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# Probing Lorentz and CPT violation in a magnetized iron detector using atmospheric neutrinos

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**ABSTRACT:** We study the sensitivity of the Iron Calorimeter (ICAL) at the India-Based Neutrino Observatory (INO) to Lorentz and CPT violation in the neutrino sector. Its ability to identify the charge of muons in addition to their direction and energy makes ICAL a useful tool in putting constraints on these fundamental symmetries. Using resolution, efficiencies, errors and uncertainties obtained from ICAL detector simulations, we determine sensitivities to  $\delta b_{31}$ , which parametrizes the violations in the muon neutrino sector. We carry out calculations for three generic cases representing mixing in the CPT violating part of the hamiltonian, specifically, when the mixing is 1) small, 2) large, 3) the same as that in the PMNS matrix. We find that for both types of hierarchy, ICAL at INO should be sensitive to  $\delta b_{31} \gtrsim 4 \times 10^{-23}$  GeV at 99% C.L. for 500 kt-yr exposure, unless the mixing in the CPT violation sector is small.

**KEYWORDS:** Neutrino Physics, Solar and Atmospheric Neutrinos

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**1 Introduction**

Invariance under the product of charge conjugation (C), parity (P) and time reversal (T), i.e. the CPT theorem [1–3], is a linchpin of present-day quantum field theories underlying particle physics. It is noteworthy that this invariance under the product of purely discrete symmetries is actually a consequence of the invariance of the Lagrangian ( $\mathcal{L}$ ) under a connected continuous group, namely, proper Lorentz transformations. Additionally, it follows from the requirement that  $\mathcal{L}$  be hermitian and the interactions in the underlying field theory be local, with the fields obeying the commutation relations dictated by the spin-statistics theorem.

Theories attempting to unify gravity and quantum mechanics may, however, break such seemingly solid pillars of low energy effective field theories (like the Standard Model (SM)) via new physics associated with the Planck scale. For a general mechanism for the breaking of Lorentz symmetry in string theories, see, for instance, [4]. Other scenarios for such breaking have been discussed in [5–7] and [8]. Also, as shown in [9], a violation of CPT always breaks Lorentz invariance, while the converse is not true. A framework for incorporating CPT and Lorentz violation into a general relativistically extended version of the SM has been formulated in [10, 11]. It is termed as the Standard Model Extension (SME), and our discussion in this paper will utilize the effective CPT violating (CPTV) terms that it introduces. Such terms, given the impressive agreement of the (CPT and Lorentz invariant) SM with all present day experiments, must of necessity be small.

A characteristic attribute of neutrino oscillations is the amplification, via interference, of the effects of certain small parameters (e.g. neutrino masses) in the underlying SM lagrangian. As discussed in [12] and [13], the CPT violation manifest in an effective SME hamiltonian can also be rendered measurable in neutrino oscillation experiments. Its non-observation, on the other hand, can be used to set impressive limits on CPT and Lorentz

violation. This has been done, to cite a few recent examples, by IceCube [14], Double Chooz [15, 16], LSND [17], MiniBooNE [18], MINOS [19, 20] and Super Kamiokande [21]. Additionally, many authors have studied how best to parametrize and/or use neutrino oscillations and neutrino interactions to perform tests of CPT and Lorentz symmetry breaking in different contexts, ranging from neutrino factories and telescopes to long baseline, atmospheric, solar and reactor experiments, including those looking for supernova neutrinos [22–47].

In this article we study the sensitivity of a large atmospheric magnetized iron calorimeter to CPTV using a three flavour analysis with matter effects. As a reference detector for our study, we use the ICAL, the proposed detector for the India-Based Neutrino Observatory(INO) [48]. The main physics objective of ICAL is the determination of the neutrino mass hierarchy through the observation of earth matter effects in atmospheric neutrinos, as discussed, for instance, in [49–53]. However, its lepton charge identification capability renders it useful in measurements related to our purpose here.

In what follows, in section 2 we review the perturbative phenomenological approach that allows us to introduce the effects of CPT violation in the neutrino oscillation probability, based on the SME. We examine effects at the probability level, in order to get a better understanding of the physics that drives the bounds we obtain using our event rate calculations. In section 3 we describe our method of analysis, and in section 4 we discuss our results, in the form the bounds on CPT violating terms. Section 5 summarizes our conclusions.

## 2 CPTV effects at the probability level

The effective Lagrangian for a single fermion field, with Lorentz violation [54] induced by new physics at higher energies can be written as

$$\mathcal{L} = i\bar{\psi}\partial_\mu\gamma^\mu\psi - m\bar{\psi}\psi - A_\mu\bar{\psi}\gamma^\mu\psi - B_\mu\bar{\psi}\gamma_5\gamma^\mu\psi, \tag{2.1}$$

where  $A_\mu$  and  $B_\mu$  are real numbers, hence  $A_\mu$  and  $B_\mu$  necessarily induce Lorentz violation, being invariant under boosts and rotations, for instance. Such violation under the group of proper Lorentz transformations then leads to CPT violation [9].<sup>1</sup>

