2014) 088 [arXiv:1311.0305] [INSPIRE].

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