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Can quantum statistics help distinguish Dirac from Majorana neutrinos?

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Abstract: Finding out if neutrinos are Dirac or Majorana particles is known to be extremely difficult due to the smallness of neutrino mass and the fact that in the limit $m_\nu = 0$ both Dirac and Majorana neutrinos become Weyl fermions, i.e. are indistinguishable. There have been suggestions in the literature that in the case of processes with production of a neutrino-antineutrino pair (if neutrinos are Dirac particles) or two neutrinos (if they are of Majorana nature) quantum statistics may be of help. This is because for Majorana neutrinos quantum indistinguishability of identical particles requires the amplitude of the process to be antisymmetrized with respect to the interchange of the final-state neutrinos, whereas no such antisymmetrization must be done for Dirac neutrinos. It has been claimed that the resulting differences between the cross sections for Dirac and Majorana neutrinos persist even for arbitrarily small but not exactly vanishing neutrino mass. We demonstrate that, at least in the framework of the Standard Model, this is not the case. We also give a general proof that within the Standard Model quantum statistics does not help tell Dirac and Majorana neutrinos apart in the limit of negligibly small m_ν/E.

Keywords: Non-Standard Neutrino Properties, Neutrino Interactions

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Contents

1 Introduction

One of the most fundamental questions of neutrino physics is whether neutrinos are different from their antiparticles, i.e. are Dirac fermions, or are their own antiparticles, which would mean that they are of Majorana nature. Despite significant experimental effort, we still do not know the answer to this question. It is generally believed that the tremendous experimental difficulties in finding out the Dirac *vs.* Majorana nature of neutrinos are related to the extreme smallness of the neutrino mass m_ν compared to typical neutrino energies, as in the limit of vanishing m_ν the difference between Dirac and Majorana neutrinos disappears.

The main distinction between neutrinos of Dirac and Majorana nature is that Dirac neutrinos are described by four-component spinor fields and possess a conserved lepton number, whereas Majorana neutrinos are described by two-component fields and no conserved lepton number can be ascribed to them. In the limit of vanishing neutrino mass the left-handed and right-handed components of both Dirac and Majorana neutrinos decouple and become two-component Weyl fields. As the right-handed components of Dirac neutrinos are sterile in the Standard Model, any distinction between the Dirac and Majorana neutrinos then disappears [1–4]: in both cases, the states that can be produced or absorbed by weak interactions are left-handed neutrinos ν_L and their right-handed CPT conjugates $(\nu_L)^c = \nu_R^c$. This, of course, need not be the case beyond the Standard Model, where the right-handed components of Dirac neutrinos ν_R may not be sterile (see, e.g., [5]).

The difficulties with telling apart Dirac and Majorana neutrinos suggest looking for processes which, even if rare, are strictly forbidden for neutrinos of one of the two types.

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Currently the most promising candidates appear to be the neutrinoless double β-decay and related processes, which can only occur if neutrinos are Majorana particles [6] (see e.g. [7] for a recent review).

The statement that, at least within the framework of the Standard Model, the smallness of neutrino mass makes Dirac and Majorana neutrinos practically indistinguishable was dubbed the Practical Dirac-Majorana Confusion Theorem [3, 4]. Crucial to it is the fact that both charged-current (CC) and neutral-current (NC) interactions of neutrinos in the Standard Model are purely chiral: $j^{\mu}_{\rm CC}(x) = \bar{l}(x)\gamma^{\mu}(1-\gamma_5)\nu_l(x)$, $j^{\mu}_{\rm NC} = \bar{\nu}_l(x)\gamma^{\mu}(1-\gamma_5)\nu_l(x)$.[1]

There have been suggestions in the literature that there may exist exceptions to this theorem; in particular, it has been argued that in the case of processes with production of a neutrino-antineutrino pair (if neutrinos are Dirac particles) or two neutrinos (if they are of Majorana nature) quantum statistics may help discriminate between the two neutrino types [8–12]. The argument is based on the fact that for Majorana neutrinos quantum indistinguishability of identical particles requires the amplitude of the process to be antisymmetrized with respect to the interchange of the final-state neutrinos, whereas no such antisymmetrization must be done if neutrinos are Dirac particles, as the produced ν and $\bar{\nu}$ are distinct. It has been claimed that the resulting differences between the cross sections for Dirac and Majorana neutrinos survive even for arbitrarily small but not exactly vanishing neutrino mass, though they disappear when $m_\nu = 0$.

Such a lack of smooth behaviour of the cross sections in the limit $m_\nu \to 0$ is very counterintuitive and unsettling, as one naturally expects physical observables to be continuous functions of the masses of the involved fermions. In the present paper we examine this issue in detail. We demonstrate that the claims of the absence of smooth behaviour in the limit $m_\nu \to 0$ do not hold, as far as observable quantities are concerned. We also prove that this result is not limited to the processes considered in [8–12], but holds true for all Standard Model processes.[2] Our conclusion is thus that quantum statistics does not lead to any exceptions to the Practical Dirac-Majorana Confusion Theorem.

The paper is organized as follows. In section 2 we first review the well known examples illustrating how the Practical Dirac-Majorana Confusion Theorem works in the cases of decay/inverse decay and neutrino scattering processes. These examples demonstrate that for Majorana neutrinos the role of the (nearly) conserved lepton number is played by chirality, which is approximately conserved for relativistic neutrinos. We then give very general arguments for why this breaks the indistinguishability of Majorana neutrinos in processes with their pair-production. In sections 3 and 4 we discuss the existing claims that for processes with two neutrinos in the final state quantum statistics leads to differences between the cross sections for Dirac and Majorana neutrinos that do not disappear in the limit $m_\nu \to 0$ and

[1] Strictly speaking, the neutral current is chiral only for Dirac neutrinos, whereas for Majorana neutrinos it is purely axial-vector. However, in the limit $m_\nu/E \to 0$ this makes no difference, see sections 2.2 and 4.2 below.

[2] A qualification is in order. To be precise, neutrinos are massless in the minimal Standard Model; what we consider in this paper is actually "the Standard Model plus neutrino mass", i.e. we assume that neutrino mass is generated by an unspecified new physics (possibly at a high energy scale) that does not affect low-energy processes, except through the neutrino mass itself. We also note that leptonic mixing (and more generally the existence of more than one neutrino flavor) is not directly relevant to the issues we consider, and so we disregard it in our discussion.

demonstrate that these claims are erroneous. In section 5 we present a general proof that quantum statistics does not violate the Practical Dirac-Majorana Confusion Theorem. The reader content with general arguments and not interested in details of our analysis may skip sections 3, 4 and 5 and go directly to section 6, where we summarize our results and conclude.

2 General arguments

2.1 Single neutrino production, absorption and scattering in CC processes

One well-known example of how the Practical Dirac-Majorana Confusion Theorem works is given by β-decay and inverse β-decay processes, which are induced by the CC weak interactions (see, e.g., ref. [13]).