The effective contribution to the neutrino Lagrangian can then be parametrized [22] as

$$\mathcal{L}_\nu^{CPTV} = \bar{\nu}_L^\alpha b_\mu^{\alpha\beta} \gamma^\mu \nu_L^\beta \tag{2.2}$$

where  $b_\mu^{\alpha\beta}$  are four Hermitian  $3 \times 3$  matrices corresponding to the four Dirac indices  $\mu$  and  $\alpha, \beta$  are flavor indices.<sup>2</sup> Therefore the effective Hamiltonian in the vacuum for ultra-relativistic neutrinos with definite momentum  $p$  is

$$H \equiv \frac{MM^\dagger}{2p} + b_0 \tag{2.3}$$

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<sup>1</sup>CPT violation may also occur if particle and anti-particle masses are different. Such violation, however, also breaks the locality assumption of quantum field theories [9]. We do not consider this mode of CPT breaking in our work.

<sup>2</sup>In the literature the CPT and Lorentz violating term  $b_\mu^{\alpha\beta}$  is also frequently denoted as  $(a_L)^\mu_{\alpha\beta}$  which combines the vector and axial couplings in equation (2.1). See, for example, [13].

where  $M$  is the neutrino mass matrix in the CPT conserving limit. As mentioned above, the  $b_\mu$  parameterize the extent of CPT violation.

In going to equation (2.3) from equation (2.2) above, we have made an implicit transition to a preferred frame in which the cosmic microwave background is isotropic. We note that this will result in small anisotropic correction terms in the laboratory frame.

We can also obtain the same effective Hamiltonian using modified dispersion relation. We generally write the Lorentz Invariance(LI) dispersion relation as  $E^2 = m^2 + p^2$ . But to violate LI, we use the modified dispersion relation  $E^2 = F(p, m)$ . We note that our world is almost LI, and at rest with respect to the CMB, hence any modified dispersion relation must reduce to the LI dispersion at lower energies. The dispersion relation using rotational invariance in CMB frame can be written as [55]

$$E^2 = m^2 + p^2 + E_{pl}f^{(1)}|p| + f^{(2)}p^2 + \dots \quad (2.4)$$

where  $f^n$ ,s are dimensionless and  $E_{pl}$  is the Planck energy at which the LI is expected to be broken in quantum gravity. Thus the effective Hamiltonian in the vacuum for ultra-relativistic neutrinos with definite momentum  $p$  is

$$H \equiv \frac{MM^\dagger}{2p} + b \quad (2.5)$$

where  $b = \frac{E_{pl}f^{(1)}}{2}$  is the leading term which contributes to the CPT violation.

In many experimental situations, the neutrino passes through appreciable amounts of matter. Accounting for this, the Hamiltonian with CPT violation in the flavour basis<sup>3</sup> becomes

$$H_f = \frac{1}{2E} \cdot U_0 \cdot D(0, \Delta m_{21}^2, \Delta m_{31}^2) \cdot U_0^\dagger + U_b \cdot D_b(0, \delta b_{21}, \delta b_{31}) \cdot U_b^\dagger + D_m(V_e, 0, 0) \quad (2.6)$$

where  $U_0$  &  $U_b$  are unitary matrices and  $V_e = \sqrt{2}G_F N_e$ , where  $G_F$  is the fermi coupling constant and  $N_e$  is the electron number density. Value of  $V_e = 0.76 \times 10^{-4} \times Y_e [\frac{\rho}{g/cc}]$  eV, where  $Y_e$  is the fraction of electron, which is  $\approx 0.5$  for earth matter and  $\rho$  is matter density inside earth.  $D$ ,  $D_m$  and  $D_b$  are diagonal matrices with their elements as listed. Here  $\delta b_{i1} \equiv b_i - b_1$  for  $i = 2, 3$ , where  $b_1$ ,  $b_2$  and  $b_3$  are the eigenvalues of  $b$ .

As is well-known, in standard neutrino oscillations,  $U_0$  is parametrized by three mixing angles ( $\theta_{12}$ ,  $\theta_{23}$ ,  $\theta_{13}$ ) and one phase  $\delta_{CP}$ .<sup>4</sup> Similarly (see footnote), any parametrization of the matrix  $U_b$ , needs three angles ( $\theta_{b12}$ ,  $\theta_{b23}$ ,  $\theta_{b13}$ ) and six phases. Thus  $H_f$  contains six mixing angles ( $\theta_{12}$ ,  $\theta_{23}$ ,  $\theta_{13}$ ,  $\theta_{b12}$ ,  $\theta_{b23}$ ,  $\theta_{b13}$ ) and seven phases.

It is clear that the results will depend on the mixing angles in the CPT violation sector. In what follows, we examine the effects of three different representative sets of mixing

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<sup>3</sup>We note that the matrices appearing in the three terms in eq. (2.3) can in principle be diagonal in different bases, one in which the neutrino mass matrix is diagonal, a second one in which the Lorentz and CPT violating interactions are diagonal, and a third flavour basis.