It is known that electron neutrinos ν_e produced in β^+-decays, (e.g. solar neutrinos) are different from those produced in β^--decays, which are usually called $\bar{\nu}_e$. The former can be detected through the reaction $\nu_e + {}^{37}\mathrm{Cl} \to {}^{37}\mathrm{Ar} + e^-$, whereas the detection of these neutrinos through the inverse β-decay on protons has never been observed. On the other hand, the latter (produced e.g. in β-decays of heavy nuclei in nuclear reactors) can be (and usually are) detected through the inverse β-decay on protons, whereas the attempts to detect them through the Cl-Ar reaction mentioned above have failed. These facts can be easily explained if neutrinos are Dirac particles and possess conserved lepton number. In this case ν_e and $\bar{\nu}_e$ are electron neutrinos and antineutrinos, respectively, and the selection rules discussed above are just a consequence of lepton number conservation.

Does this mean that we already have an experimental proof that neutrinos are different from their antiparticles, i.e. they are Dirac fermions? The answer is no, of course. The point is that the chiral structure of the weak currents means that leptons participate in CC weak interactions only by their left-handed chirality components, and antileptons by their right-handed chirality components. Chirality is not a good quantum number for fermions of nonzero mass, but for free relativistic particles chirality nearly coincides with helicity, which *is* conserved; the difference between the two is of the order of m_ν/E. For u-type and v-type spinors describing, respectively, the positive-energy and negative-energy solutions of the Dirac equation, we have

$$u_L(p) \simeq u_-(p) + \mathcal{O}\left(\frac{m_\nu}{2E}\right), \qquad v_R(p) \simeq v_+(p) + \mathcal{O}\left(\frac{m_\nu}{2E}\right),$$
$$u_R(p) \simeq u_+(p) + \mathcal{O}\left(\frac{m_\nu}{2E}\right), \qquad v_L(p) \simeq v_-(p) + \mathcal{O}\left(\frac{m_\nu}{2E}\right), \tag{2.1}$$

where $u_{L,R} = P_{L,R}u$, $v_{L,R} = P_{R,L}v$ with $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$, and the subscripts $\pm$ stand for positive and negative helicities. Eq. (2.1) means that relativistic neutrinos produced together with positively charged leptons (or absorbed in reactions with production of negatively charged leptons) are predominantly negative helicity states, whereas neutrinos produced together with negatively charged leptons (or absorbed in reactions with production of positively charged leptons) are predominantly states of positive helicity. Thus, the chirality selection rules of CC weak interactions play essentially the same role for relativistic Majorana neutrinos as lepton number conservation plays for Dirac neutrinos. For example, what we call $\bar{\nu}_e$ is an electron antineutrino in the Dirac case and an electron neutrino of nearly positive helicity if neutrinos are Majorana particles. The difference is that chirality is only approximately conserved for relativistic neutrinos with $m_\nu \neq 0$, whereas for Dirac neutrinos lepton number is conserved exactly. Therefore, in the Majorana case the processes like detection of solar neutrinos through inverse β-decay on protons (and other "wrong-helicity" processes) are not strictly forbidden, but are strongly suppressed; the suppression factors are $\mathcal{O}(m_\nu/(2E))^2 \lesssim 10^{-14}$, which explains why such processes have never been observed.

Obviously, the same arguments apply also to other decay and inverse decay CC processes, including those with participation of ν_μ or ν_τ. They are also valid for CC contributions to $\nu_\ell \ell$ scattering, where the same chirality selection rules are at work.

2.2 Neutral current induced scattering processes

What about neutral-current neutrino interactions? There is a peculiarity in this case. For Majorana particles the vector neutral current vanishes identically: $\bar{\psi}(x)\gamma^\mu\psi(x) \equiv 0$. This can be easily proven by making use of the self-conjugacy property of Majorana fields $(\psi)^c \equiv C\bar{\psi}^T = \psi$, where C is the charge conjugation matrix. As a result, for Majorana neutrinos the neutral current is purely axial-vector:

$$\bar{\nu}(x)\gamma^\mu(g_V - g_A\gamma_5)\nu(x) = -g_A\bar{\nu}(x)\gamma^\mu\gamma_5\nu(x)\,, \tag{2.2}$$

where we have introduced the vector and axial-vector coupling constants g_V and g_A for future convenience. The measurement of the NC $\nu_\mu e \to \nu_\mu e$ and $\bar{\nu}_\mu e \to \bar{\nu}_\mu e$ scattering cross sections by the CHARM-II collaboration [14, 15] was interpreted in [16] as an experimental evidence for nonzero vector neutrino NC coupling g_V. The author then concluded that neutrinos cannot be Majorana particles. It has been subsequently demonstrated in [17–21] that this interpretation was incorrect. The amplitude of the process depends on the NC matrix element $\langle\nu(p')|j^\mu_{\rm NC}(0)|\nu(p)\rangle$, for which in the Majorana case one finds

$$\begin{aligned}&\bar{u}(p')\gamma^\mu(g_V - g_A\gamma_5)u(p) - \bar{v}(p)\gamma^\mu(g_V - g_A\gamma_5)v(p') =\\ &\bar{u}(p')\gamma^\mu(g_V - g_A\gamma_5)u(p) - \bar{u}(p')\gamma^\mu(g_V + g_A\gamma_5)u(p) = -2g_A\bar{u}(p')\gamma^\mu\gamma_5 u(p)\,.\end{aligned} \tag{2.3}$$

Here p and p' are the 4-momenta of the initial- and final-state neutrinos, and in going from the first to the second line use has been made of the identities[3]

$$u_s(p) = C\bar{v}_s^T(p)\,, \qquad v_s(p) = C\bar{u}_s^T(p)\,. \tag{2.4}$$

Thus, for Majorana neutrinos the NC matrix element is purely axial-vector, as expected.

In the Dirac case, the matrix element of the neutrino neutral current is

$$\bar{u}(p')\gamma^\mu(g_V - g_A\gamma_5)u(p) = -(g_V + g_A)\bar{u}(p')\gamma^\mu\gamma_5 u(p) + \mathcal{O}\left(\frac{m_\nu}{2E}\right)\,, \tag{2.5}$$

where it was taken into account that for relativistic Dirac neutrinos $\gamma_5 u \simeq -u$. Because in the Standard Model $g_V = g_A$, the right-hand sides of eqs. (2.3) and (2.5) coincide up to terms of the order of $m_\nu/(2E)$, which proves the Practical Dirac-Majorana Confusion

[3]Transformations similar to that in eq. (2.3) are also made in a number of equations appearing below.

Theorem for NC νe-scattering [3, 17–21]. It is easy to see that this result also applies to other NC induced neutrino scattering processes.

Important to the above argument was the fact that the incident ν_μ and what we call $\bar{\nu}_\mu$ in the NC neutrino-electron scattering experiments were born in CC processes ($\pi^\pm$-decays) as left-chiral and right-chiral states, respectively [3].[4] Because chirality is nearly conserved for free relativistic fermions, and the axial-vector interaction does not flip it, the scattered neutrinos in the final state have essentially the same chirality as the incident ones for Majorana neutrinos, just as is the case for the Dirac ones.

Thus, similarly to the case of CC reactions discussed in section 2.1, the crucial role in practical indistinguishability of Dirac and Majorana neutrinos in the NC scattering experiments with incident neutrinos produced in CC processes is played by the chiral nature of neutrino interactions. In section 4.2 we shall discuss NC-induced scattering in the case when the incident neutrinos are produced in NC processes (which means that in the Majorana case their chirality is in general undefined), and we shall show that the confusion theorem remains valid in that case as well.