<sup>4</sup>In general for an  $N$  dimensional unitary matrix, there are  $N$  independent rotation angles (i.e. real numbers) and  $N(N+1)/2$  imaginary quantities (phases) which define it. For Dirac fields,  $(2N-1)$  of these may be absorbed into the representative spinor, while for Majorana fields this can be done for  $N$  phases. In the latter case, the  $N-1$  additional phases in  $U_0$  become irrelevant when the product  $MM^\dagger$  is taken.

angles, 1) small mixing ( $\theta_{b_{12}} = 6^\circ$ ,  $\theta_{b_{23}} = 9^\circ$ ,  $\theta_{b_{13}} = 3^\circ$ ), 2) large mixing ( $\theta_{b_{12}} = 38^\circ$ ,  $\theta_{b_{23}} = 45^\circ$ ,  $\theta_{b_{13}} = 30^\circ$ ) and the third set 3) uses the same values as the mixing in neutrino UPMNS, ( $\theta_{b_{12}} = \theta_{12}$ ,  $\theta_{b_{23}} = \theta_{23}$ ,  $\theta_{b_{13}} = \theta_{13}$ ). We use the recent best fit neutrino oscillation parameters in our calculation as mentioned in table 1.

Before going into the detailed numerical calculations, we can roughly estimate the bound on CPTV term. As an example, let us assume case 3) i.e. when  $U_b = U_0$ , then  $\delta b$  can effectively be added to  $\frac{\Delta m^2}{2E}$ . If we take 10 GeV for a typical neutrino energy, the value of  $\frac{\Delta m^2}{2E}$  will be about  $10^{-22}$  GeV. Assuming the CPT violating term to be of the same order, and assuming that neutrino mass splitting can be measured at ICAL to 10% accuracy, we expect sensitivity to  $\delta b$  values of approximately around  $10^{-23}$  GeV.

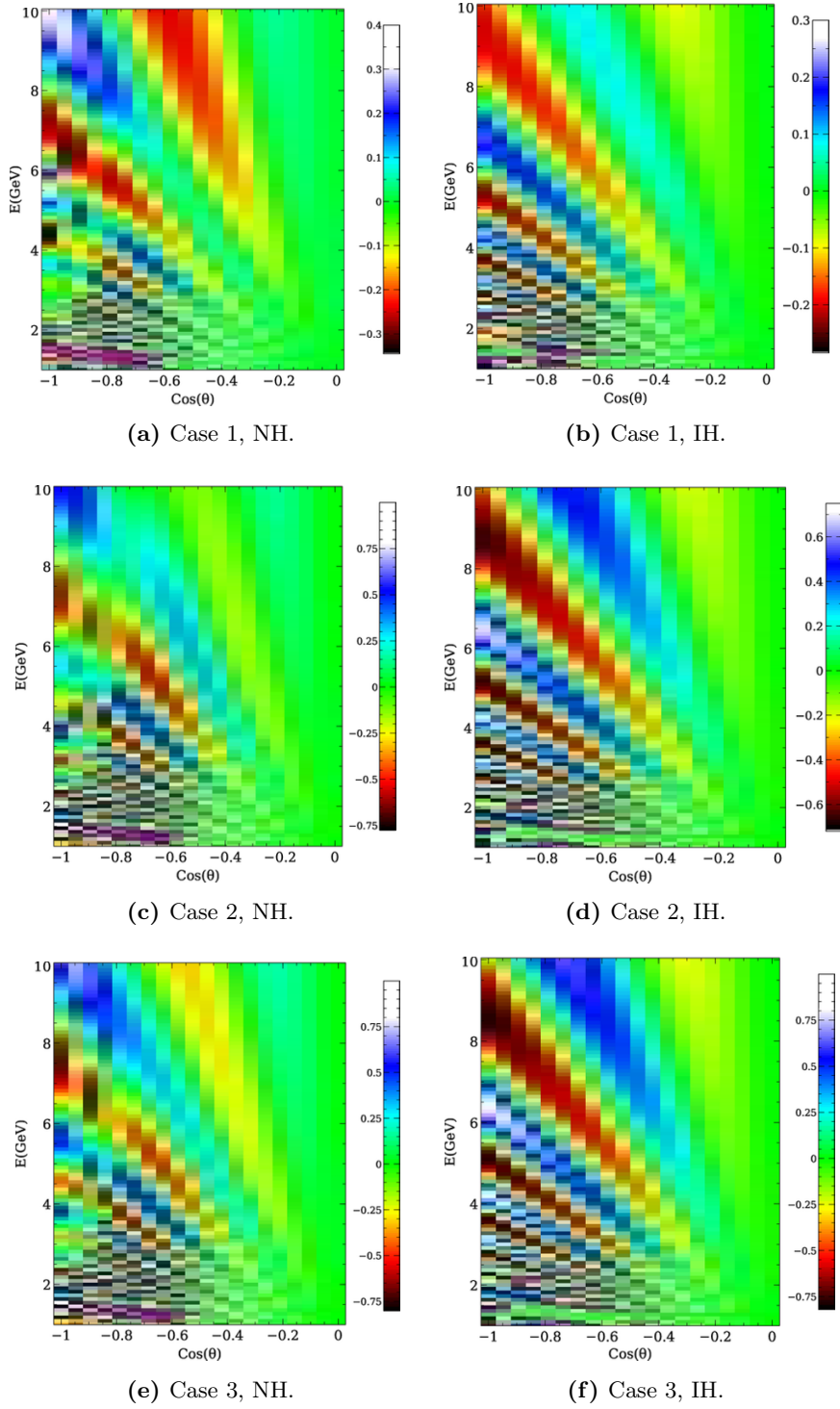
From case 3 above, we note further that in its probability expressions,  $\delta b_{21}$  will always appear with the smaller (by a factor of 30) mass squared difference  $\Delta m_{21}^2$ . Thus we expect its effects on oscillations will be subdominant in general, limiting the capability of atmospheric neutrinos to constrain it, and in our work we have not been able to put useful constraints on  $\delta b_{21}$ . Thus, our effort has been to find a method which will give the most stringent bounds on CPT violation as parametrized by  $\delta b_{31}$ . For simplicity, all phases are set to zero, hence the distinction between Dirac and Majorana neutrinos with regard to the number of non-trivial phases does not play a role in what follows. We note here that phases are contained in the imaginary part of the CPT violating matrix in the flavour basis, and hence setting them to zero allows an emphasis on CPT as opposed to the pure CP effects. Moreover, in the approximation where the effects of  $\delta b_{21}$  are much smaller than those of  $\delta b_{31}$ , the impact of at least some of the nontrivial phases anyway will be negligible. We also study the effect and impact of hierarchy in putting constraints on CPT violating terms.