2.3 Processes with two or more neutrinos in the final state

Above, we discussed processes with no more than one neutrino in the final state. Processes with pair-production of $\nu\bar{\nu}$ or $\nu\nu$ (for Dirac or Majorana neutrinos, respectively) may require special consideration because of quantum indistinguishability of the two neutrinos in the latter case. We shall argue now that this will not lead to any exceptions to the Practical Dirac-Majorana Confusion Theorem. An explicit proof of this statement will be given in sections 4 and 5.

It is true that Majorana neutrinos born in pair-production processes (such as e.g. $\ell^+\ell^- \to \nu\nu$) are identical, no matter how small their mass is; therefore, strictly speaking, the amplitude of the process must always be antisymmetrized with respect to their interchange. However, one can expect that with decreasing m_ν the *observable* effects of this antisymmetrization will decrease, and they will become unmeasurable for arbitrarily small neutrino mass. Indeed, with decreasing m_ν the left-handed and right-handed components of the Majorana neutrino field, ν_L and $(\nu_L)^c \equiv \nu_R^c$, become less strongly coupled to each other, and the transitions between them get suppressed. In the limit $m_\nu/E \to 0$ they decouple and behave effectively as distinct particles, and the amplitude of their pair production need not be antisymmetrized.Technically, this should manifest itself as the suppression of the observable effects of the antisymmetrization by positive powers of m_ν/E. This is actually related to the fact that for relativistic Majorana neutrinos chirality plays the role of an approximately conserved lepton number, as was pointed out above. All effects of chirality nonconservation should be suppressed for small m_ν/E; manifestations of the identical nature of Majorana neutrinos in $\nu\nu$ pair production is just one of these effects. Clearly, the same argument also applies to processes with production of more than two Majorana neutrinos.

[4] Actually, in *all* neutrino detection experiments carried out so far (except possibly in supernova neutrino experiments) the incident neutrinos were produced in CC reactions.

3 Confusion over the Dirac-Majorana confusion

As was mentioned above, there had been claims in the literature that in processes with pair-production of neutrinos the differences between the cross sections for Dirac and Majorana neutrinos survive in the limit $m_\nu \to 0$, contrary to the general arguments given in the previous section. In refs. [8–10], this was found to occur for the NC processes $e^+e^- \to Z^* \to \nu\bar{\nu}(\nu\nu)$. It was pointed out, however, that the Dirac/Majorana differences do smoothly disappear in the limit $m_\nu \to 0$ if the produced neutrinos are not detected (or, more generally, if their spins are not measured).

In refs. [11, 12] the four-body decays $B^0 \to \mu^+\mu^-\nu_\mu\bar{\nu}_\mu$ and $B^0 \to \mu^+\mu^-\nu_\mu\nu_\mu$ mediated by second order CC interactions were considered. The authors concluded that for the special case of back-to-back kinematics for the produced muons in the rest frame of the parent B^0-meson (which by momentum conservation also implies back-to-back kinematics for the produced neutrinos) the differences between the differential decay rates in the Dirac and Majorana cases do not disappear for arbitrarily small but nonzero neutrino mass. Surprisingly, this was found to happen even though the authors studied the case when the final-state neutrinos are not detected. This is in contrast with the mentioned above results of [8–10] for neutrino pair production in NC processes. The authors of [11, 12] argued that it was precisely their use of a CC process that led to this result, because for NC-induced processes the summation over the spins of unobserved neutrinos would kill the Dirac/Majorana difference in the limit $m_\nu/E \to 0$ [12].

We shall now comment on the above results, starting with those of refs. [11, 12].

3.1 Charged-current decays $B^0 \to \mu^+\mu^-\nu_\mu\bar{\nu}_\mu$ and $B^0 \to \mu^+\mu^-\nu_\mu\nu_\mu$

Unfortunately, the papers [11, 12], where these processes were considered, are incorrect. This can be seen from eqs. (48a) and (48b) of [11], from which the authors deduce their main conclusions. These expressions present their results for triply differential B^0-meson decay rates for the back-to-back kinematics (and for negligibly small but nonzero m_ν) in the cases of Dirac and Majorana neutrinos, respectively. Their final results are then obtained upon the integration over the unobservable angle θ between the directions of the produced neutrinos and muons. However, the denominators of the left-hand sides of eqs. (48a) and (48b) erroneously contain $\mathrm{d}\sin\theta$ instead of $\mathrm{d}\cos\theta$. The authors arrived at this result by first defining all the angles characterizing the process in general kinematics and then expressing the angle θ, corresponding to the back-to-back kinematics, through one of these angles, θ_m. However, the transition to this special case means that the angles θ_m and θ_n, defined for general kinematics, become unphysical. This can be seen from figure 4 of [11]: back-to-back kinematics corresponds to vanishing vectors $\vec{q}_m$ and $\vec{q}_n$, which means that their directions are undefined. To overcome this difficulty, the authors define the physical angle θ through a limiting procedure, which, however, is both ambiguous and unnecessary.

One can consider the problem in the back-to-back kinematics from the outset, without any limiting procedures. In this case the process is fully characterized by just two physical variables — the muon energy E_μ and the angle θ between the muon and neutrino directions. Then, since the neutrinos are not observed, one must integrate over the solid angle of the neutrino direction, which involves the integration over $\mathrm{d}\cos\theta$, not over $\mathrm{d}\sin\theta$. Once the correct angular integration of eqs. (48a) and (48b) of [11] has been carried out, their right-hand sides yield identical results. This means that for negligibly small m_ν the differential decay rates for Dirac and Majorana neutrinos coincide, in full agreement with the practical confusion theorem. The limit $m_\nu \to 0$ is smooth, as it should be.

3.2 Neutrino pair production in neutral-current processes

We now turn to refs. [8–10], in which the processes $e^+e^- \to Z^* \to \nu\bar{\nu}(\nu\nu)$ were considered. These papers are technically correct,[5] but they contain some questionable and confusing statements, see below. The main focus in these papers was on the production of hypothetical heavy neutrinos and their subsequent decay; however, a large part of their results also applies to the production of the usual light neutrinos of the Standard Model, and the authors did briefly discuss the limit of very small neutrino masses.

For pair-production of Dirac neutrinos, it was found in [8–10] that the differential cross section of the process contained, along with the usual terms that were either neutrino spin independent or proportional to the longitudinal components of the spins of the produced ν and $\bar{\nu}$, also terms that were proportional to their transverse spin components. However, the latter entered with the factors m_ν/E or $(m_\nu/E)^2$, and in the limit $m_\nu \to 0$ the standard expression for massless neutrinos was recovered:

$$\left.\frac{d\sigma^{\mathrm{D}}}{d\Omega}\right|_{m_\nu \to 0} = \frac{\sigma_0}{2}\left[f_1(1+\cos^2\theta) + 2f_2\cos\theta\right](1-n_z)(1-n'_z)\,. \tag{3.1}$$

Here the coordinates are chosen such that the neutrino and antineutrino momenta point in the positive and negative directions of the z-axis, respectively (in the c.m. frame), n_z and n'_z are the z-components of the neutrino and antineutrino unit spin vectors defined in their respective rest frames, and θ is the angle which the momentum of the incident e^-, chosen to lie in the xz-plane, makes with the z-axis. The parameters σ_0, f_1 and f_2 are defined in [8–10] and are unimportant to us here. The expression in eq. (3.1) is in full agreement with the fact that in the massless limit neutrinos become helicity eigenstates, i.e. their spins have only longitudinal components. Moreover, as expected, the cross section is only different from zero when the produced neutrino has negative helicity and the antineutrino has positive helicity, so that they both have spin projection $n_z = n'_z = -1$ on the direction of the neutrino momentum (which is the positive z-axis in our convention).

At the same time, in the Majorana neutrino case it was found that, for arbitrary m_ν,

$$\frac{d\sigma^{\mathrm{M}}}{d\Omega} = \frac{\sigma_0}{2}\beta^3\Big\{f_1[(1+n_z n'_z)(1+\cos^2\theta) - (n_x n'_x - n_y n'_y)\sin^2\theta] - 2f_2(n_z+n'_z)\cos\theta\Big\}\,, \tag{3.2}$$

where β is the neutrino velocity (see eqs. (2), (4B) or (A6) of refs. [8, 9] or [10], respectively). The coordinate convention here is such that the produced neutrinos fly away in opposite

[5]The expression for the differential cross section in the Dirac case $d\sigma^{\mathrm{D}}/d\Omega$ in eq. (3) of [8] contains some inaccuracies, which have been corrected in [9, 10] (see eq. (4A) of [9] or eq. (A5) of [10]). Note that the limit $m_\nu \to 0$ of this cross section given in eq. (4) of [8] is correct.

directions along the z-axis.[6] The expression in eq. (3.2) contains the term $\propto n_x n'_x - n_y n'_y$ that depends on the transverse spin components of the two produced neutrinos; its origin can be traced back to the antisymmetrization of the amplitude of the process required by quantum statistics in the Majorana neutrino case. This term is not suppressed by positive powers of m_ν/E and therefore survives in the limit $m_\nu \to 0$. Since massless fermions can only have longitudinal spin components, the authors argue that for $m_\nu = 0$ this term is unphysical and must be dropped. Eqs. (3.1) and (3.2) then yield identical results.[7]

The presence of an unsuppressed term depending on the transverse neutrino spin components even in the case of arbitrarily small neutrino mass is quite disturbing, and the prescription of dropping this (or actually any) term by hand in the case $m_\nu = 0$ is unsatisfactory in our opinion. One naturally expects the vanishing of the contributions of the transverse neutrino spin components to be an automatic outcome of the $m_\nu \to 0$ limit, just as it happens in the Dirac neutrino case.

4 Neutral-current neutrino pair production: solution to the problem

In fact, a hint of how this problem can be resolved is already contained in refs. [8–10]. The "anomalous" term $\propto n_x n'_x - n_y n'_y$ in eq. (3.2) may only manifest itself when both of the final-state neutrinos are observed and their spins are measured. This is because the summation over even one of the spins makes this term vanish. Therefore, to test the effects of such a term one must include the neutrino observation process into the consideration. The authors of [8–10] do consider the detection of the produced neutrinos — through their decay; it is known that decay processes can discriminate between Dirac and Majorana neutrinos (see, e.g., ref. [22] and references therein). However, observation of neutrinos through their decay cannot resolve the problem of non-smooth behavior of the cross sections in the $m_\nu \to 0$ limit, as for vanishingly small m_ν neutrinos become essentially stable. Therefore, one should consider other processes of neutrino detection, those that do not require finite neutrino mass. Within the Standard Model, these are only the processes related to neutrino gauge interactions, either CC or NC.

4.1 Neutrino detection through CC processes

We start with neutrino detection through a CC process, for which we choose neutrino-nucleon (or neutrino-nucleus) reaction with the production of a charged lepton. To be specific, we consider the case of ν_μ detection processes with $\mu^\pm$ in the final states (see figure 1(b)). We only discuss the production of opposite-sign muons, the reason being that the same-sign charged lepton production is a $\Delta L = 2$ process, which means that its amplitude is explicitly

[6]Note that because of the indistinguishability of Majorana neutrinos one cannot choose the z-axis using the same convention as in the Dirac neutrino case. However, for the same reason it does not matter which of the produced Majorana neutrinos goes in the positive and which in the negative z-direction. It is easy to see that eq. (3.2) is invariant with respect to the interchange of the two neutrinos (which amounts to the coordinate transformation $z \to -z$), as it should.

[7]To see this, one should remember the mentioned above difference in the definitions of the z-axis in the Dirac and Majorana cases, which implies that one has to flip the z-axis ($z \to -z$) for $n_z = n'_z = 1$ in the Majorana case in order to compare it to the Dirac case. See the discussion below eq. (5) of ref. [8].

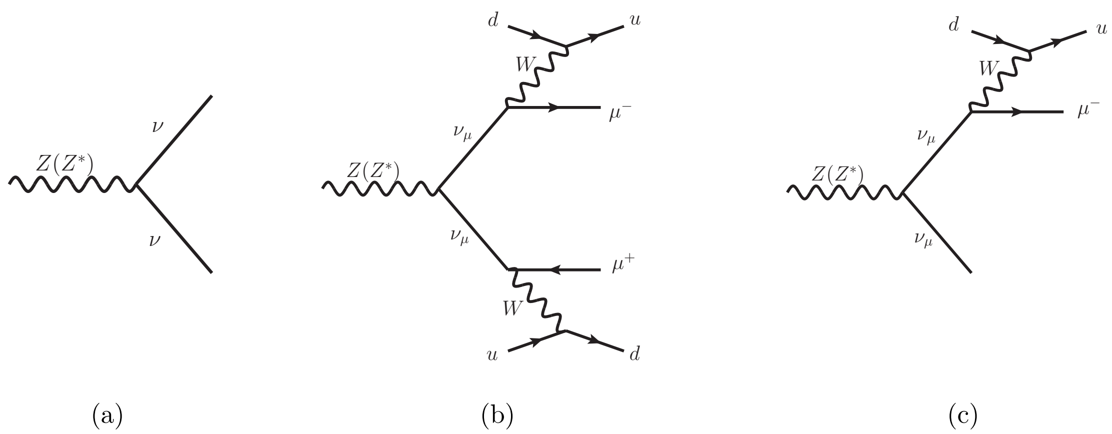

Figure 1. Feynman diagrams for some of the discussed processes. (a): production of a pair of Majorana neutrinos in decay of on-shell or off-shell Z^0-boson; (b) same as in (a) but with detection of both produced neutrinos (assumed to be of muon flavor) through CC reactions with production of μ^- and μ^+; (c) same as in (b) but with detection of only one of the produced neutrinos.

proportional to the Majorana neutrino mass and vanishes in the limit $m_\nu \to 0$. We ignore here the experimental difficulties of simultaneous detection of two neutrinos from the decay of the same on-shell or off-shell Z-boson; because we are interested in fundamental questions of distinguishability of Dirac and Majorana neutrinos, it suffices for the purposes of our study that such a detection is possible in principle. We shall also consider the situation in which only one of the produced neutrinos is detected, whereas the other escapes unobserved (see figure 1(c)).