We also note here previously obtained limits on the parameters of  $U_b$ . The solar and KamLAND data [23] gives the bound  $\delta b \lesssim 1.6 \times 10^{-21}$  GeV. In [24] by studying the ratio of total atmospheric muon neutrino survival rates, (i.e. two flavour approach different from the one in the present paper), it was shown that, for a 50 kt magnetized iron calorimeter,  $\delta b \lesssim 3 \times 10^{-23}$  GeV should be attainable with a 400 kT-yr exposure. Using a two-flavor analysis, it was noted in [25] that a long baseline ( $L = 735$  km) experiment with a high energy neutrino factory can constrain  $\delta b$  to  $\lesssim 10^{-23}$  GeV. A formalism for a three flavour analysis was presented in [26] and bounds of the order of  $\delta b \lesssim 3 \times 10^{-23}$  GeV were calculated for the upcoming NO $\nu$ A experiment and for neutrino factories. It has also been shown in [29], that a bound of  $\delta b_{31} \lesssim 6 \times 10^{-24}$  GeV at 99% CL can be obtained with a 1 Mt-yr magnetized iron detector. Global two-flavor analysis of the full atmospheric data and long baseline K2K data puts the bound  $\delta b \lesssim 10^{-23}$  GeV [56].<sup>5</sup>

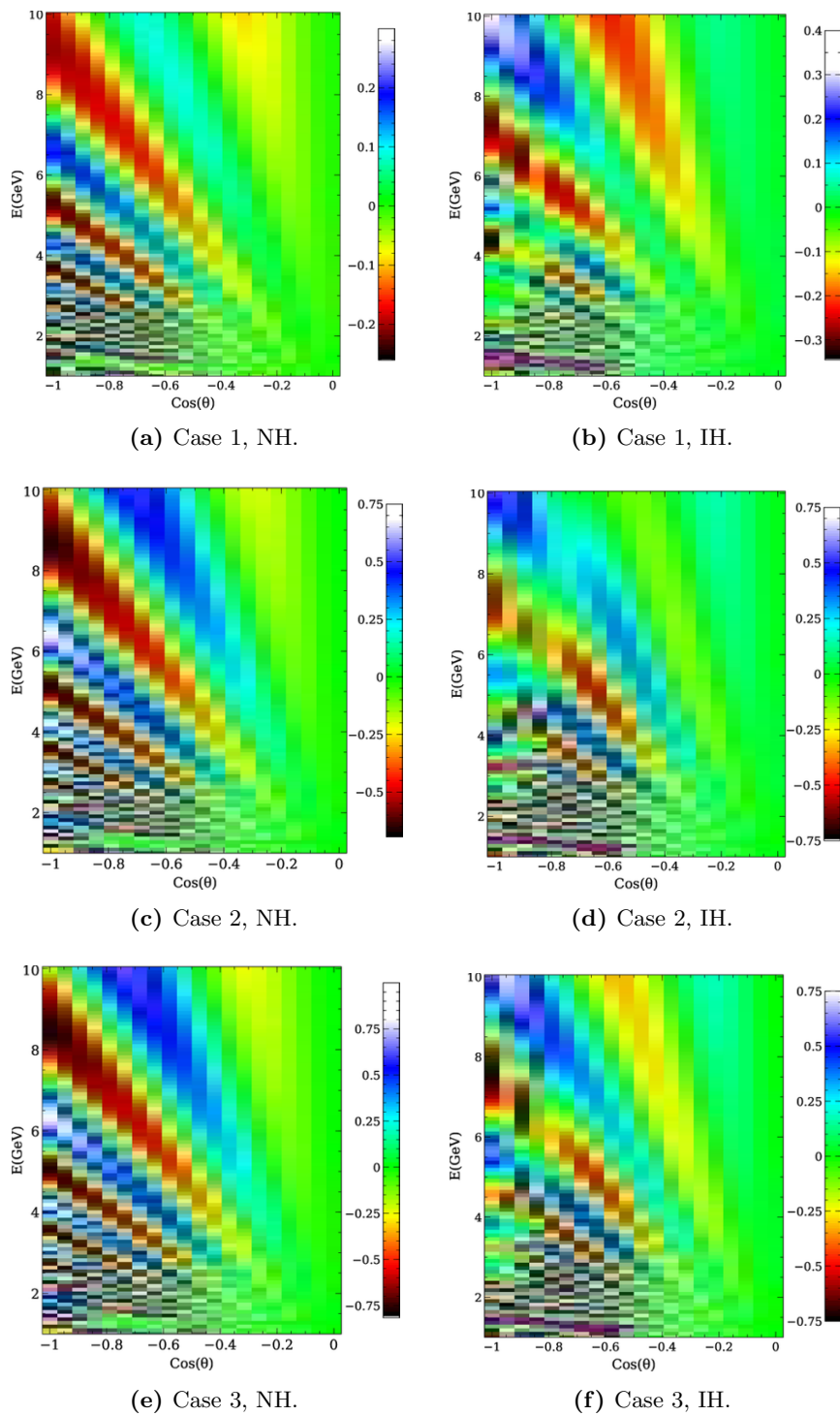
Prior to discussing the results of our numerical simulations, it is useful to examine these effects at the level of probabilities. We note that the matter target in our case is CP asymmetric, which will automatically lead to effects similar to those induced by  $\mathbf{U}_b$ . In order to separate effects arises due to dynamical CPT violation from those originating due to the CP asymmetry of the earth, it helps to consider the difference in the disappearance

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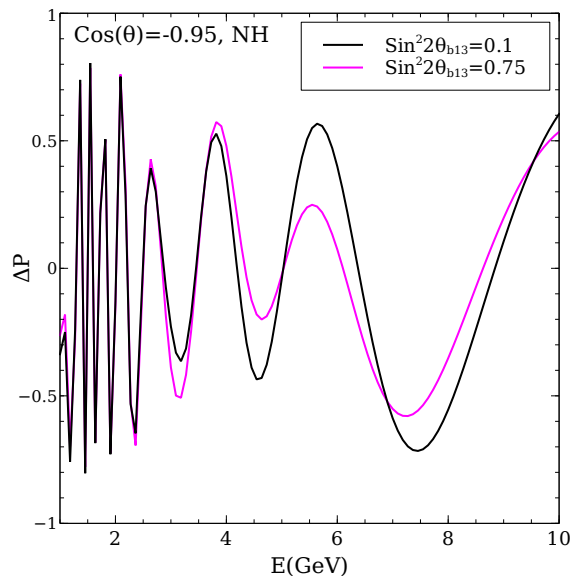
<sup>5</sup>Note that the bounds obtained in these papers, and the bounds that we will obtain below, are on the absolute value of  $\delta b$ , since in principle this quantity can be either positive or negative in the same way the  $\Delta m_{ij}^2$  can be positive or negative. In our plots, where necessary, we assume it to be positive for simplicity.



**Figure 1.** The oscillograms of  $\Delta P = (P_{\nu_\mu \nu_\mu}^{U_b \neq 0} - P_{\nu_\mu \nu_\mu}^{U_b = 0})$  for 3 different mixing cases have been shown. The value of  $\delta b_{31} = 3 \times 10^{-23}$  GeV is taken for  $U_b \neq 0$ . Left and right panels are for Normal and Inverted hierarchy respectively.



**Figure 2.** The oscillograms of  $\Delta P = (P_{\nu_\mu \nu_\mu}^{U_b \neq 0} - P_{\nu_\mu \nu_\mu}^{U_b = 0})$  for 3 different mixing cases have been shown. The value of  $\delta b_{31} = 3 \times 10^{-23}$  GeV is taken for  $U_b \neq 0$ . Left and right panels are for Normal and Inverted hierarchy respectively.



**Figure 3.** The difference of the  $P_{\nu_\mu\nu_\mu}$  with and without CPTV for  $\delta b_{31} = 3 \times 10^{-23} GeV$  for two  $\theta_{b13}$  values as a function of energy for a specific value of zenith angle. All other oscillation parameters are same.

probabilities with  $\mathbf{U}_b$  effects turned on and off, respectively. We use the difference in probabilities

$$\Delta P = P_{\nu_\mu\nu_\mu}^{U_b \neq 0} - P_{\nu_\mu\nu_\mu}^{U_b = 0}. \quad (2.7)$$

We do this separately for  $\nu_\mu$  and  $\bar{\nu}_\mu$  events with NH and IH assumed as the true hierarchy. The results are shown in the figures 1–2. (We note that at the event level, the total muon events receive contributions from both the  $P_{\nu_\mu\nu_\mu}$  disappearance and  $P_{\nu_e\nu_\mu}$  appearance channels, and the same is true for anti-neutrinos. In our final numerical results, we have taken both these contributions into account).