As can be seen from the diagrams of figures 1(b) and 1(c), the difficulty with the "anomalous" term in the cross section in eq. (3.2), which was an outcome of the antisymmetrization of the amplitude in the Majorana case, is immediately resolved by neutrino detection: for the processes shown in these figures there are either no neutrinos or only one neutrino in the final state, so there is nothing to antisymmetrize.

Thus, there are two possibilities: (1) The neutrinos produced in the $e^+e^- \to Z^* \to \nu\nu$ process are not detected. The "anomalous" term in eq. (3.2) is then averaged away because of the summation of the spins of the unobserved neutrinos. (2) Either one or both of the final-state neutrinos are detected. The "anomalous" term, which originates from the antisymmetrization procedure, then does not appear at all because there are no pairs of identical neutrinos in the final state. Thus, we come to the conclusion that terms in the cross sections of the $e^+e^- \to Z^* \to \nu\bar{\nu}(\nu\nu)$ process that are different in the case of Dirac and Majorana neutrinos and are not suppressed by powers of m_ν/E never appear, as far as observable quantities are concerned. It is not necessary to drop any terms from the cross sections by hand in the case of exactly vanishing m_ν. There are, of course, still terms that are different in the cases of Dirac and Majorana neutrinos, but they are all suppressed by factors m_ν/E and $(m_\nu/E)^2$ and thus do not violate the Practical Dirac-Majorana Confusion Theorem.

The above argument is very simple and is valid independently of the distance between the neutrino production and detection points. It is, however, instructive to consider separately the situation when the production and detection processes are separated by macroscopically

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large distances and therefore can be considered as independent.[8] This will allow us to find out how the "anomalous" term in eq. (3.2) actually becomes inoperative when at least one of the produced neutrinos is detected.

Let us first note that the pure axial-vector nature of the neutrino NC in the Majorana case is actually a result of the coherent superposition of the left-chiral and right-chiral contributions with equal weights. Indeed, the amplitude of the $e^+e^- \to Z^* \to \nu\nu$ process depends on the matrix element $\langle \nu_{s_1}(p_1)\nu_{s_2}(p_2)|j^{\mu}_{\rm NC}(0)|0\rangle$, which is given by[9]

$$\begin{aligned}
&\frac{1}{\sqrt{2}}\left[\bar{u}_{s_1}(p_1)\gamma^\mu(1-\gamma_5)v_{s_2}(p_2) - \bar{u}_{s_2}(p_2)\gamma^\mu(1-\gamma_5)v_{s_1}(p_1)\right] = \qquad (4.1)\\
&\frac{1}{\sqrt{2}}\left[\bar{u}_{s_1}(p_1)\gamma^\mu(1-\gamma_5)v_{s_2}(p_2) - \bar{u}_{s_1}(p_1)\gamma^\mu(1+\gamma_5)v_{s_2}(p_2)\right] = -\sqrt{2}\bar{u}_{s_1}(p_1)\gamma^\mu\gamma_5 v_{s_2}(p_2)\,.
\end{aligned}$$

Here p_1, s_1 and p_2, s_2 are the four-momenta and the spin indices of the two produced neutrinos, and the factor $1/\sqrt{2}$ comes from the identical nature of the two Majorana neutrinos in the final state.

Let us now return to the process shown in figure 1(b). From the chirality selection rules of CC interactions discussed in section 2.1 it follows that the neutrino producing μ^- must be of predominantly negative-helicity, while the neutrino producing μ^+ must have predominantly positive helicity; in the limit $m_\nu \to 0$ they become pure helicity eigenstates. Thus, in this limit the considered neutrino detection process breaks the indistinguishability of the two Majorana neutrinos produced in the $e^+e^- \to Z^* \to \nu\nu$ reaction. This, in particular, means that for negligibly small neutrino mass there will be no contribution to the squared matrix element of the process from the interference of the two terms in the first line of eq. (4.1) (or, equivalently, of the two terms on the left-hand side of the second line). It should be noted that the $(n_x n'_x - n_y n'_y)$ term in eq. (3.2) comes precisely from this interference. Therefore, for $m_\nu \to 0$ there will be no contribution of the transverse neutrino spin components to the observables, and one does not need to drop this term by hand. For $m_\nu \neq 0$, the contribution of the interference term to the cross section of the process will be different from zero to the same extent to which chirality deviates from helicity, i.e. it will be suppressed by positive powers of m_ν/E (see eqs. (4.8)–(4.11) below and the discussion around them).

Keeping only the squared moduli of the two terms in the second line of eq. (4.1) would reproduce the squared matrix element of the overall process of figure 1(b) for Dirac neutrinos, with μ^- detection in a given detector and μ^+ production in the other one. In the Majorana case, there is an additional factor of 2 due to the fact that each of the two detectors can observe μ^-, with the other observing μ^+;[10] with the factor $1/\sqrt{2}$ in eq. (4.1) taken into account, this means that the cross sections for Dirac and Majorana neutrinos coincide in the massless neutrino limit.

Thus, the detection process breaks the indistinguishability of the produced Majorana neutrinos in this case, which means that no antisymmetrization needs to be done (or, more precisely, that the antisymmetrization does not lead to any observable effects) in the case of

[8] See e.g. the discussion in section 4 of [23].

[9] Note the similarity with eq. (2.3).

[10] There is no such freedom in the Dirac case due to the specific choice of the positive direction of the z axis that led to eq. (3.1).

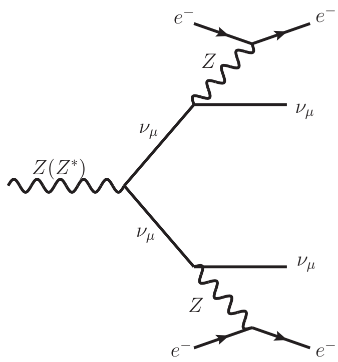

Figure 2. Same as in figure 1(b) but for neutrino detection through NC $\nu_\mu e$ scattering.

negligible neutrino mass. Moreover, the indistinguishability of the two Majorana neutrinos produced in $e^+e^- \to Z^* \to \nu\nu$ reaction is broken even if only one of them is detected (see figure 1(c)): this would single out the predominant helicity of the detected neutrino, and the other one would automatically have the opposite predominant helicity due to their entanglement. This can be immediately seen from eq. (4.1): since $P_L = \frac{1}{2}(1-\gamma_5)$ projects out left-handed components of u-type spinors and right-handed components of v-type spinors, the first term in the first line of (4.1) corresponds to neutrino of momentum p_1 being left-chiral and that of momentum p_2 right-chiral, while the second term corresponds to the opposite situation. Thus the detection process, by selecting the chirality of the observed neutrino, determines also the chirality of the other one.

4.2 Neutrino detection through NC $\nu_\mu e$ scattering

We now turn to neutrino detection via NC process, for which we consider $\nu_\mu e$ scattering (figure 2). In this case, the final state of the process contains two neutrinos (if they are Majorana particles) or a $\nu\bar{\nu}$ pair (if they are of Dirac nature), so in the Majorana case the antisymmetrization has to be carried out. However, just as in the case of the neutrino pair production in $e^+e^- \to Z^* \to \nu\nu$ reaction, no differences between the cross sections in the Dirac and Majorana cases arise in the limit $m_\nu \to 0$ if the final-state neutrinos are not detected. Thus, we are back to the situation one had for the neutrino pair production in $e^+e^- \to Z^* \to \nu\nu$ reaction: Dirac *vs.* Majorana neutrino nature could only be probed by detection of final-state neutrinos. CC detection shows that for negligibly small neutrino mass the Dirac/Majorana differences disappear; NC detection leads to the same conclusion if the final-state neutrinos in NC processes are not observed.