Several general features are apparent in figures 1–2. First, effects are uniformly small at shorter baselines irrespective of the value of the energy. From the 2 flavour analysis, e.g. [24] we recall that the survival probability difference in vacuum is proportional to  $\sin(\frac{\delta m^2 L}{2E}) \sin(\delta b L)$ . The qualitative feature that CPT effects are larger at long baselines continues to be manifest even when one incorporates three flavour mixing and the presence of matter, and this is brought out in all the figures.

Secondly, as is well-known, matter effects are large and resonant for neutrinos and NH, and for anti-neutrinos with IH. Thus in both these cases, they mask the (smaller) effect of CPT stemming from  $U_b$ . Hence for neutrino events, CPT sensitivity is significantly higher if the hierarchy is inverted as opposed to normal, and the converse is true for anti-neutrino events. Finally, effects are largest for cases 3) and 2), and smaller for case 1). The effect is smaller for case 1) is due to the fact that mixing is very small compared to other two. The origin of the difference for the case 2) and 3) is likely due to the fact that CPT violating effects are smaller when  $\theta_{b13}$  is large, as shown in figure 3.

We carry through this mode of looking at the difference between the case when  $\mathbf{U}_b$  is non-zero and zero respectively to the event and  $\chi^2$  levels in our calculations below. To use



the lepton charge identification capability of a magnetized iron calorimeter optimally, we calculate  $\chi^2$  from  $\mu^-$  and  $\mu^+$  events separately. Following this procedure, the contribution arising through matter being CPT asymmetric will expectedly cancel for any given zenith angle and energy. The numerical procedure adopted and the details of our calculation are provided in the following section.

### 3 Numerical procedure

Our work uses the ICAL detector as a reference configuration, but the qualitative content of the results will hold for any similar detector. Magnetized iron calorimeters typically have very good energy and direction resolution for reconstructing  $\mu^+$  and  $\mu^-$  events. The analysis proceeds in two steps: (1) Event simulation (2) Statistical procedure and the  $\chi^2$  analysis.

#### 3.1 Event simulation

We use the NUANCE [57] neutrino generator to generate events. The ICAL detector composition and geometry are incorporated within NUANCE and atmospheric neutrino fluxes (Honda et al. [58]) have been used. In order to reduce the Monte Carlo (MC) fluctuations in the number of events given by NUANCE, we generate a very large number of neutrino events with an exposure of 50 kt  $\times$  1000 years and then finally normalize to 500 kt-yr.

Each changed-current neutrino event is characterized by neutrino energy and neutrino zenith angle, as well as by a muon energy and muon zenith angle. In order to save on computational time, we use a re-weighting algorithm to generate oscillated events. This algorithm, takes the neutrino energy and angle for each event and calculates probabilities  $P_{\nu_\mu\nu_\mu}$  and  $P_{\nu_\mu\nu_e}$  for any given set of oscillation parameters. It then compares it with a random number  $r$  between 0 to 1. If  $r < P_{\nu_\mu\nu_e}$ , then it is classified as a  $\nu_e$  event. If  $r > (P_{\nu_\mu\nu_e} + P_{\nu_\mu\nu_\mu})$ , it classified as a  $\nu_\tau$  event. If  $P_{\nu_\mu\nu_e} \leq r \leq (P_{\nu_\mu\nu_e} + P_{\nu_\mu\nu_\mu})$ , then it is considered to come from an atmospheric  $\nu_\mu$  which has survived as a  $\nu_\mu$ . Similarly muon neutrinos from the oscillation of  $\nu_e$  to  $\nu_\mu$  are also calculated using this reweighting method.

Oscillated muon events are binned as a function of muon energy and muon zenith angle. We have divided each of the ten energy bins into 40 zenith angle bins. These binned data are folded with detector efficiencies and resolution functions as described in equation (2.6) to simulate reconstructed muon events. In this work we have used the (i) muon reconstruction efficiency (ii) muon charge identification efficiency (iii) muon energy resolution (iv) muon zenith angle resolution, obtained by the INO collaboration [59], separately for  $\mu^+$  and  $\mu^-$ .

The measured muon events after implementing efficiencies and resolution are

$$N(\mu^-) = \int dE_\mu \int d\theta_\mu [R_{E_\mu} R_{\theta_\mu} (R_{\text{eff}} C_{\text{eff}} N_{\text{osc}}(\mu^-) + \bar{R}_{\text{eff}} (1 - \bar{C}_{\text{eff}}) N_{\text{osc}}(\mu^+))] \quad (3.1)$$

where  $R_{\text{eff}}$ ,  $C_{\text{eff}}$ ,  $\bar{R}_{\text{eff}}$ ,  $\bar{C}_{\text{eff}}$  are reconstruction and charge identification efficiencies for  $\mu^-$  and  $\mu^+$  respectively,  $N_{\text{osc}}$  is the number of oscillated muons in each true muon energy and zenith angle bin and  $R_{E_\mu}$ ,  $R_{\theta_\mu}$  are energy and angular resolution functions.