Let us now study the NC detection process of figure 2 in more detail. As in the case of CC detection, it is instructive to examine the situation when the neutrino production and detection processes are separated by macroscopically large distances and therefore can be considered as independent. Let us first discuss neutrino detection in the Dirac case. The matrix elements describing the scattering of muon neutrinos and antineutrinos on electrons in one of the detectors are given, up to a constant factor, by

$$\mathcal{M}^{\rm D}_{\nu_\mu e} = \mathcal{J}_\mu\big[\bar{u}_{s_1'}(p_1')\gamma^\mu(1-\gamma_5)u_{s_1}(p_1)\big]\,, \tag{4.2}$$

$$\mathcal{M}^{\rm D}_{\bar{\nu}_\mu e} = \mathcal{J}_\mu\big[\bar{v}_{s_1}(p_1)\gamma^\mu(1-\gamma_5)v_{s_1'}(p_1')\big] = \mathcal{J}_\mu\big[\bar{u}_{s_1'}(p_1')\gamma^\mu(1+\gamma_5)u_{s_1}(p_1)\big]\,. \tag{4.3}$$

Here p_1, s_1 and p_1', s_1' denote the four-momenta and spins of the initial-state and final-state neutrinos or antineutrinos, and $\mathcal{J}_\mu$ is the convolution of the electron NC matrix element j_e^ν and the Z^0-boson propagator $D^Z_{\nu\mu}$: $\mathcal{J}_\mu = j_e^\nu D^Z_{\nu\mu}$. It is not possible to find out on a case-by-case basis whether a given neutrino detector has observed ν_μ or $\bar\nu_\mu$; however, if one of them detects ν_μ, the other will detect $\bar\nu_\mu$, and vice versa. Therefore, the number of scattering events in both detectors in a simultaneous detection experiment is proportional to $\left(\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\nu_\mu e}}{\mathrm{d}T} + \frac{\mathrm{d}\sigma^{\mathrm{D}}_{\bar\nu_\mu e}}{\mathrm{d}T}\right)$, where T is the kinetic energy of the recoil electron.

Let us now consider the case of Majorana neutrinos. The matrix element of $\nu_\mu e$ scattering in one of the detectors is, up to a constant factor,

$$\begin{aligned}\mathcal{M}^{\mathrm{M}}_{\nu_\mu e} &= \mathcal{J}_\mu\left[\bar u_{s_1'}(p_1')\gamma^\mu(1-\gamma_5)u_{s_1}(p_1) - \bar v_{s_1}(p_1)\gamma^\mu(1-\gamma_5)v_{s_1'}(p_1')\right]\\ &= \mathcal{J}_\mu\left[\bar u_{s_1'}(p_1')\gamma^\mu(1-\gamma_5)u_{s_1}(p_1) - \bar u_{s_1'}(p_1')\gamma^\mu(1+\gamma_5)u_{s_1}(p_1)\right], \end{aligned}\tag{4.4}$$

and similarly for the other detector.[11] The two terms in the square brackets in the second line sum up to the pure axial vector $-2\bar u_{s_1'}(p_1')\gamma^\mu\gamma_5 u_{s_1}(p_1)$; however, for our purposes it will be more convenient to use the expression for $\mathcal{M}^{\mathrm{M}}_{\nu_\mu e}$ as it is given in the second line of eq. (4.4). Comparing (4.4) with eqs. (4.2) and (4.3), we find

$$\mathcal{M}^{\mathrm{M}}_{\nu_\mu e} = \mathcal{M}^{\mathrm{D}}_{\nu_\mu e} - \mathcal{M}^{\mathrm{D}}_{\bar\nu_\mu e}\,. \tag{4.5}$$

The squared matrix element of the scattering process can be written as $L_{\mu\nu}N^{\mu\nu}$, where $L_{\mu\nu} = \mathcal{J}_\mu\mathcal{J}_\nu^*$, and the bilinear neutrino current products $N^{\mu\nu}$ depend on the neutrino nature and on the scattering process. In the Dirac case, one has, for $\nu_\mu e$ and $\bar\nu_\mu e$ scattering,

$$N^{(\mathrm{D})\mu\nu}_{\nu_\mu e} = \left[\bar u_{s_1'}(p_1')\gamma^\mu(1-\gamma_5)u_{s_1}(p_1)\right]\left[\bar u_{s_1}(p_1)\gamma^\nu(1-\gamma_5)u_{s_1'}(p_1')\right], \tag{4.6}$$

$$\begin{aligned}N^{(\mathrm{D})\mu\nu}_{\bar\nu_\mu e} &= \left[\bar v_{s_1}(p_1)\gamma^\mu(1-\gamma_5)v_{s_1'}(p_1')\right]\left[\bar v_{s_1'}(p_1')\gamma^\nu(1-\gamma_5)v_{s_1}(p_1)\right]\\ &= \left[\bar u_{s_1'}(p_1')\gamma^\mu(1+\gamma_5)u_{s_1}(p_1)\right]\left[\bar u_{s_1}(p_1)\gamma^\nu(1+\gamma_5)u_{s_1'}(p_1')\right]. \end{aligned}\tag{4.7}$$

In the Majorana case, we find

$$N^{(\mathrm{M})\mu\nu}_{\nu_\mu e} = N^{(\mathrm{D})\mu\nu}_{\nu_\mu e} + N^{(\mathrm{D})\mu\nu}_{\bar\nu_\mu e} - T^{\mu\nu}\,, \tag{4.8}$$

where use has been made of the relation in eq. (4.5). The quantity $T^{\mu\nu}$ comes from the interference of the two terms in eq. (4.5) and is given by

$$\begin{aligned}T^{\mu\nu} &= \left[\bar u_{s_1'}(p_1')\gamma^\mu(1-\gamma_5)u_{s_1}(p_1)\right]\left[\bar u_{s_1}(p_1)\gamma^\nu(1+\gamma_5)u_{s_1'}(p_1')\right]\\ &\quad + \left[\bar u_{s_1'}(p_1')\gamma^\mu(1+\gamma_5)u_{s_1}(p_1)\right]\left[\bar u_{s_1}(p_1)\gamma^\nu(1-\gamma_5)u_{s_1'}(p_1')\right]\\ &= -\mathrm{tr}\{\not{p}_1'\not{s}_1'\gamma^\mu\not{p}_1\not{s}_1\gamma^\nu - m_\nu(\not{p}_1'\not{s}_1'\gamma^\mu\gamma^\nu - \gamma^\mu\not{p}_1\not{s}_1\gamma^\nu)\gamma_5 - m_\nu^2\gamma^\mu\gamma^\nu\}\,. \end{aligned}\tag{4.9}$$

Here s_1 and s_1' are the spin four-vectors of the incident and scattered neutrinos,

$$s_1^\mu = \left(\frac{\vec p_1\cdot\vec n_1}{m_\nu},\, \vec n_1 + \frac{(\vec p_1\cdot\vec n_1)\vec p_1}{m_\nu(E_1+m_\nu)}\right), \tag{4.10}$$

[11] The neutrino NC matrix element here is actually that given in eq. (2.3), where it has now been taken into account that $g_V = g_A$.

with $\vec{n}_1$ being the unit spin vector of the incoming neutrino in its rest frame, and similarly for $s_1'^{\mu}$. Next, we note that the detection of the neutrinos produced in $e^+e^- \to Z^* \to \nu\nu$ reaction is achieved through the measurement of electron recoil in $\nu_\mu e$ scattering, i.e. the scattered neutrinos in the final state are not observed. Therefore, the summation over the spin s_1' (which amounts to integration over $\vec{n}_1'$) has to be carried out and we find

$$\sum_{s'} T^{\mu\nu} = \mathrm{tr}\{ -m_\nu \gamma^\mu \not{p}_1 \not{s}_1 \gamma^\nu \gamma_5 + m_\nu^2 \gamma^\mu \gamma^\nu \} \,. \tag{4.11}$$

Thus, the interference term $T^{\mu\nu}$ is suppressed for relativistic neutrinos at least as m_ν/E. From eq. (4.8) we then find that for negligibly small neutrino mass the number of events in one neutrino detector in the Majorana case is proportional to the sum of the Dirac neutrino cross sections of $\nu_\mu e$ and $\bar{\nu}_\mu e$ scattering events. This has to be multiplied by a factor of two for two detectors, but on the other hand there is a factor 1/2 because of the identical nature of the two neutrinos in the reaction $e^+e^- \to Z^* \to \nu\nu$. The two factors compensate each other, and the total number of events in the simultaneous neutrino detection experiment remains proportional to $\left(\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\nu_\mu e}}{\mathrm{d}T} + \frac{\mathrm{d}\sigma^{\mathrm{D}}_{\bar{\nu}_\mu e}}{\mathrm{d}T}\right)$, with the same proportionality factor as in the Dirac neutrino case. Thus, also in the case of neutrino detection through NC processes, the difference between the cross sections for Dirac and Majorana neutrinos smoothly disappears in the limit $m_\nu \to 0$, with no need to drop any terms in the squared matrix element by hand.

A key conclusion from the above calculation is that in the limit $m_\nu \to 0$ Majorana neutrinos produced or destroyed by their NC can be considered to be in states of definite chirality, despite the purely axial-vector form of the current. This is a consequence of the fact that the interference between the opposite chirality amplitudes vanishes in this limit.

It is obvious that the confusion theorem holds also true when one of the neutrinos from $Z^* \to \nu\nu$ decay is detected through a CC process and another through a NC process. The CC interaction selects a neutrino of one predominant helicity, either positive or negative, depending on the process; such detection modes satisfy the Practical Dirac-Majorana Confusion Theorem, as discussed in section 2.1. The other neutrino will then have the opposite predominant helicity, and its detection through a NC interaction will have, in the $m_\nu \to 0$ limit, the same cross section for Dirac and Majorana neutrinos, as discussed in section 2.2.

Thus, irrespectively of the detection processes, in the case of negligibly small neutrino mass observation of both neutrinos produced in the reaction $e^+e^- \to Z^* \to \nu\bar{\nu}(\nu\nu)$ would result in the same cross section for Dirac and Majorana neutrinos. If at least one of the produced neutrinos escapes unobserved, the inherent summation over its spin leads to the same result. Our conclusion is obviously also valid for reactions of neutrino pair creation and annihilation, which are related to neutrino scattering processes by crossing symmetry.

5 General proof

In refs. [11] and [12] it has been asserted that no general proof of the Practical Dirac-Majorana Confusion Theorem exists, and its validity had only been demonstrated for a few processes. This was a motivation for the authors to search for possible exceptions to this theorem. In our opinion, the very general argument [1–4] that the theorem should be universally valid within the Standard Model because both Dirac and Majorana neutrinos become Weyl fermions in the $m_\nu \to 0$ limit and the right-handed components of Dirac neutrinos are sterile, is by itself a sufficient proof of the confusion theorem. Still, some people prefer explicit proofs to general arguments; we therefore present here what we believe constitutes such a proof.

Consider a generic neutrino process due to n CC and k NC interactions ($n + k \geq 1$).[12]

I. *CC processes.* As discussed in section 2.1, the chiral structure of CC interactions implies that in the limit $m_\nu \to 0$ the corresponding matrix elements for Dirac and Majorana neutrinos coincide, except possibly for antisymmetrization due to identical nature of some final-state neutrinos, which may in principle be different in the Dirac and Majorana cases and which will be discussed later on.

II. *NC processes.* These may be either of scattering or of neutrino pair production or annihilation type. In scattering processes one initial-state neutrino or antineutrino is destroyed and one is produced in the final state; in pair production and annihilation processes, a $\nu\bar{\nu}$ or $\nu\nu$ pair is either produced or annihilated by each NC interaction.

(i) *NC scattering processes.* In the Dirac case, neutrino NC interaction has chiral structure, while for Majorana neutrinos, it is purely axial-vector. The analysis of Dirac/Majorana differences depends on the production mechanism of incident neutrinos in the Majorana case. There are two possibilities:

(a) An incident Majorana neutrino has been previously produced in a CC process and therefore participates in the process under consideration as a state of (nearly) definite chirality. Because axial-vector interactions do not flip chirality, the scattered neutrino will then be in the same chirality state. As shown in [3, 17–21] (see also section 2.2 above), in this case the difference between the NC matrix elements in the Dirac and Majorana cases vanishes in the limit $m_\nu \to 0$.

(b) An incident Majorana neutrino in the NC scattering process was produced in a NC pair production reaction. This case was considered in section 4.2. Being caused by a purely axial-vector interaction, this process in general leads to the production of a pair of neutrinos of no definite chirality. Indeed, for Majorana neutrinos the NC matrix element is always given by a difference of two amplitudes of opposite chirality, regardless of whether neutrino scattering or pair-production is considered (cf. eqs. (2.3) and (4.1)). This is related to the fact that the NC contains one $\nu(x)$ and one $\bar{\nu}(x)$ field operator, and in the Majorana case each of these operators can both create and annihilate a neutrino. As was shown in section 4.2, these two terms do not interfere in the limit $m_\nu \to 0$ if the neutrino spins are not measured, which means that in this limit Majorana neutrinos originating from pair-production NC processes will also participate in the NC scattering process as states of definite chirality. As discussed in point (a) above, this means that to leading order in m_ν/E there will be no differences between the corresponding cross sections in the Dirac and Majorana neutrino cases.

[12]We do not include in our discussion neutrino oscillations, which are well known to be independent of Dirac *vs.* Majorana neutrino nature. Not included here are also processes caused by neutrino electromagnetic interactions; we comment on them at the end of this section.

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(ii) *NC neutrino pair production or annihilation.* The conclusion in section 4.2 regarding the chiral nature of Majorana neutrinos born in NC pair-production processes applies not only to production of incident neutrinos, but also to pair-production of final-state neutrinos and to annihilation of initial-state ones in the generic process we are discussing here. As was stressed above, the differences between the Dirac and Majorana matrix elements disappear in the limit $m_\nu \to 0$ when neutrinos are in chiral states.