Oscillation parameter	Best fit values	Oscillation parameter	Best fit values
$\sin^2 2\theta_{12}$	0.86	$\delta_{CP}$	0.0
$\sin^2 2\theta_{23}$	1.0	$\sin^2 2\theta_{b12}$	1) 0.043, 2) 0.94, 3) 0.86
$\sin^2 2\theta_{13}$	0.1	$\sin^2 2\theta_{b23}$	1) 0.095, 2) 1.0, 3) 1.0
$\Delta m_{21}^2$ (eV <sup>2</sup> )	$7.5 \times 10^{-5}$	$\sin^2 2\theta_{b13}$	1) 0.011, 2) 0.75, 3) 0.1
$ \Delta m_{32}^2 $ (eV <sup>2</sup> )	$2.4 \times 10^{-3}$	$\delta_b, \phi_{b2}, \phi_{b3}$	0.0, 0.0, 0.0

**Table 1.** True values of the oscillation parameters used in the analysis.

The energy and angular resolution function in Gaussian form are given by

$$R_E = \frac{1}{\sqrt{2\pi}\sigma_E} \exp \left[ -\frac{(E_m - E_t)^2}{2\sigma_E^2} \right] \quad (3.2)$$

$$R_\theta = N_\theta \exp \left[ -\frac{(\theta_t - \theta_m)^2}{2(\sigma_\theta)^2} \right]. \quad (3.3)$$

Here  $E_m$ ,  $E_t$  and  $\theta_m$ ,  $\theta_t$  are measured and true energy and angle respectively.  $N_\theta$  is the normalization constant,  $\sigma_\theta$ ,  $\sigma_E$  are angular and energy smearing of muons.  $\sigma_\theta$ ,  $\sigma_E$  are obtained from ICAL simulations [59].

### 3.2 Statistical procedure and the $\chi^2$ analysis

We have generated event rate data with the true values of oscillation parameters given in table 1 and assuming no CPT violation, these are defined as  $N^{ex}$ . They are then fitted with another set of data, labelled as ( $N^{th}$ ), where we have assumed CPT violation. The statistical significance of the difference between these two sets of data will provide constraints on the CPT violation parameters.

We define  $\chi^2$  for the data as

$$\chi_{pull}^2 = \min_{\xi_k} [2(N^{th'} - N^{ex} - N^{ex} \ln(\frac{N^{th'}}{N^{ex}})) + \sum_{k=1}^{npull} \xi_k^2] \quad (3.4)$$

where

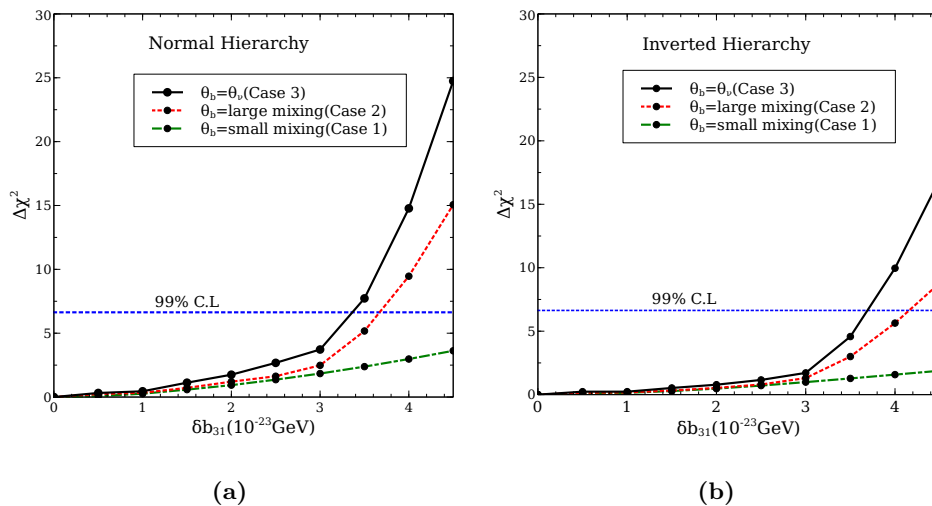
$$N^{th'} = N^{th} + \sum_{k=1}^{npull} c^k \xi_k \quad (3.5)$$

$npull$  is the number of pull variable, in our analysis we have taken  $npull = 5$ .  $\xi_k$  is the pull variable and  $c^k$  are the systematic uncertainties. We have used 5 systematic uncertainties in this analysis as mentioned in table 2 as generally used in the other analysis of the collaboration. We have assumed a Poissonian distribution for  $\chi^2$  because for higher energy bins the number of atmospheric events will be small.

Since ICAL can discriminate charge of the particle, it is useful to calculate  $\chi^2(\mu^-)$  and  $\chi^2(\mu^+)$  separately for  $\mu^-$  and  $\mu^+$  events and then added to get total  $\chi^2$ . We have marginalized the total  $\chi^2$  within a 3 $\sigma$  range of the best fit value.  $\chi^2$  has been marginalized over the oscillation parameters  $\Delta m_{31}^2$ ,  $\theta_{23}$ ,  $\theta_{13}$ ,  $\delta_{CP}$ ,  $\delta_{b21}$  for both normal and inverted hierarchy with  $\mu^+$  and  $\mu^-$  separately for given set of input data.

Uncertainties	Values
Flux Normalization	20%
Tilt Factor	5%
Zenith angle dependence	5%
Overall cross section	10%
Overall systematic	5%

**Table 2.** Systematic uncertainties used in the  $\chi^2$  analysis.



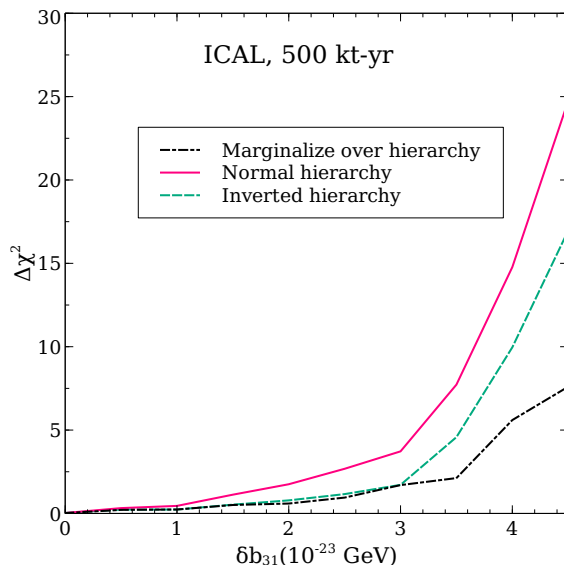
**Figure 4.**  $\Delta\chi^2$  as a function of  $\delta b_{31}$  for different mixing of  $\theta_b$ . Best fit oscillation parameters used as mentioned in table 1. The results are marginalized over  $\theta_{23}$ ,  $\theta_{13}$ ,  $\delta_{CP}$ ,  $\Delta m_{31}^2$  and  $\delta b_{21}$ . Left and right panel are for Normal and Inverted hierarchy respectively.

## 4 Results

Figure 4 illustrates  $\Delta\chi^2$  analysis performed by marginalizing over all the oscillation parameters  $\Delta m_{31}^2$ ,  $\theta_{23}$ ,  $\theta_{13}$  within a  $3\sigma$  range of their best fit values as given in table 1.  $\delta_{CP}$  is marginalized over 0 to  $2\pi$ .  $\delta b_{21}$  is marginalized over the range 0 to  $5 \times 10^{-23}$  GeV. Left and right panel are for the Normal and Inverted Hierarchy respectively. While from figure 4 we see that the best bounds arise for case 3) for both the hierarchy, where mixing in the CPTV sector is the same as in the case of neutrino mixing, good bounds are also obtainable for large mixing. Since  $\theta_{b12}$  and  $\theta_{b23}$  in both cases 2) and 3) are large, the origin of this difference is likely due to the fact that CPT violating effects are smaller when  $\theta_{b13}$  is large, as shown in figure 3.

From figure 4 we see that 99% C.L. or better constraints on the CPT violating parameter,<sup>6</sup>  $\delta b_{31}$  are possible for both hierarchies if it is  $\gtrsim 4 \times 10^{-23}$  GeV, if the mixing in the CPTV sector is not small.

<sup>6</sup>constraints are also expressible in the flavour basis, which is done in some references in literature. One could then use the data tables of [60] to make a comparison. We have chosen to express them in the diagonal basis here.



**Figure 5.**  $\Delta\chi^2$  as a function of  $\delta b_{31}$  for case 3) is shown. The Red curve represents that both sets of data has Normal hierarchy as true hierarchy, and the green curve for Inverted hierarchy respectively. Black curve shows the bounds where hierarchy is marginalized over.

It is clear from the figure 5 that if marginalization over the hierarchy is carried out, the constraints are considerably weaker. Hence a knowledge of the hierarchy certainly helps in getting useful constraints on CPT. The sensitivities obtained are comparable to those anticipated from other types of experiments and estimates in the literature [23–26, 29].

## 5 Conclusions

A magnetized iron calorimeter like ICAL, with its attributes of good energy and angular resolution for muons and its charge identification capability can be a useful tool in investigating Lorentz and CPT violation stemming from physics at higher energy scales, even though its main physics objective may be hierarchy determination. Using resolutions, efficiencies, errors and uncertainties generated by ICAL detector simulation, we have calculated reliable sensitivities to the presence of CPTV in such a detector.

It is, of course, clear that the exact value of the constraint on CPT violation depends on the choice of mixing angles in  $U_b$ . We have carried out our calculations for three representative cases, those for which the mixing is 1) small, 2) large and 3) similar to the PMNS mixing. We find that for both types of hierarchy, ICAL should be sensitive to  $\delta b_{31} \gtrsim 4 \times 10^{-23}$  GeV at 99% C.L. with 500 kt-yr of exposure, unless the mixing in the CPTV sector is small. As discussed earlier, CP (and CPT) effects due to earth matter asymmetry are subtracted out.

Our study pertains to the type of CPTV that may be parametrized by equation (2.1), which stems from explicit Lorentz violation, and to the muon detection channel. We have not considered the CPTV that arises from differing masses for particles and anti-particles. Finally, we note that in order to obtain good sensitivity to CPTV, knowing the hierarchy will be an important asset, which will anyway be done in a detector like ICAL.

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