This exhausts all possible neutrino processes induced by CC and NC weak interactions. The only issue that has so far been left out of our general proof is the antisymmetrization of the amplitudes in the Dirac and Majorana cases. We consider it next.

III. *Antisymmetrization.* For Dirac neutrinos, antisymmetrization should be done between each pair of neutrinos in the final state, and similarly for each pair of antineutrinos.[13] In the Majorana case, the antisymmetrization should in principle be carried out for each pair of produced neutrinos. As follows from the discussion above in this section, all finite-state neutrinos and/or antineutrinos in the generic process we discuss can be considered as emitted in states of definite chirality, irrespectively of their Dirac/Majorana nature and of whether they are produced by CC or NC interactions. We have demonstrated in section 4.2 that to leading order in m_ν the interference between terms of opposite chiralities in the transition amplitude disappears, and that for this reason the antisymmetrization in the Majorana case to this order should only be done between pairs of neutrinos of same chirality. Recall now that Dirac neutrinos and antineutrinos are produced as states of, respectively, negative and positive chiralities. Thus, to leading order in m_ν/E, antisymmetrization should only be done in the Majorana case between the same neutrinos for which it should be performed in the Dirac case; all effects of "additional" antisymmetrization related to the Majorana nature of neutrinos will be suppressed at least as m_ν/E. This completes our proof.

Note that the cases when some of neutrinos are Dirac particles while others are of Majorana nature are obviously also covered by the generic case discussed above.

We did not discuss here processes induced by electromagnetic interactions of neutrinos. The validity of the Practical Dirac-Majorana Confusion Theorem for such processes was proven in refs. [2, 4, 24–26] (see also [27]).

6 Summary and conclusion

We have studied in detail the question of whether antisymmetrization of the amplitudes of the processes with more than one neutrino in the final state required by quantum statistics can help distinguish Dirac from Majorana neutrinos. We examined the existing claims in the literature that the differences between the cross sections of Dirac and Majorana neutrinos due to this antisymmetrization survive even for arbitrarily small but not exactly vanishing neutrino mass and demonstrated that they do not hold true. We also analyzed the implications of quantum statistics for generic neutrino processes and presented a general proof of the Practical Dirac-Majorana Confusion Theorem with the antisymmetrization issue explicitly taken into account. The key point in our analysis was the observation that for Majorana

[13] Recall that in this paper we consider all neutrinos to be of same flavor.

neutrinos chirality, which is nearly conserved for relativistic fermions, plays essentially the same role as lepton number conservation plays for Dirac neutrinos. Because the difference between chirality and (exactly conserved) helicity is suppressed as $\mathcal{O}(m_\nu/E)$, so are the differences between the observables in the Dirac and Majorana neutrino cases.

Leaving aside lepton number violating processes whose probabilities are explicitly proportional to Majorana neutrino mass and therefore vanish when it goes to zero, there are two main differences between the matrix elements for Dirac and Majorana neutrinos:

A. In the Dirac neutrino case, NC is chiral, whereas for Majorana neutrinos it is purely axial-vector;

B. In the Dirac case, antisymmetrization of the amplitudes has to be done with respect to the interchange of each neutrino pair in the final state, and similarly for each antineutrino pair; at the same time, since for Majorana neutrinos there is no difference between neutrinos and antineutrinos, the antisymmetrization should be carried out with respect to the interchange of each pair of neutrinos in the final state.

It was proven long ago that for NC-induced neutrino scattering processes, such as $\nu_\mu e$ scattering or deuteron disintegration $\nu + d \to p + n + \nu$, the axial vector nature of the NC of Majorana neutrinos does not violate the confusion theorem provided that the incident neutrinos have been produced in a CC process and therefore are (nearly) chiral. This is because the axial-vector current, though does not project out states of definite chirality, does not flip the chirality either; the scattered neutrino is therefore essentially in the same chirality state as the incoming neutrino.

The situation when the incident neutrino in a NC scattering process was previously produced in another NC process was for the first time considered in this paper. The subtlety is that in the Majorana case, due to the axial-vector nature of the NC, the produced neutrinos are in general born in states of no definite chirality. Rather, the production amplitude is a coherent sum of terms corresponding to emission of neutrinos of opposite chirality. We have demonstrated that the interference of these two terms is suppressed for relativistic neutrinos at least as m_ν/E, and therefore to leading order it can be neglected. This means that in the limit $m_\nu/E \to 0$ Majorana neutrinos produced (destroyed) in pair-production (annihilation) processes can also be considered to be in states of definite chirality. Thus, the peculiarity of Majorana neutrinos mentioned in point A above does not lead to any exception to the practical confusion theorem.

The conclusion that to leading order in m_ν/E neutrinos can always be considered as being in states of definite chirality regardless of their origin, Dirac/Majorana nature and of the process in which they participate has also direct bearing on the antisymmetrization issue mentioned in point B. Because in the limit $m_\nu \to 0$ chirality is a good quantum number, it plays in this limit essentially the role of lepton number for Majorana neutrinos. The antisymmetrization in the Majorana case should then only be done for those neutrinos for which it should be done in the Dirac neutrino case. All corrections to this rule are suppressed at least as m_ν/E and therefore do not violate the practical confusion theorem.

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In conclusion, we have shown that the Practical Dirac-Majorana Confusion Theorem applies without restrictions within the Standard Model, and in particular that effects of quantum statistics do not lead to any exceptions to it.

The results of our study are in agreement with Hinchliffe's rule [28].

Notes added. After this paper was submitted to the e-print archive, our attention was drawn to ref. [29], where the leptonic radiative four-body decay $\ell \to \ell' \nu_\ell \bar{\nu}_{\ell'} \gamma$ was considered. Although this process is different from that studied in [11], their back-to-back kinematics is essentially the same. The authors of [29] came to the conclusion that the integration over the unobservable angle θ between the neutrino and $\ell'\gamma$ directions eliminates all the Dirac/Majorana differences in the differential decay rates to leading order in m_ν/E, and that the same applies to the B^0-decay process considered in [11], where θ is the angle between the neutrino and muon directions. This result is in full agreement with that in our section 3.1. It should be stressed, however, that in the case when no integration over θ is carried out, the differences between neutrino energy and angular distributions of Dirac and Majorana neutrinos in the limit $m_\nu/E \to 0$, shown in figure 5 of ref. [29], are actually unobservable within the Standard Model. As was demonstrated in the general case in section 5 of our paper and also discussed in detail for the special case of NC processes in sections 4.1 and 4.2, to analyze the dependence of these observables on neutrino nature one has to include the neutrino detection processes in the consideration, and this would eliminate the Dirac/Majorana differences in the limit $m_\nu/E \to 0$.

In a recent note [30] the authors of [11] attempted at a rebuttal of our criticism of their paper. However, they did not answer our critical remarks on the limiting procedure that was used in [11] to derive the expression for the angle θ (which actually led to their main conclusions) and just reiterated that their analysis was thorough and accurate. We maintain the validity of our criticism of the results of ref. [11].

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